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A New Family of Blind Adaptive Equalization Algorithms

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Abstract—New blind adaptive channel equalization techniques based on a deterministic optimization criterion are presented. A family of nonlinear functions is proposed which constitutes a generic class of blind algorithms. They have been shown to have better performance than the conventional constant modulus algorithm (CMA)-like approaches. The advantages include the relaxed stability range on the step size and that an automatic gain control unit which estimates the gain of the channel, is no longer of crucial importance.

I. INTRODUCTION

THE soft-constraint-satisfaction (SCS)-based algorithms have been proven to have better performance than the conventional CMA-like approaches [1]. The SCS approach in [1] includes a particular nonlinearity. In this letter, a general family of nonlinear functions have been postulated to maintain the improved performance of SCS-based approach thereby providing an infinite number of functions for use.

II. DERIVATION OF SCS ALGORITHMS

The rationale behind the SCS approach, contrary to the previous research of stochastic cost functions, is to use a deterministic cost function with appropriate constraints. Consider the following optimization problem, also known as the principle of minimum disturbance [2]: Determine the tap-weight vector of dimension N at time k, W_k, given the tap-input vectors X_k, X_{k-1}, $\cdots$, and desired responses d_k, d_{k-1}, $\cdots$, so as to minimize the squared Euclidean norm of the change in the tap-weight vector W_k, i.e.,

$$\min_{W_k} \|W_k - W_{k-1}\|^2 \qquad \text{subject to } X_k^H W_k = d_k \quad (1)$$

where d_k is the training sequence. In blind equalization the training sequence is not known to receiver, and hence the desired response must be obtained from pertinent quantities in the receiver. The *a priori* equalizer output $y_k = X_k^H W_{k-1}$ can be used for our purposes. Hence, the following choice for

TABLE I
THE ESSENTIAL FEATURES OF CM NONLINEAR
FUNCTIONS $\psi(y)$: $R^1 \to R^1$, $y \in R^1$, cf. FIG. 1

$\psi(y)$ is continuous and odd	$\psi(y)$ has finite derivative at the origin
$\psi(y) > y$ for $0 < y < 1$	$\psi(y)$ intersects y solely at 1 and -1
$\psi(y) - y$ must be decreasing for $y > 1$	$\psi'(y)$ must be decreasing for $0 < y < 1$

d_k is a viable one:

$$d_k = \psi(y_k) \qquad (2)$$

where $\psi(.)$ is a nonlinear function which satisfies certain conditions. It is possible to include multiple constraints in the derivation of algorithms, but it is beyond the scope of this letter. In [1] the particular SCS-1 nonlinearity $\psi(y) = (2 - |y|)y$ has been derived indirectly. Herein, a general approach is presented to include a family of functions.

The solution of the above optimization problem via Lagrange multipliers yields

$$W_k = W_{k-1} + \frac{1}{X_k^H X_k} X_k e_k \qquad (3)$$

where $e_k \triangleq d_k - X_k^H W_{k-1}$. A stepsize is also introduced to maintain the stability of the algorithm. Hence, we can obtain the update of the SCS algorithms as

$$W_k = W_{k-1} + \mu \frac{1}{X_k^H X_k} X_k e_k. \qquad (4)$$

The principle of minimal disturbance can be modified for use in fractionally spaced equalizers as well.

A. Conditions on the Nonlinear Function $\psi(\cdot)$

The choice of nonlinear functions $\psi(\cdot)$ is of crucial importance in the development. These functions restore the constant modulus (CM) of the communication signals. For the sake of simplicity, we assume that the communication system uses real modulation schemes.[1] The essential features of CM nonlinear functions are tabulated in Table I. If the characteristics of the functions are investigated with reference to Fig. 1, it is seen that the functions should be symmetric with respect to origin (odd) because these type of equalizers cannot resolve 180° of phase ambiguity. The sole intersection points ∓ 1 correspond to the desired CM of the equalizer output. When the equalizer output is close to CM, so is the nonlinear function and the correction term in (4) is minimal. The continuity and finite derivative properties come from the smoothness

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[1] The functions can be used as well for complex modulation schemes but care should be taken not to neglect modulus operators.
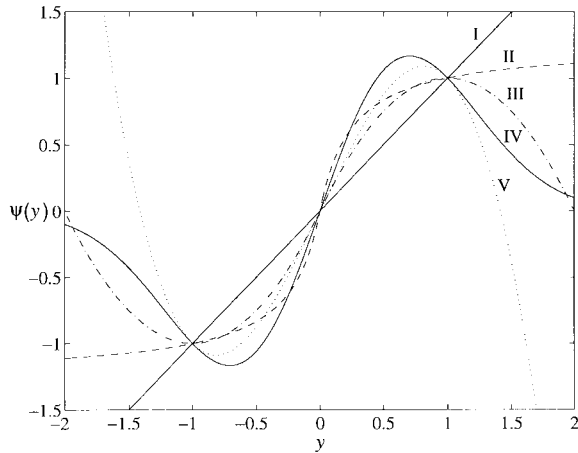
Fig. 1. Examples of CM functions for SCS algorithms: I) y (equalizer output), II) $(y/1 - 0.8(1 - |y|))$, III) SCS-1: $(2 - |y|)y$, IV) $ye^{-|y|^2+1}$, and V) Godard: $(2 - |y|^2)y$.

and differentiability restrictions. The other properties about the range of y define the requirements for the algorithm to converge to the CM. For instance, $\psi(y) > y$ for $0 < y < 1$ guarantees that the correction term in (4) pushes the algorithm toward CM when $0 < y < 1$.

An infinite number of functions can be found to satisfy the properties in Table I, which provide flexibility in the design of blind equalizers. For instance, in limited precision arithmetic processors higher powers of the input signal can be avoided. Convergence speed and estimation error are dependent on the choice of the nonlinear function. At low signal-to-noise ratio (SNR) values, the shape of the nonlinear function is less critical.

III. SIMULATIONS

We simulated a T-spaced equalizer with three taps in a fading scenario with three algorithms which result from the nonlinearities III–V in Fig. 1. Also, we have included the unnormalized versions, i.e., the normalization in (4) have been removed. The open-eye-measure (OEM)[2] [1] for each

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[2] The OEM for channel equalizers is defined as $\mathrm{OEM}(k) \triangleq (\|T_k\|_1 - \|T_k\|_\infty / \|T_k\|_\infty)$ where T_k represents the combined channel and equalizer space. If $\mathrm{OEM}(k) < 0$ dB then the eye is open and ISI has no effect in the decision process. If $\mathrm{OEM}(k) > 0$ dB, the eye is closed and hence the ISI left after equalization will cause decision errors. The lower the OEM, the better approximation of the inverse channel the equalizer provides.

TABLE II
STEP SIZED (TOP ROW) AND MEAN OEM'S (dB) (BOTTOM ROW) IN EACH SLOT. QAM APPROXIMATION OF GMSK, THREE-RAY CHANNEL MODEL WITH AVERAGE POWERS OF 0.1, 0.8, AND 0.3 AT INTEGER DELAYS, SNR OF 40 dB, BIT RATE OF 271 kb/s, 950 MHz CARRIER FREQUENCY AND MOBILE SPEED OF 150 km/h

Unnormalized IV	0.01	0.02	0.03	0.04	0.05	0.06	0.07
	-4.618	-5.374	-5.108	-7.050	-7.179	-4.609	-5.372
IV (c.f. Figure 1)	0.07	0.08	0.09	0.1	0.125	0.15	0.175
	-6.776	-6.929	-7.026	-7.088	-7.166	-6.923	-6.914
Unnormalized V (Godard Algorithm [2])	0.009	0.01	0.02	0.03	0.04	0.05	0.06
	-4.689	-5.806	-6.636	-6.698	-6.956	NaN	NaN
V (c.f. Figure 1)	0.07	0.08	0.09	0.1	0.125	0.15	0.175
	-7.642	-7.846	-7.930	-7.911	-7.841	-7.826	NaN
Unnormalized III	0.01	0.02	0.03	0.04	0.05	0.06	0.07
	-5.987	-4.657	-6.813	-5.695	-5.761	-7.159	-6.913
III (SCS-1 Algorithm [2])	0.125	0.15	0.175	0.2	0.225	0.25	0.275
	-7.556	-7.697	-7.771	-7.826	-7.641	-7.729	7.745

algorithm is averaged over 30 000 samples in dB scale. The runs are identical for each of the algorithms. The initial convergence periods have been excluded in the computations. The results in Table II demonstrate the improved tracking capability of the SCS algorithms in fading environments. Time-varying channel taps cause singularities frequently in the channel, i.e., zeros of the channel cross the unit circle, which means that it is impossible to equalize the channel. Equalizer taps fluctuate during singularities. Good tracking algorithms recover quickly and reconverge to optimum values. In that sense the SCS algorithms have a faster response to changes in the channel. The lower the OEM, the better the algorithms track the time-varying channel on average. Also, the SCS algorithms maintain tracking capability over a wider range of stepsizes, whereas the unnormalized counterparts have their best performance in a limited range.

IV. CONCLUSIONS

A family of nonlinear functions is proposed as an extension of the SCS approach [1] which form a new set of blind adaptive algorithms. They are observed to maintain the advantages of the SCS approach.

REFERENCES

[1] O. Tanrikulu, B. Baykal, A. Constantinides, and J. Chambers, "Soft constraint satisfaction (SCS) blind channel equalization algorithm," *Int. J. Adaptive Contr. Signal Processing,* vol. 12, no. 2, pp. 117–134, 1998.
[2] B. Widrow and M. Lehr, "30 years of adaptive neural networks: Perceptron, madaline and backpropagation," *Proc. IEEE,* vol. 78, pp. 1415–1442, 1990.