Component Contributions to the Failure of Systems Undergoing Phased Missions

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Abstract

The way that many systems are utilised can be expressed in terms of missions which are split into a sequence of contiguous phases. Mission success is only achieved if each of the phases is successful and each phase is required to achieve a different objective and use different elements of the system.

The reliability analysis of a phased mission system will produce the probability of failure during each of the phases together with the overall mission failure likelihood. In the event that the system performance does not meet with the acceptance requirement, weaknesses in the design are identified and improvements made to rectify the deficiencies. In conventional system assessments, importance measures can be predicted which provide a numerical indicator of the significance that each component plays in the system failure. Through the development of appropriate importance measures this paper provides ways of identifying the contribution made by each component failure to each phase failure and the overall mission failure. In addition a means to update the system performance prediction and the importance measures as phases of the mission are successfully completed is given.

1. Introduction

A phased mission is used to describe the situation where the system functional requirements change throughout the period of operation. The periods of operation between the transition points, where the system functional requirements change, are refereed to as phases. The mission can then be defined as a sequence of phases all of which need to be completed successfully for the mission success. Many systems can be seen to operate in this way, typical examples are aircraft, satellites and spacecraft.

The unreliability assessment of phased mission systems produces the likelihood of failure during each of the individual phases and also the overall mission failure probability. For some systems it may be possible for maintenance to be performed during the mission to rectify faults which have occurred. The categorisation of a mission to be non-repairable or repairable influences the reliability modelling techniques which can be used. Fault tree approaches are appropriate for non-repairable phased missions (refs 1-3) and Markov methods when some degree of repair is possible (refs 4-6). This paper focuses on non-repairable missions.

Recent work has extended the basic mission unreliability modelling methods to indicate the contribution that individual components can make to the mission failure (refs 7,8). The development of these importance measures has extended the concepts of the classical Birnbaum measure of importance and the Criticality measure of importance (refs 9,10). The contributions made to each phase failure are calculated accounting for the fact that failure in a phase can only occur providing all previous phases have successfully completed. The phase failure contributions are then combined to give an indication of the contribution that each component failure makes when considering the entire mission.

As the mission progresses and each phase is successfully completed the predictions for the phase and mission failure probability can be updated. The method to perform the revised predictions is given.

2. Phased Mission Definition

In the modelling presented in this paper the following assumptions are made for the phased mission:

- a mission is defined in terms of phases carried out consecutively.
- each phase accomplishes a specified task. It has different functional requirements and therefore the failure criteria are different for each phase.
- for a mission success each phase must be completed successfully.
- the time duration for each phase is known.
- the mission is non-repairable and component failures will exist for the remainder of the mission once they occur.
- all components are in the working state at the start of the mission.

3. Phased Mission Unreliability Quantification

The phased mission is represented as a series of fault trees, each one expressing the conditions which will lead to the failure of a specific phase. The duration of each phase is also provided in terms of the times, following the mission initiation, at which each phase is entered. A method to calculate the failure likelihood of such a phased mission, the phased mission unreliability Q_{sys} is presented in reference 3. It provides both qualitative and quantitative information regarding phase and mission failure. The method presented in this paper breaks down the phase failure modes to identify where the significant contributions occur.

Component failures are considered as separate, dependent, events in each phase. The notation used is: A_i represents the failure of component A during phase i and $A_{i,j}$ represents the component failing at some point between the start of phase i and the end of phase j. Therefore the event that the component exists in the failed state at the end of phase i is:

$$A_{1,i} = A_1 + A_2 + ... + A_i \tag{1}$$

In all logic equations '+' is used to represent OR and '.' is used to represent AND.

The first stage of the method is to establish the phase failure modes. These are the prime implicants (minimal combinations of the component states, working or failed) which will result in a particular phase failure and accounts for the successful completion of all previous phases. Considering each phase in turn, the method constructs the phase failure fault tree as shown in figure 1. Boolean reduction of the fault trees constructed in this way determines the phase failure modes. In performing the reduction for a phased mission fault tree it is possible to take advantage of the non-repairable nature of the component failures. A special phase algebra has been developed which uses the fact that once failed a component remains that way for the rest of the mission. The algebra uses the following rules:

$$
\begin{aligned}
&A_i.A_j = 0 \\
&A_i.A_i = A_i \\
&A_i.A_{i,j} = A_i \\
&A_i.\overline{A_i} = 0 \\
&\overline{A_i}.A_{i,j} = A_{i+1,j} \\
&A_i.\overline{A_j} = 0 \qquad \text{if } i < j
\end{aligned}
\tag{2}
$$

In a phased mission there are two ways that a phase can experience a failure:

- The failure can occur during the phase as a result of a component failure which occurs during the phase whose occurrence then fulfils the conditions for phase and mission failure.
- Alternatively the system can be in a state which already satisfies the conditions for phase failure before the phase is entered. Phase failure will then result as soon as the transition into the phase takes place. In this latter case the component failure events in the phase failure mode have all occurred in a previous phase but have not satisfied a previous phase failure conditions.

The phase failure modes can be split into the causes of these two categories: in-phase failure and phase transition failure.

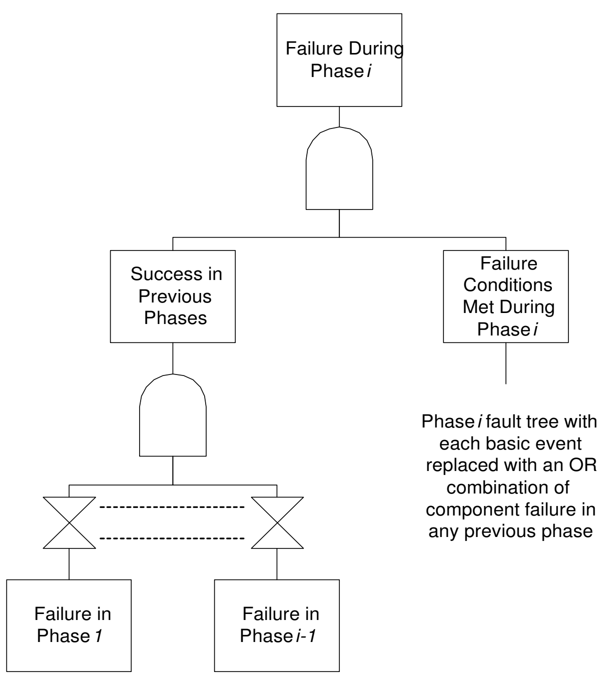

Figure 1 Fault tree for mission failure during phase i

The likelihood of the component failure events which appear in the prime implicants obtained for each phase fault tree can be determined by integrating their failure time density function over the appropriate time period:

$$q_{A_{i,j}} = \int_{t_{i-1}}^{t_j} f_A(t)dt \tag{3}$$

From the component failure probabilities together with the prime implicant sets the inclusion –exclusion expansion (equation 4) can be used to determine the phase unreliabilities (conditional on all previous phase success), Q_i^P and Q_i^T, the in-phase and phase transition failure probabilities respectively.

$$Q_i^{P/T} = \sum_{i=1}^{N_p} P(C_i) - \sum_{i=1}^{N_p}\sum_{j=1}^{i-1} P(C_i \cap C_j)..........+(-1)^{N_p-1} P(C_1 \cap C_2 \cap .... \cap C_{N_p}) \tag{4}$$

where C_i is prime implicant i.

The phase unreliability is then obtained from:

$$Q_i = Q_i^T + Q_i^P \tag{5}$$

As will be discussed in later sections, to evaluate the component importance contributions a particular formulation of equation 4 (refs 11,12) offers some advantages. Whilst the form of the equation is unimportant for phase failure likelihood quantification, the form that will be used later in the importance calculations will be described and all expressions used in the examples will be formed this way. Prime implicant terms, C_i, will be of a form which can contain components failing and functioning through particular phases. This form will also describe the combinations of the prime implicants whose likelihood is to be determined in equation 4. The terms of the inclusion-exclusion expansion are formed using two independent variables for the likelihood that any component, i, works (p_i) and that it fails (q_i). The relationship that $p_i+q_i=1$ is not used to express the whole equation in terms of either one of these variables. For example, if it is required to evaluate the probability of the combination of component failure events $A_3.\overline{B_3}.C_2$ then the form for its probability would be: $q_{A_3} p_{B_3} q_{C_2}$. Clearly the Boolean reduction carried out will prevent situations where the same component exists in both its working and failed state.

Summing the phase unreliabilities yields the mission unreliability:

$$Q_{Miss} = \sum Q_i \tag{6}$$

The phase and mission quantification is illustrated using a simple example system provided in the next section. This example will be used throughout the paper to demonstrate the computation of the component importance measures developed in the later sections.

4. Example System Analysis

An example four-phased mission system is illustrated in figure 2. The failure conditions for each of the four phases, in terms of the four components A, B, C and D on which this mission depends, are represented by fault trees. The time durations in each phase i are (t_{i-1}, t_i)

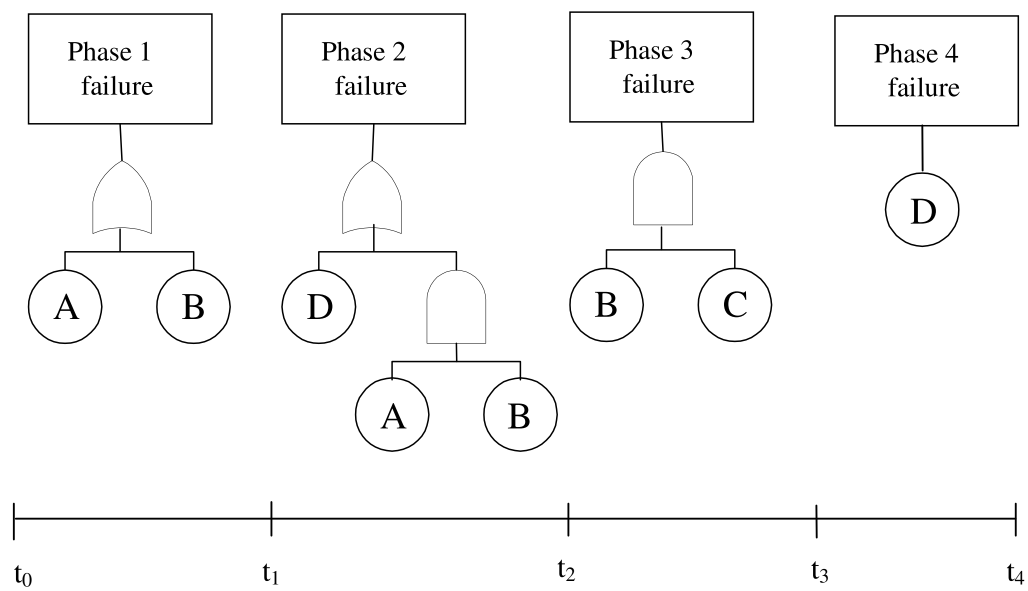

Figure 2 Example system phased mission fault trees

In system analysis which follows, $\overline{A_i}$ represents the functioning of component A throughout phase i and extending this across several phases $\overline{A_{i,j}}$ represents the functioning of component A throughout phases i to j inclusive.

Prior to obtaining the phase failure probabilities, Q_i, the in-phase failure probability Q_i^P, and the phase transition failure probability Q_i^T must be quantified. This requires the phase failure modes for each of these events.

Reduction of the fault tree shown in figure 1will yield all causes of phase failure, this will include both in-phase and phase transition causes combined in the phase failure modes. The approach taken in this paper determines the in-phase and phase transition failure modes separately. First the combined failure modes are produced by applying Boolean reduction to the fault tree of the form shown in figure 1. Then the phase transition failure modes are developed. By removing the phase transition causes from the combined phase causes will provide the in-phase failure modes.

Combined failure modes

Phase 1

There are no previous phases and so this phase can be treated as a normal non-phased mission system. The logic expression for the causes of failure in phase 1, Ph_1 and the likelihood of this event, Q_1, are given by:

$$Ph_1 = A_1 + B_1$$
$$Q_1 = q_{A_1} + q_{B_1} - q_{A_1} q_{B_1} \tag{7}$$

Phase 2
In constructing the fault tree for mission failure during phase 2 it will combine the causes of success in phase 1 ($\overline{A_1 + B_1} = \overline{A_1}.\overline{B_1}$) and the failure conditions for phase 2 being met in phase 2 ($A_{1,2}.B_{1,2} + D_{1,2}$). The failure logic expression and failure likelihood during phase 2 is:

$$\begin{aligned} Ph_2 &= \overline{A_1}.\overline{B_1}.[A_{1,2}.B_{1,2} + D_{1,2}] \\ &= A_2.B_2 + \overline{A_1}.\overline{B_1}.D_{1,2} \\ Q_2 &= q_{A_2} q_{B_2} + p_{A_1} p_{B_1} q_{D_{1,2}} - q_{A_2} q_{B_2} q_{D_{1,2}} \end{aligned} \quad (8)$$

Phase 3
The logic expression for phase 3 is constructed using the causes of successful completion of phase 1, successful completion of phase 2 and the failure conditions of phase 3 being met in phase 3:

$$\begin{aligned} Ph_3 &= \overline{A_1}.\overline{B_1}.(\overline{A_{1,2}.B_{1,2} + D_{1,2}}).[B_{1,3}.C_{1,3}] \\ &= \overline{A_{1,2}}.B_{2,3}.C_{1,3}.\overline{D_{1,2}} + \overline{A_1} B_3 C_{1,3} \overline{D_{1,2}} \\ Q_3 &= p_{A_{1,2}} q_{B_{2,3}} q_{C_{1,3}} p_{D_{1,2}} + p_{A_1} q_{B_3} q_{C_{1,3}} p_{D_{1,2}} - p_{A_{1,2}} q_{B_3} q_{C_{1,3}} p_{D_{1,2}} \end{aligned} \quad (9)$$

Phase 4
Repeating the procedure for phase 4 gives, for completeness, the results:

$$\begin{aligned} Ph_4 &= \overline{A_{1,2}}.\overline{B_1}.\overline{C_{1,3}}.D_{3,4} + \overline{A_1}.\overline{B_{1,3}}.D_{3,4} + \overline{A_1}.\overline{B_{1,2}}.\overline{C_{1,3}}.D_{3,4} \\ Q_4 &= p_{A_{1,2}} p_{B_1} p_{C_{1,3}} q_{D_{3,4}} + p_{A_1} p_{B_{1,3}} q_{D_{3,4}} + p_{A_1} p_{B_{1,2}} p_{C_{1,3}} q_{D_{3,4}} \\ &\quad - p_{A_{1,2}} p_{B_{1,2}} p_{C_{1,3}} q_{D_{3,4}} - p_{A_1} p_{B_{1,3}} p_{C_{1,3}} q_{D_{3,4}} \end{aligned} \quad (10)$$

Phase transition failure modes and probabilities

The phase transition failure modes can be obtained by modifying the top event in the fault tree structure shown in figure 1. Failure on transition to phase i requires success in the previous phases (as in the original structure) AND the failure conditions for phase i are met prior to phase i. This latter input branch to the top event fault tree is formed in the same way as before except component failures are only expanded up to the previous phase ie: $A_1 + A_2 + ... + A_{i-1}$.

The phase transition failure modes, Ph_i^T and probabilities Q_i^T for the simple example system shown in figure 2 are:

Phase 2 transition failure:

$$\begin{aligned} Ph_2^T &= \overline{A_1}.\overline{B_1}.(A_1.B_1 + D_1) = \overline{A_1}.\overline{B_1}.D_1 \\ Q_2^T &= p_{A_1} p_{B_1} q_{D_1} \end{aligned} \quad (11)$$

Phase 3 transition failure

$$\begin{aligned} Ph_3^T &= \overline{A_1}.\overline{B_1}.(\overline{A_{1,2}}+\overline{B_{1,2}}).\overline{D_{1,2}}.[B_{1,2}.C_{1,2}] \\ &= \overline{A_{1,2}}.B_2.C_{1,2}.\overline{D_{1,2}} \\ Q_3^T &= p_{A_{1,2}} q_{B_2} q_{C_{1,2}} p_{D_{1,2}} \end{aligned} \tag{12}$$

Phase 4 transition failure

$$\begin{aligned} Ph_4^T &= \overline{A_{1,2}}.\overline{B_1}.\overline{C_{1,3}}.D_3 + \overline{A_1}.\overline{B_{1,3}}.D_3 + \overline{A_1}.\overline{B_{1,2}}.\overline{C_{1,3}}.D_3 \\ Q_4^T &= p_{A_{1,2}} p_{B_1} p_{C_{1,3}} q_{D_3} + p_{A_1} p_{B_{1,3}} q_{D_3} + p_{A_1} p_{B_{1,2}} p_{C_{1,3}} q_{D_3} \\ &\quad - p_{A_{1,2}} p_{B_{1,2}} p_{C_{1,3}} q_{D_3} - p_{A_1} p_{B_{1,3}} p_{C_{1,3}} q_{D_3} \end{aligned} \tag{13}$$

In-phase failure modes and probabilities

For each phase the in-phase failure modes are obtained by removing the phase transition failure modes from the combined failure modes. These are given, together with the associated probabilities, below:

$$\begin{aligned} Ph_2^P &= (A_2.B_2 + \overline{A_1}.\overline{B_1}.D_{1,2}) - \overline{A_1}.\overline{B_1}.D_1 \\ &= A_2.B_2 + \overline{A_1}.\overline{B_1}.D_2 \end{aligned} \tag{14}$$

$$Q_2^P = q_{A_2} q_{B_2} + p_{A_1} p_{B_1} q_{D_2} - q_{A_2} q_{B_2} q_{D_2} \tag{15}$$

$$\begin{aligned} Ph_3^P &= \overline{A_{1,2}}.B_{2,3}.C_{1,3}.\overline{D_{1,2}} + \overline{A_1}.B_3.C_{1,3}.\overline{D_{1,2}} - \overline{A_{1,2}}.B_2.C_{1,2}.\overline{D_{1,2}} \\ &= \overline{A_{1,2}}.B_3.C_{1,3}.\overline{D_{1,2}} + \overline{A_{1,2}}.B_2.C_3.\overline{D_{1,2}} + \overline{A_1}.B_3.C_{1,3}.\overline{D_{1,2}} \end{aligned} \tag{16}$$

$$Q_3^P = p_{A_1}.q_{B_3}.q_{C_{1,3}}.p_{D_{1,2}} + p_{A_{1,2}}.q_{B_2}.q_{C_3}.p_{D_{1,2}} \tag{17}$$

$$\begin{aligned} Ph_4^P &= [\overline{A_{1,2}}.\overline{B_1}.\overline{C_{1,3}}.D_{3,4} + \overline{A_1}.\overline{B_{1,3}}.D_{3,4} + \overline{A_1}.\overline{B_{1,2}}.\overline{C_{1,3}}.D_{3,4}] \\ &\quad -[\overline{A_{1,2}}.\overline{B_1}.\overline{C_{1,3}}.D_3 + \overline{A_1}.\overline{B_{1,3}}.D_3 + \overline{A_1}.\overline{B_{1,2}}.\overline{C_{1,3}}.D_3] \\ &= \overline{A_{1,2}}.\overline{B_1}.\overline{C_{1,3}}.D_4 + \overline{A_1}.\overline{B_{1,3}}.D_4 + \overline{A_1}.\overline{B_{1,2}}.\overline{C_{1,3}}.D_4 \end{aligned} \tag{18}$$

$$\begin{aligned} Q_4 &= p_{A_{1,2}} p_{B_1} p_{C_{1,3}} q_{D_4} + p_{A_1} p_{B_{1,3}} q_{D_4} + p_{A_1} p_{B_{1,2}} p_{C_{1,3}} q_{D_4} \\ &\quad - p_{A_{1,2}} p_{B_{1,2}} p_{C_{1,3}} q_{D_4} - p_{A_1} p_{B_{1,3}} p_{C_{1,3}} q_{D_4} \end{aligned} \tag{19}$$

In equations 14 and 18 the algebra required for the failure mode subtraction is obvious. In equation 16 the process is not as transparent and each term is first expanded to its fundamental failure modes expressed in terms of single phase variables prior to performing the subtraction.

5. Component Importance Measures

The Criticality Measure of importance identifies the contribution that each component makes to the system failure. This concept will be extended to the phased mission context to produce the importance measures for components for both phase and mission failure. In order to calculate the criticality measure the likelihood of the system being critical for each component needs to calculated (Birbaum's measure of importance). This needs the concept of a critical system state which for non-phased missions is defines as:

A ***Critical System state*** for component i is a state of the remaining components in the system such that the failure of component i will cause the system to make a transition from the working state to the failed state.

The probability that the system is in a critical system state for any component is Birnbaum's measure of importance, G_i and can be calculated from:

$$G_i = \frac{\partial Q_{sys}}{\partial q_i} \quad (20)$$

From this the criticality measure of importance can be determined. It is the probability that the system is in a critical state for component i and that component i has failed. This is normalised by dividing by the system failure probability. This calculates the likelihood that component i has caused the system failure. The Criticality measure of importance for component i is given by:

$$I_i = \frac{G_i q_i}{Q_{sys}} \quad (21)$$

6. Critical Phase States

In-phase failure

For multi-phased missions the possible component states in any phase are dependent upon which failures have occurred during all the preceding phases up to and including phase j.

A ***critical phase state*** for component i in phase j is a state of the remaining components through the previous and current phases such that the system is working on entry to phase j and failure of component i during phase j will cause the phase (and mission) failure.

For this to happen:

i) all phases up to phase j must have completed successfully, and
ii) component i must be in the working state on entry to phase j

As an example consider the critical phase states for component A in phase 2 for the simple example system shown in figure 2. In phase 2 the system state is determined by the state of components A, B and D. In evaluating the critical phase states for A we need to consider the states of components B and D through phase 2 and the preceding phase 1. There are three options for each of the components; they can fail in phase 1, fail in phase 2 or work throughout both phases. For the two components this gives 9 states to consider, which are listed in the first column of table 1. Any combination of states which includes B failed in phase one will result in phase 1 failure and do not need to be considered in phase 2. In phase 2 if D has failed then the phase 2 fails regardless of the state of component A and so these combinations are not critical for A. It is only when D is working and B has already failed that it is critical for A which is just the one combination in row 9. The probability of this combination is the criticality for component A in phase 2 ie:

$$G_{A,2} = q_{B_2} p_{D_{1,2}} \tag{22}$$

Where $G_{i,j}$ denotes the criticality function of component i in phase j.

As the number of components and the number of phases increases this tabular approach soon becomes impractical and another derivation of the criticality function is required.

OTHER COMPONENT STATES	FAILS IN PHASE 1?	CRITICAL FOR COMPONENT A IN PHASE 2?	PROBABILITY
$\overline{B_{1,2}}.D_1$	No	No	-
$\overline{B_{1,2}}.D_2$	No	No	-
$\overline{B_{1,2}}.\overline{D_{1,2}}$	No	No	-
$B_1.D_1$	Yes	-	-
$B_1.D_2$	Yes	-	-
$B_1.\overline{D_{1,2}}$	Yes	-	-
$B_2.D_1$	No	No	-
$B_2.D_2$	No	No	-
$B_2.\overline{D_{1,2}}$	No	Yes	$q_{B_2} p_{D_{1,2}}$

Table 1 Criticality of component A in phase 2.

$$\begin{aligned} G_{i,j} &= \text{P(system is critical for component i in phase j and the system has survived to phase j} \\ &\qquad \text{and component i is working)} \\ &= \text{P(system is failed in phase j with component i failed in phase j and system survives to phase j)} \\ &\quad - \text{P(system is failed in phase j with component i working throughout phase j} \\ &\qquad \text{and system survives to phase j)} \\ &= Q_j(\underline{q}, q_{i_j} = 1) - Q_j(\underline{q}, q_{i_j} = 0) \\ &= \frac{\partial Q_j}{\partial q_{i_j}} \end{aligned} \tag{23}$$

The last step above being true as Q_j is a linear function of q_{i_j}. Note the expression for Q_j derived from the combined phase failure modes is used. For example:

$$\frac{\partial Q_2}{\partial q_{A_2}} = q_{B_2} - q_{B_2} q_{D_{1,2}} = q_{B_2}(1 - q_{D_{1,2}}) = q_{B_2} p_{D_{1,2}} \tag{24}$$

This agrees with the expression derived from table 1.

All derivatives which give the criticality for each component i in each phase j are given in table 2.

	A	***B***	***C***	***D***
Phase 1	p_{B_1}	p_{A_1}	0	0
Phase 2	$q_{B_2} p_{D_{1,2}}$	$q_{A_2} p_{D_{1,2}}$	0	$p_{A_1} p_{B_1} - q_{A_2} q_{B_2}$
Phase 3	0	$p_{A_1} q_{C_{1,3}} p_{D_{1,2}}$	$p_{A_{1,2}} q_{B_2} p_{D_{1,2}} +$ $p_{A_1} q_{B_3} p_{D_{1,2}}$	0
Phase 4	0	0	0	$p_{A_{1,2}} p_{B_1} p_{C_{1,3}} + p_{A_1} p_{B_{1,3}}$ $+ p_{A_1} p_{B_{1,2}} p_{C_{1,3}} - p_{A_{1,2}} p_{B_{1,2}} p_{C_{1,3}}$ $- p_{A_1} p_{B_{1,3}} p_{C_{1,3}}$

Table 2 Birnbaums measure of importance for each component in each phase

Phase transition failure

The phase transition function is likelihood of failure on transition to each phase, Q_j^T . For each component i that contributes to this phase transition failure there will a criticality function, $G_{i,j,k}^T$ expressing the probability that the system is in a critical condition such that the failure of the component i in a phase k prior to phase j will cause the phase transition failure. This is given by:

$$G_{i,j,k}^T = \frac{\partial Q_j^T}{\partial q_{i_k}} \tag{25}$$

The transition criticality function (equation 25) for each component in the simple phased mission example presented in figure 2 for each of the 4 phases is given in table 3.

j	***Phase of component failure k***	***i***			
		A	***B***	***C***	***D***
Phase 2	1	0	0	0	$p_{A_1} p_{B_1}$
Phase 3	1	0	0	$p_{A_{1,2}} q_{B_2} q_{D_{1,2}}$	0
	2	0	$p_{A_{1,2}} q_{C_{1,2}} p_{D_{1,2}}$	$p_{A_{1,2}} q_{B_2} p_{D_{1,2}}$	0
Phase 4	1	0	0	0	0
	2	0	0	0	0
	3	0	0	0	$p_{A_{1,2}} p_{B_1} p_{C_{1,3}} + p_{A_1} p_{B_{1,3}}$ $+ p_{A_1} p_{B_{1,2}} p_{C_{1,3}} - p_{A_{1,2}} p_{B_{1,2}} p_{C_{1,3}}$ $- p_{A_1} p_{B_{1,3}} p_{C_{1,3}}$

Table 3 Transition criticality function for the phase j failure for component i in phase k

7. Phase Importance Measures

In-phase importance

Equation 20 gives the criticality measure of importance for component i in a non-phase mission. Extending this to give the importance contribution to the failure of component i in phase j is:

$$I_{i,j}^{P} = \frac{G_{i,j} q_{i_j}}{Q_j} \tag{26}$$

This in-phase importance measure provides the contribution that the failure of component i has in causing a failure of the mission during phase j. The phase failure can also occur due to a failure on transition to the phase. Both in-phase and transition failures will contribute to Q_j. The two importance contributions can however be considered individually.

Phase transition failure

Phase transition failure requires that the failure conditions for phase j have occurred in some phase k prior to phase j and that these conditions do not result in any previous phase failure. The transition importance measure, $I_{i,j}^{T}$, is the failure contribution that component i makes to the transition failure in phase j as a proportion of the total phase failure. The contribution of the component i is summed over all the preceding phases to phase j ie:

$$I_{i,j}^{T} = \frac{\sum_{k=1}^{j-1} G_{i,j,k}^{T} q_{i_k}}{Q_j} = \frac{\sum_{k=1}^{j-1} \frac{\partial Q_j^T}{\partial q_{i_k}} q_{i_k}}{Q_j} \tag{27}$$

The total importance contribution of component i to the phase j failure is:

$$I_{i,j} = I_{i,j}^{P} + I_{i,j}^{T} \tag{28}$$

8. Mission Importance Measures

When a system does not reach the required level of performance over a mission decisions need to be made as to how the system can be improved in order to achieve a better performance. Weak aspects of the system design need to be identified and rectified. Importance measure can aid in the process of identifying the most significant contributions to the system failure. For a phased mission system it is necessary to identify the contribution to failure made by the components with regard to the entire mission, not just any signal phase. The criticality measure over the mission for each component i is given by the proportion of mission failures to which component i contributes:

$$I_i^M = \frac{\sum_{j=1}^{n}\left\{\frac{\partial Q_j}{\partial q_{i_j}} q_{i_j} + \left(\sum_{k=1}^{j-1}\frac{\partial Q_j^T}{\partial q_{i_k}} q_{i_k}\right)\right\}}{Q_{Miss}} \tag{29}$$

9. Mission Progression

Depending of the mission duration, for some systems it is possible to track the progress of the mission, for example a satellite mission. As the mission successfully completes its sequence of phases the mission or phase failure likelihoods can be updated conditional on the successful completion of each phase.

If the mission has progressed successfully to the end of phase k then the probability of failure, Q_j during phases $1 \leq j \leq k$ is known to be zero. For predictions on the phase j failure probability conditional on having successfully completed phases 1…k, $Q_{j|\bar{k}}$ we have:

$$\begin{aligned} Q_{j|\bar{k}} &= P(\text{system failure in phase j given successful completion of phase k}) \\ &= P(Ph_j | \overline{Ph_1}.....\overline{Ph_k}) \\ &= \frac{P(Ph_j \cap \overline{Ph_1} \cap .....\cap \overline{Ph_k})}{P(\overline{Ph_1} \cap .....\cap \overline{Ph_k})} \\ &= \frac{P(Ph_j)}{P(\overline{Ph_1} \cap .....\cap \overline{Ph_k})} = \frac{Q_j}{1-\sum_{i=1}^{k} Q_i} \end{aligned} \tag{30}$$

In the same way the probability of failure on transition to phase j and the probability of in-phase failure in phase j conditional on the successful completion of the first k phases are:

$$Q_{j|\bar{k}}^T = \frac{Q_j^T}{1-\sum_{i=1}^{k} Q_i}, \qquad Q_{j|\bar{k}}^P = \frac{Q_j^P}{1-\sum_{i=1}^{k} Q_i} \tag{31}$$

The updated mission failure probability is then:

$$Q_{Miss} = \sum_{j=k+1}^{n} Q_{j|\bar{k}} \tag{32}$$

10. System Example

Considering the example phased mission system illustrated in figure 2 and the component phase failure probabilities given in table 4, the phase failure probabilities are: Q_1=0.28, Q_2=0.11275, Q_3=0.01105 and Q_4=0.2106. These are made up of in-phase and phase transition contributions: Q_1^P=0.28, Q_2^P=0.04075, Q_2^T=0.072, Q_3^P=5.631 x 10^{-3}, Q_3^T=5.419 x 10^{-3} , Q_4^P=0.1404 and Q_4^T=0.0702. This gives an overall mission failure probability of Q_{Miss}=0.6144

	A	***B***	***C***	***D***
Phase 1	0.1	0.2	0.025	0.1
Phase 2	0.05	0.1	0.05	0.05
Phase 3	0.2	0.05	0.025	0.1
Phase 4	0.025	0.1	0.05	0.2

Table 4 Component phase failure probabilities

Using equations (26), (27), and (29) the component importance measures throughout the phased mission are given in table 5.

Component	***In-Phase 1 import***	***Trans to Phase 2 import***	***In-Phase 2 import***	***Trans to Phase 3 import***	***In-Phase 3 import***	***Trans to Phase 4 import***	***In-Phase 4 import***	***Total Mission import***
A	0.2857	0	0.0377	0	0	0	0	0.1371
B	0.6429	0	0.0377	0.4904	0.3462	0	0	0.3149
C	0	0	0	0.04904	0.25	0	0	0.1799
D	0	0.6386	0.3171	0	0	0.3333	0.6667	0.5181

Table 5 Importance Contributions

As the mission successfully completes phases equations (30) and (32) proivde the phase and overall mission failure probabilities which are given in table 6.

	Q_1	Q_2	Q_3	Q_4	Q_{Miss}
0	0.28	0.11275	0.01105	0.2106	0.6144
1	-	0.1566	0.01535	0.2925	0.4645
2	-	-	0.01820	0.3468	0.3650
3	-	-	-	0.3532	0.3532

Table 6 Phase Progression Failure Probabilities

11. Conclusions

A phased mission modelling approach has been presented. A means to evaluate the contribution made by each component to phase and mission failure has been developed. The method enables the phase and mission failure likelihood predictions to be updated as phases of the mission are successfully completed.

The work will be developed to enable importance measures to be updated as phases are completed successfully. A Binary Decision Diagram implementation is also expected to increase the efficiency of the quantification process for the updated mission unreliability and importance measure predictions so that thee can be accomplished in real time and open up the application of this sort of work to the development of decision making approaches for autonomous systems.

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