A NOVEL BLIND EQUALIZATION STRUCTURE FOR DEEP NULL COMMUNICATION CHANNELS

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ABSTRACT

A new blind equalization structure that is well suited for communication channels whose zeros are close to the unit circle is proposed. Most blind equalizers which operate at the baud rate perform poorly for channels whose maximum phase zeros are close to the unit circle. This limitation is mainly due to the inability to model the inverse of such maximum phase zeros with a finite length filter. Our proposed structure adaptively models the inverse channel, completely, without the need to transmit a training sequence. Therefore Inter Symbol Interference (ISI) is removed even if the channel has deep spectral nulls. Another attractive feature of this structure is that it estimates the channel parameters directly, and as such may be used with "indirect" equalization techniques. Simulation studies are included to demonstrate the performance of the scheme.

1. INTRODUCTION

Digital transmission over a band limited channel introduces ISI that can be eliminated by employing an adaptive equalizer. However, a training sequence is generally required. Equalizers which operate without such a training sequence are termed "blind", and potentially offer improved bandwidth efficiency.

A communication channel (wired/wireless) can be modelled as an FIR filter. An equalizer is a device that when cascaded with the channel ideally gives an overall impulse response of the following form

$$b * c = A\ \delta(k-d) \qquad (1)$$

where $*$ denotes discrete convolution, b and c are respectively the impulse responses of the channel and the equalizer, A is a non zero positive constant, and d is the overall delay. Taking the z-transform on both sides of eqn. (1), and rearranging, yields

$$C(z^{-1}) = \frac{A\ z^{-d}}{B(z^{-1})} \qquad (2)$$

The equalizer, $C(z^{-1})$, is therefore the inverse of the channel, $B(z^{-1})$. Even though $B(z^{-1})$ is a finite length polynomial , $C(z^{-1})$ is generally infinite length and can only be approximated by a finite length polynomial in z^{-1}. As the zeros of $B(z^{-1})$ approach the unit circle, the length of $C(z^{-1})$ required to approximate the inverse increases. In addition to the difficulty of modelling an inverse channel with a finite length filter, a blind equalizer also has to be designed automatically without a training sequence. The CMA algorithm [1], a self recovering algorithm, that is widely used in practice, performs poorly when the zeros of the channel are close to the unit circle. This limitation is also due to the difficulty of modelling the inverse channel with an FIR filter. We examine this problem, and propose a new equalization structure that models the inverse channel completely and removes ISI. Other advantages of using this structure will be demonstrated later in this paper.

2. THE BLIND EQUALIZER BASED UPON AN IIR FILTER AND AN ALL PASS FILTER

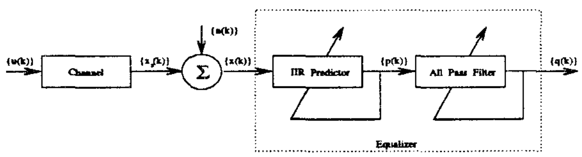

Figure 1: The base band model of the channel and the equalizer

If the FIR channel has all of its zeros inside the unit circle (i.e. a minimum phase channel), then the inverse can be completely modelled by an IIR filter, where the zeros of the channel are exactly cancelled by the poles of the IIR filter. If the channel, however,

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has at least one zero which lies outside the unit circle, then the poles of the IIR filter can no longer cancel this maximum phase zero, as a pole which lies outside the unit circle leads to an unstable causal predictor. In [2], a blind equalizer was proposed for a non minimum phase channel as in figure (1). The IIR filter coefficients are adapted to minimise $E\{p^2(k)\}$ while the all pass filter coefficients are adapted to minimise $E\{q^4(k)\}$. Let the transfer function of the channel be

$$B(z^{-1}) = \alpha \prod_{i=1}^{d}(1-\xi_i^l z^{-1}) \prod_{j=1}^{s}((\xi_j^o)^{-1} - z^{-1}) \quad (3)$$

where $n_c = d+s$ is the order of the channel, α is a scalar constant, $|\xi_i^l| < 1,\ i = 1, ..., d$ correspond to the minimum phase zeros, and $|\xi_j^o| > 1,\ j = 1, ..., s$ are the maximum phase zeros. No zeros are assumed to lie on the unit circle. After adaptation, the transfer function of the IIR filter is the Spectrally Equivalent Minimum Phase (SEMP) of the channel [2],[5], that is given as

$$\begin{aligned} I(z^{-1}) &= \frac{1}{R(z^{-1})} \quad (4) \\ &= \frac{1}{\prod_{i=1}^{d}(1-\xi_i^l z^{-1}) \prod_{j=1}^{s}(1-(\xi_j^o z)^{-1})} \end{aligned}$$

The cascade of the channel and predictor, therefore, has an all pass z-domain transfer function that is written as

$$H(z^{-1}) = \frac{B(z^{-1})}{R(z^{-1})} = \alpha \frac{\prod_{j=1}^{s}((\xi_j^o)^{-1} - z^{-1})}{\prod_{j=1}^{s}(1-(\xi_j^o z)^{-1})} \quad (5)$$

The all pass filter in figure (1) linearises the phase response of the all pass filter, $H(z^{-1})$. Again the length of the all pass filter required, increases dramatically as the maximum phase zeros of the channel approach the unit circle. In [5], we showed that by employing two time reversers, one before and one after the all pass filter, the phase distortion caused by the combination of the channel and the IIR filter can be removed with only a finite length all pass filter, as given in the theorem of [5].

Theorem 1: If $\{p(k)\}$ is the output, assumed for convenience to be deterministic, of an all pass filter, whose transfer function is $H(z^{-1})$, to the input $\{u(k)\}$, then the time reversed sequence of $\{u(k)\}$ can be reconstructed exactly by sending the time reversed sequence of $\{p(k)\}$ through an all pass filter whose transfer function is $H(z^{-1})$.

□

The length of the all pass filter required in this case is same as that of the maximum phase component of the channel.

3. THE IIR^2 FIR EQUALIZATION STRUCTURE

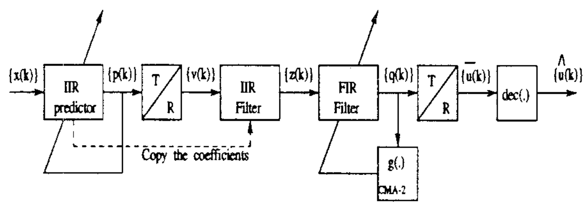

Figure 2: The blind equalizer structure based on two IIR filters and an FIR filter

The new equalizer structure is shown in figure (2) in which we decompose the all pass filter into an IIR filter whose transfer function is identical to that of the previous IIR filter and an FIR filter [6]. The first IIR filter is adapted with a prediction algorithm to minimise $E\{p^2(k)\}$. The FIR filter is adapted using a normalised CMA algorithm to minimise $E\{(q^2(k) - r)^2\}$, $r = \frac{E\{u^4(k)\}}{E\{u^2(k)\}}$, [6]. At its optimal settings, the transfer function of the FIR filter is the same as that of the channel as given in the corollary of [6].

The following results capture the characteristics of the proposed structure,

i) This equalizer removes the magnitude and phase distortion of the channel and reconstructs the transmitted sequence.

ii) The length of the IIR filters and the FIR filter that models completely the inverse channel are finite, hence the equalizer has reduced computational complexity.

iii) The order of the channel can also be estimated directly from this structure.

iv) This equalizer not only reconstructs the transmitted sequence, but also estimates the channel parameters directly. At the optimum, the impulse response of the FIR filter and the channel are identical.

The most beautiful feature of this structure is the ability to estimate the channel parameters directly while reconstructing the transmitted sequence. This property was used in [7], to initialise a Radial Basis Function (RBF) equalizer and to reconstruct the transmitted binary sequence for a rapidly time varying channel.

Let us present a simulation to show the ability of this structure to estimate a deep null communication channel, and then establish correspondence between this structure and a decision feedback equalizer.

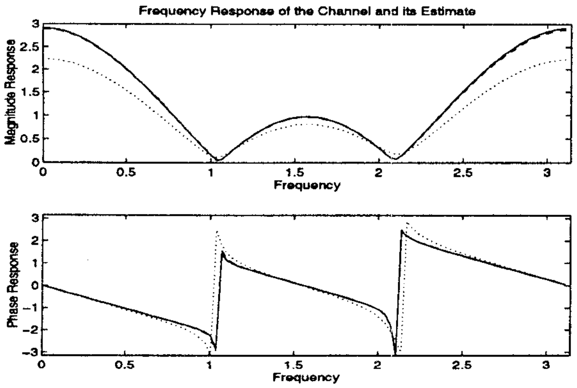

Figure 3: The estimate of the magnitude and the phase of the channel (dotted: estimation after 1500 samples; dashed: estimation after 3000 samples)

A fourth order maximum phase channel is employed $0.9413 + 0.0097z^{-1} + 0.9703z^{-2} - 0.0100z^{-3} + 1.0000z^{-4}$ that has complex zeros at $(1/0.99)\ exp(\pm j\pi/3)$ and $(1/0.98)\ exp(\pm j2\pi/3)$. The IIR filter and the FIR filter are respectively adapted with a prediction algorithm and a normalised CMA algorithm as in [6]. The magnitude and the phase response of the channel and its estimate (FIR filter) are depicted in figure (3). The estimates were averaged over 25 independent trials.

4. ERROR ACCUMULATION ANALYSIS

After adaptation, the positions of the IIR filter and the FIR filter can be reversed to produce an improved equalization structure by introducing a non linearity in the feedback loop of the IIR filter. A similar technique has been independently developed in [4] where the required all pass filter is approximated by an FIR filter and the positions of the IIR filter and the FIR filter is reversed. In order to analyse our new structure, let the transfer function of the combined channel and the IIR filter, that is an all pass filter, be

$$H(z^{-1}) = \frac{\sum_{i=0}^{n_c} h_i z^{-n_c+i}}{\sum_{i=0}^{n_c} h_i z^{-i}} \qquad (6)$$

in which $h_0 = 1$ and n_c is the order of the all pass filter.

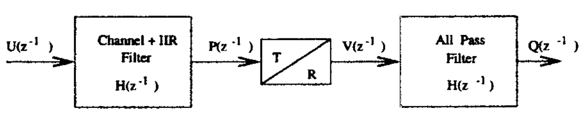

Figure 4: The structure of the proposed equalizer

The input and the output relationships of figure (4) are written as

$$P(z^{-1}) = H(z^{-1})U(z^{-1}) \qquad (7)$$

$$V(z^{-1}) = P(z) \qquad (8)$$

$$Q(z^{-1}) = H(z^{-1})V(z^{-1}) \qquad (9)$$

Denoting $T(z) = U(z^{-1})$, and applying eqn.(7) and eqn.(8), $V(z^{-1})$ is written as

$$V(z^{-1}) = H(z)T(z^{-1}) \qquad (10)$$

Substituting eqn.(6), eqn.(10) can be rewritten as

$$\sum_{i=0}^{n_c} h_i z^{n_c-i}\, T(z^{-1}) = \sum_{i=0}^{n_c} h_i z^{i}\, V(z^{-1}) \qquad (11)$$

Combination of eqn.(6) and eqn.(9) yields

$$\sum_{i=0}^{n_c} h_i z^{-i}\, Q(z^{-1}) = \sum_{i=0}^{n_c} h_i z^{-n_c+i}\, V(z^{-1}) \qquad (12)$$

The last two equations can be combined to form the following equation

$$q(k) = t(k) + \sum_{i=1}^{n_c} h_i (t(k-i) - q(k-i)) \qquad (13)$$

where $q(k)$ and $t(k)$ are respectively the inverse z transforms of $Q(z^{-1})$ and $T(z^{-1})$. Now the error sequence $\{e(k)\}$ can be written as the difference between $\{t(k)\}$ and $\{q(k)\}$ as follows

$$e(k) = -\sum_{i=1}^{n_c} h_i e(k-i) = \boldsymbol{h}^t(\boldsymbol{t}_{k-1} - \boldsymbol{q}_{k-1}) = \boldsymbol{h}^t \boldsymbol{e}_{k-1} \qquad (14)$$

where the vectors $\boldsymbol{h}$, $\boldsymbol{t}_{k-1}$, $\boldsymbol{q}_{k-1}$ and $\boldsymbol{e}_{k-1}$ are clear from the equation. If $\boldsymbol{t}_{k-1} = \boldsymbol{q}_{k-1}$ then $q(k) = t(k)$ and hence

$$q(k+j) = t(k+j) \qquad for\ all\ positive\ j \quad (15)$$

and therefore a perfect reconstruction is achieved.

When the all pass filter is initialised by a new set of data, $\boldsymbol{t}_{k-1}$ will no longer be equal to $\boldsymbol{q}_{k-1}$, and $e(k)$ will be a function of its past. The error is therefore an exponentially decaying function. However, from eqn.(14), if we set the vector $\boldsymbol{q}_{k-1} = \boldsymbol{t}_{k-1}$, we can force the error $e(k)$ to zero, and it continues at that value (i.e., zero), regardless of the weight vector of the all pass filter. Fortunately, we can do this, because, we know a priori that the transmitted sequence can have only certain discrete levels. Therefore a non linearity, the $dec(.)$ function, which is a $sgn(.)$ function for a binary sequence, is placed in the feedback loop of the all pass filter after a number of iterations. Due to this non linearity, the vector $\boldsymbol{q}_{k-1}$ will be equal to $\boldsymbol{t}_{k-1}$, and any error accumulation is stopped. A soft transition [3] can be made in the feedback loop of the all pass filter as shown in figure (5), in which the mixing parameter λ is gradually increased from zero to unity, as the iterations progress. The λ is again set to zero and increased to one whenever the block is initialised with a new set of data.

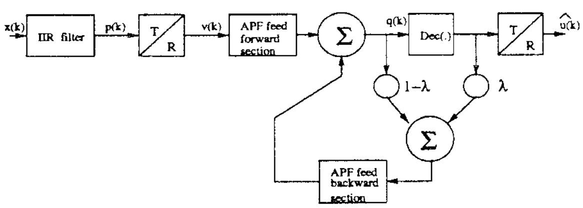

Figure 5: The modified model of the block based time reversal equalizer

Another interesting property of this structure is the all pass filtering effect of the equalizer (at full decision feedback, i.e. $\lambda = 1$) to the noise that is present at channel output, i.e. the combination of the IIR filter, the first time reverser and the feed forward path of the all pass filter (FIR filter) behaves as an all pass filter. Therefore, if the noise at the channel output is white Gaussian, then the noise at the equalizer output is also white Gaussian, but only with a scalar multiplication $(1/\alpha)$. Therefore, the Bit Error Rate (BER) of a binary transmission of $\{\pm d\}$, after the equalization using this structure, and assuming no error propagation can be written as

$$BER = \frac{1}{2}\, erfc(\frac{\alpha d}{\sqrt{2}\sigma_n}) \qquad (16)$$

where σ_n^2 is the noise variance at the channel output and $erfc(.)$ is the complementary error function given by

$$erfc(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} exp(-z^2)dz \qquad (17)$$

5. CONCLUSION

A new blind equalization structure was proposed for communication channels whose zeros are close to the unit circle (i.e. deep spectral nulls). Apart from equalizing the channel without the need for a training sequence, this structure also exhibits many useful properties such as identifying the channel parameters directly, channel order estimation and modelling the inverse channel completely with only finite length filters, hence reduced computational complexity. In addition, we showed how this structure can be modified to form a decision feedback equalizer. As far as the noise present at the channel output is concerned, this structure behaves as an all pass filter, and hence there is effectively no noise amplification due to filtering except a scalar multiplication $1/\alpha$.

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