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# CHANNEL ESTIMATORS FOR HF RADIO LINKS 

\author{


#### Abstract

A Doctoral Thesis submitted in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy of the Loughborough University of Technology


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December 1988

Supervisor: Professor A. P. Clark<br>Department of Electronic and Electrical Engineering

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#### Abstract

The thesis is concerned with the estimation of the sampled impulse-response (SIR), of a time-varying HF channel, where the estimators are used in the receiver of a 4800 bits/s, quaternary phase shift keyed (QPSK) system, operating at 2400 bauds with an 1800 Hz carrier.

HF modems employing maximum-likelihood detectors at the receiver require accurate knowledge of the SIR of the channel. With this objective in view, the thesis considers a number of channel estimation techniques, using an idealised model of the data transmission system. The thesis briefly describes the ionospheric propagation medium and the factors affecting the data transmission over HF radio. It then presents an equivalent baseband model of the HF channel, that has three separate Rayleigh fading paths (sky waves), with a 2 Hz frequency spread and transmission delays of $0,1.1$ and 3 milliseconds relative to the first sky wave.

Estimation techniques studied are, the Gradient estimator, the Recursive leastsquares (RLS) Kalman estimator, the Adaptive channel estimators, the Efficient channel estimator ( that takes into account prior knowledge of the number of fading paths in the channel ), and the Fast Transversal Filter (FTF), estimator (which is a simplified form of the Kalman estimator). Several new algorithms based on the above mentioned estimation techniques are also proposed.

Results of the computer simulation tests on the performance of the estimators, over a typical worst channel, are then presented. The estimators are reasonably optimized to achieve the minimum mean-square estimation error and adequate allowance has been made for stabilization before the commencement of actual measurements. The results, therefore, represent the steady-state performance of the estimators.

The most significant result, obtained in this study, is the performance of the Adaptive estimator. When the characteristics of the channel are known, the Efficient estimators have the best performance and the Gradient estimators the poorest. Kalman estimators are the most complex and Gradient estimators are the simplest. Kalman estimators have a performance rather similar to that of Gradient estimators. In terms of both performance and complexity, the Adaptive estimator lies between the Kalman and Efficient estimators. FTF estimators are known to exhibit numerical


instability, for which an effective stabilization technique is proposed. Simulation tests have shown that the mean squared estimation error is an adequate measurement for comparison of the performance of the estimators.

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## GLOSSARY OF SYMBOLS AND TERMS

$\mathrm{a}(\mathrm{t})$ Impulse-response of a filter
A(f) Frequency résponse of a filter
$|A(f)|$ Absolute value of $A(f)$
$a(t) * b(t)$ Convolution between $a(t)$ and $b(t)$
$C_{i}$ Weighted squared error in the $\left\{r_{b}\right\}$
$e_{i}$ Error in the estimated value of $r_{i}$
E [.] Expectation operator
$\mathrm{g}+1$ Number of samples in the sampled impulse-response of linearbaseband channel
j When not used as a subscript, $\mathrm{j}=\sqrt{-1}$
$\mathrm{K}_{\mathrm{i}}$ Kalman gain vector
$\mathrm{n}(\mathrm{t})$ White Gaussian noise with zero mean and two-sided power spectral density of $\frac{1}{2} \mathrm{~N}_{0}$
$\frac{1}{2} \mathrm{~N}_{0}$ Power spectral density of $\mathrm{n}(\mathrm{t})$
$\mathrm{q}_{\mathrm{b}}(\mathrm{t})$ Statistically independent random processes
$\left\{q_{h i}\right\}$ Sequence obtained by sampling $q_{h}(t)$
$\mathfrak{R}$ [.] Real part of a complex number
r(t) Received signal
$\left\{r_{i}\right\}$ Sequence of received signal samples
$r_{i}^{\prime}$ Estimated received signal sample
$\mathrm{s}_{\mathrm{i}}$ Data symbol$s_{i}^{\prime}$ Detected data symbol
$\bar{S}_{h}$ Complex conjugate of the vector $\mathrm{S}_{\mathrm{b}}$
Superscript * Complex conjugate
Superscript T Matrix (or Vector) transpose
T Sampling interval
$V_{i}^{*}$ Conjugate transpose of the vector $\mathrm{V}_{\mathrm{i}}$
$\mathrm{w}(\mathrm{t})$ Gaussian random process with zero mean
$y(t)$ Impulse-response of linear baseband channel
$Y_{i}$ Sampled impulse-response of linear baseband channel$Y_{i}^{\prime}$ Estimate of $\mathrm{Y}_{\mathrm{i}}$ at time $\mathrm{t}=\mathrm{i} \mathrm{T}$
$Y_{i+1, i}^{\prime}$ One-step prediction of $Y_{i+1}$ at time $t=i T$
$Y_{i+1, i}^{\prime \prime}$ Estimate (prediction) of the rate of change with respect to i of $Y_{i+1}$$\xi_{1}$ Mean square error in the estimate (prediction) of $Y_{i}$$\xi_{2}$ Mean square normalized error in the estimate (prediction) of $\mathrm{Y}_{\mathrm{i}}$$\xi_{i}$ Square of the error in the estimate (prediction) of $\mathrm{Y}_{\mathrm{i}}$
$\sigma^{2}$ Variance of $w(t)$ or $\left\{w_{i}\right\}$
$\phi_{h}$ Transition matrix
$\psi$ Signal/noise ratio

## CHAPTER 1

## INTRODUCTION

### 1.1 BACKGROUND.

The radio frequency band, between $3-30 \mathrm{MHz}$, called the High frequency (HF) band [1], has for many years been used as a transmitting medium, for communication over long distances. Despite the advent of satellite communication systems, HF still continues to be used owing to the fact that it is economical and flexible. HF has been extensively used for point - to - point communication, for commercial shipping, aircraft communication and for military applications. The ionosphere acts as a good reflector of HF radio waves and propagation of HF is achieved by ionospheric refraction [1]. However, HF transmission is unpredictable due to the existence of multiple transmission paths caused by reflection from different layers of the ionosphere [1-20]. Advances in modem design and availability of high speed signal processing chips has renewed interest in data transmission over HF radio links.

In the past, the HF radio medium was successfully used for low data rate telegraphy, such as manually transmitted and received Morse code. The signal element duration here, with serial transmission, is very much greater than the multipath spread and thus the detection of such a signal could be achieved through simple means. However, with the increasing demand for HF communication, it has become necessary to increase the data transmission rate, and as a result the multipath spread can extend over the duration of several signal elements. The system is now subject to inter-symbol interference (ISI) [2-4]. As the speed of transmission increases, so do the bit errors, affecting both modem performance and reliability. Until recently, the preferred method of transmitting digital data at medium to high speed, (greater than $1200 \mathrm{bits} / \mathrm{s}$ [67]), has been to employ a number of low speed channels in parallel in order to avoid ISI. The transmission takes place at a fairly low rate, over each of a number of subchannels within a 3 KHz band. An alternative approach is to use serial transmission and employ some form of adaptive signal processing at the receiver. Comparison of the two transmission techniques at a speed of $2400 \mathrm{bits} / \mathrm{s}$
has shown that the performance of a parallel HF modem is generally inferior to that of the serial modem [33, 52]. The present trend is, therefore, towards serial modems.

HF radio links are time-varying channels which can introduce considerable levels of attenuation and delay distortions [1-8, 12-18, 29, 42]. Therefore, for the correct operation of the data-transmission system, at all times, over such a link, the system should be designed for the optimum tolerance to attenuation and delay distortions. Considerable advances have been achieved in the design of serial modems for HF radio links [5, 29, 31, 42-43, 46-48, 50-52, 67-69, 71-72]. This has made it possible to achieve an increase in the highest practically obtainable transmission rate over a voiceband HF channel from 2400 to 9600 bits/s [5, 29, 31, 42-43, 46-48, 50-52, 67-69, 71-72]. The increase has been achieved through the development of more effective techniques for tracking the sampled impulse-response of the time-varying baseband channel [53-64, 73-89, 92-97, 99-116], together with the development of more effective detection processes for handling the severe signal distortion introduced by the HF radio link [31, 47-49, 51, 90].

Increasing the data rate over a bandlimited channel results in an increase in the amount of inter-symbol interference (ISI). Detectors used to combat inter-symbol interference can be classed into two separate groups. The first group of detectors employ an equalizer, which has a knowledge of the interfering components. The received signal passes through an equalizer before arriving at the detector input and the detection process is now a simple threshold comparator. The detector makes a decision on the value of a transmitted data symbol, by comparing the corresponding sample value with the appropriate threshold level (or levels). Equalization techniques used are the linear equalizer and the non-linear equalizer (or the deci-sion-feedback equalizer) $[20,36,42-43,56]$. The tap gains of a linear or non-linear equalizer can be adjusted adaptively for a time varying channel, using the gradient algorithm [42, 75] or the Kalman algorithm [75, 78-79] or a lattice algorithm [86] to minimize the mean square error in the equalized signal at its output. Alternatively, an equalizer can be adjusted from an estimate of the sampled impulse-response of the channel.

The second method available to overcome the problem of inter-symbol interference is to modify the detection process itself, to take account of the signal distortion that has been introduced by the channel. These detectors perform processes of maximum
likelihood detection or maximum likelihood sequence estimation (MLSE), and are the optimum detection processes for a sequence of data symbols transmitted over a non ideal bandlimited channel which introduces ISI and additive white Gaussian noise (AWGN), and where the transmitted symbols are equally likely to have any one of their possible values [42]. In maximum-likelihood detection, the detector, instead of removing the ISI, takes full account of it, thus using the entire transmitted energy in the detection process.

The Viterbi algorithm is used to implement MLSE [20, 36-37, 42-43, 47]. It is not feasible to implement the MLSE in its true form because of the enormous memory requirement and equipment complexity. However, the Viterbi algorithm achieves the same tolerance to noise as that of MLSE [20, 36-37, 42-43, 47]. A Viterbi detector operates by storing a complete set of possible sequences (vectors) of transmitted data symbol values together with the costs of the vector. The cost of a vector is taken as the square of the unitary distance between the corresponding received signal vector, for the given signal distortion but in the absence of noise, and the signal vector actually received. The detected message is selected as the particular sequence or vector, which has the minimum cost.

For a Viterbi detector, the amount of storage required and computational complexity increases exponentially as the number of components in the sampled impulse-response of the channel increases. In order to overcome this problem, the detector is further modified and also an allpass linear feedforward transversal filter network is employed ahead of the detector [49]. The detector now limits the number of vectors held in the receiver at any time instant to a small value, regardless of the number of components in the sampled impulse-response of the channel, but without reducing unduly the tolerance of the detector to noise. This type of detector is now referred to as a near-maximum likelihood (NML) detector [36-37, 42, 47-48]. It has been shown in [51] that NML detectors are not significantly inferior to true Viterbi detectors in terms of their tolerances to additive white Gaussian noise, especially when binary and quaternary signals are transmitted.

Decision-feedback equalizers have a poor performance over a time varying HF channel [37, 47]. At low error rates, they are 1-3 dB inferior to the corresponding NML detectors depending on severity of fades [47]. The errors here are caused predominantly by the deepest fades and, in addition, the equalizers suffer from inherent error propagation tendencies.

A NML detector is, therefore, the most suitable detector for a time varying HF channel. However, these detectors require a knowledge of the sampled impulse-response of the channel. The adaptive filter used ahead of the detector also requires this information. Under the condition when the detector and adaptive filter is provided with the correct channel estimate and when perfect operation of the adaptive filter can be assumed, the performance of the NML detector gives an upper bound to the performance obtained when the channel response must be estimated. Any error in the estimation of the channel directly affects the performance of the detector. Further more, an incorrect channel estimate leads to an incorrect adjustment of the adaptive filter. It is, therefore, very essential, for the good performance of the detector (and hence the HF modem), that the channel estimator is able to make an accurate estimate of the sampled impulse-response of the channel.

The channel characteristics of a telephone circuit do not vary (or vary only very slowly) with time. These channels, therefore, come under the category of time-invariant channels. The estimate of the impulse-response of such a channel can be made quite accurately, and usually remains correct over the period of the following hundred or even thousand data symbols. One method, for fast start up, is to transmit a training sequence before actual data [91]. However, with an HF channel, the channel characteristics vary considerably with time and season of the year, geographical location, sunspot number etc. [1-8, 12-18, 20-22, 25]. The received signal is continuously varying randomly in amplitude due to fading and, therefore, the sampled impulse-response of the channel must be estimated continuously at the receiver from the received data signal.

A receiver employing a NML detector can, therefore, be considered to consist of a detector and an estimator connected back to back. The input to the channel estimator is the current detected data and the received sample and its output is an estimate/prediction of the channel sample impulse-response, ready for use by the detector at the next sampling instant. A channel estimator is basically a tapped delay line finite impulse-response (FIR) filter with the filter tap coefficients forming the channel sampled impulse-response. The tap coefficients of the filter are adjusted adaptively, according to a particular algorithm, in order to track a time varying channel.

The algorithms used for the adaptive adjustment of the filter can be broadly classified into least mean-squares (LMS) and recursive least-squares (RLS) algo-
rithm. In LMS algorithm the tap coefficients of the filter are determined using the method of steepest descent [20,27, 35, 58-61, 101]. The algorithm is simple and works adequately in a variety of applications but suffers from the disadvantage of having a slow convergence rate.

The RLS algorithm, on the other hand, makes use of the input information to the channel estimator in such a way as to ensure optimality at every time instant. A Kalman filter has been used as a means of holding the receiver correctly adjusted to the channel in [53, 55, 73-75, 77-88]. Computer-simulation tests, however, have shown that the conventional Kalman filter [53, 73-75, 77-81], together with the more recent developments [83-88] are not optimum for a typical HF channel [88]. Both RLS and Kalman algorithms offer improved convergence, but at the expense of increased computation. New fast RLS algorithms have been developed [59, 84, 87, $97,101,104-116]$, but these exhibit numerical instability [111-116].

Another form of estimation technique, known as the Improved channel estimator [89], that makes use of the prior knowledge of the number of different paths (separate sky waves) present in the HF channel, has shown very much improved performance, compared to the conventional LMS and RLS algorithms. It, however, has sub-optimum performance when the channel is modelled incorrectly [100]. It has been demonstrated in [70] that the LMS algorithm will perform as well as (if not better than) the RLS algorithm over a fading HF channel. An useful improvement in performance is achieved if a predictor is also incorporated in the system [33, 54]. This simple modification, however, is not enough to match the performance of an improved channel estimator. The main source of complexity in the improved channel estimator is the requirement of modelling of the multipath propagation in the HF radio link. This suggests that the best approach towards a simpler but adequate estimator would be to develop the simple LMS algorithm [54] which does not require any prior knowledge of the channel.

### 1.2 OUTLINE OF THE INVESTIGATION

The investigation is concerned with the estimation of the sampled impulse response of a time-varying HF channel, where the estimators are used in the receiver of a 4800 bits/s, quaternary phase shift keyed (QPSK) system operating at 2400 bauds
with an 1800 Hz carrier. Computer-simulation has been used to test the performance of estimators over a model of the HF data transmission system. Several novel estimation algorithms are proposed and their performance compared.

Chapter 2 contains a description of the HF radio channel. It contains a brief description of the structure of the ionosphere followed by the propagation mechanism in the ionosphere. Finally it presents a model of the HF channel.

Chapter 3 describes a model of a synchronous serial QAM digital data transmission system, and presents an equivalent baseband model of a three sky wave data transmission system.

Chapter 4 considers a simple gradient estimator [33, 53-54], operating as an HF channel estimator. Four variations of the estimation technique are also described

Chapter 5 considers RLS Kalman estimators for HF channel estimation. Three variations of the RLS Kalman estimation technique are also described.

The class of estimators considered in Chapter 6 are called adaptive channel estimators. These estimators are adaptive in the sense that the step sizes of the gradient algorithm are here adjusted to suit the channel.

In Chapter 7, an improved channel estimator [89] is used to estimate the sampled impulse-response of an HF channel. All the estimators described in this chapter assume prior knowledge of the basic structure of the channel.

Chapter 8 considers Fast transversal filter (FTF), for HF channel estimation. This is a fast RLS algorithm [84] and is computationally efficient.

At the end of each chapter the results of the computer simulation tests on a model of a data transmission system are presented. The systems have been approximately optimized within the available computer time, for best performance and the results represent the steady-state performance of the estimators. Finally, in Chapter 9, some of the best estimators developed in the thesis have been compared.

## CHAPTER 2

## HF CHANNEL

### 2.1 INTRODUCTION.

An HF channel as a transmission medium is still of great importance even after the introduction of several other kinds of transmission media such as satellites, optical fibres, coaxial cables etc. When cable links are used as a transmission medium, the properties of the transmission route can be quite accurately defined and reproduced independently of time factors, but for HF radio links this is not the case. For sky wave propagation, in particular, the transmission conditions are constantly changing and this results in a received signal that follows the changes in the transmission medium. In estimating such a channel, major processes in the ionosphere and their effect on HF propagation must be well understood. This chapter looks in detail at the structure of the ionosphere, the distortions introduced by a time varying channel and the modelling of the channel for use in the testing of data modems.

### 2.2 STRUCTURE OF THE IONOSPHERE.

The ionosphere extends between 50 and 2000 km above the earth's surface, and is composed of molecules and atoms of nitrogen and oxygen [8, 9]. These are ionized principally, by the electromagnetic radiation from the sun, into free electrons and ions [9]. The ionosphere consists of several ionized layers. These can be classified into three main groups, named $D, E$ and $F$ layers $[1,8-9,12]$. The ionization and the density of ions present in these layers vary with time, as the ionization rate is a function of the intensity of solar radiation and this in turn varies considerably with the time of day, the season and the sunspot activity. As the solar radiation becomes stronger so does the capacity of the individual layers to reflect high frequency waves. However, the attenuation of the radio waves increases at the same time.

The D layer, at a height of 60 to 90 km above the earth, is the lowest layer. The critical frequency for the D layer, defined as the highest carrier frequency of a vertically incident ray that can be reflected by the layer [13], is of the order of 100 to

700 KHz [12]. Thus for HF radio waves, the D layer acts principally as an attenuator. The D region appears after sunrise and during night time, in the absence of solar radiation, the D layer virtually vanishes and, therefore, does not interfere with HF radio propagation.

The E layer is between 90 to 130 km above the earth's surface and is the next highest layer [1, 7-9, 12-13]. The E layers has a critical frequency of about 4 MHz [7-8]. As with the D layer, ionization begins at sunrise and maximum density occurs near noon with the seasonal maximum occurring in summer. After sunset the layer gradually breaks down. Thin ionized layers with a maximum electron density, greater than that of the Elayer, are often found between 90 and 150 Km above the earth [13]. These layers are called sporadic E layers because they arise only occasionally $[1,7-9,12-14]$. These are capable of reflecting high frequencies as they have a high critical frequency.

人 The next layer, called the F layer, is very important for the propagation of short waves. The lower region of the F layer shows a different variation characteristic than the upper region of the F layer, hence they have been sub-divided into two layers called the F1 and F2 layers. The F1 layer which exists only during daytime is located between 130 and 210 km above the earth [8-9]. Like the E layer, the F1 layer is strongly influenced by the solar radiation. The maximum ionization occurs about one hour after midday, with the seasonal maximum occurring during summer. The F1 layer is not generally used for long distance communications [6]. At night the two layers, F1 and F2, merge and are termed, simply, the F layer [1, 7-9, 12-14].

The F2 region is the highest ionospheric region and is located between 225 to 450 km above the earth's surface [1]. The critical frequency for the F2 layer is between 5 to 10 MHz . During night time and sometimes during the day time, particularly in winter, there is only a single F layer as the two layers F1 and F2 merge, and the critical frequency drops to 3 to 5 MHz . The F2 layer is an important part of the ionosphere for HF radio communication both during day and night time. Since the F2 layer is at a considerable height, it can support single hop propagations over a distance as great as 4000 km .

Fig. 2.2.1, [9], shows the ionospheric regions as a function of height above the earth's surface. Fig. 2.2.2, [13], shows the electron density profile for summer noon and midnight at middle latitudes.


Fig. 2.2.1 - lonospheric regions as a function of height above the earth's surface


Fig. 2.2.2 - Typical electron density distribution for summer noon \& midnight conditions at mid-latitudes

### 2.3 PROPAGATION MECHANISM IN IONOSPHERE.

HF radio waves are returned to earth from the ionosphere through a process known as refractive bending. The refractive index, $\eta$, of the ionospheric layer changes continuously with its height as $\eta$ is a function of the electron density in the ionised medium.

The refractive bending of a radio wave is demonstrated in Fig. 2.3.1 and Fig. 2.3.2. For a given angle of incidence of a radio wave meeting a reflecting layer, total internal reflection occurs when [12, 14]

$$
\sin \theta_{i}=\left[1-\frac{81 . N}{f^{2}}\right]^{\frac{1}{2}}
$$

where f is the frequency of the radio wave in Hz , and N is electron density in electrons per cubic metre. At vertical incidence $\left(\sin \theta_{i}=0\right)$, the wave will be completely refracted back towards the earth if the frequency of the wave is equal to or less than the crtical frequency, $\mathrm{f}_{c}$,

$$
f_{c}=9 \sqrt{N_{\max }}
$$

where $\mathrm{N}_{\max }$ is the maximum electron density in the ionosphere. For a given angle of incidence, $\theta_{i}$, the maximum frequency at which reflection takes place is called the maximum usable frequency (m.u.f) and is related to the critical frequency as

$$
\text { m.u. } f=f_{c} \sec \theta_{i}
$$

As seen in Fig. 2.3.2, at the (m.u.f), the radio wave takes the critical path which is the shortest distance back to earth. This is called the skip distance.

The refraction processes via a flat ionised region at some height " $b$ " above the earth's surface is equivalent to a mirror like reflection from a reflector located at a height " a " above the earth (Fig. 2.3.3) [12, 14]. This height " a " is called the virtual


Fig. 2.3.1 - lonospheric Reflection Mechanism


Fig. 2.3.2 - Critical Conditions for Reflection


Fig. 2.3.3 - Virtual Path
height [14]. Thus the actual ray path can be replaced by the virtual ray path in a medium of unit refractive index and reflected from a plane located at the virtual height.

For long radio path lengths several hops are necessary, with the number of hops depending upon the transmission conditions existing in the individual sections of the path. Fig. 2.3.4 shows some of the possibilities for a multi-hop link.

### 2.4 DISTORTION INTRODUCED BY THE HF CHANNEL

### 2.4.1 TIME DISPERSION.

A radio wave may reach a remote receiver via several routes (as illustrated in Fig. 2.3.4). All these routes will have different path lengths and hence the radio waves will take different times to traverse them. Time dispersion is due to multipath propagation. The earth's magnetic field splits the waves into two magneto-ionic components called the ordinary and extraordinary waves and the propagation conditions in the ionosphere are different for the two waves. Thus the ordinary and extraordinary rays appear as multiple rays and this results in time dispersion. Different modes of propagation have different group delays and this difference in the group delays also results in time dispersion. Time dispersion gives rise to inter-symbol interference, when the data transmission rate becomes comparable to the relative multipath delay. It is, thus, a function of frequency, path length, local time, season and also geographical location.

### 2.4.2 FREQUENCY DISPERSION.

Frequency dispersion arises on a single propagation path due to the Doppler effect introduced by the change in the altitude of the ionospheric layers. The upper regions of the atmosphere at high altitude are ionized first when the sun rises. The height of this ionization level reduces as the sun rises further and further, until noon when it reaches a minimum. Thus the path traversed by a particular radio wave keeps


Fig. 2.3.4 - Examples of possible radiation paths for Multipath links.
decreasing and the emitted frequency appears to have increased by an amount $+\Delta f$. Exactly the opposite phenomenon takes place when the sun sets whereby, now, the radio wave takes a longer path to reach the receiver and the emitted frequency now appears to have decreased by an amount $-\Delta f$. During night time when the ionosphere is calm there is no Doppler effect. When the ionosphere is calm the value of $\Delta f$ is between 1 and 2 Hz [1] and at other times it can be as high as 6 Hz .

### 2.4.3 FADING

Random variations of the signal strength at the receiver are referred to as fading. Fading phenomena can be classified as follows [1, 4, 14].

### 2.4.3.1 INTERFERENCE OR SELECTIVE FADING.

An HF signal arriving at a remote receiver is composed of a large number of different rays, after having travelled via the ionosphere over paths of different lengths. The total field strength of the received signal is the phasor sum of all waves arriving at the receiver. Due to random variation of the ionospheric conditions, the phase and the field strength of different rays, and hence that of the received signal, vary in a random manner.

A modulated carrier has, within its bandwidth, a large number of frequency components that are exposed to randomly varying multipath propagation conditions. This can result in selective blackouts or fading of a small section of the bandwidth. This fading effect is called selective fading. Interference fading can also occur if at the receiver the sky wave signals are also superimposed by the ground wave signals. When the radio links are exposed to severe ionospheric disturbances, there is another type of interference called flutter fading. In this the variation in signal strength takes the form of a fast rhythmic beat, as though a low frequency oscillation is superimposed on the modulated carrier. This represents a considerable source of disturbance for radio reception.

### 2.4.3.2 POLARIZATION FADING

This is due to the effect of the earth's magnetic field splitting the radio waves into ordinary and extraordinary waves (Section 2.4.1). The combined effect of the phase and amplitude of these waves is to change the polarisation of the received signal to be elliptically polarised. The phase and dimensions of the axes of the ellipse are constantly changing as the ordinary and extraordinary waves are subjected to random variations in the propagation conditions. This results in a type of fading called polarization fading.

### 2.4.3.3 ABSORPTION FADING.

This type of fading occurs due to the variation in the absorption characteristics of the ionosphere with time. The attenuation characteristic of the D layer slowly changes and can last longer than an hour [1] and is usually the greatest during sunrise and sunset [4]. The depth of fading can be as high as 10 dB below the mean value [1].

### 2.4.3.4 SKIP FADING.

For a specific distance between two short wave stations, the highest frequency to be reflected is called the maximum usable frequency (MUF). Skip fading is caused by the continuous variation of the MUF. The operating frequency, which at one particular instant, is definitely below the MUF, may no longer be so at another instant and so penetrates the reflecting layer for a short period of time. During this time radio communication is interrupted at the receiver.

### 2.5 STATISTICAL DISTRIBUTION OF THE RECEIVED SIGNAL

Consider a transmitted signal that is represented in general form as.

$$
s(t)=\mathscr{R}\left\{u(t) \cdot \exp ^{\left.j 2 \pi f_{c} t\right\}}\right.
$$

where $\mathscr{R}\{$.$\} is the real part of the complex-valued quantity in brackets, f_{c}$ is the frequency of the carrier and,

$$
u(t)=a(t) e^{j \theta(t)}
$$

$\mathrm{a}(\mathrm{t})$ denotes the amplitude (envelope) of $\mathrm{s}(\mathrm{t})$, and $\theta(t)$ denotes the phase of $\mathrm{s}(\mathrm{t})$. Due to multipath the received signal is of the form [18,20]

$$
r(t)=\sum_{n} \alpha_{n}(t) s\left[t-\tau_{n}(t)\right]
$$

where $\alpha_{n}(t)$ is the amplitude of the signal received via the $\mathrm{n}^{\text {th }}$ path at time t and $\tau_{n}(t)$ is the propagation delay for the $\mathrm{n}^{\text {th }}$ path. Considering the received signal as consisting of a continuum of multipath components, the summation in Eqn. 2.5.2 can be replaced by integral, and can be written as $[18,20]$

$$
r(t)=\int_{-\infty}^{\infty} \alpha(\tau ; t) s(t-\tau) d \tau
$$

where $\alpha(\tau ; t) d \tau$ represents the amplitude at time $t$ of all rays arriving with relative delay times in the range $(\tau, \tau+d \tau)$. Combining Eqns. 2.5 . 1 and 2.5.3

$$
r(t)=\mathfrak{R}\left\{\int_{-\infty}^{\infty} \alpha(\tau ; t) u(t-\tau) e^{j 2 \pi f_{c}(t-\tau)} d \tau\right\}
$$

or

$$
r(t)=\Re\left\{\left[\int_{-\infty}^{\infty} \alpha(\tau ; t) e^{-j 2 \pi f_{c} \tau} u(t-\tau) d \tau\right] e^{j 2 \pi f_{c} t}\right\}
$$

Let

$$
h(\tau ; t)=\alpha(\tau ; t) e^{-j 2 \pi f_{c} \tau}
$$

The integral in Eqn. 2.5.5 represents the convolution of $u(t)$ with an equivalent low-pass time-variant channel impulse-response $h(\tau ; t)$.

Thus when an unmodulated carrier, at frequency $\mathrm{f}_{\mathrm{c}}$, is transmitted, the equivalent low-pass received signal is [20]

$$
r(t)=\sum_{n} \alpha_{n}(t) e^{-j 2 \pi f_{c} \tau_{n}(t)}=\sum_{n} \alpha_{n}(t) e^{-j \theta_{n}(t)}
$$

where $\theta_{n}(t)=2 \pi f_{c} \tau_{n}(t)$. Thus the received signal consists of the sum of a number of time-variant phasors having amplitudes $\alpha_{n}(t)$ and phases $\theta_{n}(t)$. A large dynamic change in the medium is required for $\alpha_{n}(t)$ to change sufficiently to cause a significant change in the received signal. On the other hand, $\theta_{n}(t)$ changes by $2 \pi$ radians whenever $\tau_{n}(t)$ changes by ( $1 / \mathrm{f}_{\mathrm{c}}$ ). But ( $1 / \mathrm{f}_{c}$ ) is a small quantity, and, hence, $\theta_{n}(t)$ can change by $2 \pi$ radians with relatively small motions of the medium. Owing to irregularity of the ionised media, the variation in $\tau_{n}(t)$ is random and, therefore, variation in $\theta_{n}(t)$ is also random $[18,20]$. The multipath propagation model for the channel, in Eqn. 2.5.7, results in fading of the received signal. The fading is caused primarily by variation in the relative phases of the individual $\left\{\theta_{n}(t)\right\}[20]$.

When there are a sufficiently large number of phases, i.e for large value of $n$, of roughly equal size and their phases changing randomly and independently of each other, then by the central limit theorem, the two quadrature components of the resultant signal will each tend to be distributed as a zero-mean Gaussian random variable, with equal variance and independent fluctuations [18, 20].

The received waveform thus has all the characteristics of a very narrow band complex-valued Gaussian random process, characterised by a power spectral density of non-zero width, and with the envelope having a Rayleigh distribution and the phase uniformly distributed between 0 and $2 \pi$ radians [18-21].

A single non-fading ("specular") dominant component may also be received, giving a Nakagami-Rice or Ricean amplitude distribution [3, 9, 20-21]. The Rayleigh fading model need not be valid for all HF channels. A specular component can be present on high rays and ground waves on short links which are again, essentially non-fading. However, the majority of ionospheric media exhibit Rayleigh fading, and thus, based on the present knowledge of ionospheric characteristics, it appears that the Rayleigh fading model best describes most HF channels [19].

The value of the envelope of the received signal at any time, can be taken to be Rayleigh distributed. It is a continuous random variable, derived from two independent Gaussian random variables X and Y , and its probability density function is given by [1, $3,15,21-23]$

$$
p_{R}(r)=\left\{\begin{array}{lc}
\frac{r}{\sigma^{2}} e^{-\frac{r^{2}}{2 \sigma^{2}}} & 0 \leq r \leq \infty \\
0 & r<0
\end{array}\right.
$$

The mean values of X and $\mathrm{Y}, \mathrm{m}_{\mathrm{x}}$ and $\mathrm{m}_{\mathrm{y}}$ respectively are zero and their variance $\sigma^{2}$ is such that

$$
\sigma_{X}^{2}=\sigma_{Y}^{2}=\sigma^{2}
$$

The envelope, $R=+\sqrt{X^{2}+Y^{2}}$, has a Rayleigh distribution [21,24] and has a probability density function given by Eqn. 2.5.8.

Since R cannot be negative, by definition, it must have a non-zero mean value, even though X and Y have zero means. Fig. 2.5.1 shows the plot of probability density function as a function of $r$. The curve attains a maximum value of $1 / \sigma \sqrt{e}$ at $r=\sigma$.

The Cumulative distribution function, of the Rayleigh distribution, is given by

$$
\begin{align*}
f(r) & =\int_{0}^{r} \frac{u}{\sigma^{2}} e^{-\frac{u^{2}}{2 \sigma^{2}}} d u \\
& =1-e^{-\frac{r^{2}}{2 \sigma^{2}}}
\end{align*}
$$

for $\mathrm{r} \geq 0$.

Fig. 2.5.2 shows the plot of the cumulative distribution function. The mean value of $R$ is given by


Fig. 2.5.1 - Rayleigh Probability Density Function


Fig. 2.5.2 - Rayleigh Cumulative Distribution Function

$$
\not \subset \bar{r}=\int_{0}^{\infty} r f(r) d r=\sqrt{\frac{\pi}{2} \sigma}
$$

and the second moment (mean-square value) of R is given by

$$
E\left[R^{2}\right]=E\left[X^{2}+Y^{2}\right]=E\left[X^{2}\right]+E\left[Y^{2}\right]
$$

where $E\left[X^{2}\right]$ and $E\left[Y^{2}\right]$ are given by

$$
\begin{align*}
& E\left[X^{2}\right]=\sigma^{2}+m_{x}^{2} \\
& E\left[Y^{2}\right]=\sigma^{2}+m_{y}^{2}
\end{align*}
$$

Substituting Eqn. 2.5.13 in Eqn. 2.5.12 and noting that the mean values of X and Y , ( $m_{x}$ and $m_{y}$ respectively) are zero, the mean square value of $R(i . e r$ ) is given by

$$
\bar{r}^{2}=2 \sigma^{2}
$$

The variance of $R$ is given by

$$
\begin{align*}
\sigma_{r}^{2} & =E\left[R^{2}\right]-m_{r}^{2} \\
& =\bar{r}^{2}-(\bar{r})^{2}
\end{align*}
$$

From Eqns. 2.5.11, 2.5.14 and 2.5.15,

$$
\sigma_{r}^{2}=\left(2-\frac{\pi}{2}\right) \sigma^{2}
$$

The median value of the Rayleigh distribution occurs, at $r=r_{m}$, at the point where the cumulative distribution function $\mathrm{f}(\mathrm{r})$ (in Eqn. 2.5.10) is equal to 0.5 . Therefore, from Eqn. 2.5.10,

$$
f\left(r_{m}\right)=0.5=1-e^{-\frac{r_{m}^{2}}{2 \sigma^{2}}}
$$

Solving Eqn. 2.5.17 for $\mathrm{r}_{\mathrm{m}}$

$$
r_{m}=(\sqrt{2 \ln 2}) \sigma
$$

were $\sigma^{2}$ is the variance of Gaussian random variables used in the derivation of Rayleigh fading.

### 2.6 SIMULATION OF AN HF CHANNEL

An accurate assessment, of the performance of an HF digital radio system, can be made through repeated tests of the system over an actual channel. However, when comparison is to be made between two or more systems over a real channel, then they must all be tested simultaneously, because propagation or channel conditions vary uncontrollably and cannot be accurately repeated at other times or over other links. Moreover, it is not possible to repeat a test on a system for the same channel conditions.

The use of a channel simulator for evaluating the performance of a digital communication system offers several advantages. They are accurate and a large range of channel conditions can be simulated in a controlled manner. It is possible to compare the performances of several systems under the same channel conditions using a channel simulator, and tests can be repeated any number of times with consistent results.

The most commonly used channel simulator and the one recommended by the International Radio Consultative Committee(CCIR) is that proposed by Watterson et al., in reference [26]. This channel simulator is based on the tapped delay line model. This is the model used in the HF channel simulation, albeit with the omission of constant Doppler shift. The block diagram of the HF ionospheric channel model is shown in Fig. 2.6.1

The input signal is fed to the adjustable tapped delay line. There are as many taps as there are modes of propagation. At each tap the delayed signal is modulated in amplitude and phase by an appropriate complex-valued random tap gain function $\mathrm{Q}_{\mathrm{n}}(\mathrm{t})$. The delayed and modulated signals are summed with additive noise. The additive noise has a Gaussian probability density although in actual channels, the type of additive noise is usually from several sources such as galactic, man made,
solar and radio, and can be highly impulsive. However, in HF radio links the mostly common additive noise is atmospheric noise, which is Gaussian in nature [29]. Hence a good tolerance to additive white Gaussian noise almost certainly means a good tolerance to atmospheric noise.

A single Rayleigh fading propagation path is modelled as in Fig. 2.6.2. $q_{1}(t)$ and $\mathrm{q}_{2}(\mathrm{t})$, in Fig.2.6.2, are two random processes. In simulating a Rayleigh fading sky wave these random processes should be Gaussian with zero mean and the same variance. They should be statistically independent and the shape of their power spectra must be Gaussian, having same rms frequency, $f_{m}$. Thus the power spectrum of $q_{1}(t)$ and $q_{2}(t)$ are given by

$$
\left|Q_{1}(f)\right|^{2}=\left|Q_{2}(f)\right|^{2}=\exp \left(-\frac{f^{2}}{2 f_{r m s}^{2}}\right)
$$

The fading rate can be controlled by the bandwidth of the power spectra of the Gaussian variables $\mathrm{q}_{1}(\mathrm{t})$ and $\mathrm{q}_{2}\left(\mathrm{t}\right.$ ). The frequency (Doppler) spread, $\mathrm{f}_{\mathrm{sp}}$, introduced by $\mathrm{q}_{1}(\mathrm{t})$ and $\mathrm{q}_{2}(\mathrm{t})$ into an unmodulated carrier is defined as the width of the power spectrum [25] and is given by,

$$
f_{s p}=2 f_{r m s}
$$

The rms frequency is related to the fading rate, $\mathrm{f}_{\mathrm{e}}$, which is defined (for a single carrier) as the average number of downward crossings per unit time, of the envelope through the median value, according to the equation:

$$
f_{r m s}=\frac{f_{e}}{1.475}
$$

from Eqns. 2.6.2 and 2.6.3 $\mathrm{f}_{\mathrm{sp}}$ is related to $\mathrm{f}_{\mathrm{c}}$ by:

$$
f_{s p}=1.356 f_{e}
$$

Practical measurements of the channel multipath structure have shown that there are usually two to four distinct paths present [31] but each associated with a different delay in transmission. The delay spread is usually upto about 5 milliseconds $[3,30]$. Doppler spread often is under 0.01 Hz (very slow fading), whereas for a more


Fig. 2.6.1 - Block Diagram of HF Ionospheric channel model


Fig. 2.6.2 - Rayleigh fading introduced by one sky wave
notorious HF channel, the Doppler spread can usually be in the range $1-2 \mathrm{~Hz}$ [3]. Reference [19] has recommended testing of HF modems on HF channels classified as Good, Moderate, Poor and Flutter conditions. Table 2.6.1 lists the parameters for these four channel conditions. Long sky wave paths are the ones for which most HF modems are designed, usually for a multipath spread up to about 3 milliseconds and a Doppler spread of $1-2 \mathrm{~Hz}$ [3]. The chosen model is a 3 sky wave channel with a frequency spread of 2 Hz and transmission delays of the three sky waves, measured relative to that of the first sky wave, being 1.1 , and 3 milliseconds. These parameters represent a poor channel as per the classifications of reference [19] (Table 6.2.1).

The random process $\mathrm{q}_{1}(\mathrm{t})$ is generated by filtering a zero mean white Gaussian noise signal $V_{1}(t)$ as shown in Fig. 2.6.3. The power spectra of $q_{1}(t)$ is Gaussian, hence the filter should also have a Gaussian frequency response matching the power spectrum of the Gaussian variable $q_{1}(t)$. The theoretical power spectrum of $q_{1}(t)$ given in Eqn. 2.6.1 is plotted in Fig. 2.6.4. The frequency response of the filter is given by,

$$
F(f)=\exp \left(-\frac{f^{2}}{4 f_{r m s}^{2}}\right)
$$

A Bessel filter is used in the channel simulator to provide the necessary shaping to the random process $q_{1}(t)$. The frequency and impulse-response of the Bessel filter approaches Gaussian, when the order of the filter is sufficiently large [32]. In the simulated channel model there are 3 sky waves and, therefore, it requires six random processes $\mathrm{q}_{\mathrm{i}}(\mathrm{t})$ for $\mathrm{i}=1,2, \ldots, 6$. Each of the random processes are similarly generated. The variance of all six variables $q_{1}(t)$ to $q_{6}(t)$ are equal to 0.167 . This value of variance for each individual process ensures that the total variance of the 3 sky wave channel is unity. Each of the values of $q_{i}(t)$ is generated from an independent source, so that all six random processes $q_{1}(t)$ to $q_{6}(t)$ are uncorrelated. Table 2.6.2 details the characteristics of the filter chosen for the model and Appendix A describes the filter design in detail.

The digital filter is implemented as shown in Fig. 2.6.5. It is a combination of a two 2-pole section and a single pole section. The single pole section has a real pole whereas the two pole sections have complex conjugate poles.


Fig. 2.6.3 - Method of generating $q_{1}(\dagger)$


Fig. 2.6.4 - Theoritical Power Spectra of $q_{i}(\dagger)$


For digital implementation of the channel model it is neither possible nor necessary to represent the random fading sequence $\mathrm{q}_{\mathrm{i}}(\mathrm{t})$ as a continuous signal. This must be represented by discrete samples in time. From Nyquist's sampling theorem, for faithful reproduction of the continuous signal, it is necessary that the sampling rate should be greater than twice the highest frequency present in the continuous signal. $\mathrm{q}_{\mathrm{i}}(\mathrm{t})$ has a Gaussian spectrum, and hence theoretically has infinite bandwidth. However, since the assumed model has an rms bandwidth of only 1 Hz , it is adequate if the signal is sampled at 10 samples per second. For testing a 2400 baud digital data modem on the channel, however, it is necessary that the channel samples are also obtained at 2400 Hz . This means that $\mathrm{q}_{\mathrm{i}}(\mathrm{t})$ must be sampled at 2400 Hz . This gross over sampling has an adverse effect on the digital filters having the required narrow-band Gaussian shape. In order to be consistent with the sampling frequency the roots of the digital filter must correspond to this sampling frequency and at the same time it is necessary to see that these roots are not too close to the unit circle in the Z-plane. Unfortunately at this high sampling frequency the pole locations of such filters in the Z-plane are pushed very close to the unit circle and the tap values become very large. The filter coefficients must now be specified with very high precision, otherwise there can be instability in the operation of the of the filter. This problem can be overcome by employing a reduced sampling frequency in the digital filters and then interpolating between samples in order to obtain the required sampling rate. Thus $\mathrm{q}_{\mathrm{i}}(\mathrm{t})$ has been sampled at 100 Hz as a compromise between the requirements for the Nyquist sampling criterion, the need to limit the degree of interpolation used and the need to have the pole locations in the Z-plane at an adequate distance from the unit circle.

In the simulation of a 3 sky wave channel, two types of interpolation have been studied, namely the linear interpolation technique and the non-linear interpolation technique. Non-linear interpolation uses the NAG (Numerical Algorithms Group) routine, E 01 ABF [38]. This routine uses Everett's central difference form for the interpolating polynomial [39-41]. The simplest method of interpolation is linear interpolation. Linear interpolation is considered adequate as $q_{i}(t)$ is considerably oversampled [36].

TABLE 2.6.1 HF CHANNEL PARAMETERS

| Conditions | Diff. Time Delay | Frequency Spread |
| :---: | :---: | :---: |
| Good | 0.5 ms | 0.1 Hz |
| Moderate | 1 ms | 0.5 Hz |
| Poor | 2 ms | 1 Hz |
| Flutter fading | 0.5 ms | 10 Hz |

TABLE 2.6.2 CHARACTERISTICS OF THE FIFTH ORDER BESSEL FILTER

| Order of the filter, L | 5 |
| ---: | :---: |
| Frequency spread, $\mathrm{f}_{\mathrm{p}}(\mathrm{Hz})$ | 2 |
| Cutoff frequency, $\mathrm{f}_{\mathrm{c}}(\mathrm{Hz})$ | 1.1774 |
| Filter poles in the S-plane |  |
| pr |  |
| $p_{2}^{\prime}$, | $p_{3}^{\prime}$ |



Fig. 2.6.6 - Model of a 3 Sky wave HF Radio Link.

### 2.7 RESULTS OF THE TESTS ON THE SIMULATED FADING CHANNEL.

The results of the tests carried out on the simulated channel are summarized in Figs. 2.7.1-2.7.9 and in Table 2.7.1. Fig. 2.7.1 compares the frequency response of the 5-pole Bessel filter used to generate the Gaussian random sequence, with the theoretical Gaussian response given by Eqn. 2.6.5. As can be seen in Fig. 2.7.1 the 5-pole Bessel filter response agrees very well with the theoretical response in the frequency band of interest.

Figs. 2.7.3 and 2.7.2 show the amplitude and phase variations of the random sequences, $\mathrm{Q}_{\mathrm{n}}(\mathrm{t}), \mathrm{n}=1, \ldots, 3$, used in the channel simulation. The amplitude variation of $Q_{n}(t)$ is random and the phase variation is uniform, which is an essential requirement for the sequence to have a Rayleigh distribution.

Figs. 2.7.4 and 2.7 .5 compares the probability density function and the cumulative distribution function of the sequences $\mathrm{Q}_{\mathrm{n}}(\mathrm{t}), \mathrm{n}=1, \ldots, 3$, with that of the corresponding theoretical curves, given by Eqns. 2.5.8 and 2.5.10 respectively (Figs. 2.5.1 and 2.5.2). Though the sequences ( 2500 elements) are not long enough to make any statistical predictions, the results are in complete agreements with the theoretical probability density function and cumulative distribution function.

Figs. 2.7.6-2.7.9 compare the linear and non-linear interpolations. Fig. 2.7.6 shows the linear interpolated sequence superimposed on the non-interpolated sequence. Amplified versions of the positive peak and the negative peak, in Fig. 2.7.6, are depicted in Fig. 2.7.7. The error in the interpolation, though not very significant, can be clearly seen in the Fig. 2.7.7. The non-interpolated sequence, in Figs. 2.7.62.7.9, has been plotted using the curve fitting routine (spline interpolation) of computer graphics software.

Figs. 2.7.8 and 2.7.9 show the non-linear interpolated sequence superimposed on the non-interpolated sequence. The non-linear interpolation is obtained using the NAG routine, E 01 ABF . Amplified versions of the positive peak and the negative peak, in Fig. 2.7.8, are depicted in Fig. 2.7.9. Clearly, the non-linear interpolated sequence produces a much smoother curve, compared with the linear interpolated sequence.

However, non-linear interpolation is computationally complex and since the error in linear interpolation is only marginal, the latter has been adopted in the computer-simulation of the 3 sky wave HF channel.

Table 2.7 .1 shows the measured mean value, variance and the number of fades relative to the median value of the sequences $Q_{n}$, for $n=1, \ldots, 3$, for six different values of seed integer for the random number generator. From Eqn. 2.5.18, the median value corresponding to a variance of 0.167 , of the Gaussian random variables used in the derivation of a three sky wave Rayleigh fading channel, is 0.4811 . The measurement of the number of fades in Table 2.7.1 have been made relative to this median value over a duration of 25 seconds of the fading channel, sampled at 2400 Hz . The theoretical number of fades can be obtained from Eqn. 2.6.4 and for this duration, it is about 37 fades. As can be seen from the Table 2.7.1, the measured results compare very well with the theoretical value.


Fig. 2.7.1 - Frequency Response of Bessel Filter


Fig. 2.7.2 - Variation of Phase of $Q_{n}(\dagger)$ in the Complex Number Plane



Fig. 2.7.4 - PDF of $Q_{n}(t)$


Fig. 2.7.5 - CDF of $Q_{n}\left({ }^{( }\right)$


Fig. 2.7.6 - Linear Interpolation of Fading Sequence


Fig. 2.7.7 - Linear Interpolation of Fading Sequence


Fig. 2.7.8 - Non-Linear Interpolation of The Rayleigh fading Sequence



Fig. 2.7.9 - Non-Linear Interpolation of Rayleigh fading Sequence

TABLE 2.7.1 MEASURED CHARACTERISTICS OF THE FADING SEQUENCES USED TO MODEL A 3 SKY WAVE CHANNEL.

| SEED INTEGER | MEAN VALUE <br> OF <br> $q_{1}$ | VARIANCE OF $q_{1}$ | NO. OF FADES MEASURED RELATIVE TO THE MEDIAN VALUE |
| :---: | :---: | :---: | :---: |
| 9 | $\begin{array}{r} -0.003 \\ 0.015 \\ -0.007 \\ 0.014 \\ -0.054 \\ -0.031 \end{array}$ | $\begin{aligned} & 0.182 \\ & 0.129 \\ & 0.212 \\ & 0.158 \\ & 0.169 \\ & 0.166 \end{aligned}$ | 44 <br> 46 <br> 44 |
| 55 | $\begin{array}{r} -0.012 \\ 0.027 \\ 0.002 \\ -0.014 \\ -0.016 \\ 0.008 \end{array}$ | $\begin{aligned} & 0.166 \\ & 0.218 \\ & 0.165 \\ & 0.158 \\ & 0.163 \\ & 0.176 \end{aligned}$ | $\begin{aligned} & 33 \\ & 49 \\ & 48 \end{aligned}$ |
| 107 | $\begin{array}{r} -0.011 \\ -0.034 \\ -0.006 \\ 0.048 \\ -0.021 \\ 0.033 \end{array}$ | $\begin{aligned} & 0.188 \\ & 0.176 \\ & 0.153 \\ & 0.156 \\ & 0.175 \\ & 0.152 \end{aligned}$ | $\begin{aligned} & 42 \\ & 44 \\ & 40 \end{aligned}$ |
| 158 | $\begin{array}{r} 0.025 \\ 0.035 \\ -0.066 \\ -0.031 \\ 0.000 \\ -0.040 \end{array}$ | $\begin{aligned} & 0.126 \\ & 0.144 \\ & 0.152 \\ & 0.195 \\ & 0.186 \\ & 0.131 \end{aligned}$ | $\begin{aligned} & 39 \\ & 43 \\ & 38 \end{aligned}$ |
| 197 | $\begin{array}{r} \hline 0.036 \\ -0.035 \\ -0.045 \\ -0.018 \\ -0.002 \\ 0.018 \end{array}$ | $\begin{aligned} & 0.190 \\ & 0.177 \\ & 0.158 \\ & 0.200 \\ & 0.175 \\ & 0.153 \end{aligned}$ | $\begin{aligned} & 42 \\ & 41 \\ & 46 \end{aligned}$ |
| 500 | $\begin{array}{r} -0.029 \\ 0.086 \\ 0.003 \\ 0.078 \\ -0.054 \\ 0.039 \end{array}$ | $\begin{aligned} & 0.171 \\ & 0.187 \\ & 0.148 \\ & 0.189 \\ & 0.147 \\ & 0.140 \end{aligned}$ | $\begin{aligned} & 45 \\ & 41 \\ & 43 \end{aligned}$ |

## CHAPTER 3

## MODEL OF THE DATA TRANSMISSION SYSTEM

### 3.1 INTRODUCTION

A general communication system or for that matter a digital communication system consists of a transmitter, a transmission path and a receiver. The signal waveform $\mathrm{s}(\mathrm{t})$, at the input to the transmitter, carries the information to be transmitted. The interference and distortion in the transmission path ( an HF radio link in this thesis ), modifies the transmitted signal. The role of the receiver is to faithfully reproduce the transmitted information from the distorted received signal at the input to the receiver. These data transmission systems can be broadly classified as, serial data transmission systems and parallel data transmission systems.

In a serial data transmission system, the signal elements are transmitted as a sequential stream whose frequency spectrum occupies the entire available bandwidth. In a parallel data transmission system two or more sequential streams of signal elements are sent simultaneously, and the spectrum of an individual data stream occupies only a part of the available bandwidth [29]. In a serial system the signal elements are normally transmitted at a steady rate of a given number of elements per second (bauds). The receiver extracts the element timing information from the received signal and operates in time synchronism with the received signal. Such a system is called a synchronous serial system.

A serial transmission system is less complex than a parallel transmission system as the latter needs several demodulators to process the different signals. In applications where a relatively high transmission rate is required over a given channel, a synchronous serial system is the most commonly used system [42] and is the one assumed.

### 3.2 DATA TRANSMISSION OVER A MODEL OF AN HF CHANNEL USING QAM.

Fig. 3.2.1 shows the model of a data transmission system [30, 34, 36]. The input to the system is the stream of data elements $\sum_{i} s_{i} \delta(t-i T)$, where

$$
s_{i}=s_{0, i}+j s_{1, i}
$$

where $\mathrm{j}=\sqrt{-1}$ and $\left\{\mathrm{s}_{0, i}\right\}$ and $\left\{\mathrm{s}_{1, i}\right\}$ are statistically independent and equally likely to have any one of their possible values $\pm 1 \pm \mathrm{j}$.

Each of the two lowpass filters in the transmitter has a real-valued response $a^{\prime}(t)$ and transfer function $A^{\prime}(f)$. In the HF radio link, the voiceband is translated to the HF band at the transmitter and a corresponding demodulator translates it back to the voiceband at the receiver. The modulation and demodulation processes are linear and the only distortion introduced into the signal is that due to the radio equipment filters and the HF channel.

The white Gaussian noise in Fig. 3.2.1 is real-valued and has a two sided power spectral density of $\frac{1}{2} N_{0}$. The bandpass filter at the output of the demodulator removes the noise outside the data signal band without excessively distorting the signal. This filter has the impulse-response given by $c(t)$. The distorted QAM signal is now fed to two coherent demodulators whose reference carriers are in synchronism with the average instantaneous carrier frequency of the received signal. The output of the coherent demodulator is filtered by a lowpass filter before being fed to the detector. Each of the two lowpass filters in the receiver has an impulse-response $b^{\prime}(t)$ and the transfer function $B^{\prime}(f)$. In Fig. 3.2.1, the in-phase and quadrature channels are real-valued. An equivalent model of the data transmission system is shown in Fig. 3.2.2, for the case where a QAM signal is transmitted over an equivalent linear baseband channel [30, 34, 36].

The signals at the output of the two lowpass filters in the transmitter of Fig. 3.2.1, are

$$
\sum_{i} s_{0, i} a^{\prime}(t-i T) \quad \& \quad \sum_{i} s_{1, i} a^{\prime}(t-i T)
$$


Fig. 3.2.1 - Model of the Data Transmission System over HF Transmission Media.
TIME VARYING LINEAR BASEBAND CHANNEL

Fig. 3.2.2 - Equivalent Model of The Data Transmission System
and the signal $\mathrm{x}_{2}(\mathrm{t})$ at the output of the adder is a real-valued waveform and is given by

$$
\begin{align*}
x_{2}(t)= & \sqrt{2} \sum_{i} s_{0, i} a^{\prime}(t-i T) \cos \left(2 \pi f_{c} t\right)- \\
& \sqrt{2} \sum_{i} s_{1, i} a^{\prime}(t-i T) \sin \left(2 \pi f_{c} t\right)
\end{align*}
$$

The factor $\sqrt{2}$ in Eqn. 3.2.3 ensures that the average power level is unity for each of the two signals, $\sqrt{2} \cos 2 \pi f_{c} t$ and $-\sqrt{2} \sin 2 \pi f_{c} t$, when transmitted over an infinite period [30]. Therefore, the modulation process introduces no change in the signal level.

Eqn. 3.2.3 can be alternatively expressed as $[34,36]$

$$
x_{2}(t)=\sqrt{2} \Re\left[\sum_{i} s_{i} a^{\prime}(t-i T) e^{j 2 \pi f_{c} t}\right]
$$

where

$$
e^{j 2 \pi f_{c} t}=\cos \left(2 \pi f_{c} t\right)+j \sin \left(2 \pi f_{c} t\right)
$$

$x_{2}(t)$ is fed to the radio equipment transmitter filter $G$ in Fig. 3.2.2. Filter $G$ has an impulse-response of $g(t)$ and a transfer function of $G(f)$. The output of this filter, $x(t)$ is real-valued and is given by

$$
x(t)=\Re\left[\sqrt{2} \sum_{i} s_{i} a^{\prime}(t-i T) e^{j 2 \pi f_{c} t}\right] * g(t)
$$

where * represents convolution.

Eqn. 3.2.5 can be written as

$$
\begin{align*}
& x(t)= \frac{1}{\sqrt{2}}\left\{\sum_{i} s_{i} a^{\prime}(t-i T) e^{j 2 \pi f_{c} t}+\right. \\
&\left.\sum_{i} s_{i}^{*} a^{\prime}(t-i T) e^{-j 2 \pi f_{c} t}\right\} * g(t)
\end{align*}
$$

Consider the convolution

$$
\left(u_{1}(t) e^{-j 2 \pi f_{c} t}\right) *\left(u_{2}(t) e^{-j 2 \pi f_{c} t}\right)
$$

By definition Eqn. 3.2.7 can be written as

$$
\begin{align*}
\int_{-\infty}^{\infty}\left[u_{1}(\tau) e^{-j 2 \pi f_{c} \tau}\right] & {\left[u_{2}(t-\tau) e^{-j 2 \pi f_{c}(t-\tau)}\right] d \tau } \\
= & \int_{-\infty}^{\infty} u_{1}(\tau) u_{2}(t-\tau) e^{-j 2 \pi f_{c} t} d \tau
\end{align*}
$$

Therefore, from Eqns. 3.2.7 and 3.2.8,

$$
\left[u_{1}(t) * u_{2}(t)\right] e^{-j 2 \pi f_{c} t}=\left[u_{1}(t) e^{-j 2 \pi f_{c} t}\right] *\left[u_{2}(t) e^{-j 2 \pi f_{c} t}\right]
$$

From Eqns. 3.2.6 and 3.2.9

$$
\begin{align*}
x(t)= & \frac{1}{\sqrt{2}}\left\{\sum_{i} s_{i} a(t-i T) e^{j 2 \pi f_{c} t}+\right. \\
& \left.\sum_{i} s_{i}^{*} a^{*}(t-i T) e^{-j 2 \pi f_{c} t}\right\}
\end{align*}
$$

where

$$
a(t-i T)=a^{\prime}(t-i T)^{*}\left[g(t) e^{\left.-j 2 \pi f_{c}\right]}\right]
$$

Eqn. 3.2.11 represents the overall filtering at the transmitter end, which includes the lowpass filter and the radio transmitter equipment filter which is a bandpass filter.
$a^{\prime}(t-i T)$ and $g(t)$ are real-valued in Eqn. 3.2.11 and, therefore, the complex conjugate of $a(t-i T)$ is simply given by

$$
a^{*}(t-i T)=a^{\prime}(t-i T) *\left[g(t) e^{j 2 \pi f_{c} t}\right]
$$

Fig. 2.6.2 shows the Rayleigh fading introduced by a single sky wave HF channel. Thus, when $\mathrm{x}(\mathrm{t})$ is fed into a single Rayleigh fading channel, the output from it would be

$$
z^{\prime}(t)=x(t) q_{1}(t)+\hat{x}(t) q_{2}(t)
$$

where $q_{1}(t)$ and $q_{2}(t)$ are statistically independent Gaussian random processes that generate the fading and $\hat{x}(t)$ represents the Hilbert transform of $x(t)$. The Hilbert transform of $x(t)$ is given by the convolution of $x(t)$ with the Hilbert transform filter i.e

$$
\hat{x}(t)=x(t) * f(t)
$$

where $f(t)$ is the impulse-response of a Hilbert transform filter, whose Fourier transform is $\mathrm{F}(\mathrm{f})$ and is given by

$$
\begin{array}{rlrl}
F(f) & =j & f<0 \\
& =0 & f=0 \\
& =-j & f>0
\end{array}
$$

From Eqns. 3.2.10 and 3.2.14

$$
\begin{align*}
& \hat{x}(t)= \frac{1}{\sqrt{2}}\{ \\
& \sum_{i} s_{i} a(t-i T) e^{j 2 \pi f_{c} t}+ \\
&\left.\sum_{i} s_{i}^{*}(a(t-i T))^{*} e^{-j 2 \pi f_{c} t}\right\} * f(t)
\end{align*}
$$

and from Eqns. 3.2.9 and 3.2.16

$$
\begin{align*}
\hat{x}(t)= & \frac{1}{\sqrt{2}} \sum_{i} s_{i}\left[a(t-i T) * f(t) e^{-j 2 \pi f_{c} t}\right] e^{j 2 \pi f_{c} t}+ \\
& \frac{1}{\sqrt{2}} \sum_{i} s_{i}^{*}\left[(a(t-i T))^{*} * f(t) e^{j 2 \pi f_{c} t}\right] e^{-j 2 \pi f_{c} t}
\end{align*}
$$

The Fourier transform of $f\left(t e^{-2 v e}\right.$ is $\mathrm{F}\left(\mathrm{f}+\mathrm{f}_{c}\right)$ and from Eqn. 3.2.15 this has a value of -j over the frequency band $-\mathrm{f}_{\mathrm{c}}$ to $+\mathrm{f}_{\mathrm{c}}$. On the other hand, the Fourier transform of $f(t)^{\operatorname{lax} / d}$
is $F\left(f-f_{c}\right)$ and this has a value of $+j$ in the frequency band $-f_{c}$ to $+f_{c}$. Moreover $a(t)$ is bandlimited to that of $A^{\prime}(f)$. Therefore, after taking the Fourier transform of Eqn. 3.2.17, substituting the values for $F(f+f)$ and $F\left(f-f_{c}\right)$ from Eqn. 3.2.15 and then taking the inverse Fourier transform of the resultant relation, Eqn. 3.2.17 reduces to [34-37]

$$
\begin{array}{r}
\hat{x}(t)=\frac{1}{\sqrt{2}}\left\{\sum_{i}-j s_{i} a(t-i T) e^{j 2 \pi f_{c} t}+\right. \\
\left.j s_{i}^{*}(a(t-i T))^{*} e^{-j 2 \pi f_{c} t}\right)
\end{array}
$$

where $s_{i}^{*}$ and $(\mathrm{a}(\mathrm{t}))^{*}$ are the complex conjugates of $\mathrm{s}_{\mathrm{i}}$ and $\mathrm{a}(\mathrm{t})$ respectively. For the sake of simplicity, the HF channel is now considered to be composed of two independent Rayleigh-fading sky waves. The explanation can, however, be logically extended to any number of sky waves. For the two sky wave channel, the relative delay between the two sky waves is taken to be $\tau$ seconds. $x(t)$ is now fed to the HF channel and the output from the channel is given by,

$$
\begin{align*}
z(t)= & {\left[x(t) q_{1}(t)+\hat{x}(t) q_{2}(t)\right]+} \\
& {\left[x(t-\tau) q_{3}(t)+\hat{x}(t-\tau) q_{4}(t)\right] }
\end{align*}
$$

For a three sky wave channel, $\mathrm{z}(\mathrm{t})$ can be written as

$$
\begin{align*}
z(t)= & {\left[x(t) q_{1}(t)+\hat{x}(t) q_{2}(t)\right]+} \\
& {\left[x\left(t-\tau_{1}\right) q_{3}(t)+\hat{x}\left(t-\tau_{1}\right) q_{4}(t)\right]+} \\
& {\left[x\left(t-\tau_{2}\right) q_{4}(t)+\hat{x}\left(t-\tau_{2}\right) q_{6}(t)\right] }
\end{align*}
$$

where $\tau_{1}$ and $\tau_{2}$ are the delays in transmission of the second and third sky waves, respectively, relative to the first sky wave.

From Eqns. 3.2.10, 3.2.18 and 3.2.19

$$
\begin{align*}
z(t)=\frac{1}{\sqrt{2}}\left\{\sum_{i} s_{i}^{*}(a(t-i T))^{*}\left[q_{1}(t)+j q_{2}(t)\right] e^{-j 2 \pi f_{c} t}+\right. \\
s_{i} a(t-i T)\left[q_{1}(t)-j q_{2}(t)\right] e^{j 2 \pi f_{c} t}+ \\
s_{i}^{*}(a(t-\tau-i T))^{*}\left[q_{3}(t)+j q_{4}(t)\right] e^{-j 2 \pi f_{c}(t-\tau)}+ \\
\left.s_{i} a(t-\tau-i T)\left[q_{3}(t)-j q_{4}(t)\right] e^{j 2 \pi f_{c}(t-\tau)}\right\}
\end{align*}
$$

Let

$$
\begin{align*}
h_{i}(t-i T)= & a(t-i T)\left[q_{1}(t)-j q_{2}(t)\right]+ \\
& a(t-\tau-i T)\left[q_{3}(t)-j q_{4}(t)\right] e^{-j 2 \pi f_{c} \tau}
\end{align*}
$$

Then, from Eqns. 3.2.21 and 3.2.22,

$$
\begin{array}{r}
z(t)=\frac{1}{\sqrt{2}}\left\{\sum _ { i } \left(s_{i} h_{i}(t-i T) e^{j 2 \pi f_{c} t}+\right.\right. \\
\left.\left.s_{i}^{*}\left(h_{i}(t-i T)\right)^{*} e^{-j 2 \pi f_{c} t}\right)\right\}
\end{array}
$$

If $\tau$ is assumed constant then $e^{-j 2 \pi \sigma_{s}}$ is a fixed complex-valued scalar quantity with absolute value of 1 and, therefore, would not affect the statistical properties of $\left[q_{g}(t)-j q_{4}(t)\right] e^{-j 2 \sigma_{\varepsilon}}$, bearing in mind that $q_{i}(t)$ 's are statistically independent with zero mean Gaussian random processes. Therefore, Eqn. 3.2.22 can be simplified as

$$
\begin{align*}
h_{i}(t-i T)= & a(t-i T)\left[q_{1}(t)-j q_{2}(t)\right]+ \\
& a(t-\tau-i T)\left[q_{3}(t)-j q_{4}(t)\right]
\end{align*}
$$

The output from the radio receiver filter, ( Fig. 3.2.2), whose sampled impulse-response is $d(t)$, is

$$
z^{\prime}(t)=z(t) * d(t)+n(t) * d(t)
$$

and the output of the linear demodulator in Fig. 3.2.2 is

$$
\begin{align*}
r(t)= & \sqrt{2}\left\{\left[z(t)^{*} d(t)^{*} c(t)\right] e^{-j 2 \pi c_{c}^{\prime} t}\right\} * b^{\prime}(t)+ \\
& \sqrt{2}\left\{[n(t) * d(t) * c(t)] e^{-j 2 \pi f_{c}^{\prime} t}\right\} b^{\prime}(t)
\end{align*}
$$

Let

$$
b(t)=\left\{\left[d(t)^{*} c(t)\right] e^{-j 2 \pi c_{c}^{\prime} c^{l} t}\right\}^{\prime}(t)
$$

and

$$
w(t)=\sqrt{2}\left\{[n(t) * d(t) * c(t)] e^{-j 2 \pi f_{c}^{\prime} t}\right\} b^{\prime}(t)
$$

where $n(t)$ is a real-valued additive white Gaussian noise waveform comprising a two sided power spectral density of $\frac{1}{2} N_{0}$. w(t) in Eqn. 3.2.28 represents a band-limited, complex-valued Gaussian noise waveform.

Combining Eqns. 3.2.26, 3.2.27 and 3.2.28 we have

$$
r(t)=\sqrt{2}\left[z(t) e^{-j 2 \pi f_{c}^{f}}\right] * b(t)+w(t)
$$

Eqn. 3.2.27 represents the overall filtering carried out on the signal at the receiver. Also it is assumed that the receiver is operating in synchronism with the transmitter and any constant phase difference between the reference carrier and the received signal is neglected (i.e. $f_{c}=f_{c}^{\prime}$ ). Then from Eqns. 3.2.23 and 3.2.29,

$$
\begin{align*}
r(t)=\sum_{i}\left[s_{i} h_{i}(t-i T)\right. & \left.+s_{i}^{*}\left(h_{i}(t-i T)\right)^{*} e^{-j 4 \pi f_{c} t}\right] * b(t) \\
& +w(t)
\end{align*}
$$

The Gaussian shaped filter used to generate $q_{i}(t)$, has a frequency response that decreases sharply with $f$ (Fig. 2.7.1). $\mathrm{h}_{\mathrm{i}}(\mathrm{t}-\mathrm{i} \mathrm{T})$, which consists of the time invariant impulse-response $a(t)$ and the random components $q_{i}(t)$ 's can, therefore, be considered to be strictly bandlimited. i.e.

$$
|H(f)|=0 \quad|f|>f_{c}
$$

The second term in Eqn. 3.2.30,

$$
\left[s_{i}^{*}\left(h_{i}(t-i T)\right)^{*} e^{-j 4 \pi f_{c} t}\right]
$$

is, therefore, outside the pass band of the low pass filter with an impulse-response b(t). Hence

$$
r(t)=\sum_{i} s_{i} h_{i}(t-i T) * b(t)+w(t)
$$

Let

$$
Y_{i}(t-i T)=h_{i}(t-i T)^{*} b(t)
$$

Then

$$
r(t)=\sum_{i} s_{i} Y_{i}(t-i T)+w(t)
$$

Combining Eqns. 3.2.24 and 3.2.33, $\mathrm{Y}_{\mathrm{i}}(\mathrm{t}-\mathrm{iT})$ can be written as,

$$
\begin{align*}
Y_{i}(t-i T)= & \left\{a(t-i T)\left[q_{1}(t)-j q_{2}(t)\right]+\right. \\
& \left.a(t-\tau-i T)\left[q_{3}(t)-j q_{4}(t)\right]\right\}^{*} b(t)
\end{align*}
$$

and for a three sky wave channel

$$
\begin{align*}
Y_{i}(t-i T)= & \left\{a(t-i T)\left[q_{1}(t)-j q_{2}(t)\right]+\right. \\
& a\left(t-\tau_{1}-i T\right)\left[q_{3}(t)-j q_{4}(t)\right]+ \\
& \left.a\left(t-\tau_{2}-i T\right)\left[q_{5}(t)-j q_{6}(t)\right]\right\} * b(t)
\end{align*}
$$

$Y_{i}(t-\mathrm{iT})$ is the impulse-response of the equivalent time varying linear baseband channel. Fig. 3.2.3 shows the baseband model of the QAM system over a two sky wave HF radio link.

The average transmitted energy at the output of the transmitter filter in Fig. 3.2.3 is given by

$$
E_{t}=E\left[\int_{-\infty}^{\infty}\left|s_{i} a(t-i T)\right|^{2} d t\right]
$$

Where $\mathrm{E}[$.] represents the expected value of the quantity within the square brackets.

Let

$$
\vec{s}_{i}^{2}=E\left[\left|s_{i}\right|^{2}\right]
$$

From Parseval's Theorem, Eqn. 3.2.37 can be written as

$$
E_{t}=\vec{s}_{i} \int_{-\infty}^{\infty}|A(f)|^{2} d f
$$

The average energy per signal element at the input of the receiver filter in Fig. 3.2.3 is given by

$$
\begin{aligned}
& E_{r}=E \int_{-\infty}^{\infty}\left\{s_{i} a(t-i T)\left[q_{1}(t)-j q_{2}(t)\right]+\right. \\
&\left.a(t-\tau-i T)\left[q_{3}(t)-j q_{4}(t)\right]\right\}^{2} d t
\end{aligned}
$$

or

$$
\begin{align*}
& E_{r}=\vec{s}_{i}^{2}\left[\bar{q}_{1}^{2}(t)+\vec{q}_{2}^{2}(t)+\bar{q}_{3}^{2}(t)+\bar{q}_{4}^{2}(t)\right] \\
& \int_{-\infty}^{\infty}|A(f)|^{2} d(f)
\end{align*}
$$


Fig. 3.2.3- Baseband Model of The QAM System Over

If $\vec{q}_{i}^{2}(t)=\vec{q}_{j}^{2}(t)$, for $\mathrm{i}=1, \ldots, 4$, and for $\mathrm{j}=1, \ldots, 4$, and the sum of their variances is equal to unity, then the average energy per signal element at the output of the transmitter filter and at the input to the receiver filter in Fig. 3.2.3 are equal and the HF channel, on average, does not introduce any attenuation or gain to the transmitted data signal and hence does not affect the signal/noise ratio of the system.

The signal/noise ratio is defined as

$$
\psi=\frac{\text { Transmitted energy per bit }}{\text { Two sided noise power spectral density }}
$$

or

$$
\psi=10.0 \quad \log _{10}\left(\frac{E_{b}}{\frac{1}{2} N_{0}}\right)
$$

It has been shown in references [34, 37] that, for a QPSK signal,

$$
\frac{E_{b}}{\frac{1}{2} N_{0}}=\frac{s_{i}^{2}}{2 \sigma_{u}^{2}}
$$

where $\sigma_{u}^{2}$ is the variance of the additive Gaussian noise.

### 3.3 EQUIPMENT FILTERS USED IN THE CHANNEL MODEL.

The baseband model of the data transmission system over a three sky wave HF radio link is shown in Fig. 3.3.1. The impulse-response of the linear baseband channel is time varying and for a three sky wave channel, it is given by

$$
\begin{align*}
Y_{i}(t-i T)=\{ & a(t-i T)\left[q_{1}(t)-j q_{2}(t)\right]+ \\
& a\left(t-\tau_{1}-i T\right)\left[q_{3}(t)-j q_{4}(t)\right]+ \\
& \left.a\left(t-\tau_{2}-i t\right)\left[q_{5}(t)-j q_{6}(t)\right]\right\}^{*} b(t)
\end{align*}
$$

Time varying linear baseband channel

where

$$
a(t)=a^{\prime}(t) *\left(g(t) e^{-j 2 \pi f_{c} t}\right)
$$

and

$$
b(t)=\left\{\left[d(t)^{*} c(t)\right] e^{-j 2 \pi f_{c} t}\right\} \quad * \quad b^{\prime}(t)
$$

$a(t)$ is the impulse-response of the overall transmitter filter $A$ and $b(t)$ is the impulse-response of the overall receiver filter B in Fig. 3.3.1. $a^{\prime}(t), \mathrm{g}(\mathrm{t}), \mathrm{d}(\mathrm{t}), \mathrm{c}(\mathrm{t})$ and $b^{\prime}(t)$ are the impulse-responses of filters $A^{\prime}, \mathrm{G}, \mathrm{D}, \mathrm{C}$ and $B^{\prime}$ respectively in Fig. 3.2.2. Filters G and D are the radio transmitter and radio receiver filters respectively. The details of the practical filters, used in the channel model, are given elsewhere [44-45]. As is clear from Fig. 3.2.2, there are four other filters besides the radio filters G and D . The digital lowpass filters $A^{\prime}$ and $B^{\prime}$ are used to prevent aliasing and have an approximately sinusoidal roll-off in amplitude [45]. Under the condition when the HF link does not introduce any fading, attenuation or group delay distortion and where there are no multipath effects, then Eqn. 3.3.1 reduces to

$$
Y_{i}(t)=[a(t) \quad * \quad b(t)]
$$

For optimum performance of the detection process $Y_{i}(\mathrm{t})$ in Eqn. 3.3.4 should be minimum phase [46] and $\mathrm{a}(\mathrm{t})$ and $\mathrm{b}(\mathrm{t})$ should be such that $|A(f)|=|B(f)|[29]$.* Figs. 3.3.2-3.3.4 show a combination of the equipment bandpass filters operating on the voiceband signal [34, 36-37]. Fig. 3.3.2 shows the frequency characteristics of the radio filters G and D in cascade over the positive frequencies and Table 3.3.1 shows the attenuation and group delay samples of the radio filters in cascade taken at 50 Hz frequency intervals. The radio filters used are the Clansman VRC 321 type, this being a typical radio filter generally used in a practical system. Fig. 3.3.3 shows the frequency characteristics of the modem transmitter and receiver filters in cascade and in the pass band of the QAM signal over positive frequencies. Table 3.3.2 shows the sampled values of the same characteristics taken at a frequency interval of 50 Hz . The frequency characteristics in Fig. 3.3.3 corresponds to the impulse-response

$$
\left\{a^{\prime}(t) *\left[c(t) e^{-j 2 \pi f_{c} t}\right] * b^{\prime}(t)\right\} e^{j 2 \pi f_{c} t}
$$

* Although this has been assumed, it is not in fact necessary for the estimation process.
(a) - Attenuation Characteristics

(b) - Group Delay Characteristics


Fig. 3.3.2 - Frequency Characteristics of the Radio filters G and D in cascade over the positive Frequency
(a) - Attenuation Characteristics

(b) - Group Delay Characteristics


Fig. 3.3.3 - Filter Characteristics Corresponding to the impulse response $\left\{a^{\prime}(t) *\left[c(t) e^{-12 \pi t o t}\right] * b^{\prime}(t) e^{12 \pi t c t}\right\}$


Fig. 3.3.4 - Filter Characteristics Corresponding To The Impulse Response $\{a(t) * b(t)\} e^{2 \pi t t}$

TABLE 3.3.1 ATTENUATION AND GROUP DELAY CHARACTERISTICS OF RADIO FILTER IN CASCADE.

| FREQUENCY <br> (Hz) | ATT. <br> (dB) | G.D. <br> (msec) | FREQUENCY <br> (Hz) | ATT. <br> (dB) | G.D. <br> (msec) |
| :---: | ---: | :---: | :---: | :---: | :---: |
| 50 | 50.00 | 9.00 | 1950 | 0.00 | 1.18 |
| 100 | 21.00 | 7.00 | 2000 | 0.00 | 1.15 |
| 150 | 16.50 | 6.50 | 2050 | 0.00 | 1.13 |
| 200 | 13.00 | 5.30 | 2100 | 0.00 | 1.10 |
| 250 | 10.00 | 4.50 | 2150 | 0.00 | 1.10 |
| 300 | 7.60 | 3.90 | 2200 | 0.00 | 1.10 |
| 350 | 5.60 | 3.40 | 2250 | 0.00 | 1.12 |
| 400 | 4.10 | 2.90 | 2300 | 0.00 | 1.15 |
| 450 | 2.75 | 2.60 | 2350 | 0.00 | 1.18 |
| 500 | 2.00 | 2.35 | 2400 | 0.00 | 1.23 |
| 550 | 1.50 | 2.05 | 2450 | 0.00 | 1.25 |
| 600 | 1.25 | 1.90 | 2500 | 0.05 | 1.27 |
| 650 | 1.05 | 1.75 | 2550 | 1.10 | 1.29 |
| 700 | 0.95 | 1.65 | 2600 | 0.15 | 1.32 |
| 750 | 0.80 | 1.60 | 2650 | 0.30 | 1.35 |
| 800 | 0.70 | 1.55 | 2700 | 0.45 | 1.35 |
| 850 | 0.60 | 1.50 | 2750 | 0.85 | 1.35 |
| 900 | 0.50 | 1.15 | 2800 | 1.35 |  |
| 950 | 0.40 | 1.50 | 2850 | 1.02 | 1.35 |
| 1000 | 0.30 | 1.50 | 2900 | 1.20 | 1.35 |
| 1050 | 0.25 | 1.50 | 2950 | 1.42 | 1.35 |
| 1100 | 0.20 | 1.50 | 3000 | 1.38 |  |
| 1150 | 0.15 | 1.50 | 3050 | 1.90 | 1.40 |
| 1200 | 0.10 | 1.50 | 3100 | 2.20 | 1.50 |
| 1250 | 0.05 | 1.50 | 3150 | 1.58 |  |
| 1300 | 0.00 | 1.50 | 3200 | 3.60 | 1.66 |
| 1350 | 0.00 | 1.50 | 3250 | 3.50 | 1.75 |
| 1400 | 0.00 | 1.50 | 3300 | 4.00 | 1.83 |
| 1450 | 0.00 | 1.45 | 3350 | 5.25 | 1.92 |
| 1500 | 0.00 | 1.45 | 3400 | 6.50 | 2.00 |
| 1550 | 0.00 | 1.42 | 3450 | 2.25 | 2.08 |
| 1600 | 0.00 | 1.39 | 3500 | 10.00 | 2.16 |
| 1650 | 0.00 | 1.36 | 3550 | 12.00 | 2.25 |
| 1700 | 0.00 | 1.33 | 3600 | 14.00 | 2.33 |
| 1750 | 0.00 | 1.30 | 3650 | 20.00 | 2.41 |
| 1800 | 0.00 | 1.27 | 3700 | 30.00 | 2.50 |
| 1850 | 0.00 | 1.24 | 3750 | 2.58 |  |
| 1900 | 0.00 | 1.21 |  |  |  |
|  |  |  |  |  |  |

## TABLE 3.3.2 ATTENUATION AND GROUP DELAY CHARACTERISTICS OF EQUIPMENT FILTER.

| $\underset{(\mathrm{Hz})}{\text { FREQUENCY }}$ | $\underset{(\mathrm{dB})}{\mathrm{ATT}}$ | $\begin{gathered} \text { G.D. } \\ \text { (msec) } \end{gathered}$ | $\underset{(\mathrm{Hz})}{\text { FREQUENCY }}$ | $\begin{array}{r} \text { ATT. } \\ (\mathrm{dB}) \end{array}$ | $\begin{gathered} \text { G.D. } \\ \text { (msec) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | 99.99 | 1.51 | 1950 | 0.00 | 2.88 |
| 100 | 93.79 | 1.83 | 2000 | 0.01 | 2.89 |
| 150 | 77.62 | 2.12 | 2050 | 0.05 | 2.90 |
| 200 | 64.73 | 2.37 | 2100 | 0.13 | 2.92 |
| 250 | 53.94 | 2.58 | 2150 | 0.23 | 2.95 |
| 300 | 44.70 | 2.76 | 2200 | 0.35 | 2.97 |
| 350 | 36.70 | 2.91 | 2250 | 0.40 | 3.00 |
| 400 | 30.40 | 3.03 | 2300 | 0.45 | 3.03 |
| 450 | 24.40 | 3.15 | 2350 | 0.57 | 3.05 |
| 500 | 19.50 | 3.26 | 2400 | 0.76 | 3.10 |
| 550 | 15.65 | 3.37 | 2450 | 0.93 | 3.15 |
| 600 | 12.30 | 3.47 | 2500 | 1.45 | 3.19 |
| 650 | 9.55 | 3.48 | 2550 | 1.97 | 3.25 |
| 700 | 7.30 | 3.48 | 2600 | 2.64 | 3.30 |
| 750 | 5.50 | 3.47 | 2650 | 3.25 | 3.35 |
| 800 | 4.10 | 3.43 | 2700 | 4.05 | 3.39 |
| 850 | 3.10 | 3.41 | 2750 | 5.20 | 3.42 |
| 900 | 2.20 | 3.37 | 2800 | 6.72 | 3.44 |
| 950 | 1.65 | 3.32 | 2850 | 8.20 | 3.47 |
| 1000 | 1.25 | 3.26 | 2900 | 10.25 | 3.49 |
| 1050 | 0.75 | 3.19 | 2950 | 12.45 | 3.50 |
| 1100 | 0.35 | 3.14 | 3000 | 14.95 | 3.51 |
| 1150 | 0.02 | 3.09 | 3050 | 17.70 | 3.51 |
| 1200 | 0.00 | 3.04 | 3100 | 21.10 | 3.49 |
| 1250 | 0.00 | 3.01 | 3150 | 24.80 | 3.45 |
| 1300 | 0.00 | 2.98 | 3200 | 28.60 | 3.41 |
| 1350 | 0.00 | 2.95 | 3250 | 32.83 | 3.33 |
| 1400 | 0.00 | 2.93 | 3300 | 37.63 | 3.22 |
| 1450 | 0.00 | 2.90 | 3350 | 43.10 | 3.08 |
| 1500 | 0.00 | 2.88 | 3400 | 49.15 | 2.89 |
| 1550 | 0.00 | 2.87 | 3450 | 55.55 | 2.65 |
| 1600 | 0.00 | 2.87 | 3500 | 62.30 | 2.36 |
| 1650 | 0.00 | 2.86 | 3550 | 69.55 | 2.05 1.69 |
| 1700 | 0.00 | 2.86 | 3600 | 76.75 | 1.69 |
| 1750 | 0.00 | 2.85 | 3650 | 84.05 | 1.33 |
| 1800 | 0.00 | 2.85 | 3700 3750 | 90.85 96.80 | 1.03 0.79 |
| 1850 1900 | 0.00 0.00 | 2.85 2.88 | 3750 | 96.80 | 0.79 |

Fig. 3.3.4 shows the frequency characteristics corresponding to the impulse-response

$$
\{a(t) * b(t)\} e^{j 2 \pi f_{c} t}
$$

The attenuation and group delay characteristics corresponding to each of $a(t)$ and $b(t)$, in Eqn. 3.3.1, are obtained by shifting the frequency characteristics in Fig. 3.3.4 by $\mathrm{f}_{\mathrm{c}}=1800 \mathrm{~Hz}$, to the left, and dividing them by 2 . A sequence $\left\{a_{k}^{\prime}\right\}$ is obtained by taking the inverse DFT of the frequency characteristics at a sampling rate of 4800 samples $/ \mathrm{sec}$. This sequence is made minimum phase by replacing the roots which are outside the unit circle in the z plane, by the complex conjugate of their reciprocals, giving the sequence $\left\{a_{k}^{\prime \prime}\right\}$ which, is at a sampling rate of 4800 samples $/ \mathrm{sec}$. The method by which the minimum phase sequence is obtained can be found elsewhere $[43,49]$. The DFT of $\left\{a_{k}^{\prime \prime}\right\}$ has been obtained with a sampling interval of 50 Hz and since the sampling rate is 4800 samples per second, there are 96 components in the DFT of $\left\{a_{k}^{\prime \prime}\right\}$. In order to obtain different sampling phases the $\left\{a_{k}^{\prime \prime}\right\}$ have been oversampled at 20 times the original sampling rate, i.e at a sampling rate of 96000 samples per second. This is done by injecting 1824 zero-valued components, in the middle of the DFT of $\left\{a_{k}^{\prime \prime}\right\}$, thus increasing the number of components from 96 to 1920. This injection process is equivalent to increasing the sampling rate from 4800 samples $/ \mathrm{sec}$. to $4800 \times 20=96000$ samples $/ \mathrm{sec}$. The inverse DFT of this expanded sequence, $\left\{\hat{a}_{k}\right\}$, gives the minimum phase impulse-response $a(t)$ sampled at 96000 samples $/$ sec. The transmitter filter impulse-response ${ }_{\chi \times}$ $\left\{\mathrm{a}_{1, k}\right\}$ corresponding to $\mathrm{a}(\mathrm{t}-\mathrm{iT})$ is obtained by taking every $20^{\text {h }}$ sample of $\left\{\hat{a}_{k}\right\} .\left\{\mathrm{a}_{2, k}\right\}$ and $\left\{\mathrm{a}_{3, k}\right\}$ corresponding to $\mathrm{a}\left(\mathrm{t}-\tau_{1}-\mathrm{i} \mathrm{T}\right)$ and $\mathrm{a}\left(\mathrm{t}-\tau_{2}-\mathrm{i} \mathrm{T}\right)$ respectively, are also obtained by taking every $20^{\text {di }}$ sample from the same sequence $\left\{\hat{a}_{k}\right\}$. However, they are delayed $\tau_{1}$ and $\tau_{2}$ sec. with respect to $\left\{\mathrm{a}_{1, k}\right\}$.

Table 3.3.3 gives the sampled impulse-response of the minimum phase transmitter filters used in the channel model. Transmitter filters A2 and A3, in Table 3.3.3, corresponds to a delay of 1.1 msec . and 3 msec . with respect to A 1 , respectively. Appendix B gives the oversampled sequence $\left\{\hat{a}_{k}\right\}$ and explains the way in which the filters A2 and A3 are obtained. The receiver filter $b(k)$ has also been obtained by taking every $20^{\text {d }}$ sample of the oversampled sequence $\left\{\hat{b}_{k}\right\}$. $\left\{\hat{b}_{k}\right\}$ has been obtained exactly as $\left\{\hat{a}_{k}\right\}$, but at a different sampling phase, so that the model does not assume any particular sampling phase. Table 3.3.4 shows the sampled impulse-response of the minimum phase receiver filter used in the channel model and Appendix B gives the oversampled sequence $\left\{\hat{b}_{k}\right\}$.

TABLE 3.3.3 THE SAMPLED IMPULSE RESPONSE OF THE TRANSMITTER FILTERS FOR A THREE SKY WAVE CHANNEL.

| TRANSMITTER FILTER <br> A1 |  |
| ---: | ---: |
|  |  |
| REAL PART | IMAGINARY <br> PART |
|  |  |
| -0.179590 | 2.353941 |
| -3.077346 | 20.759024 |
| -9.940902 | 45.58459 |
| -11.786947 | 41.490998 |
| -3.461827 | 8.704583 |
| 4.443815 | -11.786982 |
| 3.064254 | -5.581905 |
| -1.359658 | 3.158213 |
| -1.497353 | 1.36546 |
| 0.292560 | -0.777689 |
| 0.518083 | -0.129256 |
| -0.184279 | 0.288030 |
| -0.316778 | -0.232482 |
| 0.002190 | -0.210755 |
| -0.044381 | 0.039206 |
| 0.051553 | 0.009851 |

TRANSMITTER FILTER
A2

REAL PART IMAGINARY PART
0.000000 0.000000 0.000000 0.000000 0.000000 -1.669437 $-7.849215$
-12.388708
-6.602316
2.940855
4.300508
-0.336838
-1.901434
-0.143359
0.624260
0.027858
-0.382007
-0.041691
-0.043971 0.074933
$-0.059413$

TRANSMITTER FILTER A3

| REAL PART | IMAGINARY <br> PART |
| ---: | ---: |
| 0.000000 | 0.000000 |
| 0.000000 | 0.000000 |
| 0.000000 | 0.000000 |
| 0.000000 | 0.000000 |
| 0.000000 | 0.000000 |
| 0.000000 | 0.000000 |
| 0.000000 | 0.000000 |
| 0.000000 | 0.000000 |
| 0.000000 | 0.000000 |
| 0.000000 | 0.000000 |
| 0.000000 | 0.000000 |
| 0.000000 | 0.000000 |
| 0.000000 | 0.000000 |
| 0.000000 | 0.000000 |
| -1.313654 | 11.068896 |
| -7.110405 | 37.213660 |
| -12.346972 | 47.957516 |
| -7.584870 | 22.826248 |
| 2.235385 | -7.249859 |
| 4.593861 | -10.002670 |
| 0.093164 | 0.869544 |
| -1.970418 | 3.107280 |
| -0.323370 | -0.226110 |
| 0.631324 | -0.555291 |
| 0.103572 | 0.288210 |
| -0.386594 | -0.015670 |
| -0.073453 | -0.321577 |
| -0.038647 | -0.010771 |
| 0.060805 | 0.014091 |
| -0.071350 | 0.013571 |

TABLE 3.3.4 THE SAMPLED IMPULSE RESPONSE OF THE RECEIVER FILTER

| REAL PART | IMAGINARY PART |
| :---: | :---: |
| -1.941769 | 1.362559 |
| -15.979786 | 11.594104 |
| -35.141773 | 27.334294 |
| -34.478872 | 28.087009 |
| -11.234198 | 7.271462 |
| 7.815516 | -9.260247 |
| 7.512406 | -5.095446 |
| -0.505751 | 3.232650 |
| -3.370713 | 1.897535 |
| -0.675917 | -1.281360 |
| 1.048266 | -0.483031 |
| 0.362188 | 0.761480 |
| -0.310590 | 0.197901 |
| 0.043841 | -0.153267 |
| 0.073895 | 0.094033 |
| -0.064694 | -0.031213 |

### 3.4 COMPUTER SIMULATION OF THE HF CHANNEL

Fig. 3.3.1 shows the baseband model of the data transmission system over a three sky wave HF radio link. The transmitter and receiver filters used in the channel simulation are shown in Figs. 3.3.2-3.3.4. The sampled impulse-response of the minimum phase transmitter and receiver filters, at a sampling rate of 4800 samples/sec., are given in Tables 3.3.3-3.3.4. From Eqn. 3.3.1, the sampled impulse-response of the linear baseband channel is

$$
\begin{align*}
Y_{i}(t-i T)= & \left\{a^{\prime \prime}(t-i T)\left[q_{1}(t)-j q_{2}(t)\right]+\right. \\
& a^{\prime \prime}\left(t-\tau_{1}-i T\right)\left[q_{3}(t)-j q_{4}(t)\right]+ \\
& \left.a^{\prime \prime}\left(t-\tau_{2}-i T\right)\left[q_{5}(t)-j q_{6}(t)\right]\right\}^{*} b^{\prime \prime}(t)
\end{align*}
$$

where $a^{\prime \prime}(t)$ and $b^{\prime \prime}(t)$ are minimum phase sampled impulse-responses of filters $\mathrm{a}(\mathrm{t})$ and $b(t)$ respectively. The demodulated baseband signal $r(t)$ at the output of the QAM system model, is given by Eqn. 3.2.34. The waveform $r(t)$ is sampled once per data symbol $\mathrm{s}_{\mathrm{i}}$, at the time instant iT. Assuming correct sampling at the receiver and the fact that the delay in transmission is such that the first potentially non-zero sample of a received signal element arrives without any delay, the complex-valued sample $r(t)$ at time $t=\mathrm{iT}$ is given by

$$
\begin{align*}
r_{i} & =\sum_{h=0}^{g} s_{i-h} y_{i, h}+w_{i} \\
& =Y_{i} S_{i}^{T}+w_{i}
\end{align*}
$$

where

$$
Y_{i}=\left[\begin{array}{lllll} 
& y_{i, 0} & y_{i, 1} & \ldots & y_{i, g}
\end{array}\right]
$$

and

$$
S_{i}=\left[\begin{array}{lllll}
s_{i} & s_{i-1} & \ldots \ldots & s_{i-g}
\end{array}\right]
$$

$Y_{i}$ and $S_{i}$ are $(g+1)$ - component row vectors, and $S_{i}^{\top}$ is the transpose of $S_{i}$. The $\left\{r_{i}\right\}$, $\left\{y_{i, h}\right\}$ and $\left\{w_{i}\right\}$ are complex-valued. $y_{i, h}=0$ for $h<0$ and $h>g$ for practical purpose. $Y_{i}$ is taken to be the sampled impulse-response of the linear baseband channel at time $t=i T$. The vector $Y_{i}$ is obtained by sampling the $\left\{Y_{i}(t-i T)\right\}$ at a sampling rate of 2400 samples per second. The convolution process in Eqn. 3.4.1 is carried out in the discrete time domain. In order to avoid any aliasing, when any one of the sequences $q_{i}(t)$, for $i=1,2, \ldots, 6$, changes rapidly, the sampling rate of the convolution is set to 4800 samples/sec., which is well above the Nyquist rate for filters A and B. $q_{i}(t) s, i=1,2, \ldots, 6$, have also been generated at a sampling rate of 4800 samples $/ \mathrm{sec}$., as described in Section 2.6, and the corresponding resultant samples up to time $t=i T$ are represented by the following six sequences.

$$
\begin{aligned}
Q Q_{1, i} & =\left[\begin{array}{lllll} 
& q_{1,1} & q_{1,2} & \ldots & q_{1,2 i}
\end{array}\right] \\
Q Q_{2, i} & =\left[\begin{array}{lllll} 
& q_{2,1} & q_{2,2} & \ldots . & q_{2,2 i}
\end{array}\right]
\end{aligned}
$$

$$
Q Q_{6, i}=\left[\begin{array}{llll} 
& q_{6,1} & q_{6,2} & \cdots
\end{array} q_{6,2 i}\right]
$$

Let

$$
\begin{align*}
A 1 & =\left[\begin{array}{lllll}
a_{1,0}^{\prime \prime} & a_{1,1}^{\prime \prime} & \ldots . & a_{1, p}^{\prime \prime}
\end{array}\right] \\
A 2 & =\left[\begin{array}{lllll} 
& a_{2,0}^{\prime \prime} & a_{2,1}^{\prime \prime} & \ldots . & a_{2, p}^{\prime \prime}
\end{array}\right] \\
A 3 & =\left[\begin{array}{lllll}
\prime \prime & a_{3,0}^{\prime \prime} & a_{3,1}^{\prime \prime} & \ldots . & a_{1, \mathrm{p}}
\end{array}\right]
\end{align*}
$$

and

$$
B 1=\left[\begin{array}{lllll} 
& b_{0} & b_{1} & \ldots & b_{\rho}
\end{array}\right]
$$

where

$$
\begin{align*}
& a_{1, k}^{\prime \prime}=a^{\prime \prime}\left(k \frac{T}{2}\right) \\
& a_{2, k}^{\prime \prime}=a^{\prime \prime}\left(k \frac{T}{2}-\tau_{1}\right)
\end{align*}
$$

$$
\begin{align*}
& a_{3, k}^{\prime \prime}=a^{\prime \prime}\left(k \frac{T}{2}-\tau_{2}\right) \\
& b_{k}^{\prime \prime}=b^{\prime \prime}\left(k \frac{T}{2}\right)
\end{align*}
$$

It is assumed that there are only ( $1+1$ ) significant components in the sampled impulse-response of the filters and, therefore, for all practical purpose

$$
\begin{align*}
& a(t)=b(t)=0 \\
& \text { for } \mathrm{t}<0 \text { and } \mathrm{t}>1
\end{align*}
$$

$\rho$ in Eqns. 3.4.7-3.4.10 is related to the maximum delay between sky waves ( assumed to be $\tau_{2}$ ) as $[34,36]$

$$
\rho=l+\tau_{2} \frac{2}{T}
$$

In Eqns. 3.4.7-3.4.14, $1 / \mathrm{T}$ is the data symbol rate of 2400 symbols/sec. A1, A2 and A3 are the three transmitter filters used for the modelling of a three sky wave HF channel, with impulse-responses $a^{\prime \prime}(t), a^{\prime \prime}\left(t-\tau_{1}\right)$ and $a^{\prime \prime}\left(t-\tau_{2}\right)$ respectively, sampled at 4800 samples $/ \mathrm{sec}$. B1 is the receiver filter with an impulse-response $b(t)$ sampled at 4800 samples $/ \mathrm{sec}$.

From Eqns. 3.4.6-3.4.14, the components of the vector $Y_{i}$, in Eqn. 3.4.4, at time $\mathrm{t}=\mathrm{i}$, are given by [34],

$$
\begin{array}{r}
y_{i, h}=\left(\frac{T}{2}\right) \sum_{k=0}^{2 h}\left[a_{1, k}^{\prime \prime}\left(q_{1,2(i-h)+k}-j q_{2,2(i-h)+k}\right)+\right. \\
a_{2, k}^{\prime \prime}\left(q_{3,2(i-h)+k}-j q_{4,2(i-h)+k}\right)+ \\
\left.a_{3, k}^{\prime \prime}\left(q_{5,2(i-h)+k}-j q_{6,2(i-h)+k}\right) \quad\right] b_{2 h-k}^{\prime \prime}
\end{array}
$$

for $h=0,1, \ldots, g$.
where g is related to $\tau_{2}$, by the following relation [36].

$$
g=\frac{2 l+\rho_{2}+1}{2}
$$

Thus for $\tau_{2}=3 \mathrm{msec}$. and $\mathrm{l}+1=16$ (Table 3.4.3) g is 22 . $\left\{\mathrm{y}_{\mathrm{i} h}\right\}$ are obtained at a sampling rate of 2400 samples $/$ sec., by taking every alternate sample from the convolution process.

### 3.5 MODEL OF THE SYSTEM USED IN THE TESTING OF ESTIMATORS.

The model of the data transmission system used in the tests is shown in Fig. 3.5.1. This model is consistent with Figures 3.2.2 and 3.3.1, but it shows in greater detail the receiver configuration. The output signal from the linear modulator is a serial stream of real-valued QPSK signal elements, with a carrier frequency of 1800 Hz and an element rate of 2400 bauds. Each signal element itself comprises the sum of two binary double sideband suppressed carrier amplitude modulated elements, with their carriers in phase quadrature, the binary values of the in-phase and quadrature elements being determined respectively by the real and imaginary parts ( $\mathrm{s}_{0, i}$ and $\mathrm{s}_{1, i}$ ) of the corresponding data-symbol $\mathrm{s}_{\mathrm{i}}$. Thus the QPSK signal is handled as a quadrature amplitude modulated (QAM) signal.

The HF radio link is modelled as having three independent Rayleigh fading paths with the transmission delays being $0,1.1$ and 3 milliseconds relative to the first sky waves. Stationary white Gaussian noise, with zero mean and a two-sided power spectral density $\frac{1}{2} N_{0}$, is added to the data signal at the output of the HF radio link.

The six Gaussian waveforms involved in the three sky waves have the same variance and the same root-mean-square bandwidth which is 1 Hz in every case. Thus the signal received over each sky wave has the same mean-square value and the same frequency spread of 2 Hz . The selected time delays of the three sky waves ensure a different sampled impulse-response for each path and are such that one of the relative (differential) time delays is not an integral multiple of the other.

The channel model used in this thesis is based on the CCIR recommended model for poor conditions [19]. A constant value of frequency offset (Doppler shift) is not
considered here, since this is taken care off by Doppler-shift correction circuits which operate both ahead of and independently of the channel estimator. The channel estimator, therefore, operates on a signal that is essentially free from any constant (or very slowly varying) frequency offset.

The vector $Y_{i}$ (Eqn. 3.4.4) is taken to be the sampled impulse-response of the linear baseband channel in Fig. 3.5.1. The received samples $\left\{\mathrm{r}_{\mathrm{i}}\right\}$ are fed to an adaptive linear feedforward transversal filter. The latter is an allpass network that adjusts the sampled impulse-response of the channel and filter to be minimum phase, without changing any amplitude distortion in the received signal [49]. The filter, in fact, maximises the ratio of the magnitude of the first few components of the resultant sampled impulse-response to the output noise variance, when the noise components are statistically independent [43]. With the aid of the adaptive filter, a near-optimum tolerance to noise can be achieved by means of a relatively simple detector, leading to a potentially cost-effective system [51].

The received samples $\left\{\mathrm{r}_{\mathrm{i}}\right\}$ are also fed to the channel estimator, after being suitably delayed. The channel estimator uses the received samples

$$
\begin{array}{llll}
r_{i-g} & r_{i-g+1}, & \ldots, & r_{i}
\end{array}
$$

together with the "early" detected data-symbols

$$
s_{i-g}^{\prime \prime}, \quad s_{i-g+1}^{\prime \prime}, \quad \ldots, \quad s_{i}^{\prime \prime}
$$

and the one-step prediction of $Y_{i}$, given by

$$
Y_{i, i-1}^{\prime}=\left[\begin{array}{llll}
y_{i, i-1,0}^{\prime} & y_{i, i-1,1}^{\prime} & \ldots & y_{i, i-1, g}^{\prime}
\end{array}\right]
$$

to form the updated estimate of $Y_{i}$, given by

$$
Y_{i}^{\prime}=\left[\begin{array}{llll}
y_{i, 0}^{\prime} & y_{i, 1}^{\prime} & \ldots & y_{i, g}^{\prime}
\end{array}\right]
$$

and then the one-step prediction of $\mathrm{Y}_{\mathrm{i}+1}$, given by $Y_{i+1, i,}^{\prime}$. The latter is fed to the detector, ready for the next detection process that gives $s_{i+1}^{\prime \prime}$, and so on. Clearly, any error in $Y_{i+1, i}^{\prime}$ correspondingly degrades the detection of $\mathrm{s}_{\mathrm{i}+1}$.

The "early" detected data symbols have no delay in detection. This minimizes the period over which prediction must be carried out but increases somewhat the error rate in the $\left\{s_{i}^{\prime \prime}\right\}$. The detected data-symbols $\left\{s_{i}^{\prime}\right\}$ at the output of the detector, in Fig. 3.5.1, have a delay in detection of 32 sampling intervals, no significant reduction in error rate being achieved by any further increase in the delay in detection.

In practice, a small improvement in tolerance to noise is usually achieved by using a prediction $Y_{i+m, i-1}^{\prime}$ of the sampled impulse-response of the channel, over $\mathrm{m}+1$ sampling intervals, where typically $1 \leq m \leq 8$, the value of $m$ depending upon the relative transmission delays of the different sky waves. However, since the relative performances of the different estimators is not significantly affected by the precise period over which prediction is carried out, it is assumed here that one-step prediction is used for the detector (Eqn. 3.5.1).

The important advantage gained by using the adaptive filter in Fig. 3.5.1 is that it avoids the need for prediction over many sampling intervals, such as must be used in the absence of the filter [50,52]. Prediction over many sampling intervals can considerably increase the error in the prediction [50, 54]. Further details of the adaptive filter and detector are given elsewhere [37, 43, 49, 51].

Since this study is only about the operation of the channel estimators and not about the detectors, the correct detection of all data symbols is assumed, even at low signal/noise ratios, so that

$$
s_{i}^{\prime \prime}=s_{i}
$$

for all \{i\}.

In any practical application of the system the data signal is divided into separate blocks, each preceded by a training signal whose data-symbol values are known at the receiver. Under fading conditions, most errors in detection occur during the deeper fades and generally in long bursts. Often, during an error burst, the channel estimate becomes significantly degraded, leading to more errors in the $\left\{s_{i}^{\prime \prime}\right\}$, which in turn further degrades the channel estimate, and so on, until there is a complete failure of the system. The error burst is now extended to the end of the block of data symbols, but the following training signals restore correct operation of the channel estimator, ready for the next block of data symbols. When more than a few errors
have occurred in the $\left\{s_{i}^{\prime}\right\}$, for any block of data symbols, the whole block of the detected data-symbols $\left\{s_{i}^{\prime}\right\}$ is usually rendered invalid and is rejected by the receiver. Furthermore, any large burst of errors in the $\left\{s_{i}^{\prime \prime}\right\}$ is usually accompanied by a substantial burst of errors in the $\left\{s_{i}^{\prime}\right\}$. It follows that, for the most reliable operation of the system, the channel estimator must give the most accurate possible estimate (prediction) of the channel when the $\left\{s_{i}^{\prime \prime}\right\}$ are correct. Once an appreciable burst of errors has occurred in the $\left\{s_{i}^{\prime \prime}\right\}$, the chances are that the corresponding block of $\left\{s_{i}^{\prime}\right\}$ are invalid, and no advantage is gained by improving the channel estimate under these conditions.

Tests have indicated that the performance of the channel estimate is only likely to be significantly affected by errors in the $\left\{s_{i}^{\prime \prime}\right\}$ at the higher error rates (above $10^{-2}$ ) [50, 52].

Figures 3.5.2-3.5.4 show the 3- sky wave channel characteristics over a duration of 25 seconds of transmission. The channel characteristics have been plotted for six different values of seed integer for the random number generator. A typical worst fading sequence, obtained using a seed integer value of 500 , has been chosen to test the performance of the estimators in this thesis. Table 3.5 . 2 shows the number of fades measured, over the duration of transmission, obtained for different values of seed integer. The fades have been measured relative to 0 dBm , as opposed to the median value, since the channel characteristics represent a 3- sky wave channel. The number of fades measured are generally consistent with the theoretical values (Section 2.7)

(a) - Seed Integer $=107$




TABLE 3.5.1 5-POLE BESSEL FILTER CHARACTERISTICS

| Frequency Spread | 2 Hz |  |
| ---: | :--- | :--- |
| Unit circle Roots | 0.8948131 |  |
|  | 0.9016149 | -0.0479304 |
|  | 0.9016149 | 0.0479304 |
|  | 0.9260914 | -0.1011889 |
|  | 0.9260914 | 0.1011889 |
| Abs.Value of Roots | 0.8948131 |  |
|  | 0.9028880 |  |
|  | 0.9316032 |  |
| Co-Efficients of Filter | -1.8032297225 |  |
|  | 0.8152066804 |  |
|  | -1.8521828825 |  |
|  | 0.8678845458 |  |
|  | -0.8948130729 |  |
|  | 19378.0 |  |

TABLE 3.5.2 MEASURED CHARACTERISTICS OF THE FADING CHANNEL FOR DIFFERENT VALUES OF SEED INTEGERS. THE FADES HAVE BEEN MEASURED RELATIVE TO 0 dBm .

| SEED INTEGER | NO. OF FADES | MEAN LENGTH OF <br> THE CHANNEL |
| :---: | :---: | :---: |
| 9 | 34 | 1.0321 |
| 55 | 31 | 1.0420 |
| 107 | 33 | 1.0367 |
| 158 | 28 | 1.0145 |
| 195 | 34 | 1.0653 |
| 500 | 31 | 1.0405 |

## CHAPTER 4

## LINEAR FEEDFORWARD ESTIMATOR

### 4.1 INTRODUCTION

It has been shown in [54] that the simple linear "feedforward" estimator originally proposed for use with a maximum-likelihood detector [55], is likely to form the basis of the most cost effective estimator, for a randomly varying channel or where the receiver has only a limited knowledge of the correct model of the channel. A simple estimator designed for a $2400 \mathrm{bits} / \mathrm{s}$ modem [54] is a development of the conventional gradient estimator [55], and employs a polynomial filter that gives a prediction of the channel response. This chapter describes four channel estimators for use in HF radio links and these are called as systems 4.1-4.4. System 4.2 is the simple estimator described in $[33,35,46,50,54,57]$ and forms a basis for comparison of all the estimators developed in this thesis. System 4.1 does not use a predictor and, therefore, is a simple linear "feedforward" estimator. Systems 4.3 and 4.4 are developments of the simple estimator but make no use of any knowledge of the number of sky waves. Results of the computer-simulation tests on the estimators, over a model of a data transmission system, are presented at the end of the chapter.

### 4.2 MODEL OF DATA TRANSMISSION SYSTEM USED IN THE TESTS

The model of the data transmission system used in the tests, is shown in Fig. 3.5.1. Further details on the model of the channel and the data transmission system are given in Chapters 2 and 3, respectively.

The received sample at time $\mathrm{t}=\mathrm{i} \mathrm{T}$ is given by (Eqn. 3.4.2)

$$
\begin{align*}
r_{i} & =\sum_{h=0}^{g} s_{i-h} y_{i, h}+w_{i} \\
& =Y_{i} S_{i}^{T}+w_{i}
\end{align*}
$$

$r_{i}$ is sample value of the complex-valued resultant baseband signal $r(t)$ at time $t=i T$. $\mathrm{Y}_{\mathrm{i}}$ and $\mathrm{S}_{\mathrm{i}}$ are ( $\mathrm{g}+1$ )- component row vectors, and $s_{i}^{T}$ is the transpose of $\mathrm{S}_{\mathrm{i}}$ (Eqns. 3.4.4 and 3.4.5).

$$
\begin{align*}
Y_{i} & =\left[\begin{array}{llllll} 
& y_{i, 0} & y_{i, 1} & \ldots \ldots & y_{i, g}
\end{array}\right] \\
S_{i} & =\left[\begin{array}{llllll} 
& s_{i} & s_{i-1} & \ldots . . & s_{i-g}
\end{array}\right]
\end{align*}
$$

The vector $Y_{i}$ is taken to be the sampled impulse-response of the linear baseband channel. The noise samples $\left\{w_{i}\right\}$ have zero mean and variance that is dependent on $\frac{1}{2} N_{0}$ and neighbouring $\left\{w_{i}\right\}$ being slightly correlated [46,50]. The detection process is assumed perfect, even at low signal/noise ratios and, therefore, the detected value of $\mathrm{s}_{\mathrm{i}}$ designated as $s_{i}^{\prime}$ is equal to $\mathrm{s}_{\mathrm{i}}$ for all values of \{i\} (Eqn. 3.5.3). The signals $\mathrm{r}_{\mathrm{i}}$ and $s_{i}^{\prime}$ are fed to the channel estimator to give an estimate of the channel sampled impulse-response $Y_{i}^{\prime}$ at time $\mathrm{t}=\mathrm{i} \mathrm{T}$, where

$$
Y_{i}^{\prime}=\left[\begin{array}{lllll}
y_{i, 0}^{\prime} & y_{i, 1}^{\prime} & \ldots . . & y_{i, g}^{\prime}
\end{array}\right]
$$

This estimate of the channel sampled impulse-response, $Y_{i}^{\prime}$, is fed to the detector to detect $s_{i+1}^{\prime \prime}$ when the next received signal $\mathrm{r}_{\mathrm{i}+1}$ is received by the detector.

### 4.3 SYSTEM 4.1

This system is also called the Linear feedforward estimator and has been developed by Magee and Proakis [55] and is also called the simple estimator in [54]. The channel estimator originally proposed for use with a maximum-likelihood detector employing the Viterbi algorithm, [55], after modification for use with complexvalued symbols, becomes the linear feedforward estimator. Fig. 4.3.1 shows the linear feedforward transversal filter estimator. Each square marked T, in the figure, is a store that holds the corresponding detected data symbol $\mathrm{s}_{\mathrm{i}-\mathrm{b}}$, and they are like a shift register. Each time the stores are triggered on receipt of a received sample $r_{i}$,

Fig. 4.3.1. - Linear Feedforward Transversal Filter
the stored values are shifted one place to the right. There are $(\mathrm{g}+1)$ stores corresponding to the total number of components in the sampled impulse-response of the channel. Each symbol $\mathrm{s}_{\mathrm{inh}}$ is multiplied by the corresponding tap weight $y_{i-1, k}^{\prime}$ and the resulting products are added to give the estimate $r_{i}^{\prime}$ of the received sample $r_{i}$.

To determine $Y_{i}^{\prime}$, the estimator forms an estimate of the received sample $r_{i}$, such that

$$
\begin{align*}
r_{i}^{\prime} & =\sum_{h=0}^{g} s_{i-h} y_{i-1, h}^{\prime} \\
& =Y_{i-1}^{\prime} S_{i}^{T}
\end{align*}
$$

The vector $S_{i}$ here (Eqn. 4.2.4) is determined from the values of the corresponding $\left\{s_{i-h}^{\prime \prime}\right\}$, assuming that Eqn. 3.5.3 holds. The estimator next forms the error signal

$$
e_{i}=r_{i}-r_{i}^{\prime}
$$

The estimation problem is to determine $Y_{i}^{\prime}$ in such a way that $\mathrm{e}_{\mathrm{i}}$ given by Eqn. 4.3.3 is made as small as possible.

From Eqns. 4.2.2 and 4.3.2

$$
e_{i}=\left(Y_{i}-Y_{i-1}^{\prime}\right) S_{i}^{T}+w_{i}
$$

The estimator uses $\mathrm{e}_{\mathrm{i}}$ to form a correction vector $\mathrm{X}_{\mathrm{i}}$, which is added to $Y_{i-1}^{\prime}$ to give the next estimate of the channel

$$
Y_{i}^{\prime}=Y_{i-1}^{\prime}+X_{i} \quad x_{1}=Y_{i}^{\prime}-Y_{i-1}^{\prime}
$$

where $X_{i}$ is a (g+1)- component row vector and is chosen such that

$$
Y_{i}^{\prime} S_{i}^{T}=r_{i}
$$

From Eqns. 4.3.2-4.3.6

$$
X_{i} S_{i}^{T}=e_{i}
$$

It is thus clear, from Eqns. 4.3.5 and 4.3.6, that both $Y_{i}^{\prime} S_{i}^{T}$ and $Y_{i-1}^{\prime} S_{i}^{T}$ are the estimates of $\mathrm{r}_{\mathrm{i}}$. $\mathrm{X}_{\mathrm{i}}$ is added to $Y_{i-1}^{\prime}$ in Eqn. 4.3.5, in such a way that $Y_{i}^{\prime}$ is more close to the actual channel impulse-response $Y_{i}$ and reduces to zero the error in the corresponding estimate of $\mathrm{r}_{\mathrm{i}}$.

Let $Z_{i}$ be any ( $\mathrm{g}+1$ )- component row vector such that

$$
Z_{i} S_{i}^{T}=0
$$

which means that the vectors $\mathrm{Z}_{\mathrm{i}}$ and $s_{i}^{*}$ are orthogonal, where $s_{i}^{*}$ is the complex conjugate of $S_{i}$. The vector $X_{i}$ can now be represented as

$$
X_{i}=a S_{i}^{*}+Z_{i}
$$

where a is the appropriate complex-valued scalar quantity. From Eqns. 4.3.8 and 4.3.9

$$
\begin{align*}
X_{i} S_{i}^{T} & =a S_{i}^{*} S_{i}^{T}+Z_{i} S_{i}^{T} \\
& =a\left|S_{i}\right|^{2}
\end{align*}
$$

where $\left|S_{i}\right|$ is the unitary length of $S_{i}$, and from Eqn. 4.3.7

$$
a=e_{i}\left|S_{i}\right|^{-2}
$$

Therefore, in the absence of noise, $\mathrm{e}_{\mathrm{i}}$ represents the deviation from the ideal or the error in the value of the orthogonal projection of $Y_{i-1}^{\prime}$ on the one- dimensional subspace spanned by $s_{i}^{*}$.

In the gradient or steepest-descent algorithm for estimating $\mathrm{Y}_{\mathrm{i}}$, the vector $\mathrm{X}_{\mathrm{i}}$, that is added to $Y_{i-1}^{\prime}$ to give the estimate $Y_{i}^{\prime}$ satisfying (Eqn. 4.3.6) is in the direction for which $\left|X_{i}\right|$ is minimized. All vectors here lie in a $(\mathrm{g}+1)$ - dimensional unitary vector space [46].

From Eqns. 4.3.8 and 4.3.9

$$
\left|X_{i}\right|^{2}=\left|a S_{i}^{*}\right|^{2}+\left|Z_{i}\right|^{2}
$$

and the component $\mathrm{a} S_{i}^{*}$ of $X_{i}$ is fixed by Eqns. 4.3.7 and 4.3.10. Thus $\left|X_{i}\right|$ is minimum when $Z_{i}=0$. Now

$$
X_{i}=a S_{i}^{*}=e_{i} \mid S_{i} \vdash^{-2} S_{i}^{*}
$$

so that, for Eqn. 4.3.6 to be satisfied,

$$
Y_{i}^{\prime}=Y_{i-1}^{\prime}+e_{i} \mid S_{i} \vdash^{-2} S_{i}^{*}
$$

To reduce the effect of noise, the magnitude of the change $\mathrm{X}_{\mathrm{i}}$ in Eqn. 4.3.5 is scaled, without changing its direction. Now

$$
Y_{i}^{\prime}=Y_{i-1}^{\prime}+b e_{i} S_{i}^{*}
$$

where b is an appropriate small positive real-valued constant, such that $b \ll \mid s_{i} t^{2}$. Eqn. 4.3.6 is no longer satisfied. Eqn. 4.3.15 is the conventional gradient algorithm, which for convenience is now referred to as system 4.1. This estimator assumes that the sampled impulse-response of the channel varies only very slowly with time. The algorithm for system 4.1 is, in fact, a recursive solution to the least squares estimation problem, also termed as the least mean-square (LMS) error algorithm [35,121, 58-59]. The estimator starts with an initial estimate $Y_{0}^{\prime}$ and measures the gradient of the mean square error function that is to be minimized, and updates the estimate according to the gradient. The error in the estimate is successively reduced and the estimate converges to the optimum value of the sampled impulse-response of the channel.

### 4.4 SYSTEM 4.2

This estimator, called the system 4.2, is a simple modification to system 4.1. It has been shown [57] that a linear feedforward estimator has a good overall performance, to track a time invariant or a slowly time-varying channel. However, the characteristics of a HF channel vary rapidly and in order to track such a channel it is necessary to adopt sophisticated techniques. It has been shown that an useful improvement in the performance of system 4.1 can be achieved if a predictor is also incorporated with the system [54]. Eqn. 4.3.15 now gets modified to

$$
Y_{i}^{\prime}=Y_{i, i-1}^{\prime}+b e_{i} S_{i}^{*}
$$

where $Y_{i, i-1}^{\prime}$ is the prediction of $Y_{i}$ at time $\mathrm{t}=(\mathrm{i}-1) \mathrm{T}$.

With a receiver employing a maximum-likelihood detector (Fig. 4.4.1), there is an inherent delay of several sampling interval. Thus the detected data-symbol $\mathrm{s}_{\mathrm{i}}$, designated as $\left\{s_{i}^{\prime}\right\}$, is detected after the reception $\mathrm{r}_{\mathrm{i}+\mathrm{n}-1}$, where $(\mathrm{n}-1)$ is the delay in detection [62]. Thus a feedforward estimator makes an estimate $Y_{i}^{\prime}$ of $Y_{i}$ and is only available to the detector on the receipt of $r_{i+n}$. Therefore, there is a delay of $n$ sampling intervals in the estimation of $Y_{i}$. The use of $Y_{i+n}^{\prime}$ in place of $Y_{i}^{\prime}$ reduces the error in the detection of $\mathrm{s}_{\mathrm{i}+1}$. Not only is there a need for n -step prediction of $Y_{i++, i}^{\prime}$, a one-step prediction $Y_{i+1, i}^{\prime}$ of $Y_{i+1}$ is also required so that $Y_{i}^{\prime}$ can be replaced by $Y_{i+1, i}^{\prime}$ when forming the updated estimate $Y_{i+1}^{\prime}$ in the channel estimator.

However, with the detector arrangement as shown in Fig. 4.4.1, there is an important advantage gained in using the adaptive filter. It avoids the need for prediction over many sampling intervals such as must be used in the absence of the filter [50, 52]. Prediction over many sampling intervals can increase considerably the error in prediction [53-54]. The adaptive filter is used to make the sampled impulse-response of the channel and filter to be minimum phase [49]. Fig. 4.4.1 shows the most cost effective detection method, wherein the estimator is fed the early detected data, $\left\{s_{i}^{\prime \prime}\right\}$, and the estimator need to do only a one-step prediction of the sampled impulse-response of the channel [51].

Least-Squares fading memory prediction is used to make a one-step prediction of the channel sampled impulse-response. This is done by determining a set of ( $\mathrm{g}+1$ ) polynomial of degree-1, from the sequence of vectors $Y_{i}^{\prime}, Y_{i-1}^{\prime}, \ldots$. , each of which gives weighted least-squares fit to the components in the corresponding locations in the vectors $Y_{i}^{\prime}, Y_{i-1}^{\prime}, \ldots$. , and then using the values of the polynomial at time $t=(i+1) \mathrm{T}$. $\nless$ Extensive tests on the different versions of the prediction process have shown that a degree-1 polynomial gives the best overall performance [52, 54]. The chosen polynomial is such that it gives the best fit to the sequence of past observations and the exponentially weighted sum of the squares of the error function is minimized [53]. In [53], the technique is applied to the prediction of the value of a variable parameter, derived from past observations which are either inaccurate or are corrupted in noise, the observations being unaffected by the prediction process. The technique is now applied to make a prediction based on the past updated estimates of

the parameter and the prediction has an influence on the subsequent updated estimation. Extensive tests have shown that this technique has improved the overall performance of the estimator without any sign of instability [54].

The prediction process carried out by the estimator is now considered. The estimator uses the updated estimate of $Y_{i}$, given by $Y_{i}^{\prime}$ in Eqn. 4.2.5, and the one-step prediction of $Y_{i}$, given by $Y_{i, i-1}^{\prime}$ in Eqn. 3.5.1, to determine an estimate of the error in prediction, which is

$$
X_{i}=Y_{i}^{\prime}-Y_{i, i-1}^{\prime}
$$

the actual error in $Y_{i, i-1}^{\prime}$ being

$$
Y_{i}-Y_{i, i-1}^{\prime}
$$

The prediction of $Y_{i+1}$ is now determined by means of a polynomial filter [53] that operates as follows

$$
\begin{align*}
& Y_{i+1, i}^{\prime \prime}=Y_{i, i-1}^{\prime \prime}+(1-\theta)^{2} X_{i} \\
& Y_{i+1, i}^{\prime}=Y_{i, i-1}^{\prime}+Y_{i+1, i}^{\prime \prime}+\left(1-\theta^{2}\right) X_{i}
\end{align*}
$$

The vector $Y_{i+1, i}^{\prime}$ is the degree -1 least squares fading memory prediction of $Y_{i+1}$ [53-54], and the vector $Y_{i+1, i}^{\prime \prime}$ is a prediction of the rate of change with respect to i of $\mathrm{Y}_{\mathrm{i}+1}$. The symbol $\theta$ is a real-valued constant in the range 0 to 1 and is usually close to 1. $\theta$ in Eqns. 4.4.3 and 4.4.4 and b in Eqn. 4.4.1 are optimized in combination so that the error in the one-step prediction of the sampled impulse-response of the channel is minimized.

$$
-e^{2} x_{i}
$$

At the start of the process,

$$
Y_{1,0}^{\prime \prime}=0
$$

and

$$
Y_{1,0}^{\prime}=Y_{0}^{\prime}
$$

where $Y_{0}^{\prime}$ is determined from an appropriate training sequence transmitted ahead of the transmission of actual data [91]. The results of the computer-simulation tests are given at the end of the chapter.

### 4.5 SYSTEM 4.3

This estimator known as system 4.3 operates by moving from $Y_{i, i-1}^{\prime}$ to $Y_{i}^{\prime}$ in a direction closer to that given by the correct direction $Y_{i}-Y_{i, i-1}^{\prime}$ than that given by the gradient algorithm, which is be $s_{i}^{*}$. The process uses no prior knowledge of $Y_{i}$, and operates entirely from the $\left\{r_{i}\right\}$ and $\left\{s_{i}^{\prime \prime}\right\}$, just like system 4.1 and system 4.2.

Assume that

$$
Y_{i}=Y_{i, i-1}^{\prime}+V_{i}
$$

so that $\mathrm{V}_{\mathrm{i}}$ is the actual error in $Y_{i, i-1}^{\prime}$. In the gradient algorithm, the receiver does not attempt to estimate $\mathrm{V}_{\mathrm{i}}$ itself but instead determines $\mathrm{X}_{\mathrm{i}}$ from Eqn. 4.3.13. A better estimate of $\mathrm{V}_{\mathrm{i}}$ is determined as follows.

Suppose that

$$
V_{i-h} \simeq V_{i}
$$

for $\mathrm{h}=1,2, \ldots, \mathrm{~m}$, where m is not too large. This is the case when $Y_{i, i-1}^{\prime}$ is tracking $Y_{i}$ with an error $\mathrm{V}_{\mathrm{i}}$ that varies only slowly with i . Eqn. 4.5 .2 should usually hold when $m \leq 4$. Now, in the absence of noise, $X_{i}$ in Eqn. 4.3.13 is the orthogonal projection of $\mathrm{V}_{\mathrm{i}}$ on to the one-dimensional subspace spanned by $s_{i}^{*}$. A better estimate of $\mathrm{V}_{\mathrm{i}}$, than that given by $X_{i}$, should normally be given by the orthogonal projection of $V_{i}$ on to the subspace spanned by the $\mathrm{m}+1$ vectors $\left\{s_{i-h}^{*}\right\}$, for $\mathrm{h}=0,1, \ldots, \mathrm{~m}$. Indeed, the more of the $\left\{S_{i-n}^{*}\right\}$ that are linearly independent and hence the higher the dimensionality of the subspace, the better is likely to be the resulting estimate of $\mathrm{V}_{\mathrm{i}}$, at least at high signal/noise ratios.

Thus, to achieve a better estimate of $\mathrm{V}_{\mathrm{i}}$, the receiver uses the one-step prediction $Y_{i, i-1}^{\prime}$ that has previously been determined to evaluate the corresponding estimates $r_{i, 0}^{\prime}, r_{i, 1}^{\prime}, \ldots ., r_{i, m}^{\prime}$ of the received samples $r_{i}, r_{i-1}, \ldots ., r_{i-m}$, respectively, such that

$$
r_{i, h}^{\prime}=Y_{i, i-1}^{\prime} S_{i-h}^{T}
$$

for $h=0,1, \ldots, m$. It is assumed here that $Y_{h}$ itself does not vary significantly with $h$, for $i-m \leq h \leq i$. The receiver next determines the error $\mathrm{e}_{\mathrm{i}, \mathrm{h}}$ in each $r_{i, h}^{\prime}$, as given by

$$
e_{i, h}=r_{i-h}-r_{i, h}^{\prime}
$$

Assume that $\mathrm{m}+1$ vectors $\left\{s_{i-h}^{*}\right\}$, for $\mathrm{h}=0,1, \ldots, \mathrm{~m}$, are linearly independent, and let the $(g+1)$ - component row vector $P_{i}$ be the orthogonal projection of $V_{i}$ on to the $(\mathrm{m}+1)$ - dimensional subspace spanned by the $\left\{S_{i-k}^{*}\right\}$. Since $P_{i}$ lies in the given subspace, it must be a linear combination of the $m+1$ vectors $\left\{S_{i-n}^{*}\right\}$, such that

$$
P_{i}=L_{i} Q_{i}^{*}
$$

where $L_{i}$ is an ( $m+1$ )-component row vector and $Q_{i}$ is an ( $m+1$ ) $x(g+1)$ matrix whose $(h+1)^{t h}$ row is $\mathrm{S}_{\mathrm{i}-\mathrm{h}}$. The vector $\mathrm{V}_{\mathrm{i}}-\mathrm{P}_{\mathrm{i}}$ is now orthogonal to the given subspace and hence to each vector $s_{i-k}^{*}$. Thus

$$
\left(V_{i}-P_{i}\right) S_{i-h}^{T}=0
$$

for $h=0,1, \ldots, m$, and

$$
\left(V_{i}-P_{i}\right) Q_{i}^{T}=0
$$

From Eqns. 4.5.5 and 4.5.7,

$$
L_{i} Q_{i}^{*} Q_{i}^{T}=V_{i} Q_{i}^{T}
$$

where $Q_{i}^{*} Q_{i}^{T}$ is a $(\mathrm{m}+1) \mathrm{x}(\mathrm{m}+1)$ nonsingular matrix, so that

$$
L_{i}=V_{i} Q_{i}^{T}\left(Q_{i}^{*} Q_{i}^{T}\right)^{-1}
$$

and

$$
P_{i}=V_{i} Q_{i}^{T}\left(Q_{i}^{*} Q_{i}^{T}\right)^{-1} Q_{i}^{*}
$$

Now, from Eqns. 4.3.4 and 4.5.1 to 4.5.4,

$$
e_{i, h} \approx V_{i} S_{i-h}^{T}+w_{i-h}
$$

Clearly, $\mathrm{e}_{\mathrm{i}, \mathrm{h}}$ is an unbiased estimate of $v_{i} S_{i-h}^{T}$. Let

$$
E_{i}=\left[\begin{array}{llll}
e_{i, 0} & e_{i, 1} & \ldots & e_{i, m}
\end{array}\right]
$$

so that, from Eqn. 4.5.11, $\mathrm{E}_{\mathrm{i}}$ is an unbiased estimate of $V_{i} Q_{i}^{T}$. It follows that

$$
P_{i}^{\prime}=E_{i}\left(Q_{i}^{*} Q_{i}^{T}\right)^{-1} Q_{i}^{*}
$$

is an unbiased estimate of $P_{i}$ and can be evaluated from the knowledge of $E_{i}$ and $Q_{i}$.

The updated estimate of $Y_{i}$ is next evaluated as

$$
Y_{i}^{\prime}=Y_{i, i-1}^{\prime}+b P_{i}^{\prime}
$$

where b is a positive constant.
A small value of $b$ reduces the effects of additive noise. Finally, the prediction of $Y_{i+1}$ is determined using the degree-1 least square fading memory prediction, as explained in Section 4.4, using Eqns. 4.4.2 to 4.4.4, where $\mathrm{X}_{\mathrm{i}}=\mathrm{bP} P_{i}^{\prime}$. At high signal/noise ratios, the updated estimate of $Y_{i}$ given by Eqn. 4.5.14 is usually closer to $Y_{i}$ than that given by Eqn. 4.3.15. However, as m increases so there is a corresponding increase in the number of noise components $\left\{\mathrm{w}_{i-\mathrm{h}}\right\}$ introduced into $P_{i}^{\prime}$, with the result that at low signal/noise ratios, Eqn. 4.3.15 could well give a better estimate of $Y_{i}$ than Eqn. 4.5.14.

In principle, $Y_{i}^{\prime}$ in Eqn. 4.5.14 can be determined directly from Eqns. 4.5.12 and 4.5.13, provided that $Q_{i} Q_{i}^{T}$ is nonsingular. When $Q_{i} Q_{i}^{T}$ is singular, the last $j$ rows of both $Q_{i}^{*}$ and $\mathrm{Q}_{\mathrm{i}}$ are removed, without changing the remaining rows, to leave each matrix with $\mathrm{m}-\mathrm{j}+1$ rows. The integer j here has the smallest possible value such that the resultant $(\mathrm{m}-\mathrm{j}+1) \mathrm{x}(\mathrm{m}-\mathrm{j}+1)$ matrix $Q_{i} Q_{i}^{T}$ is nonsingular. The ( $\mathrm{m}+1$ )-component row vectors $E_{i}$ is similarly reduced to $m-j+1$ components, by discarding its last $j$
components, without changing its remaining components and the corresponding ( $\mathrm{m}-\mathrm{j}+1$ )-components row vector $P_{i}^{\prime}$ is then evaluated from Eqn. 4.5.13 using the reduced forms of $\mathrm{E}_{\mathrm{i}}, \mathrm{Q}_{\mathrm{i}}^{*}$ and $Q_{i}^{T}$. The vectors $Y_{i}^{\prime}$ and $Y_{i+1, i}^{\prime}$ are determined as before.

On the receipt of $\mathrm{r}_{\mathrm{i}+1}$, the receiver determines the error signal

$$
e_{i+1, h}=r_{i+1-h}-Y_{i+1, i}^{\prime} S_{i+1-h}^{T}
$$

for $\mathrm{h}=0,1, \ldots, \mathrm{~m}$, to give $\mathrm{E}_{\mathrm{i}+1}$, and it is now ready to determine $P_{i+1}^{\prime}, Y_{i+1}^{\prime}$, and $Y_{i+2, i+1}^{\prime}$

Much of the complexity of system 4.3 is involved with the inversion of the matrix $Q_{i}^{*} Q_{i}^{T}$ in Eqn. 4.5.13, together with the determination of the value of j when $Q_{i} Q_{i}^{T}$ is singular. Many different techniques, including various iterative processes, are available for the matrix inversion, and some of these are ideally suited to the given application [42, 63-66].

### 4.6 SYSTEM 4.4

The estimator to be discussed in this section is called the system 4.4. The receiver now operates as system 4.3, but it assumes always that $\mathrm{e}_{\mathrm{i} h}=0$ for $\mathrm{h}=1,2, \ldots, \mathrm{~m}$, regardless of whether or not that is, in fact, the case. Thus ( $\mathrm{m}+1$ )-component vector $\mathrm{E}_{\mathrm{i}}$ in Eqn. 4.5.12 is now taken to be

$$
E_{i}=\left[\begin{array}{llllll}
e_{i, 0} & 0 & 0 & \ldots & 0
\end{array}\right]
$$

where

$$
e_{i, 0}=e_{i}=r_{i}-r_{i}^{\prime}
$$

as before. This assumption is not perhaps as arbitrary as it may seem at first sight, since it is equivalent to applying the algorithm of system 4.3 such that this operates only to reduce the magnitude of $\mathrm{e}_{\mathrm{i}, 0}$, the magnitude of the $\left\{\mathrm{e}_{\mathrm{i}, \mathrm{h}}\right\}$, for $\mathrm{h}=1,2, \ldots, \mathrm{~m}$, having been reduced by the corresponding previous operations.

It follows from Eqn. 4.5.13 that

$$
P_{i}^{\prime}=e_{i, 0} F_{i} Q_{i}^{*}
$$

where the $(m+1)$-component row vector $F_{i}$ is given by the first row of the $(\mathrm{m}+1) \mathrm{x}(\mathrm{m}+1)$ nonsingular matrix $\left(Q_{i} Q_{i}^{T}\right)^{-1}$. Now the first row of $\left.\varphi_{i} Q_{i}^{T}\right)^{-1}$ must be orthogonal to each of the columns of $Q_{i}\left(Q_{i}^{*}\right)^{T}$ other than the first, since $\left.Q_{i} Q_{i}^{T}\right)^{-1} \varphi_{i} Q_{i}^{T}$ is an identity matrix. Let the $(\mathrm{h}+1)^{\text {ih }}$ column of $Q_{i}\left(Q_{i}\right)^{T}$ be $G_{i, h}^{T}$, so that $\left\{\mathrm{G}_{i, h}\right\}$, for $\mathrm{h}=0,1, \ldots$, m , are linearly independent $(\mathrm{m}+1)$-component row vectors. The Gram-Schmidt orthonormalization process, ( see Appendix E ), is now applied to the $\mathrm{m}+1$ vectors $\left\{\mathrm{G}_{\mathrm{i}, \mathrm{h}}\right\}$, in order, starting with $\mathrm{G}_{\mathrm{i}, \mathrm{m}}$ and ending with $\mathrm{G}_{\mathrm{i}, 0}$. The orthogonal vectors $\left\{\mathrm{H}_{\mathrm{i}, \mathrm{m}}\right\}$ obtained from this process are as follows

$$
\begin{align*}
& H_{i, m}=G_{i, m} \\
& H_{i, m-1}=G_{i, m-1}-\left|H_{i, m}\right|^{-2} G_{i, m-1}\left(H_{i, m}^{*}\right)^{T} H_{i, m}
\end{align*}
$$

and so on to

$$
\begin{align*}
H_{i, 0}=G_{i, 0} & -\mid H_{i, 1} \vdash^{2} G_{i, 0}\left(H_{i, 1}^{*}\right)^{T} H_{i, 1} \\
& -\ldots \ldots \\
& -\mid H_{i, m} \vdash^{2} G_{i, 0}\left(H_{i, m}^{*}\right)^{T} H_{i, m}
\end{align*}
$$

But $\mathrm{H}_{\mathrm{i}, 0}$ is orthogonal to $\mathrm{H}_{\mathrm{i}, \mathrm{m}}, \mathrm{H}_{\mathrm{i}, \mathrm{m}-1}, \ldots, \mathrm{H}_{\mathrm{i}, 1}$, so that it is also orthogonal to $\mathrm{G}_{\mathrm{i}, \mathrm{m}}, \mathrm{G}_{\mathrm{i}, \mathrm{m}-1}, \ldots$, $\mathrm{G}_{\mathrm{i}, 1}$. This means that $\mathrm{H}_{\mathrm{i}, 0}$ lies in the same one-dimensional subspace as $\mathrm{G}_{\mathrm{i}, 0}$, since $\left\{\mathrm{H}_{\mathrm{i}, \mathrm{h}}\right\}$ and $\left\{\mathrm{G}_{\mathrm{i}, \mathrm{h}}\right\}$, for $\mathrm{h}=1,2, \ldots, \mathrm{~m}$, span the same m-dimensional subspace of the $(m+1)$-dimensional vector space containing all $\left\{\mathrm{H}_{\mathrm{i}, \mathrm{h}}\right\}$ and $\left\{\mathrm{G}_{\mathrm{i}, \mathrm{h}}\right\}$. Consequently $\mathrm{F}_{\mathrm{i}}$ (in Eqn. 4.6.2) must be such that

$$
F_{i}=f_{i} H_{i, 0}
$$

where $f_{i}$ is an appropriate scalar, and

$$
F_{i}\left(G_{i, 0}^{*}\right)^{T}=1
$$

since $\left(Q_{i}^{*} Q_{i}^{T}\right)^{-1}\left(Q_{i}^{*} Q_{i}^{\tau}\right)$ is an identity matrix. Thus

$$
f_{i}=\left(H_{i, 0}\left(G_{i, 0}^{*}\right)^{T}\right)^{-1}
$$

and the $(m+1)$-component vector $\mathrm{F}_{\mathrm{i}}$ is now given by (4.6.5), (4.6.6) and (4.6.8).

The ( $\mathrm{g}+1$ )-component vector $P_{i}^{\prime}$ is finally determined from Eqn. 4.6.2, to give $Y_{i}^{\prime}$ from Eqn. 4.5.14 and $Y_{i+1, i}^{\prime}$ obtained by using the one-step least square fading memory prediction, using Eqns. 4.4.2 to 4.4.4, where $X_{i}=b P_{i}^{\prime}$. On the receipt of $r_{i+1}, e_{i+1,0}$ is determined from Eqn. 4.5.15, where $\mathrm{h}=0$, and the whole procedure just described is repeated.

If $Q_{i}^{*} Q_{i}^{T}$ is singular, $\mathrm{H}_{\mathrm{i}, \mathrm{h}}=0$ for some integer h , and the Gram-Schmidt process is terminated. When this occurs the last row of $Q_{i}^{*}$ and the last row of $Q_{i}$ are discarded, so that $Q_{i}^{*}$ and $Q_{i}$ have $m$ rows and $Q_{i}^{*} Q_{i}^{T}$ is an $m \times m$ matrix. The Gram-Schmidt orthonormalization process is then applied to the columns of the reduced matrix $Q_{i}\left(Q_{i}^{*}\right)^{T}$, starting with the $\mathrm{m}^{\mathrm{h}}$ column. If again $\mathrm{H}_{\mathrm{i}, \mathrm{h}}=0$ for some integer h , the process is terminated, and each of the matrices $Q_{i}^{*}$ and $\mathrm{Q}_{\mathrm{i}}$ is reduced by omitting its last row, to give (m-1) $x(m-1)$ matrix $Q_{i} Q_{i}^{T}$. The procedure is continued as described until an $(\mathrm{m}-\mathrm{j}+1) \times(\mathrm{m}-\mathrm{j}+1)$ nonsingular matrix $Q_{i}^{*} Q_{i}^{T}$ is obtained. The last j rows of both $Q_{i}^{\cdot}$ and $Q_{i}$ have now been removed, without changing the remaining rows, to leave each matrix with $\mathrm{m}-\mathrm{j}+1$ rows. Finally, the Gram-Schmidt orthonormalization process is applied to the columns of the $(m-j+1) \times(m-j+1)$ matrix $Q_{i}\left(Q_{i}\right)^{T}$, starting with the $(m-j+1)^{\text {th }}$ column, and the $(m-j+1)$-component row vector $F_{i}$ is derived from Eqns. 4.6.5 to 4.6 .8 to give $P_{i}^{\prime}$ in Eqn. 4.6.2. The process then continues as before.

### 4.7 RESULTS AND ANALYSIS OF COMPUTER-SIMULATION TESTS.

Computer-simulation tests have been carried out on the channel estimators described in Section 4.3-4.6. The results of the tests are compiled in Tables 4.7.1-4.7.4 and in Figs. 4.7.1-4.7.2. The error measurement is

$$
\xi_{1}=10 \log _{10}\left(\frac{1}{54000} \sum_{i=6001}^{60000}\left|Y_{i}-Y_{i, i-1}^{\prime}\right|^{2}\right)
$$

and

$$
\xi_{2}=10 \log _{10}\left(\frac{1}{54000} \sum_{i=6001}^{60000} \frac{\left|Y_{i}-Y_{i, i-1}^{\prime}\right|^{2}}{\left|Y_{i}\right|^{2}}\right)
$$

The parameter $\xi_{1}$ is called the mean square estimation error and is a measure of the actual error in $Y_{i, i-1}^{\prime}$. The parameter $\xi_{2}$ is called the mean square normalized estimation error and is a measure of the normalized or relative error in $Y_{i, i-1}^{\prime}$.

During the first 6000 received samples the estimation process operates as described in Sections 4.3-4.6, but no measurements are carried out. This stabilizes the fading, additive noise and the estimation process, thus eliminating any transient behaviour of the estimator at start up. Measurements are thus carried out on the estimators during their stable operation, over the next 54000 received samples. The results, in Tables 4.7.1-4.7.4 are, therefore, the steady state performances of the estimators under test.

The signal/noise ratio, is measured as $\psi$, where

$$
\psi=10 \quad \log _{10}\left(\frac{E_{b}}{\frac{1}{2} N_{0}}\right)
$$

where $E_{b}$, the average transmitted energy per bit at the input and output of the HF radio link, is unity and the two sided power spectral density of the white Gaussian noise at the output of the HF radio link is $(1 / 2) N_{0}$.

In all tests, ( $\mathrm{g}+1$ ), the total number of components in the sampled impulse-response is taken to be 32. At the start of the estimation process, $Y_{1,0}^{\prime}=Y_{0}$, the first actual channel sampled impulse-response. In each of the Tables 4.7.1 to 4.7.4, the scalar constants, such as, b in Tables 4.7.1 and b and $\theta$ in Tables 4.7.2 to 4.7.4, have been optimized as accurately as possible so that the error in the estimation/prediction of the sampled impulse-response of the channel, defined by Eqn. 4.7.1, is minimized.

Eqns. 4.7.1 and 4.7.2 measures the unitary distance between the vectors $Y_{i}$ and $Y_{i, i-1}^{\prime}$ in dBs . In Eqn. 4.7.2, this unitary distance has been normalized with the length of the vector $Y_{i}$. In Fig. 4.7.1, systems 4.1 and 4.2 have been compared using measures $\xi_{1}$ and $\xi_{2}$. A comparison of the systems 4.1 and 4.2, in Fig. 4.7.1 and Tables 4.7.1 and 4.7.2, show that the relative performances of the two systems are not significantly affected by whether $\xi_{1}$ or $\xi_{2}$ is used as a measurement criteria.

A degree-one predictor with system 4.1 has significantly improved the performance of system 4.2. This is evident from Tables 4.7.1 and 4.7.2. The results of the simulation tests on systems 4.1 and 4.2 with statistically independent noise component $\left\{w_{i}\right\}$ in Eqn. 4.2.1, in place of the slightly correlated noise components actually obtained at the output of the receiver filter, show only a negligibly small differences in performance (Tables 4.7.1 and 4.7.2). Thus correlation in the noise components does not appear to have any significant effect.

Table 4.7.3 shows the mean square error in the estimates of channel sampled impulse-response given by system 4.3 and the results from system 4.4 are compiled in Table 4.7.4. Simulation tests were carried out on systems 4.3 and 4.4 for a maximum value of $\mathrm{m}=4$. From the results of the computer-simulation tests, it is evident that not much advantage is gained in the use of sophisticated estimation processes of systems 4.3 and 4.4 , at least at low signal/noise ratio. However, the systems show improved performance at high signal/noise ratios. At low signal/noise ratios, increasing m has only marginal improvement in the performance of systems 4.3 and 4.4. The performance of system 4.3 is more or less the same as system 4.4. System 4.3 is the most complex of the four systems considered in this chapter as it involves inversion of a $(\mathrm{m}+1) \mathrm{x}(\mathrm{m}+1)$ matrix. No case was observed, during simulation test on systems 4.3 and 4.4 , when the matrix $Q_{i}^{*} Q_{i}^{T}$ was singular.

Fig. 4.7.2 shows the steady state performance of systems 4.1 and 4.2 at 30 dB signal/noise ratio. The parameter in Fig. 4.7.2, is here the square of the error in $Y_{i, i-1}^{\prime}$ measured in dB , relative unity, and is

$$
\xi_{i}=10 \log _{10}\left(\left|Y_{i}-Y_{i, i-1}^{\prime}\right|^{2}\right)
$$

The relatively better performance of system 4.2 over systems 4.3 and 4.4 , when $\psi=$ 20 or 30 dB , confirms that this is basically a good estimation process for a time varying channel of the type tested. A degree-one predictor with system 4.1 (system 4.2) provides a useful overall improvement in the performance at all signal/noise ratios. System 4.2 is considerably less complex than systems 4.3 and 4.4, it is by far the most cost effective of the four system.

TABLE 4.7.1 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATE OF THE CHANNEL SAMPLED IMPULSE-RESPONSE GIVEN BY SYSTEM 4.1 FOR A 3 SKY WAVE CHANNEL.

| $\psi$ <br> $(d B)$ | b | Correlated noise | Uncorrelated noise |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\xi_{2}$ |  |  |
| $(d B)$ | $(d B)$ | $\xi_{1}$ | $\xi_{2}$ |  |  |
| $(d B)$ | $(d B)$ |  |  |  |  |
| 10 | 0.009 | -11.848 | -10.590 | -11.814 | -10.581 |
| 20 | 0.010 | -15.967 | -14.895 | -15.919 | -14.889 |
| 30 | 0.020 | -19.891 | -18.816 | -19.853 | -18.839 |
| 40 | 0.021 | -20.649 | -19.644 | -20.598 | -19.671 |
| 60 | 0.020 | -20.718 | -19.719 | -20.664 | -19.750 |

TABLE 4.7.2 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATE OF THE CHANNEL SAMPLED IMPULSE-RESPONSE GIVEN BY SYSTEM 4.2 FOR A 3 SKY WAVE CHANNEL.

| $\psi$ | $\mathbf{d B}$ |  |  | Correlated noise |  | Uncorrelated noise |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ |  |
| 10 | 0.139 | 0.980 | -11.925 | -10.640 | -12.013 | -10.709 |  |
| 20 | 0.113 | 0.966 | -18.942 | -17.660 | -18.918 | -17.572 |  |
| 30 | 0.091 | 0.949 | -25.052 | -23.825 | -25.072 | -23.797 |  |
| 40 | 0.070 | 0.930 | -29.012 | -27.944 | -29.016 | -27.955 |  |
| 60 | 0.087 | 0.933 | -30.990 | -30.116 | -30.952 | -30.112 |  |

TABLE 4.7.3 MEAN SQUARE ERROR IN THE ESTIMATE OF THE CHANNEL SAMPLED IMPULSE-RESPONSE GIVEN BY SYSTEM 4.3 FOR A 3 SKY WAVE HF CHANNEL.

| $\begin{gathered} \psi \\ (\mathrm{dB}) \end{gathered}$ | M | $\theta$ | b |  |
| :---: | :---: | :---: | :---: | :---: |
| 20 | 1 | 0.970 | 4.76 | -19.068 |
|  | 2 | 0.970 | 3.34 | -19.111 |
|  | 3 | 0.970 | 2.56 | -19.140 |
|  | 4 | 0.970 | 2.07 | -19.163 |
| 30 | 1 | 0.960 | 5.36 | -25.339 |
|  | 2 | 0.960 | 3.88 | -25.461 |
|  | 3 | 0.960 | 3.06 | -25.554 |
|  | 4 | 0.960 | 2.52 | -25.630 |
| 60 | 1 | 0.940 | 4.60 | -31.567 |
|  | 2 | 0.930 | 2.65 | -32.008 |
|  | 3 | 0.930 | 2.24 | -32.450 |
|  | 4 | 0.930 | 1.98 | -32.785 |

TABLE 4.7.4 MEAN SQUARE ERROR IN THE ESTIMATE OF THE CHANNEL SAMPLED IMPULSE-RESPONSE GIVEN BY SYSTEM 4.4 FOR A 3 SKY WAVE HF CHANNEL.

| $\begin{gathered} \psi \\ (\mathrm{dB}) \end{gathered}$ | M | $\theta$ | b | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 20 | 1 | 0.970 | 8.11 | -19.010 |
|  | 2 | 0.970 | 7.97 | -19.025 |
|  | 3 | 0.970 | 7.86 | -19.026 |
|  | 4 | 0.970 | 7.75 | -19.028 |
| 30 | 1 | 0.960 | 8.35 | -25.290 |
|  | 2 | 0.960 | 8.25 | -25.405 |
|  | 3 | 0.960 | 8.15 | -25.490 |
|  | 4 | 0.960 | 8.06 | -25.588 |
| 60 | 1 | 0.940 | 6.74 | -31.557 |
|  | 2 | 0.940 | 6.88 | -32.081 |
|  | 3 | 0.930 | 5.45 | -32.644 |
|  | 4 | 0.930 | 5.62 | -33.231 |



Fig. 4.7.1 - Performance of Systems $4.1 \& 4.2$

## CHAPTER 5

## RLS KALMAN ESTIMATOR.

### 5.1 INTRODUCTION

The Kalman filter has become one of the most investigated estimation algorithm in many areas and in particular for the HF channel estimation [35, 53, 70, 73-79, 81-88, 92-99, 101-110], following the first publication of the theory of the Kalman filter [73]. A Kalman estimator gives the least-squares estimate of the sampled impul-se-response of a time invariant channel that introduces additive Gaussian noise [53, 77, 95-96] and that it has the most rapid rate of convergence, when the estimation process is started with a completely unknown channel estimate. This has motivated the study of Kalman filter algorithm for use in HF channel estimation. The algorithm as a HF channel estimator has been extensively studied elsewhere [35, 88]. This chapter considers a particular form of the algorithm called the Recursive least-squares (RLS) algorithm for the application to the HF channel estimation. The method is very closely related to the Kalman algorithm, by virtue of its similarity to the state-space stochastic filter approach of the Kalman algorithm. The algorithm is referred to as the RLS Kalman algorithm or, more simply, as the Kalman algorithm [59, 99, 101, 103].

The increased rate of convergence of the RLS Kalman algorithm, as compared to the LMS algorithm, is at the expense of increased computational complexity. This has led to the development of computationally efficient Kalman algorithms, called the Fast Kalman algorithms. A class of Fast Kalman algorithm, called the Fast Transversal Filter (FTF) algorithm, for HF channel estimation, is considered in detail in Chapter 8. In this chapter three types of Kalman algorithms, referred to as systems 5.1-5.3, are considered. Systems 5.1 and 5.2 assume that the channel varies linearly with time, or in other words the rate of change in the channel is constant, whereas system 5.3 assumes that the channel is time invariant or varies very slowly with time, so that the rate of change in the channel can be neglected.

### 5.2 MODEL OF DATA TRANSMISSION SYSTEM USED IN THE TESTS

Fig. 3.5.1 shows the model of the data transmission system used in the tests and has been considered in detail in Chapter 3. The HF radio link and the model of the data transmission system are given in detail in Chapters 2 and 3, respectively. The linear baseband channel has a sampled impulse-response, given by the ( $\mathrm{g}+1$ )- component row vector, $\mathrm{Y}_{\mathrm{i}}$, where, (Eqn. 3.4.4)

$$
Y_{i}=\left[\begin{array}{lllll}
{\left[\begin{array}{llll}
i, 0 & y_{i, 1} & \ldots & y_{i, g}
\end{array}\right]}
\end{array}\right.
$$

where $y_{i \mathrm{~h}}=0$ for $\mathrm{h}<0$ and $\mathrm{h}>\mathrm{g}$.

The received sample at time $\mathrm{t}=\mathrm{iT}$, is given by (Eqn. 3.4.2)

$$
\begin{align*}
r_{i} & =\sum_{h=0}^{g} s_{i-h} y_{i, h}+w_{i} \\
& =Y_{i} S_{i}^{T}+w_{i}
\end{align*}
$$

where $S_{i}$, the input data vector, is a ( $\mathrm{g}+1$ )- component row vector, given by (Eqn. 3.4.5)

$$
S_{i}=\left[\begin{array}{lllll}
s_{i} & s_{i-1} & s_{i-2} & \ldots . & s_{i-g}
\end{array}\right]
$$

$S_{i}^{T}$ is the transpose of $\mathrm{S}_{\mathrm{i}}$. The scalar quantity $\mathrm{w}_{\mathrm{i}}$ in Eqns. 5.2.2 and 5.2.3 is a noise component originating from the white Gaussian noise. The signals $r_{i}$ and $s_{i}$ are fed to the channel estimator to give an estimate of the channel sampled impulse-response, $Y_{i}^{\prime}$ , at time $\mathrm{t}=\mathrm{iT}$, where

$$
Y_{i}^{\prime}=\left[\begin{array}{lllll}
y_{i, 0}^{\prime} & y_{i, 1}^{\prime} & \ldots . & y_{i, g}^{\prime}
\end{array}\right]
$$

$Y_{i}^{\prime}$ is fed to the detector, ready to detect $\mathrm{s}_{\mathrm{i}+1}$, when $\mathrm{r}_{\mathrm{i}+1}$ is received by the detector.

### 5.3 SYSTEM 5.1

The first of the RLS Kalman estimators considered here is called the system 5.1. It assumes that the channel is varying linearly with time, such that

$$
Y_{i+1}-Y_{i}=Y_{i}-Y_{i-1}
$$

which means that $Y_{i+1}-Y_{i}$ is a constant vector that is independent of $i$. It is assumed that the receiver has prior knowledge of Eqn. 5.3.1 but has no knowledge of the vector $Y_{i+1}-Y_{i}$.

System 5.1 operates with a channel-estimation vector, for time $\mathrm{t}=\mathrm{i} \mathrm{T}$, which is

$$
V_{i}=\left[\begin{array}{lllllll} 
& y_{i, 0}^{\prime} & y_{i, 1}^{\prime} & \ldots . & y_{i, g}^{\prime} & y_{i, 0}^{\prime \prime} & \ldots . \\
y_{i, g}^{\prime \prime}
\end{array}\right]
$$

where $y_{i, h}^{\prime}$ is an estimate of $y_{i, h}$, (Eqn. 5.2.1), for $h=0,1, \ldots, g$, and $y_{i, h}^{\prime \prime}$ is an estimate of the rate of change of $y_{i, b}$ with $i$. Thus $V_{i}$ is a $(2 g+2)$ - component row vector. The data vector, for time $\mathrm{t}=\mathrm{iT}$, is the $(2 \mathrm{~g}+2)$ - component row vector

$$
S_{i}=\left[\begin{array}{lllllllll}
s_{i} & s_{i-1} & \ldots . . & s_{i-g} & 0 & 0 & \ldots & 0
\end{array}\right]
$$

The estimate of $r_{i}$ formed by the channel estimator, is now

$$
r_{i}^{\prime}=V_{i} S_{i}^{T}
$$

The quantity $r_{i}^{\prime}$ is the updated estimate of $r_{i}$. Similarly $V_{i}$ is an updated estimate of the corresponding channel vector.

The error in $r_{i}^{\prime}$ is

$$
e_{i}=r_{i}-r_{i}^{\prime}
$$

The vector $\left\{\mathrm{V}_{\mathrm{i}}\right\}$ determined by system 5.1 is such as to minimize the weighted least-squares cost function

$$
C_{i}=\sum_{h=0}^{i} \omega^{i-h}\left|e_{h}\right|^{2}
$$

where $\omega$ is a real-valued constant in the range 0 to 1 . The quantity $\mathrm{C}_{\mathrm{i}}$ is the weighted squared error in the $\left\{r_{h}^{\prime}\right\}$. On receipt of $r_{i}^{\prime}$, the algorithm of system 5.1 repeats a sequence of operations to update the channel-estimation vector in such a manner as to minimize $\mathrm{C}_{\mathrm{i}}$. Hence the algorithm is recursive and is least squares as well. The quantity that is minimized by the gradient algorithm is the expected value of the squared error, whereas here it is the weighted squared error that is minimized.

Now, consider the $(2 g+2) \times(2 g+2)$ matrix

$$
\begin{align*}
& \phi_{h}=\left[\begin{array}{cccccccccccccc}
1 & 0 & 0 & \ldots & . & 0 & 0 & 0 & 0 & \ldots & . & 0 & 0 & 0 \\
0 & 1 & 0 & \ldots & . & 0 & 0 & 0 & 0 & \ldots & . & 0 & 0 & 0 \\
0 & 0 & 1 & \ldots & . & 0 & 0 & 0 & 0 & \ldots & . & 0 & 0 & 0 \\
. & . & . & & & & & & & & & & & \\
. & . & . & & & & & & & & & & & \\
0 & 0 & 0 & \ldots & . & 0 & 1 & 0 & 0 & \ldots & . & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & . & 0 & 0 & 1 & 0 & \ldots & . & 0 & 0 & 0 \\
h & 0 & 0 & \ldots & . & 0 & 0 & 0 & 1 & \ldots & . & 0 & 0 & 0 \\
0 & h & 0 & \ldots & . & 0 & 0 & 0 & 0 & \ldots & . & 0 & 0 & 0 \\
0 & 0 & h & \ldots & . & 0 & 0 & 0 & 0 & \ldots & . & 0 & 0 & 0 \\
. & . & . & & & & & & & & & & & \\
. & . & . & & & & & & & & & & & \\
0 & 0 & 0 & \ldots & . & h & 0 & 0 & 0 & \ldots & . & 1 & 0 & 0 \\
0 & 0 & 0 & \ldots & . & 0 & h & 0 & 0 & \ldots & . & 0 & 1 & 0 \\
0 & 0 & 0 & \ldots & . & 0 & 0 & h & 0 & \ldots & . & 0 & 0 & 1
\end{array}\right] \\
& \phi_{h}=\left[\begin{array}{cc}
I & O \\
h I
\end{array}\right]
\end{align*}
$$

where I is a $(\mathrm{g}+1) \mathrm{x}(\mathrm{g}+1)$ identity matrix, O is a $(\mathrm{g}+1) \mathrm{x}(\mathrm{g}+1)$ zero matrix, and h is any positive or negative integer or zero. From Eqns. 5.3.1 and 5.3.7, it is evident that

$$
V_{i+h}=V_{i} \phi_{h}
$$

is the best channel estimation vector that can be determined for time $t=(i+h) T$, given the channel estimation vector $V_{i}$, for time $t=i T$. This is because it makes full use of all the available prior knowledge of the channel. If $\mathrm{V}_{\mathrm{i}}$ is taken to be the actual channel vector instead of an estimate of this vector, Eqn. 5.3.8 holds exactly for all positive and negative values of integer $\{\mathrm{h}\}$. The matrix $\phi_{h}$ is known as a transition
matrix, and it can be used to convert the channel vector or estimate of this vector, at time iT, to the channel vector or estimate of this vector, at time ( $\mathrm{i}+\mathrm{h}$ )T. Except where otherwise stated, $\mathrm{V}_{\mathrm{i}}$ is taken to be the estimate of the channel vector at time $\mathrm{t}=\mathrm{i} \mathrm{T}$, as in Eqn. 5.3.2. Finally, from Eqn. 5.3.7

$$
\phi_{h}=\phi^{h}
$$

and $\phi_{0}$ is the $(2 g+2) x(2 g+2)$ identity matrix.

From Eqns. 5.3.4-5.3.6, the least-squares cost function at time $t=i \mathrm{~T}$ becomes

$$
C_{i}=\sum_{h=0}^{i} \omega^{i-h}\left|r_{h}-V_{h} S_{h}^{T}\right|^{2}
$$

At time $t=\mathrm{i} \mathrm{T}$, the channel estimation vector is $\mathrm{V}_{\mathrm{i}}$ and from Eqn. 5.3.8, $\mathrm{V}_{\mathrm{i}}$, is related to $V_{h}$, as

$$
V_{h}=V_{i} \phi_{h-i}
$$

Combining Eqns. 5.3.10 and 5.3.11, $\mathrm{C}_{\mathrm{i}}$ is given by

$$
C_{i}=\sum_{h=0}^{i} \omega^{i-h}\left|r_{h}-V_{i} \phi_{h-i} S_{h}^{T}\right|^{2}
$$

The estimator of system 5.1, determines the channel estimation vector $V_{i}$ at time $\mathrm{t}=\mathrm{i} \mathrm{T}$, which together with the given transition-matrix $\phi_{h}$, minimizes $\mathrm{C}_{\mathrm{i}}$.

Now

$$
C_{i}=\sum_{h=0}^{i} \omega^{i-h}\left(r_{h}-V_{i} \phi_{h-i} S_{h}^{T}\right)\left(r_{h}-V_{i} \phi_{h-i} S_{h}^{T}\right)^{*}
$$

or

$$
\begin{align*}
C_{i}=\sum_{h=0}^{i} \omega^{i-h}\left(r_{h} r_{h}^{*}-r_{h} \bar{S}_{h} \phi_{h-i}^{T} V_{i}^{*}\right. & -r_{h}^{*} V_{i} \phi_{h-i} S_{h}^{T} \\
& \left.+V_{i} \phi_{h-i} S_{h}^{T} S_{h} \phi_{h-i}^{T} V_{i}^{*}\right)
\end{align*}
$$

where $r_{k}^{\bullet}$ is the complex conjugate of $\mathrm{r}_{\mathrm{h}}$ and so is $\bar{r}_{h}$. Vector $\bar{S}_{h}$ is the complex conjugate of the vector $\mathrm{S}_{\mathrm{h}}$, and the vector $v_{i}^{*}$ is the conjugate transpose of the vector $\mathrm{V}_{\mathrm{i}}$, that is $\left.\bar{v}_{1}\right)^{\text {r }}$.

The parameter $\mathrm{C}_{\mathrm{i}}$ in Eqn. 5.3.13 is real and positive and with all the parameters (except $\mathrm{V}_{\mathrm{i}}$ ) remaining constant, $\mathrm{C}_{\mathrm{i}}$ is a convex function of the channel estimation vector $\mathrm{V}_{\mathrm{i}}[59,103]$. The quantity $\mathrm{C}_{\mathrm{i}}$ and the elements of the vector $\mathrm{V}_{\mathrm{i}}$ can then be seen to have a bowl-shaped surface with a unique minimum. At the bottom or minimum point the gradient of $C_{i}$ with respect to $V_{i}$ is zero. The gradient of $C_{i}$ with respect to $V_{i}$ is

$$
\nabla C_{i}=\left[\begin{array}{lllll}
\frac{\partial C_{i}}{\partial y_{i, 0}^{\prime}} \frac{\partial C_{i}}{\partial y_{i, 1}^{\prime}} & . . & \frac{\partial C_{i}}{\partial y_{i, g}^{\prime}} \frac{\partial C_{i}}{\partial y_{i, 0}^{\prime \prime}} \frac{\partial C_{i}}{\partial y_{i, 1}^{\prime \prime}} & . & \frac{\partial C_{i}}{\partial y_{i, g}^{\prime \prime}}
\end{array}\right]
$$

For $\nabla C_{i}$ to be zero, each component of the vector must also be zero. Differentiating Eqn. 5.3.14 with respect to $V_{i}$, the gradient of $C_{i}$ with respect to $V_{i}$ is (Appendix C) [59, 103]

$$
\sum_{h=0}^{i} \omega^{i-h}\left(-2 r_{h} \bar{S}_{h} \phi_{h-i}^{T}+2 V_{i} \phi_{h-i} S_{h}^{T} \bar{S}_{h} \phi_{h-i}^{T}\right)
$$

Under the condition when $\nabla C_{i}$ is zero,

$$
\sum_{h=0}^{i} \omega^{i-h} 2 V_{i} \phi_{h-i} S_{h}^{T} \bar{S}_{h} \phi_{h-i}^{T}=\sum_{h=0}^{i} \omega^{i-h} 2 r_{h} \bar{S}_{h} \phi_{h-i}^{T}
$$

or

$$
V_{i} \sum_{h=0}^{i} \omega^{i-h} \phi_{h-i} S_{h}^{T} \bar{S}_{h} \phi_{h-i}^{T}=\sum_{h=0}^{i} \omega^{i-h} r_{h} \bar{S}_{h} \phi_{h-i}^{T}
$$

Let

$$
R_{i}=\sum_{h=0}^{i} \omega^{i-h} \phi_{h-i} S_{h}^{T} S_{h} \phi_{h-i}^{T}
$$

and

$$
Q_{i}=\sum_{h=0}^{i} \omega^{i-h} r_{h} \bar{S}_{h} \phi_{h-i}^{T}
$$

where $R_{i}$ is a $(2 g+2) \times(2 g+2)$ component square matrix and $Q_{i}$ is a $(2 g+2)$ component row vector. Combining, Eqns. 5.3.17-5.3.19,

$$
V_{i} R_{i}=Q_{i}
$$

Therefore,

$$
V_{i}=Q_{i} R_{i}^{-1}
$$

$\downarrow$ It is assumed in Eqn. 5.3.21 that the matrix $\mathrm{R}_{\mathrm{i}}$ is nonsingular and there exists an inverse of the matrix $R_{i}$. Eqn. 5.3.21 gives the weighted least-squares estimate of the desired channel-estimation vector at time $t=i \mathrm{~T}$. To determine $\mathrm{V}_{\mathrm{i}}$ from Eqn. 5.3.21, would mean enormous computational complexity. It is, therefore, necessary to modify Eqn. 5.3.20 in such a way that $V_{i}, R_{i}$ and $Q_{i}$, at time $t=i T$, can be obtained recursively from $V_{i-1}, R_{i-1}$ and $Q_{i-1}$, at time $t=(i-1) T$.

From Eqn. 5.3.18

$$
R_{i}=\omega \phi_{-1} R_{i-1} \phi_{-1}^{T}+S_{i}^{T} \bar{S}_{i}
$$

and, from Eqn. 5.3.19

$$
Q_{i}=\omega Q_{i-1} \phi_{-1}^{T}+r_{i} \bar{S}_{i}
$$

From Eqns. 5.3.20 and 5.3.23

$$
V_{i} R_{i}=\omega V_{i-1} R_{i-1} \phi_{-1}^{T}+r_{i} \bar{S}_{i}
$$

Eqn. 5.3.24 gives a relationship between $V_{i}$ and $V_{i-1}$ which form the basis of the required recursive algorithm to determine $V_{i}$.

From Eqn. 5.3.22,

$$
\omega R_{i-1}=\phi_{1} R_{i} \phi_{1}^{T}-\phi_{1} S_{i}^{T} \bar{S}_{i} \phi_{1}^{T}
$$

Substituting Eqn. 5.3.25 in Eqn. 5.3.24

$$
\begin{align*}
V_{i} R_{i} & =V_{i-1} \phi_{1} R_{i} \phi_{1}^{T} \phi_{-1}^{T}-V_{i-1} \phi_{1} S_{i}^{T} \bar{S}_{i} \phi_{1}^{T} \phi_{-1}^{T}+r_{i} \bar{S}_{i} \\
& =V_{i-1} \phi_{1} R_{i}-V_{i-1} \phi_{1} S_{i}^{T} \bar{S}_{i}+r_{i} \bar{S}_{i}
\end{align*}
$$

or

$$
\begin{align*}
V_{i} & =V_{i-1} \phi_{1}-V_{i-1} \phi_{1} S_{i}^{T} \bar{S}_{i} R_{i}^{-1}+r_{i} \bar{S}_{i} R_{i}^{-1} \\
& =V_{i-1} \phi_{1}+\left(r_{i}-V_{i-1} \phi_{1} S_{i}^{T}\right) \bar{S}_{i} R_{i}^{-1}
\end{align*}
$$

Let

$$
\begin{align*}
& P_{i}=R_{i}^{-1} \\
& V_{i, i-1}=V_{i-1} \phi_{1}
\end{align*}
$$

and

$$
P_{i, i-1}=\phi_{1}^{T} P_{i-1} \phi_{1}
$$

In Eqns. 5.3.30 and 5.3.31, the transition matrix $\phi_{1}$ shifts the time to which the corresponding estimate applies by one sampling interval from time (i-1)T to time iT. Hence, the updated estimate for time (i-1)T is converted into the corresponding prediction for time iT.

Combining Eqns. 5.3.28-5.3.30,

$$
V_{i}=V_{i, i-1}+\left(r_{i}-V_{i, i-1} S_{i}^{T}\right) \bar{S}_{i} P_{i}
$$

It may be noted that, in Eqn. 5.3.32, the term $\left(r_{i}-V_{i, i-1} s_{i}^{T}\right)$ represents the error in the estimation of the received signal $r_{i}$.

From Eqns. 5.3.22 and 5.3.29

$$
P_{i}^{-1}=\omega \phi_{-1} P_{i-1}^{-1} \phi_{-1}^{T}+S_{i}^{T} \bar{S}_{i}
$$

and from Eqn. 5.3.31

$$
\begin{align*}
P_{i, i-1}^{-1} & =\left(\phi_{1}^{T} P_{i-1} \phi_{1}\right)^{-1} \\
& =\phi_{-1} P_{i-1}^{-1} \phi_{-1}^{T}
\end{align*}
$$

Therefore, from Eqns. 5.3.33 and 5.3.34,

$$
P_{i}^{-1}=\omega P_{i, i-1}^{-1}+S_{i}^{T} \bar{S}_{i}
$$

or

$$
R_{i}=\omega R_{i, i-1}+S_{i}^{T} \bar{S}_{i}
$$

Eqn. 5.3.36 gives the recursive relationship between $\mathrm{R}_{\mathrm{i}}$ and $\mathrm{R}_{\mathrm{i}, \mathrm{i}-\mathrm{l}}$. However, in order to obtain the updated estimate of $\mathrm{V}_{\mathrm{i}}$ from the one-step prediction $\mathrm{V}_{\mathrm{i},-1}$ using Eqn. 5.3.32, it is necessary to evaluate $\mathrm{P}_{\mathrm{i}}$.

Applying matrix inverse identity (Appendix D) to Eqn. 5.3.36, the following relationship is obtained.

$$
R_{i}^{-1}=\frac{1}{\omega}\left[R_{i, i-1}^{-1}-\frac{R_{i, i-1}^{-1} S_{i}^{T} \bar{S}_{i} R_{i, i-1}^{-1}}{\omega+\bar{S}_{i} R_{i, i-1}^{-1} S_{i}^{T}}\right]
$$

or

$$
P_{i}=\frac{1}{\omega}\left[P_{i, i-1}-\frac{P_{i, i-1} S_{i}^{T} \bar{S}_{i} P_{i, i-1}}{\omega+\bar{S}_{i} P_{i, i-1} S_{i}^{T}}\right]
$$

Now let

$$
K_{i}=\left(\omega+\bar{S}_{i} P_{i, i-1} S_{i}^{T}\right)^{-1} \bar{S}_{i} P_{i, i-1}
$$

Thus, from Eqns. 5.3.38 and 5.3.39

$$
P_{i}=\omega^{-1} P_{i, i-1}\left[I-S_{i}^{T} K_{i}\right]
$$

Thus $P_{i}$ may be computed recursively according to Eqn. 5.3.40.

Premultiplying both sides of Eqn. 5.3 .40 by $\bar{s}_{i}$, then

$$
\bar{S}_{i} P_{i}=\omega^{-1}\left[\bar{S}_{i} P_{i, i-1}-\bar{S}_{i} P_{i, i-1} S_{i}^{T} K_{i}\right]
$$

and from Eqn. 5.3.39

$$
\bar{S}_{i} P_{i, i-1}=\left[\omega+\bar{S}_{i} P_{i, i-1} S_{i}^{T}\right] K_{i}
$$

Combining Eqns. 5.3.41 and 5.3.42,

$$
\begin{align*}
\bar{S}_{i} P_{i} & =\omega^{-1}\left[\left(\omega+\bar{S}_{i} P_{i, i-1} S_{i}^{T}\right) K_{i}-\bar{S}_{i} P_{i, i-1} S_{i}^{T} K_{i}\right] \\
& =K_{i}
\end{align*}
$$

From Eqns. 5.3.32 and 5.3.43,

$$
V_{i}=V_{i, i-1}+\left(r_{i}-V_{i, i-1} S_{i}^{T}\right) K_{i}
$$

This completes the derivation of the algorithm for system 5.1. The algorithm is consistent with the corresponding algorithm in [20, 83, 103]. Eqn. 5.3.44 is the desired update recursion for the vector $\mathrm{V}_{\mathrm{i}}$. The complete algorithm for system 5.1 is given by Eqns. 5.3.30, 5.3.31, 5.3.39, 5.3.40 and 5.3.44.

### 5.4 SYSTEM 5.2

System 5.2 is a simple modification of system 5.1 and just as system 5.1, it assumes that the channel is varying linearly with time and that Eqn. 5.3.1 is satisfied. System 5.2 operates with a channel-estimation vector, for time $t=i T$, which is

$$
V_{i}=\left[\begin{array}{lllll} 
& y_{i, 0}^{\prime} & y_{i, 1}^{\prime} & \ldots . . & y_{i, g}^{\prime}
\end{array}\right]
$$

where $\mathrm{V}_{\mathrm{i}}$ is a $(\mathrm{g}+1)$ - component row vector and $y_{i, h}^{\prime}$ is an estimate of $\mathrm{y}_{\mathrm{i}, \mathrm{h}}$ (Eqn. 5.2.1) for $\mathrm{h}=0,1, \ldots ., \mathrm{g}$. The data vector, for time $\mathrm{t}=\mathrm{iT}$, is the $(\mathrm{g}+1)$ - component row vector, and is now given by

$$
S_{i}=\left[\begin{array}{llll}
s_{i} & s_{i-1} & \ldots & s_{i-g}
\end{array}\right]
$$

The transition matrix $\phi_{1}$, is no longer given by Eqn. 5.3.30, and is now replaced by

$$
\phi_{i}=\left[\begin{array}{ccccc}
q_{0, i} & 0 & \ldots & . & 0 \\
0 & q_{1, i} & \ldots & . & 0 \\
\cdot & & & & \\
\cdot & & & & \\
\cdot & & & & \\
0 & 0 & \ldots & . & q_{g, i}
\end{array}\right]
$$

where

$$
\begin{align*}
& q_{h, i}=\frac{y_{i, i-1, h}^{\prime}}{y_{i-1, h}^{\prime}} \\
& \text { for } \mathrm{h}=0,1, \ldots \ldots, \mathrm{~g}
\end{align*}
$$

The transition matrix, $\phi_{i}$, is therefore, no longer a constant but varies with time.

An update of the vector $\mathrm{V}_{\mathrm{i}}$ is determined using the RLS Kalman filter algorithm. All the vectors here are $(\mathrm{g}+1)$ - component row vectors and all the matricies are $(\mathrm{g}+1) \mathrm{x}$ $(\mathrm{g}+1)$. System 5.2, then makes a one-step prediction of the vector $\mathrm{V}_{\mathrm{i}}$ using the least-squares fading memory prediction [53], in the following manner.

An error in the update of the vector $\mathrm{V}_{\mathrm{i}}$ is given by

$$
E_{i}=V_{i}-V_{i, i-1}
$$

A one-step prediction of $V_{i}$, is now given by a polynomial filter which is described by the following two equations.

$$
\begin{align*}
& V_{i+1, i}^{\prime}=V_{i, i-1}^{\prime}+\theta_{1} E_{i} \\
& V_{i+1, i}=V_{i, i-1}+V_{i+1, i}^{\prime}+\theta_{2} E_{i}
\end{align*}
$$

where $\theta_{1}$ and $\theta_{2}$ are positive real-valued scalar constants. $\theta_{1}$ and $\theta_{2}$ are optimized to minimize the error in the prediction of the channel impulse-response. In Eqns. 5.4.6 and 5.4.7, $v_{i+1, i}^{\prime}$ is the estimate of the rate of change with $i$ of the vector $V_{i}$. At the start of the estimation process

$$
V_{i+1, i}^{\prime}=0
$$

and

$$
V_{i, i-1}=V_{0}
$$

where $\mathrm{V}_{0}$ is determined from the appropriate training sequence that precedes the transmission of data [91]. The one-step prediction of system 5.2 (Eqns. 5.4.5-5.4.7) is slightly different from that used by system 4.2 (Eqns. 4.4.2-4.4.4). $\theta_{1}$ and $\theta_{2}$, in Eqns. 5.4.6 and 5.4.7 respectively, no longer bear any fixed relationship. Com-puter-simulation tests, have shown some useful improvement in the performance of system 5.2. Eqns. 5.4.6 and 5.4.7 allows greater flexibility and improved performance of system 5.2 and tests have not shown any kind of instability in the algorithm.

Having obtained the one-step prediction of the vector $\mathrm{V}_{\mathrm{i}}$, given by Eqn. 5.4.7, the transition matrix $\phi_{i}$ can now be determined using Eqn. 5.4.3, ready for determining the next updated vector $\mathrm{V}_{\mathrm{i}+1}$, for time $\mathrm{t}=(\mathrm{i}+1) \mathrm{T}$.

### 5.5 SYSTEM 5.3

System 5.3 is the conventional RLS Kalman algorithm and assumes that the channel is time invariant or it varies very slowly with time. The algorithm of system 5.3 is a simple modification of that for system 5.1. Eqn. 5.3.1 is now replaced by

$$
Y_{i+1}-Y_{i}=0
$$

It is, therefore, assumed that the rate of change in the channel estimate is zero. The channel-estimation vector, for time $\mathrm{t}=\mathrm{iT}$, now becomes

$$
V_{i}=\left[\begin{array}{lllll}
y_{i, 0}^{\prime} & y_{i, 1}^{\prime} & \ldots & y_{i, g}^{\prime}
\end{array}\right]
$$

where $y_{i, h}^{\prime}$ is an estimate of $y_{i, h}$ for $h=0,1, \ldots ., g . V_{i}$ is now a ( $\mathrm{g}+1$ )- component row vector. The data vector, for time $t=i T$, is the $(g+1)$ - component row vector

$$
S_{i}=\left[\begin{array}{llll}
s_{i} & s_{i-1} & \ldots & s_{i-g}
\end{array}\right]
$$

The remainder of the algorithm for system 5.3 is exactly the same as that for system 5.1, except that the transition matrix $\phi_{1}$, in Eqns. 5.3.30 and 5.3.31, is now given by the $(\mathrm{g}+1) \mathrm{x}(\mathrm{g}+1)$ component identity matrix.

Thus

$$
\phi_{1}=\left[\begin{array}{cccccc}
1 & 0 & \ldots & . & 0 & 0 \\
0 & 1 & \ldots & . & 0 & 0 \\
. & & & & & \\
. & & & & & \\
. & & & & & \\
0 & 0 & \ldots & . & 1 & 0 \\
0 & 0 & \ldots & . & 0 & 1
\end{array}\right]
$$

All the vectors here are $(\mathrm{g}+1)$ components vectors and the matrix is $(\mathrm{g}+1) \mathrm{x}(\mathrm{g}+1)$. The Kalman filter is now the conventional arrangement with exponential window (fading memory).

### 5.6 RESULTS AND ANALYSIS OF COMPUTER SIMULATION TESTS

Computer-simulation tests have been carried out on the systems 5.1-5.3 over a model of the receiver of a 4800 bits/s QPSK system, operating at 2400 bauds, with an 1800 Hz carrier. The results of the tests are given in Tables 5.6.1-5.6.3 and in Figs. 5.6.1-5.6.2. Two different measures of the average error in $Y_{i, i-1}^{\prime}$, have been used in the tests. These are

$$
\begin{align*}
& \xi_{1}=10 \log _{10}\left(\frac{1}{54000} \sum_{i=6001}^{60000}\left|Y_{i}-Y_{i, i-1}^{\prime}\right|^{2}\right) \\
& \xi_{2}=10 \log _{10}\left(\frac{1}{54000} \sum_{i=6001}^{60000} \frac{\mid Y_{i}-Y_{i, i-1}^{\prime} P^{2}}{\left|Y_{i}\right|^{2}}\right)
\end{align*}
$$

where the mean-square value of $\left|Y_{i}\right|$ is close to unity. The parameter $\xi_{1}$, termed as the mean-squared estimation error, measures the actual error in $Y_{i, i-1}^{\prime}$, whereas the parameter $\xi_{2}$, termed as the mean-squared normalized error, measures the normalized or relative error in $Y_{i, i-1}^{\prime}$. The first 1000 of the received samples in any test, are ignored to allow the stabilization of the fading and additive noise processes. During the next 5000 received samples the estimation process is allowed to stabilize and no measurement of error is carried out. This eliminates the effect of any transient behaviour of the estimators at the start-up. In fact computer-simulation tests were carried out to see the effect of stabilization length on the performance of the systems $5.1-5.3$, and 5000 received samples were found to be more than adequate for the estimators to stabilize. Over the next 54000 received samples, $\xi_{1}$ and $\xi_{2}$ are evaluated according to Eqns. 5.6.1-5.6.2. Thus $\xi_{1}$ and $\xi_{2}$ give a measure of the steady-state performance of the estimators.

At the start of the estimation process, the matrix $P_{i}$, in Eqn. 5.3.40, is taken as an identity matrix and the estimate of the channel is set to its actual value, for all the systems. In system 5.1, the vector $V_{i}$, in Eqn. 5.3.2, is taken as

$$
V_{0}=\left[\begin{array}{lllllll}
y_{0,0} & y_{0,1} & \ldots & y_{0,8} & 0 & \ldots & 0
\end{array}\right]
$$

at the start of estimation process. The initial estimate is obtained by means of a training sequence that precedes before the start of the actual transmission of data. In the tests, however, the estimate of the channel is set to its actual value.

Simulation tests were carried out on all systems with different initial settings of the $P_{i}$ matrix and the results show that it did not make any difference to the performance of the systems. The transition matrix $\phi_{i}$, in system 5.2 , is set to identity matrix at the beginning of the transmission.

The signal/noise ratio is measured as $\psi \mathrm{dB}$, where

$$
\psi=10 \log _{10}\left(\frac{E_{b}}{\frac{1}{2} N_{0}}\right)
$$

where $\mathrm{E}_{\mathrm{b}}$ is the average transmitted energy per bit at the input to the HF radio link, and is unity, while ${ }_{2}^{2} N_{0}$ is the two-sided power spectral density of the additive white Gaussian noise at the output of the HF radio link.

At every signal/noise ratio, the scalar constants $\omega$ in systems 5.1 and 5.3 and $\omega, \theta_{1}$ and $\theta_{2}$ in system 5.2 have been approximately optimized, so that the error in the estimation/prediction of the sampled impulse-response of the channel, defined by Eqn. 5.6.1, is minimized.

Tables 5.6.1-5.6.3 and Figs. 5.6.1-5.6.2 summarize the results of extensive computer-simulation tests. Fig. 5.6.1 compares the performance of systems 5.1 5.3. The results show a significant improvement in the performance of systems 5.1 and 5.2, compared with system 5.3. The results also show that the relative performance of systems 5.1 - 5.3 are not significantly affected by the error measurement used. Thus, for the purpose of comparison, both $\xi_{1}$ and $\xi_{2}$ give a reliable measure of the effectiveness of an estimator. Computer-simulation tests on systems 5.1-5.3, to study the effect of noise statistics on the performance of the estimators, have shown that only negligibly small differences in the performance of the systems occur with either correlated or uncorrelated noise introduced in the channel in Eqn. 5.2.3.

Fig. 5.6.2 show the steady state performance of systems 5.1 and 5.3 , at 60 dB signal/noise ratio. The parameter in Fig. 5.6.2, is here, the square of the error in $Y_{i, i-1}^{\prime}$ measured in dB , relative to unity, and is

$$
\xi_{i}=10 \log _{10}\left(\left|Y_{i}-Y_{i, i-1}^{\prime}\right|^{2}\right)
$$

The measurement in Fig. 5.6.2, is taken during the stable operation of the estimator. The channel estimators, listed in order of increasing complexity are systems 5.3, 5.2 and 5.1. System 5.2 is, however, comparatively a simpler system, compared with system 5.1, as the former determines the transition matrix $\phi_{i}$ using a degree-one predictor (Section 5.5) and, all vectors are having ( $\mathrm{g}+1$ ) components and all matrices are $(\mathrm{g}+1) \mathrm{x}(\mathrm{g}+1)$.

TABLE 5.6.1 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATE OF THE CHANNEL SAMPLE IMPULSE-RESPONSE GIVEN BY SYSTEM 5.1 FOR A 3 SKY WAVES CHANNEL.

| $\psi$ <br> $(\mathrm{dB})$ | $\omega$ | Correlated noise |  | Uncorrelated noise |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ |
| 10 | 0.990 | -12.270 | -10.975 | -12.273 | -10.955 |
| 20 | 0.985 | -19.418 | -18.214 | -19.435 | -18.289 |
| 30 | 0.970 | -26.195 | -24.928 | -26.229 | -24.939 |
| 40 | 0.960 | -32.437 | -31.361 | -32.477 | -31.381 |
| 60 | 0.880 | -40.191 | -39.347 | -40.185 | -39.342 |

TABLE 5.6.2 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATE OF THE CHANNEL SAMPLE IMPULSE-RESPONSE GIVEN BY SYSTEM 5.2 FOR A 3 SKY WAVES CHANNEL.

| $\psi$ <br> $(\mathrm{dB})$ | $\omega$ | $\theta_{1}$ | $\theta_{2}$ | Correlated noise |  | Uncorrelated noise |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ | $\xi_{1}$ <br> dB | $\xi_{2}$ <br> $(\mathrm{~dB})$ |
| 10 | 0.975 | 0.002 | 0.800 | -12.168 | -10.878 | -12.153 | -10.853 |
| 20 | 0.965 | 0.008 | 0.825 | -18.944 | -17.659 | -18.961 | -17.656 |
| 30 | 0.950 | 0.014 | 0.800 | -25.764 | -24.520 | -25.826 | -24.572 |
| 40 | 0.930 | 0.020 | 0.850 | -31.744 | -30.607 | -31.796 | -30.656 |
| 60 | 0.850 | 0.038 | 0.925 | -38.057 | -37.132 | -38.131 | -37.219 |

TABLE 5.6.3 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR in the estimate of the channel sample impulse reSPONSE GIVEN BY SYSTEM 5.3 FOR A 3 SKY WAVES CHANNEL.

| $\psi$ <br> $(\mathrm{dB})$ |  | Correlated noise |  | Uncorrelated noise |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\xi_{1}$ <br> $(d B)$ | $\xi_{2}$ <br> $(d B)$ | $\xi_{1}$ <br> $(d B)$ | $\xi_{2}$ <br> $(d B)$ |
| 10 | 0.980 | -12.170 | -10.917 | -12.137 | -10.894 |
| 20 | 0.950 | -17.634 | -16.391 | -17.624 | -16.379 |
| 30 | 0.910 | -21.695 | -20.587 | -21.693 | -20.578 |
| 40 | 0.870 | -23.163 | -22.147 | -23.172 | -22.150 |
| 60 | 0.860 | -23.403 | -22.407 | -23.421 | -22.424 |



Fig. 5.7.1 - Performance of Systems 5.1, $5.2 \& 5.3$

## CHAPTER 6

## ADAPTIVE CHANNEL ESTIMATORS

### 6.1 INTRODUCTION

A simple estimator designed for a 4800 bits/s modem and employing a polynomial filter that gives a prediction of the channel sampled impulse-response has already been considered in Chapter 4. This estimator is a development of the conventional gradient estimator [33, 35-36, 52, 54, 57, 62, 99-100]. The class of estimators, referred to as systems $6.1-6.5$, considered in this chapter are called the Adaptive channel estimators. These estimators are developments of the simple gradient estimator. They are adaptive because they make no use of any prior knowledge of the channel and are able to track effectively an HF channel irrespective of the number of sky waves present in the fading channel. The adaptive channel estimators are studied for use in a QPSK modem that operates at 4800 bits/s over a voiceband HF radio link. Results of the computer-simulation tests are presented, comparing the accuracies of the channel estimates given by different estimators, at the end of this chapter.

### 6.2 MODEL OF DATA TRANSMISSION SYSTEM USED IN THE TESTS

The model of the data transmission system used in tests is shown in Fig. 3.5.1. The received signal, at time $t=i T$, is given by (Eqn. 3.4.2)

$$
\begin{align*}
r_{i} & =\sum_{h=0}^{g} s_{i-h} y_{i, h}+w_{i} \\
& =Y_{i} S_{i}^{T}+w_{i}
\end{align*}
$$

$r_{i}$ is sample value of the complex-valued resultant baseband signal $r(t)$ at time $t=i T$, as can be seen from Fig. 3.5.1. $\mathrm{Y}_{\mathrm{i}}$ and $\mathrm{S}_{\mathrm{i}}$ are $(\mathrm{g}+1)$ - component row vectors, and $s_{i}^{\tau}$ is the transpose of $S_{i}$.
$Y_{i}$ and $S_{i}$ are given by (Eqns. 3.4.4 and 3.4.5)

$$
\begin{align*}
Y_{i} & =\left[\begin{array}{llllll}
y_{i, 0} & y_{i, 1} & y_{i, 2} & \ldots & y_{i, g}
\end{array}\right] \\
S_{i} & =\left[\begin{array}{llllll}
s_{i} & s_{i-1} & s_{i-2} & \ldots & s_{i-g}
\end{array}\right]
\end{align*}
$$

The vector $\mathrm{Y}_{\mathrm{i}}$, represents the sampled impulse-response of the linear baseband channel, at time $t=\mathrm{iT}$. The HF channel is assumed to have $(\mathrm{g}+1)$ components in its sampled impulse-response. The scalar quantity $w_{i}$ in Eqns. 6.2 .1 and 6.2 .2 is the noise sample at time $t=i T$. The signal $r_{i}$ and $s_{i}$ are fed to the channel estimator to give an estimate of the channel sampled impulse-response $Y_{i}^{\prime}$, at time $\mathrm{t}=\mathrm{i} \mathrm{T}$, where

$$
Y_{i}^{\prime}=\left[\begin{array}{llllll}
y_{i, 0}^{\prime} & y_{i, 1}^{\prime} & y_{i, 2}^{\prime} & \ldots & y_{i, g}^{\prime}
\end{array}\right]
$$

This estimate of the channel sampled impulse-response, $Y_{i}^{\prime}$, is fed to the detector to detect $s_{i+1}$ when the next received signal $r_{i+1}$ is received by the detector. Details regarding the channel and the channel model can be found in Chapters 2 and 3 respectively.

### 6.3 SYSTEM 6.1

System 6.1 is a development of the Feedforward transversal filter estimator of system 4.2 [54]. The estimator of system 6.1 uses the same linear feedforward transversal filter (Fig. 4.3.1, [54]). As is shown in Fig. 4.3.1, the estimator holds in store the detected data symbols

$$
S_{i}^{\prime}=\left[\begin{array}{llllll} 
& s_{i}^{\prime} & s_{i-1}^{\prime} & s_{i-2}^{\prime} & \ldots & s_{i-g}^{\prime}
\end{array}\right]
$$

following the detection of $\mathrm{s}_{\mathrm{i}}$. Correct detection is assumed and, therefore, Eqn. 6.3.1 can be written as

$$
S_{i}=\left[\begin{array}{lllll}
s_{i} & s_{i-1} & s_{i-2} & \cdots & s_{i-g}
\end{array}\right]
$$

The estimator forms an estimate of $r_{i}^{\prime}$ of the received sample $r_{i}$, such that

$$
\begin{align*}
r_{i}^{\prime} & =\sum_{h=0}^{g} s_{i-h} y_{i, i-1, h}^{\prime} \\
& =Y_{i, i-1}^{\prime} S_{i}^{T}
\end{align*}
$$

where $Y_{i, i-1}^{\prime}$ is the one-step prediction of $Y_{i}$, given by

$$
Y_{i, i-1}^{\prime}=\left[\begin{array}{llllll} 
& y_{i, i-1,0}^{\prime} & y_{i, i-1,1}^{\prime} & \cdots & y_{i, i-1, g}^{\prime}
\end{array}\right]
$$

$Y_{i, i-1}^{\prime}$ is a $(g+1)$ - component row vector. The error in the estimation of received signal is, therefore,

$$
e_{i}=r_{i}-r_{i}^{\prime}
$$

The one-step prediction of $Y_{i}$, given by $Y_{i, i-1}^{\prime}$, in Eqn. 6.3.5, is obtained from the updated estimate of $Y_{i}$, given by

$$
Y_{i}^{\prime}=\left[\begin{array}{lllll} 
& y_{i, 0}^{\prime} & y_{i, 1}^{\prime} & \cdots \cdots & y_{i, g}^{\prime}
\end{array}\right]
$$

in the following manner.

An estimate of the error in prediction is

$$
X_{i}=Y_{i}^{\prime}-Y_{i, i-1}^{\prime}
$$

The actual error in $Y_{i, i-1}^{\prime}$ is

$$
Y_{i}-Y_{i, i-1}^{\prime}
$$

The prediction of $Y_{i+1}$ is now determined by means of a polynomial filter [53] that operates as follows

$$
Y_{i+1, i}^{\prime \prime}=Y_{i, i-1}^{\prime \prime}+(1-\theta)^{2} X_{i}
$$

$$
Y_{i+1, i}^{\prime}=Y_{i, i-1}^{\prime}+Y_{i+1, i}^{\prime \prime}+\left(1-\theta^{2}\right) X_{i}
$$

The vector $Y_{i+1, i}^{\prime}$ is a degree-1 least-squares fading memory prediction of $Y_{i+1}$ [53-54], and the vector $Y_{i+1, i}^{\prime \prime}$ is a prediction of the rate of change with $i$ of $Y_{i+1}$. The scalar constant $\theta$ is real-valued and is in the range 0 to 1 (usually close to 1 ).

An estimate $X_{i}^{\prime}$ of the actual error in $Y_{i, i-1}^{\prime}$ given by Eqn. 6.3.9, can in principle, be derived from the fact that the prediction given by Eqns. 6.3.8-6.3.11 employs a degree-1 least-squares fading memory polynomial filter [53-54]. The latter assumes that the rate of change of $Y_{i}$ with $i$ is constant or only slowly varies with $i$. Thus a significant source of error in a prediction $Y_{i, i-1}^{\prime}$ is likely to be the acceleration (variation in rate of change) in $Y_{i}$. If the only source of error in $Y_{i, i-1}^{\prime}$ is due to the acceleration in $\mathrm{Y}_{\mathrm{i}}$, then

$$
Y_{i}=Y_{i, i-1}^{\prime}+c_{i} A_{i}
$$

where $c_{i}$ is a complex-valued scalar and

$$
\begin{align*}
A_{i} & =\left(Y_{i+1}-Y_{i}\right)-\left(Y_{i}-Y_{i-1}\right) \\
& =Y_{i+1}-2 Y_{i}+Y_{i-1}
\end{align*}
$$

such that $X_{i}=c_{i} A_{i}$ (from Eqn. 6.3.9). An estimate of $A_{i}$ is given by

$$
A_{i}^{\prime}=Y_{i+1, i}^{\prime}-2 Y_{i, i-1}^{\prime}+Y_{i-1, i-2}^{\prime}
$$

The weakness of $A_{i}^{\prime}$ in Eqn. 6.3.14 is its relatively high noise level, bearing in mind that $Y_{i-1, i-2}^{\prime}, Y_{i, i-1}^{\prime}$ and $Y_{i+1, i}^{\prime}$ do not differ greatly, much of the difference between them being due to the noise. Thus, instead of using $A_{i}^{\prime}$, the estimator uses the corresponding vector

$$
Z_{i}=z_{i, 0} \quad z_{i, 1} \quad \ldots . \quad z_{i, g}
$$

which is derived from $A_{i}^{\prime}$ as follows. First, let the absolute value (modulus) of the $(\mathrm{h}+1)^{\text {th }}$ component of $A_{i}^{\prime}$ be $\alpha_{i, h}$, for $\mathrm{h}=0,1, \ldots, \mathrm{~g}$, and suppose that $A_{1}^{\prime}$ is the first of the $\left\{A_{i}^{\prime}\right\}$ to be processed. Now $z_{i, h}$ is a measure of the average value of $\alpha_{i, h}$, which may be either the growing-memory average, given by

$$
z_{i, h}=i^{-1} \sum_{j=1}^{i} \alpha_{j, h}
$$

or else the fading-memory average, given by

$$
z_{i, h}=a \sum_{j=1}^{i}(1-a)^{i-j} \alpha_{j, h}
$$

where a is a real-valued constant such that $0<\mathrm{a}<1$, and j is an integer. Eqn. 6.3.16 can be implemented sequentially as

$$
\begin{align*}
z_{i, h} & =\left(1-i^{-1}\right) z_{i-1, h}+i^{-1} \alpha_{i, h} \\
& =z_{i-1, h}+i^{-1}\left(\alpha_{i, h}-z_{i-1, h}\right)
\end{align*}
$$

and Eqn. 6.3.17 can be implemented sequentially as

$$
\begin{align*}
z_{i, h} & =(1-a) z_{i-1, h}+a \alpha_{i, h} \\
& =z_{i-1, h}+a\left(\alpha_{i, h}-z_{i-1, h}\right)
\end{align*}
$$

where

$$
z_{0, h}=\alpha_{0, h}=0
$$

for $h=0,1, \ldots ., g$.

Since all components of $Z_{i}$ are real-valued whereas the components of $A_{i}$ in Eqn. 6.3.13 are in general complex-valued, neither $Y_{i, i-1}^{\prime}+Z_{i}$ nor $Y_{i, i-1}^{\prime}+c_{i} Z_{i}$ could be used as a satisfactory updated estimate of $Y_{i}$ in Eqn. 6.3.12. Nevertheless, $z_{i, ~}$ gives a measure of the magnitude $\left|y_{i, h}-y_{i, i-1, h}^{\prime}\right|$ of the error in the component $y_{i, i-1, h}^{\prime}$ of $Y_{i, i-1}^{\prime}$. Furthermore, for the most accurate tracking of a time-varying channel, the step size employed in the gradient algorithm of Eqn. 4.4 .1 should be permitted to vary from one component of $Y_{i, i-1}^{\prime}$ to another, and should increase with the likely magnitude of the error in that component. These considerations suggest that Eqn. 4.4.1 should be replaced by

$$
y_{i, h}^{\prime}=y_{i, i-1, h}^{\prime}+b u_{i, h} e_{i} s_{i-h}^{*}
$$

for $\mathrm{h}=0,1, \ldots ., \mathrm{g}$, where b is an appropriate small positive real-valued constant, and

$$
u_{i, h}=p\left(z_{i, h}\right)
$$

$p\left(z_{i, h}\right)$ is a monotonically non-decreasing positive real-valued function of $z_{i b}$. The parameter $u_{i, h}$ in Eqn. 6.3 .23 cannot be replaced by $z_{i, h}$ itself, for the following reasons. Firstly, no $u_{i, ~}$ must be permitted to remain at zero for any significant period, since, if this occurs, the corresponding component of $Y_{i, i-1}^{\prime \prime}$ in Eqn. 6.3.10 may become locked at zero, thus preventing any further change in the corresponding $y_{i, i-1, h}^{\prime}$. Secondly, no $u_{i, h}$ should be permitted to become too large, in order to avoid possible instability of the algorithm given by Eqn. 6.3.23. Thus the value of $u_{i, p}$ should be constrained such that

$$
k_{1}<u_{i, h}<k_{2}
$$

where $k_{1}$ and $k_{2}$ are appropriate positive real-valued constants. Finally, tests have shown that, for the best performance, $u_{i, h}$ must vary non-linearly with $z_{i, d}$ over the range $k_{1}$ and $k_{2}$. In the most effective arrangement that has been found, $u_{i, h}$ varies with $\mathrm{z}_{\mathrm{i}, \mathrm{b}}$ as shown in Fig. 6.3.1, where $-\mathrm{k}_{1}=10^{-6}$ and $\mathrm{k}_{2}=\infty$. The quantity $\mathrm{k}_{0}$ is a small positive real-valued constant, such that

$$
d=k_{0}^{4}
$$

and, when $\mathrm{z}_{\mathrm{i}, \mathrm{h}}>\mathrm{d}$,

$$
u_{i, h}=z_{i, h}^{0.25}
$$

The nonlinear variation of $u_{i, h}$ with $z_{i, h}$ here prevents $u_{i, h}$ from becoming too large, so that it is not, in fact, necessary to limit the maximum value of $u_{i, b}$. The prediction of $\mathrm{Y}_{\mathrm{i}+1}$ is determined by Eqns. 6.3.8-6.3.11.

### 6.4 SYSTEM 6.2

System 6.2, instead of attempting to measure the acceleration in $Y_{i}$ directly, uses the fact that the greater the maximum magnitude of any $y_{i b}$, the greater is likely to be its maximum acceleration and hence the greater the probable value of the largest error in the corresponding prediction $y_{i, i-1, h}^{\prime}$. System 6.2 operates on estimates of the magnitude of the $\left\{y_{i, h}\right\}$.

The estimator forms an estimate $r_{i}^{\prime}$ of the received sample $r_{i}$, given by Eqns. 6.3.36.3.4. It then forms the error in the estimate of the received signal, given by Eqn. 6.3.6. The estimator forms either the growing-memory or the fading-memory average $x_{i, h}^{2}$ of the mean square absolute value of $y_{i, i-1, h}^{\prime}$, for $\mathrm{h}=0,1, \ldots$, g . In particular, the growing memory average $x_{i, h}^{2}$ is now given by

$$
x_{i, h}^{2}=x_{i-1, h}^{2}+i^{-1}\left(\left|y_{i, i-1, h}^{\prime}\right|^{2}-x_{i-1, h}^{2}\right)
$$

and the fading memory average $x_{i, h}^{2}$ is given by

$$
x_{i, h}^{2}=x_{i-1, h}^{2}+a\left(\left|y_{i, i-1, h}^{\prime}\right|_{-}^{2}-x_{i-1, h}^{2}\right)
$$

where a is positive real-valued constant such that $0<\mathrm{a}<1$, and

$$
x_{0, h}^{2}=\left|y_{0,-1, h}\right|^{2}
$$

for $\mathrm{h}=0,1, \ldots, \mathrm{~g} . \quad \mathrm{Y}_{0.1}$ is determined by a training signal that precedes the transmission of data.

The estimator next forms an update of $y_{i, i-1, h}^{\prime}$ using Eqn. 6.3.23,

$$
y_{i, h}^{\prime}=y_{i, i-1, h}^{\prime}+b u_{i, h} e_{i} s_{i-h}^{*}
$$

where

$$
u_{i, h}=p\left(x_{i, h}^{2}\right)
$$

$p\left(x_{i, h}^{2}\right)$, as before, is a monotonically non-decreasing positive real-valued function of $x_{i, h}^{2}$. In system 6.2, $\mathrm{u}_{\mathrm{i}, \mathrm{h}}$ varies with $x_{i, h}^{2}$ according to Fig. 6.4.1. In all the tests, $(\mathrm{g}+1)$, the total number of components in the sampled impulse-response of the channel is taken as 32. However, with the particular HF radio link tested, the last ten components of $Y_{i}$ are all ideally equal to zero. This has lead to the non-adaptive version of system 6.2, in which the number of components of $Y_{i, i-1}^{\prime}$ is reduced to 22, by simply setting to zero its last ten components and operating system 4.2 with the corresponding 22 component vector $Y_{i, i-1}^{\prime}$. Eqn. 4.4.1 is now used in place of Eqn. 6.4.4 for the gradient algorithm. The prediction of $Y_{i+1}$ is determined, as before, by means of a degree-1 least-squares fading memory prediction, using Eqns. 6.3.86.3.11. The results of the computer-simulation tests on a model of a data transmission system are presented at the end of the chapter.

### 6.5 SYSTEM 6.3

System 6.3 is a simple modification of system 6.2. Just as system 6.2, system 6.3 operates on the estimates of the magnitude of the $\left\{y_{i, h}\right\}$.

Just before the receipt of the received-signal $\mathrm{r}_{\mathrm{i}}$, the estimator has in store the one-step prediction $Y_{i, i-1}^{\prime}$ of the vector $Y_{i}$. The estimator first forms the growing memory average using Eqn. 6.4.1, or the fading memory average using Eqn. 6.4.2, $x_{i, h}^{2}$, of the mean square absolute value of $y_{i, i-1, h}^{\prime}$ for $h=0,1, \ldots ., g$. At the start of the estimation process

$$
x_{a, h}^{2}=\left|y_{0,-1, h}\right|^{2}
$$

$\mathrm{Y}_{0.1}$ is determined by a training signal that proceeds the transmission of data.

The estimator next forms an estimate $r_{i}^{\prime}$ of the received sample $r_{i}$, using Eqn. 6.3.3. It then measures the error in the estimation of the received signal using Eqn. 6.3.6.

The estimator next forms an update of $y_{i, i-1, k}^{\prime}$ using

$$
y_{i, h}^{\prime}=y_{i, i-1, h}^{\prime}+b u_{i, h} e_{i} s_{i-h}^{*}
$$



Fig. 6.3.1 - Variation of $u_{i, h}$ with $z_{i, h}$


Fig. 6.4.1 - Variation of $u_{i, h}$ with $x_{i, h}^{2}$
where $b$ is an appropriate real-valued scalar constant and

$$
u_{i, h}=p\left(x_{i, h}^{2}\right)
$$

$p\left(x_{i, h}^{2}\right)$ is a monotonically non-decreasing positive real-valued function of $x_{i, k}^{2}$. In system 6.3, $\mathrm{u}_{\mathrm{i}, \mathrm{h}}$ and $\mathrm{x}_{\mathrm{i}, \mathrm{h}}$ are related according to Fig. 6.5.1. Over the linear region $\mathrm{u}_{\mathrm{i}, \mathrm{h}}$ and $\mathrm{x}_{\mathrm{i} \mathrm{h}}$ satisfies the relation

$$
u_{i, h}=c x_{i, h}
$$

and $\mathrm{u}_{\mathrm{i}, \mathrm{p}}$ is such that

$$
K_{1}<u_{i, h}<K_{2}
$$

In Eqn. 6.5.4, c is an appropriate positive real-valued constant.

An interesting arrangement of system 6.3 is that a in Eqn. 6.4 .2 is set to unity so that

$$
x_{i, h}^{2}=\left|y_{i, i-1, h}^{\prime}\right|^{2}
$$

and no averaging is in fact carried out. The one-step prediction of $Y_{i+1}$ is determined using Eqns. 6.3.8-6.3.11.

### 6.6 SYSTEM 6.4

System 6.4 also makes use of the estimate of the magnitude of the $\left\{y_{i, h}\right\}$.

The estimator holds in store the one-step prediction $Y_{i, i-1}^{\prime}$ of the vector $Y_{i}$ just before the receipt of $\mathrm{r}_{\mathrm{i}}$. It forms the growing memory average, Eqn. 6.4.1, or the fading memory average, Eqn. 6.4.2, of the mean square absolute value of $y_{i, i,-1, h}^{\prime}$, for $\mathrm{h}=0,1$, $\ldots ., \mathrm{g}$. At the start of the estimation process, it is assumed that $\mathrm{Y}_{0,1}$, is known, being determined by a training signal that precedes the transmission of data. Thus

$$
x_{0, h}^{2}=\left|y_{0,-1, h}\right|^{2}
$$

The estimator forms an estimate of the received signal using Eqn. 6.3.3 and measures the error in the estimation of the received signal using Eqn. 6.3.6. It then forms an update of $y_{i, i-1, n}^{\prime}$ using Eqn. 6.4.4. One-step prediction of $Y_{i+1}$ is then made by the estimator using Eqns. 6.3.8-6.3.11. In Eqn. 6.4.4, $\mathrm{u}_{\mathrm{i}, \mathrm{h}}$ and $x_{i, k}^{2}$ are now related according to Fig. 6.6.1, for system 6.4. Over the curved portion of the relationship in Fig. 6.6.1,

$$
u_{i, h}=c x_{i, h}^{0.5}
$$

where c is an appropriate positive real-valued scalar constant.

The value of $\mathrm{u}_{\mathrm{i}, \mathrm{h}}$ is constrained such that

$$
K_{1}<u_{i, h}<K_{2}
$$

where $K_{1}$ and $K_{2}$ are appropriate positive real-valued constants. The nonlinear variations of $u_{i, h}$ with $x_{i, h}^{2}$ here prevents $u_{i, h}$ from becoming too large so that it is not, in fact, necessary to limit the maximum value of $u_{i, h}$. System 6.4 has been studied for both fading-memory averaging and growing-memory averaging of $x_{i, k}^{2}$ and the results of the computer-simulation tests on a model of a digital data transmission system are presented in Section 6.8.

### 6.7 SYSTEM 6.5

System 6.5 is a simple modification of system 6.4. On receipt of $r_{i}$, the estimator holds in store the one-step prediction $Y_{i, i-1}^{\prime}$ of the vector $Y_{i}$. System 6.5 first forms the fading-memory average using Eqn. 6.4.2, or the growing-memory average using Eqn. 6.4.1, $x_{i, h}^{2}$, of the mean squared absolute value of $y_{i, i-1, h}^{\prime}$ for $h=0,1, \ldots ., \mathrm{g}$. An estimate of the received signal is formed using Eqn. 6.3.3 and a measure of the error in the estimation of the received signal obtained using Eqn. 6.3.6. Now system 6.5 forms an update of $Y_{i, i,-1, h}^{\prime}$ using Eqn. 6.4.4. Here in Eqn. 6.4.4, b is an appropriate scalar constant and $u_{i, h}$ and $x_{i, h}^{2}$ are related according to Fig. 6.7.1. Over the curved portion of the relationship in Fig. 6.7.1


Fig. 6.5.1 - Variation of $u_{i, h}$ with $x_{i, h}$


Fig. 6.6.1 - Variation of $u_{i, h}$ with $x_{i, h}^{2}$

$$
u_{i, h}=\left(x_{i, h}^{2}\right)^{\frac{1}{4}}=x_{i, h}^{\frac{1}{2}}
$$

The value of $u_{i, h}$ is constrained such that

$$
K_{1}<u_{i, h}<K_{2}
$$

where $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ are appropriate positive real-valued constants. It is not necessary to limit the maximum value of $u_{i, h}$, owing to the nonlinear variation of $u_{i, h}$ and $x_{i, h}^{2}$, as can been seen form Fig. 6.7.1. This prevents $u_{i, f}$ from becoming too large.

System 6.5 makes a one-step prediction of $Y_{i+1}$ using Eqns. 6.3.8-6.3.11. As a variation to the algorithm of system 6.5 , no prediction is made and the updated estimates given by Eqn. 6.4 .4 is now used. This arrangement is therefore called system 6.5 (Degree-zero), whereas the former is called the system 6.5 (Degree-one).

### 6.8 RESULTS AND ANALYSIS OF COMPUTER-SIMULATION TESTS

Computer-simulation tests have been carried out on the systems 6.1-6.5. The results of the tests are given in Tables 6.8.1-6.8.9 and in Figs. 6.8.1-6.8.3. The mean-square error in $Y_{i, i-1}^{\prime}$ is measured in dB relative to unity, and is given by

$$
\xi_{1}=10 \log _{10}\left(\frac{1}{56000} \sum_{i=4001}^{60000}\left|Y_{i}-Y_{i, i-1}^{\prime}\right|^{2}\right)
$$

where the mean-square value of $\left|Y_{i}\right|$ is unity. The first 1000 of the received sample in any test are ignored to allow the stabilization of the fading and additive noise processes. During the next 3000 received samples the estimation process operates as described, with a good starting-up procedure, but no measurements are carried out. This eliminates the effect of any transient behaviour of the estimator at start up. Over the following 56000 received samples, $\xi_{1}$ is evaluated according to Eqn. 6.8.1. Thus $\xi_{1}$ gives a measure of the steady-state performance of the estimator, which is here taken to be its performance during the prolonged and uninterrupted transmission of the data signal.


Fig. 6.7.1 - Variation of $u_{i, h}$ with $x_{i, h}^{2}$

The signal/noise ratio is measured as $\psi \mathrm{dB}$, where

$$
\psi=10 \quad \log _{10}\left(\frac{1}{\frac{1}{2} N_{0}}\right)
$$

Eqn. 6.8.2 uses the fact that the average transmitted energy per bit at the input and output of the HF radio link is unity, and the two-sided power spectral density of the additive white Gaussian noise at the output of the HF radio link is $\frac{1}{2} N_{0}$.

In each of the Tables 6.8.1 and 6.8.9, the adjustable scalar parameters have been optimized as far as possible to minimize $\xi_{1}$. In a particular case, for each of systems 6.1 and 6.3 (Tables 6.8.1 and 6.8.3), no averaging is applied in the evaluation of $\mathrm{z}_{\mathrm{i}, \mathrm{h}}$ and $\mathrm{x}_{\mathrm{i},}$, respectively, such that $\mathrm{a}=1$ in Eqns. 6.3.17 and 6.4.2. Each system is now approximately optimized, subject to the condition $\mathrm{a}=1$ in the fading-memory algorithm. Again, for the first half of the results in Table 6.8.1, b is fixed at unity.

Three different values of $\psi(20,30$ and 60 dB$)$ have been used in the tests, where the values 20 and 30 dB are such that a significant number of errors in detection of the received data symbols are likely to be caused from time to time by the additive noise, whereas the value 60 dB represents a high signal/noise, where the fading predominates over the noise. System 6.5 has, however, been tested at additional values of $\psi(10$ and 40 dB$)$.

Table 6.8 .6 shows the performance of system 6.5 incorporated with a degree-one predictor, whereas, Table 6.8 .7 shows the performance of system 6.5 without the predictor. In Tables 6.8.6-6.8.9, the parameter $\xi_{1}$, is a measure of the actual error in $Y_{i, i-1}^{\prime}$, given Eqn. 6.8.1, whereas, the parameter $\xi_{2}$ is a measure of the normalized or relative error in $Y_{i, i-1}^{\prime}$. $\xi_{2}$ is given by

$$
\xi_{2}=10 \log _{10}\left(\frac{1}{56000} \sum_{i=4001}^{60000} \frac{\left|Y_{i}-Y_{i, i-1}^{\prime}\right|^{2}}{\left|Y_{i}\right|^{2}}\right)
$$

System 6.5, in Tables 6.8.6 and 6.8.7, uses the fading memory average $x_{i, h}^{2}$ of the mean squared absolute value of $y_{i, i-1, h}^{\prime}$, for $\mathrm{h}=0,1, \ldots ., \mathrm{g}$, given by Eqn. 6.4.2.

Tables 6.8.8 and 6.8.9 show the performance of system 6.5 with a four Rayleigh fading sky waves. The scalar constants, for system 6.5 , have been optimized for a
three sky wave channel. Three of the sky waves are exactly as previously assumed, and the fourth sky wave has a frequency spread of 2 Hz and a transmission delay of 4 ms . relative to that of the first sky wave. The average signal power received from the fourth sky wave is 20 dB below that received from each of the others, so that the average energy per bit at the output of the HF radio link is now a little above unity. The parameters measured in Tables 6.8.8 and 6.8.9, are $\xi_{1}$ and $\xi_{2}$, using Eqns. 6.8.1 and 6.8 .3 respectively. Table 6.8 .9 shows the performance of system 6.5 when the channel is introducing uncorrelated noise, in Eqn. 6.2.1.

The good performance achieved by system 6.1 suggests that the basic mechanism behind the improvement in performance of systems 6.2-6.5 over system 4.2 (Chapter 4), is, at least in part, due to the fact that systems 6.2-6.5 are better able to correct an error in $Y_{i, i-1}^{\prime}$ caused by an acceleration in $Y_{i}$. In system 6.2 there are a series of local minima in the value of $\xi_{1}$, as the parameters are varied, instead of a single global minimum. This has led to some difficulty in the selection of the parameter values in the Table 6.8.2.

The rather similar performances of system 6.2-6.5 suggest that the precise relationship between $u_{i, h}$ and $x_{i, h}$ is not critical, so long as the general form of the relationship does not differ too much from that for system 6.5. In a practical implementation of any of these systems, $u_{i, h}$ would be determined from $x_{i, h}$ by means of a look-up table, so that the complexity of the relationship between $u_{i, t}$ and $x_{i, b}$ is of no great practical significance.

The growing-memory averages would not be suitable for a practical application of the system, since a drift in phase of the timing waveform at the receiver could introduce considerable changes into the relative peak magnitudes of the different components of $Y_{i}$, and contrary to the case of fading-memory averages, these would not be tracked by the growing-memory averages. The latter have, however, been studied as a check for the effectiveness of the former, because, in the absence of any shift in timing phase or change in fading statistics, the growing-memory averages can be taken to be optimum.

Tests have been carried out with system 6.5, for two different values of $K_{1}$ and also two different values of a (Table 6.8.5). A very near good performance is obtained here, particularly when $K_{1}=10^{-6}$ and $a=0.01$. System 6.5 has similar performances with a three sky wave and with a four sky wave channel, as can be seen from Tables
6.8.6 and 6.8.9. Fig. 6.8.1 shows the variation of $\xi_{1}$ and $\xi_{2}$ with $\psi$ for system 6.5 (Degree-zero) and system 6.5 (Degree-one) and Fig. 6.8.2 compares system 6.5 for the two different measurement of errors given by Eqns. 6.8.1 and 6.8.3. From the plots it can be seen that the relative performance of the systems are not significantly affected by whether $\xi_{1}$ or $\xi_{2}$ is used as a measurement criteria. Further tests have been carried out with statistically independent noise components $\left\{w_{i}\right\}$ in Eqn. 6.2.1, in place of the slightly correlated noise components actually obtained at the output of the receiver filters. Tests have shown that there is only a negligibly small difference in the observations. Thus the correlation in the noise components does not appear to have any significant effect. Fig. 6.8 .3 shows the steady state performance of system 6.5 at $\psi=60 \mathrm{~dB}$.

The most promising of the various systems studied here is system 6.5 , which gains considerable advantage over system 4.2 in tolerance to additive white Gaussian noise at all signal/noise ratios. The fact that system 6.1 has a performance almost as good as that of system 6.5 suggests that at least a part of the basic mechanism behind the good performance of system 6.5 is its ability to track the accelerations in $Y_{i}$ more accurately than can system 4.2.

TABLE 6.8.1 MEAN SQUARE ERROR IN THE ESTIMATED SAMPLED IMPUL-SE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 6.1

| Averaging | $\psi$ | b | $\theta$ | $K_{0}$ | $K_{1}$ | $K_{2}$ | a | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Growing memory | 20 | 1.0 | 0.97 | 0.043 | $10^{-6}$ | $\infty$ | - | -23.1 |
|  | 30 | 1.0 | 0.96 | 0.035 | $10^{-6}$ | $\infty$ | - | -30.2 |
|  | 60 | 1.0 | 0.92 | 0.030 | $10^{-6}$ | $\infty$ | - | -41.3 |
| Fading memory | 20 | 1.0 | 0.97 | 0.043 | $10^{-6}$ | $\infty$ | 0.25 | -23.1 |
|  | 30 | 1.0 | 0.96 | 0.035 | $10^{6}$ | $\infty$ | 0.11 | -30.2 |
|  | 60 | 1.0 | 0.92 | 0.030 | $10^{-6}$ | $\infty$ | 0.42 | -41.2 |
| Fading memory | 20 | 1.0 | 0.97 | 0.043 | $10^{-6}$ | $\infty$ | 1.00 | -22.3 |
|  | 30 | 1.0 | 0.96 | 0.035 | $10^{-6}$ | $\infty$ | 1.00 | -29.8 |
|  | 60 | 1.0 | 0.92 | 0.030 | $10^{-6}$ | $\infty$ | 1.00 | -40.9 |
| Growing memory | 20 | 0.9 | 0.97 | 0.043 | $10^{-6}$ | $\infty$ | - | -23.3 |
|  | 30 | 1.1 | 0.96 | 0.035 | $10^{-6}$ | $\infty$ | - | -30.2 |
|  | 60 | 1.2 | 0.92 | 0.030 | $10^{-6}$ | $\infty$ | - | -41.7 |
| Fading memory | 20 | 1.0 | 0.97 | 0.043 | $10^{-6}$ | $\infty$ | 0.25 | -23.1 |
|  | 30 | 1.1 | 0.96 | 0.035 | $10^{-6}$ | $\infty$ | 0.11 | -30.2 |
|  | 60 | 1.2 | 0.92 | 0.030 | $10^{-6}$ | $\infty$ | 0.42 | -41.6 |
| Fading memory | 20 | 0.8 | 0.97 | 0.043 | $10^{-6}$ | $\infty$ | 1.00 | -23.3 |
|  | 30 | 1.1 | 0.96 | 0.035 | $10^{6}$ | $\infty$ | 1.00 | -29.8 |
|  | 60 | 1.2 | 0.92 | 0.030 | $10^{-6}$ | $\infty$ | 1.00 | -41.0 |

TABLE 6.8.2 MEAN SQUARE ERROR IN THE ESTIMATED SAMPLED IMPUL-SE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 6.2

| Averaging | $\psi$ | b | $\theta$ | $d_{0}$ | $K_{1}$ | $K_{2}$ | a | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Growing memory$\mathrm{g}=31$ | 20 | 1.0 | 0.960 | $4 \times 10^{-5}$ | $10^{-6}$ | 0.120 | - | -23.3 |
|  | 30 | 1.0 | 0.947 | $4 \times 10^{-5}$ | $10^{-6}$ | 0.086 | - | -29.0 |
|  | 60 | 1.0 | 0.900 | $10^{-5}$ | $10^{-6}$ | 0.130 | - | -40.6 |
| Fading memory$g=31$ | 20 | 1.0 | 0.960 | $4 \times 10^{-6}$ | $10^{-6}$ | 0.170 | 0.04 | -23.1 |
|  | 30 | 1.0 | 0.947 | $3 \times 10^{-6}$ | $10^{-6}$ | 0.086 | 0.02 | -29.0 |
|  | 60 | 1.0 | 0.940 | $2 \times 10^{-6}$ | $10^{-6}$ | 0.240 | 0.02 | -37.2 |
| NonAdaptive$\begin{aligned} & \mathrm{u}_{\mathrm{i}, \mathrm{l}}=1 \\ & \mathrm{~g}=21 \end{aligned}$ | 20 | 0.15 | 0.970 | - | - |  | - | -21.1 |
|  | 30 | 0.12 | 0.950 | - | - |  | - | -27.8 |
|  | 60 | 0.15 | 0.930 | - | - |  | - | -36.8 |

TABLE 6.8.3 MEAN SQUARE ERROR IN THE ESTIMATED SAMPLED IMPUL-SE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 6.3

| AVERAG- <br> ING | $\Psi$ | $\mathbf{b}$ | $\theta$ | $\mathbf{c}$ | $K_{1}$ | $K_{2}$ | $\mathbf{a}$ | $\xi_{1}$ <br> $(\mathbf{d B})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Growing <br> memory | 20 | 1.0 | 0.985 | 2.8 | $10^{-5}$ | 0.72 | - | -22.4 |
|  | 30 | 1.0 | 0.977 | 2.8 | $10^{-5}$ | 0.64 | - | -29.2 |
|  | 60 | 1.0 | 0.954 | 19.4 | 0.02 | 0.48 | - | -40.3 |
| Fading <br> memory | 20 | 1.0 | 0.985 | 2.8 | $10^{-5}$ | 0.72 | 0.95 | -21.9 |
|  | 30 | 1.0 | 0.977 | 2.8 | $10^{-5}$ | 0.64 | 0.95 | -28.1 |
|  | 60 | 1.0 | 0.954 | 19.4 | 0.02 | 0.48 | 1.00 | -38.0 |
| Fading <br> memory | 20 | 1.0 | 0.985 | 2.8 | $10^{-5}$ | 0.72 | 1.00 | -21.9 |
|  | 30 | 1.0 | 0.977 | 2.8 | $10^{-5}$ | 0.64 | 1.00 | -28.1 |
|  | 60 | 1.0 | 0.954 | 19.4 | 0.02 | 0.48 | 1.00 | -38.0 |

TABLE 6.8.4 MEAN SQUARE ERROR IN THE ESTIMATED SAMPLED IMPULSE RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 6.4

| Averaging | $\psi$ | $\mathbf{b}$ | $\theta$ | $\mathbf{c}$ | $K_{1}$ | $K_{2}$ | $\mathbf{a}$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Growing <br> memory | 20 | 1.0 | 0.985 | 0.90 | 0.001 | $\infty$ | - | -21.6 |
|  | 30 | 1.0 | 0.976 | 0.80 | 0.001 | $\infty$ | - | -28.7 |
|  | 60 | 1.0 | 0.951 | 0.72 | 0.003 | $\infty$ | - | -40.3 |
|  | 30 | 1.0 | 0.985 | 0.90 | 0.001 | $\infty$ | 1.00 | -22.0 |
|  | 60 | 1.0 | 0.976 | 0.80 | 0.001 | $\infty$ | 1.00 | -28.8 |

TABLE 6.8.5 MEAN SQUARE ERROR IN THE ESTIMATED SAMPLED IMPUL-SE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 6.5

| AVERAG- <br> ING | $\psi$ | $\mathbf{b}$ | $\theta$ | $K_{0}$ | $K_{1}$ | $K_{2}$ | $\mathbf{a}$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Growing <br> memory | 20 | 1.0 | 0.985 | 0.063 | $10^{-6}$ | $\infty$ | - | -23.6 |
|  | 30 | 1.0 | 0.976 | 0.064 | $10^{-6}$ | $\infty$ | - | -30.6 |
|  | 60 | 1.0 | 0.950 | 0.024 | $10^{-6}$ | $\infty$ | - | -43.7 |
| Fading <br> memory | 20 | 1.0 | 0.985 | 0.063 | $10^{-6}$ | $\infty$ | 0.01 | -23.2 |
|  | 30 | 1.0 | 0.976 | 0.064 | $10^{-6}$ | $\infty$ | 0.01 | -30.3 |
|  | 60 | 1.0 | 0.950 | 0.024 | $10^{-6}$ | $\infty$ | 0.01 | -43.4 |
| Fading <br> memory | 20 | 0.86 | 0.985 | 0.063 | 0.001 | $\infty$ | 0.01 | -22.3 |
|  | 30 | 0.86 | 0.976 | 0.064 | 0.001 | $\infty$ | 0.01 | -28.8 |
|  | 60 | 0.78 | 0.950 | 0.024 | 0.001 | $\infty$ | 0.01 | -40.0 |
| Fading <br> memory | 20 | 1.0 | 0.985 | 0.063 | $10^{-6}$ | $\infty$ | 1.0 | -23.1 |
|  | 30 | 1.0 | 0.976 | 0.064 | $10^{-6}$ | $\infty$ | 1.0 | -29.5 |
|  | 60 | 1.0 | 0.950 | 0.024 | $10^{-6}$ | $\infty$ | 1.0 | -42.2 |

TABLE 6.8.6 MEAN SQUARE ERROR AND MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 3 SKy WAVE CHANNEL FROM SYSTEM 6.5 (DEGREE ONE )

| $\psi$ | $\mathbf{b}$ | $\theta$ | $K_{0}$ | $K_{1}$ | $K_{2}$ | $\mathbf{a}$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 1.0 | 0.990 | 0.063 | $10^{-6}$ | $\infty$ | 0.01 | -16.2 | -14.9 |
| 20 | 1.0 | 0.985 | 0.063 | $10^{-6}$ | $\infty$ | 0.01 | -23.2 | -21.9 |
| 30 | 1.0 | 0.976 | 0.064 | $10^{-6}$ | $\infty$ | 0.01 | -30.3 | -29.0 |
| 40 | 1.0 | 0.970 | 0.064 | $10^{-6}$ | $\infty$ | 0.01 | -35.8 | -34.4 |
| 60 | 1.0 | 0.950 | 0.024 | $10^{-6}$ | $\infty$ | 0.01 | -43.4 | -42.7 |

TABLE 6.8.7 MEAN SQUARE ERROR AND MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 6.5 (DEGREE ZERO )

| $\psi$ | $\mathbf{b}$ | $K_{0}$ | $K_{1}$ | $K_{2}$ | $\mathbf{a}$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 0.02 | 0.001 | $10^{-6}$ | $\infty$ | 0.01 | -15.595 | -14.299 |
| 20 | 0.05 | 0.003 | $10^{-6}$ | $\infty$ | 0.01 | -21.835 | -20.646 |
| 30 | 0.08 | 0.002 | $10^{-6}$ | $\infty$ | 0.01 | -26.737 | -25.522 |
| 40 | 0.11 | 0.003 | $10^{-6}$ | $\infty$ | 0.01 | -29.625 | -28.553 |
| 60 | 0.11 | 0.002 | $10^{-6}$ | $\infty$ | 0.01 | -30.297 | -29.098 |

TABLE 6.8.8 MEAN SQUARE ERROR AND MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 4 SKY WAVE CHANNEL FROM SYSTEM 6.5 (DEGREE ONE ).

| $\psi$ | $\mathbf{b}$ | $\theta$ | $K_{0}$ | $K_{1}$ | $K_{2}$ | $\mathbf{a}$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 1.0 | 0.990 | 0.063 | $10^{-6}$ | $\infty$ | 0.01 | -16.3 | -15.0 |
| 20 | 1.0 | 0.985 | 0.063 | $10^{-6}$ | $\infty$ | 0.01 | -23.4 | -22.1 |
| 30 | 1.0 | 0.976 | 0.064 | $10^{-6}$ | $\infty$ | 0.01 | -30.4 | -29.1 |
| 40 | 1.0 | 0.970 | 0.064 | $10^{-6}$ | $\infty$ | 0.01 | -35.4 | -34.2 |
| 60 | 1.0 | 0.950 | 0.024 | $10^{-6}$ | $\infty$ | 0.01 | -42.8 | -42.1 |

TABLE 6.8.9 MEAN SQUARE ERROR AND MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 4 SKY WAVE CHANNEL FROM SYSTEM 6.5 ( DEGREE ONE ). THE CHANNEL INTRODUCING UNCORRELATED NOISE.

| $\psi$ | $\mathbf{b}$ | $\theta$ | $K_{0}$ | $K_{1}$ | $K_{2}$ | $\mathbf{a}$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 1.0 | 0.990 | 0.063 | $10^{-6}$ | $\infty$ | 0.01 | -16.4 | -15.1 |
| 20 | 1.0 | 0.985 | 0.063 | $10^{-6}$ | $\infty$ | 0.01 | -23.5 | -22.1 |
| 30 | 1.0 | 0.976 | 0.064 | $10^{-6}$ | $\infty$ | 0.01 | -30.4 | -29.1 |
| 40 | 1.0 | 0.970 | 0.064 | $10^{-6}$ | $\infty$ | 0.01 | -35.5 | -34.2 |
| 60 | 1.0 | 0.950 | 0.024 | $10^{-6}$ | $\infty$ | 0.01 | -42.8 | -42.2 |



Fig. 6.8.1 - Performance of System 6.5


Fig. 6.8.2 - Performance of System 6.5

## CHAPTER 7

## EFFICIENT CHANNEL ESTIMATORS

### 7.1 INTRODUCTION

An estimation technique, that takes into account the knowledge of the basic structure of the HF channel, specifically the number of sky waves present in the channel and that the relative delays in transmission between the sky waves are fixed, is proposed in [89]. Computer-simulation tests over various fading channels, on the new estimation technique in [89], have shown that it gains a considerable improvement in performance over some more conventional estimators [20, 35-36, 50-59, 62, 77, 83-84, 87-89, 103]. The much greater accuracy in the channel estimate given by the technique, enables satisfactory operation to be achieved over a model of an HF radio link, at substantially higher transmission rates than is possible with more conventional estimators [20, 35-36, 50, 52-55, 57, 77, 89]. All the estimators discussed in this chapter take into account the prior knowledge of the channel. Several estimation techniques are considered and in some of these a Kalman filter is incorporated into the system in such a way that the filters operates on only a few variable quantities and is, therefore, considerably less complex than conventional Kalman filters. Three different Kalman filters are studied, two of these being designed for a channel varying linearly (at a constant rate) with time and the third one being a conventional Kalman filter that is designed for a time invariant or very slowly time varying channel. All of these employ an exponential window and, therefore, operate with a fading memory. Performance of these estimators are compared through series of computer simulation tests and these are presented at the end of the chapter. Comparison of the results of this estimator with that of the feedforward estimator with prediction, shows a considerable superiority for the efficient channel estimator.

### 7.2 MODEL OF DATA TRANSMISSION SYSTEM USED IN THE TESTS

Fig. 3.5.1 shows the model of the data transmission system used in the tests. The
type of equipment filters used in the computer-simulation of the channel and the channel model are described in detail in Chapter 3. Chapter 2 describes the modelling of the channel and the particular type of channel used for the test.

The received signal sample, at time $t=i T$, is given by (Eqn. 3.4.2)

$$
\begin{align*}
r_{i} & =\sum_{h=0}^{g} s_{i-h} y_{i, h}+w_{i} \\
& =Y_{i} S_{i}^{T}+w_{i}
\end{align*}
$$

where (Eqn. 3.4.4)

$$
Y_{i}=\left[\begin{array}{llll}
y_{i, 0} & y_{i, 1} & \cdots & y_{i, g}
\end{array}\right]
$$

and (Eqn. 3.4.5)

$$
S_{i}=\left[\begin{array}{llll}
s_{i} & s_{i-1} & \ldots & s_{i-g}
\end{array}\right]
$$

$\mathrm{Y}_{\mathrm{i}}$ and $\mathrm{S}_{\mathrm{i}}$ are $(\mathrm{g}+1)$ - component row vectors, and $s_{i}^{T}$ is the transpose of $\mathrm{S}_{\mathrm{i}},(\mathrm{g}+1=32)$.
The vector $Y_{i}$, represents the sampled impulse-response of the channel, at time $t=i T$. The signals $r_{i}$ and the detected data are fed to the channel estimator to give an estimate of the channel sampled impulse-response $Y_{i}^{\prime}$ at time $\mathrm{t}=\mathrm{i} \mathrm{T}$. Where

$$
Y_{i}^{\prime}=\left[\begin{array}{lllll}
y_{i, 0}^{\prime} & y_{i, 1}^{\prime} & \ldots & y_{i, g}^{\prime}
\end{array}\right]
$$

The channel estimate obtained from the estimator is now fed to the detector to detect $S_{i+1}$ when the next received signal $r_{i+1}$ is received by the detector.

### 7.3 SYSTEM 7.1

The estimator to be described, called the system 7.1, is the original estimator called the Improved channel estimator in [89]. Data signal is transmitted via 3 independent sky waves and that the time delay introduced in transmission over each sky wave is taken to be fixed over the duration of the data signal. It is assumed that there is no
drift in phase of the timing sampling waveform at the receiver relative to the received signal. The resulting impulse-response of the combined transmitter and receiver filter extends over only a few sampling intervals and the rate of fading on the received data signal is very small compared with the signal element rate. Under the assumed conditions the sampled impulse-response $Y_{i}$ of the linear baseband channel, at time $t=\mathrm{iT}$, approximately satisfies [89]

$$
Y_{i}=\lambda_{i} L+\mu_{i} M+v_{i} N
$$

where $\mathrm{L}, \mathrm{M}$ and N are fixed ( $\mathrm{g}+1$ )- component row vectors, with complex-valued components, and $\lambda_{i}, \mu_{i}$ and $v_{i}$ are complex-valued scalars that vary with i. Each of the vectors $\lambda_{i} L, \mu_{i} M$ and $v_{i} N$ is the sampled impulse-response of the channel, at time $\mathrm{t}=\mathrm{iT}$, when the corresponding one of the three sky wave is received in the absence of the other two. For any given value of $i$, the quantities $\lambda_{i}, \mu_{i}$ and $v_{i}$ are statistically independent Gaussian random variables with zero mean and the same fixed variance. However, for neighbouring values of $i$, the $\left\{\lambda_{i}\right\},\left\{\mu_{i}\right\}$ and $\left\{v_{i}\right\}$ are highly correlated. The vectors $\mathrm{L}, \mathrm{M}$ and N are linearly independent and, therefore, span a three-dimensional of the $(\mathrm{g}+1)$ dimensional unitary vector space containing all possible $(\mathrm{g}+1)$ component vectors $\left\{Y_{i}\right\}$ [89]. Since $Y_{i}$ is a linear combination of $L, M$ and $N$, it must be in the three-dimensional subspace spanned by these.

Thus if the receiver can determine the time invariant vectors $L, M$, and $N$, then by just estimating the variables $\lambda_{i}, \mu_{i}$ and $v_{i}$ the estimate of the sampled impulse-response of the channel $Y_{i}$ can easily be obtained. They are not normally orthogonal and do not bear a simple relationship. However, the vector $Y_{i}$ must lie in a three dimensional subspace spanned by $L, M$, and $N$, in the ( $g+1$ )- dimensional unitary vector space containing all ( $\mathrm{g}+1$ )- component vectors over the complex field. Since $L, M$, and $N$ are fixed, subspace spanned by the vectors is also fixed, so that the receiver needs only the estimate of the subspace. Consider three orthonormal $(g+1)$ - component vectors $A, B$, and $C$ that form a basis of the given subspace, such that

$$
Y_{i}=a_{i} A+b_{i} B+c_{i} C
$$

and, for the given vectors $\mathrm{A}, \mathrm{B}$, and $\mathrm{C}, Y_{i}$ is uniquely determined by $\mathrm{a}_{\mathrm{i}}, \mathrm{b}_{\mathrm{i}}$ and $\mathrm{c}_{\mathrm{i}}$. The scalars $a_{i}, b_{i}$ and $c_{i}$ and the components of $A, B$, and $C$ are all complex-valued. The orthonormal vectors $\mathrm{A}, \mathrm{B}$, and C are all of unit length and may be selected quite arbitrarily, just so long as they lie in the given subspace.

Just before the received signal $r_{i}$ is fed to the estimator, the latter has formed the one-step prediction $Y_{i, i-1}^{\prime}$ of the vector $Y_{i}$. The estimator also holds estimate of $\mathrm{A}, \mathrm{B}$, and $C$, which are the $(g+1)$ - component row vectors $A_{i}, B_{i}$ and $C_{i}$. These are orthonormal vectors which span a three-dimensional subspace close to that spanned by $\mathrm{A}, \mathrm{B}$, and C . The estimator now forms an estimate (prediction) of the received signal $r_{i}$ given by

$$
r_{i}^{\prime}=Y_{i, i-1}^{\prime} S_{i}^{T}
$$

On receipt of $r_{i}$, the estimator forms the error signal

$$
e_{i}=r_{i}-r_{i}^{\prime}
$$

and then the updated estimate of $Y_{i}$, given by

$$
Y_{i}^{\prime}=Y_{i, i-1}^{\prime}+b e_{i} \bar{S}_{i}
$$

where b is a small positive real-valued constant and $\bar{S}_{i}$ is the complex conjugate of $S_{i}$. Although $Y_{i, i-1}^{\prime}$ lies in the subspace spanned by $\mathrm{A}_{\mathrm{i}}, \mathrm{B}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{i}}, Y_{\mathrm{i}}^{\prime}$ does not usually lie in the given subspace. The estimator, therefore, forms the ( $\mathrm{g}+1$ )- component row vector $F_{i}$ that lie in the subspace, at the minimum unitary distance form $Y_{i}^{\prime}$. From the projection theorem [43], $\mathrm{F}_{\mathrm{i}}$ is the orthogonal projection of $Y_{i}^{\prime}$ on to the given subspace. It has been shown [89] that

$$
F_{i}=\alpha_{i} A_{i}+\beta_{i} B_{i}+\gamma_{i} C_{i}
$$

where $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$ are obtained as follows

$$
\begin{align*}
& \alpha_{i}=Y_{i}^{\prime} A_{i}^{*} \\
& \beta_{i}=Y_{i}^{\prime} B_{i}^{*} \\
& \gamma_{i}=Y_{i}^{\prime} C_{i}^{*}
\end{align*}
$$

$A_{i}, B_{i}$ and $C_{i}$ are not likely to lie exactly in the three- dimensional subspace containing $\mathrm{Y}_{\mathrm{i}}$, moreover, the vectors $\mathrm{L}, \mathrm{M}$ and N can vary slowly with time and, therefore, the subspace is unlikely to be stationary. It is for this reason that for the satisfactory operation of the estimator, the subspace spanned by $A_{i}, B_{i}$ and $C_{i}$ must be adjusted adaptively to track the received signal in such a way that the new subspace spanned by the new vectors $\mathrm{A}_{\mathrm{i}+1}, \mathrm{~B}_{\mathrm{i}+1}$ and $\mathrm{C}_{\mathrm{i}+1}$ is more closer to $Y_{i}^{\prime}$. The adjustment of the subspace is done in the following manner

The vector

$$
E_{i}=Y_{i}^{\prime}-F_{i}
$$

is now taken to represent the error in the subspace spanned by $A_{i}, B_{i}$ and $C_{i}$, such that a three-dimensional subspace closer to that containing $Y_{i}$ is spanned by $A_{i+1}, B_{i+1}$ and $\mathrm{C}_{\mathrm{i}+1}$, where

$$
\begin{align*}
A_{i+1} & =A_{i}+\eta \alpha_{i}^{*} E_{i} \\
B_{i+1} & =B_{i}+\eta \beta_{i}^{*} E_{i} \\
C_{i+1} & =C_{i}+\eta \gamma_{i}^{*} E_{i}
\end{align*}
$$

where $\eta$ is a small positive real-valued constant, and $\alpha_{i}^{*}, \beta_{i}^{*}$ and $\gamma_{i}^{*}$ are the complex conjugate of $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$, respectively. Clearly, $\mathrm{E}_{\mathrm{i}}$ is orthogonal to the given subspace and, therefore, to each of $A_{i}, B_{i}$ and $C_{i}[89]$.

The estimator next determines the one- step predictions $\alpha_{i+1, i}, \beta_{i+1, i}$ and $\gamma_{i+1, i}$ of $a_{i+1}$, $b_{i+1}$ and $c_{i+1}$ in Eqn. 7.3.2, for the case $A=A_{i+1}, B=B_{i+1}$ and $C=C_{i+1}$. Degree- 1 least square fading memory prediction is employed here [53-54], and $\alpha_{i+1, i}$ is determined from the following equations.

$$
\begin{align*}
& \varepsilon_{\alpha, i}=\alpha_{i}-\alpha_{i, i-1} \\
& \alpha_{i+1, i}^{\prime}=\alpha_{i, i-1}^{\prime}+(1-\theta)^{2} \varepsilon_{\alpha, i}
\end{align*}
$$

and

$$
\alpha_{i+1, i}=\alpha_{i, i-1}+\alpha_{i+1, i}^{\prime}+\left(1-\theta^{2}\right) \varepsilon_{\alpha, i}
$$

$\varepsilon_{\alpha, i}$ is the measured error in $\alpha_{i, i-1}, \theta$ is a real-valued constant in the range 0 to 1 , and $\alpha_{i+1, i}^{\prime}$ is the one-step prediction of the rate of change with $i$ of $a_{i+1}$. The estimator here assumes that $a_{i}, b_{i}$ and $c_{i}$ in Eqn. 7.3.2 vary linearly (at a constant rate) with i. Finally the estimator forms

$$
Y_{i+1, i}^{\prime}=\alpha_{i+1, i} A_{i+1}+\beta_{i+1, i} B_{i+1}+\gamma_{i+1, i} C_{i+1}
$$

which is the one-step prediction of $Y_{i+1}$, ready for the next estimation process on receipt of $r_{i+1}$. $Y_{i, i-1}^{\prime}$ in Eqns. 7.3.3 and 7.3.5, is of course given by Eqn. 7.3.17 with i replaced by $\mathrm{i}-1$.

A retraining process is normally carried out after every one or two thousand received samples $\left\{r_{i}\right\}$ using an appropriate training signal [51, 91], and, during this operation, the Gram-Schmidt orthonormalization process (Appendix E) is applied to $\mathrm{A}_{\mathrm{i}}, \mathrm{B}_{\mathrm{i}}$ and $C_{i}$ to hold them accurately orthonormal [89]. It is not, however, necessary to orthonormalize the vectors more frequently, and the results of computer-simulation tests in fact suggest that substantially less frequent orthonormalization could well be used [89]. The basic algorithm for system 7.1 is given by Eqns. 7.3.3-7.3.17.

### 7.4 SYSTEM 7.2

System 7.2 is a simple modification of system 7.1 in which equations 7.3.14-7.3.16 are replaced by

$$
\alpha_{i, i-1}=\alpha_{i-1}
$$

and similarly for $\beta_{i, i-1}$ and $\gamma_{i, i-1}$. The remainder of the algorithm of system 7.2 is the same as for system 7.1. The assumption made here is that $a_{i}, b_{i}$ and $c_{i}$ are time invariant or are varying very slowly with time [55, 57]. Thus prediction algorithm is not applied to the scalar quantities $\alpha, \beta$ and $\gamma$. System 7.2 can, therefore, be called as system 7.1 without prediction.

### 7.5 SYSTEM 7.3

System 7.3 is a development of system 7.1 in which a number of changes are made to the algorithm. The estimator first forms the fading memory average $x_{i, k}^{2}$ of the mean square absolute value of $y_{i, i-1, h}^{\prime}$ for $h=0,1, \ldots ., \mathrm{g}$, given by (as explained in Chapter 6 and in [62]).

$$
x_{i, h}^{2}=x_{i-1, h}^{2}+a\left(\left|y_{i, i-1, h}^{\prime}\right|^{2}-x_{i-1, h}^{2}\right)
$$

and at the start of the estimation process

$$
x_{0, h}^{2}=\left|y_{0,-1, h}\right|^{2}
$$

where a is a real-valued constant, which has the value 0.01 here and $Y_{0,1}$ is determined by a training signal that precedes the transmission of data. Eqn. 7.3.5 is then replaced by the following equation:

$$
y_{i, h}^{\prime}=y_{i, i-1, h}^{\prime}+b u_{i, h} e_{i} s_{i-h}^{*}
$$

for $h=0,1, \ldots, g$, where $u_{i, h}$ is related to $x_{i, h}$ according to Fig. 6.7.1. Over the curved portion of the curve in Fig. 6.7.1,

$$
u_{i, h}=\left(x_{i, h}^{2}\right)^{\frac{1}{4}}=x_{i, h}^{\frac{1}{2}}
$$

Further details of Eqns. 7.5.1-7.5.4 are given in Chapter 6. The estimator next forms the $(\mathrm{g}+1)$ - component row vector

$$
F_{i}=Y_{i}^{\prime} A_{i}^{*} A_{i}+Y_{i}^{\prime} B_{i}^{*} B_{i}+Y_{i}^{\prime} C_{i}^{*} C_{i}
$$

which is the orthogonal projection of $Y_{i}^{\prime}$ on to the subspace spanned by $\mathrm{A}_{\mathrm{i}}, \mathrm{B}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{i}}$, as can be seen from Eqns. 7.3.6-7.3.9. Again from equations 7.3.7-7.3.13, the estimator forms

$$
\begin{align*}
& E_{i}=Y_{i}^{\prime}-F_{i} \\
& A_{i+1}=A_{i}+\eta A_{i}\left(Y_{i}^{\prime}\right)^{*} E_{i} \\
& B_{i+1}=B_{i}+\eta B_{i}\left(Y_{i}^{\prime} E_{i} E_{i}\right. \\
& C_{i+1}=C_{i}+\eta C_{i}\left(Y_{i}^{\prime}\right)^{*} E_{i}
\end{align*}
$$

as before. However, the quantities $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$ are not given by equations 7.3.77.3.9, for which reason they have been omitted from Eqn. 7.5.5 and equations 7.5.7 -7.5.9. They are, instead, determined independently by a technique that is derived as follows

Suppose that $\mathrm{S}_{\mathrm{i}}$ in Eqn. 7.2.4 satisfies

$$
S_{i}=s_{a, i} \bar{A}_{i}+s_{b, i} \bar{B}_{i}+s_{c, i} \bar{C}_{i}+\bar{U}_{i}
$$

where $\mathrm{s}_{\mathrm{a}, \mathrm{i}}, \mathrm{s}_{\mathrm{b}, \mathrm{i}}$ and $\mathrm{s}_{\mathrm{c}, \mathrm{i}}$ are appropriate scalars, and

$$
A_{i} U_{i}^{*}=B_{i} U_{i}^{*}=C_{i} U_{i}^{*}=0
$$

$\bar{U}_{i}$ and $U_{i}^{*}$ are the complex conjugate and conjugate transpose, respectively, of $U_{i}$, and similarly for $A_{i}, B_{i}$ and $C_{i}$. It is evident that $U_{i}$ is orthogonal to each of $A_{i}, B_{i}$ and $C_{i}$, so that it is also orthogonal to

$$
Y_{i, i-1}^{\prime}=\alpha_{i, i-1} A_{i}+\beta_{i, i-1} B_{i}+\gamma_{i, i-1} C_{i}
$$

which is the one-step prediction of $\mathrm{Y}_{\mathrm{i}}$. Therefore, from Eqns. 7.5.10-7.5.12

$$
\begin{aligned}
Y_{i, i-1}^{\prime} S_{i}^{T}=\alpha_{i, i-1} s_{a, i}\left|A_{i}\right|^{2} & +\beta_{i, i-1} s_{b, i}\left|B_{i}\right|^{2} \\
& +\gamma_{i, i-1} s_{c, i}\left|C_{i}\right|^{2}
\end{aligned}
$$

or

$$
Y_{i, i-1}^{\prime} S_{i}^{T}=\alpha_{i, i-1} S_{a, i}+\beta_{i, i-1} S_{b, i}+\gamma_{i, i-1} S_{c, i}
$$

bearing in mind that $A_{i}, B_{i}$ and $C_{i}$ form an orthogonal set of vectors in a unitary vector space. Now let

$$
F_{i, i-1}^{\prime}=\left[\begin{array}{llll}
\alpha_{i, i-1} & \beta_{i, i-1} & \gamma_{i, i-1}
\end{array}\right]
$$

and

$$
S_{i}^{\prime}=\left[\begin{array}{lll}
s_{a, i} & s_{b, i} & s_{c, i}
\end{array}\right]
$$

From equations 7.5.10 and 7.5.11, the following relations can be obtained

$$
\begin{align*}
s_{a, i} & =S_{i} A_{i}^{T} \\
s_{b, i} & =S_{i} B_{i}^{T} \\
s_{c, i} & =S_{i} C_{i}^{T}
\end{align*}
$$

The estimate of the received signal from Eqns. 7.3.3, 7.5.13-7.5.18, is given by

$$
r_{i}^{\prime}=F_{i, i-1}^{\prime}\left(S_{i}^{\prime}\right)^{T}
$$

The error in the estimation of the received signal is

$$
e_{i}=r_{i}-r_{i}^{\prime}
$$

and the updated estimate

$$
F_{i}^{\prime}=\left[\begin{array}{llll}
\alpha_{i} & \beta_{i} & \gamma_{i}
\end{array}\right]
$$

is given by

$$
F_{i}^{\prime}=F_{i, i-1}^{\prime}+b^{\prime} e_{i}\left(\overline{S_{i}^{\prime}}\right)
$$

according to the gradient algorithm, as in Eqn. 7.3.5. The parameter $b^{\prime}$ is a small positive real-valued constant.

The estimator next determines the one-step prediction $F_{i+1, i}^{\prime}$ using degree-1 least square fading memory prediction [53-54], in the following manner

$$
\begin{align*}
& \varepsilon_{i}=F_{i}^{\prime}-F_{i, i-1}^{\prime} \\
& F_{i+1, i}^{\prime \prime}=F_{i, i-1}^{\prime \prime}+(1-\theta)^{2} \varepsilon_{i} \\
& F_{i+1, i}^{\prime}=F_{i, i-1}^{\prime}+F_{i+1, i}^{\prime \prime}+\left(1-\theta^{2}\right) \varepsilon_{i}
\end{align*}
$$

where the vector $\varepsilon_{i}$ is the measured error in $F_{i, i-1}^{\prime}, \theta$ is a real valued constant in the range 0 to 1 and the vector $F_{i+1, i}^{\prime \prime}$ is the one-step prediction of the rate of change with i of $a_{i+1}, b_{i+1}$ and $c_{i+1}$ respectively in Eqn. 7.3.2. Finally the estimator forms

$$
Y_{i+1, i}^{\prime}=\alpha_{i+1, i} A_{i+1}+\beta_{i+1, i} B_{i+1}+\gamma_{i+1, i} C_{i+1}
$$

where $\alpha_{i+1, i}, \beta_{i+1, i}$ and $\gamma_{i+1, i}$ are the elements of the vector $F_{i+1, i}^{\prime}$. The basic algorithm of system 7.3 is, therefore, given by Eqns. 7.5.12, 7.5.14-7.5.22, 7.5.1-7.5.9, 7.5.237.5.26.

An important difference between systems 7.1 and 7.3 , is that system 7.3 uses an additional set of operations, given by equations 7.5.14-7.5.25, to evaluate $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$, instead of evaluating these from $Y_{i}^{\prime}$ in (Eqns. 7.3.7-7.3.9). Thus, in system 7.3, the quantities $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$ are determined independently of $Y_{i}^{\prime}$. The modification introduces greater flexibility into the algorithm and provides additional decoupling (isolation) between scalars $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$ and the vectors $A_{i}, B_{i}$ and $C_{i}$. Another important difference between systems 7.1 and 7.3 , is that a more effective algorithm, given by equations 7.5.1-7.5.4, is used in system 7.3 to determine $Y_{i}^{\prime}$ and hence to adjust the orthogonal vectors $\mathrm{A}_{\mathrm{i}}, \mathrm{B}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{i}}$ in place of Eqn. 7.3.5 in system 7.1.

### 7.6 SYSTEM 7.4

This estimator is a simple modification of system 7.3. System 7.4 assumes that the sampled impulse-response of the channel is time invariant or varies only slowly with time. Thus it is assumed that

$$
F_{i, i-1}^{\prime}=F_{i-1}^{\prime}
$$

and the prediction algorithm, given by Eqns. 7.5.23-7.5.25, is not used by system 7.4. The remainder of the algorithm, for system 7.4, are the same as for system 7.3.

### 7.7 SYSTEM 7.5

System 5 is a development of system 7.3. The estimator first forms the fadingmemory average $x_{i, k}^{2}$ of the mean-square absolute value of $y_{i, i-1,1}$, for $h=0,1, \ldots ., \mathrm{g}$, as given by [62].

$$
x_{i, h}^{2}=x_{i-1, h}^{2}+a\left(\left|y_{i, i-1, h}^{\prime}\right|^{2}-x_{i-1, h}^{2}\right)
$$

and

$$
x_{0, h}^{2}=\left|y_{0,-1, h}\right|^{2}
$$

where a is a real-valued constant, which has a value 0.01 here. An update of $y_{i, i-1, k}^{\prime}$ is formed using equation

$$
y_{i, h}^{\prime}=y_{i, i-1, h}^{\prime}+b u_{i, h}\left(r_{i}-Y_{i, i-1}^{\prime} S_{i}^{T}\right) s_{i-h}^{*}
$$

for $\mathrm{h}=0,1, \ldots ., \mathrm{g}$, where $\mathrm{u}_{\mathrm{i}, \mathrm{h}}$ is related to $\mathrm{x}_{\mathrm{i}, \mathrm{h}}$ according to Fig. 6.7.1, and Eqn. 7.5.4. The one-step prediction $Y_{i+1, i}^{\prime}$ of $Y_{i}$, using the degree- 1 least squares fading memory prediction [53-54], is given by

$$
\begin{align*}
& E_{i}^{\prime}=Y_{i}^{\prime}-Y_{i, i-1}^{\prime} \\
& Y_{i+1, i}^{\prime \prime}=Y_{i, i-1}^{\prime \prime}+(1-\kappa)^{2} E_{i}^{\prime} \\
& Y_{i+1, i}^{\prime}=Y_{i, i-1}^{\prime}+Y_{i+1, i}^{\prime \prime}+\left(1-\kappa^{2}\right) E_{i}^{\prime}
\end{align*}
$$

$E_{i}^{\prime}$ is the measured error in the update of $Y_{i, i-1}^{\prime}, \mathrm{K}$ is a real-valued constant in the range 0 to 1 , and $Y_{i+1, i}^{\prime \prime}$ is the one-step prediction of the rate of change with i of $Y_{i+1}$.

The estimator next forms the $(g+1)$ - component row vector $F_{i}$.

$$
F_{i}=Y_{i+1, i}^{\prime} A_{i}^{*} A_{i}+Y_{i+1, i}^{\prime} B_{i}^{*} B_{i}+Y_{i+1, i}^{\prime} C_{i}^{*} C_{i}
$$

An update of the subspace spanned by $A_{i}, B_{i}$ and $C_{i}$ is next determined using equations

$$
\begin{array}{lr}
E_{i}=Y_{i+1, i}^{\prime}-F_{i} & \ldots .7 .7 .8 \\
A_{i+1}=A_{i}+\eta A_{i}\left(Y_{i+1, i}^{\prime}\right)^{*} E_{i} & \ldots .7 .7 .9 \\
B_{i+1}=B_{i}+\eta B_{i}\left(Y_{i+1, i}^{\prime}\right)^{*} E_{i} & \ldots .7 .7 .10 \\
C_{i+1}=C_{i}+\eta C_{i}\left(Y_{i+1, i}^{\prime}\right)^{*} E_{i} & \ldots .7 .7 .11
\end{array}
$$

From equations 7.5.10-7.5.19, the estimate of the received signal, $r_{i}^{\prime}$, is

$$
r_{i}^{\prime}=F_{i, i-1}^{\prime}\left(S_{i}^{\prime}\right)^{T}
$$

where, $F_{i, i-1}^{\prime}$ and $s_{i}^{\prime}$ are given by Eqns. 7.5.14 and 7.5.15 respectively.

The estimator next forms the updated estimate, $F_{i}^{\prime}$, given by ( from Eqns. 7.5.20 7.5.22)

$$
F_{i}^{\prime}=F_{i, i-1}^{\prime}+b^{\prime} e_{i}\left(\bar{S}_{i}^{\prime}\right)
$$

The estimation algorithm of system 5, next determines the one-step prediction of $F_{i}^{\prime}$, using a degree- 1 least squares fading memory prediction, given by

$$
\begin{align*}
F_{i+1, i}^{\prime \prime} & =F_{i, i-1}^{\prime \prime}+(1-\theta)^{2}\left[F_{i}^{\prime}-F_{i, i-1}^{\prime}\right] \\
F_{i+1, i}^{\prime} & =F_{i, i-1}^{\prime}+F_{i+1, i}^{\prime \prime}+\left(1-\theta^{2}\right)\left[F_{i}^{\prime}-F_{i, i-1}^{\prime}\right]
\end{align*}
$$

Finally the estimator forms

$$
Z_{i+1, i}=\alpha_{i+1, i} A_{i+1}+\beta_{i+1, i} B_{i+1}+\gamma_{i+1, i} C_{i+1}
$$

which is the same as Eqn. 7.3 .17 but $\mathrm{Z}_{\mathrm{i}+1, i}$ used in place of $Y_{i+1, i}^{\prime}$, to avoid confusion with $Y_{i+1, i}^{\prime}$ in Eqn. 7.7.6. The algorithm given by (7.7.1) - (7.7.6) and (7.7.7) (7.7.16) is now quite independent of the rest of the algorithm of system 5. It is evident that $Y_{i+1, i}^{\prime}$ is no longer constrained to lie in the three-dimensional subspace. Use of Eqns. 7.7.1-7.7.6 should further improve the ability of the vectors $A_{i}, B_{i}$ and $C_{i}$ to track variations in the three-dimensional subspace, particularly when the estimator employs an incorrect model of the HF radio link.

### 7.8 SYSTEM 7.6

This system is based on the application of Kalman estimation technique to system 7.3. An important property of systems 7.3 and 7.4 is that the evaluation of $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$ from $\alpha_{i-1}, \beta_{i-1}$ and $\gamma_{i-1}$ involves only the use of three-component vectors, which means that the gradient estimator here can be replaced by a Kalman-filter, without resulting in any undue increase in equipment complexity. System 7.6, employs a recursive least-squares (RLS) Kalman filter [103] to determine $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$ in place of the gradient algorithm with prediction used in system 7.3. Thus Eqns. 7.5.227.5.25 in system 7.3 are now replaced by the Kalman-filter algorithm. The assumption made here is that $a_{i}, b_{i}$ and $c_{i}$ in Eqn. 7.3 .2 vary linearly (at a constant rate) with i. Furthermore, it uses an exponential window (a fading memory) and attempts to minimize the quantity

$$
C_{h}=\sum_{i=0}^{h} \omega^{h-i}\left|r_{i}-V_{i} X_{i}^{T}\right|^{2}
$$

where

$$
\begin{align*}
V_{i} & =\left[\begin{array}{lllllll}
\alpha_{i} & \beta_{i} & \gamma_{i} & \alpha_{i}^{\prime} & \beta_{i}^{\prime} & \gamma_{i}
\end{array}\right] \\
X_{i} & =\left[\begin{array}{lllllll} 
& s_{a, i} & s_{b, i} & s_{c, i} & 0 & 0 & 0
\end{array}\right]
\end{align*}
$$

and $\omega$ is a positive real-valued constant in the range 0 to $1 . V_{i} X_{i}^{T}$ in Eqn. 7.8.1 is an estimate of $r_{i}$. The parameter $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$ in Eqn. 7.8.2, are the estimates of $a_{i}, b_{i}$ and $c_{i}$ respectively, as before, and $\alpha_{i}^{\prime}, \beta_{i}^{\prime}$ and $\gamma_{i}$ are the estimate of the rate of change with $i$ of $a_{i}, b_{i}$ and $c_{i}$, respectively. It is assumed in Eqn. 7.8.1, that the estimator started operation on the receipt of $r_{0}$ and that $r_{h}$ has just been received. Consider now the 6 x 6 matrix

$$
\phi_{i}=\left[\begin{array}{llllll}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
i & 0 & 0 & 1 & 0 & 0 \\
0 & i & 0 & 0 & 1 & 0 \\
0 & 0 & i & 0 & 0 & 1
\end{array}\right]
$$

where i is a positive or negative integer or zero. The parameter $\mathrm{C}_{\mathrm{b}}$ in Eqn. 7.8.1 can then be taken to be

$$
C_{h}=\sum_{i=0}^{h} \omega^{i-h}\left|r_{i}-V_{h} \phi_{i-h} X_{i}^{T}\right|^{2}
$$

If $P_{i-1}$ is the appropriate $6 \times 6$ positive-definite Hermitian matrix, the Kalman-filter algorithm becomes

$$
\begin{array}{lr}
P_{i, i-1}=\phi_{1}^{*} P_{i-1} \phi_{1} & \ldots .7 .8 .6 \\
K_{i}=\left(\omega+\bar{X}_{i} P_{i, i-1} X_{i}^{T}\right)^{-1} \bar{X}_{i} P_{i, i-1} & \ldots .7 .8 .7 \\
P_{i}=\omega^{-1} P_{i, i-1}\left(I-X_{i}^{T} K_{i}\right) & \ldots .7 .8 .8 \\
V_{i}=V_{i, i-1}+\left(r_{i}-V_{i, i-1} X_{i}^{T}\right) K_{i} & \ldots .7 .8 .9 \\
V_{i+1, i}=V_{i} \phi_{1} & \ldots .7 .8 .10
\end{array}
$$

where $\mathrm{K}_{\mathrm{i}}$ is a row vector, I is a identity matrix, $\phi_{i}$ is the conjugate transpose of $\phi_{1}, \bar{X}_{i}$ is the complex conjugate of $\mathrm{X}_{\mathrm{i}}$, and $X_{i}^{T}$ is the transpose of $\mathrm{X}_{\mathrm{i}}$. All the vectors here have six components and all matrices are $6 \times 6$. The algorithm given by Eqns. 7.8.67.8.10 is derived from first principle in Chapter 5 [103] and is consistent with the corresponding algorithms in [20] and [83]. The basic algorithm for system 7.6 is now given by Eqns. 7.5.12, 7.5.16-7.5.18, 7.8.2-7.8.4, 7.8.6-7.8.10, 7.5.1-7.5.9 and 7.5.26, where 7.5.26 is determined from Eqn. 7.8.10 and from Eqns. 7.5.7-7.5.9.

### 7.9 SYSTEM 7.7

System 7.7 is a modification of system 7.6. Here in system 7.7 as well, the scalars $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$ are determined using the RLS Kalman-filter algorithm using Eqns. 7.8.6 to 7.8.10. All the vectors here are, however, 3-component vectors and all matrices are $3 \times 3$. The vector $\mathrm{V}_{\mathrm{i}}$ is now given by

$$
V_{i}=\left[\begin{array}{lll}
\alpha_{i, i-1} & \beta_{i, i-1} & \gamma_{i, i-1}
\end{array}\right]
$$

and the vector $\mathrm{X}_{\mathrm{i}}$ is now replaced by

$$
X_{i}=\left[\begin{array}{lll}
s_{a, i} & s_{b, i} & s_{c, i}
\end{array}\right]
$$

The transition matrix $\phi_{1}$ is no longer given by Eqn. 7.8.4, and is now replaced by

$$
\phi_{i}=\left[\begin{array}{ccc}
q_{\alpha, i} & 0 & 0 \\
0 & q_{\beta, i} & 0 \\
0 & 0 & q_{\gamma, i}
\end{array}\right]
$$

where

$$
q_{\alpha, i}=\frac{\alpha_{i, i-1}}{\alpha_{i-1}}
$$

and similarly for $q_{\beta, i}$ and $q_{\gamma, i}$. The quantity $\alpha_{i, i-1}$ is here determined from $\alpha_{i-1}$ and $\alpha_{i-1, i-2}$ by Eqns. 7.3.14-7.3.16, and similarly for $\beta_{i, i-1}$ and $\gamma_{i, i-1}$. System 7.7 otherwise operates in the same way as does system 7.6. The basic algorithm of system 7.7 is, therefore, given by Eqns. 7.5.12, 7.5.16-7.5.18, 7.9.1-7.9.4, 7.8.6-7.8.10, 7.5.1-7.5.9 and 7.3.14-7.3.17. System 7.7 assumes that the channel varies linearly with time.

### 7.10 SYSTEM 7.8

System 7.8 is a very simple modification of system 7.7, in which the transition matrix $\phi_{1}$, in Eqn. 7.9.3, is replaced by the $3 \times 3$ identity matrix given

$$
\phi_{1}=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right]
$$

The system assumes that the channel is time invariant or varies very slowly with time so that the rate of change of $a_{i}, b_{i}$ and $c_{i}$ in Eqn. 7.3.2 is assumed zero. Thus

$$
\alpha_{i, i-1}=\alpha_{i-1}
$$

Similar changes are applied also to $\beta_{i, i-1}$ and $\gamma_{i, i-1}$. The remainder of the algorithm of system 7.7 is left unchanged. The Kalman filter is now the conventional arrangement with an exponential window (fading memory).

### 7.11 STARTING UP PROCEDURE FOR THE ESTIMATORS

Systems 7.1 to 7.8 require the knowledge of the initial subspace, spanned by $\mathrm{A}_{0}, \mathrm{~B}_{0}$ and $C_{0}$. In addition, the systems also require the scalars $\alpha_{0,-1}, \beta_{0,-1}$ and $\gamma_{0,-1}$ and their rate of change $\alpha_{0,-1}^{\prime}, \beta_{0,-1}^{\prime}$ and $\gamma_{0,-1}$ and the initial estimate of the sampled impulse response of the channel $Y_{0,-1}^{\prime}$. Of course, the rate of change of scalars would not be essential for systems 7.2, 7.4 and 7.8 , as these systems assume that the channel is time invariant or varies only slowly with time. In Reference [36], a number of techniques have been suggested to obtain the initial subspace. The initial subspace spanned by $A_{0}, B_{0}$ and $C_{0}$ may be determined as follows. Using a conventional estimation method, estimates of the sampled impulse response of the channel are obtained at three well spaced time instances, $\mathrm{t}=-2 \mathrm{kT}, \mathrm{t}=-\mathrm{kT}$ and $\mathrm{t}=0$ and let these estimates be $Y_{-2 k}^{\prime}, Y_{-k}^{\prime}$ and $Y_{0}^{\prime}$, respectively. The constant k is a reasonably large positive integer, so that the estimates are significantly different and non-collinear. It is assumed that these estimates are reasonably correct and the constant k is chosen in such way that the HF channel is not in deep fade at that time instant. The estimates $Y_{-2 k}^{\prime}, Y_{-k}^{\prime}$ and $Y_{0}^{\prime}$ are then orthonormalized, using Gram-Schmidt orthonormalization procedure (see Appendix E), to give the orthonormal vector

$$
\begin{align*}
& A_{0}=\left|Y_{-2 k}^{\prime}\right|^{-1} Y_{-2 k}^{\prime} \\
& B_{0}^{\prime}=Y_{-k}^{\prime}-Y_{-k}^{\prime} A_{0}^{*} A_{0} \\
& B_{0}=\left|B_{0}^{\prime}\right|^{-1} B_{0}^{\prime} \\
& C_{0}^{\prime}=Y_{0}^{\prime}-Y_{0}^{\prime} B_{0}^{*} B_{0}-Y_{0}^{\prime} A_{0}^{*} A_{0} \\
& C_{0}=\left|C_{0}^{\prime}\right|^{-1} C_{0}^{\prime}
\end{align*}
$$

where the vectors $A_{0}^{*}, B_{0}^{*}$ and $C_{0}^{*}$ are the conjugate transpose of the vectors $\mathrm{A}_{0}, \mathrm{~B}_{0}$ and $\mathrm{C}_{0}$ respectively. The vectors $\mathrm{A}_{0}, \mathrm{~B}_{0}$ and $\mathrm{C}_{0}$, therefore, form an orthogonal basis of the three-dimensional subspace spanned by $Y_{-2 k}^{\prime}, Y_{-k}^{\prime}$ and $Y_{0}^{\prime}$. The initial values of scalars $\alpha_{i, i-1}, \beta_{i, i-1}, \gamma_{i, i-1}, \alpha_{i}, \beta_{i}$ and $\gamma_{i}$ are then given by

$$
\begin{align*}
& \alpha_{0,-1}=\alpha_{i}=Y_{0}^{\prime} A_{0}^{*} \\
& \beta_{0,-1}=\beta_{i}=Y_{0}^{\prime} B_{0}^{*} \\
& \gamma_{0,-1}=\gamma_{i}=Y_{0}^{\prime} C_{0}^{*}
\end{align*}
$$

and the rate of change $\alpha_{0,-1}^{\prime}, \beta_{0,-1}^{\prime}$ and $\gamma_{0,-1}^{\prime}$ are set to

$$
\begin{align*}
& \alpha_{0,-1}^{\prime}=0 \\
& \beta_{0,-1}^{\prime}=0 \\
& \gamma_{0,-1}^{\prime}=0
\end{align*}
$$

The initial estimate of the sampled impulse response of the channel, $Y_{0,-1}^{\prime}$, is set to $Y_{0}$

### 7.12 RESULTS AND ANALYSIS OF COMPUTER-SIMULATION TESTS.

Computer-simulation tests have been carried out on the systems 7.1 to 7.8 . The results of the tests are given in Tables 7.12.1-7.12.14 and in Figs. 7.12.1-7.12.11. Two different measures of the average error in $Y_{i, i-1}^{\prime}$, have been used in the tests. These are

$$
\begin{align*}
& \xi_{1}=10 \log _{10}\left(\left.\frac{1}{54000} \sum_{i=6001}^{60000} \right\rvert\, Y_{i}-Y_{i, i-1}^{\prime} P^{2}\right) \ldots .7 .12 .1 \\
& \xi_{2}=10 \log _{10}\left(\frac{1}{54000} \sum_{i=6001}^{60000} \frac{Y_{i}-Y_{i, i-1}^{\prime} P^{2}}{\left|Y_{i}\right|^{2}}\right)
\end{align*}
$$

where the mean-square value of $\left|Y_{i}\right|$ is close to unity. The parameter $\xi_{1}$ is a measure of the actual error in $Y_{i, i-1}^{\prime}$, whereas the parameter $\xi_{2}$ is a measure of the normalized or relative error in $Y_{i, i-1}^{\prime}$. During the first 6000 of the received samples in any test, starting up procedure is carried out according to Section 7.11. During the start-up the vectors $A_{i}, B_{i}$ and $C_{i}$ are adjusted to be orthonormal by means of the Gram-Schmidt orthonormalization process [89], (Appendix E), which is not repeated over the remainder of the test. The starting-up procedure is followed by an appropriate period with no measurements, to ensure that there are no transients introduced during the start-up. Over the following 54,000 received samples, measurement of errors according to Eqns. 7.12.1 and 7.12.2 are carried out.

In all the tests, except where stated, the estimates $Y_{-2 k}^{\prime}, Y_{-k}^{\prime}$ and $Y_{0}^{\prime}$ are taken as their actual values $\mathrm{Y}_{-2 k}, \mathrm{Y}_{-k}$ and $\mathrm{Y}_{0}$, respectively. Where -2 kT , -KT and T in the tests were $2000^{\text {d }}, 3500^{\text {h }}$ and $5000^{\text {th }}$ sampling instants. Thus, the orthogonal vectors $A_{0}, B_{0}$ and $C_{0}$ derived from these estimates span the correct subspace containing $Y_{-2 k}, Y_{-k}$ and $Y_{0}$. In the tests the magnitude of $Y_{-2 k}, Y_{-k}$ and $Y_{0}$ are

$$
\begin{align*}
& \left|Y_{-2 k}\right|=0.898 \\
& \left|Y_{-k}\right|=0.617 \\
& \left|Y_{0}\right|=1.024
\end{align*}
$$

and the angle between the vectors, being

$$
\begin{align*}
& Y_{-2 k} \& Y_{0}=72.38^{\circ} \\
& Y_{-k} \& Y_{0}=62.98^{\circ} \\
& Y_{-2 k} \& Y_{-k}=64.56^{\circ}
\end{align*}
$$

A duration of 1000 sampling interval was considered sufficient for the stabilization, following start-up, after studying the results from the simulation tests and hence the actual measurements starts after the receipt of the first 6000 received samples. Thus $\xi_{1}$ and $\xi_{2}$ give a measure of the steady-state performance of the systems.

The signal/noise ratio is measured as $\psi \mathrm{dB}$, where

$$
\psi=10 \quad \log _{10}\left(\frac{1}{\frac{1}{2} N_{0}}\right)
$$

Eqn. 7.12.5 uses the fact that the average transmitted energy per bit, at the input and output of the HF radio link is unity, and the two-sided power spectral density of the additive white Gaussian noise at the output of the HF radio link is $\frac{1}{2} N_{0}$.

Tables 7.12.1-7.12.14 and Figs. 7.12.1-7.12.11 summarise the results of extensive computer-simulation tests. At every point on each curve, in Figs. 7.12.1-7.12.5, the appropriate parameters b and $\theta$ or $\mathrm{b}^{\prime}$ and $\theta$ are adjusted, as closely as has been possible to determine within the available computer time, to their optimum value, which vary steadily over each curve. The three-dimensional subspace spanned by the vectors $A_{i}, B_{i}$ and $C_{i}$ is held fixed as the subspace containing $Y_{i}$ over the initial training signal at the start of the test, the parameter $\eta$ being set to zero, during this period. During the period when the actual error measurements are done, however, two values of $\eta, 0.0$ and 0.01 , have been considered in the tests, unless otherwise stated. $\eta=0.0$, represents the condition when the three-dimensional subspace, $A_{i}, B_{i}$ and $C_{i}$ remain unchanged during the period of error measurements, whereas $\eta=0.01$ represents the condition when the subspace is changed to neutralize the small drift in the subspace. The value of 0.01 is about as large as would be desirable for $\eta$, in an
application where there are no more than three sky waves, and it permits adequate tracking of the expected drift in $Y_{i}$ with $i$ caused by an error in the receiver timing sampling frequency [89, 36]. A comparison of the Tables 7.12.1-7.12.14 and Figures 7.12.1-7.12.5 show that a substantial advantage in performance is gained by systems $7.1,7.3,7.6$ and 7.7 , which assume that the channel is varying linearly with time, over systems $7.2,7.4$ and 7.8 , which assume that the channel is time-invariant or varies only very slowly with time.

Figs. 7.12.1-7.12.2 and Tables 7.12.1 and 7.12.3, show that, with $\eta=0$, system 7.1 has a slightly better performance than system 7.3 , whereas, with $\eta=0.01$, system 7.3 has better performance than system 7.1 over the whole range of signal/noise ratios tested, with a typical advantage of about 1 dB . Again, when $0<\psi<40$, system 7.1 has a noticeably better performance with $\eta=0.0$ than with $\eta=0.01$, with the difference in performance as much as 1.7 dB at 10 dB signal/noise ratio, whereas the performance of system 7.3 is effectively the same with the two values of $\eta$ and here the difference is only of the order of 0.3 dB at 10 dB signal/noise ratio. The best performance here is given by system 7.1 with $\eta=0.0$. When $\psi>40$, system 7.1 and 7.3 show a much improved performance when $\eta$ is set to 0.01 in place of 0.0 . The reason for this is that Eqn. 7.3.1 only holds exactly if all shaping of the data signal in the demodulated waveform $r(t)$ is introduced at the transmitter [89]. Since the shaping of data signal is in fact shared approximately equally between the transmitter and receiver in Fig. 3.5.1, Eqn. 7.3.1 does not hold exactly. However, the discrepancy is quite small [89]. Checks on the operation of system 7.3 have confirmed that when, $\eta=0.01$ and $\psi=60$, the typical or average distance of $Y_{i}$ to the three-dimensional subspace is substantially smaller than when $\eta=0.0$ and $\psi=60$, and this appears to account for much of the improvement gained in setting $\eta$. Thus when $\eta=0.01$, the subspace spanned by $A_{i}, B_{i}$ and $C_{i}$ approximately tracks the small variations in the corresponding subspace containing $\mathrm{Y}_{\mathrm{i}}$. With systems 7.2 and 7.4, the performances are more or less the same with the two values of $\eta$.

Fig. 7.12.5 shows the performance of systems $7.1,7.3$ and 7.5 with four Rayleigh fading sky waves. Three of the sky waves are exactly as previously assumed, and the fourth sky wave has a frequency spread of 2 Hz and a transmission delay of 4 ms relative to that of the first sky wave. The average signal power received from the fourth sky wave is 20 dB below that received from each of the others, so that the average energy per bit at the output of the HF radio link is now a little above unity.

Each of the systems 7.1, 7.3 and 7.5 here assume that there are just three sky waves, which are as previously described, and this means that the systems now employ an incorrect model of the HF radio link.

The two curves, in Fig. 7.12.5, marked "System 7.1 not optimized" and "System 7.3 not optimized" show the performances of systems 7.1 and 7.3 , using a three-dimensional subspace as before, and with $\eta=0.01$ and their other parameters having the corresponding values used in Fig. 7.12.1 and Fig. 7.12.2, respectively. A comparison of these curves with those in Figures 7.12 .1 and 7.12 .2 shows the serious degradation in performance that is introduced by the fourth sky wave.

The two curves, in Fig. 7.12.5, marked "System 7.1 optimized" and "System 7.3 optimized" show the performances of systems 7.1 and 7.3 , with four sky waves, where the parameter $\eta, b, b^{\prime}$ and $\theta$ are appropriately optimized at each signal/noise ratios. However, a three-dimensional subspace is again used, so that the systems are, in fact, far from being fully optimized. A substantial improvement in performance is clearly achieved by the optimization process.

The best performance in Fig. 7.12.5, particularly at high signal/noise ratios, is achieved by system 7.5. The parameters $\eta, b, b, \theta$ and $\kappa$ are here appropriately optimized at each point of the curve, but a three-dimensional subspace is again used, so that the system is tested under the conditions equivalent to those for the previous two curves in Fig. 7.12.5.

Fig. 7.12.3 shows the performance of system 7.5 over a three sky wave channel. Comparing the performances of system 7.5 with that of systems 7.1 and 7.3, in Figs. 7.12.1 and 7.12.2, it is clear that when $\eta=0.00$, all the three systems have a very similar performance. However, when $\eta=0.01$, then the performance system 7.5 deteriorates, at least at high signal/noise ratios. The reason for this is that in the algorithm for system 7.5, Eqn. 7.7.6 used for the adjustment of the three-orthogonal subspace is obtained from an independent estimation process and Eqn. 7.7.16 provides the channel estimate formed from the knowledge of the subspace. Thus as long as the subspace formed is correct, the adjustment of the subspace is better done in systems 7.1 and 7.3 and, therefore, provide the better channel estimate as compared to system 7.5. However, under incorrect start-up condition system 7.5 provides better channel estimate, as is evident from Fig. 7.12.5.

Fig. 7.12.4 shows the performance of systems 7.6-7.8. These algorithm use RLS Kalman estimation technique. Tables 7.12.6-7.12.8 show the error measure and the optimum value of the parameters used in the algorithm. It is evident from Fig. 7.12.4 that the performances of systems 7.6-7.8 are very similar to those of systems 7.1 and 7.3.

Table 7.12.14 shows the performance of system 7.5 over a three sky wave channel. The orthogonal vector $A_{0}, B_{0}$ and $C_{0}$ are, however, not determined from the actual sample values, $Y_{-2 k}, Y_{-k}$ and $Y_{0}$, as described earlier, and are instead determined from the estimates $Y_{-2 k}^{\prime}, Y_{-k}^{\prime}$ and $Y_{0}^{\prime}$, given by Eqn. 7.7.6. The three-dimensional vector space is now formed from the noisy channel estimate. Under this condition, it is clear from Table 7.12.14, that there is a considerable improvement in the performance of system 7.5 , when $\eta$ is set to 0.01 , than when it is 0.0 .

Fig. 7.12.6 shows the steady-state performance of systems 7.3 and 7.4 at $\psi=60$ and $\eta=0.01$. The $y$-axis parameter, Estimation error in $\mathrm{dB}, \xi_{i}$, is the square of the error in the channel estimate $Y_{i, i-1}^{\prime}$ in dB and is given by

$$
\xi_{i}=10.0 \log _{10}\left(\left|Y_{i}-Y_{i, i-1}^{\prime}\right|^{2}\right)
$$

Fig. 7.12.7 shows the steady-state performance of systems $7.1,7.3$ and 7.5 over a 4 sky wave channel. Figs. 7.12.8-7.12.11 give a measure of the degree to which the vectors $A_{i}, B_{i}$ and $C_{i}$ remain orthogonal over the duration of a test for systems 7.1 and 7.3 at $\psi=10$ and $\eta=0.01$. Figs. 7.12.8 and 7.12.10 plot the variation with $i$ of each of

$$
\left|A_{i} B_{i}^{*}\right|, \quad\left|A_{i} C_{i}^{*}\right| \&\left|B_{i} C_{i}^{*}\right|
$$

for systems 7.1 and 7.3 respectively, the three curves being superimposed to show for each i , the maximum magnitude of the corresponding three inner products. Departure of the vectors $\mathrm{A}_{\mathrm{i}}, \mathrm{B}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{i}}$ from orthogonality for systems 7.3 is less as compared to system 7.1. Thus the slightly inferior performance of system 7.1 compared to system 7.3 can be attributed to this phenomena. However, there is no evidence of any tendency for the magnitude of any inner product to increase with i , for any system. The maximum magnitude of the inner product is of the order of $55 \times 10^{-6}$ for system 7.3 and $27.5 \times 10^{-5}$ for system 7.1.

Figs. 7.12.9 and 7.12.11 plots the variation with i of each of,

$$
\left(\left|A_{i}\right|-1\right), \quad\left(\left|B_{i}\right|-1\right) \&\left(C_{i} \mid-1\right)
$$

for systems 7.1 and 7.3 , respectively, the three curves being superimposed, to show, for each $i$, the maximum value of the corresponding three errors. As before, the tests were carried out on the systems, operating with $\psi=10 \mathrm{~dB}$ and $\eta=0.01$. It is clear, from the figures, that no error exceeds, $55 \times 10^{-6}$ for system 7.3 and $52 \times 10^{-5}$ for system 7.1, over the duration of the test. Here again, it is clear that for system 7.1, the departure of the vectors $A_{i}, B_{i}$ and $C_{i}$ from unit length is far greater than that for systems 7.3. There is, however, no evidence of any tendency for the error to increase with i .

It can be seen from Figures 7.12.8-7.12.11, that the vectors $A_{i}, B_{i}$ and $C_{i}$ remain orthonormal to a remarkable degree of accuracy, even under the unfavourable conditions tested. System 7.1 show substantially greater departure from the ideal, but again there is no evidence of any instability or steady deterioration in the system. Systems 7.3 uses a much better update of $Y_{i}$, given by Eqn. 7.5.3, to adjust the three-dimensional orthogonal vector space and hence has the superior orthonormality property.

Tests have been carried out on modifications of systems 7.3 and 7.4 in which $F_{i}$ is given by Eqn. 7.3.6, where $\alpha_{i}, \beta_{i}$ and $\gamma_{i}$ are now determined from $F_{i}^{\prime}$ in Eqn. 7.5.22. Thus Eqn 7.3.6 replaces Eqn. 7.5.5 and Eqns. 7.5.7-7.5.9 are replaced by Eqns. 7.3.11-7.3.13. The system is much simpler than the corresponding system 7.3 or 7.4 and it operates well over a sequence of 54,000 received symbols, at typical signal/noise ratios but there is now a steady and significant drift in $A_{i}, B_{i}$ and $C_{i}$ from an ideal orthonormal set. Tests at high noise levels have shown that a catastrophic failure in operating of the system can occur. The reason for this effect is that $E_{i}$ in Eqn. 7.5.6 is no longer necessarily orthogonal to the subspace spanned by $A_{i}, B_{i}$ and $\mathrm{C}_{\mathrm{i}}$, the orthogonality of $\mathrm{E}_{\mathrm{i}}$ being a basic assumption on which the algorithms of systems 7.1 - 7.8 are based [89]. These modifications have, therefore, not been considered in the thesis.

Figures 7.12.1-7.12.5 compare the performances of systems 7.1-7.8 using the error measure defined by Eqns. 7.12.1 and 7.12.2. The figures, however, reveal that the
relative performances of the systems have not changed with the type of measurement used. It is, therefore, evident that for purposes of comparison, both $\xi_{1}$ and $\xi_{2}$ give a reliable measure of the effectiveness of an estimator.

Tests were also carried out on systems $7.1-7.8$ with statistically independent noise components $\left\{w_{i}\right\}$ in Eqn. 7.2.2, in place of the slightly correlated noise components actually obtained at the output of the receiver filter. The results, however, show that only negligibly small difference in performances of the systems with the two types of noise. Thus the correlation in the noise component does not appear to have any significant effect.

Systems 7.1-7.8 achieve a considerable improvement in performance, over the conventional Kalman (systems 5.1-5.3) and gradient estimators (systems 4.1-4.1), this being due to the additional prior knowledge of the channel that is used by the systems. When the number of dimensions of the subspace is too small (so that the estimator assumes too small a number of separate fading paths), the system which has potentially the best overall performance is system 7.5. When the number of dimensions of the subspace is correct or nearly so, but there may be significant drifts in the subspace (due, say, to drifts in the timing phase), system 7.3 is the most promising system. When the number of dimensions of the subspace is correct and there is a negligible drift in the subspace, system 7.1 is the preferred system, being less complex than system 7.3 , which, in turn, is less complex than system 7.5.

TABLE 7.12.1 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 7.1

| $\begin{gathered} \psi \\ (\mathrm{dB}) \end{gathered}$ | $\theta$ | b | $\eta=0.00$ |  |  |  | $\eta=0.01$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Correlated noise |  | Uncorrelated noise |  | Correlated noise |  | Uncorrelated noise |  |
|  |  |  | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \\ \hline \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \\ \hline \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ |
| 10 | 0.960 | 0.095 | -20.843 | -19.503 | -21.361 | -20.044 | -19.098 | -18.277 | -19.539 | -18.768 |
| 20 | 0.946 | 0.112 | -28.715 | -27.465 | -29.197 | -27.913 | -27.382 | -26.534 | -27.817 | -26.959 |
| 30 | 0.922 | 0.130 | -36.473 | -35.240 | -36.886 | -35.621 | -35.484 | -34.554 | -35.865 | -34.920 |
| 40 | 0.865 | 0.110 | -43.713 | -42.585 | -44.080 | -42.902 | -43.470 | -42.418 | -43.840 | -42.743 |
| 60 | 0.788 | 0.156 | -51.445 | -51.224 | -51.514 | -51.279 | -55.366 | -54.268 | -55.529 | -54.361 |

TABLE 7.12.2 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 7.2.

| $\Psi$ <br> $(\mathrm{dB})$ | $\mathbf{b}$ | $\eta=0.00$ |  |  |  | $\eta=0.01$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Correlated noise |  | Uncorrelated noise |  | Correlated noise |  | Uncorrelated noise |  |
|  | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ |  |  |
| 10 | 0.020 | -19.265 | -18.088 | -19.703 | -18.425 | -18.991 | -17.891 | -19.424 | -18.263 |  |
| 20 | 0.040 | -25.543 | -24.366 | -25.911 | -24.653 | -25.262 | -24.165 | -25.623 | -24.459 |  |
| 30 | 0.075 | -31.329 | -30.152 | -31.576 | -30.363 | -31.036 | -29.946 | -31.281 | -30.161 |  |
| 40 | 0.140 | -35.975 | -34.806 | -36.065 | -34.913 | -35.584 | -34.544 | -35.673 | -34.646 |  |
| 60 | 0.185 | -38.479 | -37.445 | -38.495 | -37.514 | -38.097 | -37.246 | -38.056 | -37.286 |  |

TABLE 7.12.3 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 7.3

| $\psi$ <br> (dB) | b | $b^{\prime}$ | $\theta$ | $\eta=0.00$ |  |  |  | $\eta=0.01$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Correlated noise |  | Uncorrelated noise |  | Correlated noise |  | Uncorrelated noise |  |
|  |  |  |  | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\overline{\xi_{1}}$ <br> (dB) | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \\ \hline \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ |
| 10 | 0.1 | 0.09 | 0.960 | -20.811 | -19.479 | -21.315 | -20.018 | -20.545 | -19.298 | -20.998 | -19.808 |
| 20 | 0.1 | 0.11 | 0.945 | -28.719 | -27.464 | -29.208 | -27.917 | -28.566 | -27.359 | -29.029 | -27.800 |
| 30 | 0.1 | 0.13 | 0.920 | -36.495 | -35.249 | -36.921 | -35.637 | -36.446 | -35.210 | -36.854 | -35.592 |
| 40 | 1.0 | 0.10 | 0.860 | -43.678 | -42.553 | -44.044 | -42.870 | -43.965 | -42.741 | -44.341 | -43.064 |
| 60 | 1.0 | 0.14 | 0.770 | -51.444 | -51.227 | -51.521 | -51.290 | -56.172 | -54.850 | -56.335 | -54.937 |

TABLE 7.12.4 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 7.4

| $\begin{gathered} \psi \\ (\mathrm{dB}) \end{gathered}$ | b | $b^{\prime}$ | $\eta=0.00$ |  |  |  | $\eta=0.01$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Correlated noise |  | Uncorrelated noise |  | Correlated noise |  | Uncorrelated noise |  |
|  |  |  | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\xi_{2}$ (dB) | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{array}{r} \xi_{2} \\ (\mathrm{~dB}) \\ \hline \end{array}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ |
| 10 | 0.10 | 0.02 | -19.265 | -18.088 | -19.703 | -18.425 | -19.032 | -17.925 | -19.424 | -18.236 |
| 20 | 0.10 | 0.04 | -25.543 | -24.366 | -25.911 | -24.653 | -25.444 | -24.298 | -25.780 | -24.580 |
| 30 | 0.10 | 0.09 | -31.393 | -30.153 | -31.655 | -30.380 | -31.350 | -30.120 | -31.614 | -30.349 |
| 40 | 1.00 | 0.15 | -35.967 | -34.778 | -36.042 | -34.870 | -36.008 | -34.797 | -36.091 | -34.896 |
| 60 | 1.00 | 0.18 | -38.450 | -37.413 | -38.467 | -37.484 | -38.285 | -37.363 | -38.295 | -37.409 |

TABLE 7.12.5 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 7.5.

| $\Psi$ <br> (dB) | $\kappa$ | $b^{\prime}$ | $\theta$ | $\eta \underset{b=1.0}{=} 0.00$ |  |  |  | $\eta \underset{b=1.0}{=0.01}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Correlated noise |  | Uncorrelated noise |  | Correlated noise |  | Uncorrelated noise |  |
|  |  |  |  | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \\ \hline \end{gathered}$ | $\xi_{2}$ <br> (dB) | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \\ \hline \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{array}{r} \xi_{2} \\ (\mathrm{~dB}) \\ \hline \end{array}$ |
| 10 | . 990 | . 09 | . 960 | -20.757 | -19.593 | -21.398 | -20.054 | -18.621 | -17.813 | -18.911 | -18.076 |
| 20 | . 985 | . 11 | . 945 | -28.685 | -27.486 | -29.319 | -27.985 | -26.596 | -25.777 | -26.947 | -26.141 |
| 30 | . 976 | . 13 | . 920 | -36.484 | -35.256 | -37.062 | -35.780 | -34.125 | -33.387 | -34.482 | -33.782 |
| 40 | . 970 | . 10 | . 860 | -43.672 | -42.602 | -44.179 | -43.051 | -40.185 | -39.611 | -40.551 | -40.036 |
| 60 | . 950 | . 14 | . 770 | -51.460 | -51.268 | -51.557 | -51.318 | -49.622 | -50.010 | -49.752 | -50.199 |

TABLE 7.12.6 MEAN SQUARE ERROR \& NORMALIZED MEAN SQUARE ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 7.6

| $\begin{gathered} \psi \\ (\mathrm{dB}) \end{gathered}$ | b | $\omega$ | $\eta=0.00$ |  |  |  | $\eta=0.01$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Correlated noise |  | Uncorrelated noise |  | Correlated noise |  | Uncorrelated noise |  |
|  |  |  | $\begin{array}{r} \hline \xi_{1} \\ (\mathrm{~dB}) \\ \hline \end{array}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\overline{\xi_{1}}$ (dB) | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{array}{r} \xi_{1} \\ (\mathrm{~dB}) \\ \hline \end{array}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \\ \hline \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ |
| 10 | 0.10 | 0.986 | -20.510 | -19.229 | -20.903 | -19.630 | -20.264 | -19.062 | -20.614 | -19.436 |
| 20 | 0.10 | 0.974 | -28.281 | -26.971 | -28.723 | -27.387 | -28.143 | -26.877 | -28.561 | -27.282 |
| 30 | 0.10 | 0.962 | -35.976 | -34.739 | -36.325 | -35.077 | -35.918 | -34.695 | -36.256 | -35.029 |
| 40 | 1.00 | 0.932 | -43.344 | -42.155 | -43.653 | -42.454 | -43.608 | -42.322 | -43.940 | -42.640 |
| 60 | 1.00 | 0.880 | -51.258 | -51.043 | -51.307 | -51.084 | -55.946 | -54.618 | -56.050 | -54.711 |

TABLE 7.12.7 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 7.7

| $\begin{gathered} \Psi \\ (\mathrm{dB}) \end{gathered}$ | b | $\omega$ | $\theta$ | $\eta=0.00$ |  |  |  | $\eta=0.01$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Correlated noise |  | Uncorrelated noise |  | Correlated noise |  | Uncorrelated noise |  |
|  |  |  |  | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ |
| 10 | 0.1 | 0.860 | 0.955 | -19.313 | -17.745 | -20.250 | -18.774 | -19.242 | -17.723 | -19.997 | -18.593 |
| 20 | 0.1 | 0.830 | 0.935 | -28.091 | -26.773 | -28.593 | -27.266 | -27.944 | -26.675 | -28.430 | -27.161 |
| 30 | 0.1 | 0.815 | 0.900 | -36.175 | -34.861 | -36.542 | -35.225 | -36.122 | -34.820 | -36.472 | -35.180 |
| 40 | 1.0 | 0.820 | 0.840 | -43.722 | -42.532 | -44.051 | -42.886 | -43.383 | -42.339 | -43.662 | -42.662 |
| 60 | 1.0 | 0.800 | 0.710 | -51.699 | -51.507 | -51.799 | -51.591 | -56.809 | -55.367 | -56.970 | -55.522 |

TABLE 7.12.8 MEAN SQUARE ERROR \& NORMALIZED MEAN SQUARE ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 7.8.

| $\begin{gathered} \psi \\ (\mathrm{dB}) \end{gathered}$ | b | $\omega$ | $\eta=0.00$ |  |  |  | $\eta=0.01$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Correlated noise |  | Uncorrelated noise |  | Correlated noise |  | Uncorrelated noise |  |
|  |  |  | $\xi_{1}$ (dB) | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \\ \hline \end{gathered}$ | $\xi_{1}$ $(\mathrm{dB})$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \\ \hline \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \\ \hline \end{gathered}$ | $\xi_{2}$ (dB) | $\xi_{1}$ <br> (dB) | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ |
| 10 | 0.10 | 0.958 | -19.332 | -18.145 | -19.746 | -18.468 | -19.096 | -17.978 | -19.462 | -18.275 |
| 20 | 0.10 | 0.906 | -25.720 | -24.499 | -26.080 | -24.792 | -25.617 | -24.429 | -25.962 | -24.717 |
| 30 | 0.10 | 0.807 | -31.810 | -30.590 | -32.036 | -30.784 | -31.759 | -30.554 | -31.980 | -30.747 |
| 40 | 1.00 | 0.616 | -37.008 | -35.846 | -37.051 | -35.879 | -37.046 | -35.868 | -37.089 | -35.902 |
| 60 | 1.00 | 0.330 | -40.996 | -40.151 | -40.990 | -40.167 | -39.979 | -39.743 | -40.374 | -39.849 |

TABLE 7.12.9 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 4 SKY WAVE CHANNEL FROM SYSTEM 7.1. SYSTEM 7.1 PARAMETERS OPTIMIZED FOR THE ESTIMATION OF A 3 SKY WAVE CHANNEL.

| $(\underset{d B}{\psi})$ | $\theta$ | b | $\eta=0.00$ |  | $\eta=0.01$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\xi_{1}$ dB | $\begin{gathered} \xi_{2} \\ \mathrm{~dB} \end{gathered}$ | $\begin{gathered} \xi_{1} \\ \mathrm{~dB} \end{gathered}$ | $\begin{gathered} \xi_{2} \\ \mathrm{~dB} \end{gathered}$ |
| 10 | 0.960 | 0.095 | -19.538 | -18.359 | -18.603 | -17.725 |
| 20 | 0.946 | 0.112 | -23.655 | -22.754 | -24.760 | -23.581 |
| 30 | 0.922 | 0.130 | -24.738 | -23.948 | -27.452 | -25.909 |
| 40 | 0.865 | 0.110 | -24.539 | -23.779 | -27.208 | -25.714 |
| 60 | 0.788 | 0.156 | -23.422 | -22.648 | -26.780 | -25.085 |

TABLE 7.12.10 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 4 SKY WAVE CHANNEL FROM SYSTEM 7.1. SYSTEM 7.1 PARAMETERS OPTIMIZED FOR THE ESTIMATION OF A 4 SKY WAVE CHANNEL.

| $(\underset{(d B)}{\Psi}$ | $\theta$ | b | $\eta=0.00$ |  | Optimized $\eta$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | Correlated noise |  | Uncorrelated noise |  |
|  |  |  | $\overline{\xi_{1}}$ (dB) | $\xi_{2}$ (dB) | $\eta$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \\ \hline \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ |
| 10 | 0.960 | 0.095 | -19.538 | -18.359 | 0.00 | -19.538 | -18.359 | -19.935 | -18.762 |
| 20 | 0.950 | 0.120 | -23.652 | -22.756 | 0.01 | -24.775 | -23.605 | -24.920 | -23.743 |
| 30 | 0.940 | 0.135 | -24.798 | -24.013 | 0.03 | -29.149 | -27.353 | -29.150 | -27.386 |
| 40 | 0.940 | 0.145 | -24.956 | -24.187 | 0.03 | -30.637 | -28.334 | -30.579 | -28.312 |
| 60 | 0.935 | 0.125 | -24.974 | -24.205 | 0.04 | -31.038 | -28.707 | -30.994 | -28.679 |

TABLE 7.12.11 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 4 SKY WAVE CHANNEL FROM SYSTEM 7.3. SYSTEM 7.3 PARAMETERS OPTIMIZED FOR THE ESTIMATION OF A 3 SKY WAVE CHANNEL.

| $\underset{\mathrm{dB}}{\Psi}$ | b | $b^{\prime}$ | $\theta$ | $\eta=0.00$ |  | $\eta=0.01$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\xi_{1}$ dB | $\begin{gathered} \xi_{2} \\ \mathrm{~dB} \end{gathered}$ | $\begin{gathered} \xi_{1} \\ \mathrm{~dB} \end{gathered}$ | $\begin{gathered} \xi_{2} \\ \mathrm{~dB} \end{gathered}$ |
| 10 | 0.1 | 0.09 | 0.960 | -19.517 | -18.241 | -19.528 | -18.348 |
| 20 | 0.1 | 0.11 | 0.945 | -23.654 | -22.751 | -24.123 | -23.056 |
| 30 | 0.1 | 0.13 | 0.920 | -24.722 | -23.932 | -25.405 | -24.382 |
| 40 | 1.0 | 0.10 | 0.860 | -24.543 | -23.784 | -25.264 | -24.267 |
| 60 | 1.0 | 0.14 | 0.770 | -23.338 | -22.562 | -28.351 | -26.086 |

TABLE 7.12.12 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 4 SKY WAVE CHANNEL FROM SYSTEM 7.3. SYSTEM 7.3 PARAMETERS OPTIMIZED FOR THE ESTIMATION OF A 4 SKY WAVE CHANNEL.

| $(\mathrm{dB})$ | b | $b^{\prime}$ | $\theta$ | $\eta=0.00$ |  | Optimized $\eta$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | Correlated noise |  | Uncorrelated noise |  |
|  |  |  |  | $\xi_{1}$ <br> (dB) | $\xi_{2}$ (dB) | $\eta$ | $\xi_{1}$ <br> (dB) | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ |
| 10 | 0.1 | 0.095 | 0.960 | -19.538 | -18.359 | 0.00 | -19.538 | -18.359 | -19.935 | -18.762 |
| 20 | 0.3 | 0.110 | 0.945 | -23.654 | -22.751 | 0.02 | -25.369 | -23.998 | -25.495 | -24.137 |
| 30 | 1.0 | 0.115 | 0.935 | -24.796 | -24.010 | 0.02 | -30.036 | -27.928 | -30.036 | -27.961 |
| 40 | 1.0 | 0.125 | 0.935 | -24.956 | -24.187 | 0.02 | -31.861 | -29.327 | -31.835 | -29.315 |
| 60 | 1.0 | 0.130 | 0.935 | -24.978 | -24.209 | 0.02 | -32.226 | -29.535 | -32.227 | -29.526 |

TABLE 7.12.13 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 4 SKY WAVE CHANNEL FROM SYSTEM 7.5. SYSTEM 7.5 PARAMETERS OPTIMIZED FOR THE ESTIMATION OF A 4 SKY WAVE CHANNEL.

| $\boldsymbol{( d B})$ | $\mathbf{b}$ | $\kappa$ | $b^{\prime}$ | $\theta$ | $\eta$ | Correlated noise |  | Uncorrelated noise |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ |
| 10 | 1.0 | 0.990 | 0.09 | 0.960 | 0.00 | -19.538 | -16.321 | -19.935 | -16.323 |
| 20 | 1.0 | 0.985 | 0.11 | 0.945 | 0.01 | -25.572 | -23.359 | -25.679 | -23.375 |
| 30 | 1.0 | 0.976 | 0.13 | 0.920 | 0.02 | -30.915 | -30.285 | -30.929 | -30.320 |
| 40 | 1.0 | 0.970 | 0.10 | 0.860 | 0.15 | -35.899 | -35.822 | -35.965 | -35.911 |
| 60 | 1.0 | 0.950 | 0.14 | 0.770 | 0.16 | -38.243 | -42.369 | -38.388 | -42.832 |

TABLE 7.12.14 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 7.5. THE ORTHOGONAL SUB-SPACE FORMED FROM THE ESTIMATES.

| $(\underset{d B}{*})$ | b | к | $b^{\prime}$ | $\theta$ | $\eta$ | $\eta .00$ |  | $\eta .01$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{1} \\ (\mathrm{~dB}) \end{gathered}$ | $\begin{gathered} \xi_{2} \\ (\mathrm{~dB}) \end{gathered}$ |
| 10 | 1.0 | 0.990 | 0.09 | 0.960 | 0.00 | -13.772 | -13.886 | -18.600 | -17.795 |
| 20 | 1.0 | 0.985 | 0.11 | 0.945 | 0.00 | -22.025 | -22.071 | -26.584 | -25.765 |
| 30 | 1.0 | 0.976 | 0.13 | 0.920 | 0.00 | -29.139 | -29.225 | -34.114 | -33.375 |
| 40 | 1.0 | 0.970 | 0.10 | 0.860 | 0.00 | -32.963 | -33.286 | -40.158 | -39.577 |
| 60 | 1.0 | 0.950 | 0.14 | 0.770 | 0.00 | -39.888 | -40.441 | -49.562 | -49.923 |



Fig. 7.12.1 - Performance of Systems $7.1 \& 7.2$


Fig. 7.12.2 - Performance of Systems $7.3 \& 7.4$


Fig. 7.12.3 - Performance of System 7.5


Fig. 7.12.4 - Performance of Systems 7.6, 7.7 \& 7.8









## CHAPTER 8

## FAST TRANSVERSAL FILTER ALGORITHM FOR HF CHANNEL ESTIMATION

### 8.1 INTRODUCTION

When a channel estimator has no prior knowledge of the channel, a recursive least-squares (RLS) algorithm gives a convergence rate that is far superior to that of the least mean squares (LMS) algorithm [20, 59, 101]. However, this superior convergence rate of the RLS algorithm is at the expense of increased computation. Adaptive and computationally efficient RLS algorithms have been introduced in transversal filter form [20,59, 97, 101, 104-106] as well as in lattice filter form [20, $59,101,108-110]$. These algorithm are computationally efficient, requiring a number of arithmetic operations per iteration that is proportional to the number of variable parameters in the adaptive filter. However, they are still very much more complex than the LMS algorithm [20, 59, 101]. Fast Transversal Filter (FTF) implementations of the RLS adaptive filtering algorithm are presented in [84, 87]. This technique is the most promising of all the Fast RLS algorithms. The FTF algorithm is particularly suited to the application of channel estimation, as most of the computations involve only the detected data symbols that have possible values of $\pm 1 \pm \mathrm{j}$ for a QPSK system. This chapter studies the application of the FTF algorithm to HF channel estimation. The FTF implementation of the RLS algorithm exploits the shifting property of serialized data, thereby resulting in a substantial reduction in computational complexity.

The FTF algorithm in its original form [84, 87] is known to exhibit an unstable behaviour and a sudden divergence due to accumulation of round-off errors in finite precision computation [111-116]. Methods to overcome these round-off errors have been suggested in [116-118]. These introduce a redundant equation to measure a particular parameter in the algorithm which, however, only prolongs the stable operation of the estimator [113]. An alternative method to overcome the round-off
error accumulation is suggested in this chapter, and a one-step prediction is incorporated into the FTF algorithm that takes into account the rate of change in the estimates of the sampled impulse-response of the channel.

### 8.2 MODEL OF DATA TRANSMISSION SYSTEM USED IN THE TESTS

Fig. 3.5.1 shows the model of the data transmission system used in the tests. The particular application studied is the transmission of digital data at 4800 bits/sec. employing a serial quaternary phase shift keyed (QPSK) signal with a carrier frequency of 1800 Hz and an element rate of 2400 bauds. Stationary Gaussian noise, with zero mean and a two-sided power spectral density of $\frac{1}{2} N_{0}$ is added to the data signal at the output of the HF radio link.

The received signal sample at time $\mathrm{t}=\mathrm{i} \mathrm{T}$ is given by (Eqn. 3.4.2)

$$
\begin{align*}
r_{i} & =\sum_{h=0}^{g} s_{i-h} y_{i, h}+w_{i} \\
& =Y_{i} S_{i}^{T}+w_{i}
\end{align*}
$$

where (Eqn. 3.4.4)

$$
Y_{i}=\left[\begin{array}{lllll} 
& y_{i, 0} & y_{i, 1} & \ldots . & y_{i, g}
\end{array}\right]
$$

and (Eqn. 3.4.5)

$$
S_{i}=\left[\begin{array}{llll}
s_{i} & s_{i-1} & \ldots . & s_{i-g}
\end{array}\right]
$$

$Y_{i}$ and $S_{i}$ are ( $\mathrm{g}+1$ )- component row vectors, and $S_{i}^{T}$ is the transpose of $\mathrm{S}_{\mathrm{i}}$. In all the tests here $g=31$, so that the sampled impulse-response of the channel has 32 components. As is shown in Fig. 3.5.1, the signal $r_{i}$ and the "early" detected data-symbol $s_{i}^{\prime \prime}$ (see Section 3.5) are fed to the channel estimator to give an estimate of the channel sampled impulse-response $Y_{i}^{\prime}$ at time $\mathrm{t}=\mathrm{iT}$, given by

$$
Y_{i}^{\prime}=\left[\begin{array}{lllll} 
& y_{i, 0}^{\prime} & y_{i, 1}^{\prime} & \ldots & y_{i, 8}^{\prime}
\end{array}\right]
$$

Correct detection is assumed so that (Eqn. 3.5.3)

$$
s_{i}^{\prime \prime}=s_{i}
$$

Since the prime concern is only with the performance of the channel estimator. Further details on the channel model are given in Chapter 3.

### 8.3 SYSTEM 8.1

System 8.1 is the conventional FTF algorithm [84]. The channel estimator operates with a channel estimation vector, for time $t=i T$, which is

$$
Y_{i}^{\prime}=\left[\begin{array}{lllll}
y_{i, 0}^{\prime} & y_{i, 1}^{\prime} & \cdots & y_{i, g}^{\prime}
\end{array}\right]
$$

where $y_{i, h}^{\prime}$ is an estimate of $y_{i, h}$, for $\mathrm{h}=0,1, \ldots, \mathrm{~g}$. The data vector, for time $\mathrm{t}=\mathrm{i} \mathrm{T}$, is a ( $\mathrm{g}+1$ )- component row vector given by Eqn. 8.2.4.

The estimator forms an estimate of the received sample, $\mathrm{r}_{\mathrm{i}}$, given by

$$
r_{i}^{\prime}=Y_{i}^{\prime} S_{i}^{T}
$$

where $S_{i}^{T}$ is the transpose of $\mathrm{S}_{\mathrm{i}}$. $\mathrm{S}_{\mathrm{i}}$ is determined from the corresponding $\left\{S_{i-n}^{\prime}\right\}$, assuming that Eqn. 8.2.6 holds. The estimator next forms the error signal

$$
e_{i}=r_{i}-r_{i}^{\prime}
$$

The estimate of $Y_{i}$ is obtained recursively in such way that the cumulative squared error measure

$$
C_{i}=\sum_{h=0}^{i} \lambda^{i-h}\left|e_{h}\right|^{2}
$$

is minimized. The parameter $\lambda$ is a real valued constant in the range 0 to 1 . The quantity $\mathrm{C}_{\mathrm{i}}$ is the cumulative sum of the weighted squared error in the $\left\{r_{n}^{\prime}\right\}$, bearing in mind that $r_{0}$ is the first received sample that is operated on by the estimator. $\lambda$ is a weighting factor that introduces an exponential window into the processed samples and is, therefore, sometimes called the fade factor or the forgetting factor for the filter.

At time $\mathrm{t}=\mathrm{i} \mathrm{T}$, the channel estimation vector $Y_{i}^{\prime}$ that minimizes $\mathrm{C}_{\mathrm{i}}$ is given by $[20,59$, 83, 101, 103],

$$
Y_{i}^{\prime}=Q_{i} R_{i}^{-1}
$$

where

$$
R_{i}=\sum_{h=0}^{i} \lambda^{i-h} S_{h}^{T} \bar{S}_{h}
$$

and

$$
Q_{i}=\sum_{h=0}^{i} \lambda^{i-h} r_{h} \bar{S}_{h}
$$

where $\bar{S}_{h}$ is the complex conjugate of $\mathrm{S}_{\mathrm{h}}$ and $S_{h}^{T}$ is the transpose of $\mathrm{S}_{\mathrm{h}} . \mathrm{R}_{\mathrm{i}}$, called the sample autocorrelation matrix, is $(\mathrm{g}+1) \mathrm{x}(\mathrm{g}+1)$ square matrix and $\mathrm{Q}_{\mathrm{i}}$, called the sample cross-correlation vector, is a ( $\mathrm{g}+1$ )- component row vector.
$Y_{i}^{\prime}$ is given recursively as (see Eqn. 5.3.44) $[20,59,83,101,103]$

$$
\begin{align*}
Y_{i}^{\prime} & =Y_{i-1}^{\prime}+\left(r_{i}-Y_{i-1}^{\prime} S_{i}^{T}\right) K_{i} \\
& =Y_{i-1}^{\prime}+K_{i} e_{i}
\end{align*}
$$

where $\mathrm{K}_{\mathrm{i}}$ is called the Kalman gain vector. Eqn. 8.3.7 is derived from first principle in Chapter 5 and in [103] and is consistent with [20,83].

The FTF algorithm uses four transversal filters in order to obtain the channel estimate [59, 84, 87, 101]. One filter gives an estimate of the sampled impulse-res-
ponse of the HF channel. Three other filters called the forward linear predictor, the backward linear predictor and the gain transversal filter are used in the estimation of so called Kalman gain vector ( $\mathrm{K}_{\mathrm{i}}$ in Eqn. 8.3.7) necessary for updating the channel estimate. For each of the four filters an estimation error is first evaluated followed by the updating of the tap coefficients of the filter.

### 8.3.1 ADAPTIVE FORWARD LINEAR PREDICTOR

The set of data symbols $\mathrm{s}_{\mathrm{i}-1}, \mathrm{~s}_{\mathrm{i}-2}, \ldots ., \mathrm{s}_{\mathrm{i}(\mathrm{s}+1)}$ is used to make a prediction of the symbol $\mathrm{s}_{\mathrm{i}}$ at time $t=i T$. The operation corresponds to one-step prediction into the future, measured with respect to the time $t=(i-1) \mathrm{T}$. This form of prediction is referred to as one-step prediction in the forward direction or simply forward prediction.

The one-step forward predictor is as shown in Fig. 8.3.1. It consists of a linear filter with $(\mathrm{g}+1)$ taps. The tap co-efficients of the forward predictor are given by the (g+1)- component vector

$$
F_{g+1, i}=\left[\begin{array}{lllll}
f_{i, 1} & f_{i, 2} & \cdots & f_{i, g+1}
\end{array}\right]
$$

and the data symbols held in the forward predictor are given by the ( $\mathrm{g}+1$ )component vector

$$
S_{g+1, i-1}=\left[\begin{array}{llll}
s_{i-1} & s_{i-2} & \cdots & s_{i-(g+1)}
\end{array}\right]
$$

In Eqns. 8.3.8 and 8.3.9, and henceforth in this Chapter, the first subscript of any vector or matrix represents the order of the vector or matrix. The forward prediction error produced by the predictor at time $t=i T$ in response to the input vector $S_{g+1, i-1}$ is given by

$$
e_{f, i}=s_{i}-\sum_{h=1}^{8+1} f_{i, h} s_{i-h}
$$

where $e_{f, i}$ is referred to as the a-posteriori forward error prediction since its computation is based on the current value of the predictor tap weight vector. The
subscript $f$, in $e_{f i}$, denotes that the error is from forward predictor. Thus the one-step forward predictor can also be represented as a one-step forward prediction error filter, as shown in Fig. 8.3.2, with ( $\mathrm{g}+2$ ) taps. Comparing Figs. 8.3.1 and 8.3.2 with reference to Eqn. 8.3.10, the tap co-efficients of the two filters can be related as follows

$$
\begin{align*}
A_{g+2, i} & =\left[\begin{array}{llllll} 
& a_{i, 0} & a_{i, 1} & a_{i, 2} & \ldots & a_{i, g+1}
\end{array}\right] \\
& =\left[\begin{array}{llllll}
1 & -f_{i, 1} & -f_{i, 2} & \ldots & -f_{i, g+1}
\end{array}\right]
\end{align*}
$$

In $A_{g+2 i}(g+2)$ represents the order, and $i$ represents the time instant.

The $\mathrm{e}_{\mathrm{f}, \mathrm{i}}$ can now be written as

$$
e_{f, i}=A_{g+2, i} S_{g+2, i}^{T}
$$

where

$$
S_{g+2, i}=\left[\begin{array}{lllll}
s_{i} & s_{i-1} & \ldots & s_{i-(g+1)}
\end{array}\right]
$$

Now let

$$
\phi_{g+2, i}=\sum_{h=0}^{i} \lambda^{i-h} S_{g+2, h}^{T} \bar{S}_{g+2, h}
$$

$\phi_{g+2, i}$ is a square matrix of dimension ( $\mathrm{g}+2$ ) $\mathrm{x}(\mathrm{g}+2)$ and is called the auto correlation matrix of the input vector $\mathrm{S}_{\mathrm{g}+2 \mathrm{i}}$ to the forward prediction error filter. The matrix, $\phi_{\delta+2 i,}$, can be partitioned as

$$
\phi_{g+2, i}=\left[\begin{array}{cc}
x_{1, i} & \delta_{1, i} \\
\delta_{1, i}^{*} & \phi_{g+1, i-1}
\end{array}\right]
$$

where the scalar $\mathrm{X}_{1, i}$ is the first element of the matrix $\phi_{8+2, i}$ and is given by

$$
x_{1, i}=\sum_{h=0}^{i} \lambda^{i-h}\left|s_{h}\right|^{2}
$$



Fig. 8.3.1 - One Step Forward Predictor


Fig. 8.3.2 - Forward Prediction Error Filter
and the ( $\mathrm{g}+1$ )- component row vector, $\delta_{1, i}$, in Eqn. 8.3.16, is given by

$$
\delta_{1, i}=\sum_{h=0}^{i} \lambda^{i-h} s_{h} \bar{S}_{g+1, h-1}
$$

$\delta_{1, i}$ is the complex conjugate transpose of $\delta_{1, i} \cdot \phi_{z_{+1, i-1}}$ is $(\mathrm{g}+1) \mathrm{x}(\mathrm{g}+1)$ square matrix and is given by

$$
\phi_{g+1, i-1}=\sum_{h=0}^{i-1} \lambda^{i-1-h} S_{g+1, h}^{T} \bar{S}_{g+1, h}
$$

At time $\mathrm{t}=\mathrm{iT}$, the vector $\mathrm{A}_{g^{+2} 2}$ is obtained in such a way that the cumulative sum of the weighted squared errors from the forward prediction error filter,

$$
\alpha_{i}=\sum_{h=0}^{i} \lambda^{i-h}\left|e_{f, h}\right|^{2}
$$

is minimized, subject to the constraint that the first component of $A_{8+2 i}$ equals unity. $\mathrm{A}_{\mathrm{g}+2 \mathrm{i}}$ now satisfies the relation

$$
A_{g+2, i} \phi_{g+2, i}=\left[\begin{array}{llll}
\alpha_{i} & 0 & \ldots . & 0
\end{array}\right]
$$

Eqn. 8.3.21 is called the augmented normal equation for a forward linear predictor [59].

Combining equations 8.3.12, 8.3.16 and 8.3.21 we have

$$
\left[\begin{array}{ccc}
1 & \left.-F_{g+1, i}\right]
\end{array}\left[\begin{array}{lc}
x_{1, i} & \delta_{1, i} \\
\delta_{1, i}^{*} & \phi_{g+1, i-1}
\end{array}\right]=\left[\begin{array}{cccc}
\alpha_{i} & 0 & \ldots . & 0
\end{array}\right]\right.
$$

The updated estimate of $\mathrm{F}_{\mathrm{g}+1, i}$, is given by

$$
F_{g+1, i}=F_{g+1, i-1}+K_{g+1, i-1} e_{f, p, i}
$$

where $e_{f, p, i}$ is the forward a-priori prediction error and is given by

$$
e_{f, p, i}=s_{i}-F_{g+1, i-1} S_{g+1, i-1}^{T}
$$

$K_{8+1, i-1}$ is the gain vector for the forward prediction filter of order ( $\mathrm{g}+1$ ) and is given by (see Eqn. 5.3.43)

$$
K_{g+1, i-1}=\bar{S}_{g+1, i-1} \phi_{g+1, i-1}^{-1}
$$

Combining equations $8.3 .11,8.3 .12$ and 8.3.23

$$
A_{g+2, i}=A_{g+2, i-1}-\left[\begin{array}{ll}
0 & K_{g+1, i-1}
\end{array}\right] e_{f, p, i}
$$

where

$$
\begin{align*}
e_{f, p, i} & =\left[\begin{array}{ll}
1 & -F_{g+1, i-1}
\end{array}\right]\left[\begin{array}{ll}
s_{i} & S_{g+1, i-1}
\end{array}\right]^{T} \\
& =A_{g+2, i-1} S_{g+2, i}^{T}
\end{align*}
$$

The minimum value of the sum of squares of weighted forward prediction error is given by

$$
\alpha_{i}=\lambda \alpha_{i-1}+e_{f, p, i} e_{f, i}^{*}
$$

In Eqn. 8.3.29, the second term is always a real valued scalar, i.e.

$$
e_{f, p, i} e_{f, i}^{*}=e_{f, p, i}^{*} e_{f, i}
$$

### 8.3.2 ADAPTIVE BACKWARD LINEAR PREDICTOR

In the backward linear predictor, the set of data symbols $\mathrm{s}_{\mathrm{i}}, \mathrm{s}_{\mathrm{i}-1}, \ldots, \mathrm{~s}_{\mathrm{i}, \mathrm{g}}$ is used to make a prediction of the symbol $\mathrm{s}_{\mathrm{i}_{(\mathrm{g}+1)} \text {. }}$. This is a one-step backward prediction. Fig 8.3.3 shows the one-step backward predictor and Fig. 8.3.4 shows the corresponding backward prediction error filter.

The backward prediction error at time $t=i T$ is given by


Fig. 8.3.3 - One Step Backward Predictor


Fig. 8.3.4-Backward Prediction Error Filter

$$
e_{b, i}=s_{i-(g+1)}-\sum_{h=0}^{g} p_{i, h} s_{i-h}
$$

$e_{b, i}$ is referred to as the backward a-posteriori prediction error. The subscript $b$, in $e_{b, j}$, denotes that the error is from the backward predictor. Eqn. 8.3.31 is equivalent to

$$
e_{b, i}=s_{i-(g+1)}-P_{g+1, i} S_{g+1, i}^{T}
$$

The vector

$$
P_{g+1, i}=\left[\begin{array}{llll}
p_{i, 0} & p_{i, 1} & \cdots & p_{i, g}
\end{array}\right]
$$

gives the tap coefficients of the backward predictor and the data symbols held in the backward predictor is given by the ( $\mathrm{g}+1$ )-component vector

$$
S_{g+1, i}=\left[\begin{array}{lllll}
s_{i} & s_{i-1} & s_{i-2} & \cdots & s_{i-g}
\end{array}\right]
$$

The backward predictor, of Fig. 8.3.3, gives an estimate of $\mathrm{s}_{\mathrm{i}-(\xi+1)}$, on the other hand, the output from the backward prediction error filter, of Fig. 8.3.4, is the error in estimating $\mathrm{s}_{\mathrm{i}(-(\mathrm{z}+1)}$. From Figs. 8.3.3 and 8.3.4, the tap coefficients of the backward prediction error filter, are given by the ( $\mathrm{g}+2$ )- component vector

$$
\begin{align*}
B_{g+2, i} & =\left[\begin{array}{lllll}
-P_{g+1, i} & 1 & ]
\end{array}\right. \\
& =\left[\begin{array}{llllll} 
& b_{i, 0} & b_{i, 1} & \ldots \ldots & b_{i, g+1} &
\end{array}\right] \\
& =\left[\begin{array}{llllll}
-p_{i, 0} & -p_{i, 1} & \ldots & -p_{i, g} & 1
\end{array}\right]
\end{align*}
$$

From equations 8.3.32 and 8.3.35, $\mathrm{e}_{\mathrm{b}, \mathrm{i}}$ is now given by

$$
e_{b, i}=B_{g+2, i} S_{g+2, i}^{T}
$$

The desired vector $\mathrm{B}_{\mathrm{g}+2 \mathrm{i}}$ is obtained by minimizing the cumulative sum squares of the weighted backward prediction error, up to the time instant $t=i T$, subjected to the constraint that the last component of $\mathrm{B}_{\mathrm{g}+2 \mathrm{i}}$ equals unity.

The elements of the matrix $\phi_{\boldsymbol{z}+2, i}$, (Eqn. 8.3.15), can now be written in partition form as

$$
\phi_{g+2, i}=\left[\begin{array}{cc}
\phi_{g+1, i} & \delta_{2, i}^{*} \\
\delta_{2, i} & x_{2, i}
\end{array}\right]
$$

where $\phi_{g+1, i}$ is a $(g+1) x(g+1)$ element matrix and is given by,

$$
\phi_{g+1, i}=\sum_{h=0}^{i} \lambda^{i-h} S_{g+1, h}^{T} \bar{S}_{g+1, h}
$$

$\delta_{2 i}$ is a ( $\mathrm{g}+1$ )- component row vector given by

$$
\delta_{2, i}=\sum_{h=0}^{i} \lambda^{i-h} s_{h-(g+1)} \bar{S}_{g+1, h}
$$

$\delta_{2 i}^{*}$ is the complex conjugate transpose of the vector $\delta_{2, i}$. Finally the scalar $x_{2, i}$, in Eqn. 8.3.37, is the last element of the matrix $\phi_{8+2, i}$ and is the weighted sum of the squares of the desired response form the backward predictor, and is given by

$$
x_{2, i}=\sum_{h=0}^{i} \lambda^{i-h}\left|s_{h-(g+1)}\right|^{2}
$$

The vector $\mathrm{B}_{\mathrm{g}+2 \mathrm{i}}$ (Eqn. 8.3.35) giving the minimum sum of weighted backward prediction error squares,

$$
\beta_{i}=\sum_{h=0}^{i} \lambda^{i-h}\left|e_{b, h}\right|^{2}
$$

satisfies

$$
B_{g+2, i} \phi_{g+2, i}=\left[\begin{array}{lllll}
0 & \ldots . & 0 & \beta_{i}
\end{array}\right]
$$

Eqn. 8.3.42 is called the augmented normal equation for a backward predictor [59].

The updated estimate of $\mathrm{P}_{\mathrm{g}+1, \mathrm{i}}$ is given by

$$
P_{g+1, i}=P_{g+1, i-1}+K_{g+1, i} e_{b, p, i}
$$

where $e_{b, p, i}$ is the a-priori prediction error for the backward predictor and is given by

$$
e_{b, p, i}=s_{i-(g+1)}-P_{g+1, i-1} S_{g+1, i}^{T}
$$

$\mathrm{K}_{\mathrm{g}+1, \mathrm{i}}$ is the gain vector for the backward prediction filter and is given by

$$
K_{g+1, i}=\bar{S}_{g+1, i} \phi_{g+1, i}^{-1}
$$

Combining equations 8.3.35 and 8.3.43,

$$
B_{g+2, i}=B_{g+2, i-1}-\left[\begin{array}{ll}
K_{g+1, i} & 0
\end{array}\right] e_{b, p, i}
$$

Also

$$
e_{b, p, i}=\left[\begin{array}{ll}
-P_{g+1, i-1} & 1
\end{array}\right]\left[\begin{array}{ll}
S_{g+1, i} & S_{i-(g+1)}
\end{array}\right]^{T}
$$

or

$$
e_{b, p, i}=B_{g+2, i-1} S_{g+2, i}^{T}
$$

The recursive weighted sum of the square of backward prediction error is given by

$$
\beta_{i}=\lambda \beta_{i-1}+e_{b, p, i} e_{b, i}^{*}
$$

It may be noted that the last term in Eqn. 8.3.48, is real valued scalar so that

$$
e_{b, p, i} i_{b, i}^{*}=e_{b, p, i}^{*} e_{b, i}
$$

It can thus be seen that, $\mathrm{K}_{\mathrm{g}+1, \mathrm{i}}$ is used in updating the backward prediction filter, whereas $\mathrm{K}_{\mathrm{k}_{\mathrm{+1}, \mathrm{i}-1}}$ is used in updating the forward prediction filter. Another transversal filter called the gain transversal filter is used to obtain $\mathrm{K}_{\mathrm{g}+1, \mathrm{i}}$ from $\mathrm{K}_{\mathrm{g}+1, \mathrm{j} 1}$.

### 8.3.3 GAIN TRANSVERSAL FILTER

The Gain vectors for the forward and backward prediction filter are given by Eqn. 8.3.25 and Eqn. 8.3.45 respectively. $\mathrm{K}_{\mathrm{k}+2 \mathrm{i}}$, is given by

$$
K_{g+2, i}=\bar{S}_{g+2, i} \Phi_{g+2, i}^{-1}
$$

The inverse of the matrix $\phi_{8+2, i}$, (Eqn.8.3.15), can be expressed as, [59],

$$
\phi_{g+2, i}^{-1}=\left[\begin{array}{cc}
O & O_{g+1} \\
O_{g+1}^{T} & \phi_{g+1, i-1}^{-1}
\end{array}\right]+\frac{1}{\alpha_{i}} A_{g+2, i}^{T} \bar{A}_{g+2, i}
$$

where $\mathrm{O}_{\mathrm{g}+1}$ is a $(\mathrm{g}+1)$ - component zero vector. Pre-multiplying Eqn. 8.3.50 by $\bar{s}_{\mathrm{s}+2 \mathrm{i}}$ and simplifying,

$$
K_{g+2, i}=\left[\begin{array}{ll}
0 & K_{g+1, i-1}
\end{array}\right]+\frac{e_{f, i}^{*}}{\alpha_{i}} A_{g+2, i}
$$

since

$$
e_{f, i}^{*}=\bar{A}_{g+2, i} S_{g+2, i}^{*}
$$

$S_{8+2 i}^{*}$ is the complex conjugate transpose of the vector $\mathrm{S}_{g^{2}+2 i}$.
Similarly the inverse of $\phi_{g+2 i}$ can be expressed as, [59]

$$
\phi_{g+2, i}^{-1}=\left[\begin{array}{cc}
\phi_{g+1, i}^{-1} & O_{g+1} \\
O_{g+1}^{T} & O
\end{array}\right]+\frac{1}{\beta_{i}} B_{g+2, i}^{T} \bar{B}_{g+2, i}
$$

Pre-multiplying Eqn. 8.3.53 by $\bar{S}_{8+2 ; i}$ and simplifying,

$$
\left.K_{g+2, i}=\begin{array}{ll}
K_{g+1, i} & 0
\end{array}\right]+\frac{e_{b, i}^{*}}{\beta_{i}} B_{g+2, i}
$$

Since $e_{b, i}^{\circ}$, in Eqn. 8.3.54, is given by

$$
e_{b, i}^{*}=\bar{B}_{g+2, i} S_{g+2, i}^{*}
$$

The gain vector $\mathrm{K}_{\mathrm{g}+1, \mathrm{i}}$, (Eqn. 8.3.45), can be considered to be the tap coefficients of a transversal filter with $\mathrm{g}+1$ taps, and the data symbols held in the filter being given
by the vector $\mathrm{S}_{\mathrm{s}+1 \mathrm{j}}$, Eqn. 8.3.34 ( see Fig. 8.3.5 ). The output from the filter is the least-squares estimate of the desired response, $\mathrm{d}_{\mathrm{i}}[4]$, where $\mathrm{d}_{\mathrm{i}}$ has a value of unity at time $t=\mathrm{iT}$ and is zero elsewhere [59].

The error in estimating $\mathrm{d}_{\mathrm{i}}$, in Fig. 8.3.5 is, therefore,

$$
\gamma_{i}=1-K_{g+1, i} S_{g+1, i}^{T}
$$

Combining equations 8.3.45 and 8.3.56

$$
\gamma_{i}=1-\bar{S}_{g+1, i} \phi_{g+1, i}^{-1} S_{g+1, i}^{T}
$$

Since $\phi_{8}^{-1}+1, i$ is an Hermitian matrix, it is evident from Eqn. 8.3.57 that $\gamma_{i}$ is real valued, and has the limits given by [59, 84, 87, 101]

$$
0 \leq \gamma_{i} \leq 1
$$

Post-multiplying both sides of Eqn. 8.3.26 by $S_{8+2 i}^{T}$,

$$
A_{g+2, i} S_{g+2, i}^{T}=A_{g+2, i-1} S_{g+2, i}^{T}-\left[0 \quad K_{g+1, i-1}\right] S_{g+2, i}^{T} e_{f, p, i}
$$

or

$$
\begin{align*}
e_{f, i} & =e_{f, p, i}-\left[\begin{array}{ll}
0 & K_{g+1, i-1}
\end{array}\right]\left[\begin{array}{ll}
s_{i} & S_{g+1, i-1}
\end{array}\right]^{T} e_{f, p, i} \\
& =e_{f, p, i}-\left[\begin{array}{l}
1 \\
-
\end{array} \gamma_{i-1}\right] e_{f, p, i}
\end{align*}
$$

or

$$
\gamma_{i-1}=\frac{e_{f, i}}{e_{f, p, i}}
$$

Similarly the following equations can be obtained.

$$
\gamma_{i}=\frac{e_{b, i}}{e_{b, p, i}}
$$


Fig. 8.3.5-Gain Transversal Filter.

$$
\gamma_{i}=\frac{e_{i}}{e_{p, i}}
$$

where $e_{p, i}$ and $e_{i}$ are the a-priori and a-posteriori error, respectively, in the estimation of the received signal, $\mathrm{r}_{\mathrm{i}}$.

Post-multiplying Eqn. 8.3 .51 by $s_{s+2 i}^{T}$,

$$
\begin{aligned}
K_{g+2, i} S_{g+2, i}^{T} & =\left[\begin{array}{ll}
0 & K_{g+1, i-1}
\end{array}\right] S_{g+2, i}^{T} \\
& +\frac{e_{f, i}^{*}}{\alpha_{i}} A_{g+2, i} S_{g+2, i}^{T}
\end{aligned}
$$

Using Eqn. 8.3.56, this reduces to

$$
\left(1-\gamma_{1, i}\right)=\left(1-\gamma_{i-1}\right)+\frac{e_{f, i}^{*}}{\alpha_{i}} e_{f, i}
$$

or

$$
\gamma_{1, i}=\gamma_{i-1}-\frac{\left|e_{f, i}\right|^{2}}{\alpha_{i}}
$$

where $\gamma_{1, i}$ is the error in the estimation of $d_{i}$, from the extended gain transversal filter, of order $(\mathrm{g}+2)$, with tap coefficient, $\mathrm{K}_{\mathrm{g}+2 \mathrm{i}}$.

Similarly post-multiplying Eqn. 8.3.54 by $S_{8+2 i}^{T}$ and simplifying,

$$
\gamma_{1, i}=\gamma_{i}-\frac{\left|e_{b, i}\right|^{2}}{\beta_{i}}
$$

From Eqn. 8.3.29,

$$
\frac{\lambda \alpha_{i-1}}{\alpha_{i}}=1-\frac{e_{f, p, i} e_{f, i}^{*}}{\alpha_{i}}
$$

Using equations 8.3.60 and 8.3.63

$$
\frac{\lambda \alpha_{i-1}}{\alpha_{i}}=1-\frac{\left|e_{f, i}\right|}{\gamma_{i-1} \alpha_{i}}=\frac{\gamma_{1, i}}{\gamma_{i-1}}
$$

Therefore,

$$
\gamma_{1, i}=\lambda \frac{\alpha_{i-1}}{\alpha_{i}} \gamma_{i-1}
$$

Similarly using equations $8.3 .48,8.3 .61$ and 8.3 .64 , it can be shown that

$$
\gamma_{1, i}=\lambda \frac{\beta_{i-1}}{\beta_{i}} \gamma_{i}
$$

Thus, the three filters that provide the necessary inputs to solve the RLS problem, defined in Eqn. 8.3.4, have now been defined. Fig. 8.3.6 shows the parameters evaluated by the transversal filter.

The update of the tap coefficients of all the Transversal filters will now be considered. The four transversal filters in Fig. 8.3.6 hold a common ( $\mathrm{g}+2$ )components data vector (Eqn. 8.3.14).

The update of the extended gain vector is obtained as follows. Substituting Eqn. 8.3.26 in Eqn. 8.2.51,

$$
\begin{align*}
K_{g+2, i} & =\left[\begin{array}{ll}
0 & K_{g+1, i-1}
\end{array}\right]+\frac{e_{f, i}^{*}}{\alpha_{i}}\left\{A_{g+2, i-1}-\left[\begin{array}{ll}
0 & K_{g+1, i-1}
\end{array}\right] e_{f, p, i}\right\} \\
& =\left\{1-\frac{e_{f, i}^{*} e_{f, p, i}}{\alpha_{i}}\right\}\left[\begin{array}{ll}
0 & K_{g+1, i-1}
\end{array}\right]+\frac{e_{f, i}^{*}}{\alpha_{i}} A_{g+2, i-1}
\end{align*}
$$

From Eqns. 8.3.29 and 8.3.65

$$
\left\{1-\frac{e_{f, i}^{*} e_{f, p, i}}{\alpha_{i}}\right\}=\frac{\lambda \alpha_{i-1}}{\alpha_{i}}=\frac{\gamma_{1, i}}{\gamma_{i-1}}
$$

Therefore, Eqn. 8.3.67 can be written as


Fig. 8.3.6-Transversal Filter
Computation Of RLS Variables

$$
K_{g+2, i}=\frac{\gamma_{1, i}}{\gamma_{i-1}}\left[\begin{array}{ll}
0 & K_{g+1, i-1}
\end{array}\right]+\frac{e_{f, i}^{*}}{\alpha_{i}} A_{g+2, i-1}
$$

Let

$$
\tilde{K}_{g+2, i}=\frac{K_{g+2, i}}{\gamma_{1, i}}
$$

$\bar{K}_{g+2, i}$ is called the normalized gain vector. Therefore, from Eqns. 8.3.68 and 8.3.69

$$
\bar{K}_{g+2, i}=\left[\begin{array}{ll}
0 & \bar{K}_{g+1, i-1}
\end{array}\right]+\frac{e_{f, i}^{*}}{\gamma_{1, i} \alpha_{i}} A_{g+2, i-1}
$$

From Eqns. 8.3.60, 8.3.65 and 8.3.70

$$
\tilde{K}_{g+2, i}=\left[\begin{array}{ll}
0 & \tilde{K}_{g+1, i-1}
\end{array}\right]+\lambda^{-1} \frac{e_{f, p, i}^{*}}{\alpha_{i-1}} A_{g+2, i-1}
$$

An update of $\mathrm{A}_{8+2, i}$ is obtained using Eqn. 8.3.26. Substituting $\bar{K}_{8+1, i-1}$, (using the definition in Eqn. 8.3.69), in place of $\mathrm{K}_{\mathrm{g}+1, \mathrm{i}-1}$ in Eqn. 8.3.26,

$$
A_{g+2, i}=A_{g+2, i-1}-\left[\begin{array}{ll}
0 & \tilde{K}_{g+1, i-1}
\end{array}\right] \gamma_{i-1} e_{f, p, i}
$$

or

$$
A_{g+2, i}=A_{g+2, i-1}-\left[\begin{array}{ll}
0 & \tilde{K}_{g+1, i-1}
\end{array}\right] e_{f, i}
$$

Consider Eqn. 8.3.54

$$
K_{g+2, i}=\left[\begin{array}{ll}
K_{g+1, i} & 0
\end{array}\right]+\frac{e_{b, i}^{*}}{\beta_{i}} B_{g+2, i}
$$

Substituting Eqn. 8.3.46, for $\mathrm{B}_{\mathrm{g}^{+2 i}}$.

$$
\begin{align*}
K_{g+2, i} & =\left[\begin{array}{ll}
K_{g+1, i} & 0
\end{array}\right]+\frac{e_{b, i}^{*}}{\beta_{i}} \begin{cases}\left.B_{g+2, i-1}-\left[\begin{array}{ll}
K_{g+1, i} & 0
\end{array}\right] e_{b, p, i}\right\} \\
& =\left\{\begin{array}{ll}
1-\frac{e_{b, p, i} e_{b, i}^{*}}{\beta_{i}}
\end{array}\right\}\left[\begin{array}{ll}
K_{g+1, i} & 0
\end{array}\right]+\frac{e_{b, i}^{*}}{\beta_{i}} B_{g+2, i-1}\end{cases}
\end{align*}
$$

But from Eqns. 8.3.48 and 8.3.66

$$
\left\{1-\frac{e_{b, p, i} e_{b, i}^{*}}{\beta_{i}}\right\}=\frac{\lambda \beta_{i-1}}{\beta_{i}}=\frac{\gamma_{1, i}}{\gamma_{i}}
$$

Therefore, from Eqn. 8.3.73 and 8.3.74

$$
K_{g+2, i}=\left[\begin{array}{ll}
K_{g+1, i} & 0
\end{array}\right] \frac{\gamma_{1, i}}{\gamma_{i}}+\frac{e_{b, i}^{*}}{\beta_{i}} B_{g+2, i-1}
$$

or (using Eqn. 8.3.69)

$$
\begin{align*}
\tilde{K}_{g+2, i} & =\left[\begin{array}{ll}
\bar{K}_{g+1, i} & 0
\end{array}\right]+\frac{e_{b, i}^{*}}{\gamma_{1, i} \beta_{i}} B_{g+2, i-1} \\
& =\left[\begin{array}{ll}
\tilde{K}_{g+1, i} & 0
\end{array}\right]+\frac{e_{b, p, i}^{*} \gamma_{i}}{\gamma_{1, i} \beta_{i}} B_{g+2, i-1}
\end{align*}
$$

and from Eqns. 8.3.66 and 8.3.75

$$
\bar{K}_{g+2, i}=\left[\begin{array}{ll}
\bar{K}_{g+1, i} & 0
\end{array}\right]+\lambda^{-1} \frac{e_{b, p, i}^{*}}{\beta_{i-1}} B_{g+2, i-1}
$$

or

$$
\left[\begin{array}{ll}
\tilde{K}_{g+1, i} & 0
\end{array}\right]=\bar{K}_{g+2, i}-\lambda^{-1} \frac{e_{b, p, i}^{*}}{\beta_{i-1}} B_{g+2, i-1}
$$

The last element of $\tilde{K}_{g+2, i}$ is given by (from Eqns. 8.3.75 and 8.3.77),

$$
\tilde{k}_{g+2, g+2}=\frac{\lambda^{-1} e_{b, p, i}^{*}}{\beta_{i-1}}=\frac{e_{b, i}^{*}}{\gamma_{1, i} \beta_{i}}
$$

since the last term of $\mathrm{B}_{\mathrm{k}+2 \cdot \mathrm{i}-1}$ is to be equal to unity.

Therefore,

$$
\left[\begin{array}{cc}
\tilde{K}_{g+1, i} & 0
\end{array}\right]=\tilde{K}_{g+2, i}-\tilde{K}_{g+2, g+2} B_{g+2, i-1}
$$

Also from Eqn. 8.3.78,

$$
e_{b, p, i}=\lambda \beta_{i-1} \tilde{k}_{g+2, g+2}^{*}
$$

It is thus seen that $\mathrm{e}_{\mathrm{b}, \mathrm{p}, \mathrm{i}}$ can be obtained either using Eqn. 8.3.47 or using Eqn. 8.3.80. In the computer-simulation tests, however, $\mathrm{e}_{\mathrm{b}, \mathrm{p}, \mathrm{i}}$ is obtained from Eqn. 8.3.47, the reasons for which are discussed in detail in Section 8.4.

From Eqn. 8.3.66

$$
\gamma_{i}=\frac{\beta_{i}}{\lambda \beta_{i-1}} \gamma_{1, i}
$$

and from Eqn. 8.3.48

$$
\frac{\lambda \beta_{i-1}}{\beta_{i}}=1-\frac{e_{b, p, i} e_{b, i}^{*}}{\beta_{i}}
$$

Combining Eqns. 8.3.78 and 8.3.82

$$
\frac{\lambda \beta_{i-1}}{\beta_{i}}=1-e_{b, p, i} \gamma_{1, i} \bar{k}_{g+2, g+2}
$$

Therefore, from Eqns. 8.3.81 and 8.3.83

$$
\gamma_{i}=\left[1-e_{b, p, i} \gamma_{1, i} \bar{k}_{g+2, g+2}\right]^{-1} \gamma_{1, i}
$$

From Eqn. 8.3.46, the update of $\mathrm{B}_{8+2 i}$ is

$$
\begin{align*}
B_{g+2, i} & =B_{g+2, i-1}-\left[\begin{array}{ll}
K_{g+1, i} & 0
\end{array}\right] e_{b, p, i} \\
& =B_{g+2, i-1}-\left[\begin{array}{ll}
\bar{K}_{g+1, i} & 0
\end{array}\right] \gamma_{i} e_{b, p, i} \\
& =B_{g+2, i-1}-\left[\begin{array}{ll}
\bar{K}_{g+1, i} & 0
\end{array}\right] e_{b, i}
\end{align*}
$$

Finally the update of the estimate of the sampled impulse-response of the HF channel is obtained as follows. The estimate of the received signal is (Eqn. 8.3.2),

$$
r_{i}^{\prime}=Y_{g+1, i-1}^{\prime} S_{g+1, i}^{T}
$$

The actual received signal is given in Eqn. 8.2.1. The $\mathrm{e}_{\mathrm{p}, \mathrm{i}}$ is given by

$$
e_{p, i}=r_{i}-Y_{g+1, i-1}^{\prime} S_{g+1, i}^{T}
$$

and $\mathrm{e}_{\mathrm{i}}$ is

$$
e_{i}=e_{p, i} \gamma_{i}
$$

The update of $Y_{s+1 ;}^{\prime}$ is given by

$$
Y_{g+1, i}^{\prime}=Y_{g+1, i-1}^{\prime}+\tilde{K}_{g+1, i} e_{i}
$$

A complete summary of the steady state algorithm is given in Table 8.3.1.

### 8.4 STABILIZATION OF FTF ALGORITHM.

FTF algorithms are known to exhibit numerical instability [111-116] and this restricts the use of FTF algorithms to only a limited period of time, beyond which the estimation of the sampled impulse-response of the channel is incorrect. Instability occurs when, either, the value of $\gamma_{i}$ exceeds unity (its theoretical maximum value) or when it diverges towards zero. When $\gamma_{i}$ exceeds unity the co-efficients of the four transversal filters diverges to infinity [84, 112-114], and

$$
\begin{aligned}
& e_{f, p, i}=A_{g+2, i-1} S_{g+2, i}^{T} \\
& e_{f, i}=\gamma_{i-1} e_{f, p, i} \\
& \alpha_{i}=\lambda \alpha_{i-1}+e_{f, p, i} e_{f, i}^{*} \\
& \gamma_{1, i}=\lambda \frac{\alpha_{i-1}}{\alpha_{i}} \gamma_{i-1} \\
& \tilde{K}_{g+2, i}=\left[0 \quad \tilde{K}_{g+1, i-1}\right]+\lambda^{-1} \frac{e_{f, p, i}^{*}}{\alpha_{i-1}} A_{g+2, i-1} \\
& A_{g+2, i}=A_{g+2, i-1}-\left[0 \quad \tilde{K}_{g+1, i-1}\right] e_{f, i} \\
& e_{b, p, i}=B_{g+2, i-1} S_{g+2, i}^{T} \\
& \zeta_{i}=B_{g+2, i-1} S_{g+2, i}^{T}-\lambda \beta_{i-1} \tilde{k}_{g+2, g+2}^{*} \\
& \gamma_{i}=\left(1-e_{b, p, i} \gamma_{1, i} \tilde{k}_{g+2, g+2}\right)^{-1} \gamma_{1, i} \\
& e_{b, i}=\gamma_{i} e_{b, p, i} \\
& \beta_{i}=\lambda \beta_{i-1}+e_{b, p, i} e_{b, i}^{*} \\
& {\left[\tilde{K}_{g+1, i}\right.} \\
& \left.B_{g+2, i}=0\right]=\tilde{K}_{g+2, i}-\tilde{k}_{g+2, g+2} B_{g+2, i-1} \\
& e_{p, i}=r_{i}-Y_{g+1, i-1}^{\prime} S_{g+1, i}^{T} \\
& e_{i}=\gamma_{i} e_{p, i} \\
& Y_{g+1, i}^{\prime}=Y_{g+1, i-1}^{\prime}+\tilde{K}_{g+1, i} e_{i}
\end{aligned}
$$

when $\gamma_{i}$ diverges to zero, the gain vector $\mathrm{K}_{\mathrm{g}+1, \mathrm{i}}$ also drifts to zero. Under the former condition, the estimation process fails, and under the latter condition, the tracking ability of the algorithm is degraded [112-114].

Several techniques have been proposed to overcome the problem of instability [59, 84, 112-115]. A periodic reinitialization procedure is proposed in [59], were the operation of the FTF algorithm is interrupted and then restarted at periodic intervals. Immediately following such a restart, a simple LMS algorithm provides an estimate of the desired response, with the coefficients of the feedforward filter set to the value attained by the FTF algorithm just before the restart was initiated. This procedure is depicted in Fig. 8.4.1. This method is not particularly suitable for estimating a time varying channel as the LMS algorithm does not quickly adapt to the transition from the FTF algorithm, owing to its slow rate of convergence. Thus change-overs, between FTF and LMS algorithms, introduce large errors into the estimation of the channel sampled impulse-response [113].

The parameter $\mathrm{e}_{\mathrm{b}, \mathrm{p}, \mathrm{i}}$, can be obtained either using a simple but numerically unstable relation $\lambda \cdot \beta_{i-1} \cdot \tilde{k}_{g+2,8+2}$, (Eqn. 8.3.80), requiring only two multiplications or using a more complicated but more stable relation $B_{8+2, i-1} S_{8+2, i}^{T}$, (Eqn. 8.3.47), which requires ( $\mathrm{g}+2$ ) multiplications. [59, 84, 101, 111-114].

Let

$$
\zeta_{i}=B_{g+2, i-1} S_{g+2, i}^{T}-\lambda \beta_{i-1} \tilde{k}_{g+2, g+1}^{*}
$$

$\zeta_{i}$ is exactly equal to zero when the equations of the algorithm, in Table 8.3.1, are evaluated with infinite precision. However, simulation tests have shown that, due to the occurrence of numerical round-off errors, $\left|\zeta_{i}\right|$ grows exponentially with time [111-114]. $\zeta_{i}$, therefore, gives a measure of the round-off error [111-114]. In [84], a version of a stable algorithm, $\mathrm{e}_{\mathrm{b}, \mathrm{p}, \mathrm{i}}$ is given by $B_{8+2, i-1} S_{8+2 ; \cdot}^{T}$. There is, however, only a marginal increase in the period of stable operation.

The stabilization algorithm of [112-114] attempts to minimize $\chi_{i}$, given by [111114],

$$
\begin{align*}
\chi_{i}=\sum_{h=0}^{i} \lambda^{i-h} & {\left[\left\{s_{i-(g+1)}-B_{g+2, i-1} S_{g+2, i}^{T}\right\}\right.} \\
& \left.-\left\{s_{i-(g+1)}-B_{g+2, i-1}^{\prime} S_{g+1, i}^{T}\right\}\right]^{2} \\
& +\rho \zeta_{i}^{\prime 2}
\end{align*}
$$

where

$$
\zeta_{i}^{\prime}=e_{b, p, i}^{\prime}-\lambda \beta_{i-1} \bar{k}_{g+2, g+2}^{*}
$$

and

$$
e_{b, p, i}^{\prime}=B_{g+2, i-1}^{\prime} S_{g+2, i}^{T}
$$

The algorithm finds a modified set of tap coefficients, $B_{8+2 i-1}^{\prime}$, for the backward prediction error filter, in place of $\mathrm{B}_{\mathrm{g}^{2} 2 \cdot \mathrm{i} 1}$, and attempts to force $\zeta_{i}$ to zero. $\rho$ is a scalar constant in the range 0 to 1 . Computer-simulation tests on the application of the algorithm to acoustic echo cancellation have shown an improved stability [ 113 ] as compared to the original FTF algorithm [84, 87]. However, contrary to claims in [113], instability has been observed in moderately severe environments [111].

The new stabilization technique proposed here, makes only passive use of the parameter $\zeta_{i}$ defined in Eqn. 8.4.1, and does not attempt to modify $\mathrm{B}_{\mathrm{g}+2, i-1}$, as has been the case in [113]. As a matter of fact, this technique takes advantage of the instability measure, Eqn. 8.4.1, proposed in [113] and $e_{b, p, i}$ is measured using $B_{g+2, i-1} S_{z+2, i}^{T}$. When the absolute value of the control variable, $\zeta_{i}$, exceeds a certain threshold, a parallel FTF algorithm is initialized, while the original FTF algorithm is still operational and still providing an estimate of the sampled impulse-response of the channel. The threshold value is decided in such a way that the parallel FTF algorithm has sufficient time to start up, while at the same time the original FTF algorithm has not completely failed during this period. When the parallel FTF algorithm is fully operational it takes over from the original FTF algorithm, to provide the estimate of the channel impulse-response. The arrangement is shown in Fig. 8.4.2. The process continues, as in Fig. 8.4.2, every time the control variable exceeds the threshold limit. The algorithm is only slightly more complex than the original FTF algorithm [84], since during the period when the parallel FTF algorithm is operational, most of the tap co-efficients of the forward and backward prediction


> Initiation of restart cycle $t=(i-N) T$

Transition
from LMS to FTF algorithm $\mathrm{t}=\mathrm{i} \mathrm{T}$

Initiation of next restart cycle $\mathrm{t}=(\mathrm{i}+\mathrm{M}) \mathrm{T}$

Fig 8.4.1 - One cycle of the periodic reinitialization procedure of the FTF algorithm


Fig 8.4.2 - One cycle of the stabilization technique for the FTF algorithm
filters are zero. The input vectors to both the original and parallel FTF algorithm is $\mathrm{S}_{\mathrm{g}+2 \mathrm{i}}$, (Eqn. 8.3.14), and so, at the time of take over from the parallel FTF algorithm, the normalized Kalman gain vector, $\bar{K}_{g+1, i}$, is nearly the same from the two algorithm. This is due to the fact that FTF algorithm has a fast tracking ability and, therefore, the period of operation of the parallel FTF algorithm generally need not be very large.

Tests have shown that the period over which the FTF algorithm is stable is, to a large extent, a function of the factor $\lambda$, in Eqn. 8.3.4, which in turn is a function of the system signal/noise ratio. The closer the optimum value of $\lambda$ is equal to 1 , the longer is the period of stable operation. For the lower signal/noise ratios, the optimum value of $\lambda$ is much closer to 1 . Simulation tests on the new stabilization technique, for HF channel estimation, have shown that this technique provides adequate stabilization, at the required signal/noise ratios.

### 8.5 INITIALIZATION OF THE ALGORITHM

Prior to the actual transmission of data, training signal, comprising of a particular known sequence of data symbols, is transmitted. Since the data symbols are known at the receiver, the latter can estimate quite accurately, the initial sampled impulse-response of the channel [91]. This prior knowledge of the channel estimate, at the start of the estimation process using the FTF algorithm, permits the use of non zero initial conditions [59, 84]. At the start of operation of the FTF algorithm, the vectors $\mathrm{A}_{8+2,0}, \mathrm{~B}_{8+2,20}, \bar{K}_{g+1,0}$ and $Y_{8+1,0}^{\prime}$ are set to the following values

$$
\begin{align*}
& A_{g+2,0}=\left[\begin{array}{llllll}
1 & 0 & \ldots . & 0 & ] \\
B_{g+2,0} & =\left[\begin{array}{llllll}
0 & \ldots . . & 0 & 1
\end{array}\right] \\
K_{g+1,0} & =\left[\begin{array}{llllll} 
& 0 & 0 & \ldots . & 0
\end{array}\right] \\
Y_{g+1,0}^{\prime} & =Y_{g+1,0}
\end{array}\right.
\end{align*}
$$

$Y_{8+1,0}^{\prime}$ is determined from the training signal at the start of transmission, and (in Table 8.3.1)

$$
\begin{align*}
& \alpha_{0}=\lambda^{g+1} \mu \\
& \beta_{0}=\mu \\
& \gamma_{0}=1
\end{align*}
$$

The parameter $\mu$ is a scalar constant that adjusts the tracking ability of the algorithm [59, 84].

### 8.6 SYSTEM 8.2

System 8.2 uses the algorithm of system 8.1 , modified by the assumption that the sampled impulse-response of the channel varies linearly with time. It has been shown in $[99,103]$ that a useful improvement in the performance of the RLS Kalman algorithm can be achieved by the given assumption. Eqn. 8.3.89 is now modified to

$$
Y_{g+1, i}^{\prime}=Y_{g+1, i, i-1}^{\prime}+\tilde{K}_{g+1, i} e_{i}
$$

where $Y_{\xi+1, i, i-1}^{\prime}$ is the prediction of $Y_{i}$ made at time $\mathrm{t}=(\mathrm{i}-1) \mathrm{T}$ and $r_{i}^{\prime} \mathrm{in}$ Eqn. 8.3.2 is now given by

$$
r_{i}^{\prime}=Y_{g+1, i, i-1}^{\prime} S_{i}^{T}
$$

$Y_{\xi+1, i}^{\prime}$ is the updated estimate of the channel sampled impulse-response at time $\mathrm{t}=\mathrm{i} \mathrm{T}$. System 8.2 makes a one-step prediction of the channel sampled-impulse response using a least-squares fading memory prediction. This is achieved by $g+1$ separate degree-1 least squares fading memory polynomial filters, each operating on the corresponding component of $Y_{8+1, i, i-1}^{\prime}[33,53]$. Further details of these filters are given elsewhere [33, 53].

The estimator of system 8.2 uses the updated estimate of $Y_{z+1, i}^{\prime}$ given by Eqn. 8.6.1 and the one step prediction of $Y_{\xi+1, i, i-1}^{\prime}$ to determine an estimate of the error in the prediction which is

$$
X_{i}=Y_{g+1, i}^{\prime}-Y_{g+1, i, i-1}^{\prime}
$$

A one-step prediction is now given by a polynomial filter, which is described by the following two equations

$$
\begin{align*}
& Y_{g+1, i+1, i}^{\prime \prime}=Y_{g+1, i, i-1}^{\prime \prime}+\theta_{1} X_{i} \\
& Y_{g+1, i+1, i}^{\prime}=Y_{g+1, i, i-1}^{\prime}+Y_{g+1, i+1, i}^{\prime \prime}+\theta_{2} X_{i}
\end{align*}
$$

where $\theta_{1}$ and $\theta_{2}$ are scalar constants that may be selected as required and are, therefore, adjusted to minimize the error in the prediction of the channel impul-se-response. In the original prediction algorithm $[19,53] \theta_{1}$ and $\theta_{2}$ are given by $(1-\theta)^{2}$ and $\left(1-\theta^{2}\right)$, where $0<\theta<1$. A further development of the algorithm, studied here, is to allow $\theta_{1}$ and $\theta_{2}$ to be optimized independently. At the start of the prediction process

$$
Y_{g+1,1,0}^{\prime \prime}=0
$$

and

$$
Y_{g+1,1,0}^{\prime}=Y_{g+1,0}^{\prime}
$$

where $Y_{\xi+1,0}^{\prime}$ is determined from the training signal that precedes the transmission of data [91]. The initialization of the estimation process is carried out according to Section 8.5.

Computer-simulation tests, on the accuracy of the one-step prediction given by Eqns. 8.6 .4 and 8.6 .5 , for use with the FTF algorithm, have shown a useful improvement in the performance of the estimator without any sign of instability. Table 8.6.1 summarises the complete algorithm for system 8.2 and the results of the com-puter-simulation tests are presented in Section 8.7.

## UPDATING ALGORITHM

$$
\begin{aligned}
& e_{f, p, i}=A_{g+2, i-1} S_{g+2, i}^{T} \\
& e_{f, i}=\gamma_{i-1} e_{f, p, i} \\
& \alpha_{i}=\lambda \alpha_{i-1}+e_{f, p, i} e_{f, i}^{*} \\
& \gamma_{1, i}=\lambda \frac{\alpha_{i-1}}{\alpha_{i}} \gamma_{i-1} \\
& \bar{K}_{g+2, i}=\left[0 \quad \tilde{K}_{g+1, i-1}\right]+\lambda^{-1} \frac{e_{f, p, i}^{*}}{\alpha_{i-1}} A_{g+2, i-1} \\
& A_{g+2, i}=A_{g+2, i-1}-\left[0 \quad \tilde{K}_{g+1, i-1}\right] e_{f, i} \\
& e_{b, p, i}=B_{g+2, i-1} S_{g+2, i}^{T} \\
& \zeta_{i}=B_{g+2, i-1} S_{g+2, i}^{T}-\lambda \beta_{i-1} \tilde{k}_{g+2, g+2}^{*} \\
& \gamma_{i}=\left(1-e_{b, p, i} \gamma_{1, i} \tilde{k}_{g+2, g+2}\right)^{-1} \gamma_{1, i} \\
& e_{b, i}=\gamma_{i} e_{b, p, i} \\
& \beta_{i}=\lambda \beta_{i-1}+e_{b, p, i} e_{b, i}^{*} \\
& {\left[\tilde{K}_{g+1, i}\right.} \\
& \left.B_{g+2, i}=0\right]=\tilde{K}_{g+2, i}-\tilde{k}_{g+2, g+2} B_{g+2, i-1} \\
& e_{p, i}=r_{i}-Y_{g+1, i-1}^{\prime} S_{g+1, i}^{T} \\
& e_{i}=\gamma_{i} e_{p, i} \\
& Y_{g+1, i}^{\prime}=Y_{g+1, i, i-1}^{\prime}+\tilde{K}_{g+1, i} \\
& 0] e_{b, i} \\
& l_{g+1, i} e_{i}
\end{aligned}
$$

## PREDICTION ALGORITHM

$$
\begin{aligned}
& X_{i}=Y_{g+1, i}^{\prime}-Y_{g+1, i, i-1}^{\prime} \\
& Y_{g+1, i+1, i}^{\prime \prime}=Y_{g+1, i, i-1}^{\prime \prime}+\theta_{1} X_{i} \\
& Y_{g+1, i+1, i}^{\prime}=Y_{g+1, i, i-1}^{\prime}+Y_{g+1, i+1, i}^{\prime \prime}+\theta_{2} X_{i}
\end{aligned}
$$

### 8.7 RESULTS OF THE COMPUTER-SIMULATION TEST.

Computer-simulation tests have been carried out on systems 8.1 and 8.2. The results of the tests are given in Tables 8.7.1 to 8.7.3 and in Figs. 8.7.1 to 8.7.5. The error measurements are

$$
\xi_{1}=10 \log _{10}\left(\frac{1}{54000} \sum_{i=6001}^{60000}\left|Y_{g+1, i}-Y_{g+1, i, i-1}^{\prime}\right|^{2}\right)
$$

and

$$
\xi_{2}=10 \log _{10}\left(\frac{1}{54000} \sum_{i=6001}^{60000} \frac{\left|Y_{g+1, i}-Y_{g+1, i, i-1}^{\prime}\right|^{\prime}}{\left|Y_{g+1, i}\right|^{2}}\right)
$$

In the case of system $8.2 Y_{\xi+1, i}^{\prime}=Y_{\xi+1, i, i-1}^{\prime}$. The parameter $\xi_{1}$ is a measure of the actual error in $Y_{8+1, i, i-1}^{\prime}$, whereas, the parameter $\xi_{2}$ is a measure of the normalized or relative error in $Y_{\xi+1, i, i-1}^{\prime}$. During the first 6000 received samples the estimation process operates as described in Sections 8.3 and 8.6, but no measurements are carried out. This stabilizes the fading, additive noise and the estimation process, thus eliminating the effect of any transient behaviour of the estimator at start up. Measurements are carried out according to Eqns. 8.7.1 and 8.7.2 over the next 54000 received samples. Thus $\xi_{1}$ and $\xi_{2}$ measure the steady-state performance of an estimator. In Eqns. 8.7.1 and 8.7.2, $\left|Y_{g+1, i}-Y_{g+1, i, i-1}^{\prime}\right|$ is the unitary length of the vector $Y_{g+1, i}-Y_{g+1, i, i-1}^{\prime}$ and so is the unitary distance between the vectors $Y_{g+1, i}$ and $Y_{\xi+1, i, i-1}^{\prime}$. In Eqn. 8.7.2, this unitary distance has been normalized by the length of the vector $Y_{g+1, i}$.

In all the tests the signal/noise ratio is measured as $\psi \mathrm{dB}$, where

$$
\psi=10 \quad \log _{10}\left(\frac{E_{b}}{\frac{1}{2} N_{0}}\right)
$$

and $E_{b}$ is the average transmitted energy per bit at the input to the HF radio link and is arranged to be unity. The two sided power spectral density of the white Gaussian noise at the output of the HF radio link is $(1 / 2) \mathrm{N}_{0}$.

In each of the Tables 8.7.1 and 8.7.2, the scalar constants, such as $\lambda$ in Table 8.7.1 and $\lambda, \theta_{1}$ and $\theta_{2}$ in Table 8.7.2, have been approximately optimized to minimize the error in the estimation/prediction of the sampled impulse-response of the channel. The value of $\mu$ in Eqns. 8.5.5 and 8.5.6 has been set to 0.1. Stabilization of the algorithm is carried according to Section 8.4 and Fig. 8.4.2, when the control variable, $\left|\zeta_{i}^{\mid}\right|^{2}$, defined in Eqn. 8.4.3 exceeds a threshold level of 0.0001.

Fig. 8.7.1 shows the variation of $\xi_{1}$ and $\xi_{2}$ with $\psi$, for systems 8.1 and 8.2 . It can be seen from Fig. 8.7.1 that the relative performances of systems 8.1 and 8.2 are not significantly affected by whether Eqn. 8.7.1 or Eqn. 8.7.2 is used as a measurement criteria. This is further demonstrated in Tables 8.7.1 to 8.7.2. Therefore, it does not make any difference whether $\xi_{1}$ or $\xi_{2}$, is used to give a relative measure of the effectiveness of an estimator [99]. Comparing the performances of systems 8.1 and 8.2, in Fig. 8.7.1 and Tables 8.7.1 to 8.7.2, it is clear that the one-step predictor has considerably improved the estimation process.

Fig. 8.7.2 is a plot of $\xi_{1}$ against $\psi$ and compares systems 8.1 and 8.2 with the corresponding RLS Kalman estimators, [99], referred to as system 5.3 and 5.1 respectively, of Chapter 5. As would be expected, the performance of system 8.1 is essentially the same as that of system 5.3, the difference being hardly noticeable, since the two estimators are basically the same and differ only in their implementation. However, system 8.2 shows a marked improvement over system 5.1, over the entire range of signal/noise ratios tested. The difference between system 8.2 and system 5.1 is in the way the one-step predictor has been implemented.

Figs. 8.7.3 and 8.7.4 show the variation with time of $\gamma_{i}$, (Eqn. 8.3.84), and $10 \log _{10}$ $\mid \zeta_{i}{ }^{2}$, (Eqn. 8.4.3), respectively at $\psi=10$, for the transmission of data at 2400 bauds over 25 seconds. The two plots clearly show the transitions when reinitialization (according to Fig. 8.4.2) is taking place following a build-up of error. Although the parallel FTF algorithm is brought into use as many as 18 times, the estimation process does not show any sign of instability or collapse. This is the case at all signal/noise ratios tested.

Fig. 8.7.5 show the steady state performance of systems 8.1 and 8.2 at $\psi=30$. The parameter estimation error, $\xi_{i}$, in Fig. 8.7.5 is here the square of the error in $Y_{i+1, i, i-1}^{\prime}$, measured in dB relative to unity, and is

$$
\xi_{i}=10 \log _{10}\left|Y_{g+1, i}-Y_{g+1, i, i-1}^{\prime}\right|^{2} \quad \text {...8.7.4 }
$$

Computer-simulation tests on the estimators with statistically independent noise components $\left\{w_{i}\right\}$ in Eqn. 8.2.2, in place of the slightly correlated noise components actually obtained at the output of the receiver filter, show only a negligibly small difference in performance. Thus the correlation in the noise components does not appear to have any significant effect.

System 8.2 achieves a considerable improvement in performance over a conventional FTF algorithm (system 8.1). The new stabilization technique is adequate to counter the built-up of round-off errors and only marginally increases the computational complexity of the algorithm. In view of the good performance by system 8.2 it is clearly the most cost-effective of the two estimators studied in this chapter and is well worth further study.

TABLE 8.7.1 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 8.1.

| $\psi$ <br> $(\mathrm{dB})$ | $\lambda$ | Correlated noise |  | Uncorrelated noise |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ |
| 10 | 0.98 | -12.153 | -10.916 | -12.119 | -10.879 |
| 20 | 0.96 | -17.628 | -16.438 | -17.617 | -16.434 |
| 30 | 0.92 | -21.638 | -20.546 | -21.637 | -20.536 |
| 40 | 0.88 | -23.126 | -22.115 | -23.130 | -22.107 |
| 60 | 0.88 | -23.348 | -22.354 | -23.360 | -22.361 |

TABLE 8.7.2 MEAN SQUARE ERROR \& MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF A 3 SKY WAVE CHANNEL FROM SYSTEM 8.2.

| $\psi$ | $\theta_{1}$ | $\theta_{2}$ | $\lambda$ | Correlated noise |  | Uncorrelated noise |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(\mathrm{dB})$ |  |  |  | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ | $\xi_{1}$ <br> $(\mathrm{~dB})$ | $\xi_{2}$ <br> $(\mathrm{~dB})$ |
| 10 | 0.008 | 1.100 | 0.988 | -12.458 | -11.190 | -12.436 | -11.197 |
| 20 | 0.015 | 0.925 | 0.980 | -19.938 | -18.627 | -19.920 | -18.683 |
| 30 | 0.020 | 0.850 | 0.965 | -27.141 | -25.873 | -27.123 | -25.886 |
| 40 | 0.030 | 0.900 | 0.950 | -33.698 | -32.506 | -33.622 | -32.449 |
| 60 | 0.048 | 0.900 | 0.896 | -42.269 | -41.385 | -42.223 | -41.326 |

TABLE 8.7.3 COMPARISON OF MEAN SQUARE ERROR IN THE ESTIMATED SAMPLED IMPULSE-RESPONSE OF 3 SKY WAVE CHANNEL FROM FTF \& RLS KALMAN ALGORITHM.

| $\Psi$ <br> $(\mathrm{dB})$ | SYSTEM 8.1 <br> $\xi_{1}$ <br> $(\mathrm{~dB})$ | SYSTEM 5.3 <br> $\boldsymbol{\xi}_{1}$ <br> $(\mathrm{~dB})$ | SYSTEM 8.2 <br> $\xi_{1}$ <br> $(\mathrm{~dB})$ | SYSTEM 5.1 <br> $\xi_{1}$ <br> $(\mathrm{~dB})$ |
| :---: | :---: | :---: | :---: | :---: |
| 10 dB | -12.153 | -12.186 | -12.458 | -12.270 |
| 20 dB | -17.628 | -17.650 | -19.938 | -19.418 |
| 30 dB | -21.638 | -21.719 | -27.141 | -26.195 |
| 40 dB | -23.126 | -23.191 | -33.698 | -32.437 |
| 60 dB | -23.348 | -23.432 | -42.269 | -40.191 |



Fig. 8.7.1 - Performance of Systems 8.1 and 8.2


Fig. 8.7.2 - Performance comparison of Systems 8.1, 5.3, 8.2 and 5.1




## CHAPTER 9

## COMMENTS ON THE RESEARCH PROJECT

### 9.1 COMPARISON OF THE CHANNEL ESTIMATORS.

Results of the computer-simulation tests on different estimators considered in this thesis have already been presented at the end of their respective chapters. Tables 9.1.1 and 9.1.2 and Fig. 9.1.1, however, compare the performances of systems 4.2, $5.1,6.5,7.5$ and 8.2. These systems have the best performance in their respective class of estimators. As can be seen from the Figure 9.1.1, system 7.5 has the best overall performance. This system however, utilizes some prior knowledge of the basic structure of the channel in the estimation process. Compared with the gradient estimator (system 4.2), the improvement in performance of system 7.5 is of the order of 6.5 dB at 10 dB signal/noise ratio and 19.5 dB at 60 dB signal/noise ratio.

At high signal/noise ratios, a Kalman estimator (system 5.1 and system 8.2) has a significantly better performance than the corresponding gradient estimator (system 4.2). However, the improvement is only marginal at low signal/noise ratios. With the assumption that the channel varies linearly with time and taking into account the rate of change in the channel sampled impulse-response (system 5.1), a considerable improvement in the performance can be achieved, at least at high signal/noise ratios, at the expense of increased computation.

The best of the estimators considered in this thesis is system 6.5 which require only a marginal increase in computational complexity compared to system 4.2. This has a particularly good relative performance at the lower signal/noise ratios, which is where a good performance is of greatest practical value. Systems 6.5 and 7.5 have comparable performances at high signal/noise ratios when the latter assumes an incorrect model of the channel. However, the former is a much simpler estimator than system 7.5.

TABLE 9.1.1 MEAN SQUARE ERROR IN THE ESTIMATES OF THE CHANNEL SAMPLED IMPULSE-RESPONSE.

| SNR <br> $\mathbf{d B}$ | SYSTEM <br> $\mathbf{4 . 2}$ <br> $\mathbf{d B}$ | SYSTEM <br> $\mathbf{5 . 1}$ <br> $\mathbf{d B}$ | SYSTEM <br> $\mathbf{6 . 5}$ <br> $\mathbf{d B}$ | SYSTEM <br> $\mathbf{7 . 5}$ <br> $\mathbf{d B}$ | SYSTEM <br> $\mathbf{8 . 2}$ <br> $\mathbf{d B}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | -11.925 | -12.270 | -16.200 | -18.621 | -12.458 |
| 20 | -18.942 | -19.418 | -23.200 | -26.596 | -19.938 |
| 30 | -25.052 | -26.195 | -30.300 | -34.125 | -27.141 |
| 40 | -29.012 | -32.437 | -35.800 | -40.185 | -33.698 |
| 60 | -30.990 | -40.191 | -43.400 | -49.622 | -42.269 |

TABLE 9.1.2 MEAN SQUARE NORMALIZED ERROR IN THE ESTIMATES OF THE CHANNEL SAMPLED IMPULSE-RESPONSE.

| SNR <br> $\mathbf{d B}$ | SYSTEM <br> $\mathbf{4 . 2}$ <br> $\mathbf{d B}$ | SYSTEM <br> $\mathbf{5 . 1}$ <br> $\mathbf{d B}$ | SYSTEM <br> $\mathbf{6 . 5}$ <br> $\mathbf{d B}$ | SYSTEM <br> $\mathbf{7 . 5}$ <br> $\mathbf{d B}$ | SYSTEM <br> $\mathbf{8 . 2}$ <br> $\mathbf{d B}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | -10.640 | -10.975 | -14.900 | -17.813 | -11.190 |
| 20 | -17.660 | -18.214 | -21.900 | -25.777 | -18.627 |
| 30 | -23.825 | -24.928 | -29.000 | -33.387 | -25.873 |
| 40 | -27.944 | -31.361 | -34.400 | -39.611 | -32.506 |
| 60 | -30.116 | -39.347 | -42.700 | -50.010 | -41.385 |




Fig. 9.1.1 - Performance of Systems 4.2, 5.1, $6.5,7.5$ and 8.2

All the estimators in Tables 9.1.1 and 9.1.2 uses a one-step least-squares fadingmemory prediction algorithm. Simulation tests have shown that the one-step predictor offers a considerable improvement in the performance of the estimators compared to the case where prediction is not used.

System 8.2, together with the stabilization algorithm proposed in Chapter 8, offers a most efficient way to implement the RLS Kalman algorithm and is computationally less complex compared with system 5.1. Fig. 9.1.1 and Tables 9.1.1-9.1.2 compare the estimators using two different measures, namely, the mean square estimation error and the mean square normalized estimation error. The two measures do not change the relative performance of the estimators. The channel estimators listed in order of complexity and starting with the simplest estimator, are system $4.2,6.5,7.5$, 8.2 and 5.1.

### 9.2 CONCLUSION

In this thesis several novel estimation techniques have been developed and tested on a model of a data transmission system over an HF radio link. The channel estimators are either based on the RLS Kalman filter algorithm or the feedforward transversal filter algorithm. When the characteristics of the channel, such as the number of sky waves etc., are known then the efficient estimators (system 7.1-7.8) offer the best solution for channel estimation. The advent of fast Kalman algorithms have not reduced the complexity of the Kalman estimators compared to the Feedforward estimators. They have, in addition, the problem of numerical instablity in the algorithm. A degree-one Kalman algorithm (systems 5.1 and 5.2) which takes into account the rate of change in the channel sampled impulse-response has improved the estimator performance only at high signal/noise ratios. A feedforward estimator together with a degree-one fading memory predictor (system 4.2) is the simplest of all the estimators. System 6.5 is only slightly more complex than system 4.2 and has very good performance in the entire range of signal/noise ratios. The former does not take into account any prior knowledge of the channel. System 6.5 appears to be potentially the most cost-effective of all the estimators, considered in this thesis, for the given application in this thesis.

### 9.3 SUGGESTIONS FOR FUTURE WORK

The most promising of all the estimators have been systems $7.1-7.8$, which require a prior knowledge of the number of sky waves present in the HF channel. With this information and correct start-up, the systems 7.1-7.8 have near optimum performance. In practice, however, the number of sky waves present in the HF channel cannot be accurately forecasted and, moreover, they constantly change with time. Computer-simulation tests have shown (Chapter 7) that, incorrect assumption or incorrect start-up leads to an inefficient estimation process. In some of the systems, the algorithm uses a more effective updating process (systems 7.3-7.8) and this has considerably reduced the estimation error due to an incorrect start-up or an incorrect channel model. This suggests that, perhaps, the use of a better updating algorithm can lead to an even better estimation process. This needs further investigation.

The efficient estimators (Chapter 7) become increasingly complex as the number of sky waves in the channel increases, since the existing algorithm requires an adequate start-up procedure. This thesis, however, has proposed a number of changes to the original algorithm (system 7.1) whereby it would only be required to track a reduced number of variable quantities (systems $7.3-7.8$ ), where these are numerically equal to the number of sky waves present in the channel. This has resulted in an much simpler estimation algorithm. However, the problem of determining the $n$-dimensional subspace at start-up still remains. Here n is the number of sky waves present in the HF channel. So a suitable, adaptive (and a very much simpler) starting-up procedure is still to be developed.

It is evident from the results of the computer-simulation tests (Fig. 9.1.1), that system 6.5 has performance intermediate between systems 7.5 (Efficient estimator) and 4.2 (Simple gradient estimator). An important feature of system 6.5 is that it applies error correction, in the channel update algorithm, in accordance with the modulus value of the channel components. Although this thesis has proposed a number of novel techniques to give corrections to the channel components, this, however, needs further theoretical and experimental investigation in order to ascertain the precise relationship between the channel components and the required
error correction. A simple mathematical relationship would simplify the hardware implementation of the algorithm. It would be of interest also to see the performance of the adaptive channel estimators on a frequency domain channel response.

Computer-simulation results of the Kalman estimators have shown that they have a performance comparable to that of the simple gradient estimator. However, modifications which take account of the variation in the channel sampled impul-se-response have improved the performance only at high signal/noise ratios (system 5.1, Fig. 9.1.1) and also this algorithm, however, is computationally complex. System 8.2 is much simpler than system 5.1 and requires very much less computation. In recent times, their has been considerable interest in Recursive least-squares estimation using systolic arrays. This new technique is not likely to improve the performance of the estimation process. However, the algorithm can be very easily implemented in hardware. Moreover, there appears to be a possibility of obtaining the minimum phase version of the channel together with the estimate of the channel sampled impulse-response thus reducing the overall computational complexity of the modem.

## APPENDIX A

## RAYLEIGH FADING FILTER

A single Rayleigh fading propagation path is modelled as in Fig. 2.6.2, where $q_{1}(\mathrm{t})$ and $q_{2}(\mathrm{t})$ are two Gaussian random processes with zero mean and the same variance. The shape of their power spectrum is Gaussian having the same rms frequency, $\mathrm{f}_{\mathrm{m}}$ : The power spectrum of $q_{1}(\mathrm{t})$ and $q_{2}(\mathrm{t})$ are given by, Eqn. 2.6.1, as

$$
\left|Q_{1}(f)\right|^{2}=\left|Q_{2}(f)\right|^{2}=\exp \left(-\frac{f^{2}}{2 f_{r m s}^{2}}\right)
$$

As is shown in Fig. 2.6.3, the random process $\Psi_{i}(t)$ is generated by filtering a zero mean white Gaussian noise signal $\mathrm{V}_{\mathrm{i}}(\mathrm{t})$. The filter used in Fig. 2.6.3 has a Gaussian frequency response and is given by, Eqn. 2.6.5, as

$$
F(f)=\exp \left(-\frac{f^{2}}{4 f_{r m s}^{2}}\right)
$$

and the 3 dB cut-off frequency of the filter is

$$
f_{c}=1.17741 f_{r m s}
$$

The rms frequency, $\mathrm{f}_{\mathrm{mm}}$, and the frequency spread, $\mathrm{f}_{\mathrm{sp}}$, are related as follows, (Eqn. 2.6.2),

$$
\begin{equation*}
f_{s p}=2 f_{r m s} \tag{A 1.4}
\end{equation*}
$$

Therefore, from Eqns. A1.3 and A1.4,

$$
f_{c}=0.588705 f_{s p}
$$

The impulse-response and the magnitude-response of a Bessel filter tends towards Gaussian as the order of the filter is increased [32]. A Bessel filter has, therefore, been used to obtain the Rayleigh fading filter in Fig. 2.6.3.

$$
H(s)=\frac{d_{0}}{B_{n}(s)}
$$

where $B_{n}(s)$ is the $n^{\text {th }}$ - order Bessel Polynomial and $d_{0}$ is a normalizing constant of the form

$$
d_{0}=\frac{(2 n)!}{2^{2} n!}
$$

$\mathrm{B}_{\mathrm{n}}(\mathrm{s})$ can be put in the form [32]

$$
B_{n}(s)=\sum_{k=0}^{n} d_{k} s^{k}
$$

where

$$
\begin{aligned}
& d_{k}=\frac{(2 n-k)!}{2^{n-k} k!(n-k)!} \\
& \text { for } \mathrm{k}=0,1, \ldots \ldots, \mathrm{n}
\end{aligned}
$$

A $5^{\text {th }}$ order Bessel filter has been chosen as a practical choice and thus $n=5$. Fig. 2.7.1 compares the frequency response of this filter with that of the desired theoretical frequency response (Gaussian). It can be seen that a $5^{\text {th }}$ order Bessel filter has a frequency response that is Gaussian, at least in the range of interest.

Eqn. A1. 6 becomes

$$
H(s)=\frac{945}{s^{5}+15 s^{4}+105 s^{3}+420 s^{2}+945 s+945}
$$

Eqn. A1.10 can be expressed as

$$
H(s)=\frac{945}{\prod_{i=1}^{s}\left(s-P_{i}\right)}
$$

where $\mathrm{P}_{\mathrm{i}}$ are known as the poles of $\mathrm{H}(\mathrm{s})$ and are given by [117]

$$
\begin{align*}
& P_{1}=-3.64674 \\
& P_{2}, P_{3}=-3.35196 \pm j 1.74266 \\
& P_{4}, P_{5}=-2.32467 \pm \mathrm{j} 3.57102
\end{align*}
$$

Substituting $\mathrm{s}=\mathrm{j} \Omega$, in Eqn. A1.11, the frequency response of the Bessel filter is

$$
H(j \Omega)=\frac{945}{\prod_{i=1}^{5}\left(j \Omega-P_{i}\right)}
$$

where, in Eqn. A1.13, $\Omega$ is the angular frequency and $\mathrm{j}=\sqrt{-1}$. When $\Omega=\Omega_{c}$ $\mathrm{rad} / \mathrm{sec}$., the amplitude response of the $5^{\text {h }}$ order Bessel filter, drops by 3 dB from its peak value. $\Omega_{c}$ is called the 3 dB cut-off angular frequency and is given by [34]

$$
\Omega_{c}=2.4274 \mathrm{rad} / \mathrm{sec} .
$$

One of the parameter of importance in the characterization of a channel is the frequency spread, $\mathrm{f}_{\mathrm{sp}}$. Therefore, it is desirable to express the cut-off frequency of the Bessel filter in terms of the frequency spread.

Let

$$
\omega=C_{0} \Omega
$$

where

$$
\begin{equation*}
C_{0}=\frac{\omega_{c}}{\Omega_{c}}=\frac{2 \pi f_{c}}{\Omega_{c}} \tag{A 1.16}
\end{equation*}
$$

where $f_{c}$, from Eqn. A1.3, is the cut-off frequency of the desired filter.

Therefore, from Eqns. A1.14 and A1.16,

$$
C_{0}=2.58844 f_{c}
$$

Substituting the value of $\Omega$, from Eqn. A1.15, in Eqn. A1.13

$$
H(j \omega)=\frac{945}{\prod_{i=1}^{5}\left(j \frac{\omega}{c_{0}}-P_{i}\right)}
$$

Let

$$
P_{i}^{\prime}=C_{0} P_{i}
$$

Then, from Eqns. A1.18 and A1.19,

$$
H(j \omega)=\frac{945 C_{0}^{5}}{\prod_{i=1}^{5}\left(j \omega-P_{i}^{\prime}\right)}
$$

and, from Eqns. A1.17 and A1.20,

$$
\begin{align*}
H(j \omega) & =\frac{109805.0518 f_{c}^{5}}{\prod_{i=1}^{5}\left(j \omega-P_{i}^{\prime}\right)} \\
& =\frac{d_{0}^{\prime}}{\prod_{i=1}^{5}\left(s-P_{i}^{\prime}\right)}
\end{align*}
$$

where, in Eqn. A1.22,
$s=j w$
$d_{0}^{\prime}=109805.0518 f_{c}^{5}$
and

$$
P_{i}^{\prime}=2.58844 f_{c} P_{i}
$$

$$
\text { for } \mathrm{i}=1,2, \ldots, 5
$$

Table A1.1 summarizes all the parameters of the Bessel filter for a frequency spread of 2 Hz .

Eqn. A1.22 is the transfer function of a $5^{\text {th }}$ order Bessel filter. This analog filter is to be digitized for use in computer-simulation. The method used for this is called the impulse-invariant transformation method [32]. The important feature of this transformation is that, the impulse-response of the resulting digital filter is a sampled version of the impulse-response of the analog filter. In this technique the poles $\left\{P_{i}^{\prime}\right\}$, in the s-plane, of Eqn. A1.22, are transformed to poles at $\left\{e^{p+4}\right\}$, in the $z$-plane [32], where T is the sampling interval.

Therefore, using the impulse-invariant transformation method, Eqn. A1.22 can be written as

$$
\begin{align*}
H(z) & =\frac{K}{\prod_{i=1}^{5}\left(1-e^{P_{i}^{\prime} T} z^{-1}\right)} \\
& =\frac{K}{\prod_{i=1}^{5}\left(1-q_{i} z^{-1}\right)}
\end{align*}
$$

where in Eqn. A1.24, K is the DC gain of the filter, $\mathrm{q}_{\mathrm{i}} \mathrm{s}$ are the poles and are equal to

$$
q_{i}=e^{P_{i}^{\prime} T}
$$

$q_{i}(t) s$ have Gaussian spectra, and so contain all the frequency components. However, for a frequency spread of 2 Hz , the 3 dB bandwidth of the analog filter is 2.35 Hz , as can be seen from Eqns. A1.3 and A1.4. So frequencies above about 25 Hz have negligibly small amplitude as can be seen from Fig. 2.6.4 and, therefore, a sampling rate of 100 samples $/ \mathrm{sec}$. is adequate enough for accurate representation of $q_{i}(t)$. The z-plane poles obtained from Eqns. A1.24 for a frequency spread of 2 Hz are

$$
\begin{aligned}
q_{1} & =0.8948 \\
q_{2}, q_{3} & =0.9016 \pm j 0.047 \vartheta \\
q_{4}, q_{5} & =0.9261 \pm j 0.1012
\end{aligned}
$$

The digital filter is implemented as shown in Fig. 2.6.5 [34-36]. It comprises of a cascade of two 2-pole section and a single pole section. Each of the 2-pole section has a complex conjugate poles and the single pole section has a real pole. The transfer function of the filter in Fig. 2.6.5 is, therefore,
$H(z)=\frac{K}{\left(1-q_{1} z^{-1}\right)\left\{\left(1-q_{2} z^{-1}\right)\left(1-q_{3} z^{-1}\right)\right\}\left\{\left(1-q_{4} z^{-1}\right)\left(1-q_{5} z^{-1}\right)\right\}}$
..A 1.27
$=\frac{K}{\left(1-q_{1} z^{-1}\right)\left\{1-\left(q_{2}+q_{3}\right) z^{-1}+\left(q_{2} q_{3}\right) z^{-2}\right\}\left\{1-\left(q_{4}+q_{5}\right) z^{-1}+\left(q_{4} q_{5}\right) z^{-2}\right\}}$
.A 1.28
where $q_{2}$ and $q_{3}$ and $q_{4}$ and $q_{5}$ are complex conjugate pairs. Therefore, from Eqn. A1.28 and Fig. 2.6.5, the filter co-efficients $\left\{\mathrm{C}_{\mathrm{i}}\right\}$ are given by

$$
\begin{aligned}
& \mathrm{C}_{1}=-\mathrm{q}_{1} \\
& \mathrm{C}_{2}=-\left(\mathrm{q}_{2}+\mathrm{q}_{3}\right) \\
& \mathrm{C}_{3}=\mathrm{q}_{2} \mathrm{q}_{3} \\
& \mathrm{C}_{4}=-\left(\mathrm{q}_{4}+\mathrm{q}_{5}\right) \\
& \mathrm{C}_{5}=\mathrm{q}_{4} \mathrm{q}_{5}
\end{aligned}
$$

The filter co-efficients obtained for a frequency spread of 2 Hz are listed in Table 2.6.2. The value of $K$, called the gain of the filter, in Eqn. A1.28, is chosen such that the $\left\{\mathscr{q}_{i}(t)\right\}$ s have a variance corresponding to $1 / 2 n_{s}$, where $n_{s}$ represents the number of sky waves. This ensures that the mean length of the channel sampled impulse-response vector is equal to unity. Theoretically the value of K can be obtained as follows. The energy in the waveform $\mathrm{H}(\mathrm{f})$, Eqn. A1.22, given by

$$
\begin{equation*}
E_{h}=\int_{-\infty}^{\infty} \mid H(F)^{p} d f \tag{A 1.29}
\end{equation*}
$$

is determined. $\mathrm{E}_{\mathrm{b}}$ is normalized by a scalar, K , such that the energy in the waveform is equal to $1 / 2 n_{s}$. However, a simpler method is to pass a sequence of digital data, whose first element is a 1 and the rest of the elements are zero, through the 5 pole digital filter. For a sufficiently long sequence, the sum of the squares of the output of the digital filter, $\mathrm{E}_{\mathrm{sum}}$, is very close to $\mathrm{E}_{\mathrm{h}}$, particularly since the sampling frequency of the filter is considerably larger than the bandwidth of the filter. $\mathrm{E}_{\text {sum }}$ is now normalized by K such that the energy in the waveform is equal to $1 / 2 n_{s}$. The gain of the 5 -pole digital filter obtained in this way for a 2 Hz frequency spread is equal to 19378. With this value of K , in Eqn. A1.28, the resultant channel sampled impulse-response vector length is very close to unity as can be seen from Table 3.5.2.

TABLE A1.1 FIFTH ORDER ANALOG BESSEL FILTER FOR A FREQUENCY SPREAD OF $2 \mathbf{H z}$.

| Frequency spread, $\mathrm{f}_{\mathrm{sp}}(\mathrm{Hz})$ | 2 |  |
| :--- | ---: | :--- |
| Cut-off frequency, $\mathrm{f}_{\mathrm{c}}(\mathrm{Hz})$ | 1.1774 |  |
| Constant $d_{0}^{\prime}$ | 248451.99 |  |
| Filter poles in the s-plane |  |  |
|  | $P_{1}^{\prime}$ | $-11.1139+\mathrm{j} 0$ |
|  | $P_{2}^{\prime}, P_{3}^{\prime}$ | $-10.2155 \pm \mathrm{j} 5.3110$ |
|  | $P_{4}^{\prime} P_{5}^{\prime}$ | $-7.0847 \pm \mathrm{j} 10.8831$ |
|  |  |  |

## APPENDIX B

## TRANSMITTER \& RECEIVER FILTERS

Fig. 3.3.4 shows the frequency characteristics of the combined equipment and radio filters. In order to obtain different sampling phases, the filter sampled impulse-response has been obtained at a sampling rate that is 20 times higher (i.e., at a sampling rate of 96000 samples $/ \mathrm{sec}$.) than the original sampling rate. This oversampled transmitter and receiver filter responses are given in Table B1.1. Section 3.3 explains in detail, the method by which the oversampled filters are obtained. The sampled impulse-responses $a_{1, k}, a_{2, k}$ and $a_{3, k}$ corresponding to $a(t-i T), a\left(t-\tau_{1}-i T\right)$ and $\mathrm{a}\left(\mathrm{t}-\tau_{2}-\mathrm{iT}\right)$ (for generating a three sky wave channel) have been obtained by taking every $20^{\text {h }}$ sample from the oversampled transmitter filter. The three filters are, therefore, at a sampling rate of 4800 samples/sec. $\mathrm{a}_{1, \mathrm{k}}$ has $(-0.179590+\mathrm{j} 2.353941)$ as its first sample and the other samples are obtained by picking every $20^{\text {th }}$ sample from the first sample from Table B1.1 (Table 3.3.3).
$\mathrm{a}_{2, \mathrm{k}}$ is delayed 1.1 millisecond with respect to $\mathrm{a}_{1, k}$. Expressing this delay as a fraction of the number of samples, $\rho^{\prime}$, gives

$$
\rho^{\prime}=\frac{1.1 x 10^{-3}}{(1 / 4800)}=5.28
$$

In other words the first sample of $\mathrm{a}_{2, \mathrm{k}}$ is delayed by 5.28 samples with respect to the first sample of $a_{1, k}$. It is, however, necessary to obtain the samples of the delayed filters at the sampling instants of the non-delayed filter. This delay can be expressed as a whole number of samples and a fractional part (i.e. $5+0.28$ ). The first component of $a_{2, k}$ is thus added to the $(5+1) 6^{\text {th }}$ component of $a_{1, k}$. This leaves a discrepancy of ( $6-5.28$ ) 0.72 sampling intervals. This discrepancy is taken care of by choosing (from the oversampled version) the sample that is $(0.72 \times 96000 / 4800=$ 14.4) 14 samples ahead of $(-0.179590+\mathrm{j} 2.353941)$. Thus the sample that is chosen as the first sample of $a_{2, k}$ is $(-1.669437+j 13.237271)$. The remaining sample of $a_{2, k}$ are, of course, chosen as every $20^{\mathrm{m}}$ sample from the sampled version, starting with $(-1.669437+\mathrm{j} 13.237271)$. The 3 millisecond delayed filter $\mathrm{a}_{3, \mathrm{k}}$ has been similarly obtained and is shown in Table 3.3.3. The oversampled version of the receiver filter in Table B1.1 has been obtained as described in Section 3.3, but at a different
sampling phase. Table 3.3 .4 shows the sample impulse-response of the receiver filter at 4800 samples $/ \mathrm{sec}$. and has been obtained from Table B1.1 with $(-1.941764+$ j 1.362559) as its first sample.

TABLE B1.1. - OVERSAMPLED TRANSMITTER \& RECEIVER FILTER

## (A) - IN PHASE RESPONSE OF TRANSMITTER FILTER SAMPLED AT 96000 SAMPLES/SEC.

| -0.0002096 |
| ---: |
| -0.0002893 |
| 0.0002107 |
| 0.0002904 |
| -0.0002113 |
| -0.0002909 |
| 0.0002113 |
| 0.0002905 |
| -0.0002106 |
| -0.0002889 |
| 0.0002089 |
| 0.0002858 |
| -0.0002059 |
| -0.0002805 |
| 0.0002011 |
| 0.0002722 |
| -0.0001937 |
| -0.0002596 |
| 0.0001825 |
| 0.0002407 |
| -0.0001655 |
| -0.0002118 |
| 0.0001394 |
| 0.0001666 |
| -0.0000975 |
| -0.0000918 |
| 0.0000253 |
| -0.0000440 |
| 0.0001148 |
| 0.0003319 |
| -0.0004487 |
| -0.0011355 |
| 0.0016070 |
| 0.0049406 |
| -0.0105240 |
| -0.0841076 |
| -0.2740073 |
| -0.6560838 |
| -1.3136537 |
| -2.3098150 |
| -3.6554823 |
| -5.2949561 |
| -7.1104051 |
| -8.9314755 |
| -10.5497287 |
| -11.7490864 |
| -12.3469721 |
| -12.2252865 |
| -11.3445819 |
| -9.7521911 |
| -7.5848703 |
| -5.0528272 |
| -2.4056875 |
| 0.1015622 |
| 2.2353854 |
| 3.8123224 |
| 4.7276308 |

-0.0002887
-0.0002103
0.0002901
0.0002111
-0.0002908
-0.0002114
0.0002908
0.0002110
-0.0002897
-0.0002098
0.0002873
0.0002073
-0.0002829
-0.0002033
0.0002760
0.0001970
-0.0002653
-0.0001875
0.0002492
0.0001731
-0.0002249
-0.0001513
0.0001873
0.0001168
-0.0001267
-0.0000594
0.0000212
-0.0000461
0.0001868
0.0002744
-0.0006966
-0.0009363
0.0025614
0.0043280
-0.0189960
-0.1105917
-0.3323647
-0.7630237
-1.4845339
-2.5518017
-3.9628099
-5.6487240
-7.4804261
-9.2784116
-10.8296895
-11.9216180
-12.3826494
-12.1096509
-11.0799204
-9.3588701
-7.0999776
-4.5243415
-1.8851016
0.5649945
2.5999552
4.0500667
4.8284836

| -0.0003572 | -0.0003399 |
| ---: | ---: |
| 0.0000000 | 0.0001107 |
| 0.0003588 | 0.0003413 |
| 0.0000000 | -0.0001110 |
| -0.0003596 | -0.0003420 |
| 0.0000000 | -0.0001111 |
| 0.0003593 | 0.0003416 |
| 0.0000000 | -0.0001108 |
| -0.0003577 | -0.0003399 |
| 0.0000000 | 0.0001100 |
| 0.0003542 | 0.0003364 |
| 0.0000000 | -0.0001085 |
| -0.0003483 | -0.0003305 |
| 0.0000000 | 0.0001060 |
| 0.0003389 | 0.0003212 |
| 0.0000000 | -0.0001023 |
| -0.0003246 | -0.0003070 |
| 0.0000000 | 0.0000966 |
| 0.0003030 | 0.0002856 |
| 0.0000000 | -0.0000881 |
| -0.0002703 | -0.0002531 |
| 0.0000000 | 0.0000750 |
| 0.0002194 | 0.0002024 |
| 0.0000000 | -0.0000540 |
| -0.0001362 | -0.0001190 |
| 0.0000000 | 0.0000182 |
| -0.0000113 | -0.0000305 |
| 0.000000 | 0.0000502 |
| 0.0003124 | 0.0003414 |
| 0.0000000 | -0.0002091 |
| -0.0010966 | -0.0011788 |
| 0.0000000 | 0.0007335 |
| 0.0043277 | 0.0048672 |
| 0.0000000 | -0.0043177 |
| -0.044391835 | -0.0622160 |
| $-0.1795896(1)$ | -0.2233037 |
| -0.4747796 | -0.5602256 |
| -1.0126609 | -1.1564934 |
| -1.8685694 | -2.0820312 |
| $-3.07734553)$ | -3.3601064 |
| -4.6101723 | -4.9483961 |
| -6.3729500 | -6.7407393 |
| -8.2151244 | -8.5764574 |
| -9.9409021 | -10.2528991 |
| -11.3324982 | -11.5522808 |
| -12.1892192 | -12.2822589 |
| -12.3646935 | -12.3102829 |
| -11.7869473 | -11.5804806 |
| -10.4677274 | -10.1222704 |
| -8.5086316 | -8.0550490 |
| -6.0938784 | -5.5766967 |
| -3.4618271 | -2.9319357 |
| -0.8688202 | -0.3769656 |
| 1.4399026 | 1.8482796 |
| 3.2570916 | 3.5478141 |
| 4.4438154 | 4.5994647 |
| 4.9495636 | 4.9707467 |


| 4.9665164 | 4.9376234 |
| ---: | ---: |
| 4.5938614 | 4.4562826 |
| 3.7379757 | 3.5236767 |
| 2.5744110 | 2.3220794 |
| 1.2992157 | 1.0473111 |
| 0.0931639 | -0.1265567 |
| -0.9032046 | -1.0682722 |
| -1.5969945 | -1.6948431 |
| -1.9472954 | -1.9765922 |
| -1.9704176 | -1.9408745 |
| -1.7308727 | -1.6590449 |
| -1.3164980 | -1.2207005 |
| -0.8183711 | -0.7164903 |
| -0.3233694 | -0.2315715 |
| 0.0960762 | 0.1665695 |
| 0.3981238 | 0.4432363 |
| 0.5738109 | 0.5943870 |
| 0.6313238 | 0.6298011 |
| 0.5854067 | 0.5657863 |
| 0.4609658 | 0.4295803 |
| 0.2911328 | 0.2543641 |
| 0.1035718 | 0.0656237 |
| -0.0826035 | -0.1177907 |
| -0.2438612 | -0.2704365 |
| -0.3506899 | -0.3636354 |
| -0.3865939 | -0.3855371 |
| -0.3580021 | -0.3459950 |
| -0.2815380 | -0.2620450 |
| 0.1763861 | -0.1543700 |
| -0.0734526 | -0.0566187 |
| -0.0098828 | -0.0037687 |
| 0.0011207 | -0.0013998 |
| -0.0181736 | -0.0228239 |
| -0.0386471 | -0.0415585 |
| -0.0477136 | -0.0475377 |
| -0.0366473 | -0.0308152 |
| 0.0053171 | 0.0167220 |
| 0.0608046 | 0.0689425 |
| 0.0765135 | 0.0710176 |
| 0.0234435 | 0.0076085 |
| -0.0508838 | -0.0609485 |
| -0.0713496 | -0.0665586 |
| -0.0295126 | -0.0188364 |
| 0.0126510 | 0.0160815 |
| 0.0122893 | 0.0084275 |
| -0.0068719 | -0.0090381 |
| -0.0076666 | -0.0053654 |
| 0.0046743 | 0.0062320 |
| 0.0055350 | 0.0039116 |
| -0.0035251 | -0.0047346 |
| -0.0043159 | -0.0030676 |
| 0.0028221 | 0.0038080 |
| 0.0035297 | 0.0025184 |
| -0.0023491 | -0.0031798 |
| -0.0029819 | -0.0021333 |
| 0.0020098 | 0.0027267 |
| 0.0025790 | 0.0018487 |
|  |  |

4.8849186
4.3005084
3.2985708
2.0668985
0.7995878
-0.3368383
-1.2205644
-1.7787073
-1.9930423
-1.8014342
-1.5809597
-1.1222815
-0.6154345
-0.1433592
0.2320973
0.4832902
0.6103202
0.6242601
0.5432531
0.3966679
0.2170838
0.0278577
-0.1517680
-0.2945709
-0.3736477
-0.3820071
-0.3322094
-0.2415492
-0.1543700
-0.0416905
0.0002018
-0.0048610
-0.0272931
-0.0439705
-0.0464696
-0.0236212
0.0284445
0.0749333
0.0626009
-0.0084686
-0.0681863
-0.0594132
-0.0088283
0.0175578
0.0041943
-0.0101718
-0.0027205
0.0071031
0.0020017
-0.0054343
-0.0015785
0.0043901
0.0013006
-0.0036771
-0.0011046
0.0031602
0.0009591

| 4.8093491 | 4.7119541 |
| ---: | ---: |
| 4.1279029 | 3.9398962 |
| 3.0642536 | 2.8223321 |
| 1.8103897 | 1.5540244 |
| 0.5572496 | 0.3214244 |
| -0.5368582 | -0.7258700 |
| -1.3596576 | -1.4852158 |
| -1.8486294 | -1.9047481 |
| -1.9971243 | -1.9893827 |
| -1.8528030 | -1.7957047 |
| -1.4973528 | -1.4089564 |
| -1.0219528 | -0.9204182 |
| -0.5158362 | -0.4182949 |
| -0.0591323 | 0.0207712 |
| 10.2925598 | 0.3479056 |
| 0.5183509 | 0.5484959 |
| 0.6217122 | 0.6286729 |
| 0.6148771 | 0.6018517 |
| 0.5180829 | 0.4905592 |
| 0.3624759 | 0.3272300 |
| 0.1794357 | 0.1415545 |
| -0.0095622 | -0.0464514 |
| -0.1842786 | -0.2150615 |
| -0.3160803 | -0.3348201 |
| -0.3807599 | -0.3850420 |
| -0.3761463 | -0.3680978 |
| -0.3167778 | -0.2998377 |
| -0.2202693 | -0.1984549 |
| -0.1118143 | -0.0919475 |
| -0.0288560 | -0.0182370 |
| 10.0021899 | 0.0024109 |
| -0.0089801 | -0.0134947 |
| -0.0314685 | -0.0352709 |
| -0.0458416 | -0.0471151 |
| -0.0443806 | -0.0411434 |
| -0.0151063 | -0.0053910 |
| 0.0400358 | 0.0509968 |
| 0.0783720 | 0.0789564 |
| 0.0515533 | 0.0383126 |
| -0.0240446 | -0.0384004 |
| -0.0723509 | -0.0733706 |
| -0.0504447 | -0.0402683 |
| 0.0000000 | 0.0072489 |
| 0.0172292 | 0.0153572 |
| 0.0000000 | -0.0037884 |
| -0.0102567 | -0.0093694 |
| 0.0000000 | 0.0025394 |
| 0.0072476 | 0.0066944 |
| 0.0000000 | -0.0019003 |
| -0.0055818 | -0.0051885 |
| 0.0000000 | 0.0015140 |
| 0.0045286 | 0.0042267 |
| 0.0000000 | -0.0012561 |
| -0.0038043 | -0.0035609 |
| 0.0000000 | 0.0010721 |
| 0.0032766 | 0.0030735 |
| 0.0000000 | -0.0009344 |

## (B) - QUADRATURE RESPONSE OF TRANSMITTER FILTER SAMPLED AT 96000 SAMPLES/SEC.

-0.0018960
-0.0026938
0.0020224
0.0028795
-0.0021666
-0.0030923
0.0023328
0.0033386
-0.0025262
-0.0036272
0.0027542
0.0039696
-0.0030268
-0.0043825
0.0033584
0.0048899
-0.0037705
-0.0055278
0.0042956
0.0063535
-0.0049871
-0.0074620
0.0059365
0.0090242
-0.0073159
-0.0113770
0.0094846
0.0152746
-0.0133169
-0.0227450
0.0214730
0.0410111
-0.0456111
-0.1123821
0.1932385
1.2493078
3.3108456
6.5455425
11.0688962
16.8321857
23.5042446
30.5187820
37.2136597
42.9059887
46.9257109
48.7146794
47.9575159
44.6307293
38.9744463
31.4713441
22.8262482
13.8752357
5.4416399
-1.7735266
-7.2498590
-10.7020093
-12.1218489
-11.7524642
-10.0026703
-7.3680608
-4.3741695
-1.5058820
$-1.5058820$
-0.0026261
-0.0019699
0.0028023
0.0021066
-0.0030035
-0.0022634
0.0032356
0.0024451
-0.0035060
-0.0026583
0.0038253
0.0029116
-0.0042076
-0.0032175
0.0046736
0.0035943
-0.0052539
-0.0040692
0.0059957
0.0046859
-0.0069761
-0.0055173
0.0083288
0.0066959
-0.0103073
-0.0084850
0.0134480
0.0114840
-0.0190802
-0.0173412
0.0313657
0.0321468
-0.0694746
-0.0942916
0.3342662
1.5751498
3.8591911
7.3452371
12.1286245
18.1095118
24.9003445
31.9050405
38.4543095
43.8641239
47.4762122
48.7718190
47.4936015
43.6738546
37.6019147
29.8079459
21.0330921
12.1224692
3.8815751
-3.0207994
-8.1065690
-11.1439499
-12.1798142
-11.4979467
-9.5297832
6.7813882
-3.7766193
-0.9822748
-0.0031067
-0.0010424
0.0033165
0.0011152
-0.0035564
-0.0011988
0.0038332
0.0012959
-0.0041563
-0.0014098
0.0045381
0.0015454
-0.0049962
-0.0017095
0.0055556
0.0019120
-0.0062539
-0.0021679
0.0071491
0.0025013
-0.0083367
-0.0029528
0.0099837
0.0035966
-0.0124094
-0.0045820
0.0162988
0.0062543
-0.0233823
-0.0095856
0.0392410
0.0183105
-0.0909950
-0.0579434
0.5082941
1.9429154
4.4557341
8.1974720
13.2372707
19.4199272
26.3051808
33.2731191
39.6493461
44.7499103
47.9331639
48.7250491
46.9272219
42.6261192
36.1597834
28.1049601
19.2346609
10.3969514
2.3756530
-4.1946128
-8.8804125
-11.5050480
-12.1686116
-11.1917489
-9.0256163
-6.1846416
-3.1879262
-0.4810978

| -0.0032874 | -0.0031465 |
| :---: | :---: |
| 0.0000000 | 0.0010562 |
| 0.0035109 | 0.0033619 |
| 0.0000000 | -0.0011310 |
| -0.0037666 | -0.0036085 |
| 0.0000000 | 0.0012171 |
| 0.0040621 | 0.0038938 |
| 0.0000000 | -0.0013172 |
| -0.0044073 | -0.0042275 |
| 0.0000000 | 0.0014350 |
| 0.0048159 | 0.0046230 |
| 0.0000000 | -0.0015757 |
| -0.0053068 | -0.0050990 |
| 0.0000000 | 0.0017465 |
| 0.0059076 | 0.0056827 |
| 0.0000000 | -0.0019583 |
| -0.0066593 | -0.0064148 |
| 0.0000000 | 0.0022274 |
| 0.0076259 | 0.0073593 |
| 0.0000000 | -0.0025804 |
| -0.0089133 | -0.0086221 |
| 0.0000000 | 0.0030629 |
| 0.0107080 | 0.0103921 |
| 0.0000000 | -0.0037594 |
| -0.0133702 | -0.0130372 |
| 0.0000000 | 0.0048442 |
| 0.0176836 | 0.0173697 |
| 0.0000000 | -0.0067345 |
| -0.0256677 | -0.0255267 |
| 0.0000000 | 0.0106649 |
| 0.0440432 | 0.0448589 |
| 0.0000000 | -0.0218003 |
| -0.1074007 | -0.1156220 |
| 0.0000000 | 0.0827311 |
| 0.7175366 | 0.9639589 |
| 2.3539405 | 2.8095026 |
| 5.1016214 | 5.7979150 |
| 9.1023946 | 10.0597638 |
| 14.3926361 | 15.5920086 |
| 20.7590237 | 22.1221047 |
| 27.7134459 | 29.1197780 |
| 34.6175658 | 35.9329096 |
| 40.7932681 | 41.8806123 |
| 45.5584592 | 46.2851556 |
| 48.2935224 | 48.5547200 |
| 48.5737616 | 48.3178150 |
| 46.2599267 | 45.4936549 |
| 41.4909978 | 40.2723551 |
| 34.6534397 | 33.0886132 |
| 26.3691066 | 24.6072306 |
| 17.4377717 | 15.6491184 |
| 8.7045826 | 7.0510150 |
| 0.9284061 | -0.4559482 |
| -5.2922114 | -6.3112590 |
| -9.5708828 | -10.1779415 |
| -11.7869820 | -11.9917925 |
| -12.0913914 | -11.9515022 |
| -10.8377827 | -10.4400583 |
| -8.4944258 | -7.9404850 |
| -5.5819054 | -4.9771435 |
| -2.6113002 | -2.0497120 |
| -0.0043047 | 0.4463982 |


| 0.8695437 | 1.2638933 | 1.6284281 | 1.9623394 | 2.2650217 |
| :---: | :---: | :---: | :---: | :---: |
| 2.5360673 | 2.7752627 | 2.9825874 | 3.1582131 | 3.3025044 |
| 3.4160186 | 3.4995038 | 3.5538956 | 3.5803102 | 3.5800345 |
| 3.5545124 | 3.5053270 | 3.4341804 | 3.3428704 | 3.2332656 |
| 3.1072800 | 2.9668481 | 2.8139013 | 2.6503465 | 2.4780490 |
| 2.2988169 | 2.1143913 | 1.9264385 | 1.7365460 | 1.5462211 |
| 1.3568901 | 1.1698997 | 0.9865176 | 0.8079318 | 0.6352500 |
| 0.4694956 | 0.3116039 | 0.1624149 | 0.0226669 | -0.1070118 |
| -0.2261096 | -0.3342379 | -0.4311352 | -0.5166680 | -0.5908297 |
| -0.6537350 | -0.7056120 | -0.7467906 | -0.7776891 | -0.7987986 |
| -0.8106669 | -0.8138825 | -0.8090599 | -0.7968257 | -0.7778083 |
| -0.7526319 | -0.7219063 | -0.6862301 | -0.6461880 | -0.6023532 |
| -0.5552906 | -0.5055596 | -0.4537174 | -0.4003201 | -0.3459233 |
| -0.2910796 | -0.2363349 | -0.1822225 | -0.1292556 | -0.0779191 |
| -0.0286612 | 0.0181148 | 0.0620573 | 0.1028714 | 0.1403215 |
| 0.1742316 | 0.2044829 | 0.2310091 | 0.2537894 | 0.2728409 |
| 0.2882096 | 0.2999617 | 0.3081762 | 0.3129383 | 0.3143350 |
| 0.3124536 | 0.3073812 | 0.2992077 | 0.2880296 | 0.2739555 |
| 0.2571122 | 0.2376502 | 0.2157487 | 0.1916186 | 0.1655045 |
| 0.1376831 | 0.1084602 | 0.0781650 | 0.0471432 | 0.0157477 |
| -0.0156703 | -0.0467706 | -0.0772327 | -0.1067625 | -0.1350979 |
| -0.1620121 | -0.1873135 | -0.2108453 | -0.2324818 | -0.2521238 |
| -0.2696933 | -0.2851283 | -0.2983775 | -0.3093963 | -0.3181453 |
| -0.3245889 | -0.3286970 | -0.3304483 | -0.3298346 | -0.3268660 |
| -0.3215770 | -0.3140314 | -0.3043271 | -0.2925984 | -0.2790169 |
| -0.2637901 | -0.2471575 | -0.2293845 | -0.2107548 | -0.1915610 |
| -0.1720951 | -0.1526387 | -0.1334542 | -0.1147769 | -0.0968097 |
| -0.0797193 | -0.0636352 | -0.0486510 | -0.0348275 | -0.0221972 |
| -0.0107706 | -0.0005416 | 0.0085057 | 0.0163927 | 0.0231422 |
| 0.0287766 | 0.0333169 | 0.0367850 | 0.0392056 | 0.0406106 |
| 0.0410431 | 0.0405616 | 0.0392435 | 0.0371870 | 0.0345120 |
| 0.0313582 | 0.0278816 | 0.0242491 | 0.0206313 | 0.0171942 |
| 0.0140909 | 0.0114524 | 0.0093809 | 0.0079429 | 0.0071662 |
| 0.0070377 | 0.0075054 | 0.0084818 | 0.0098505 | 0.0114746 |
| 0.0132057 | 0.0148949 | 0.0164020 | 0.0176054 | 0.0184091 |
| 0.0187484 | 0.0185930 | 0.0194722 | 0.0168480 | 0.0153603 |
| 0.0135711 | 0.0115816 | 0.0094992 | 0.0074293 | 0.0054678 |
| 0.0036951 | 0.0021717 | 0.0009352 | 0.0000000 | -0.0006413 |
| -0.0010149 | -0.0011618 | -0.0011323 | -0.0009808 | -0.0007607 |
| -0.0005197 | -0.0002965 | -0.0001182 | 0.0000000 | 0.0000550 |
| 0.0000538 | 0.0000111 | -0.0000540 | -0.0001207 | -0.0001701 |
| -0.0001877 | -0.0001650 | -0.0001006 | 0.0000000 | 0.0001250 |
| 0.0002584 | 0.0003820 | 0.0004777 | 0.0005303 | 0.0005291 |
| 0.0004698 | 0.0003546 | 0.0001930 | 0.0000000 | -0.0002049 |
| -0.0003999 | -0.0005637 | -0.0006772 | -0.0007265 | -0.0007039 |
| -0.0006091 | -0.0004496 | -0.0002399 | 0.0000000 | 0.0002463 |
| 0.0004742 | 0.0006599 | 0.0007839 | 0.0008322 | 0.0007987 |
| 0.0006853 | 0.0005019 | 0.0002658 | 0.0000000 | -0.0002695 |
| -0.0005158 | -0.0007142 | -0.0008442 | -0.0008922 | -0.0008527 |
| -0.0007287 | -0.0005317 | -0.0002807 | 0.0000000 | 0.0002828 |
| 0.0005397 | 0.0007452 | 0.0008787 | 0.0009265 | 0.0008835 |
| 0.0007534 | 0.0005486 | 0.0002891 | 0.0000000 | -0.0002902 |

## (C) - IN PHASE RESPONSE OF RECEIVER FILTER

## SAMPLED AT 96000 SAMPLES/SEC.

| 0.0020834 | 0.0028840 |
| ---: | ---: |
| 0.0029515 | 0.0021571 |
| -0.0022093 | -0.0030593 |
| -0.0031359 | -0.0022927 |
| 0.0023522 | 0.0032587 |
| 0.0033463 | 0.0024477 |
| -0.0025161 | -0.0034875 |
| -0.0035888 | -0.0026265 |
| 0.0027060 | 0.0037529 |
| 0.0038713 | 0.0028351 |
| -0.0029286 | -0.0040645 |
| -0.0042050 | -0.0030818 |
| 0.0031936 | 0.0044359 |
| 0.0046052 | 0.0033782 |
| -0.0035142 | -0.0048862 |
| -0.0050943 | -0.0037411 |
| 0.0039102 | 0.0054437 |
| 0.0057057 | 0.0041961 |
| -0.0044121 | -0.0061521 |
| -0.0064922 | -0.0047831 |
| 0.0050685 | 0.0070821 |
| 0.0075410 | 0.0055692 |
| -0.0059635 | -0.0083562 |
| -0.0090078 | -0.0066748 |
| 0.0072530 | 0.0102042 |
| 0.0111969 | 0.0083378 |
| -0.0092603 | -0.0131078 |
| -0.0147831 | -0.0110937 |
| 0.0127627 | 0.0182477 |
| 0.0215573 | 0.0163959 |
| -0.0200914 | -0.0292647 |
| -0.0377947 | -0.0295260 |
| 0.0412785 | 0.0626214 |
| 0.0995193 | 0.0830984 |
| -0.1668071 | -0.2869621 |
| -1.0483676 | -1.3142129 |
| -2.7013103 | -3.1322002 |
| -5.2132779 | -5.8263700 |
| -8.6610237 | -9.4639655 |
| -13.0181011 | -13.9817192 |
| -18.0503691 | -19.1041028 |
| -23.3570252 | -24.4113966 |
| -28.4829812 | -29.4457608 |
| -32.9622667 | -33.7385899 |
| -36.3227256 | -36.8220438 |
| -38.1530362 | -38.3098230 |
| -38.2009723 | -37.9881746 |
| -36.4049087 | -35.8308386 |
| -32.8685841 | -31.9726333 |
| -27.8544486 | -26.7090951 |
| -21.7865466 | -20.4926773 |
| -15.1996546 | -13.8692888 |
| -8.6472568 | -7.3866587 |
| -2.6349998 | -1.5364595 |
| 2.4143343 | 3.2797839 |
| 6.2049538 | 6.7978249 |
| 8.6112151 | 8.9274450 |
| 9.6733158 | 9.7369523 |
| 9.5513810 | 9.4048894 |
| 8.4850836 | 8.1836652 |
| 6.7664935 | 6.3716409 |
| 4.6999446 | 4.2696374 |
|  |  |


| 0.0036060 | 0.0034493 |
| ---: | ---: |
| 0.0000000 | -0.0011545 |
| -0.0038282 | -0.0036635 |
| 0.0000000 | 0.0012286 |
| 0.0040814 | 0.0039075 |
| 0.0000000 | -0.0013136 |
| -0.0043724 | -0.0041884 |
| 0.0000000 | 0.0014119 |
| 0.0047108 | 0.0045154 |
| 0.0000000 | -0.0015271 |
| -0.0051093 | -0.0049009 |
| 0.0000000 | 0.0016639 |
| 0.0055857 | 0.0053625 |
| 0.0000000 | -0.0018291 |
| -0.0061655 | -0.0059255 |
| 0.0000000 | 0.0020327 |
| 0.0068867 | 0.0066276 |
| 0.0000000 | -0.0022900 |
| -0.0078087 | -0.0075278 |
| 0.0000000 | 0.0026255 |
| 0.0090284 | 0.0087235 |
| 0.0000000 | -0.0030806 |
| -0.0107162 | -0.0103866 |
| 0.0000000 | 0.0037323 |
| 0.0131981 | 0.0128502 |
| 0.0000000 | -0.0047375 |
| -0.0171766 | -0.0168415 |
| 0.0000000 | 0.0064675 |
| 0.0244403 | 0.0242505 |
| 0.0000000 | -0.0100061 |
| -0.0408489 | -0.0414752 |
| 0.0000000 | 0.0198065 |
| 0.0959832 | 0.1028664 |
| 0.0000000 | -0.0718006 |
| -0.6090863 | -0.8135869 |
| -1.9417691 | -2.3047252 |
| -4.0997796 | -4.6379411 |
| -7.1672740 | -7.8952046 |
| -11.1762206 | -12.0820595 |
| -15.9797864 | -17.0077375 |
| -21.2303061 | -22.2954378 |
| 26.4837800 | -27.4943951 |
| -31.2786220 | -32.1410152 |
| $-35.1417733 x$ | -35.7616526 |
| -37.6250430 | -37.9243142 |
| -38.4036547 | -38.3394980 |
| -37.3414093 | -36.9088698 |
| $-34.4788717 \times$ | -33.7048744 |
| -30.0136650 | -28.9572744 |
| -24.3073586 | -23.0596138 |
| -17.8606794 | -16.5315711 |
| $-11.2301982 \times$ | -9.9297163 |
| -4.9457589 | -3.7724599 |
| 0.5325620 | 1.4979660 |
| 4.8518440 | 5.5561560 |
| $7.8155160 \times$ | 8.2408417 |
| 9.4016301 | 9.5620723 |
| 9.7283725 | 9.6599211 |
| 9.0070400 | 8.7603660 |
| 7.5124057 | 7.1475503 |
| 5.5498420 | 5.1273471 |
| 3.4078680 | 2.9799668 |


| 2.5562923 | 2.1384269 | 1.7278820 | 1.3260998 | 0.9344548 |
| :---: | :---: | :---: | :---: | :---: |
| 0.5542524 | 0.1867261 | -0.1669666 | -0.5057505 | -0.8286388 |
| -1.1347394 | -1.4232612 | -1.6935186 | -1.9449349 | -2.1770436 |
| -2.3894877 | -2.5820173 | -2.7544853 | -2.9068417 | -3.0391275 |
| -3.1514680 | -3.2440658 | -3.1371960 | -3.3712008 | -3.4064862 |
| -3.4235203 | -3.4228322 | -3.4050122 | -3.3707125 | -3.3206485 |
| -3.2555987 | -3.1764049 | -3.0839704 | -2.9792569 | -2.8632797 |
| -2.7371004 | -2.6018195 | -2.4585658 | -2.3084858 | -2.1527328 |
| -1.9924551 | -1.8287851 | -1.6628297 | -1.4956608 | -1.3283086 |
| -1.1617554 | -0.9969315 | -0.8347123 | -0.6759166 | -0.5213057 |
| -0.3715827 | -0.2273931 | -0.0893236 | 0.0420976 | 0.1664022 |
| 0.2831831 | 0.3920953 | 0.4928585 | 0.5852576 | 0.6691446 |
| 0.7444381 | 0.8111235 | 0.8692507 | 0.9189314 | 0.9603349 |
| 0.9936831 | 1.0192439 | 1.0373248 | 1.0482656 | 1.0524314 |
| 1.0502056 | 1.0419837 | 1.0281681 | 1.0091636 | 0.9853737 |
| 0.9571988 | 0.9250337 | 0.8892674 | 0.8502824 | 0.8084543 |
| 0.7641518 | 0.7177366 | 0.6695626 | 0.6199751 | 0.5693099 |
| 0.5178921 | 0.4660349 | 0.4140381 | 0.3621876 | 0.3107546 |
| 0.2599960 | 0.2101542 | 0.1614588 | 0.1141273 | 0.0683671 |
| 0.0243771 | -0.0176514 | -0.0575333 | -0.0950895 | -0.1301474 |
| -0.1625418 | -0.1921175 | -0.2187313 | -0.2422558 | -0.2625825 |
| -0.2796255 | -0.2933242 | -0.3036466 | -0.3105902 | -0.3141835 |
| -0.3144855 | -0.3115844 | -0.3055961 | -0.2966613 | -0.2849425 |
| -0.2706218 | -0.2538974 | -0.2349824 | -0.2141025 | -0.1914962 |
| -0.1674140 | -0.1421198 | -0.1158908 | -0.0890187 | -0.0618098 |
| -0.0345840 | -0.0076731 | 0.0185830 | 0.0438410 | 0.0677611 |
| 0.0900148 | 0.1102940 | 0.1283201 | 0.1438535 | 0.1567017 |
| 0.1667264 | 0.1738479 | 0.1780475 | 0.1793672 | 0.1779058 |
| 0.1738134 | 0.1672828 | 0.1585399 | 0.1478332 | 0.1354230 |
| 0.1215723 | 0.1065385 | 0.0905683 | 0.0738947 | 0.0567369 |
| 0.0393033 | 0.0217962 | 0.0044183 | -0.0126208 | -0.0290985 |
| -0.0447741 | -0.0593875 | -0.0726604 | -0.0843018 | -0.0940174 |
| -0.1015230 | -0.1065605 | -0.1089161 | -0.1084392 | -0.1050604 |
| -0.0988063 | -0.0898108 | -0.0783207 | -0.0646936 | -0.0493906 |
| -0.0329595 | -0.0160129 | 0.0007998 | 0.0168251 | 0.0314392 |
| 0.0440824 | 0.0542901 | 0.0617199 | 0.0661713 | 0.0675974 |
| 0.0661066 | 0.0619555 | 0.0555316 | 0.0473282 | 0.0379127 |
| 0.0278901 | 0.0178649 | 0.0084034 | 0.0000000 | -0.0069515 |
| -0.0121784 | -0.0155414 | -0.0170357 | -0.0167846 | -0.0150226 |
| -0.0120718 | -0.0083134 | -0.0041553 | 0.0000000 | 0.0037858 |
| 0.0068974 | 0.0091119 | 0.0103004 | 0.0104327 | 0.0095730 |
| 0.0078684 | 0.0055313 | 0.0028173 | 0.0000000 | -0.0026535 |
| -0.0049063 | -0.0065707 | -0.0075227 | -0.0077101 | -0.0071534 |
| -0.0059408 | -0.0042170 | -0.0021675 | 0.0000000 | 0.0020758 |
| 0.0038676 | 0.0052171 | 0.0060139 | 0.0062038 | 0.0057913 |
| 0.0048377 | 0.0034531 | 0.0017843 | 0.0000000 | -0.0017257 |
| -0.0032301 | -0.0043763 | -0.0050659 | -0.0052469 | -0.0049169 |
| -0.0041225 | -0.0029530 | -0.0015311 | 0.0000000 | 0.0014904 |
| 0.0027980 | 0.0038019 | 0.0044133 | 0.0045833 | 0.0043063 |
| 0.0036196 | 0.0025991 | 0.0013507 | 0.0000000 | -0.0013207 |
| -0.0024846 | -0.0033830 | -0.0039349 | -0.0040943 | -0.0038540 |
| -0.0032453 | -0.0023344 | -0.0012152 | 0.0000000 | 0.0011920 |

## (D) - QUADRATURE RESPONSE OF RECEIVER FILTER SAMPLED AT 96000 SAMPLES/SEC.

|  |
| :---: |
|  |
|  |
|  |
| -0 |
|  |
|  |
| -0.00 |
| -0. |
| 01 |
| 0.002 |
| 0.002 |
| . |
| 0.00 |
|  |
| 0. |
| 0.00 |
| 002 |
| 0.0042 |
| , |
| 0.004 |
| . 00 |
| 0.006 |
| 0.0048 |
| 0. |
| 00 |
| 0.0100 |
| 0 |
| 0.014 |
| 0.013 |
| 0.026 |
| 0.0287 |
| -0.069 |
| 0. |
| 0.734 |
| 1.89776 |
| 3.67964 |
| 6 |
| 9.3680 |
| 13.174 |
| 17.328 |
| 21.512 |
| 35 |
|  |
| 0.37 |
| 82 |
| . 6124 |
| 6 |
| 22.411 |
| 6.9650207 |
| . 93 |
| 4.89 |
|  |
| 5.05 |
|  |
| . 7021 |
| . 7994 |
|  |
|  |
|  |
|  |


| -0.0017684 | -0.0020920 |
| ---: | ---: |
| -0.0013265 | -0.0007019 |
| 0.0018869 | 0.0022331 |
| 0.0014184 | 0.0007509 |
| -0.0020222 | -0.0023944 |
| -0.0015237 | -0.0008071 |
| 0.0021780 | 0.0025803 |
| 0.0016458 | 0.0008722 |
| -0.0023595 | -0.0027970 |
| -0.0017887 | -0.0009486 |
| 0.0025734 | 0.0030529 |
| 0.0019583 | 0.0010394 |
| -0.0028292 | -0.0033593 |
| -0.0021628 | -0.0011490 |
| 0.0031405 | 0.0037328 |
| 0.0024141 | 0.0012841 |
| -0.0035271 | -0.0041980 |
| -0.0027302 | -0.0014543 |
| 0.0040200 | 0.0047926 |
| 0.0031392 | 0.0016754 |
| -0.0046689 | -0.0055784 |
| -0.0036884 | -0.0019735 |
| 0.0055601 | 0.0066628 |
| 0.0044624 | 0.0023960 |
| -0.0068550 | -0.0082493 |
| -0.0056287 | -0.0030378 |
| 0.0088926 | 0.0107699 |
| 0.0075633 | 0.0041152 |
| -0.0125015 | -0.0153018 |
| -0.0112861 | -0.0062288 |
| 0.0202348 | 0.0252628 |
| 0.0204986 | 0.0116438 |
| -0.0436259 | -0.0569346 |
| -0.0580471 | -0.0355096 |
| 0.2008434 | 0.3038040 |
| 0.9214035 | 1.1305184 |
| 2.2020347 | 2.5316323 |
| 4.1177943 | 4.5847515 |
| 6.7470708 | 7.3609602 |
| 10.0879953 | 10.8306611 |
| 13.9873413 | 14.8115810 |
| 18.1741338 | 19.0179670 |
| 22.3214508 | 23.1140945 |
| 26.0527816 | 26.7121061 |
| 28.9443490 | 29.3849631 |
| 30.5940557 | 30.7469241 |
| 30.7135164 | 30.5392595 |
| 29.1695346 | 28.6607020 |
| 25.9921623 | 25.1760495 |
| 21.3959867 | 20.3392687 |
| 15.7869608 | 14.5898947 |
| 9.7056198 | 8.4840207 |
| 3.7326488 | -2.6003648 |
| -1.5852635 | -6.46689878 |
| -5.7899250 | -8.9481726 |
| -8.5746090 | -9.9078276 |
| -9.8339173 | -9.9145327 |
| -9.6600448 | -8.2992019 |
| -6.1091243 | -3.4690010 |
| -3.5158932 | -0.9467899 |


| -0.0022137 | -0.0021188 |
| ---: | ---: |
| 0.0000000 | 0.0007112 |
| 0.0022331 | 0.0022636 |
| 0.0000000 | -0.0007615 |
| -0.0023944 | -0.0024294 |
| 0.0000000 | 0.0008193 |
| 0.0025803 | 0.0026209 |
| 0.0000000 | -0.0008865 |
| -0.0027970 | -0.0028448 |
| 0.0000000 | 0.0009655 |
| 0.0030529 | 0.0031097 |
| 0.0000000 | -0.0010596 |
| -0.0033593 | -0.0034280 |
| 0.0000000 | 0.0011738 |
| 0.0037328 | 0.0038176 |
| 0.0000000 | -0.0013149 |
| -0.0041980 | -0.0043050 |
| 0.0000000 | 0.0014938 |
| 0.0047926 | 0.0049319 |
| 0.0000000 | -0.0017277 |
| -0.0055784 | -0.0057667 |
| 0.0000000 | 0.0020459 |
| 0.0066628 | 0.0069307 |
| 0.0000000 | -0.0025024 |
| -0.0082493 | -0.0086579 |
| 0.0000000 | 0.0032077 |
| 0.0107699 | 0.0114595 |
| 0.0000000 | -0.0044223 |
| -0.0153018 | -0.0166617 |
| 0.0000000 | 0.0069065 |
| 0.0252628 | 0.0287494 |
| 0.0000000 | -0.0137805 |
| -0.0569346 | -0.0717856 |
| 0.0000000 | 0.0502174 |
| 0.3038040 | 0.5699953 |
| 1.3625952 | 1.6181574 |
| 2.5316323 | 3.2696998 |
| 5.0809810 | 5.6067709 |
| 7.3609602 | 8.6725928 |
| 11.5941040 | 12.3762123 |
| 14.8115810 | 16.4849309 |
| 19.8575654 | 20.6900291 |
| 23.1140945 | 24.6367063 |
| 27.3342937 | 27.9159203 |
| 29.3849631 | 30.1049827 |
| 30.8364944 | 30.8614495 |
| 30.5392595 | 29.9886380 |
| 28.0870086 | 27.4498335 |
| 25.1760495 | 23.3826834 |
| 19.2456627 | 18.1194377 |
| 14.5898947 | 12.1575352 |
| 7.2714615 | 6.0724514 |
| 2.6003648 | 0.4313078 |
| -3.4220063 | -4.2654451 |
| -6.4668938 | -7.6426318 |
| -9.2602472 | -9.5113244 |
| -9.9078276 | -9.8887123 |
| -9.2402193 | -8.9651555 |
| -7.9145327 | -7.0582687 |
| -5.0954462 | -4.5733089 |
| -2.9873479 | -1.9465057 |
| -0.0090979 | 0.4307804 |


| 0.8486902 | 1.2428835 |
| ---: | ---: |
| 2.5543929 | 2.8106573 |
| 3.5326443 | 3.6372023 |
| 3.7662970 | 3.7311846 |
| 3.3626719 | 3.2219288 |
| 2.5192911 | 2.3180335 |
| 1.4645527 | 1.2474814 |
| 0.4101854 | 0.2153215 |
| -0.4728898 | -0.6179151 |
| -1.0739389 | -1.1558594 |
| -1.3573908 | -1.3777569 |
| -1.3526856 | -1.3224823 |
| -1.1234453 | -1.0572163 |
| -0.7459592 | -0.6599706 |
| -0.3039922 | -0.2153002 |
| 0.1206639 | 0.1975324 |
| 0.4652678 | 0.5208688 |
| 0.6916247 | 0.7206306 |
| 0.7792146 | 0.7794833 |
| 0.7265489 | 0.7010670 |
| 0.5613013 | 0.5191920 |
| 0.3366648 | 0.2898653 |
| 0.1106709 | 0.0697030 |
| -0.0695178 | -0.0968894 |
| -0.1712422 | -0.1806410 |
| -0.1828896 | -0.1755313 |
| -0.1233136 | -0.1063995 |
| -0.0343991 | -0.0169347 |
| 0.0434430 | 0.0553573 |
| 0.0874120 | 0.0912970 |
| 0.0902695 | 0.0861192 |
| 0.0574296 | 0.0481657 |
| 0.0097832 | 0.0010815 |
| -0.0243087 | -0.0277859 |
| -0.0304985 | -0.0289175 |
| -0.0182778 | -0.0153320 |
| -0.0057484 | -0.0041414 |
| -0.0006569 | -0.0003243 |
| 0.0000920 | 0.0001053 |
| 0.0000975 | 0.0000765 |
| -0.0001082 | -0.0001611 |
| -0.0002088 | -0.0001599 |
| 0.0001903 | 0.0002715 |
| 0.0003061 | 0.0002281 |
| -0.0002485 | -0.0003484 |
| -0.0003711 | -0.0002733 |
| 0.0002867 | 0.0003987 |
| 0.0004136 | 0.0003029 |
| -0.0003116 | -0.0004316 |
| -0.0004412 | -0.0003222 |
| 0.0003278 | 0.0004529 |
| 0.0004591 | 0.0003346 |
|  |  |


|  |  |  |
| ---: | ---: | ---: |
| 1.6118081 | 1.6118081 | 2.2686229 |
| 3.0368596 | 3.2326498 | 3.3978877 |
| 3.7120534 | 3.7120534 | 3.7756159 |
| 3.6716792 | 3.5893140 | 3.4857337 |
| 3.0653490 | 3.0653490 | 2.7121606 |
| 2.1101940 | 1.8975352 | 1.6817692 |
| 1.0320863 | 1.0320863 | 0.6120931 |
| 0.0286218 | -0.1488970 | -0.3163297 |
| -0.7508719 | -0.7508719 | -0.9790951 |
| -1.2249389 | -1.2813604 | -1.3253953 |
| -1.3869546 | -1.3869546 | -1.3738767 |
| -1.2838523 | -1.2373936 | -1.1837168 |
| -0.9856814 | -0.9856814 | -0.8293693 |
| -0.5720984 | -0.4830313 | -0.3934445 |
| -0.1279595 | -0.1279595 | 0.0405073 |
| 0.2707403 | 0.3399563 | 0.4048860 |
| 0.5714807 | 0.5714807 | 0.6570132 |
| 0.7439369 | 0.7614804 | 0.7732345 |
| 0.7741548 | 0.7741548 | 0.7474360 |
| 0.6713682 | 0.6378702 | 0.6010234 |
| 0.4751887 | 0.4751887 | 0.3834522 |
| 0.2434774 | 0.1979014 | 0.1535145 |
| 0.0309219 | 0.0309219 | -0.0389437 |
| -0.1208743 | -0.1413246 | -0.1581338 |
| -0.1863757 | -0.1863757 | -0.1873154 |
| -0.1655443 | -0.1532672 | -0.1390640 |
| -0.0886997 | -0.0886997 | -0.0523731 |
| -0.0002249 | 0.0155204 | 0.0301249 |
| 0.0657745 | 0.0657745 | 0.0818470 |
| 0.0934988 | 0.0940330 | 0.0929364 |
| 0.0806009 | 0.0738595 | 0.0660688 |
| 0.0385182 | 0.0287380 | 0.0190781 |
| -0.0068255 | -0.0137700 | -0.0196251 |
| -0.0300688 | -0.0312132 | -0.0313141 |
| 0.0267365 | -0.0241266 | -0.0212551 |
| -0.0125319 | -0.0099654 | -0.0076930 |
| -0.0028609 | -0.0018799 | -0.0011601 |
| -0.0001182 | 0.0000000 | 0.0000622 |
| 0.0001109 | 0.0001122 | 0.0001085 |
| 0.0000439 | 0.0000000 | -0.0000524 |
| -0.0002036 | -0.0002290 | -0.0002318 |
| -0.0000883 | 0.0000000 | 0.0000963 |
| 0.0003299 | 0.0003578 | 0.0003503 |
| 0.0001228 | 0.0000000 | -0.0001281 |
| -0.0004166 | -0.0004453 | -0.0004300 |
| -0.0001456 | 0.0000000 | 0.0001491 |
| 0.0004734 | 0.0005024 | 0.0004821 |
| 0.0001605 | 0.0000000 | -0.0001628 |
| -0.0005104 | -0.0005397 | -0.0005161 |
| -0.0001702 | 0.0000000 | 0.0001716 |
| 0.0005344 | 0.0005638 | 0.0005380 |
| 0.0001764 | 0.0000000 | -0.0001773 |
|  |  |  |

## APPENDIX C

## DIFFERENTIATION WITH RESPECT TO A VECTOR

The quantity $\mathrm{C}_{\mathrm{i}}$ in Eqn. 5.3 .13 is real and positive and is a function of the channel estimation vector $\mathrm{V}_{\mathrm{i}}$ given by Eqn. 5.3.2. The elements of the vector $\mathrm{V}_{\mathrm{i}}$ can be written as

$$
\begin{aligned}
V_{i}=\left[\begin{array}{llll}
\left(y_{i, 0,1}^{\prime}+j y_{i, 0,2}^{\prime}\right), & \left(y_{i, 1,1}^{\prime}+j y_{i, 1,2}^{\prime}\right), & \ldots, & \left(y_{i, g, 1}^{\prime}+j y_{i, g, 2}^{\prime}\right), \\
& \left(y_{i, 0,1}^{\prime \prime}+j y_{i, 0,2}^{\prime \prime}\right), & \left(y_{i, 1,1}^{\prime \prime}+j y_{i, 1,2}^{\prime \prime}\right), & \ldots .,
\end{array}\left(y_{i, g, 1}^{\prime \prime}+j y_{i, g, 2}^{\prime \prime}\right)\right]
\end{aligned}
$$

or

$$
\begin{align*}
V_{i}=\left[\left(v_{i, 0,1}+j v_{i, 0,2}\right),\right. & \left(v_{i, 1,1}+j v_{i, 1,2}\right), \\
& \ldots, \\
& \left.\left(v_{i, 2 g+1,1}+j v_{i, 2 g+1,2}\right)\right]
\end{align*}
$$

where, for example, $\mathrm{v}_{\mathrm{i}, 0,1}$ is the real part and $\mathrm{v}_{\mathrm{i}, 0,2}$ is the imaginary part of the first element of the vector $\mathrm{V}_{\mathrm{i}}$. By the definition of differentiation with respect to a vector, $\frac{x_{c}}{w_{1}}$ is given by [59, 103],

$$
\frac{\partial C_{i}}{\partial V_{i}}=\left[\begin{array}{cccc}
\frac{\partial C_{i}}{\partial v_{i, 0,1}} & + & j & \frac{\partial C_{i}}{\partial v_{i, 0,2}} \\
\frac{\partial C_{i}}{\partial v_{i, 1,1}} & + & j & \frac{\partial C_{i}}{\partial v_{i, 1,2}} \\
& \cdot & & \\
& \cdot & & \\
\frac{\partial C_{i}}{\partial v_{i, 2 g+1,1}} & + & j & \frac{\partial C_{i}}{\partial v_{i, 2 g+1,2}}
\end{array}\right]^{T}
$$

From Eqn. 5.3.13, $\mathrm{C}_{\mathrm{i}}$ is given by

$$
\begin{align*}
C_{i}=\sum_{h=0}^{i} \omega^{i-h}\left(r_{h} r_{h}^{*}-r_{h} \bar{S}_{h} \phi_{h-i}^{T} V_{i}^{*}\right. & -r_{h}^{*} V_{i} \phi_{h-i} S_{h}^{T}+ \\
& \left.+V_{i} \phi_{h-i} S_{h}^{T} \bar{S}_{h} \phi_{h-i}^{T} V_{i}^{*}\right)
\end{align*}
$$

It is necessary to determine the gradient of $\mathrm{C}_{\mathrm{i}}$ with respect to $\mathrm{V}_{\mathrm{i}}$ (Eqn. 5.3.14). From Eqn. $\mathrm{C} 1.4, \partial C_{i} / \partial V_{i}$ is given by,

$$
\begin{align*}
\frac{\partial C_{i}}{\partial V_{i}}= & \sum_{h=0}^{i} \omega^{i-h}\left[\frac{\partial\left(r_{h} r_{h}^{*}\right)}{\partial V_{i}}+\frac{\partial\left(-r_{h} \bar{S}_{h} \phi_{h-i}^{T} V_{i}^{*}\right)}{\partial V_{i}}+\right. \\
& \left.+\frac{\partial\left(-r_{h}^{*} V_{i} \phi_{h-i} S_{h}^{T}\right)}{\partial V_{i}}+\frac{\partial\left(V_{i} \phi_{h-i} S_{h}^{T} \bar{S}_{h} \phi_{h-i}^{T} V_{i}^{*}\right)}{\partial V_{i}}\right]
\end{align*}
$$

Consider the term

$$
C_{i}^{\prime}=-r_{h} \bar{S}_{h} \phi_{h-i}^{T} V_{i}^{*}
$$

and let

$$
D=-r_{h} \bar{S}_{h} \phi_{h-i}^{T}
$$

then from Eqns. C1.6-C1.7

$$
C_{i}^{\prime}=D V_{i}^{*}=\sum_{k=0}^{2 g+1} d_{k}\left(v_{i, k, 1}-j v_{i, k, 2}\right)
$$

where $\mathrm{d}_{\mathrm{k}}$ are the elements of the vector D , for $\mathrm{k}=0,1, \ldots,(2 \mathrm{~g}+1)$.

Hence

$$
\frac{\partial C_{i}^{\prime}}{\partial v_{i, k, 1}}=d_{k}
$$

for $\mathrm{k}=0,1, \ldots .,(2 \mathrm{~g}+1)$, and

$$
\begin{array}{r}
\frac{\partial C_{i}^{\prime}}{\partial v_{i, k, 2}}=-j d_{k} \\
\text { for } \mathrm{k}=0,1, \ldots .,(2 \mathrm{~g}+1) .
\end{array}
$$

Substituting Eqns. C1.9 and C1.10 in Eqn. C1.3 and simplifying

$$
\frac{\partial C_{i}^{\prime}}{\partial V_{i}}=2 D=-2 r_{h} \bar{S}_{h} \phi_{h-i}^{T}
$$

Similarly consider the term

$$
C_{i}^{\prime \prime}=-r_{h}^{*} V_{i} \phi_{h-i} S_{h}^{T}
$$

From Eqns. C1.7 and C1.12, $c_{i}^{\prime \prime}$ can now be written as

$$
C_{i}^{\prime \prime}=\sum_{k=0}^{2 g+1} d_{k}^{*}\left(v_{i, k, 1}+j v_{i, k, 2}\right)
$$

Hence

$$
\frac{\partial C_{i}^{\prime \prime}}{\partial v_{i, k, 1}}=d_{k}^{*}
$$

for $\mathrm{k}=0,1, \ldots .,(2 \mathrm{~g}+1)$, and

$$
\frac{\partial C_{i}^{\prime \prime}}{\partial v_{i, k, 2}}=j d_{k}^{*}
$$

Substituting Eqns. C1.14 and C1.15 in Eqn. C1.3 and simplifying

$$
\frac{\partial C_{i}^{\prime \prime}}{\partial V_{i}}=0
$$

Now consider the term

$$
\begin{align*}
C_{i}^{\prime \prime \prime} & =V_{i} S_{h}^{T} \bar{S}_{h} \phi_{h-i}^{T} V_{i}^{*} \\
& =V_{i} Q V_{i}^{*}
\end{align*}
$$

where

$$
Q=\phi_{h-i} S_{h}^{T} S_{h} \phi_{h-i}^{T}
$$

is a $(2 g+1) \times(2 g+1)$ matrix.

For the sake of simplicity, let us assume that $\mathrm{V}_{\mathrm{i}}$ is a two component row vector given by

$$
V_{i}=\left[\left(v_{i, 0,1}+j v_{i, 0,2}\right) \quad\left(v_{i, 1,1}+j v_{i, 1,2}\right)\right]
$$

and Q is a $2 \times 2$ matrix given by

$$
Q=\left[\begin{array}{ll}
q_{11} & q_{12} \\
q_{21} & q_{22}
\end{array}\right]
$$

By definition (Eqn. C1.3)

$$
\frac{\partial C_{i}^{\prime \prime \prime}}{\partial V_{i}}=\left[\left(\frac{\partial C_{i}^{\prime \prime \prime}}{\partial v_{i, 0,1}}+j \frac{\partial C_{i}^{\prime \prime \prime}}{\partial v_{i, 0,2}}\right) \quad\left(\frac{\partial C_{i}^{\prime \prime \prime}}{\partial v_{i, 1,1}}+j \frac{\partial C_{i}^{\prime \prime \prime}}{\partial v_{i, 1,2}}\right)\right]
$$

Expanding Eqn. C1.18, using Eqns. C1.20 and C1.21

$$
\begin{align*}
C_{i}^{\prime \prime \prime} & =\left(v_{i, 0,1}^{2}+v_{i, 0,2}^{2}\right) q_{11}+\left(v_{i, 1,1}+j v_{i, 1,2}\right) q_{21}\left(v_{i, 0,1}-j v_{i, 0,2}\right)+ \\
& +\left(v_{i, 0,1}+j v_{i, 0,2}\right) q_{12}\left(v_{i, 1,1}-j v_{i, 1,2}\right)+\left(v_{i, 1,1}^{2}+v_{i, 1,2}^{2}\right) q_{22}
\end{align*}
$$

Therefore,

$$
\begin{align*}
\frac{\partial C_{i}^{\prime \prime \prime}}{\partial v_{i, 0,1}}= & 2 v_{i, 0,1} q_{11} \\
+ & \left(v_{i, 1,1}+j v_{i, 1,2}\right) q_{21} \\
& +\left(v_{i, 1,1}-j v_{i, 1,2}\right) q_{12} \\
\frac{\partial C_{i}^{\prime \prime \prime}}{\partial v_{i, 0,2}}= & 2 v_{i, 0,2} q_{11}-j\left(v_{i, 1,1}+j v_{i, 1,2}\right) q_{21} \\
& +j\left(v_{i, 1,1}-j v_{i, 1,2}\right) q_{12} \\
\frac{\partial C_{i}^{\prime \prime \prime}}{\partial v_{i, 1,1}}= & \left(v_{i, 0,1}-j v_{i, 0,2}\right) q_{21}+\left(v_{i, 0,1}+j v_{i, 0,2}\right) q_{12} \\
& +2 v_{i, 1,1} q_{22} \\
\frac{\partial C_{i}^{\prime \prime \prime}}{\partial v_{i, 1,2}}= & j\left(v_{i, 0,1}-j v_{i, 0,2}\right) q_{21}-j\left(v_{i, 0,1}+j v_{i, 0,2}\right) q_{12} \\
& +2 v_{i, 1,2} q_{22}
\end{align*}
$$

From Eqns. C1.24 and C1.25

$$
\begin{align*}
\left(\frac{\partial C_{i}^{\prime \prime \prime}}{\partial v_{i, 0,1}}+j \frac{\partial C_{i}^{\prime \prime \prime}}{\partial v_{i, 0,2}}\right) & =2\left(v_{i, 0,1}+j v_{i, 0,2}\right) q_{11} \\
& +2\left(v_{i, 1,1}+j v_{i, 1,2}\right) q_{21}
\end{align*}
$$

and from Eqn. C1.26 and C1.27

$$
\begin{align*}
\left(\frac{\partial C_{i}^{\prime \prime \prime}}{\partial v_{i, 1,1}}+j \frac{\partial C_{i}^{\prime \prime \prime}}{\partial v_{i, 1,2}}\right) & =2\left(v_{i, 0,1}+j v_{i, 0,2}\right) q_{12} \\
& +2\left(v_{i, 1,1}+j v_{i, 1,2}\right) q_{22}
\end{align*}
$$

Therefore, from Eqns. C1.19, C1.21, C1.22, C1.28 and C1.29

$$
\begin{align*}
\frac{\partial C_{i}^{\prime \prime \prime}}{\partial V_{i}} & =2 V_{i} Q \\
& =2 V_{i} \phi_{h-i} S_{h}^{T} S_{h} \phi_{h-i}^{T}
\end{align*}
$$

Also

$$
\frac{\partial\left(r_{h} r_{h}^{*}\right)}{\partial V_{i}}=0
$$

Thus from Eqns. C1.5, C1.11, C1.16, C1.30 and C1.31

$$
\frac{\partial C_{i}}{\partial V_{i}}=\sum_{h=0}^{i} \omega^{i-h}\left(-2 r_{h} \bar{S}_{h} \phi_{h-i}^{T}+2 V_{i} \phi_{h-i} S_{h}^{T} \bar{S}_{h} \phi_{h-i}^{T}\right)
$$

as is given in Eqn. 5.3.15.

## APPENDIX D

## MATRIX INVERSE IDENTITY

From Eqn. 5.3.36

$$
R_{i}=\omega R_{i, i-1}+S_{i}^{T} \bar{S}_{i}
$$

and from Eqn. 5.3.29

$$
P_{i}=R_{i}^{-1}
$$

In order to obtain the updated estimate of $\mathrm{V}_{\mathrm{i}}$ from the one-step prediction $\mathrm{V}_{\mathrm{i},-1}$, it is necessary to evaluate $\mathrm{P}_{\mathrm{i}}$ (see Section 5.3).
$\mathrm{R}_{\mathrm{i}}$ is assumed non-singular, so that premultiplying Eqn. D1.1 by $R_{i}^{-1}$,

$$
R_{i}^{-1} R_{i}=I=\omega R_{i}^{-1} R_{i, i-1}+R_{i}^{-1} S_{i}^{T} S_{i}
$$

Postmultiplying Eqn. D1.3 by $R_{i, i-1}^{-1}$,

$$
R_{i, i-1}^{-1}=\omega R_{i}^{-1}+R_{i}^{-1} S_{i}^{T} \bar{S}_{i} R_{i, i-1}^{-1}
$$

or

$$
R_{i}^{-1} S_{i}^{T} S_{i} R_{i, i-1}^{-1}=R_{i, i-1}^{-1}-\omega R_{i}^{-1}
$$

Postmultiplying Eqn. D1.4 by $s_{i}^{T}$

$$
\begin{align*}
R_{i, i-1}^{-1} S_{i}^{T} & =\omega R_{i}^{-1} S_{i}^{T}+R_{i}^{-1} S_{i}^{T} \bar{S}_{i} R_{i, i-1}^{-1} S_{i}^{T} \\
& =\omega R_{i}^{-1} S_{i}^{T}\left(I+\omega^{-1} \bar{S}_{i} R_{i, i-1}^{-1} S_{i}^{T}\right)
\end{align*}
$$

The matrix $\left(I+\omega^{-1} \bar{S}_{i} R_{i, i-1}^{-1} S_{i}^{T}\right)$ is also assumed non-singular, hence postmultiplying Eqn. D1.7 by the inverse of the matrix, and simplfying,

$$
\omega R_{i}^{-1} S_{i}^{T}=R_{i, i-1}^{-1} S_{i}^{T}\left(I+\omega^{-1} \bar{S}_{i} R_{i, i-1}^{-1} S_{i}^{T}\right)^{-1}
$$

Postmultiplying Eqn. D1.8 by $\bar{S}_{i} \omega^{-1} R_{i, i-1}^{-1}$,

$$
R_{i}^{-1} S_{i}^{T} \bar{S}_{i} R_{i, i-1}^{-1}=R_{i, i-1}^{-1} S_{i}^{T}\left(I+\omega^{-1} \bar{S}_{i} R_{i, i-1}^{-1} S_{i}^{T}\right)^{-1} \bar{S}_{i} \omega^{-1} R_{i, i-1}^{-1}
$$

Combining Eqns. D1.5 and D1.9

$$
R_{i, i-1}^{-1}-\omega R_{i}^{-1}=R_{i, i-1}^{-1} S_{i}^{T}\left(I+\omega^{-1} \bar{S}_{i} R_{i, i-1}^{-1} S_{i}^{T}\right)^{-1} \bar{S}_{i} \omega^{-1} R_{i, i-1}^{-1}
$$

or

$$
R_{i}^{-1}=\omega^{-1} R_{i, i-1}^{-1}-\omega^{-1} R_{i, i-1}^{-1} S_{i}^{T}\left(I+\omega^{-1} \bar{S}_{i} R_{i, i-1}^{-1} S_{i}^{T}\right)^{-1} \bar{S}_{i} \omega^{-1} R_{i, i-1}^{-1}
$$

or

$$
R_{i}^{-1}=\frac{1}{\omega}\left[R_{i, i-1}^{-1}-\frac{R_{i, i-1}^{-1} S_{i}^{T} \bar{S}_{i} R_{i, i-1}^{-1}}{\omega+\bar{S}_{i} R_{i, i-1}^{-1} S_{i}^{T}}\right]
$$

Eqn. D1.12 is also referred to as the matrix inverse lemma. This completes the derivation of Eqn. 5.3.37 from Eqn. 5.3.36.

## APPENDIX E

## GRAM SCHMIDT ORTHONORMALIZATION PROCESS

The three orthonormal (g+1)- component basis vectors A, B and C, in Chapter 7, are not likely to lie exactly in the three-dimensional subspace containing $Y_{i}$, as the subspace itself is unlikely to be stationary owing to the time varying nature of the HF channel. For satisfactory operation of the estimator, the subspace spanned by $\mathrm{A}_{\mathrm{i}}$, $\mathrm{B}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{i}}$ must be adjusted adaptively to track the received signal in such a way that the new subspace spanned by the new vectors $A_{i+1}^{\prime}, B_{i+1}^{\prime}$ and $C_{i+1}^{\prime}$ is closer to $\mathrm{Y}_{\mathrm{i}}$. These three vectors will not exactly be orthonormal and so they are orthonormalized using the Gram-Schmidt orthonormalization process [33, 35-36, 120], as follows.

First the receiver sets

$$
A_{i+1}=\mid A_{i+1}^{\prime} \Gamma^{-1} A_{i+1}^{\prime}
$$

so that

$$
\left|A_{i+1}\right|=1
$$

and then

$$
B_{i+1}^{\prime \prime}=B_{i+1}^{\prime}-B_{i+1}^{\prime} A_{i+1}^{*} A_{i+1}
$$

and

$$
B_{i+1}=\left|B_{i+1}^{\prime \prime}\right|^{1} B_{i+1}^{\prime \prime}
$$

so that

$$
\begin{aligned}
& \left|B_{i+1}\right|=1 \\
& B_{i+1} A_{i+1}^{*}=A_{i+1} B_{i+1}^{*}=0
\end{aligned}
$$

and finally

$$
C_{i+1}^{\prime \prime}=C_{i+1}^{\prime}-C_{i+1}^{\prime} B_{i+1}^{*} B_{i+1}-C_{i+1}^{\prime} A_{i+1}^{*} A_{i+1}
$$

and

$$
C_{i+1}=\left|C_{i+1}^{\prime \prime}\right|^{-1} C_{i+1}^{\prime \prime}
$$

so that

$$
\begin{aligned}
& \left|C_{i+1}\right|=1 \\
& C_{i+1} A_{i+1}^{*}=C_{i+1} B_{i+1}^{*}=A_{i+1} C_{i+1}^{*}=B_{i+1} C_{i+1}^{*}=0
\end{aligned}
$$

$A_{i+1}, B_{i+1}$ and $C_{i+1}$ now form the new three orthonormal ( $\mathrm{g}+1$ )- component basis vectors containing $Y_{i+1}$.

## APPENDIX F1

## GENERATION OF THE SAMPLED IMPULSE-RESPONSE OF A 3 SKY WAVE HF CHANNEL

```
PROGRAM HFSIR
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
REAL CF(5),Q(6,-1:3000)
REAL TXR(16),TXI (16),TXDR(16),TXDI (16)
REAL TXDDR(16),TXDDI(16)
REAL RXR(30),RXI (30),WSR(30),WSI (30)
REAL QR (30,30),QI (30,30)
REAL YR(30),YI(30)
REAL QQ (6),EQ(6),VQ (6),FMEAN (6),FVAR (6)
REAL VALI(6),VALO(7)
```

```
DATA CF /-1.80322972300000,0.81520668040000,-1.85218288200000,
0.86788454580000,-0.89481307290000/
DCG=19378
DCG=1.0/DCG
STDVN1=SQRT (1.0)
NOSAM=2600
INFD=500
OPEN (25,FILE='IMP200',FORM='UNFORMATTED')
OPEN (12,FILE='OUTPUT')
C INITIALISING
TAP1=0.0
TAP2=0.0
TAP3=0.0
TAP4=0.0
TAP5=0.0
ISEQ=50+NOSAM
ISEQ1=ISEQ+1
CALL G05CBF (INED)
C GENERATION OF Q (T)
DO 1000 I=1,6
IND=-1
TF=0.0
TVF=0.0
TFG=0.0
TVFG=0.0
DO 1500 J=1,ISEQ1
F0=G05DDF(0.0,STDVN1)
F1=F0-(TAP1*CF(1)+TAP2*CF (2))
F2=F1-(TAP3*CF (3) +TAP4*CF (4))
F3=F2-(TAP5*CF (5))
```

```
    FDCG=F3*DCG
    TAP5=F3
    TAP4=TAP3
    TAP3=F2
    TAP2=TAP1
    TAP1=F1
    IF(J.LE.49) GO TO 1500
    (I,IND)=FDCG
    IND=IND+1
    TF=TF+F3
    TVF=TVF+(F3**2)
    TFG=TFG+FDCG
    TVFG=TVFG+(FDCG**2)
EF=TF/(NOSAM+1)
VARF=TVF/(NOSAM+1)
EFG=TFG/(NOSAM+1)
VARFG=TVFG/ (NOSAM+1)
WRITE (12,1600)EF,VARF
WRITE (12,1700) EFG,VARFG
FORMAT('MEAN OF F3 =',1X,F10.5,2X,'VAR OF F3 =',1X,F10.5)
FORMAT('MEAN OF FDCG =',1X,F10.5,2X,'VAR OF FDCG='1X,F10.5)
CONTINUE
C
TRANSMITTER FILTER 3 ms DELAY
```



RECEIVER FILTER

```
    DATA RXR / -1.9417691,-15.9797864,-35.1417733,-34.4788717,
        -11.2301982, 7.8155160, 7.5124057, -0.5057505,
        -3.3707125, -0.6759166, 1.0482656, 0.3621876,
        -0.3105902, 0.0438410, 0.0738947, -0.0646936,
        0.0000000, 0.0000000, 0.0000000, 0.0000000,
        0.0000000, 0.0000000, 0.0000000, 0.0000000,
        0.0000000, 0.0000000, 0.0000000, 0.0000000,
        0.0000000, 0.0000000/
    DATA RXI / 1.3625952, 11.5941040, 27.3342937, 28.0870086,
        7.2714615, -9.2602472, -5.0954462, 3.2326498,
        1.8975352, -1.2813604, -0.4830313, 0.7614804,
        0.1979014, -0.1532672, 0.0940330, -0.0312132,
        0.0000000, 0.0000000, 0.0000000, 0.0000000,
        0.0000000, 0.0000000, 0.0000000, 0.0000000,
        0.0000000, 0.0000000, 0.0000000, 0.0000000,
        0.0000000, 0.0000000 /
    MLOOP=2500
    ISTEP=48
    STEP=1.0/ISTEP
    DEL1=1.1
    DEL2=3.0
    SAPRAT=2.4
    SFACT=1.0/(2.0*SAPRAT*1000)
    IDELI=INT (SAPRAT*2*DEL1)
    IDEL2=INT(SAPRAT*2*DEL2)
    KMPL=16
    KMP=IDEL2+KMPI
    KMP 1=KMP-1
    ICOUNT=0
    JCOUNT=0
    POS=-1.0
    DO 3010 I=1,KMP
    DO 3005 J=1,KMP
    QR (I,J) =0.0
    QI (I,J)=0.0
3005 CONTINUE
3010 CONTINUE
    DO 3020 I=1,6
    EQ(I)=0.0
    VQ(I)=0.0
CONTINUE
KVL=3
C ENTERING MAIN LOOP
DO 9000 KMAIN=1,MLOOP
\(\mathrm{COM}=0.0\)
C ENTERING SECONDARY LOOP DO 8000 KSEC=1,ISTEP
IFAIL=1
C
NON-LINEAR INTERPOLATION
```

```
DO 3100 I=1,6
```

DO 3100 I=1,6
VALI (1) =Q (I, KMAIN-2)
VALI (1) =Q (I, KMAIN-2)
VALI (2) =Q (I, KMAIN-1)
VALI (2) =Q (I, KMAIN-1)
VALI (3) =Q (I, KMAIN)
VALI (3) =Q (I, KMAIN)
VALI (4)=Q(I,KMAIN+1)
VALI (4)=Q(I,KMAIN+1)
VALI (5) =Q (I,KMAIN+2)
VALI (5) =Q (I,KMAIN+2)
VALI(6)=Q(I,KMAIN+3)
VALI(6)=Q(I,KMAIN+3)
CALL E01ABF (KVL, COM, VALI, VALO, KVL*2, KVL* $2+1$, IFAIL)

```
```

QQ (I) =VALO (KVL*2+1)

```
CONTINUE
COM=COM + STEP
ICOUNT=ICOUNT+1
COUNT=REAL (ICOUNT)
DO \(3120 \mathrm{I}=1,6\)
\(E Q(I)=E Q(I)+Q Q(I)\)
\(V Q(I)=V Q(I)+(Q Q(I) * * 2)\)
\(Q Q(2)=-Q Q(2)\)
\(Q Q(4)=-Q Q(4)\)
\(Q Q(6)=-Q Q(6)\)
SHIFTING ARRAYS FOR CONVOLUTION
DO 3140 I=1, KMP
DO \(3140 \mathrm{~J}=1, \mathrm{KMP} 1\)
\(Q R(I, K M P+1-J)=Q R(I, K M P-J)\)
QI ( \(I, K M P+1-J)=Q I(I, K M P-J)\)
CONTINUE
DO \(3160 \mathrm{I}=1\), KMP
\(\mathrm{QR}(\mathrm{I}, 1)=0.0\)
\(Q I(I, I)=0.0\)
CONTINUE
C CONVOLUTION (TO OBTAIN IMPULSE-RESPONSE OF CHANNEL), BEGINS
DO \(3180 \mathrm{I}=1\), KMPL
\(\operatorname{QR}(1,1)=T X R(I) * Q Q(1)-T X I(I) * Q Q(2)\)
\(Q I(I, 1)=T X R(I) * Q Q(2)+T X I(I) * Q Q(1)\)
CONTINUE
DO \(3200 \mathrm{I}=1, \mathrm{KMPL}\)
\(Q R(I+I D E L 1,1)=Q R(I+I D E L 1,1)+T X D R(I) * Q Q(3)-T X D I(I) * Q Q(4)\)
\(Q I(I+I D E L 1,1)=Q I(I+I D E L 1,1)+T X D R(I) * Q Q(4)+T X D I(I) * Q Q(3)\)
CONTINUE
DO 3220 I=1, KMPL
\(Q R(I+I D E L 2,1)=Q R(I+I D E L 2,1)+T X D D R(I) * Q Q(5)-T X D D I(I) * Q Q(6)\)
\(Q I(I+I D E L 2,1)=Q I(I+\operatorname{IDEL} 2,1)+\operatorname{TXDDR}(I) * Q Q(6)+\operatorname{TXDDI}(I) * Q Q(5)\)
CONTINUE
POS=-POS
IF (POS.LT.0.0) GO TO 8000
\(I O=0\)
JCOUNT=JCOUNT+1
DCOUNT=REAL (JCOUNT)
DO \(3250 \mathrm{I}=1\), \(\mathrm{KMP}, 2\)
IO \(=10+1\)
\(Y R(I O)=0.0\)
\(Y I(I O)=0.0\)
    DO \(3240 \mathrm{~J}=1\), I
    \(\mathrm{YR}(I O)=Y R(I O)+Q R(J, I+1-J) * R X R(I+1-J)-Q I(J, I+1-J) * R X I(I+1-J)\)
    \(Y I(I O)=Y I(I O)+Q I(J, I+1-J) * R X R(I+1-J)+Q R(J, I+1-J) * R X I(I+1-J)\)
    CONTINUE
    \(Y R(I O)=Y R(I O) * S F A C T\)
    \(Y I(I O)=Y I(I O) * S F A C T\)
    CONTINUE
    IF (MOD (KMP, 2) .EQ.0) THEN
GO TO 3260
ELSE
GO TO 3400
END IF
        FMEAN (I) =EQ (I)/COUNT
        FVAR (I)=VQ (I)/COUNT
        CONTINUE
        WRITE (25)(YR(J),J=1,30)
        WRITE(25)(YI (J),J=1,30)
8000 CONTINUE
9000 CONTINUE
C PRINTING RESULTS
        WRITE (12,9210)
9210 FORMAT('MEAN VALUES OF THE QQ-SIGNALS')
        WRITE (12, 9230) (FMEAN (I), I=1,6)
        WRITE (12,9220)
9220 FORMAT(/'VARIANCES OF THE QQ-SIGNALS')
        WRITE (12, 9230) (FVAR (I), I=1,6)
        FORMAT(E20.10)
    PRINT *,'COUNT=',COUNT
    PRINT *,'REAL PART OF IMPULSE RESPONSE'
    WRITE (12,9240) (YR(I), I=1,30)
    PRINT *,'IMAGINARY PART OF IMPULSE RESPONSE'
    WRITE (12,9240)(YI (I), I=1,30)
    FORMAT (5F10.5)
    STOP
    END
```


## APPENDIX F2

## COMPUTER-SIMULATION PROGRAM FOR SYSTEM 5.1

```
    PROGRAM SYS51
    IMPLICIT DOUBLE PRECISION (A-H, O-Z)
    PARAMETER (N=64,NN=32)
    REAL YR(NN),YI (NN)
    REAL SR(NN),SI(NN),PHI (N,N)
    REAL VR(N),VI(N),S1R(N),SII(N)
    REAL V1R(N/2),V1I (N/2)
    REAL PR(N,N),PI(N,N),PYR(N),PYI(N)
    REAL FPR(N,N),FPI(N,N)
    REAL YPR(N),YPI (N),GKR(N),GKI(N)
    REAL WDR(NN),WDI (NN),WFR(NN),WFI (NN)
    REAL OMEGA
    OPEN (10,FILE='CYB500',FORM='UNFORMATTED')
    OPEN (30,FILE='OUTPUT',FORM='FORMATTED')
    SNR=60.0
    OMEGA=0.88
    ERRTOT=0.0
    ERRNOM=0.0
LCOUNT=0
IQ=200
CALL G05CBF(IQ)
C INITIALISATION OF Y1 & P MATRIX
DO 1000 J=1,N
S1R(J)=0.0
SII(J)=0.0
VR(J)=0.0
VI (J) =0.0
DO 1000 I=1,N
PR(I,J)=0.0
PI(I,J)=0.0
PHI (I,J) =0.0
CONTINUE
DO 1020 I=1,N
PR(I,I)=1.0
PHI (I,I) =1.0
CONTINUE
DO 1040 J=1,NN
PHI((J+NN),J)=1.0
CONTINUE
C INITIALISATION OF W,NOISE AND DATA MATRIX
DO 1060 J=1,NN
SR(J)=1.0
SI (J)=1.0
WDR(J)=0.0
WDI(J)=0.0
CONTINUE
C VALUES TO VARIABLES AND ARRAYS
DATA WFR/-0.0280463, -0.2308071, -0.5075768, -0.4980021,
```

    1 0.0000000, 0.0000000, 0.0000000, 0.0000000,
    1
    DATA WFI/ 0.0196809, 0.1674616, 0.3948080, 0.4056800,
        0.1050267, -0.1337521, -0.0735970, 0.0466914,
        0.0274074, -0.0185076, -0.0069768, 0.0109986,
        0.0028584, -0.0022137, 0.0013582, -0.0004508,
        0.0000000, 0.0000000, 0.0000000, 0.0000000,'
        0.0000000, 0.0000000, 0.0000000, 0.0000000,
    0.0000000, 0.0000000, 0.0000000, 0.0000000',
    0.0000000, 0.0000000, 0.0000000, 0.0000000/
    STDVN=10.0**(-SNR/10.0)
    STDVN=SQRT (STDVN)
    DO 9000 ICOUNT=1,60000
    READ (10) (YR(J),J=1,NN)
    READ (10) (YI(J),J=1,NN)
    IF(ICOUNT.EQ.1) THEN
    DO 2000 J=1,NN
    VR(J)=YR(J)
    VI(J)=YI (J)
    CONTINUE
ENDIF
C SHIFTING OF ARRAYS ONCE FOR EVERY DATA SYMBOL
DO 2020 J=(NN-1),1,-1
K=J+1
SR(K)=SR(J)
SI (K)=SI (J)
2020 CONTINUE
C GENERATING QPSK DATA
XX=G05CAF (XX)
IF (XX-0.5)2100,2100,2120
2100 SR(1)=-1.0
GO TO 2150
2120 SR(1)=1.0
2150 XX=G05CAF (XX)
IF (XX-0.5)2170,2170,2190
2170 SI(1)=-1.0
GO TO 2200
2190 SI(1)=1.0
2200 CONTINUE
C GENERATING NOISE
DO 2250 LNSE=1,2
DO 2220 J=1,NN
K=J+1
WDR (J) = WDR (K)
WDI (J) =WDI (K)
CONTINUE
WDR (NN)=G05DDF (0.0,STDVN)
WDI (NN) =G05DDF (0.0,STDVN)
WNR=0.0
WNI=0.0
DO 2240 J=1,NN
K1=NN+1-J
WNR=WNR+WDR(K1)*WFR(J) -WDI (K1) *WFI (J)
WNI=WNI+WDR (K1) *WFI (J) +WDI (K1) *WFR (J)
CONTINUE

```
ENDIF
C ESTIMATING RECEIVED SIGNAL
\(\mathrm{R} 1 \mathrm{R}=0.0\)
R1I \(=0.0\)
DO \(3060 \mathrm{~J}=1\), NN
\(R 1 R=R 1 R+(S 1 R(J) * V R(J)-S 1 I(J) * V I(J))\)R1I=R1I+(S1R(J)*VI(J) +S1I(J)*VR(J))CONTINUE
C ERROR IN ESTIMATION OF RECEIVED SIGNAL RECER=RR-R1R

            RECEI=RI-R1I
C COMPUTING PHI*P*PHI MATRIX
```

DO 3080 I=1,NN
DO 3080 J=1,N
FPR(I,J)=PR(I,J)+PR((I+NN),J)
FPI(I,J)=PI(I,J)+PI((I+NN),J)

```
CONTINUE

C COMPUTING KALMAN GAIN VECTOR
```

DO 3180 I=1,N
SPMR=0.0
SPMI=0.0
DO 3160 J=1,N
SPMR=SPMR+(S1R(J)*PR(J,I) +SII (J)*PI (J,I))
SPMI=SPMI+(S1R(J)*PI(J,I)-S1I(J)*PR(J,I))
CONTINUE
PYR(I)=SPMR
PYI(I)=SPMI
CONTINUE

```
YPYR=0.0
YPYI=0.0
DO \(3200 \mathrm{~J}=1, \mathrm{~N}\)
\(Y P Y R=Y P Y R+(P Y R(J) * S 1 R(J)-P Y I(J) * S 1 I(J))\)
YPYI=YPYI+(PYI (J) *S1R(J) +PYR(J)*S1I (J))
CONTINUE
\(Y P W R=Y P Y R+O M E G A\)
YPWI=YPYI
YPM \(=Y P W R * Y P W R+Y P W I * Y P W I\)
DO 3220 J=1,N
\(\operatorname{GKR}(J)=(\operatorname{PYR}(J) * Y P W R+P Y I(J) * Y P W I) / Y P M\)
\(\operatorname{GKI}(J)=(P Y I(J) * Y P W R-P Y R(J) * Y P W I) / Y P M\)
CONTINUE

UPDATE INVERSE MATRIX
DO \(3260 \mathrm{~J}=1, \mathrm{~N}\)
SYPM=0.0
RYPM \(=0.0\)
DO \(3240 \mathrm{I}=1, \mathrm{~N}\)
\(\operatorname{SYPM}=\operatorname{SYPM}+(\operatorname{PR}(J, I) * S 1 R(I)-P I(J, I) * S 1 I(I)) \quad i, i\)
RYPM=RYPM+(PI(J,I)*S1R(I) +PR(J,I)*SII(I))
CONTINUE
YPR (J) = SYPM
YPI \((J)=\) RYPM
CONTINUE
```

    DO 3280 I=1,N
    DO 3280 J=1,N
    PR(I,J)=PR(I,J)-(YPR(I)*GKR(J) -YPI(I)*GKI(J))
    PI(I,J)=PI(I,J)-(YPR(I)*GKI (J) +YPI (I)*GKR(J))
    C UPDATING THE ESTIMATES OF THE CHANNEL
DO 4000 J=1,N
VR(J)=VR(J)+(GKR(J)*RECER-GKI (J)*RECEI)
VI(J)=VI (J) +(GKI (J)*RECER+GKR (J)*RECEI)
4 0 0 0 ~ C O N T I N U E ~
9000 CONTINUE
C PRINTING OUT RESULT
ERRTOT=10.0*LOG10 (ERRTOT/LCOUNT)
ERRNOM=10.0*LOG10(ERRNOM/LCOUNT)
PRINT *,'OMEGA=',OMEGA
PRINT *,'SNR=',SNR
PRINT *,'LCOUNT=',LCOUNT
PRINT *,'MEAN SQ ERROR=',ERRTOT
PRINT *,'NORM.MEAN SQ.ERROR=',ERRNOM
REWIND (10)
STOP
END

```

\section*{APPENDIX F3}

\section*{COMPUTER-SIMULATION PROGRAM SYSTEM 6.8}
```

PROGRAM SYS68
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
PARAMETER (IMPR=32)
DIMENSION YR(IMPR),YI(IMPR),Y1R(IMPR),YII(IMPR)
DIMENSION GRDR(IMPR),GRDI(IMPR),ERR(IMPR),ERI(IMPR)
DIMENSION UY1R(IMPR),UY1I(IMPR),SR(IMPR),SI(IMPR)
DIMENSION WDR(IMPR),WDI(IMPR),WFR(IMPR),WFI(IMPR)
DIMENSION B(IMPR),BB(IMPR)
OPEN (10,FILE='IMP500',FORM='UNFORMATTED')
OPEN (30,FILE='OUTPUT')
ERRTOT=0.0
ERRNOM=0.0
LCOUNT=0
IQ=200
CALL G05CBF(IQ)
DO 2000 J=1,IMPR
BB(J)=0.0
B(J)=0.0

| DATA WFR / | $\begin{aligned} & -0.0280463, \\ & -0.1622055, \\ & -0.0486855, \\ & -0.0044861, \\ & 0.0000000, \\ & 0.0000000, \\ & 0.0000000, \\ & 0.0000000, \end{aligned}$ | $\begin{array}{r} -0.2308071, \\ 0.1128849, \\ -0.0097627, \\ 0.0006332, \\ 0.0000000, \\ 0.0000000, \\ 0.0000000, \\ 0.0000000, \end{array}$ | $\begin{array}{r} -0.5075768, \\ 0.1085069, \\ 0.0151408, \\ 0.0010673, \\ 0.0000000, \\ 0.0000000, \\ 0.0000000, \\ 0.0000000, \end{array}$ | $\begin{array}{r} -0.4980021, \\ -0.0073049, \\ 0.0052313, \\ -0.0009344, \\ 0.0000000, \\ 0.0000000, \\ 0.0000000, \\ 0.0000000 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: |
| DATA WFI / | 0.0196809 , | 0.1674616 , | 0.3948080 , | 0.4056800, |
|  | 0.1050267 , | -0.1337521, | -0.0735970, | 0.0466914, |
|  | 0.0274074 , | -0.0185076, | -0.0069768, | 0.0109986, |
|  | 0.0028584 , | -0.0022137, | 0.0013582, | -0.0004508, |
|  | 0.0000000 , | 0.0000000 , | 0.0000000 , | 0.0000000, |
|  | 0.0000000 , | 0.0000000 , | 0.0000000 , | 0.0000000 , |
|  | 0.0000000 , | 0.0000000 , | 0.0000000 , | 0.0000000 , |
|  | 0.0000000 , | 0.0000000 , | 0.0000000 , | 0.0000000 |

```
```

        SNR=60.0
        STDVN=10.0**(-SNR/10.0)
        STDVN=SQRT (STDVN)
        DO 9000 ICOUNT=1,60000
        READ (10) (YR (J), J=1, IMPR)
        READ (10) (YI (J),J=1,IMPR)
    SR(IMPR)=1.0
XX=G05CAF (XX)
IF (XX-0.5) 2120,2120,2140
2120 SI (IMPR)=-1.0
GO TO 2150
2140 SI (IMPR)=1.0
2150 CONTINUE
C GENERATING NOISE
DO 2040 J=1,(IMPR-1)
L=J+1
SR(J)=SR(L)
SI(J)=SI (L)
CONTINUE
GENERATING QPSK DATA
XX=G0 5CAF (XX)
IF(XX-0.5) 2060,2060,2080
SR (IMPR)=-1.0
GO TO 2100
DO 2200 LNSE=1,2
DO 2170 J=1,(IMPR-1)
K=J+1
WDR (J) =WDR (K)
WDI (J) =WDI (K)
CONTINUE
WDR (IMPR)=G05DDF (0.0,STDVN)
WDI (IMPR) =G05DDF (0.0,STDVN)
WR=0.0
WI=0.0
DO 2190 J=1,IMPR
K1=IMPR+1-J
WR=WR+WDR(K1) *WFR(J) -WDI (K1) *WFI (J)
WI=WI+WDR (K1) *WFI (J) +WDI (K1) *WFR(J)
CONTINUE
CONTINUE
CALCULATION OF RECEIVED SIGNAL
RR=0.0
RI=0.0
DO 2220 J=1,IMPR
K1=IMPR+1-J
RR=RR+SR(K1)*YR(J)-SI(K1)*YI (J)
RI=RI+SR(K1)*YI (J) +SI (K1) *YR(J)
CONTINUE
RECEIVED SIGNAL WITH NOISE
RR=RR+WR
RI=RI+WI

```
```

    IF (ICOUNT.LE.2000) THEN
    GOTO 9000
    ENDIF
    DO 2240 J=1,IMPR
    YVAR=Y1R(J) *Y1R(J)+Y1I(J)*Y1I(J)
    C BB(J)=(1.0-(1.0/ICOUNT) )*BB(J)+(1.0/ICOUNT)*YVAR
    BB}(J)=(1.0-ALFA)*BB(J)+ALFA*YVAR
    CONTINUE
    DO 2240 J=1,IMPR
    B(J)=BB(J)**0.25
    IF(B(J).LT.0.024) THEN
    B(J)=0.000001
    ENDIF
    CONTINUE
    YTOT=0.0
    YERR=0.0
    DO 2260 J=1,IMPR
    YTOT=YTOT+YR(J)**2+YI(J)**2
    YERR=YERR+(YR(J) -Y1R(J))**2+(YI (J) -Y1I (J))**2
    CONTINUE
IF(ICOUNT.GT.6000) THEN
ERRTOT=ERRTOT+YERR
ERRNOM=ERRNOM+YERR/YTOT
LCOUNT=LCOUNT+1
C IF (MOD (LCOUNT, 20).EQ.0) THEN
C PRINT *,10.0*LOG10 (YERR)
C ENDIF
ENDIF
C
C ERROR IN ESTIMATION OF RECEIVED SIGNAL
RECER=RR-R1R
RECEI=RI-R1I
C UPDATING CHANNEL USING FEEDFORWARD ESTIMATOR
DO 2300 J=1,IMPR
K1=IMPR+1-J
UY1R(J)=Y1R(J) +B(J)*(RECER*SR(KI)+RECEI*SI(KI))
UY1I (J) =Y1I (J) +B(J)* (RECEI*SR(K1)-RECER*SI (K1))
CONTINUE
C ERROR IN UPDATING
DO 2320 J=1,IMPR
ERR(J)=UY1R(J)-Y1R(J)
ERI (J)=UY1I (J)-Y1I (J)
CONTINUE
C
PREDICTION USING LS GRADIENT ALGORITHM

```
```

    DO 2340 J=1,IMPR
    GRDR(J)=GRDR(J) + (DUM1 *ERR (J))
    GRDI (J) =GRDI (J) + (DUM1*ERI (J)
    Y1R(J)=Y1R(J) +GRDR(J) + (DUM2 *ERR (J))
    Y1I (J)=Y1I (J) +GRDI (J) +(DUM2*ERI (J))
    CONTINUE
    CONTINUE
    C PRINTING OUT RESULT
ERRTOT=10.0*(LOG10(ERRTOT/LCOUNT))
ERRNOM=10.0*(LOG10(ERRNOM/LCOUNT))
PRINT *,'THETA=',THETA
PRINT *,'SNR=',SNR
PRINT *,'LCOUNT=',LCOUNT
PRINT *,'MEAN SQ ERROR=',ERRTOT
PRINT *,'NOM.MEAN SQ.ERR.=',ERRNOM
REWIND (10)
STOP
END

```

\section*{APPENDIX F4}

\title{
COMPUTER-SIMULATION PROGRAM FOR SYSTEM 7.5
}
```

    PROGRAM SYS75
    IMPLICIT DOUBLE PRECISION (A-H, O-Z)
    PARAMETER(IMPR=32)
    REAL YR(IMPR),YI(IMPR)
    REAL SR(IMPR),SI(IMPR)
    REAL WDR(IMPR),WDI(IMPR),WFR(IMPR),WFI(IMPR)
    REAL Y02R(IMPR),Y02I(IMPR),Y01R(IMPR),Y01I(IMPR)
    REAL YOR(IMPR),YOI(IMPR)
    REAL AR (IMPR),AI (IMPR),BR(IMPR),BI (IMPR)
    REAL CR(IMPR),CI(IMPR),FR(IMPR),FI(IMPR)
    REAL ERRR(IMPR),ERRI(IMPR)
    REAL B(IMPR),BB(IMPR),Y2R(IMPR),Y2I(IMPR)
    REAL SLPR(IMPR),SLPI(IMPR),Y2ER(IMPR),Y2EI(IMPR)
    REAL VR(3),VI(3),S1R(3),S1I(3)
    REAL V1R(3),V1I(3),GR(IMPR),GI(IMPR)
    REAL Y1R(IMPR),Y1I(IMPR)
    REAL ERR(3),ERI (3),GRDR (3),GRDI (3)
    OPEN (10,FILE='IMP500',FORM='UNFORMATTED')
    OPEN (12,FILE='OUTPUT')
    SNR=30.0
    INFD=200
    CALL G05CBF(INFD)
    | DATA WFR / | $\begin{aligned} & -0.0280463, \\ & -0.1622055, \end{aligned}$ | -0.2308071, 0.1128849, | -0.5075768, 0.1085069, | $\begin{aligned} & -0.4980021, \\ & -0.0073049, \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | -0.0486855, | -0.0097627, | 0.0151408 , | 0.0052313 , |
|  | -0.0044861, | 0.0006332 , | 0.0010673 , | -0.0009344, |
|  | 0.0000000 , | 0.0000000 , | 0.0000000 , | 0.0000000 , |
|  | 0.0000000 , | 0.0000000 , | 0.0000000 , | 0.0000000 , |
|  | 0.0000000 , | 0.0000000 , | 0.0000000 , | 0.0000000 , |
|  | 0.0000000 , | 0.0000000 , | 0.0000000 , | $0.0000000 /$ |
| DATA WFI / | 0.0196809 , | 0.1674616 , | 0.3948080 , | 0.4056800 , |
|  | 0.1050267 , | -0.1337521, | -0.0735970, | 0.0466914 , |
|  | 0.0274074 , | -0.0185076, | -0.0069768, | 0.0109986 , |
|  | 0.0028584 , | -0.0022137, | 0.0013582, | -0.0004508, |
|  | 0.0000000 , | 0.0000000 , | 0.0000000 , | 0.0000000 , |
|  | 0.0000000 , | 0.0000000 , | 0.0000000 , | 0.0000000 , |
|  | 0.0000000 , | 0.0000000 , | 0.0000000 , | 0.0000000 , |
|  | 0.0000000 | 0.0000000 , | 0.0000000 , | 0.0000000 |

C NOISE DATA MATRIX INITIALISATION
DO 1010 J=1,IMPR
WDR(J)=0.0
WDI (J) =0.0
CONTINUE
C SETTING SR \& SI MATRIX TO 1.0
DO 1020 J=1,IMPR
SR(J)=1.0
SI(J)=1.0
B (J)=0.0
BB}(J)=0.

```
\begin{tabular}{|c|c|}
\hline \multirow[t]{2}{*}{} & \(\operatorname{SLPR}(J)=0.0\) \\
\hline & \(\operatorname{SLPI}(J)=0.0\) \\
\hline 1020 & CONTINUE \\
\hline \multirow[t]{8}{*}{C} & INITIALISING OF Y1 \& P MATRIX \\
\hline & DO \(1030 \mathrm{~J}=1,3\) \\
\hline & \(\operatorname{S1R}(\mathrm{J})=0.0\) \\
\hline & \(\operatorname{SII}(\mathrm{J})=0.0\) \\
\hline & \(\operatorname{VR}(J)=0.0\) \\
\hline & \(V I(J)=0.0\) \\
\hline & \(\operatorname{GRDR}(\mathrm{J})=0.0\) \\
\hline & \(\operatorname{GRDI}(\mathrm{J})=0.0\) \\
\hline 1030 & CONTINUE \\
\hline \multirow[t]{17}{*}{C} & DATA FOR IMPROVED CHANNEL ESTIMATOR \\
\hline & LCOUNT=0 \\
\hline & DLTA=1.0 \\
\hline & \(\mathrm{C}=0.115\) \\
\hline & THETA \(=0.935\) \\
\hline & ALFA \(=0.01\) \\
\hline & ETA=0.02 \\
\hline & EPSLON=0.976 \\
\hline & \(\operatorname{CosT}=0.064\) \\
\hline & THETA1 \(=(1.0-\) THETA \() *(1.0-\) THETA \()\) \\
\hline & THETA \(2=1.0-\) THETA*THETA \\
\hline & DUM1 \(=(1.0-E P S L O N) *(1.0-E P S L O N)\) \\
\hline & DUM2 \(=(1.0-E P S L O N * E P S L O N)\) \\
\hline & ERRSUM \(=0.0\) \\
\hline & SUMERR=0.0 \\
\hline & ERNOM \(=0.0\) \\
\hline & ERNON \(=0.0\) \\
\hline \multirow[t]{9}{*}{C} & ENTERING MAIN LOOP \\
\hline & DO 9000 JCOUNT \(=1,60000\) \\
\hline & READ (10) (YR (J) , J=1, IMPR) \\
\hline & \(\operatorname{READ}(10)(\mathrm{YI}(\mathrm{J}), \mathrm{J}=1, \mathrm{IMPR})\) \\
\hline & IF (JCOUNT.GE.5002) GO TO 4000 \\
\hline & IF (JCOUNT.EQ.5001) GO TO 1080 \\
\hline & IF (JCOUNT.EQ.3501) GO TO 1060 \\
\hline & IF (JCOUNT.EQ.2001) GO TO 1040 \\
\hline & GO TO 9000 \\
\hline \multirow[t]{6}{*}{C
1040} & StARTING UP PROCEDURE \\
\hline & DO \(1050 \mathrm{I}=1\), IMPR \\
\hline & \(\mathrm{Y} 02 \mathrm{R}(\mathrm{I})=\mathrm{YR}(\mathrm{I})\) \\
\hline & Y02I(I) = YI (I) \\
\hline & \(\operatorname{AR}(\mathrm{I})=\mathrm{YR}(\mathrm{I})\) \\
\hline & \(A I(I)=Y I(I)\) \\
\hline \multirow[t]{2}{*}{1050} & CONTINUE \\
\hline & GO TO 9000 \\
\hline \multirow[t]{5}{*}{1060} & DO 1070 I=1, IMPR \\
\hline & \(\mathrm{Y} 01 \mathrm{R}(\mathrm{I})=\mathrm{YR}(\mathrm{I})\) \\
\hline & Y01I(I) \(=\) YI (I) \\
\hline & \(B R(I)=Y R(I)\) \\
\hline & \(B I(I)=Y I(I)\) \\
\hline \multirow[t]{2}{*}{1070} & CONTINUE \\
\hline & GO TO 9000 \\
\hline \multirow[t]{7}{*}{1080} & DO \(1090 \mathrm{I}=1, \mathrm{IMPR}\) \\
\hline & \(\mathrm{YOR}(\mathrm{I})=\mathrm{YR}(\mathrm{I})\) \\
\hline & \(\mathrm{YOI}(\mathrm{I})=\mathrm{YI}(\mathrm{I})\) \\
\hline & \(C R(I)=Y R(I)\) \\
\hline & \(C I(I)=Y I(I)\) \\
\hline & Y2R(I) \(=\mathrm{YR}\) ( I ) \\
\hline & \(Y 2 I(I)=Y I(I)\) \\
\hline 1090 & CONTINUE \\
\hline
\end{tabular}
```

    Y02MAG=0.0
    Y01MAG=0.0
    YOMAG=0.0
    DO 2000 I=1,IMPR
    Y02MAG=Y02MAG+(Y02R(I)**2+Y02I(I)**2)
    Y01MAG=Y01MAG+(Y01R(I)**2+Y01I(I)**2)
    YOMAG=YOMAG+(YOR(I)**2+YOI(I)**2)
    CONTINUE
    Y02=SQRT(Y02MAG)
    Y01=SQRT (Y01MAG)
    Y0O=SQRT (YOMAG)
    WRITE (12, 2010)Y02,Y01,Y00
    $\mathrm{Y} 20 \mathrm{R}=0.0$
Y20I=0.0
Y10R=0.0
Y10I=0.0
Y21R=0.0
Y21I=0.0
DO 2020 I=1,IMPR
Y20R=Y20R+(Y02R(I)*Y0R(I) +Y02I(I)*Y0I(I))
Y20I=Y20I+(Y02I(I)*Y0R(I)-Y02R(I)*YOI(I))
Y10R=Y10R+(Y01R(I)*Y0R(I) +Y01I(I) *Y0I(I))
Y10I=Y10I+(Y01I(I)*Y0R(I)-Y01R(I) *Y0I(I))
Y21R=Y21R+(Y02R(I)*Y01R(I)+Y02I(I)*Y01I(I))
Y21I=Y21I+(Y02I(I)*Y01R(I)-Y02R(I)*Y01I(I))
CONTINUE
ANG20=((Y20R**2+Y20I**2)/(Y02MAG*YOMAG))**0.5
ANG10=((Y10R**2+Y10I**2)/(Y01MAG*Y0MAG))**0.5
ANG21=((Y21R**2+Y21I**2) /(Y02MAG*Y01MAG))**0.5
ANG20=(180.0*7.0*ACOS (ANG20))/22.0
ANG10 = (180.0*7.0*ACOS (ANG10))/22.0
ANG21=(180.0*7.0*ACOS (ANG21))/22.0
WRITE (12, 2040) ANG20, ANG10, ANG21
FORMAT(1H ,'ANGLE BETWEEN Y(-2T)\&Y(0) =',F10.4,' DEGREES',/,
1
1
CALL GRMSHM (IMPR,AR,AI,BR,BI,CR,CI)
YOCR=0.0
YOCI=0.0
YOBR=0.0
YOBI=0.0
YOAR=0.0
Y0AI=0.0
DO 1220 I=1,IMPR
Y0CR=Y0CR+Y0R(I)*CR(I) +Y0I (I)*CI (I)
YOCI=YOCI+YOI (I) *CR(I)-YOR(I) *CI (I)
Y0BR=Y0BR+Y0R(I)*BR(I) +Y0I (I)*BI (I)
YOBI=Y0BI+Y0I (I)*BR(I) -Y0R(I)*BI (I)
YOAR=Y0AR+Y0R(I)*AR(I) +Y0I (I)*AI (I)
YOAI=YOAI+YOI (I)*AR(I)-YOR(I)*AI(I)
CONTINUE
AFAR=Y0AR
AFAI=Y0AI
BTAR=YOBR
BTAI=Y0BI
GAMR=YOCR
GAMI=YOCI

```
```

    DO 1300 I=1,IMPR
    FR(I)=(AFAR*AR(I)-AFAI*AI(I))+(BTAR*BR(I) -BTAI*BI(I))
    1
    (I)
    1 CONTINUE
DO 1310 I=1,IMPR
$\operatorname{ERRR}(I)=Y 0 R(I)-F R(I)$
$\operatorname{ERRI}(I)=Y 0 I(I)-E I(I)$
CONTINUE
DO 1320 I=1, IMPR
$\operatorname{AR}(I)=\operatorname{AR}(I)+E T A *(A F A R \star E R R R(I)+A F A I * E R R I(I))$
$\mathrm{AI}(I)=A I(I)+E T A *(\operatorname{AFAR} * \operatorname{ERRI}(I)-A F A I * E R R R(I))$
$\operatorname{BR}(I)=\operatorname{BR}(I)+E T A *(B T A R * E R R R(I)+B T A I * E R R I(I))$
$B I(I)=B I(I)+E T A *(B T A R * E R R I(I)-B T A I * E R R R(I))$
$\operatorname{CR}(I)=\operatorname{CR}(I)+E T A *(G A M R * E R R R(I)+G A M I * E R R I(I))$
$C I(I)=C I(I)+E T A *(G A M R * E R R I(I)-G A M I * E R R R(I))$
CONTINUE
CALL GRMSHM (IMPR,AR,AI,BR,BI,CR,CI)
DO 1520 I=1,IMPR
$\mathrm{Y} 1 \mathrm{R}(\mathrm{I})=(\operatorname{AFAR} \star \mathrm{AR}(\mathrm{I})-\mathrm{AFAI} \star \mathrm{AI}(\mathrm{I}))+(\mathrm{BTAR} * \mathrm{BR}(\mathrm{I})-\mathrm{BTAI} * \mathrm{BI}(\mathrm{I}))$
Y1I (I) $=(A F A I * A R(I)+(G A M R * C R(I)-G A M I * C I(I))$
$Y 1 I(I)=(\operatorname{AFAI} \star \operatorname{AR}(I)+\operatorname{AFAR} \star A I(I))+(B T A I * B R(I)+B I A R \star B I(I))$
CONTINUE
$\mathrm{VR}(1)=\mathrm{AFAR}$
$\mathrm{VI}(1)=\mathrm{AFAI}$
$\mathrm{VR}(2)=\mathrm{BTAR}$
$\mathrm{VI}(2)=\mathrm{BTAI}$
VR (3) =GAMR
VI (3) =GAMI
GO TO 9000
C IMPROVED CHANNEL ESTIMATOR
CONTINUE
SQE1=0.0
SQE2=0.0
YTOT=0.0
DO 4500 I=1,IMPR
YTOT=YTOT+YR (I) **2+YI (I) **2
$\operatorname{SQE} 1=\operatorname{SQE} 1+(\mathrm{YR}(\mathrm{I})-\mathrm{Y} 1 \mathrm{R}(\mathrm{I})) * * 2+(\mathrm{YI}(\mathrm{I})-\mathrm{Y} 1 \mathrm{I}(\mathrm{I})) * * 2$
$\operatorname{SQE} 2=\operatorname{SQE} 2+(\mathrm{YR}(\mathrm{I})-Y 2 R(I)) * * 2+(Y I(I)-Y 2 I(I)) * * 2$
CONTINUE
IF (JCOUNT.GT.6000) THEN
ERRSUM=ERRSUM+SQE1
SUMERR=SUMERR+SQE2
ERNOM=ERNOM+SQE1/YTOT
ERNON=ERNON+SQE2/YTOT
LCOUNT $=$ LCOUNT +1
ENDIF
DO 4510 JK=1,IMPR
$Y V A R=Y 2 R(J K) * * 2+Y 2 I(J K) * * 2$
$\mathrm{BB}(\mathrm{JK})=(1.0-\mathrm{ALFA}) * \mathrm{BB}(\mathrm{JK})+\mathrm{ALFA} * \mathrm{YVAR}$
CONTINUE

```
\begin{tabular}{|c|c|}
\hline \multirow[t]{5}{*}{} & DO 4520 JK=1, IMPR \\
\hline & \(\mathrm{B}(\mathrm{JK})=\mathrm{BB}(\mathrm{JK}) * * 0.25\) \\
\hline & IF (B (JK).LT.COST) THEN \\
\hline & \(B(J K)=0.000001\) \\
\hline & ENDIF \\
\hline 4520 & CONTINUE \\
\hline \multirow[t]{5}{*}{C} & SHIFTING OF DATA MATRIX \\
\hline & DO \(4530 \mathrm{I}=\mathrm{IMPR}, 2,-1\) \\
\hline & \(\mathrm{J}=\mathrm{I}-1\) \\
\hline & SR (I) \(=\) SR ( \(J\) ) \\
\hline & \(S I(I)=S I(J)\) \\
\hline 4530 & CONTINUE \\
\hline \multirow[t]{3}{*}{C} & GENERATING DATA \\
\hline & XX=G05CAF (XX) \\
\hline & IF (XX-0.5) 4540, 4540,4550 \\
\hline \multirow[t]{2}{*}{4540} & \(\mathrm{SR}(1)=-1.0\) \\
\hline & GO TO 4560 \\
\hline 4550 & \(\mathrm{SR}(1)=1.0\) \\
\hline \multirow[t]{2}{*}{4560} & \(\mathrm{XX}=\mathrm{G} 05 \mathrm{CAF}(\mathrm{XX})\) \\
\hline & IF (XX-0.5) 4570,4570,4580 \\
\hline \multirow[t]{2}{*}{4570} & SI (1) = - 1.0 \\
\hline & GO TO 4600 \\
\hline 4580 & SI (1) =1.0 \\
\hline 4600 & CONTINUE \\
\hline \multirow[t]{6}{*}{C} & GENERATING NOISE \\
\hline & DO 4650 NOLUP \(=1,2\) \\
\hline & DO \(4620 \mathrm{~J}=\) IMPR,2,-1 \\
\hline & \(\mathrm{I}=\mathrm{J}-1\) \\
\hline & WDR (J) \(=\) WDR ( I ) \\
\hline & WDI (J) \(=\) WDI ( I ) \\
\hline \multirow[t]{8}{*}{4620} & CONTINUE \\
\hline & WDR (1) =G05DDF (0.0, STDVN) \\
\hline & WDI (1) =G05DDF ( \(0.0, \mathrm{STDVN}\) ) \\
\hline & \(\mathrm{WR}=0.0\) \\
\hline & \(\mathrm{WI}=0.0\) \\
\hline & DO \(4640 \mathrm{~J}=1\), IMPR \\
\hline & \(W R=W R+W D R(J) * W F R(J)-W D I ~(J) * W F I ~(J) ~\) \\
\hline & WI=WI+WDR (J) *WFI (J) +WDI (J) *WFR (J) \\
\hline 4640 & CONTINUE \\
\hline 4650 & CONTINUE \\
\hline \multirow[t]{6}{*}{C} & CALCULATING RECEIVED SIGNAL \\
\hline & \(\mathrm{RR}=0.0\) \\
\hline & \(\mathrm{RI}=0.0\) \\
\hline & DO \(4700 \mathrm{I}=1\), IMPR \\
\hline & \(R \mathrm{R}=\mathrm{RR}+\mathrm{SR}(\mathrm{I}) * \mathrm{YR}(\mathrm{I})-S I(\mathrm{I}) * \mathrm{YI}(\mathrm{I})\) \\
\hline & \(R I=R I+S I(I) * Y R(I)+S R(I) * Y I(I)\) \\
\hline \multirow[t]{14}{*}{4700} & CONTINUE \\
\hline & \(\mathrm{RR}=\mathrm{R} R+\mathrm{W} \mathrm{R}\) \\
\hline & \(R I=R I+W I\) \\
\hline & \(\operatorname{S1R}(1)=0.0\) \\
\hline & S1I (1) \(=0.0\) \\
\hline & S1R (2) \(=0.0\) \\
\hline & \(\operatorname{SII}(2)=0.0\) \\
\hline & \(\operatorname{S1R}(3)=0.0\) \\
\hline & S1I (3) \(=0.0\) \\
\hline & DO \(4720 \mathrm{~J}=1\), IMPR \\
\hline & S1R (1) =S1R (1) + (SR (J)*AR (J)-SI (J)*AI (J)) \\
\hline & S1I (1) \(=\) SII (1) + (SR (J)*AI (J) +SI (J) *AR (J) ) \\
\hline & \(\operatorname{SIR}(2)=S 1 R(2)+(S R(J) * B R(J)-S I(J) * B I(J))\) \\
\hline & S1I (2) =S1I (2)+(SR(J)*BI (J) +SI (J) *BR(J)) \\
\hline
\end{tabular}
```

S1R(3)=S1R(3)+(SR(J)*CR(J)-SI (J)*CI (J))
S1I(3)=S1I (3)+(SR(J)*CI (J)+SI(J)*CR(J))

```

C ERROR IN ESTIMATION OF REC!! SIGNAL RECER=RR-R2R
RECEI=RI-R2I
\(R E R=R R-R 1 R\)
\(R E I=R I-R 1 I\)
DO 4760 I=1,IMPR
\(\operatorname{Y2ER}(I)=B(I) *(R E R * S R(I)+R E I * S I(I)) * D L T A\) \(\mathrm{Y} 2 \mathrm{EI}(\mathrm{I})=\mathrm{B}(\mathrm{I}) *(\mathrm{REI*} \mathrm{SR}(\mathrm{I})-\mathrm{RER} * S I(\mathrm{I})) * D L T A\) CONTINUE

DO 4770 I=1, IMPR
\(\operatorname{SLPR}(I)=\operatorname{SLPR}(I)+D U M 1 * Y 2 E R(I)\)
\(\operatorname{SLPI}(I)=\operatorname{SLPI}(I)+D U M 1 * Y 2 E I(I)\)
\(\mathrm{Y} 2 \mathrm{R}(\mathrm{I})=\mathrm{Y} 2 \mathrm{R}(\mathrm{I})+\mathrm{SLPR}(\mathrm{I})+\mathrm{DUM} 2 \star \mathrm{Y} 2 \mathrm{ER}(\mathrm{I})\)
\(\mathrm{Y} 2 \mathrm{I}(\mathrm{I})=\mathrm{Y} 2 \mathrm{I}(\mathrm{I})+\mathrm{SLPI}(\mathrm{I})+\mathrm{DUM} 2 * \mathrm{Y} 2 \mathrm{EI}(\mathrm{I})\)
CONTINUE
C UPDATING OF V VECTOR USING GRADIENT ALGORITHM DO \(4780 \mathrm{I}=1,3\)
\(\operatorname{ERR}(I)=C *(\operatorname{RECER} * S 1 R(I)+R E C E I * S 1 I(I))\)
\(\operatorname{ERI}(I)=C *(R E C E I * S 1 R(I)-R E C E R * S 1 I(I))\)
\(\operatorname{V1R}(I)=\operatorname{VR}(I)+E R R(I)\)
\(\operatorname{V1I}(I)=V I(I)+E R I(I)\)
CONTINUE
DO 5080 JK=1,3
\(\operatorname{GRDR}(J K)=\operatorname{GRDR}(J K)+\) THETA1 *ERR (JK)
GRDI (JK) =GRDI (JK) +THETAI *ERI (JK)
\(\operatorname{VR}(J K)=\operatorname{VR}(J K)+G R D R(J K)+T H E T A 2 * E R R\) (JK)
\(V I(J K)=V I(J K)+G R D I(J K)+T H E T A 2 * E R I\) (JK)
CONTINUE
\(A F A R=0.0\)
\(A F A I=0.0\)
\(B T A R=0.0\)
BTAI \(=0.0\)
GAMR \(=0.0\)
GAMI \(=0.0\)
DO \(5000 \mathrm{I}=1\),IMPR
\(\operatorname{AFAR}=\operatorname{AFAR}+\mathrm{Y} 2 \mathrm{R}(\mathrm{I}) \star \operatorname{AR}(I)+Y 2 I(I) * A I(I)\)
\(A F A I=A F A I+Y 2 I(I) * A R(I)-Y 2 R(I) * A I(I)\)
\(\mathrm{BTAR}=\mathrm{BTAR}+\mathrm{Y} 2 \mathrm{R}(\mathrm{I}) * \mathrm{BR}(\mathrm{I})+\mathrm{Y} 2 \mathrm{I}(\mathrm{I}) * \mathrm{BI}(\mathrm{I})\)
\(B T A I=B T A I+Y 2 I(I) * B R(I)-Y 2 R(I) * B I(I)\)
GAMR \(=\operatorname{GAMR}+\mathrm{Y} 2 \mathrm{R}(\mathrm{I}) * \mathrm{CR}(\mathrm{I})+\mathrm{Y} 2 \mathrm{I}(\mathrm{I}) * \mathrm{CI}(\mathrm{I})\)
\(G A M I=G A M I+Y 2 I(I) * C R(I)-Y 2 R(I) * C I(I)\)
5000
```

CONTINUE

```

DO 5020 I=1,IMPR
\(\mathrm{GR}(\mathrm{I})=(\operatorname{AFAR} \star \operatorname{AR}(\mathrm{I})-\operatorname{AFAI} \star A I(I))+(\mathrm{BTAR} * \mathrm{BR}(I)-\mathrm{BTAI} * \mathrm{BI}(\mathrm{I}))\)
```

    GI(I)=(AFAI*AR(I)+AFAR*AI(I))+(BAMI*BR(I) +BTAR*CBI (I))
    ```

DO 5040 I=1, IMPR
\(\operatorname{ERRR}(I)=Y 2 R(I)-G R(I)\)
\(\operatorname{ERRI}(I)=Y 2 I(I)-G I(I)\)
CONTINUE
DO 5060 I=1,IMPR
\(\operatorname{AR}(I)=\operatorname{AR}(I)+E T A *(\operatorname{AFAR} \star \operatorname{ERRR}(I)+\operatorname{AFAI} \star \operatorname{ERRI}(I))\)
\(A I(I)=A I(I)+E T A *(A F A R * E R R I(I)-A F A I * E R R R(I))\)
\(\operatorname{BR}(I)=\operatorname{BR}(I)+E T A *(B T A R * E R R R(I)+B T A I * E R R I(I))\)
\(\mathrm{BI}(\mathrm{I})=\mathrm{BI}(\mathrm{I})+E T A *\) (BTAR*ERRI (I) -BTAI*ERRR (I))
\(\operatorname{CR}(I)=\operatorname{CR}(I)+E T A *(G A M R \star E R R R(I)+G A M I * E R R I(I))\)
\(C I(I)=C I(I)+E T A *(G A M R * E R R I(I)-G A M I * E R R R(I))\)
CONTINUE
CALL GRMSHM (IMPR,AR,AI,BR,BI,CR,CI)
DO \(6000 \mathrm{~J}=1\), IMPR
\(\operatorname{Y1R}(J)=(\operatorname{VR}(1) * \operatorname{AR}(J)-\operatorname{VI}(1) * A I(J))+(\operatorname{VR}(2) * B R(J)-\operatorname{VI}(2) * B I(J))\)
\(+(\operatorname{VR}(3) * C R(J)-V I(3) * C I(J))\)
\(\mathrm{Y} 1 \mathrm{I}(\mathrm{J})=(\mathrm{VI}(1) * \operatorname{AR}(\mathrm{~J})+\operatorname{VR}(1) * \operatorname{AI}(\mathrm{~J}))+(\mathrm{VI}(2) * \operatorname{BR}(\mathrm{~J})+\operatorname{VR}(2) * \operatorname{BI}(\mathrm{~J}))\)
1
CONTINUE
CONTINUE
C
PRINTING OUT RESULTS
ERRSUM=ERRSUM/LCOUNT
SUMERR=SUMERR/LCOUNT
ERNOM \(=10.0 *\) LOG10 (ERNOM/LCOUNT)
ERNON \(=10.0 *\) LOG10 (ERNON/LCOUNT)
ERRSUM=10.0* (LOG10 (ERRSUM))
SUMERR \(=10.0 *\) LOG10 (SUMERR)
PRINT *,' LCOUNT=', LCOUNT
PRINT *,'C=', C
PRINT *,' THETA=', THETA
PRINT *,'EPSLON=', EPSLON
PRINT *,'ETA=', ETA
PRINT *,' MEAN SQ ERROR IN ESTIMATION=', ERRSUM
PRINT *,'MEAN SQ ERROR IN ESTIMATION (GRAD EST) \(=\) ', SUMERR
PRINT *,'NORM.MEAN SQ.ERR IN EST.=', ERNOM
PRINT *,'NORM.MEAN SQ.ERR IN EST(ADPT.EST)=',ERNON
PRINT *,'SET SNR=', SNR
PRINT *,'DLTA=', DLTA
REWIND (10)
STOP
END
SUBROUTINE GRMSHM (IMPR,AR,AI,BR,BI,CR,CI)
REAL AR (IMPR), AI (IMPR), BR (IMPR), BI (IMPR)
REAL CR (IMPR), CI (IMPR)
AMAG=0.0
DO 1330 I=1, IMPR
AMAG \(=\) AMAG + AR (I) \(* * 2+A I(I) * * 2\)
CONTINUE
AMAG=SQRT (AMAG)
IF (AMAG) \(1340,1350,1340\)
AMAG=0.0
GO TO 1360
1340 AMAG \(=1.0 / \mathrm{AMAG}\)
1360 DO 1370 I=1, IMPR
```

    AR(I)=AMAG*AR (I)
    AI(I)=AMAG*AI (I)
    CONTINUE
    BAR=0.0
    BAI=0.0
    DO 1380 I=1,IMPR
    BAR=BAR+BR(I)*AR(I)+BI (I)*AI (I)
    BAI=BAI+BI(I)*AR(I) -BR(I)*AI(I)
    CONTINUE
DO 1390 I=1,IMPR
BR(I)=BR(I)-(BAR*AR(I)-BAI*AI (I))
BI (I) = BI (I) - (BAI*AR (I) +BAR*AI (I))
CONTINUE
BMAG=0.0
DO 1400 I=1,IMPR
BMAG=BMAG+BR(I)**2+BI(I)**2
1400 CONTINUE
BMAG=SQRT (BMAG)
IF (BMAG) 1410,1420,1410
BMAG=0.0
GO TO 1430
BMAG=1.0/BMAG
DO 1440 I=1,IMPR
BR(I)=BMAG*BR(I)
BI(I)=BMAG*BI (I)
CONTINUE
CBR=0.0
CBI=0.0
CAR=0.0
CAI=0.0
DO 1450 I=1,IMPR
CBR=CBR+CR(I)*BR(I)+CI (I)*BI (I)
CBI=CBI+CI(I)*BR(I) -CR(I)*BI(I)
CAR=CAR+CR(I)*AR(I)+CI(I)*AI(I)
CAI=CAI+CI (I)*AR(I) -CR(I)*AI (I)
CONTINUE
DO 1460 I=1,IMPR
CR(I)=CR(I)-(CBR*BR(I)-CBI*BI(I))-(CAR*AR(I) -CAI*AI (I))
CI(I)=CI(I) - (CBI*BR(I) +CBR*BI (I)) - (CAI*AR(I) +CAR*AI (I))
CONTINUE
CMAG=0.0
DO 1470 I=1,IMPR
CMAG=CMAG+CR(I)**2+CI(I)**2
CONTINUE
CMAG=SQRT (CMAG)
IF (CMAG) 1480,1490,1480
1490 CMAG=0.0
GO TO 1500
1480 CMAG=1.0/CMAG
1500 DO 1510 I=1,IMPR
CR(I)=CMAG*CR(I)
CI(I)=CMAG*CI (I)
1510 CONTINUE
RETURN
END

```

\section*{APPENDIX F5}

\section*{COMPUTER-SIMULATION PROGRAM FOR SYSTEM 8.2}
```

            PROGRAM STFTF
    IMPLICIT DOUBLE PRECISION (A-H, O-Z)
    REAL YR(32),YI(32)
    REAL Y1R(32),Y1I(32),SR(0:32),SI(0:32)
    REAL AR (0:32),AI (0:32),BR(0:32),BI (0:32)
    REAL CR(0:32),CI(0:32),C1R(0:32),C1I (0:32)
    REAL WDR(0:32),WDI (0:32),WFR(0:32),WFI(0:32)
    REAL GRDR(32),GRDI (32),ERR(32),ERI (32)
    REAL A1R(0:32),A1I (0:32),B1R(0:32),B1I(0:32)
    REAL CMR (0:32),CMI (0:32)
    REAL MU,LAMDA
    C FILE 'IMP500' HAS IN ITS STORE 60000 SIR
C OF THE 3 SKY WAVE HE CHANNEI
OPEN (10,FILE='IMP500',FORM='UNFORMATTED')
N=32
NN=N+1
ERRTOT=0.0
ERRNOM=0.0
LCOUNT=0
MCOUNT=0
KSTART=1000
ETAT=0.0
IQ=200
CALL G05CBF(IQ)
MU=0.1
SNR=10.0
LAMDA=0.988
THETA1=0.008
THETA2=1.10
DO 2000 J=0,N
AR (J)=0.0
AI (J)=0.0
BR (J)=0.0
BI (J)=0.0
CR(J)=0.0
CI (J)=0.0
CONTINUE
AR(0)=1.0
BR (N)=1.0
GAMA=1.0
ALFA=(LAMDA**32.0)*MU
BETA=MU
LSTART=0
C INITIALISATION OF NOISE AND DATA MATRIX
DO 2010 J=0,N
SR (J)=1.0
SI (J)=1.0
WDR (J) =0.0
WDI (J) =0.0
CONTINUE

```

C SHIFTING OF ARRAYS ONCE FOR EVERY DATA SYMBOL DO \(2160 J=N, 1,-1\)
SR (J) \(=\) SR ( \(J-1\) )
\(S I(J)=S I(J-1)\)

C GENERATING QPSK DATA
XX=G05CAF (XX)
IF (XX-0.5) 2200,2200,2220
\(\operatorname{SR}(0)=-1.0\)
GO TO 2250
\(S R(0)=1.0\)
\(\mathrm{XX}=\mathrm{G} 05 \mathrm{CAF}(\mathrm{XX})\)
IF (XX-0.5) \(2260,2260,2280\)
\(2260 \operatorname{SI}(0)=-1.0\)
```

        GO TO 2300
    2280 SI (0)=1.0
2300 CONTINUE
C GENERATING NOISE
DO 2450 LNSE=1,2
DO 2420 J=1,N
WDR (J-1) =WDR (J)
WDI (J-1) =WDI (J)
CONTINUE
WDR(N)=G05DDF (0.0,STDVN)
WDI (N)=G05DDF (0.0,STDVN)
WNR=0.0
WNI=0.0
DO 2440 J=0,N
K1=N-J
WNR=WNR+WDR (K1) *WFR (J) -WDI (K1) *WFI (J)
WNI=WNI+WDR(K1) *WFI (J) +WDI (K1) *WFR(J)
CONTINUE
CONTINUE
C CALCULATION OF RECEIVED SIGNAL
RR=0.0
RI=0.0
DO 2480 J=0,(N-1)
RR=RR+SR(J)*YR(J+1)-SI (J)*YI (J+1)
RI=RI+SR(J)*YI (J+1) +SI (J)*YR(J+1)
CONTINUE
C RECEIVED SIGNAL WITH NOISE
RR=RR+WNR
RI=RI+WNI
IF (ETAT.GT.0.0001.AND.KSTART.GT.100) THEN
KSTART=0
MCOUNT=MCOUNT+1
ENDIF
KSTART=KSTART+1
LSTART=LSTART+1
IF(KSTART.LE.50) THEN
CALL RESTART (N,SR,SI,MU,LAMDA,KSTART,
1
1
ENDIF
IF(KSTART.EQ.51) THEN
DO 2500 J=0,N
AR(J)=A1R(J)
AI(J)=A1I(J)
BR(J)=B1R(J)
BI (J)=BII (J)
CR(J)=CMR (J)
CI(J)=CMI (J)
CONTINUE
ALFA=ALFAL
BETA=BETAL
GAMA=GAMAL
ENDIF

```
```

C ESTIMATION OF IMPULSE RESPONSE
ENPR=0.0
ENPI=0.0
DO 2520 I=0,N
ENPR=ENPR+AR(I)*SR(I)-AI (I)*SI (I)
ENPI=ENPI+AR(I)*SI(I) +AI (I)*SR(I)
CONTINUE
ENR=ENPR*GAMA
ENI=ENPI*GAMA
ALFA1=ALFA
ALFA=LAMDA*ALFA1+(ENPR*ENR+ENPI*ENI)
GAMA2=GAMA
GAMA1=LAMDA* (ALFA1/ALFA) *GAMA
EN1R=ENPR/(LAMDA*ALFA1)
EN1I=ENPI/ (LAMDA*ALFAI)
DO 2540 J=1,N
C1R(J)=CR(J-1)+(EN1R*AR(J) +EN1I*AI (J))
C1I(J)=CI (J-1)+(EN1R*AI (J) -EN1I*AR(J))
CONTINUE
C1R(0)=+EN1R
C1I(0)=-EN1I
RNP1R=0.0
RNP1I=0.0
DO 2560 I=0,N
RNP1R=RNP1R+BR(I) *SR(I) -BI (I) *SI (I)
RNP1I=RNP1I+BR(I)*SI(I) +BI (I)*SR(I)
CONTINUE
IF(LSTART.LE.32) THEN
RNP1R=0.0
RNP1I=0.0
ENDIF
GAMA=(1.0/(1.0-GAMA1* (RNP1R*C1R(N)-RNP1I*C1I (N))))*GAMA1
RNPR=+LAMDA*BETA*C1R(N)
RNPI=-LAMDA*BETA*C1I(N)
ETAR=RNP1R-RNPR
ETAI=RNP1I-RNPI
ETAT=ETAR**2+ETAI**2
DO 2580 J=1,N
AR(J)=AR(J)-(ENR*CR(J-I)-ENI*CI (J-1))
AI (J) =AI (J) - (ENR*CI (J-1) +ENI*CR (J-1))
CONTINUE
RNR=RNP1R*GAMA
RNI=RNP1I*GAMA
BETA=LAMDA*BETA+(RNP1R*RNR+RNP1I*RNI)
DO 2600 J=0,(N-1)
CR(J)=C1R(J)-(C1R(N)*BR(J)-C1I (N)*BI (J))
CI(J)=C1I (J) - (C1I (N)*BR(J) +C1R(N)*BI (J))
CONTINUE
DO 2620 J=0,(N-1)
BR(J)=BR(J) - (RNR*CR(J) -RNI*CI (J))
BI (J) = BI (J) - (RNR*CI (J) +RNI*CR (J))
CONTINUE

```
```

    WER=0.0
    WEI=0.0
    DO 2680 J=0,(N-1)
    WER=WER+Y1R(J+1)*SR(J)-Y1I (J+1)*SI (J)
    WEI=WEI+Y1R(J+1)*SI (J) +Y1I (J+1)*SR(J)
    CONTINUE
    ESPR=RR-WER
    ESPI=RI-WEI
    EPSR=ESPR*GAMA
    EPSI=ESPI*GAMA
    DO 2700 J=0,(N-1)
    ERR(J+1)=EPSR*CR(J)-EPSI*CI (J)
    ERI(J+1)=EPSR*CI (J)+EPSI*CR(J)
    CONTINUE
    DO 2800 J=1,N
    GRDR (J)=GRDR (J) +THETA1*ERR (J)
    GRDI (J) =GRDI (J) +THETA1*ERI (J)
    Y1R(J)=Y1R(J) +GRDR (J) +THETA2*ERR (J)
    Y1I (J) =Y1I (J) +GRDI (J) +THETA2*ERI (J)
    CONTINUE
    9000 CONTINUE
4000 CONTINUE
C PRINTING OUT RESULT
PRINT *,'MU=',MU
PRINT *,'LAMDA=',LAMDA
PRINT *,'THETA1=',THETA1
PRINT *,'THETA2=',THETA2
PRINT *,'SNR=',SNR
PRINT *,'LCOUNT=',LCOUNT
PRINT *,'MCOUNT=',MCOUNT
PRINT *,'MEAN SQ ERROR=',10*(LOGIO(ERRTOT))
PRINT *,'NORM.MEAN SQ.ERR=',10.0*(LOG10(ERRNOM))
REWIND (10)
STOP
END
SUBROUTINE RESTART (N,SR,SI,MU,LAMDA,KSTART,
1
I
A1R,A1I,B1R,B1I,CMR,CMI,
ALFAL, BETAL,GAMAL)
REAL SR(0:32),SI (0:32)
REAL A1R(0:32),A1I (0:32),B1R(0:32),B1I (0:32)
REAL CMR (0:32),CMI (0:32), CNR (0:32), CNI (0:32)
REAL MU,LAMDA
IF(KSTART.EQ.1) THEN
DO 6000 J=0,N
A1R(J)=0.0
A1I (J) =0.0
B1R(J)=0.0
B1I (J)=0.0
CMR (J) =0.0
CMI (J)=0.0
CONTINUE
A1R(0)=1.0
B1R(N)=1.0
GAMAL=1.0
ALFAL=(LAMDA**32.0)*MU
BETAL=MU

```
```

    ENDIF
    EMPR=0.0
    EMPI=0.0
    DO 6020 I=0,N
    EMPR=EMPR+A1R(I)*SR(I) -A1I (I) *SI (I)
    EMPI=EMPI+A1R(I)*SI(I)+AII(I)*SR(I)
    6 0 2 0
6 0 4 0
6 0 6 0
ENDIF
EMPR=0.0
EMP I $=0.0$
$\operatorname{EMPI=EMPI+A1R(I)*SI(I)+AII(I)*SR(I)~}$
CONTINUE
$E M R=E M P R * G A M A L$
EMI=EMPI*GAMAL
$A L F A M=A L F A L$
$A L F A L=L A M D A * A L E A L+(E M P R * E M R+E M P I * E M I)$
$G A M A M=L A M D A *(A L F A M / A L F A L) * G A M A L$
$E M 1 R=E M P R /(L A M D A * A L F A M)$
EM1 I=EMP I / (LAMDA*ALFAM)
DO $6040 \mathrm{~J}=1, \mathrm{~N}$
CNR $(J)=\operatorname{CMR}(J-1)+(E M 1 R * A 1 R(J)+E M 1 I * A 1 I(J))$
CNI (J) $=$ CMI (J-1) $+(E M 1 R \star A 1 I(J)-E M 1 I * A 1 R(J))$
CONTINUE
$\operatorname{CNR}(0)=+E M 1 R$
CNI (0) $=-$ EM1I
DO $6060 \mathrm{~J}=1, \mathrm{~N}$
A1R (J) $=$ A1R (J) $-(\operatorname{EMR} * \operatorname{CMR}(J-1)-E M I * C M I(J-1))$
AII $(J)=A 1 I(J)-(E M R * C M I(J-1)+E M I * C M R(J-1))$
CONTINUE
$R M P R=+L A M D A * B E T A L * C N R(N)$
$R M P I=-L A M D A * B E T A L * C N I(N)$
COM1 $=1.0-$ GAMAM* (RMPR*CNR (N) $-\operatorname{RMPI*CNI~(N))~}$
GAMAL=GAMAM/COM1
$R M R=R M P R * G A M A L$
$\mathrm{RMI}=\mathrm{RMP} I * G A M A L$
$B E T A L=L A M D A * B E T A L+(R M P R * R M R+R M P I * R M I)$
DO $6080 \mathrm{~J}=0,(\mathrm{~N}-1)$
$\operatorname{CMR}(J)=\operatorname{CNR}(J)-(\operatorname{CNR}(N) * B 1 R(J)-C N I(N) * B 1 I(J))$
$\operatorname{CMI}(J)=\operatorname{CNI}(J)-(\operatorname{CNI}(N) * B 1 R(J)+\operatorname{CNR}(N) * B 1 I(J))$
CONTINUE
DO $6100 \mathrm{~J}=0,(\mathrm{~N}-1)$
$\operatorname{B1R}(J)=\operatorname{B1R}(J)-(R M R * C M R(J)-R M I * C M I(J))$
B1I (J) $=\mathrm{B} 1 \mathrm{I}(\mathrm{J})-(R M R * C M I(J)+R M I * C M R(J))$
CONTINUE
RETURN
END

```

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