APPLICATION OF NON-LINEAR OPTIMIZATION TO MULTIPURPOSE RESERVOIR SYSTEMS

by

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ABSTRACT

The aim of this research is to investigate the application of nonlinear programming techniques to multipurpose reservoir systems. A multipurpose multiple reservoir operation problem is a typical nonlinear large scale optimization problem. The currently applied techniques overcome the nonlinearity and dimensionality problems through simplification. To model the problem more closely, a successful trial is made in this study to apply the most efficient and suitable nonlinear programming techniques.

Although research in large scale nonlinear optimization has been in recent years a major subject of interest within the mathematical programming community, its application to reservoir systems is very limited. As a result of these activities software packages, as Lancelot, have been developed. Lancelot is a general purpose software package designed for solving large-scale nonlinear optimization problems. It uses Augmented Lagrangian and Conjugate Gradient methods. This software is used here successfully to solve an optimization problem formulated for a major river system, the Blue Nile in Sudan. The system has two in series reservoirs used for hydropower generation, maintaining minimum downstream flows and irrigation. For optimization, some features of the system have been modelled. These are sedimentation, evaporation, demand and flow. To represent the effect of sedimentation a model is fitted and verified. To include the effect of evaporation a model that estimates the total evaporation losses is fitted using Penman approach and verified using water balance. To cope with flow uncertainty the Blue Nile flow has been modelled. ARMA(1,1) has given the best fitting. Irrigation requirements have been estimated using Penman-Monteith approach. Efficiency of water use has been investigated and other possible demand scenarios resulting from efficient water use are obtained. The results of flow and demand modelling are used as direct input to the optimization model while sedimentation and evaporation models are incorporated in the model.

The objective of this model is to maximise power benefits on condition that certain irrigation and downstream requirements be met. To solve this problem a double precision version of Lancelot was installed in a hp-UNIX system. For the problem a

specification and a standard input format, SIF, files were written and put under the same directory with Lancelot to run the program. The problem was solved successfully in few minutes. The solution includes values for the objective function, decision variables (releases and storage volumes), penalty parameter, Lagrange multipliers and slack variables.

In reservoir operation, general operation rules are needed more than computed releases corresponding to specified flow sequences. To achieve this, the optimization model is solved repeatedly using different generated flow sequences. The optimum releases are then regressed linearly and nonlinearly on the important independent variables, flows and/or storage volumes, to derive operation rules. The derived rules have been tested successfully both statistically using R^2 criterion and simulation. To be easily used in practice the rules are presented in a graphical form.

The optimization output is affected by reservoir sedimentation. Therefore the developed optimization and sedimentation models have been linked to investigate sedimentation effect on optimization output along the course of reservoir operation. Results have shown that this approach can be used to investigate the effect of sedimentation on reservoir optimum output.

In, a multipurpose reservoir system, the optimization output for one purpose is affected by the efficiency of water use for other purposes. Therefore the effect of efficient water use in irrigation on power benefits is investigated. Results have shown an increment in benefits due to using irrigation water efficiently. This approach can be applied to systems where priority is given for one purpose over the others.

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900	conversion factor.
A	area of x-section
A	= [$a_1 a_2$..................a_l] is a vector of constants
A	m*n deterministic matrix
A	reservoir area in squared kilometre.
AIC(p,q)	Akaike Information Criterion.
$A^{(k)}$	update matrix
$\dot{a}$	= $\Psi(t)$ function of time
a,b	constants. 0.25 and 0.5 can be used for a and b respectively
a_i^0	initial content of the reservoir i.
a_i^k	final content of reservoir i
$B(R_t)$	return obtained from release R_t during period t.
BQP	quadratic problem with simple bounds
B_0 to B_6	regression model coefficients
B_d	deformable bed width
$B^{(k)}$	symmetric approximation of the hessian matrix $G(x^{(k)})$.
b,c and g	m*1, 1*n and 1*j vectors respectively
b_t	decision parameter
C	1*n vector
C_s	average spatial sediment concentration in the cross-section;
$C(S^k)$	production cost in period k.
C_p	specific heat of moist air = 1.013 [kj /kg ,^{0}C]
C_k	lag k autocovariance
C_p	power overall efficiency coefficient
C(i)	power price in month i in Sudanese dinnars / kwh
CRP	regression model variable = (QFL * STG)
COV(X,Y)	covariance
D	root depth (m)
Ds	increase in storage

D^k	demand for energy in period k
D_{ij}	coefficients
$D.S_a$,	available soil moisture in the root depth
d_r	relative distance earth - sun.
$\bar{d}$	search direction of the free variable
E	ratio molecular weight water vapour / dry air
E{ }	expectation with respect to a random vector
E_a	application efficiency
E_c	conveyance efficiency
Ed	distribution efficiency
E_0	evaporation rate in mm/day
Ep	overall project efficiency
E_t	evaporation losses in period t.
ET_a	actual crop evapotranspiration [mmd^{-1}]
ET_{crop}	crop water requirements (m^3).
ET_m	maximum crop evapotranspiration [mmd^{-1}]
ET_0	reference crop evapotranspiration [mmd^{-1}].
e_a	actual vapour pressure at the mean air temperature [mbar].
e_s	saturation vapour pressure of water at the mean air temperature [mbar].
$e_s - e_a$	vapour pressure deficit [kpa]
F	revenues from power generated in million Sudanese dinnars.
F_{alag}	Augmented Lagrangian Penalty Function
$F_P^{-1}(\alpha)$	inverse of commulative probability function at the given value of α.
$f_t(S_t, I_{t+1})$	return obtained from operating the system optimally.
f(x)	objective function.
G	vector of constants
G	soil heat flux [$MJ\ m^{-2}d^{-1}$]
G_b	bed load
G_s	suspended load
G(x)	hessian

$G(x)^{-1}$	inverse of hessian.
g	gravitational acceleration
$\bar{g}^{(k,j)}$	gradient of the free variable
$g_i(x)$	inequality constraints.
H	stage in metres.
H	$= [h_1\ h_2\ h_3 \ldots h_m]^T$ vector of constants
H_{ds}	average monthly downstream level in (m)
$HP(i)$	power generated in month i in KWh.
$H_i(x_i^{k-1}, u_i^k)$	power generated from plant i in period k.
$H^{(k)}$	symmetric positive definite approximation to $G(x^{(k)})^{-1}$
H_{us}	average monthly upstream water level in (m)
H_t	average productive head in (m)
$h_i(x)$	equality constraints
I	average annual flow
I_t, I_{t-k}	inflows during time period t and t-k respectively
K	reservoir capacity.
K_c	crop factor
K_y	yield response factor
KWH_t	hydropower in kwh
k	class intervals, each with 1/k probability.
L	monthly losses in million m^3
LGP	length of the growing period(days)
$L(x,u,v)$	Lagrangian
M1	linear regression model
M2	nonlinear regression model
M3	nonlinear regression model
m	$= f(t)$, function of time
$m^{(k)}(x)$	quadratic model
$\bar{m}^{(k,j)}(\bar{d})$	quadratic model function of free variables
N	possible hours of bright sunshine
N	number of years

N	sample size
N'	length of the generated sample
N_g	desired generated length
N_w	warm-up length
n	constant commonly varies between 2 and 3
n	actual hours of bright sunshine.
P	atmospheric pressure [kpa]
P	depletion factor
P_e	effective precipitation in m^3
P_{eff}	effective rainfall in mm\day
P_{tot}	total rainfall in mm\day
$P[I_t \| I_{t+1}]$	transition probability function.
$Pr\{\ \}$	denotes probability
$P(I_t)$	probability of (discrete) inflow I_t.
p	k*1 vector.
p	order (rate) of convergence
p	autoregrssive model order
$p(x,l,u)$	projection operator
Q	statistic
Q	transition matrix
Q	discharge
Q	streamflow in million m^3 / day
QFL	inflow in month i
QFL1	lagged inflows in month i-1
QFL2	lagged inflows in month i-2
QFL3	lagged inflows in month i-3
QFL4	lagged inflows in month i-4
Q_s	suspended sediment load in tonnes / day.
q	moving average model order.
q_i	river flow in month i, in million m^3
q_i^k	total inflow to reservoir i in period k.
q_t	average flow rate in m^3/sec

$q_t(i,j)$	conditional probability that the flow is in state i at time t, given that it was in state j at time t-1
R_a	extraterrestrial radiation[$MJm^{-2}d^{-1}$]
R_a	theoretical radiation that would be received at the ground surface in absence of atmosphere, extraterrestrial radiation, [mm/day].
R^2	coefficient of determination
R_n	net radiation [$MJm^{-2}d^{-1}$]
R_{ns}	net short-wave radiation [$MJm^{-2}d^{-1}$]
R_{nl}	net long-wave radiation[$MJm^{-2}d^{-1}$]
REL	release in month i
RH	relative humidity in %.
R_t	release during time period t.
R_t^*, S_t^*,	optimum releases and storages
r	albedo (reflection coefficient)
r	$= (i+1)^{-1}$ is the discount factor with a discount rate (i)
$r^{(k)}$	ratio of actual reduction in the objective function to that predicted by the quadratic model
r_k	autocorrelation coefficients
Stb	storage at the beginning of the time period
Ste	storage at the end of the time period
S_f	energy gradient
S_{t-1}	storage at end of time period t-1.
S_t and I_t	storage at the beginning of and inflow during period t respectively.
S^k	energy imported with some cost C.
$S(\phi_j,\theta_j)$	minimum sum of squares of errors
S_{av}	average storage in million m^3
$S_{i,j}$	storage of reservoir i at the beginning of month j
$S_{i,j+1}$	storage of reservoir i at the beginning of month j+1
$S_{(1,i)}$,$S_{(2,i)}$	storages in Roseries and Sennar, respectively, at the beginning of month i, in million m^3.
$S_{(1,i+1)}$, $S_{(2,i+1)}$	storages in Roseries and Sennar, respectively, at the end of month i, in million m^3.

SIF	Standard Input Format file
STG	storage at the beginning of month i
SUM1	regression model variable
SUM2	regression model variable
SUM3	regression model variable
T	trap efficiency
T	transpose.
T	k*n matrix.
T	number of periods within a year.
T	average air temperature in ^{0}c.
T_a	average temperature in Kelvin, ^{0}K.
T_{kx}	maximum temperature [K]
T_{kn}	minimum temperature [K]
T_{monthn}	mean temperature in months n [^{0}C]
$T_{monthn-1}$	mean temperature in months n-1 [^{0}C]
t	time
t'	time during which $ET_m = ET_a$
t	number of years in which the reservoir had been in operation
U_2	wind speed measured at 2 metre height [ms^{-1}]
U	wind speed at 2 metre level above the ground [mile/day]
u_i and v_i	Lagrange Multipliers.
$u_1, u_2, .., u_{k-1}$	values obtained from the normal probability tables corresponding to commulative probabilities 1/k, 2/k, ..., (k-1)/k.
u_1 u_2	random numbers of the uniform (0,1) distribution
$V(H_m)$	reservoir capacity upto the mean operating level H_m.
Vc	volume of water diverted or pumped from the river (m^3)
Vd	volume of water delivered to the distribution system (m^3).
V_f	volume of water furnished to the fields (m^3)
V_m	volume of irrigation water needed in m^3 (based on ET_a)
V_m'	volume of irrigation water needed in m^3 (based on ET_m)
W	k*j matrix
w	number of intervals during the year.

w_s	sunset hour angle.
X	$= [x_1 x_2 \ldots x_l]^T$ is a vector of variables
X_t	inflow in period t.
X	release through turbines in million m^3/month
X	n*1 vector of decision variables
X^k	solution of the original model f(x)
$X^{k,1}$	generalised Cauchy point
$X^{k,j}$	a point lying within the intersection of the feasible and trust region which minimises the quadratic model.
$X_{v,\tau}$	original time series with v number of years and τ number of months
$X_{(1,i)}$, $X_{(2,,i)}$	hydropower releases in Roseries and Sennar respectively, in million m^3/month
$\ddot{X}$	mean of the time series X_t, t=1,.....N
x_i^k	content of reservoir i at end of period k
x^*	minimum
x	distance along the channel bed measured in the downstream direction
$\|x\|$	norm of x
Y_t	outflow in period t.
Y	j*1 vectors of decision variables
Y	release through other gates in million m^3/month
$Y_{(1,i)}$, $Y_{(2,,i)}$	releases through other gates in Roseries and Sennar respectively in million m^3/month
y	water surface elevation
y_t	time dependent series
$y_{v,\tau}$	transformed series with v number of years and τ number of months
Z	finite reservoir state
Z	bed elevation
Z	altitude in metres
Z_t	standardised time series
$\ddot{Z}_t$	mean of the first N-k values Z_1, ..., Z_{N-k}.
$\ddot{Z}_{t+k}$	mean of the last N-k values, Z_{k+1}, ..., Z_N.
$Z_{v,\tau}$	standardised series with v number of years and τ number of months.

α	m*1 constant vector with components α_i $(0 \le \alpha_i \le 1)$.
$\alpha^{(k)}$	step length along the search direction.
χ^2	chi-square values
Δ	slope of saturation vapour pressure curve for water at mean air temperature.
$\frac{\Delta}{\Delta+\gamma}$	weighting factor to relate solar radiation to evaporation.
$\Delta^{(k)}$	scalar
δ	solar declination [rad]
ε_g	small convergence tolerance.
ε_t	time independent (uncorrelated) series
ε_t	residuals, (white noise)
$\phi_1, --, \phi_p$	autoregression coefficients
ϕ_j	autoregressive parameter
γ	asymptotic error constant (a e c).
γ	constant of the wet and dry bulb psychorometer equation.
γ	skewness coefficient
γ	psychometric constant [kpa $^0C^{-1}$]
φ	latitude [rad]
λ	latent heat of vaporisation.
μ_x	mean of random variable x.
θ_j	moving average parameter.
$\rho_{x,y}$	correlation coefficient
ρ_*	density of sediment in the bed
σ	Boltzman constant
σ^2	variance of the series Z_t,
σ_ε^2	variance of the independent series ε_t
σ_x^2	variance of the random variable x.
ξ_t	standard normal random number
$\zeta^{(k,j)}$	fixed variables on their bounds at the generalised Cauchy point.
$\nabla f(x)$	first derivative

CHAPTER I

INTRODUCTION

Summary ~ This chapter highlights the main features of the proposed research, the justification for the research and the hypotheses and objectives of the research. It also outlines methods of approach used to conduct the research and finally it shows the organisation of the thesis.

1.1 GENERAL

Reservoirs are usually built to redistribute water in time and space. They modify the pattern of natural flow by storing water when the inflow exceeds the demand so as to be released when the requirements exceed the inflow. These releases have to be carefully made. Releasing too much or too little water may result in an economic loss. Therefore water has to be released optimally to maximise the benefits from reservoirs on one hand and to meet the growing demands on the other. This growing demand is caused, especially in developing countries, by growing population and continuous and rapid urbanisation. In developing countries, the urbanisation increases demand in sectors like power and recreation. Although these sectors are not water consumptive, but they may use water in a way that may contradict satisfying the requirements in the traditional largest water user; irrigation. To meet these growing demands, reservoirs have to be operated optimally and water used efficiently by the traditional water users (e.g. irrigation). Alternatively, these demands could have been met by expansion of new facilities (reservoirs). This option is not viable with the increasing awareness about environment and preservation of natural resources and economic constraints. A country like Sudan, which is desperately in need to develop projects which would take up its remaining share of Nile Waters, couldn't achieve that mainly because of economic dislocation in recent years.

1.2 THE NEED FOR THE STUDY

A water resource system may consist of direct abstraction, underground or other sources with one or more reservoirs. Such a system may supply water for consumptive, hydroelectric and dilution purposes, may provide reservations for flood control and may maintain minimum levels for recreation. In view of the diversity of needs, it becomes necessary to seek optimal decisions in planning, design and operation of the system. The decisions are based on economic, environmental, legal and other requirements and, if implemented, would cause the greatest benefit.

An optimization problem of a reservoir system is complex. The complexity arises from the number of subproblems involved and hence the large number of parameters that need to be dealt with. For this reason most of the published work in reservoir system operation use simplified systems. This simplification made it difficult to apply such work on real world problems. Therefore, there is a real need for development of models that are applicable to real world problems.

Examples of the subproblems involved in reservoir optimization are: stochasticity of inflows, reservoir losses, demand modelling and reservoir sedimentation. Each of these factors affects the optimum operation of a reservoir system and the severity differs from one system to another.

The uncertainty in inflow has no effect in simulation models, since the computations are carried out step by step or period by period so that the future releases will not be affected. In deterministic optimization models, releases are made to maximise or minimise the objective function without knowing future flows. This limits the applicability of the optimization results. Evaluation of the optimum outcome or the potential of any reservoir system should be tested under varying flow conditions. Thus there is a need for reflecting the effects of uncertainty in knowing future stream flows. In some of the optimization techniques the stochastic nature of the flow has been accommodated in different ways, while much work to achieve this is needed in non-linear programming.

Almost in all kinds of optimization models, very little attention has been paid to the effect of sedimentation, because sedimentation is less important in climates where these models have mostly been applied. Sedimentation reduces reservoir capacity and hence its ability to meet the requirements optimally. In rivers with high sediment contents and relatively small reservoirs with no alternate potentials to build new reservoirs to replace the existing ones, the most sustainable operation policy is the one that keeps sedimentation at its lowest level. Even if this policy is followed, the effect of sedimentation cannot be neglected and has to be represented when reservoir optimization is considered. Bathemetric survey data are available, world-wide, and use can be made of this data to estimate the change in reservoir storage capacity and other reservoirs' relationships with time and hence incorporating the effect of sediment in optimization models. Estimation of the demand also affects the optimization output. Assume that a reservoir system is used for irrigation and hydropower generation. If an optimization is carried out to maximise the power generation on condition that irrigation requirements are to be met first, then the optimization results are highly affected by the efficiency of water use in irrigation. If water is not used efficiently in the sector that has been given the priority, then some of the supply, which could have been used for other purposes, would be wasted.

Inclusion of the issues discussed above in an optimization problem, will result in having a large number of variables to be dealt with (dimensionality problem). Also in reservoir system optimization problems, some functions like evaporation and power production are non-linear. Therefore an exact reservoir optimization problem is a non-linear large scale one. There are difficulties in applying non-linear programming. These difficulties arise from the fact that non-linear programming mathematics is a little bit complicated compared to linear programming. In addition to that the problem of dimensionality, which is not faced in linear programming, is faced in non-linear programming. A description of the number of variables, as big, takes on a different meaning in non-linear programming to linear programming. In linear programming, thousands of variables and constraints might be considered big, where as in non-linear programming, hundreds of variables and constraints will be generally big. These problems of dimensionality and nonlinearity in reservoir optimization are overcome by:

1) linearising the problem i.e. transforming it into a linear programming problem. In linear programming the problem of dimensionality is usually not faced, but the system is approximately represented.

2) state discretization which is practised in dynamic programming. Dynamic

programming is a powerful tool but discretization reduces the dimensionality at the expense of accuracy.

It can be noticed that all the techniques used approximate the solution. This may not be desired in environments where water is scarce and/or reservoirs have limited storage capacities. Therefore there is a need to apply a technique, like non-linear programming that represents the reservoir optimization more realistically.

Research in large-scale non-linear optimization has been in recent years a major subject of interest within the mathematical programming community. Despite this its application to reservoir systems is very limited. Therefore various non-linear programming techniques will be investigated and applied to reservoir systems.

Simulation models represent the operating rules in more details, while the optimization models compute the releases that maximise or minimise the objective function without tackling the details of the operation rules. No attempt has ever been made to derive operation rules out of the non-linear optimization results. Optimization results would become more useful and practical when expressed into operation rules. Therefore trials have to be made to represent the non-linear optimisation results into operation rules.

Usefulness of a model is measured by its application. Therefore the non-linear model to be developed has to and will be applied to a major system. This will be the Blue Nile System in Sudan. This case study is an example of a multipurpose, multiple reservoir system, located in a semiarid tropical environment. The system is composed of two in series reservoirs used for hydropower and irrigation. The system features are:

a) a short flood season and a long low flow season.
b) high fine sediment concentration occurring during the short flood season.
c) high evaporation losses.
d) the existence of large irrigation schemes.

Based on the above discussion the hypotheses are formulated and their verification could lead towards achieving the aim of the study.

1.3 HYPOTHESES AND OBJECTIVES

The output from the optimum reservoir operation is affected by the variations in inflow, amount of sediment trapped, variation in demand due to efficiency of water use for one or more purposes, evaporation losses and the optimization techniques used to reach the solution. Based on these considerations the following hypotheses are formulated and will be verified in this research.

1. In a multiple-purpose reservoir system, where water is released for irrigation and hydropower generation, inappropriate water supply to irrigation schemes can be identified and reallocated to increase provisions for power generation.

2. Sedimentation effect on reservoir storage-water level relationship can be modelled. Linking this sedimentation model to the developed optimization model, effect of sedimentation on optimum reservoir operation can be investigated.

3. The stochastic nature of inflow can be implicitly incorporated in an optimization problem by synthetically generating inflows. (This approach does not consider the impact of droughts and low flow clusters on optimization, but in the Blue Nile System droughts do not affect the filling of reservoirs which have small capacities while low flow clusters have no effect due to the operation of the system on annual basis.)

4. Evaporation losses can be modelled and incorporated in an optimization problem.

5. Non-linear programming techniques can be applied to reservoir system real problems.

6. Regression analysis can be used to derive operation rules out of the non-linear optimization results.

This research is aiming at verifying the above hypotheses with the following objectives:

1. To investigate the performance of non-linear programming techniques on reservoir systems. A non-linear optimization model for the operation of a multiple-purpose multiple-reservoir system will be developed. General-purpose software, designed for large-scale optimization, is going to be used for this purpose.

2. To test the applicability of the model by considering a case study of the Blue Nile System in Sudan. The aim is to maximise power generation revenues: subject to the conditions that specified downstream and irrigation requirements are met.

3. To test the usefulness of the non-linear optimization output in operation rules derivation.

1.4 METHOD OF APPROACH

The literature review of the work done previously, showed that the optimization techniques used in modelling reservoir systems do not represent the system realistically. Therefore the non-linear programming techniques which represent the system better will be used. A general Software package, named Lancelot, will be used. The package uses efficient non-linear optimization algorithms and is specially designed for large-scale optimization.

The literature review also showed that general operation rules are needed more than the optimum computed releases corresponding to specified stream flow sequences. Therefore different linear and non-linear regression models will be tried to derive operation rules using the optimization results.

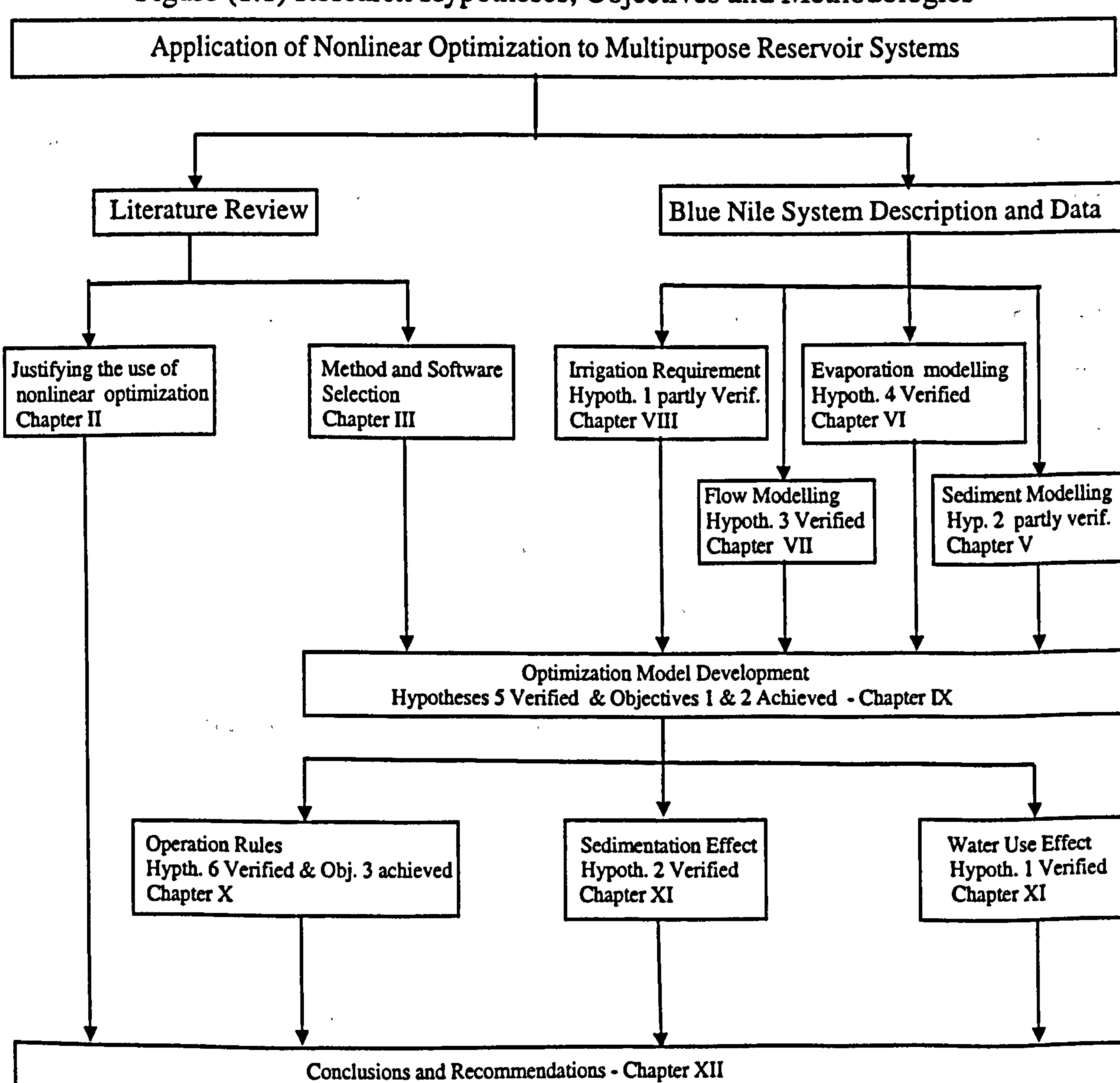
The hypotheses formulated in this study formed the basis for the development of the optimization model. The hypotheses will be tested using the case study data from the Blue Nile System, Sudan. Also data from the same system will be used to verify the applicability of the model.

1.5 ORGANISATION OF THE THESIS

The thesis consists of twelve chapters. Chapter I is the introduction where the need for the research is justified, the hypotheses and objectives are stated and the method of approach is outlined. In Chapter II and III literature is reviewed. In Chapter II the most widely used optimisation techniques and their applications are reviewed while in Chapter III the non-linear techniques to be applied in this research are investigated.

Chapter IV describes the features of the Blue Nile System. In Chapters V to VIII analyses have been carried out to form a base for the model development. In Chapter V a sedimentation model is developed and verified. In Chapter VI a model that quantifies evaporation losses is also developed and verified. In chapter VII the inflow to the system is modelled and the best model that generates flow sequences is chosen. In Chapter VIII the efficient use of irrigation water is investigated and different scenarios of irrigation demands are estimated. In Chapter IX the optimization model is developed. A problem is formulated and solved. In Chapters X and XI the usefulness of the model is tested. In Chapter X the non-linear optimization output is used for derivation of monthly operation rules. In Chapter XI the use of the non-linear model in investigating the effect of sedimentation and efficient water use is highlighted. Chapter XII concludes the findings of the study. Figure (1.1) outlines the research hypotheses, objectives and methodologies.

Figure (1.1) Research Hypotheses, Objectives and Methodologies



CHAPTER II
RESERVOIRS' MATHEMATICAL MODELLING

Summary ~ In this literature review the general issues concerned with reservoir modelling are discussed first. Then modelling techniques other than optimization i.e. simulation are reviewed before the optimization techniques are discussed in detail. From this literature review the need for developing a model as well as the model categorisation are outlined.

2.1 INTRODUCTION

2.1.1 Purpose of the Reservoir

Reservoirs are used to redistribute water quantity and quality in time and space. They modify the pattern of natural flow by storing water when inflow exceeds the demand so as to be released when the requirements exceed the inflow. In large multipurpose reservoirs (Simonovic, 1992), the storage volume can be divided into three parts, namely, flood control storage, active storage and dead storage. The active storage provides water for various purposes. These purposes can be power generation, irrigation, domestic and industrial water supply and increasing the low flow for navigation, pumping or to improve water quality. The dead storage is a consequence of the topography and design, but has some uses for sediment control and recreation.

2.1.2 Aim of Reservoir Operation Studies

The basic problem in reservoir operation studies, (Simonovic, 1992), is to find the relationship between inflow characteristics, reservoir storage, reservoir releases and reliable reservoir operation. Kreuze (1986) stated that the aim of reservoir operation studies is to determine the optimum useful output of a reservoir or a reservoir system.

2.1.3 Reservoir Operation Rules

Kreuze (1986) defined the operating rules as a set of rules for determining the volume of water to be stored or released from a reservoir under various conditions. Rules resulting in optimum output should be applied as a guideline for actual operation of the reservoir. Rules are also required to deal with extreme events, i.e. to pass peak flood or minimise damage caused by exceptional droughts.

Rule curves are often used in actual system operation (Loucks and Sigvaldason, 1982). They are commonly used in multipurpose reservoirs or single reservoirs used for hydropower or recreation. The operating policies and associated rule curves commonly define the desired storage volumes and releases at any time of the year as a function of existing storage volumes, the time of the year, demand for water or hydropower and possibly expected inflows (Loucks and Sigvaldason, 1982). According to Loucks and Sigvalddason (1982) operating policies may include one or more of the following:

a) Target storage levels or volumes.
b) Multiple zoning: Often operating rules include, in addition to target storage volumes, various storage allocation zones for conservation, flood control etc.
c) Flow range: Here releases are decided according to the zone in which the storage volume is.
d) Conditional Rule Curve: Conditional rule curves are defined for reservoir releases not only as a function of the existing storage and time of the year, but also as a function of the expected natural inflows.

2.1.4 Types of Reservoir Operation Studies

According to Yeh (1985) reservoir system studies are typically divided into planning and operation studies. Operation studies are further divided into short and long term studies. Kreuze (1986) classified reservoir operation studies into three classes and gave the following examples:

a) The first type aims at assessing the optimum output of a reservoir with long-term or seasonal storage. Determining monthly storages or releases is a typical example of this kind of studies which are called the strategic problems (Turgeon, 1981).

b) The second type aims at assessing the short-term, hourly, daily or weekly storage required for meeting fluctuations in the demand for water, e.g. a hydro-project needs to produce little power in the night and peak power in the morning and perhaps in the evening (Kreuze, 1986). These types of studies are called the tactical problems (Turgeon, 1981). The storage requirements for this purpose is superimposed on the first one, but in case of a reservoir with substantial seasonal storage, the storage requirements for the daily and weekly fluctuations in the demand are negligible. The storage capacity needed for this purpose is relatively so small, that it is called poundage rather than storage. In some cases poundage requirements are important and have to be considered in an early phase of project preparation. That is, in case of a so-called run-of-the-river project, in case of a pumped storage project and in case downstream conditions impose limits on the poundage operation.

c) The third type of studies aims at devising operation rules for the operation in times of extreme floods and extreme droughts.

2.1.5 Related Subjects to Reservoir Operation

Reservoir operation studies are closely related to and affected by the flow, water requirements, reservoir sedimentation and reservoir losses. Thus, these issues have to be studied when a reservoir operation problem is to be handled.

2.2 RESERVOIR SYSTEM SIMULATION AND OPTIMIZATION MODELLING

2.2.1 Simulation and Optimization Objectives

Reservoir-system-management and associated modelling, simulation and optimization, and analysis methods involve allocating storage capacity and stream flow between multiple uses and users in such a way that optimizes the use of water and minimises the risks and consequences of water shortage, flooding and adverse environmental impacts.

A simulation model is a representation of a system used to predict its behaviour under a given set of conditions. This representation is done on a computer, largely by a mathematical or algebraic description (Ackoff, 1961; Maass et al., 1962). In water resources, operation of the system is simulated period by period with known inflows, system characteristics and operating rules (Beard, 1972). Alternative runs of a simulation model are made to analyse the performance of the system under varying conditions, such as alternative operating rules. In 1953 US Army Corps of Engineers did an operational study for six reservoirs on the Missouri River. The result of that study was a simulation model, which is widely considered to be the first simulation model to appear in literature (Hall and Dracup, 1970).

The term optimization is commonly used in literature with mathematical programming to come up to a mathematical formulation in which a formal algorithm is used to minimise or maximise an objective function subject to constraints. Where simulation models are limited to predicting system performance for a user-specified set of variable values, optimization models automatically search for an optimum solution.

2.2.2 Considerations in Formulating Modelling Approach

Since each reservoir system is unique, several key factors are considered in formulating a modelling approach. Some of these considerations have been identified (Wurb, 1993) and will be used as a guide for modelling in this research.

a) Model Development Environment

A variety of models and general purpose commercial software are available. Therefore a choice has to be made between using or modifying an existing model or developing a new one using FORTRAN, C or a general purpose commercial software.

b) Availability and Operational Status of Generalised Models

If a model is selected for use, the degree to which it has been tested in actual reservoir/river system is an important consideration.

c) Interpolation and Communication of Results

For a model to be useful, results have to be displayed in a meaningful and understandable manner.

d) Reservoir Purposes

Reservoir purposes represent a key consideration in formulating a modelling approach. A distinction has to be made between flood control and conservation purposes, since:

1) Hydrologic analysis of flood are event oriented while it is long-term-time-series oriented for droughts.
2) Modelling flood wave-attenuation effects is important for flood control operations while other considerations such as evaporation are important for conservation operations.
3) Modelling flood control uses daily or hourly stream flow data while it is daily, weekly or monthly for conservation.
4) Simulation models are more suitable for flood-wave-attenuation modelling, while mass balance computations done in conservation operations can use either simulation or optimization.

e) Stream Flow Data

Modelling studies are based on historical gauged-stream flow data adjusted to represent past, present or future flow conditions at certain locations. Stream flow data are used, as an input to a model, in different ways:

1) Historical sequences of stream flow.
2) Synthetically generated stream flow sequences, which preserve selected statistical characteristics of the adjusted historical data.
3) Stream flow represented as probability of distribution.
4) Stochastic processes in various formats that capture the probabilistic characteristics of data. For example, the reservoir inflows in an explicit stochastic model may be represented by a transition-probability matrix that describes the discrete probability

of occurrence of a certain inflow depending on the occurrence of previous inflow.

$$Q = \begin{array}{c} \\ \text{current} \\ \text{state } i \\ \\ \end{array} \begin{array}{c} (0) \\ (1) \\ \\ (c) \end{array} \overset{\text{previous state, } j}{\overset{\begin{array}{ccc} (0) & (1) & (c) \end{array}}{\begin{bmatrix} q(0,0) & q(0,1) \ldots\ldots & q(0,c) \\ q(1,0) & q(1,1) \ldots\ldots & q(1,c) \\ & & \\ q(c,0) & q(c,1) \ldots\ldots & q(c,c) \end{bmatrix}}}$$

Q is the transition matrix and $q_t(i,j)$ is the conditional probability that the flow is in state i at time t, given that it was in state j at time t-1.

An example of this function has been given by Kottegoda (1980). The states of flow have been divided into 0 and 1 units in summer and 1 and 2 units in winter. For a given winter inflow, the conditional probabilities of inflows in the following summer are as in Table (2.1).

Table (2.1) Examples of conditional probabilities of inflow

Inflow in Following Summer	Conditional probability for	
	winter flow = 1	winter flow = 2
0	0.3	0.6
1	0.7	0.4

5) Synthesis of stream hydrograph from rainfall data using rainfall-runoff models.

f) System Representation

Simulation models represent operating rules in more detail, while optimization models compute the releases that optimize the objective function without tackling the details of operating rules.

g) Measures of System Performance

System performance can be measured by its yield or reliability. Some simulation and optimization models perform this kind of analysis. Economic-analysis models can be used to compare benefits and/or costs resulting from different operating plans.

h) Prescriptive Versus Descriptive Orientation

Descriptive models, such as simulation models, describe what will happen if a certain plan is implemented. Prescriptive models decide what plan is to be adopted to meet the decision criteria. Optimization models tend to be more prescriptive, since they exactly optimize the objective function.

l) Computational Algorithms

For the purpose of planning, construction and operation of reservoir system, simulation with deterministic stream flow sequence has been used. However researchers are enthusiastic about applying optimization and stochastic analysis to reservoir operation. Simulation models have the advantage of better representing the characteristics of the system as well as the operation rules. They also carry out the computations period by period, so that future releases are not affected by future stream flows.

The advantages of optimization models are that they allow more prescriptive analysis using a more systematic and efficient algorithm. However the following problems are encountered in optimization:

1) When operating the system, a release is made without knowing future stream flow.
2) General operating rules are needed more than computed releases corresponding to specified stream flow sequences. In optimization releases that maximise or minimise the objective function are computed. Some trials have been done, to relate dynamic programming results to the design of operating rules and to reflect the effect of uncertainty of not knowing future stream flows.

2.2.3 Studies Related to Reservoir Modelling

As outlined in Section (2.1.5) of this literature review, the reservoir operation studies are highly related to river flows, reservoir losses and reservoir sedimentation. Here different methods that can be used in modelling these issues are discussed.

2.2.3.1 River Flow Analysis

As mentioned in Section (2.2.2.e), stream flow data can be used as an input to a model in various forms. This can be a deterministic sequence of stream flow, as was used by Parikh (1966), who assumed complete knowledge of inflows. The flow can also be inputted explicitly in a form of a probability transition matrix (Section 2.2.2.e). Alternatively river flows can be modelled. The output for long-term flow modelling can take one of two possible forms (Hirsch, 1981): probability distributions for stream flow volume during some period or a set of plausible stream flow traces covering that period. The latter can take forms of models that preserve certain statistical properties.

a) Probability Distributions

This is a direct method which describes random variables by their probability distribution (Loucks et al., 1981). This would enable to cope with uncertainty and, may be, missing information. The normal distribution and its transformation, the lognormal distribution are the widely used distributions in engineering (Loucks et al., 1981). The density function of a normal variable is :

$$f_x(x) = [1/ (\sqrt{2\pi\sigma_x^2})]* \exp[-1/(2\sigma_x^2)*(x-\mu_x)^2] \quad -\infty < x < +\infty \qquad (2.1)$$

Where μ_x and σ_x^2, the mean and variance of the random variable x.

The normal distribution is symmetric about u_x and can have values from $-\infty$ to $+\infty$. This is not always suitable for modelling stream flow, since they are positive and skewed. A suitable transformation can be made for the skewed distribution. The frequently used transformations are the power and logarithmic transformations. If lnx is normally distributed, then the variable x is lognormally distributed and its density function is:

$$f_x(x) = [1/ (\sqrt{2\pi\sigma_x^2})]* \exp[-1/(2\sigma_x^2)*(\ln x-\mu_x)^2]* d(\ln x)/dx \quad 0 < x < +\infty \qquad (2.2)$$

Datta (1984) developed an optimization model in which he incorporated the probabilistic nature of flow by considering the distribution of actual stream flow.

b) Synthetic Stream flow Generation

Generated flows have been called synthetic or operational to distinguish them from historic flows. By generating a range of flow sequences that are likely to occur, the system design or policies can be tested better and understanding of the variability and range of possible future performances can be improved (Burges, 1979; Loucks et al., 1981). Two techniques are used for stream flow generation (Loucks et. al., 1981):

1) Fitting a statistical stream flow model to the historic flow. This requires the presence of a long historic record and the stream flow to be stationary, i.e. its parameters do not change with time. The fitted model can then be used in generating synthetic flows.

2) If the stream flows are not stationary, it will be assumed that rainfall is a stationary stochastic process. Out of which synthetic rainfall sequences may be generated and routed through a suitable rainfall-runoff model to produce sequences of stream flows.

Examples of the first category are the autoregressive models (AR) and autoregressive-moving average models (ARMA). They have been used extensively in hydrology and water resources (Salas et al., 1997). The mathematical formulation of AR models with constant parameters is:

$$y_t = \mu + \sum_{j=1}^{p} \phi_j (y_{(t-j)} - \mu) + \varepsilon_t \qquad (2.3)$$

Where :

y_t is the time dependent series

ε_t is the time independent (uncorrelated) series

ϕ_1, --------, ϕ_p are the autoregression coefficients

μ the mean of series y_t

Yang et al., (1995) fitted AR1 model for Bar-Sur-Seine, upstream Paris, and used the fitted model to generate flow sequences in a study to compare real time reservoir operation techniques.

2.2.3.2 Evaporation Losses

The rate of evaporation from a reservoir surface depends on a large number of factors such as solar energy, wind speed, air temperature, humidity, water temperature and the presence of floating vegetation in reservoir (e.g. water hyacinth). Usually this rate is estimated from pan evaporation data, but since the evaporation from a pan differs from that of a reservoir, a correction factor has to be applied. For a so-called class-A evaporation pan, this factor lies between 0.7 for deep reservoirs and 0.85 for shallow reservoirs(a few meters deep). In case the reservoir is covered with floating vegetation, the factor could increase to 1 for deep reservoirs and 1.15 for shallow reservoirs. These factors apply only to tropics where variations through the year are not large (Kreuze, 1986).

If no or only unreliable, as it is faced in this study, pan evaporation data are available the evaporation can be estimated on the basis of climatological data. Winter et al., (1995) evaluated the success of 11 equations for calculating evaporation from lake Williams in north central USA. It was found that Penman equation was among the best three equations that gave best results. Details of the Penman method will be given later in Chapter VI. Also evaporation losses can be estimated using water balance.

2.2.3.3 Seepage Losses

Seepage occurs mainly through the dam and the rims of the reservoir. Most reservoir banks are permeable, but the permeability and leakage are very low (Linsley and Franzini, 1972). This seepage should always be small because a reservoir with a high rate of seepage is dangerous. Therefore seepage losses can be neglected.

2.2.3.4 Loss of Storage due to Sedimentation

Every system carries some suspended sediment and moves larger solids along the stream bed as bed load. Since the specific gravity of soil material is about 2.65, the particles of suspended sediment tend to settle to the channel bottom but the upward currents in turbulent flow counteract the gravitational settling. When sediment laden water reaches a reservoir, the velocity and turbulence are greatly reduced. The larger suspended particles and most of the bed load are deposited as a delta at the head of the reservoir. Smaller particles remain in suspension longer and are deposited further down the reservoir, although the smallest particles may remain in suspension for a long time and some may pass with water discharged through sluiceways, turbines or spillway.

The suspended sediment concentration of streams is measured by sampling the water, filtering to remove the sediment, drying and weighing the filtered material. Sediment concentration is expressed in ppm, computed by dividing the weight of the sediment by the weight of sediment and water in the sample and multiplying the quotient by 10^6.

The relation between suspended sediment transport Q_s and stream flow Q is often represented by a logarithmic plot relation which may be expressed mathematically by an equation of the form:

$$Q_s = KQ^n \tag{2.4}$$

Where n commonly varies between 2 and 3, and K, the intercept when Q is unity, is usually quite small (Linsley and Franzini, 1972). A sediment-rating curve may be used to estimate suspended-sediment transport from the continuous record of stream flow in the same manner that the flow is estimated from the continuous stage record by use of a stage-discharge relation. Having upstream and downstream discharge and sediment rating curves, the amount of sediment entering, leaving and consequently sediment deposited in the reservoir can be estimated.

The volume of sediment trapped represents a loss of storage capacity, which reduces the efficiency of a reservoir to regulate the flow. Methods used to predict various aspects of reservoir sedimentation can be broadly divided into two classes: empirical methods that are founded on fairly correct understanding of the physical processes but are based on the inductive analysis of data and mathematical models that are based on an analytical treatment of hydraulic and sedimentary processes in reservoirs (Mahmood, 1987). Examples of the two classes are discussed in more detail in the following paragraphs.

a) Trap Efficiency of Reservoirs

Trap efficiency of reservoirs is defined as the proportion of incoming sediment load that is retained in the reservoir. Empirical methods to predict trap efficiency of reservoirs are represented by the graphical techniques developed by Churchill (1947), Brune (1953) and Heinemann (1981). Of these Brune's curve, is most popular in practice, mainly because it uses a rather simple and readily available predictor. The independent parameter in this method is the volume ratio of reservoir storage to annual water inflow and the dependent variable is trap efficiency. Brune median curve can be approximated by

$$T = 100(1-[1/\{222.92\log(V(H_m)/I)\}]) \tag{2.5}$$

Where T = Trap efficiency

$V(H_m)$ = reservoir capacity upto the mean operating level H_m.

I = Average annual flow

Both I and V are expressed in similar units of volume

This method, Churchill and Heineman curves, cannot be used for a duration less than a year (Mahmood, 1987).

For individual reservoirs, curves can be drawn. Trijylo (1977) carried out a study to find the trapping efficiency for the Highland Creek reservoir in USA. The trap efficiency is found by analysing data about sediment inflow and outflow and also by analysing reservoir survey and sediment inflow data. The computed trap efficiency by both methods were found to be 88% and 86% respectively.

b) Mathematical Models

In mathematical modelling the following equations are used (Mahmood, 1987):

The governing equations of motion:

$$\frac{\partial}{\partial t}(Q/(gA)) + \frac{\partial}{\partial x}(Q^2/(2gA^2) + y) + S_f = 0 \tag{2.6}$$

Equation of Continuity of Bed Materials:

$$\frac{\partial G_b}{\partial x} + \frac{\partial G_s}{\partial x} + \frac{\partial}{\partial t}(C_s A) + \rho_* \frac{\partial}{\partial t}(B_d Z) = 0 \qquad (2.7)$$

Where, Q = discharge; g = gravitational acceleration; A = area of x-section; y = water surface elevation; S_f = energy gradient; G_b = bed load; G_s = suspended load; C_s = average spatial sediment concentration in the cross-section; ρ_* = density of sediment in the bed; B_d = deformable bed width; Z = bed elevation; x = distance along the channel bed measured in the downstream direction and, t = time.

The above two equations require two supplementary equations. One relating S_f and the other relating transport quantities: G_b, G_s, and C_s to the flow and sediment size values. They also require the initial conditions and boundary conditions to be specified. In reservoir sedimentation, the accuracy of initial conditions is not very critical because they are overtaken by the deposition processes. At the downstream end, hydrograph of reservoir pool elevation provides boundary conditions and at the upstream end, the discharge and sediment inflow hydrograghs provide the necessary boundary conditions. The model results are very sensitive to the sediment inflow boundary condition and to the accuracy of supplementary equations used to compute sediment transport quantities. The above equations constitute a one-dimensional representation of sediment transients. They can be solved by one of the finite difference scheme (Mahmood, 1987).

Here are some examples of simulation of fine grained sediment transport model:

Ziegler and Nisbet (1995) carried out a 30-year simulation of cohesive sediment in Watts Bar reservoir, located in Tennesse. They concluded that the sediment model (SEDZL) can be successfully used to simulate the fine sediment transport. This model, (Ziegler and Nisbet, 1995), was successfully used in a number of aquatic systems including the Fox River, in Wisconsin (Gailani et al., 1991), the Pawtuxet River, in Rhode Island (Ziegler and Nisbet, 1994) and Lake Erie (Lich et al., 1994). This sedimentation model was used in combination with a well-established hydrodynamic model called ECOM (Blumberg, 1994). The results from the hydrodynamic model provide information about the transport field, horizontal and vertical velocities and water depth.

c) Alternative Approach

Storage volume of a reservoir, S, is a function of both the reservoir elevation, H, and time. The storage volume can be approximated by the following function (Yevdjevich 1965):

$$S = a\,H^{m} \tag{2.8}$$

With $a = \Psi(t)$ and $m = f(t)$, which are functions of time. This time function is a result of sedimentation process which is a function of time.
If surveys are carried out, their results can be used to fix a relation between S & H and obtain values for "a" and "m". These obtained values can be used to fit the relations $a = \Psi(t)$ and $m = f(t)$. These functions can be used to find the values of "a" and "m" at any time through the reservoir course of operation and hence the storage-water level relationship.

2.3 SIMULATION OF RESERVOIR SYSTEM

As outlined in Section (2.2.2) of this literature review, the simulation models represent the operation rules in more detail compared to the optimization models. They are usually used for testing the operation rules derived from the optimization results. Therefore a description of simulation basic equations and examples of simulation models are given hereafter.

2.3.1 Basic Simulation Equations

The basic equations in a reservoir operation study are the continuity equation (mass balance) and the reservoir state equation (Kreuze, 1986).
The mass balance equation states that for a time period the inflow minus the outflow equals the increase in storage, Ds, or:

$$\text{Inflow} - \text{Outflow} = Ds \tag{2.9}$$

The reservoir state equation states that the storage at the end of a time period, Ste, is equal to the storage at the beginning of the time period, Stb, plus the increase in storage, Ds.

$$Ste = Stb + Ds \tag{2.10}$$

The outflow consists of useful outflow (irrigation, hydropower) + spill + losses. With a given reservoir size the benefits from the useful outflow is to be maximised and in each period a decision is to be made to divide the outflow between useful outflow and spill. If the outflow serves more than one purpose, rules have to be devised to maximise the benefits. If the inflow is modified by an upstream reservoir, the inflow to the downstream reservoir becomes subject to the operation of the upstream reservoir and to formulate an optimum operation, both reservoirs have to be considered simultaneously.

2.3.2 Time Period for Simulation Analysis

In reservoir operation studies continuity equation has to be solved for every time period. As flood control models use an hourly or daily time intervals, conservation models use a daily, weekly or monthly time interval. The larger time interval of a week and month are more appropriate for planning purposes (Wurbs, 1993). For conservation models, Kreuze (1986) recommended to start with a period of month. If that interval is not adequate, a more detailed study can be made for months which are critical for a certain purpose.

2.3.3 Examples of Simulation Models

HEC-5 Simulation of flood control and conservation systems; is one of the more widely used reservoir-system simulation model (Mays and Tung, 1992). It was developed by Hydrologic Engineering Centre (Yeh, 1985). It has been used in studies of both proposed new projects and operational modifications of existing reservoirs. It is also used to support real-time operations.

HEC-3 was developed also by Hydrologic Engineering Centre. It can be used for reservoir-system analysis for conservation, but it does not have the comprehensive flood control capabilities of HEC-5 (Wurbs, 1993).

SWD Model Hula (1981) described SWD model that simulates the daily sequential regulations of a multiple reservoir system, performing generally the same types of hydrologic and economic simulations as HEC-5.

MITSIM Strzepek et al., (1989) provides the capability to evaluate the economic as well as hydrologic performance of a river-basin system involving hydroelectric power, irrigation and municipal and industrial water supply.

2.4 OPTIMIZATION OF RESERVOIR SYSTEMS

Optimization models are formulated to determine values for a set of decision variables that will maximise or minimise an objective function subject to constraints. The objective function and constraints are represented by mathematical expressions as a function of decision variables. For a reservoir operation problem, the decision variables, are typically release rate and end of period storage volumes. Constraints typically include storage capacities and other physical characteristics of the reservoir/stream system, diversion or stream flow requirements for various purposes and mass balance (Yeh, 1985).

2.4.1 Objectives and Objective Functions

The objective function is the heart of an optimization model. The objective function may be a penalty or a utility function. The following objectives have been reflected in the objective functions of various optimization models reported in the literature reviewed.

* Minimise pumping cost.
* Maximise hydropower production to increase correlation with demand.
* Maximise the return of a multipurpose multiple reservoir system.
* To optimise crop yield response to water deficit.
* To find least cost withdrawal and release pattern for water supply.

* Maximise long term yield.
* Maximise the benefits from hydropower generated from excess water for export.
* Minimise losses resulting from floods.
* Minimise losses resulting from droughts.
* Minimise the loss of potential energy
* Maximise the daily power output.
* Minimise economic losses resulting from not meeting a specified target.
* Minimise the purchase cost of imported power.
* Minimise shortage frequencies and/or volumes.
* Minimise shortage indices, such as the sum of the squared deviations between target and actual diversion.
* Maximise the minimum stream flow.
* Maximise reservoir storage at the end of the optimization horizon.
* Minimise spill and evaporation losses.
* Minimise average monthly storage fluctuations.
* Maximise the length of navigation season.
* Maximise firm energy
* Maximise average annual energy
* Maximise the potential energy of water stored in the system

Although several different objectives will typically be of concern in a particular reservoir system analysis study, an optimization model can normally incorporate only one objective function (Kottegoda, 1980). Multiple objectives can be combined in a single function if expressed in similar units; such as pounds. However objectives are often not in similar units. Two alternative approaches are typically adopted to analyse trade-offs between objectives (Kottegoda, 1980). One approach is to execute the optimization model with one selected objective reflected in the objective function and the other objectives are treated as constraints. For example, the model might maximise average annual energy, subject to the constraints that a specified water-supply be maintained. Alternative runs of the model could be made to show how the average annual energy is affected by changes in the user-specified water supply. In many models the benefits and losses are included in the objective function while the risk is considered as a constraint (Van-On and Helweg, 1988). An alternative approach for analysing trade-offs between similar objectives involve treating each objective as a weighted component of the objective function. The objective function is the sum of each component multiplied by a weighting factor reflecting the relative importance of that objective. The weighting factors can be arbitrary, with no physical significance other than to reflect relative weight assigned to the alternative objectives included in the objective function. The model can be executed iteratively with different sets of weighting-factor values to analyse the trade-off between the objectives with alternative operating plans.

The main three mathematical optimization models that have been applied to reservoir systems are (Lobbrecht, 1997):

* Linear Programming (LP), using a linear model.
* Dynamic Programming (DP), using a recursive model.
* Non-linear Programming (NLP), using a non-linear model.

The word programming means the selection of an optimum allocation of resources after initial description and specification. The characteristics of each method, its application in water resources, its advantages and disadvantages will be discussed:

2.4.2 Linear Programming (LP)

2.4.2.1 LP Characteristics

a) Deterministic Linear Programming

Linear programming is a relatively unsophisticated technique of system engineering. In LP the objective function to be optimised is of the form

$$U = AX \qquad (2.11)$$

Where $X = [x_1 x_2 \ldots\ldots x_l]^T$ is a vector of variables denoting, for example, units of water supplied for domestic uses, irrigation,............etc.

$A = [a_1 a_2 \ldots\ldots a_l]$ is a vector of constants representing, say, returns per unit of water released and T denotes transpose.

The optimization is subject to a set of m constraints

$$GX \geq H$$

in which the sign of inequality may be reversed and where

$$G = \begin{bmatrix} g_{11} \ldots\ldots\ldots g_{1l} \\ \\ g_{m1} \ldots\ldots\ldots g_{ml} \end{bmatrix}$$

and, $H = [h_1\ h_2\ h_3 \ldots\ldots\ldots h_m]^T$

Also there are generally, l non - negative constraints:

$$X \geq 0$$

If the function is non-linear and the linearisation is done in stages and at each stage the optimization model is solved, then this process is called successive linear optimization, SLP. Better results could be obtained if the output from each iteration is used as an input to the next iteration (Lobbrecht, 1997).

b) Stochastic Linear Programming

Deterministic models do not account for the uncertainties in flows. Uncertainties of some parameters may be dealt with through sensitivity analysis, but still this procedure does not explicitly consider the uncertainty and may not lead to satisfactory results (Yeh, 1985). The main task under uncertainty is to derive a deterministic equivalent of the stochastic program. If this step is successful, then a standard optimization solution procedure can be used (Yeh, 1985). Some of the LP stochastic procedures are discussed here.

b.1) Stochastic LP for Markov Process

The basic components of the Markov process are the "State" and the "Transition" attained by a "Decision". For reservoir operation the inflow and/or the storage at the beginning of each time period are the state. The release made in each time period is the decision and the system makes a transition from one state to another in successive time periods (Yeh, 1985). Moran's theory of reservoirs is an application of Markov process. Moran's assumed discrete time units, discrete series of inflows which are not serially correlated and neglected losses (Kottegoda, 1980). Annual river flow data is generally appropriate for these assumptions, since they are independent.

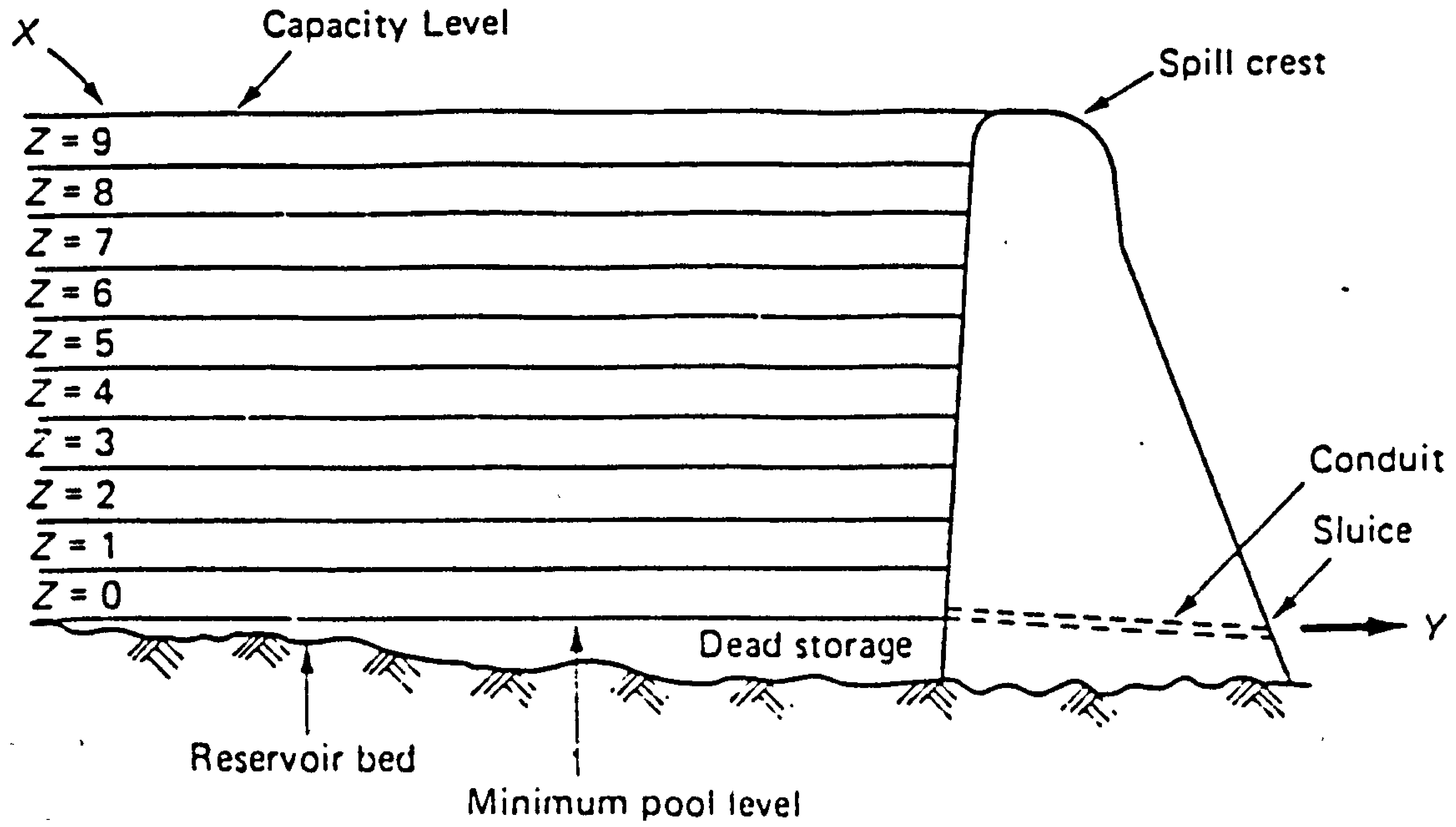


Figure (2.1) Application of Markov process to reservoirs (Kottegoda, 1980)

Z is finite reservoir state. A Z value denotes a particular reservoir state or storage between two limits. Usually volumetric increments or differences with respect to Z = 1,2,......................., c-1 are equal. When Z= 0 reservoir is empty and it is full when Z = c (c = 9 in Figure (2.1)).

X_t is inflow

Y_t outflow

$Z_t = Z_{t-1} + X_t - Y_t$ subject to $0 \leq Z_t \leq C$

The dam content process is now a Markov chain with the states Z = 1,..........,c and with a transition probability matrix (tpm).

Probability that the reservoir is in state i at time t, given it was in state j at time t-1 is denoted by:

$q_t(i,j) = pr(Z_t = i \mid Z_{t-1} = j)$

The collection of the one step transition probabilities q(i,j) forms the probability transition matrix Q.

$$
Q = \begin{array}{c} \text{current} \\ \text{state} \\ i \end{array}
\begin{array}{c} \\ (0) \\ (1) \\ \vdots \\ (c) \end{array}
\overset{\text{previous state } j}{\overset{(0)\quad (1) \ldots\ldots\ldots (c)}{\begin{bmatrix} q(0,0) & \ldots\ldots & q(0.c) \\ & & \\ \vdots & & \vdots \\ q(c,0). & \ldots\ldots & q(c,c) \end{bmatrix}}}
$$

Where $0 \le q(i,j) \le 1$, for $i,j = 0,1,2\ldots\ldots\ldots c$.

$$\sum_i q(i,j) = 1$$

Lloyd (1963) extended Moran's theory to treat serially correlated inflows, such as monthly inflows. Instead of writing the probability of transition from state Z_{t-1} to state Z_t, probabilities of transition from state (Z_{t-1},X_{t-1}) to (Z_t, X_t) are used.

Manne (1962) demonstrated the application of LP model for a single reservoir. Thomas and Watermyer (1962) extended Manne's work by defining the initial state of the system as both inflow and storage rather than storage only. Loucks (1968) applied this approach by estimating the transition probabilities of inflows from historic record.

b.2) Stochastic Programming with Recourse

In stochastic programming with recourse, the decision is made in, at least, two stages. A computational procedure was presented for a two stage LP model by Dantzig (1955). In this method, the activity levels are determined in the first stage, then a corrective action is followed in the second stage. Essentially the problem considered is to find the optimum solution of the vector X, in the following program (Wets, 1966; Prekopa, 1980):

$$\min\{CX+E[\min_Y(gY)]\} \tag{2.12}$$

Subject to

$$AX = b \tag{2.13}$$

$$TX+WY = p \tag{2.14}$$

$$X \ge 0 \ , \quad Y \ge 0 \tag{2.15}$$

E{ } is the expectation with respect to the random vector p .

A is a m*n deterministic matrix.

T is a k*n matrix.

W is k*j matrix

b,c and g are m*1, 1*n and 1*j vectors respectively;

X and Y are n*1 and j*1 vectors of decision variables;

p is a k*1 vector.

Elements of T, W and p can be random variables.

First a decision X_{feas} (feasible) is chosen (first stage) to satisfy;

$$AX_{feas} = b, \quad X \in K \tag{2.16}$$

in which K is the set of X vectors for which there is at least one Y for which (2.14) is satisfied for whatever p is realised after the decision for X is made.

Each X will lead to some cost CX, and when the random event p is observed with certainty, the chosen X will finally lead to a recourse action Y (second stage) such that

$$\min_{Y} gY \tag{2.17}$$

subject to

$$WY = p - TX_{feas} \qquad Y \geq 0 \tag{2.18}$$

At stage one a feasible solution, X_{feas}, which also feasible for stage two, is found. At stage two, the problem is solved when the random event, p, occurs. In water resources the second stage decisions may be represented by the loss of not meeting supply targets (Dorfman, 1962). It is easier to solve linear simple recourse problems with random variables being discrete, with uniform or normal distributions (Yeh, 1985).

b.3) Chance Constrained Linear Programming

This stochastic programming model is attractive to practitioners (Yeh, 1985). It reflects the probability condition on the constraints. Chance-constrained formulation can be expressed in this form of stochastic programming:

$$\min_{X} CX$$

Subject to

$$AX = b, \quad \Pr\{ TX \geq p \} \geq \alpha \quad , X \geq 0$$

where

Pr{ } denotes probability.

α is a m*1 constant vector with components α_i ($0 \leq \alpha_i \leq 1$).

C is a 1*n vector

X is a n*1 vector of decision variables

A is a m*n deterministic matrix

b is a m*1 vector

T is a k*n matrix

p is a k*1 vector (element of T and p can be random variables).

The idea of chance-constrained for LP optimization was first used, by Charnes et al., (1958), for determining refinery rates for heating oils to meet stochastic weather dependent demands. Revelle et al., (1969) first used chance-constrained LP for reservoir system optimization.

Consider the constraints $\Pr\{TX \geq p\} \geq \alpha$, α scalar, T deterministic and $0 \leq \alpha \leq 1$.

If the probability distribution function of the random variable p is known, the above probabilistic constraint can be converted to a deterministic equivalent using the commulative probability distribution function of the random variable p, F_p.

$$\Pr\{ p \leq TX \} \geq \alpha$$

$$F_p(TX) \geq \alpha$$

The resulting deterministic equivalent is

$$T(X) \geq F_p^{-1}(\alpha)$$

Where $F_P^{-1}(\alpha)$ is the inverse of the commulative probability function at the given value of α. If α is chosen to be 0.9, for example, then there will be 0.1 or 10 % probability that the constraint represented by $\Pr\{TX \geq p\} \geq \alpha$ will not be met.

Chance-constrained formulation neither penalise the constraints violation nor provide recourse action to correct realised constraint violation as a penalty (Yeh, 1985).

b.4) Linear Decision Rules (LDR)

Linear decision rules relate releases to storage, inflow and decision parameters. Revelle et al., (1969) first proposed this original LDR for reservoir design and/or operation:

$$R_t = S_{t-1} - b_t \tag{2.19}$$

Where R_t release during time period t.

S_{t-1} storage at end of time period t-1.

b_t Decision parameter to be determined by the model

This rule has the advantages that the release is determined at the beginning of the time period and it eliminates mathematical difficulties in formulating chance constraints (Sigvaldason, 1976). There are basic limitations of the rule. First, it yields conservative results. Possible explanation for this is that the rule may not take into account the complete nature of stream flow stochastically or the LDR is an additional constraint. Second limitation is that the solution from a LDR model is not guaranteed to be optimal since it reduces the number of possible operating policies and each flow in each period is considered critical (Loucks and Dorfman, 1975).

Loucks (1970) proposed the following LDR that he termed "Linear release rule"

$$R_t = S_{t-1} + I_t - b_t \qquad (2.19)$$

Where, I_t is the inflow during time period t.

He found that this rule had resulted in less conservative results than the original LDR. Here releases are not decided at the beginning of the period, but the release is adapted to match a value specified by the end of the period. Revelle and Kirby (1970) modified the original LDR to include evaporation losses using linearised storage-area curves and projected storage. Revelle and Gundelach (1975) proposed the following LDR to incorporate the stochastic nature of inflows:

$$R_t = S_{t-1} + \beta_t I_t - \beta_{t-1} I_{t-1} - \dots\dots\dots\dots - \beta_{t-k} I_{t-k} - \dots\dots + b_t. \qquad (2.21)$$

$$0 \le \beta_i \le 1 \quad , \quad i = 0, \dots\dots\dots\dots, t$$

Where I_t, I_{t-1}, I_{t-k} are the inflows during time period t, t-1 and t-k respectively.

A prior knowledge of current inflow and nonzero values of β_t are required when using this rule. By approximate choice of the parameter, Revelle and Gundelach (1975), were able to achieve a smaller release variance with this rule but it required slightly larger reservoir capacity when compared to the original LDR. It also presented mathematical complexities.

2.4.2.2 Application of Linear Programming Methods to Reservoir Systems

In this section applications of different types of Linear Programming are reviewed and shown in Table (2.2).

Table (2.2) Application of Linear Programming methods to reservoir systems

No	Researcher & year	Techniques Applied	Objectives	Place of application	Comments
1	Young (1968)	Linear Programming	Efficient use of a flood control reservoir to include water quality	-	Rule curves which related optimal releases to the amount of available water were produced
2	Houck and Cohon (1978)	Linear Programming	Design & management of multipurpose reservoir system.	two reservoirs	-
3	Houck (1979)	Linear Decision Rule (LDR)	To make LDR less conservative	Theoretical	The model incorporated explicitly the stochastic nature of the stream flow.
4	Diacon et al., (1981)	Linear Programming	To maximise the value of net energy production.	-	The model considered the variation of hydraulic efficiency with actual water levels in reservoirs.
5	Marino and Simonovic (1981)	Chance – constrained	To resolve a reservoir sizing problem	Theoretical	Minimum reservoir volumes are considered as variables in the continuity equation.
6	Shane and Gilbert (Hydrosim) (1982)	Linear Programming	Compute storages, releases and hydropower generation for a 52 week period.	42 reservoirs on Tennesse Valley, USA	A search procedure was used to handle non-linear hydropower production function.
7	Datta (1984)	Chance – constrained	To have an operational model that can be updated daily.	-	The probabilistic nature of real time forecast was incorporated by considering the distribution of actual stream flow volumes.

Application of Linear Programming,, Table (2.2) continued

No	Researcher & year	Techniques Applied	Objectives	Place of application	Comments
8	Stedinger (1984)	LDR	Reservoir screening	Theoretical	LDR models are of questionable value for reservoir screening.
9	Martin (1987) (Monitor 1)	Linear Programming	To maximise net economic benefits from a system operated for hydropower, water supply & low flow augmentation.	A system of surface water storage and conveyance system.	Decision variables were daily reservoir releases, water diversion and pipeline and canal flow.
10	Palmer and Holmes (1988)	Linear Programming	Maximise yield and minimise economic loss resulting from not meeting a specified target.	Seatle water, USA	-
11	Crawley and Dandy (1993)	Linear Programming	To maximise yield, water supply and minimising pumping cost, while maintaining satisfactory system reliability.	10 reservoirs & major supply pump line on Murray River, Australia.	-
12	Martin (1995)	Linear Programming	To maximise hydropower generating capacity without adversely affecting water supplies or lake storage levels.	Highlands Lakes	-

2.4.2.3 Advantage & Disadvantage of Linear Programming

In linear programming there are readily available solvers. Also linear programming has the advantage of having low dimensionality compared to NLP. Therefore large problems can be solved using LP. But reservoir optimization problems are non-linear when issues like hydropower and evaporation are modelled. To apply linear programming the non-linear functions should be linearised. The linearisation is usually carried out successively, as in successive linear programming SLP. In SLP the highly non-linear functions have to be linearised and the optimization problem solved in each iteration. This makes the process time consuming.
Also the linearisation of the objective function and/or the constraints may cause accuracy problems when modelling a high non-linear function such as hydropower. Therefore linear programming is less attractive, compared to non-linear programming, when modelling systems with high non-linear functions such as hydropower and evaporation and accuracy is required.

2.4.3 Dynamic Programming (DP)

2.4.3.1 Characteristics of DP

a) Deterministic DP

Dynamic programming, DP, is a method developed by Bellman (1957). DP is used to optimize multistage problems. DP has the ability to accommodate the nonlinearity of the optimized function. Also it can account for the uncertainty in flows (Yeh, 1985). When the returns are independent and additive then the recurrence function that maximises the net benefits of a single reservoir is (Loucks, 1981):

$$\text{Maximise} \sum_{t=1}^{T} NB_t\,(S_t, S_{t+1}, R_t) \tag{2.22}$$

Subject to $S_{t+1} = S_t + I_t - R_t$ for each period t.

$S_t \leq K$ for each period t.

Where S_t and S_{t+1} are initial and end of period storage volumes.

I_t and R_t are inflows and releases during period t.

K reservoir capacity.

T number of periods within a year.

Such a problem can be seen as a multistage decision process. The stages are the number of the periods and the states are the storage volumes. Moving backward in time, the general recursive equation for each period t with n stages (n>1), is:

$$f^{n}_t(S_t) = \max\ [NB_t(S_t,\ S_t + I_t - R_t,\ R_t) + f^{n-1}_{t+1}(S_t+I_t - R_t)] \quad (2.23)$$

Subject to $R_t \geq 0$

$R_t \leq S_t + I_t$

Where $S_t + I_t - R_t \leq K$

b) Stochastic Dynamic Programming (SDP)

Stochastic dynamic programming can accommodate the stochastic nature of inflow explicitly. Howard (1960) introduced the idea of returns based on the probability transition matrix. The probability transition matrix concept was discussed in Section (2.2.2.e) of this literature review. A stochastic dynamic programming applied on a reservoir is of the following form:

$$f_t(S_t,\ I_{t+1}) = \max_{Rt}\ \{\sum_{I_t=0}^{I_{tmax}} P[I_t|I_{t+1}][B(R_t) + f_{t-1}\ ]\} \quad (2.24)$$

Subject to

$$S_{t+1} = S_t + I_t - R_t - E_t$$

$$f_1\ (S_t\ ,\ I_{t+1}) = \max_{R1}\ \{\sum_{I_t=0}^{I_{1,max}} P[I_t|I_{t+1}][B(R_1)\ ]$$

Where

$f_t(S_t,\ I_{t+1})$ return obtained from operating the system optimally.

S_t and I_t storage at the beginning of and inflow during period t respectively.

$B(R_t)$ return obtained from release R_t during period t. Sometimes B may be expressed in terms of S_t as in case of power generation.

$P[I_t|I_{t+1}]$ transition probability function in which probability of inflow in period t, subject to inflow in period t+1, is found.

E_t evaporation losses in period t.

t time period index, can be months.

c) Incremental DP (IDP) and Discrete Differential DP(DDDP)

In IDP an initial feasible state trajectory is assumed which represents an initial policy and an initial value of the objective function. The DP recursive equation is used to test the states which are just above and below the assumed trajectory. If a better value of the objective function is obtained, then the first trajectory is replaced by the new one. The process is continued until convergence is obtained, i.e. when no better value of the objective function is obtained (Yeh, 1985). DDDP is a generalisation of IDP (Nopmongcal and Askew, 1976). IDP or DDDP reduces the dimensionality.

d) Incremental DP with Successive Approximation (IDPSA)

This is a technique used for curse of dimensionality alleviation. In IDPSA a multiple – state variable dynamic programming is decomposed into a series of subproblems. Each subproblem has only one state variable. The solutions of the subproblems should converge to the solution of the original problem (Yeh, 1985)

e) Reliability – Constrained or Chance Constrained DP (CCDP)

In long term reservoir operation, trade-off between returns and associated risks should be considered. In DP these considerations are formulated as a probabilistic DP with discount. The probabilistic DP recursive function is:

$$f_t(S_t) = \max_{R_t} \sum_{I_t} P(I_t) \{B(R_t) + r\, f_{t-1}(S_{t-1})\} \qquad (2.25)$$

Where:

S_t is the reservoir initial storage for period t.

$r = (i+1)^{-1}$ is the discount factor with a discount rate (i)

$B(R_t)$ is the benefit associated with release R_t.

$P(I_t)$ is the probability of (discrete) inflow I_t .

f) Differential Dynamic Programming (DDP)

For certain DP problems, as in DDP, use can be made of special properties of the objective function and constraints. Problems in DP which have linear constraints and quadratic objective function are known as LQP problems. If the constraints are linear and the objective function is quadratic, separable and convex (for minimisation), then the decision is a linear function of the current state (Yeh, 1985). Recursive formulas can be derived to obtain coefficients of the linear decision.

g) Progressive Optimality

The principle of progressive optimality can better be explained by an example given by Turgeon (1981). In progressive optimality state variables do not have to be discretized. The algorithm considered the tactical power generation of a system of hydropower plants in series. The aim was to determine the discharges u_i^k from the reservoir i in period k, where k = 1, ...,K and i = 1,................., n.

$$\min \sum_{k=1}^{K} C(S^k) \tag{2.26}$$

Subject to $x_i^k = x_i^{k-1} + q_i^k - u_i^k$: $x_i^0 = a_i^0$: $x_i^K = a_i^K$: $\sum_i H_i(x_i^{k-1}, u_i^k) + S^k = D^4$

$0 \leq x_i^{k-1} \leq x_i'$: $0 \leq u_i^k$

Where:

q_i^k is the total inflow to reservoir i in period k.

x_i^k is the content of reservoir i at end of period k, capacity x_i'

$H_i(x_i^{k-1}, u_i^k)$ power generated from plant i in period k.

S^k is the energy imported with some cost C.

D^k is the demand for energy in period k.

$C(S^k)$ production cost in period k.

a_i^0 and a_i^k initial and final contents of the reservoir i, respectively.

2.4.3.2 Application of Dynamic Programming Techniques to Reservoir Systems

Here different applications of DP are reviewed (Table 2.3).

Table (2.3) Application of Dynamic Programming techniques to reservoir systems

No	Researcher & year	Techniques Applied	Objectives	Place of application	Comments
1	Butcher and Fordham (1970)	SDP	To optimize a multipurpose single reservoir	-	-
2	Meredith (1975)	DDDP	Maximise the return of a multipurpose multiple reservoir system.	-	the stages were represented by reservoirs and state variables were represented by releases.
3	Collins (1977)	DP	To find least cost withdrawal & release patterns for water supply.	Four reservoir system, Dalas, USA	Applications were made to find least-cost operating patterns. By inclusion of a water loss penalty function, supply patterns that reduce evaporation losses were found.
4	Sargent (1979)	DP	To determine optimal reservoir releases during droughts.		DP was used in conjunction with generated streamflow. it proved to be very effective & simple in producing near optimal losses.
5	Young Moore and Yeh (1980)	DP	Economic evaluation of different alternative reservoirs for water supply	A project in Northern California	some aspects known by economists and not often used in water resources optimization, such as sensitivity of demand to prices and a willingness to pay concept were used.
6	Stedinger et al. (1984)	SDP	To define reservoir release policy and calculate expected benefits of future operations.	Aswan High Dam on River Nile	Use of a better hydrologic state variable would improve results of optimization models.

Application of Dynamic Programming techniques to reservoir systems, Table (2.3) continued

No	Researcher and year	Techniques Applied	Objectives	Place of application	Comments
7	Allen and Bridgeman (1986)	DP	Optimal monthly hydropower scheduling to minimise the purchase of imported power	-	-
8	Mereuta and Paduroiu (1986)	Progressive optimality	Correlation between demands and maximal production of hydroelectric energy.	theoretical	an improvement over simulation models is obtained.
9	Trezos and Yeh (1987)	SDP	Increase hydropower produced	case study	probabilistic forecast was used
10	Frang et al. (1989)	DP	To optimize the crop yield to water deficit	-	The reservoir was originally for irrigation with the possibility of taking hydropower and water supply as secondary objectives.
11	Kelman et al. (1990)	Implicit Stochastic Dynamic Programming	Optimization of a reservoir operation for hydropower	complex hydro-electric system on the Feather River, California, USA	Implicit stochastic dynamic programming generated efficient operating policy with less time and effort than rule curve methods.
12	Kuo et al. (1990)	DDDP	To determine optimal joint operational policy for two reservoirs	Shihman & Feitsui reservoirs in the Tanshu river basin, Taiwan	10 day operating rules were used as boundaries for daily or hourly operations.
13	Vedula and Mohan (1990)	SDP	Trade-off between irrigation, the primary function, and hydropower	Bhadra reservoir system, India	1.6 to 3.3 % irrigation shortages increase power production by 52 - 57 %

Application of Dynamic Programming techniques to reservoir systems, Table (2.3) continued

No	Researcher and year	Techniques Applied	Objectives	Place of application	Comments
14	Braga et al. (1991)	DP&SDP	To optimize hydropower production for a multiple reservoir system	Brazil	The deterministic model was used for calculating potential power while SDP was used for real time operation.
15	Piccardi and Soncini-Sess (1991)	SDP	To use advances in computers to increase accuracy of DP.	Theoretical	In DP computations were simplified at the expense of accuracy, i.e. inflow correlation was neglected & state variables were coarsely discretized. More reliable solution can be obtained by making discretization more finer and considering the flow correlation since correlated and uncorrelated inflows gave different results in situations where correlation occurred.
16	Karamouz and Vasiliadis (1992)	SDP	To reduce the effect of forecast uncertainty in reservoir operation	Loch Raven reservoir on Gunpowder River, Meryland, USA	-
17	Nivivattanon et al. (1996)	Progressive optimality	Minimise pumping costs	City of Pittsburgh's water supply network, Thailand	Model used decomposition in space and time and discretized pump discharges.

2.4.3.3 Advantages and Disadvantages of Dynamic Programming

Dynamic programming is the widely applied technique for reservoir optimization. It has the advantage of accommodating non-linear objective and constraint functions. However the solution process is very slow and a dynamic programming problem is difficult to understand and lacks a standard mathematical formulation. Therefore no general DP solver has been developed.

Dimensionality is the most significant disadvantage of dynamic programming. To reduce the dimensionality and improve the performance of dynamic programming, discretization is widely applied. Discretization reduces the dimensionality effect at the expense of accuracy. Therefore, as linear programming, dynamic programming is less attractive when accuracy is required.

2.4.4 Non-linear Programming (NLP)

2.4.4.1 NLP Characteristics

A general non-linear programming, NLP, problem can be defined as follows:

$$
\begin{aligned}
&\text{minimise: } f_i(x),\ i = 1, \ldots\ldots, n \\
&\text{subject to } g_i(x) \leq 0,\ i = 1, \ldots\ldots, l \\
&\qquad h_i(x) = 0,\ i = 1, \ldots\ldots, m \\
&\qquad x \in R^n
\end{aligned}
$$

f(x), g(x) and h(x) are the objective function, inequality and equality constraints respectively. The methods of solving the constrained and unconstrained non-linear programming problems will be dealt with in the next chapter.

2.4.4.2 Application of NLP to Reservoir Systems

In this section some NLP applications are reviewed (Table 2.4).

Table (2.4) Application of NLP to reservoir systems

No	Researcher and year	Techniques applied	Objectives	Place of application	Comments
1	Lee and Waziruddin (1970)	Gradient projection method	To maximise irrigation releases and reservoir storages.	Hypothetical reservoir system containing three reservoirs in series.	The objective function was non-linear while the constraints are linear. The method used has very low convergence.
2	Simonovic and Marino (1980)	Gradient projection method	To improve reliability.	a single reservoir	The authors included both benefits and risks in the objective function and considered random inflows and demands.
3	Lefkoff and Kendall. (1996)	Projected augmented Lagragian algorithm	Maximisation of long term yield	California state water project and the Central Valley project; USA	The algorithm requires that all non-linear functions be continuous & continuously differentiable; a 3 month time step was used.

2.4.4.3 Advantages and Disadvantages of NLP

In non-linear programming, the nonlinearity can be modelled more accurately than in linear programming and dynamic programming. Despite this its application to reservoir systems remains very limited for the following reasons:

1) The complexity of the optimization problem.
2) The large order of dimensionality and the long time taken to solve a problem which may limit its size.
3) The convergence process is slow and much computer memory is needed.
4) There is a need to find a feasible point in the solution space as a starting point in most of NLP algorithms.

However, and due to development in computers, research in mathematical programming has addressed some of these difficulties in applying non-linear programming techniques to large scale non-linear systems. These developments and the fact that non-linear programming can model the nonlinearity more accurately than linear programming and dynamic programming have made the application of non-linear programming techniques to reservoir systems more attractive than the application of linear and dynamic programming.

Lobbrecht (1997) summarised some of the characteristics of the mathematical optimization methods (Table 2.5).

Table (2.5) Comparison of characteristics of optimization methods

Technique	Solvable problem size	Solution speed	Model complexity	Solvers available
LP	Large	fast	moderate	yes
SLP	Large	moderate to fast	moderate	yes
DP	Very small	very slow	high to very high	no
NLP	moderate	slow	high to very high	yes

2.4.5 Combined Techniques

Some of the models combined the above described techniques. Table (2.6) summarises some of them.

Table (2.6) Application of combined techniques

no	Researcher & year	Techniques applied	Objectives	Place of application	Comments
1	Yeh (1981)	LP-DP	To determine multiple reservoir release schedules that minimises loss of potential energy	California Central Valley Project	An LP-DP model was used to compute monthly releases that minimises the loss of stored power. An LP model was used to find daily releases that minimises loss of potential energy. An LP-DP model was used to determine hourly releases the maximise power output.
2	Karamouz and Houck (1982)	DP; Regression & Simulation	To generate operating rules to minimise loss resulting from both floods & droughts	Applied on 12 cases to derive annual operating rules & 36 cases to derive monthly ones	DP was used to derive the optimum policy. Regression was used to derive operating rules and simulation was used to verify these rules.
3	Stedinger et al. (1983)	LDR; Chance constrained	Preliminary assessment of the cost-effectiveness of different multiple reservoir system design	Three reservoir system	Both LDR & chance constrained performed poorly compared to yield models.
4	Chung and Helweg (1985)	DP & simulation (Hec-3)	To maximise the benefits from exporting excess water	Lake Oroville & San Luis reservoir, California, USA.	Hec-3 was used to determine the excess water. DP was used to decide how to operate the reservoirs to maximise the benefits from exporting water.
5	Marino and Loaiciga (1985)	LP & Quadratic Optimizatiom	Management of reservoir for hydropower & irrigation	California Central Valley Project, USA	Both models led to a potential increase in annual power production. By adopting optimal release policy of quadratic model, irrigation water delivery was increased
6	Simonovic (1992)	Simulation, LP, DP & NLP	Various analysis for a single reservoir	General	The decision supporting system comprises 11 models.

2.5 NEED FOR MODEL DEVELOPMENT AND MODEL CATEGORISING

From the literature review done, it can be seen that the application of optimization techniques to multipurpose, multiple reservoir system represents a challenge due to:

1) High state dimensionality of the related multireservoir systems.
2) System nonlinearities which complicates the application of mathematical programming methods.
3) Dynamic or multistage character of the reservoir system operation problem which results in high dimensionality.
4) The stochastic nature of stream flow inputs which have their impacts on system reliability.
5) Not suitably interpreting optimization results into practical means, such as operating rules.
6) Difficulties in representing the effect of some related issues.

Some work was done in which trials were made to alleviate the effects associated with these characteristics. This can be summarised as follows:

1) Dimensionality

The effect of dimensionality can be alleviated by applying one of the following techniques:

a) Discretization

In discretization the continuous space of the reservoir content and/or inflow is replaced by a discrete one (Lochert and Phatarford, 1979). Moran (1959) suggested the following discretization for the reservoir content space [0,K-M]. He divided the content space into K-M+1 intervals;[0,1/2],[1/2,3/2],..........., [K-M-3/2,K-M-1/2],[K-M-1/2,K-M], (K is the reservoir content and M is the release). However the coarser the discretization the less the dimensionality obtained but this is obtained at the expense of accuracy.

b) Aggregation

The dimensionality problem can be avoided by replacing the large number of reservoirs by only one aggregated reservoir (Liang et al., 1996). Liang replaced an eight multipurpose reservoir system in Upper Colorado by only one aggregated reservoir. The aggregation should be followed by a problematic disaggregation step to determine each reservoir release targets (Pereira and Pinto, 1985).

c) Decomposition

Most of the dimensionality reduction methods decompose a system into subsystems and then use iterative methods to reach a solution (Yeh, 1985). Arunkumar and Yeh (1973) proposed a decomposition approach for a multireservoir system. For a system with m reservoirs, they fixed the operating policies of (m-1) reservoirs, (say 2 to m), and optimized w.r.t reservoir 1. The optimized policy of reservoir 1, then replaces its initial policy. Then reservoir 2 is chosen for optimization while having fixed operating policies for reservoirs 1,3,...........,m. The process is continued until the policies do not change or a certain desired level is obtained.

2) System Nonlinearity

Reservoir optimization problems are mainly non-linear since they contain non-linear functions such as evaporation or power production functions. The most common practice is to linearise these functions and solve the problem using Linear Programming. This results in an approximate solution (Lobbrecht, 1997). The direct use of non-linear programming techniques, which have not been widely used, represents the system more realistically.

3) Dynamic or Multistage Character of Reservoir System

The decisions in reservoir operation are made in different stages i.e. months. If a monthly, weekly, daily or hourly operation of a system is to be considered over a period of one year, then the number of the state variables will be large and a problem of dimensionality will be faced. To deal with such a problem, in dynamic programming, the state variables are discretized. The discretization improves the dynamic programming performance. This improvement is achieved at the expense of accuracy.

4) Stochastic Nature of the Flow

The stochastic nature of the flow can be expressed implicitly or explicitly. The implicit models incorporate the stochastic nature of the flow by incorporating synthetic stream flows while explicit models incorporate explicitly a conditional probability matrix as in SDP (Karamouz and Vasiliadis, 1992).

The explicit optimization methods are well theoretically based but they (Lund and Ferreira, 1996):

a) Suffer from computational inconvenience and limited computational feasibility. Previous treatments simplified computations at the expense of accuracy by neglecting the inflow correlation and adopting a coarse state dicretization (Piccardi and Soncini Sessa, 1991).

b) Require explicit representation of probabilistic stream flow. This representation is uncertain itself and statistically difficult.

On the contrary deterministic methods of which the implicit optimization is a sophisticated application are more detailed and can be solved quickly and easier to explain. Recent applications of implicit stochastic optimization mixed optimization, regression and simulation to derive and test operating rules (Bhaskar and Whitlatch, 1980; Karamouz and Vasiliadis, 1992; Lund and Ferreira, 1996). However comparisons of explicit and implicit stochastic techniques have found that the latter produced better results, because they represent inflows less coarsely than the explicit techniques (Karamouz and Houck, 1987).

5) Interpretation of Optimization Results

Not many of the studies done were directly concerned with the implications of the application of the optimization results to actual problems (Goulter and Tai, 1985).

Optimization results can be made more practical and useful if they are used to generate operating rules. Young (1967) first proposed how to obtain operating rules from a deterministic optimization results. He suggested doing a least squares regression of the optimal releases against any preceding characteristics of an optimal operation. These could be previous seasons' releases, inflows and storages. Karamouz and Houck (1982) derived operating rules by applying regression analysis to deterministic dynamic programming results and used simulation to test the derived rules. They used the following simple linear operating rule:

$$R_t = a\, I_t + bS_t + c \tag{2.27}$$

Where

a, b and c are constants that can be found by least squares multiple regression. R_t^* can be regressed against S_t^*, R_{t-1}^*, R_{t-2}^*, I_t, I_{t-1} or any other characteristics of the operation that precede in time the release in period t, to find the coefficients of the operating rule. R_t^*, R_{t-1}^*, S_t^*, R_{t-1}^*, R_{t-2}^*, are optimum releases and storages.

Bhaskar and Whitlach (1980) compared the results obtained using more complex non-linear and this simple linear operating rules on dynamic programming results. It has been found that the results obtained from the latter were as good as or better than those obtained from applying the former. However the results of both linear and non-linear regression models were very poor during some months.

6) Representation of the Effect of Some Reservoir's Related Issues

As mentioned in Section (2.1.5), the reservoir operation is related to many subjects such as evaporation, sedimentation and demand estimation. To simplify the reservoir optimization, some of the issues that may influence the results are not considered. Evaporation, which is non-linear, is not incorporated in many works to avoid the nonlinearity. Also no much work, if not at all, has been done to investigate the effect of sedimentation on optimum reservoir operation.

From the above discussion it can be noticed that:

a) Most of the techniques widely used now simplify and then approximate the solution of the optimization problems. Therefore there is a need for using techniques that represent the system more accurately. Non-linear optimization techniques, which have not been widely used, represent the nonlinearity of the reservoir system better than any other techniques.

b) Implicit stochastic stream flow represents the nature of the flow more realistically and easier than the explicit stochastic stream flow.

c) Not much work has been done to transfer optimization output into practical means, i.e. operating rules.

d) The effects of some issues that influence optimization results, like sedimentation, has not been investigated.

Therefore there is a need for developing a model that takes into account these considerations. The model can be categorised as:

1) The model will be developed for a system of two reservoirs in series that are used for irrigation, hydropower generation and low flow augmentation. Therefore the model can be considered a multiple reservoir multipurpose model.

2) Out of the optimization results, regression analysis will be used to derive reservoirs operating rules. Therefore the model is viewed as a " planning" rather than a "real time" model.

3) The model will incorporate the stochastic nature of the flow implicitly by incorporating synthetically generated stream flows. Therefore the model can be classified as an implicit stochastic model.

4) The model will use non-linear objective function and some non-linear constraints. This is why the model is categorised as non-linear.

2.6 CONCLUSIONS

The widely used linear and dynamic programming models approximately represent the reservoir optimization problem. The first due to the linearisation process and the second due to the widely applied disceritization technique used to reduce dimensionality. When modelling highly non-linear systems and accuracy is required, then there is a need to apply techniques, such as non-linear programming techniques investigated in Chapter III, that enhance the accuracy. Application of non-linear programming in reservoir operation is problematic. A trial will be made here to apply some of these techniques, see Chapter IX, and this forms the basis for hypothesis (5) and objectives 1 and 2.

Due to the complexity of reservoir optimization problems, usually they are simplified by not considering all the issues involved at a time. However it is claimed here that most of the issues involved such as demand modelling, sedimentation effect, flow uncertainty, evaporation losses can be incorporated in or linked to the optimization model and this forms the basis for hypothesis 1, 2, 3 and 4. The results of sedimentation, evaporation, flow and demand modelling are shown respectively in Chapters V, VI, VII and VIII respectively and the effects of sedimentation and water use are investigated in Chapter XI.

Optimization models compute releases that maximise or minimise the objective function without tackling the details of the operating rules. General operation rules are needed more than computed releases corresponding to specified stream flow sequences. No trials have been made to derive operation rules out of non-linear optimization results. Therefore a trial will be made here, see Chapter X, to derive linear and non-linear operation rules using non-linear optimization output. This forms the basis for hypothesis 6 and objectives 2 and 3.

CHAPTER III

NON-LINEAR PROGRAMMING TECHNIQUES

Summary ~ As was explained in Chapter II, there is a need for applying Non-linear Programming Techniques to reservoir systems. Therefore different NLP techniques will be reviewed in this chapter. Although reservoir optimization problems are constrained, some of the constrained optimization techniques use unconstrained optimization techniques to solve these problems. Therefore both the constrained and the unconstrained NLP techniques will be reviewed aiming at reaching the most appropriate technique for application to reservoir systems (Chapter IX). Some terms related to NLP techniques and used here are reviewed first.

3.1 DEFINITIONS

3.1.1 NLP Problem

In NLP an objective function is optimized. The variables of the function are constrained by a set of equality and inequality functions. At least one of the functions should be non-linear. A definition of the non-linear programming problem is:

$$\begin{aligned} &\text{minimise } f(x) \\ &\text{subject to } g_i(x) \leq 0, \ i = 1, \ldots\ldots, m. \\ &\qquad h_i(x) = 0, \ i = 1, \ldots\ldots, l. \\ &\qquad x \in R^n. \end{aligned}$$

Notes:

1) $f(x)$ is the objective function.

2) $g_i(x) \leq 0$, $i = 1 \ldots\ldots m$, is the inequality constraints.

3) $h_i(x) = 0$, $i = 1 \ldots\ldots l$, is the equality constraints.

4) x is a feasible vector which satisfies the constraints of NLP. The set $\{x \in R^n: g_i(x) \leq 0, i = 1, \ldots\ldots, m, h_i(x) = 0, i = 1, \ldots\ldots, l\}$ is the feasible region.

5) In NLP it is normally assumed that all the functions involved are at least twice continuously differentiable.

6) An objective function that maximise f(x) can be included in NLP since maximising f(x) is equivalent to minimising -f(-x) .

7) "≥" constraints can be included in NLP by multiplying these constraints by (-1). The "≥" constraints become "≤" ones after the multiplication.

3.1.2 Necessary and Sufficient Conditions for Optimality in Unconstrained Optimization

The necessary conditions are the ones that must be true for every local optimal solution. A point satisfying these conditions need not to be a local minimum, but is a candidate for that. A point is described as a strict local minimum if it satisfies the sufficient conditions.

Necessary Conditions

First order condition:

$$\nabla f(x^*) = 0$$

x^* is a local minimum and f is differentiable at x^*.

Second order conditions:

Suppose f is twice differentiable at x^*. If x^* is a possible local minimum, then,

$$\nabla f(x^*) = 0$$

$G(x^*)$, the hessian $\nabla^2 f(x^*)$, is positive semidefinite.

Sufficient conditions

Suppose f is twice differentiable at x^*. If x^* is a strict local minimum then:

$$\nabla f(x^*) = 0$$

$G(x^*)$, the hessian, is positive definite.

3.1.3 The Lagrangian

The Lagrangian L(x,u,v) associated with NLP is defined by

$$L(x,u,v) = f(x) + \sum_{i=1}^{m} u_i\, g_i(x) + \sum_{i=1}^{l} v_i\, h_i(x) \quad (3.1)$$

Where, u_i and v_i are called Lagrange multipliers.

3.1.4 Convergence

The order of convergence of the sequence $x^{(k)}$ is defined as p such that,

$$\lim_{k\to\infty} \frac{\left\| x^{(k+1)} - x^{*} \right\|}{\left\| x^{(k)} - x^{*} \right\|^{p}} = \gamma < \infty \quad (3.2)$$

p is order (rate) of convergence, and γ is asymptotic error constant (a e c). As p increases, then the convergence is fast.

If $p = 1$ and $0 < \gamma < 1$, then the convergence is linear.

If $p = 1$ and $\gamma = 0$: or if $p > 1$, then the convergence is superlinear.

If $p = 2$, then the convergence is quadratic.

Quadratic convergence $\Rightarrow$ superlinear convergence.

3.2 UNCONSTRAINED NON-LINEAR PROGRAMMING(UCNLP)

The reservoir problem to be dealt with later is a constrained one. The constrained methods that will be used to solve this problem may use unconstrained methods. Therefore these unconstrained methods will be discussed first. According to Rao (1996), the unconstrained minimisation methods are classified into two classes: direct search methods and descent methods (Table 3.1). The main difference between the two classes is that the former don't require the derivative of the function while the latter require the first and/or the second derivatives of the function. The direct search methods, with the exception of Rosenbrock and Simplex methods in some applications, are less efficient than the descent methods and are suitable for simple problems with a small number of variables (Rao, 1996). These methods seem to be inadequate to reservoir optimization problems, which usually have a large number of variables. Therefore these methods will not be discussed further here and the discussions will be limited to the more powerful descent methods.

Table (3.1) Unconstrained minimisation methods

Direct Search Methods	Descent Methods
Random Search Methods	Steepest Descent Method
Univariate Methods	Conjugate Gradient Methods
Pattern Search Methods	Newton's Method
Powell's Method	Quasi-Newton Methods
Hooke-Jeeves Method	(Variable Metric Methods)
Rosenbrock's Method	
Simplex Method	

For x^* to be an optimal solution of a function f(x), the sufficient condition in Section (3.1.2) should be satisfied. It is possible to find a local minimum by solving the first condition and testing the second for positive definiteness. However this is not viable, since it can be very difficult to solve the first condition. Instead, all the optimization algorithms follow an iterative scheme which produces points with decreasing values of f until a good estimate is obtained of a local minima. This general iterative scheme is shown in Figure (3.1). All the unconstrained optimization methods require an initial point, $x^{(1)}$, to start this iteration scheme. They only differ from each other in generating the new point $x^{(k+1)}$ from $x^{(k)}$, and in testing $x^{(k+1)}$ for optimality (Rao, 1996)

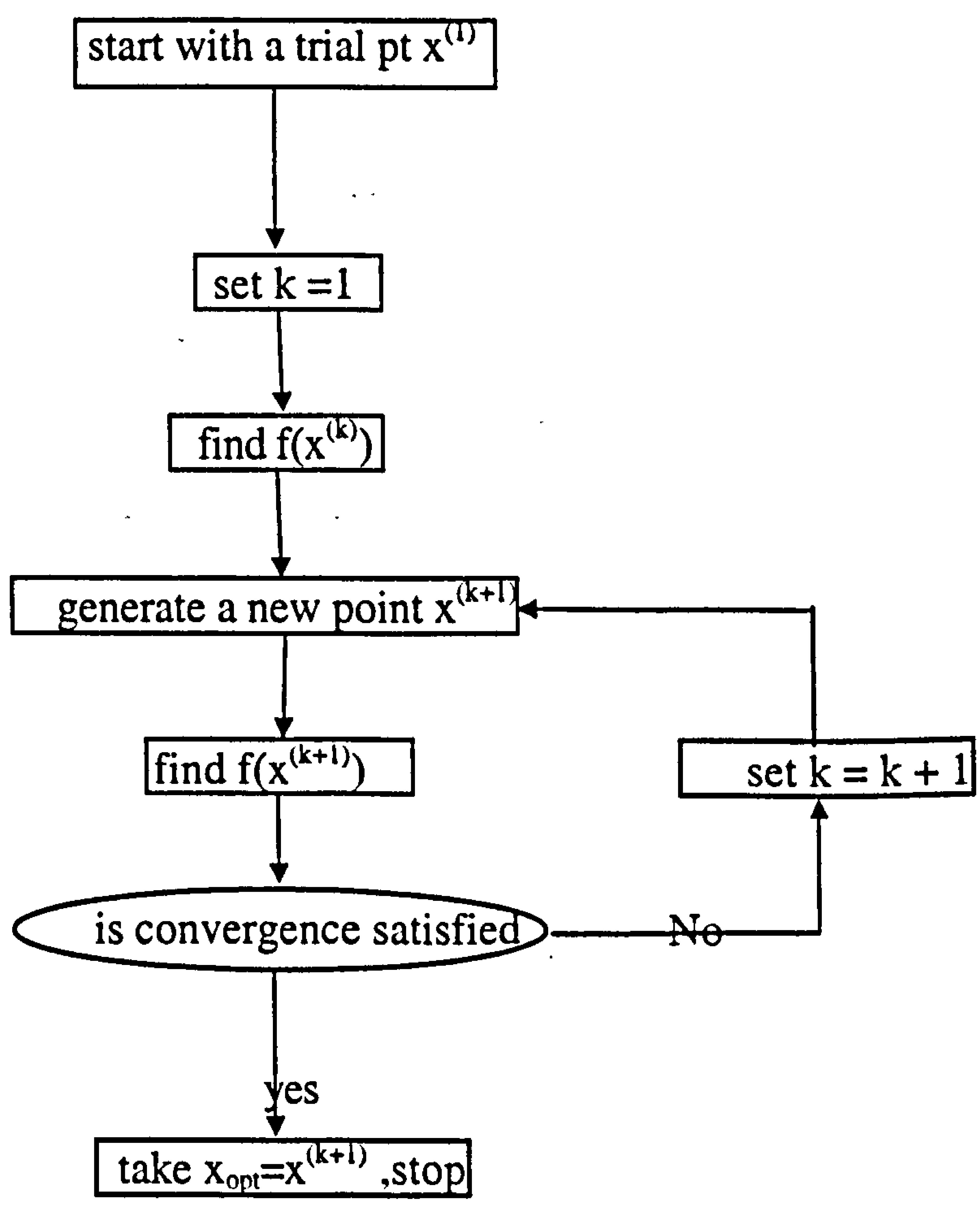


Figure (3.1) General iterative scheme of optimization

To implement this iterative scheme the descent methods follow an algorithm called the descent algorithm. The steps carried out in this algorithm are:

1) Start with initial estimate $x^{(1)}$ for some local minimum. Set $k = 1$.

2) Find a descent direction $d^{(k)}$ for $x^{(k)}$, i.e. find d such that

   $\nabla f(x^{(k)})^T d^{(k)} < 0$ (If the search is exact, then $\nabla f(x^{(k)})^T d^{(k)} = 0$) .

3) Compute α_k such that $f(x^{(k)} + \alpha_k d^{(k)}) < f(x^{(k)})$. Typically,

$$f(x^{(k)} + \alpha_k d^{(k)}) = \min_{\alpha} f(x^{(k)} + \alpha\, d^{(k)})$$

This step is known as a line search & α is the minimisation step length in direction $d^{(k)}$.

4) set $x^{k+1} = x^{(k)} + \alpha_k d^{(k)}$. If $x^{(k+1)}$ is near a local minimum then stop. This is known as a convergence test. Otherwise set $k = k+1$ and return to step 1.

3.2.1 Steepest Descent Method

This method arises from the general algorithm by selecting the search direction to be in the opposite direction to the gradient vector. i.e. $d^{(k)} = - g^{(k)}$, ($g = \nabla f$). Figure 3.2 shows the flow chart of the steepest descent method.

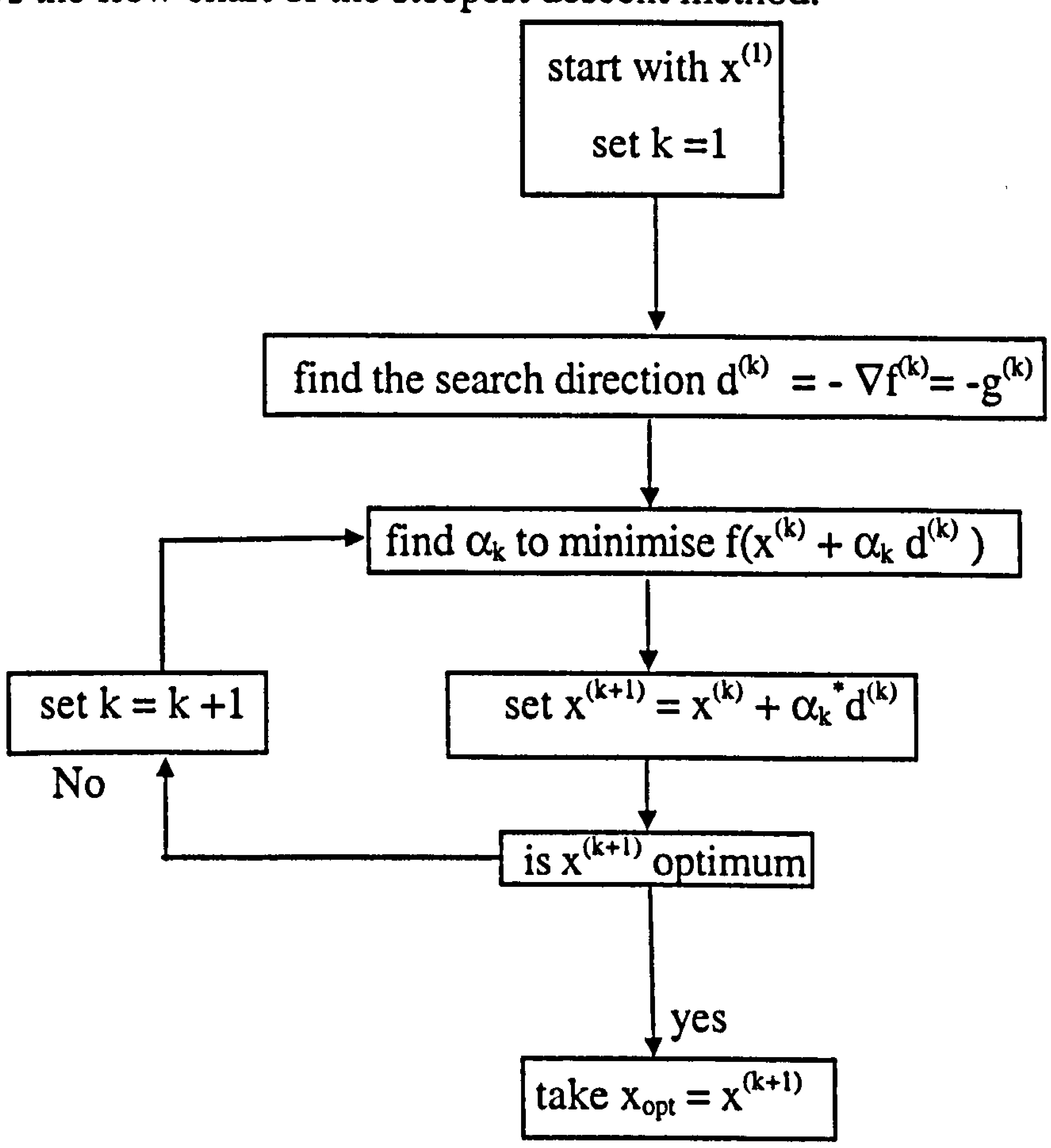


Figure (3.2) Flow chart for steepest descent method

Steepest descent method guarantees to start in a descending search direction, and this makes it to look as the best unconstrained minimisation techniques. The method is criticised for being very slow in convergence to the optimum solution (Rao, 1996).

3.2.2 Conjugate Gradient Methods

Since the steepest descent method is slow in convergence, the conjugate gradient method was developed to improve the convergence characteristics of this method. Any minimisation method that makes use of the conjugate directions is quadratically convergent. Quadratic convergence insures that the method minimises a quadratic function in n steps or less (Rao, 1996). "n" is the number of variables in the function. Therefore if any general function is approximated as a quadratic, then this method is expected to find the minimum in a finite number of iterations.

The iterative procedure of the conjugate method can be stated as follows :

1) Start with an arbitrary initial point $x^{(1)}$.

2) Set the search direction $d^{(1)} = - \nabla f(x^{(1)}) = - g^{(1)}$

3) Find the point $x^{(2)}$ according to the relation

$$x^{(2)} = x^{(1)} + \alpha_1^* d^{(1)}$$

Where α_1^* is the optimal length in the direction $d^{(1)}$. Set k = 2 and go to next step.

4) Set

$$d^k = - g^k + \beta_{(k-1)} d^{k-1}$$

Where

$$\beta_{(k-1)} = \frac{g^{(k)}(g^{(k)} - g^{(k-1)})}{d^{(k-1)} (g^{(k)} - g^{(k-1)})} \qquad (3.3)$$

5) Compute the optimum step length α_k^* in the direction $d^{(k)}$ and find the new point

$$x^{(k+1)} = x^{(k)} + \alpha_k^* d^{(k)}$$

6) Test $x^{(k+1)}$ for optimality. Stop if it is optimal, otherwise set the value of k = k +1 and go to step 4.

These methods are vastly superior to the steepest descent method but they are less efficient than Newton and Quazi-Newton methods if they are not performing on quadratic functions. These methods are very efficient when they are applied to quadratic functions (Rao, 1996).

3.2.3 Newton's Method

The steepest descent method fits a tangent plane locally to the function at the point $x^{(k)}$ and so uses the first partial derivative. This tangent plane is a linear approximation of the function at $x^{(k)}$. Alternatively the function may be quadratically approximated (using Taylor's expansion) by making use of the second partial derivative. The minimum of this quadratic could then be taken as an approximation to the minimum of the real function. This is the basis for Newton's method. The basic iteration of Newton method is,

$$x^{(k+1)} = x^{(k)} - G(x^{(k)})^{-1}\ g(x^{(k)}) \tag{3.4}$$

Where

$g(x^{(k)})$ is the first derivative of the function f.

$G(x^{(k)})^{-1}$ is the inverse of its hessian.

Newton's Method finds the minimum of a quadratic function in limited iterations. But if the function is not quadratic, the method may diverge and it may converge to a saddle point or a maximum (Rao, 1996). To avoid this a modification of the basic iteration was made by including some step length, $\alpha^{(k)}$, to be taken in the search direction $-G(x^{(k)})^{-1} * g(x^{(k)})$. The iteration becomes, Modified Newton,

$$x^{(k+1)} = x^{(k)} - \alpha^{(k)}\, G(x^{(k)})^{-1}\ g(x^{(k)}) \tag{3.5}$$

As described in the steepest descent method, $\alpha^{(k)}$ can be chosen to minimise $f(x^{k+1})$. Newton's modified method has the advantage of avoiding convergence to a saddle point or a maximum, which is faced by Newton's original method. Despite that the method remains impractical for problems involving a complicated objective function with a large number of variables. This is mainly due to the facts that :

1) The method requires the storage of the n*n matrix G.
2) It becomes very difficult and sometimes impossible to compute the elements of the matrix G.
3) The method requires the inversion of the matrix G at each step.
4) The method requires the evaluation of the quantity $G^{-1}\ g(x)$ at each step.

3.2.4 Variable Metric Methods (Quasi - Newton Methods)

A method that takes the advantages and avoids the disadvantages of both steepest descent and Newton methods was proposed by Davidon W.C. (Fletcher and Powell, 1963). The steepest descent method reduces the function value when the vector $x^{(k)}$ is away from the optimum point x^*. Newton Method, on the other hand, converges very fast when the vector $x^{(k)}$ is close to x^*. Therefore the method starts as steepest descent and changes to Newton method as the number of iterations increase. The method is known as Davidon - Fletcher - Powell, DFP, method because of the refinements made by Fletcher and Powell. However this is one method out of many that use the same principle and known collectively as variable metric or Quasi - Newton methods. The basic idea of the Quasi-Newton methods is to approximate either the hessian or the inverse hessian, in Newton's modified method, by an other matrix, H, using only the first partial derivatives. The general iteration of all these methods is

$$x^{(k+1)} = x^{(k)} - \alpha^{(k)} H^{(k)} g^k \tag{3.6}$$

Where

$H^{(k)}$ is a symmetric positive definite approximation to $G(x^{(k)})^{-1}$

$\alpha^{(k)}$ is the step length along the search direction $d^{(k)} = - H^{(k)} g^k$.

Generally $H^{(1)} = I_n$, I_n is the identity matrix. Therefore the first step follows steepest descent. As k increases $H^{(k)}$ approximates $G(x^{(k)})^{-1}$ more closely, but no matrix inversion or second partial derivative of f(x) are needed. In this way the method in the initial steepest descent like stages attempts to generate a sufficiently good estimate of the minimum x^* in order that the Newton like steps in the later iterations can speed up the convergence of $x^{(k)}$ to x^*. The different variable metric methods are obtained from the different ways of updating $H^{(k)}$ (the metric).

In order to have quadratic termination in case of a general function f(x), $H^{(k+1)}$ should satisfy the following Quasi - Newton condition,

$$H^{(k+1)} q^{(k)} = p^{(k)} \tag{3.7}$$

Where

$p^{(k)} = x^{(k+1)} - x^{(k)}$

$q^{(k)} = g^{(k+1)} - g^{(k)}$

$H^{(k+1)} = H^{(k)} + A^{(k)}$, where

$A^{(k)}$ is the update matrix.

The update of $H^{(k+1)}$ can be obtained by the use of one of the following formulae:

1) Rank One Update Formula

$$H^{(k+1)} = H^{(K)} + \frac{(P^{(k)} - H^{(k)} q^{(k)})(P^{(k)} - H^{(k)} q^{(k)})^T}{(P^{(k)} - H^{(k)} q^{(k)})^T q^{(k)}} \quad (3.8)$$

This formula has quadratic termination since it satisfies the Quasi -Newton Condition. But the formula does not preserve the positive definiteness of $H^{(k)}$ with the consequence that the search direction may not be downhill. The denominator of the update $A^{(k)}$, can become unacceptably small or even vanish, causing serious numerical difficulties. If these difficulties are not faced, rank one formula would be very efficient.

2) Broyden - Fletcher -Goldfarb-Shanno (BFGS) Formula

$$H^{(k+1)} = H^{(k)} + \left(1 + \frac{(q^{(k)})^T H^{(k)} q^{(k)}}{(p^{(k)})^T q^{(k)}}\right) \cdot \frac{p^{(k)}(p^{(k)})^T}{(p^{(k)})^T q^{(k)}} - \frac{(p^{(k)}(q^{(k)})^T H^{(k)} + H^{(k)} q^{(k)} (p^{(k)})^T)}{(P^{(k)})^T q^{(k)}} \quad (3.9)$$

3) DFP Method

$$H^{(k+1)} = H^{(k)} - \frac{H^{(k)} q^{(k)} (q^{(k)})^T H^{(k)}}{(q^{(k)})^T H^{(k)} q^{(k)}} + \frac{p^{(k)} (p^{(k)})^T}{p^{(k)} q^{(k)}} \quad (3.10)$$

BFGS and DFP methods belong to rank two update family, (rank 2 means that the update matrix has two columns). They update the inverse hessian function. According to Fletcher (1980) and Rao (1996), the BFGS method shows superlinear convergence near the optimum point x^* and that numerical experience shows that BFGS method is less influenced by errors in finding α^* compared to DFP.

The BFGS method can be described as follows (Rao, 1996):

1) Start with an initial point $x^{(1)}$ and a n*n positive definite symmetric matrix $H^{(1)}$ as an

initial estimate of the inverse of the hessian matrix of the function f. Usually the identity matrix I_n is taken for $H^{(1)}$. Compute the gradient vector $\nabla f_1 = \nabla f(x_1) = g(x^{(1)})$ and set the iteration number as $k = 1$.

2) Compute the gradient of the function, $g(x^{(k)})$; at the point $x^{(k)}$, and set

$$d^{(k)} = -H^{(k)} g(x^{(k)})$$

3) Find the optimal step length α_k^* in the direction $d^{(k)}$ and set

$$x^{(k+1)} = x^{(k)} + \alpha_k^* d^{(k)}$$

4) Test the point x^{k+1} for optimality. If $||g(x^{(k)})|| < \varepsilon$, where ε is a small quantity, take $x^* \approx x^{(k+1)}$ and stop the process. Otherwise, go to step 5. ($||x|| = \sqrt{\sum x_i^2} = \sqrt{x^T x}$).

5) Update the hessian matrix using equation (3.9)

6) Set the new iteration number as $k = k + 1$ and go to step 2.

3.3 CONSTRAINED NON-LINEAR PROGRAMMING

There are many methods to solve the constrained problem defined in Section (3.1.1). These methods, as shown in Table (3.2), can be classified in two broad categories: direct and indirect methods. In the former, the constraints are handled in an explicit manner whereas in most of the indirect methods, the constrained problem is solved as a sequence of unconstrained problems.

An indirect method, Augmented Lagrange Method (ALM), which combines Lagrange multiplier and penalty function methods,.will be applied to solve the formulated optimization problem. Reasons for this choice are discussed in Section (3.3.4).

Table (3.2) Constrained optimization techniques

Direct Methods	Indirect Methods

Random search methods
Heuristic search methods
  Complex method
Objective and constraint approximation
  Sequential linear programming method
  Sequential quadratic programming Method
Methods of feasible directions
  Zoutendijk's method
  Rosen's gradient projection method
Generalized reduced gradient method
	Transformation of variables techniques
	Sequential unconstrained minimisation techniques.
	  Interior penalty function method
	  Exterior penalty function method
	  Augmented lagrangian multiplier method.

3.3.1 Exterior Penalty Method

In the exterior penalty method, a penalty term is added to the objective function for any violation of the constraints. This method generates a sequence of feasible and infeasible points whose limit is an optimum solution of the original problem.

A suitable penalty function, $\alpha(x)$, should add positive penalty for infeasible points and no penalty for feasible points. For the general NLP problem

$$\text{Minimise } f(x)$$

$$\text{Subject to } g_i(x) \leq 0 \text{ for } i = 1, \ldots\ldots, m.$$

$$h_i(x) = 0 \text{ for } i = 1, \ldots\ldots, l.$$

a suitable penalty function is defined by

$$\alpha(x) = \sum_{i=1}^{m} \phi[g_i(x)] + \sum_{i=1}^{l} \psi[h_i(x)] \qquad (3.11)$$

Where ϕ and ψ are continuous functions satisfying

$$\phi(y) = 0 \text{ if } y \leq 0 \text{ and } \quad \phi(y) > 0 \text{ if } y > 0$$

$$\psi(y) = 0 \text{ if } y = 0 \text{ and } \quad \psi(y) > 0 \text{ if } y \neq 0.$$

Typical form of ϕ and ψ are :

$$\phi(y) = [\text{ maximum}\{0,y\}]^p$$

$$\psi(y) = |y|^p$$

Where p is a positive integer. p can take the value 2 to overcome the differentiation problems. Substituting for ϕ and ψ, the penalty function becomes,

$$\alpha(x) = \sum_{i=1}^{m} [\text{maximum}\{0,y\}]^2 + \sum_{i=1}^{l} |y|^2 \qquad (3.12)$$

Adding the penalty function and the objective function, the auxiliary function, $f(x) + \mu \alpha(x)$, is obtained. When finding the optimum solution of this function, the penalty parameter, μ, will have infinite values.

3.3.2 Barrier Function Method(Interior Penalty Method)

In the barrier method, a penalty term that prevents the points generated from leaving the feasible region is added to the objective function. The method generates a sequence of feasible points whose limit is an optimum solution of the original problem.

3.3.3 Augmented Lagrangian Penalty Function (multiplier penalty)

The Augmented Lagrangian Penalty Function, F_{alag}, finds an exact optimum for finite penalty parameters. Also F_{alag} enjoys the property of being differentiable. If only the equality constraints are considered in the general optimization problem defined earlier, then the function will be (Conn et al., 1996)

$$F_{alag}(x,v) = f(x) + \sum_{i=1}^{l} v_i h_i(x) + (1/2\mu) \sum_{i=1}^{l} h_i^{\,2}(x) \qquad (3.13)$$

3.3.4 Remarks on Constrained Optimization Methods

The following remarks can be made on direct optimization methods (Rao, 1996):

1) The direct search methods are less efficient than indirect methods.
2) The complex method becomes inefficient quickly as the number of variables increases and it cannot be used to solve problems with equality constraints.
3) Some other direct search methods approximate the objective and constraints functions. This produces very efficient techniques, but at accuracy expenses.

For these reasons, only the indirect methods will be considered further. The transformation of variables techniques method, which is an indirect method, will also be excluded, since it requires very simple constraint functions and this might not be the case when considering a reservoir optimization problem. In the interior and exterior penalty function methods, the penalty function is added to the objective function to form the auxiliary function, $f(x) + \mu\alpha(x)$. The problem with this function is that, in finding its optimal solution, the penalty parameter, μ, will have infinite values (Bazaraa et al., 1993). This problem is avoided by using the Augmented Lagrange Multiplier Method. Therefore this method has been chosen for use in this study.

3.4 LANCELOT

Lancelot is a Fortran software package designed for solving large scale non-linear problems (Conn et al., 1996). The package uses the efficient Augmented Lagrangian Multiplier Method (described in Section 3.3.3). In the package a quadratic model is built and the conjugate gradient method, which is very efficient in solving quadratic models, is used (see Section 3.2.2). Finally the package tests the agreement between the quadratic and the original models. The methods used by the package are the most efficient ones. Therefore this package has been chosen for application in this research. Full description of Lancelot algorithmic structure and how it works will be given in Chapter IX.

3.5 CONCLUSIONS

The reservoir optimization problem to be solved here, is constrained. To solve this problem it would be converted first into an unconstrained optimization problem using the Lagrangian Multiplier Method, since it is suitable and less problematic compared to other methods. To solve this unconstrained problem, the descent methods are preferred over the less efficient direct methods, which are suitable for simple problems with a small number of variables. Further a choice has to be made among the descent methods. The steepest descent method has the problem of being very slow in convergence to the optimum solution. Conjugate gradient method was developed to overcome this problem. These methods are far better than the steepest descent method, but are less efficient than Newton and Quasi-Newton Methods when applied on nonquadratic functions. Newton method on its part has the problem of converging, sometimes, to a maximum or a saddle point instead of a minimum. Quasi-Newton methods are designed to take the advantage and avoid the disadvantage of the steepest descent and Newton methods. They start their iterations similar to the steepest descent method and then become like a Newton method as the iterations progress. Therefore, it can be concluded that the reservoir optimization problem, to be formulated in Chapter IX, can be solved more efficiently by constructing the augmented Lagrangian function and then using the Quasi-Newton method, such as first rank formula or BFGS, or the conjugate gradient method if the function is quadratic. Lancelot is chosen for use in this study since it uses the Augmented Lagrangian method and the conjugate gradient method to minimise a quadratic model.

CHAPTER IV

DESCRIPTION OF THE BLUE NILE SYSTEM

4.1 INTRODUCTION

If a model is developed, then the degree to which it has been tested in actual reservoir / river system is an important consideration (Chapter 2.2.2.b). The applicability of the model to be developed here will be tested by using data from the Blue Nile system. The data will be used in modelling sedimentation (Chapter V), evaporation (Chapter VI), flow uncertainty (Chapter VII) and demand (Chapter VIII). Therefore the features of this system will be described hereafter.

4.2 THE BLUE NILE

The Blue Nile is a main tributary of River Nile and the main contributor to its flow. It originates in Ethiopia at elevation up to 3000 m and runs through Sudan to meet the White Nile at Khartoum, forming the Nile. Figure (4.1) shows the whole Nile Basin.

4.2.1 Blue Nile Catchment

The catchment of the Blue Nile has an area of 324,500 Km^2 (Howell and Allan, 1994). The length of the main stem is 1000 km and in Sudan the Blue Nile has a mean river gradient of 1 in 10000 (Sir Alexander Gibb and Partners, 1978).

4.2.2 Climate and Hydrology

a) Climate

The Blue Nile catchment lies in the tropical zone. The rainfall that causes the main run-off occurs during the period from July to September (Howell and Allan, 1994). Climatic characteristics of the Blue Nile Basin are estimated at Wad Medani (Sir

Alexander Gibb and Partners, 1978). With an average annual air temperature of 28 ^{0}C, an annual average sunshine duration of 82 % and an annual average solar radiation of 534 cal / cm^2 / day. Therefore evaporation losses are expected to be significant.

b) Hydrology

The flow in the Blue Nile is at its minimum during April or May, 100 m^3/s, and reaches its peak, about 6000 m^3/s, in late August. During October the flow falls sharply to reach about one-sixth of the peak level by early November. Following it there is a smooth recession to the minimum. Figure (5.1) shows the average annual flow for the Blue Nile at Ed Diem. This is the main measuring station which is located well-off the backwater effect of the Blue Nile upstream reservoir. A record of flow measurements dating back to early years of this century for this station is available. Howell and Allan (1994) investigated the variations in the Blue Nile flow and found that discharges have fallen consistently after mid sixties. River flow modelling is required to include the effect of such variation.

4.2.3 Sediment Transport

The upper reaches of the Blue Nile are young in geomorphological terms, so that the river beds and channels, and the catchment in general, are still being eroded (Sir Alexander Gibb and Partners, 1978). As a result of this and of increased land use, the Blue Nile carries very high sediment load during the flood season. Sediment sampling at Ed Diem were carried out by the Hydraulic Research Station, Wad Medani, Sudan, in 1993. This sampling showed that the sediment carried by the Blue Nile is mainly wash load and the concentration reaches up-to 3941 ppm in the last days of July. Also these measurements showed that most of the concentration occurs in July and August. Figure (5.1) also shows these sediment concentrations. This large amount of sediment transported have accelerated reservoir sedimentation rates. As a consequence Roseries has lost 40% of its capacity in less than 30 years while Sennar lost 56% over 70 years. These figures were obtained from bathemetric surveys carried by the Ministry of Irrigation and Water Resources, Sudan, in 1976, 1981, 1985 and 1992 for Roseries and 1986 for Sennar.

4.3 RESERVOIRS

The Sudan relies heavily on using Blue Nile waters both for agriculture and power generation. To make use of this river, two dams have been constructed. The upstream one is located at Roseries and the other is at Sennar.

4.3.1 Sennar Reservoir

The Sennar dam, 350 km upstream Khartoum, was built in the early 1920's. The main section of the dam is, masonry construction, 1600 m long with a maximum height of 30 m. It contains 80 low level sluices, each 2 m wide, which are adequate to pass the seasonal floods in most years. Maximum discharge of the sluices is 9500 m^3/s. In addition spillways are provided at the higher level to pass the peaks of exceptional floods. Spillway maximum discharge is 1500 m^3/s. The maximum combined capacity of the deep sluices and spillways is 28500 million m^3/month (MOI, 1968). Head regulators for the Gezira and Managil canals are situated at the west end of the masonry section. The combined maximum discharge of these canals is 350 m^3/s, 30.5 million m^3/ day, (Sir Alexander Gibb and Partners, 1978).

The maximum retention level of the reservoir is 421.7 m and the downstream levels range from 404 m to 414 m. Through peak floods the reservoir is kept at 417.2 m, a level corresponding to the sill of the spillways, and subsequently filled on the falling flood, when the sediment content of the river inflows has reduced (MOI, 1968).

Between 1959 and 1962 a power station with two 7.5 MW turbo generator units was built downstream of the west side of the dam replacing part of the spillway section. The maximum discharge through the two units is 330 million m^3/month (MOI, 1968). Table (4.1) shows the variation of the overall efficiency, C_p, with the net head for both Sennar and Roseries. The average overall efficiency for both reservoirs is 0.88.

A bathemetric survey, carried out in 1986, showed that the reservoir storage capacity has declined from about 1 milliard m^3 when the reservoir was commissioned to 370 million m^3 at the time of the survey.

4.3.2 Roseries Reservoir

Roseries Dam which spans the Blue Nile 630 km upstream Khartoum, was built between 1961 and 1966. The dam has a structural height of 68 m and a length of 13.5 km. The central concrete section, 1 km long, has 5 deep sluices. The deep sluices are placed at the river main channel bed, at an inverted level of 435.5 m. Each sluice is 10.5 m high and 6.0 m wide. Away from the deep sluices, an overflow spillway is provided with a crest level of 463.7 m. This has 7 radial gates, each 12.0 m high by 10.0 m wide. The design level is 480.0 m, although the reservoir is now operated to a maximum level of 481.0 m. At 480 m level, the lake is 75 km long and has a storage capacity of 3 milliards m^3. As was designed and now under implementation, a rise to 490 m level will increase the storage capacity to 7.4 milliards m^3. If the dam is heightened, its total length would be 25 km (Sir Alexander Gibb and Partners, 1978). A recent survey, carried out in 1992, showed that at level 480 the storage capacity has reduced to 1.886 milliards m^3 and to 2.104 milliards m^3 at level 481(Gismalla, 1993). At level 467.0 m, the seven spillways pass 70 million m^3 / day and each of the five deep sluices passes a discharge of 1160 m^3/s (MOI, 1968). This means that the total discharge of the spillways and deep sluices is 17250 million m^3 / month when they are fully opened and operated at 467.5 m level. The hydropower house, installed in 1971, has a discharge capacity of 2014 million m^3/month (MOI, 1968) and a total installed capacity of 275 MW (National Electricity Corporation, NEC, Sudan).

4.3.3 Reservoir Operation

At present the Blue Nile system, including the Sennar and Roseries reservoirs, is operated with the document "Regulation Rules for the working of the Reservoirs at Roseries and Sennar on the Blue Nile" prepared by the Ministry of Irrigation and Water Resources in 1968. The aim of these rules is to distribute stored water and natural river flows during the low flow season between abstraction from the river for irrigation and minimum flows at Khartoum. There is a provision for flows at Roseries and Sennar for power generation but this provision is subject to irrigation demands. The system of operation divides the year into three main periods:

1) The flood period, before filling, during which the reservoirs levels are held at low levels to reduce siltation.
2) The filling period, when reservoirs are filled.
3) The period of shortage, when storage is used to meet the requirements of irrigation and minimum flow at Khartoum.

The flood period starts from early July and continues until the filling is started in September. The aim of the operation is to maintain reservoirs' levels at 467.0 m at Roseries and 417.2 m at Sennar to provide head for power generation and irrigation canals at Sennar.

Filling is carried out on the falling flood taking into account the need to delay the filling as long as possible to reduce siltation on one hand and to guarantee the filling on the other. The starting of filling varies from year to year according to the flow at Ed Diem and then follows a day by day program. According to the current operation rules the filling period begins either on:

1) 1st September at the earliest.
2) The day after the day on which Ed Diem flow falls to 350 million m^3/day, during the period from 1st to 26th September.
3) 26th September at the latest, if the flow has not fallen sufficiently.

However, for this study the filling will be started at the 1st of September.

The reservoirs are usually kept at retention levels as far as the natural river flow satisfies irrigation requirements, minimum flow at Khartoum (3.5 million m^3/day) and all river and reservoir losses. When river flow does not satisfy these requirements, the deficiency is met by releases from storage.

4.4 IRRIGATION DEVELOPMENT

According to Alexander Gibb and Partners (1978), about 20 % of the Blue Nile water is diverted for irrigation. The total irrigated area in the system is about 2,685,383 feddan (1 ha = 2.38 feddan). The largest single scheme is the Gezira. The area of this scheme is about 2,081,692 feddan (approximately 2.1 million feddan), which represents 77.5 % of the whole Blue Nile irrigated area. The crops grown in Gezira are mainly cotton, wheat, groundnut and dura. Other schemes in the Blue Nile have the same cropping pattern with the exception of relatively very small sugar and kenaf schemes. Table (4.2) and Figure (4.2) show these schemes, their areas and their locations. It can be seen that 96 % of the irrigation water is withdrawn from upstream Sennar reservoir. The other 4 % is withdrawn from downstream Sennar reservoir and no direct irrigation withdrawals are made from Roseries reservoir.

4.5 CONCLUSION

From the description of the Blue Nile System, it can be noticed that the system is located in the tropics, therefore evaporation losses are expected to be significant. Also it can be noticed that reservoirs' storage capacities have shrunk due to sedimentation. Variations in the Blue Nile flows are observed. Therefore, the effects of these issues are to be dealt with in Chapters V, VI and VII to formulate an optimization problem for the system in Chapter IX. As the system is featured by the presence of large irrigation schemes, the benefits obtained from the system are expected to be affected by use of water in these schemes. Therefore, the efficiencies of water use in these irrigation schemes will be investigated in Chapter VIII.

Table (4.1) Variation of overall efficiency with head - Sennar and Roseries

Sennar		Roseries	
Turbine Net Head (m)	Overall efficiency (%)	Turbine Net Head (m)	Overall efficiency (%)
5.8	69.0	17.0	84.0
6.0	72.0	20.0	85.2
7.0	88.0	25.0	87.0
8.0	95.6	27.5	87.8
9.0	96.0	30.0	88.5
10.0	88.0	32.5	89.1
11.0	90.0	35.0	89.7
12.0	92.0	37.5	90.0
13.0	92.0	40.0	90.1
14.0	92.0	42.5	90.0
15.0	94.0	45.0	89.7
16.0	93.0	48.0	89.2
17.0	90.0	-	-
Average	88.0	Average	88.0

source National Electricity Corporation, NEC, Sudan.

Table (4.2) Blue Nile irrigation schemes

Scheme	Area (feddans)	Offtake location
Gezira & Managil	2,081,692	Upstream Sennar
Rahad1	300,000	"
Es Suki	88,000	"
Abu naama	30,000	"
Sennar Sugar	27,300	"
B.N.A.P.C	158,390	"
Hurga & Nureldeen	22,300	Downstream Sennar
Guneid Extension	49,000	"
Guneid Sugar	33,000	"
B.N.A.P.C	3,115	"

source: Ministry of Irrigation & Water Resources, Sudan

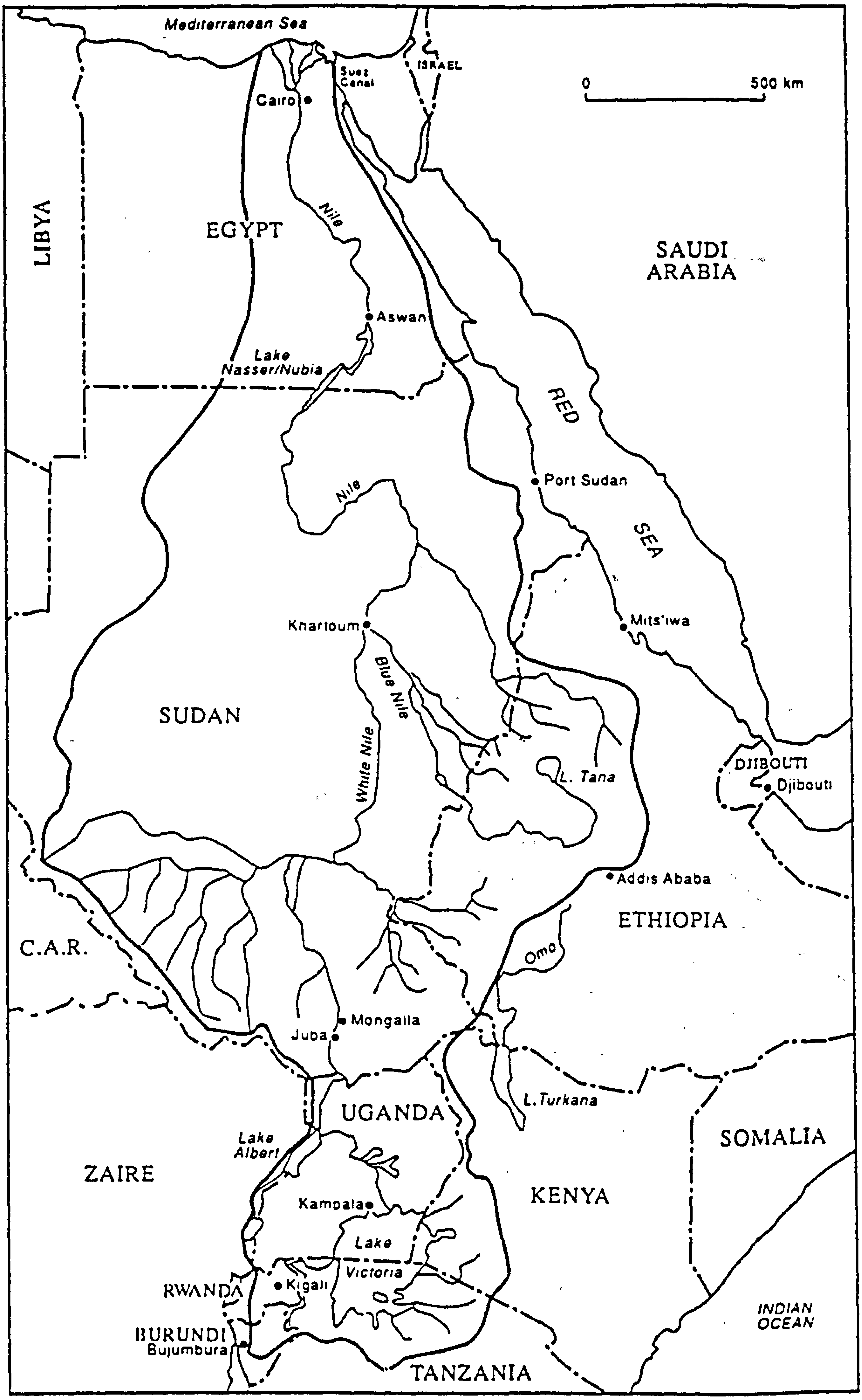


Figure (4.1): The Nile Basin

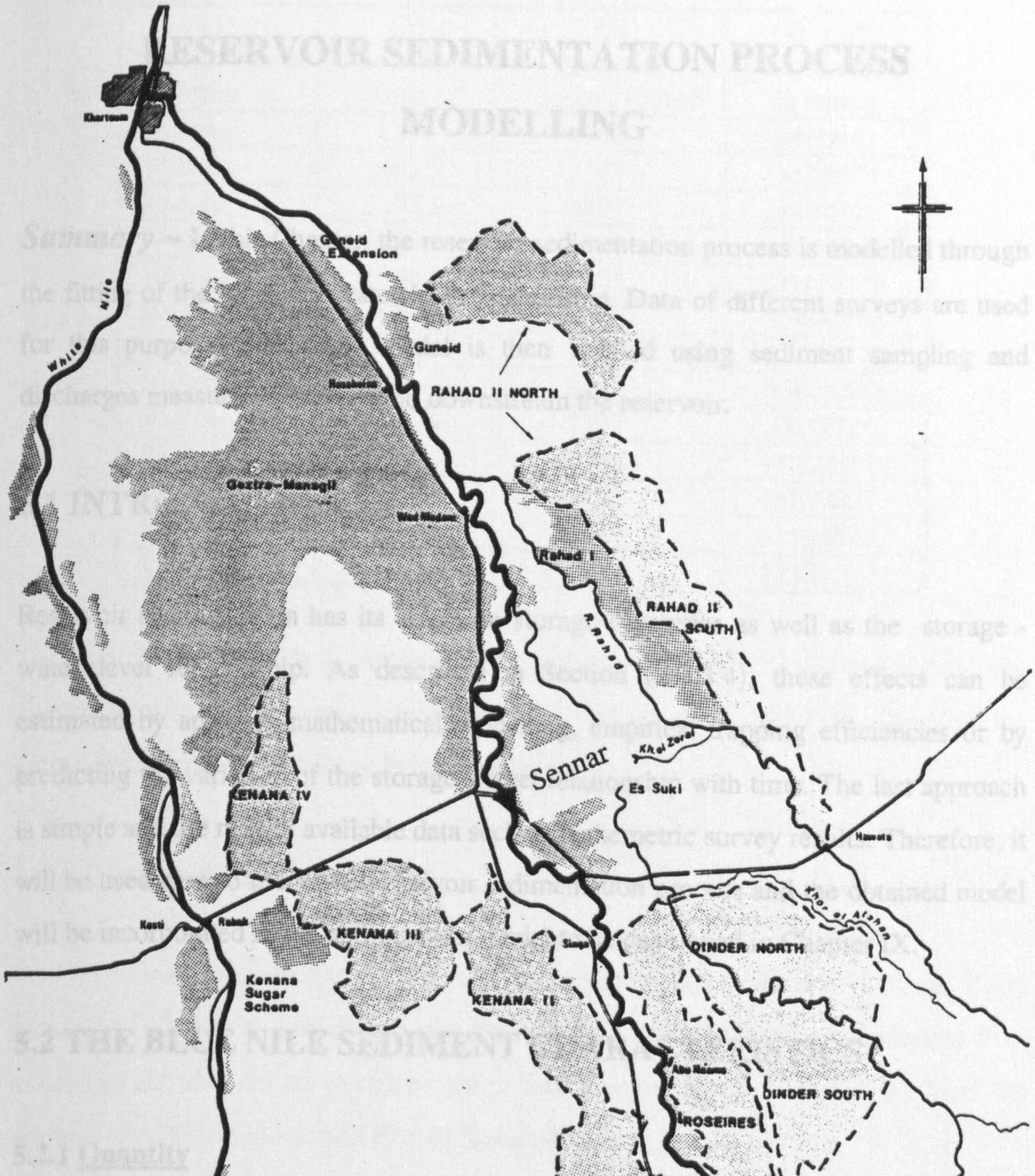


Figure (4.2): Blue Nile Region

CHAPTER V

RESERVOIR SEDIMENTATION PROCESS MODELLING

Summary ~ In this Chapter, the reservoir sedimentation process is modelled through the fitting of the storage - water level relationship. Data of different surveys are used for this purpose. The fitted model is then verified using sediment sampling and discharges measured upstream and downstream the reservoir.

5.1 INTRODUCTION

Reservoir sedimentation has its effect on storage capacities as well as the storage - water level relationship. As described in Section (2.2.3.4), these effects can be estimated by applying mathematical modelling, empirical trapping efficiencies or by predicting the variation of the storage - level relationship with time. The last approach is simple and use readily available data such as bathemetric survey results. Therefore, it will be used here to model the reservoir sedimentation process and the obtained model will be incorporated in the optimization model to be developed in Chapter IX.

5.2 THE BLUE NILE SEDIMENT CHARACTERISTICS

5.2.1 Quantity

The Blue Nile natural flow is monitored on daily basis at El Diem, a station located upstream of the two reservoirs well off the backwater curve of the upstream one. During the period June 1993 - October 1993, daily suspended sediment samples were taken at the same location. The same process described in Section (2.2.3.4) was followed in the analysis of the collected samples. Table (5.1) shows the Blue Nile discharges, sediment concentrations and sediment load during period of sampling.

Table (5.1) Blue Nile sediment yield-El Diem-1993

(1) 10 days Period ending on	(2) Discharge in million m^3	(3) Sediment content. (ppm)	(4) = 2*3 Sediment load in tonnes
30-June	1258	1956	2460648
10-July	1663.7	3387	5634952
20-July	2172.9	3897	8467791
31-July	4560.6	3941	17973325
10-August	5546.5	3687	20449946
20-August	4486.6	3433	15402498
31-August	5086.2	2948	14994118
10-September	5480.67	3590	19675593
20-September	3888.87	2324	9037726
30-September	3292.57	1734	5709311
10-October	3595	1165	4187097
20-October	2190	609	1334586
Total			125327591

The Blue Nile transports an annual amount of sediment of 125 million tonnes. The average density of core samples, taken from the system, is 1 tonnes / m^3. Therefore the annual sediment load in volume is 125 million m^3.

5.2.2 Time Span of Sediment Input

The sediment transported by the Blue Nile occurs in the short period of the flood season. As shown in Table (5.1) and Figure (5.1), the whole annual sediment load carried by the river occurs over a period of four months, July to October. 94 % of this amount occurs over the period July to September.

5.2.3 Current Sediment Control Measures

During the high sediment concentration period, July and August, water levels in reservoirs are kept to a minimum level. The level is usually the dead storage level. At the same time the deep sluices of the reservoirs are opened to sluice incoming sediment. Deep sluices are located at the bottom of the reservoir and in the main channel of the river. Figure (5.6) shows the extent of deposits and the effect of sluicing.

5.2.4 Prediction of Sediment Transport

Linsley and Franzini (1972) stated that suspended sediment transport, Q_s, and stream flow, Q, is often represented by the relationship

$$Q_s = k\,Q^n,$$

Where, n commonly varies between 2 and 3.

To find k and n, the daily suspended load is plotted against the daily stream flow, Figure (5.2). Two distinct trends can be observed and not only one as Linsley and Franzini (1972) reported. A possible interpretation of this phenomenon may be the changing conditions over the vast catchment area of the river. The rainfall that causes the main runoff occurs over the period from June to September. This rainfall follows a long period of drought. During this period of drought the top soil disintegrates and becomes easy to wash. With the runoff reaching its peak, it is expected that most of the disintegrated soil will be washed, and new conditions over the catchment will occur as the runoff starts to fall. Accepting this argument the sediment season has been divided into two parts: the first extends from the 20^{th} of June to the 10^{th} of September, i.e. the period of rising and peak flood while the second covers the rest of the season up to the end of October.

The results of curve fitting for the two periods are (Figures 5.3 and 5.4) :

Rising and peak flood

$$Q_s = 1799.5\,Q^{1.1033} \quad \text{with} \quad R^2 = 0.85 \qquad (5.1)$$

Falling Flood

$$Q_s = 0.0011\,Q^{3.4307} \quad \text{with} \quad R^2 = 0.94 \qquad (5.2)$$

Q_s is suspended sediment load in tonnes / day.

Q is streamflow in million m^3 / day.

For the same discharge, Figure (5.5), the amount of sediment transported during the rising flood far exceeds the amount transported during the falling flood. The two curves converge as they approach the peak flood.
Knowing the stream flow, equations (5.1) and (5.2) or Figure (5.5) can be used to estimate the river suspended sediment yield.

5.3 STORAGE - LEVEL RELATIONSHIP FITTING

According to Yevdjevich (1965), the relationship between the storage volume, S, and the reservoir elevation, H, can be approximated by this relation.

$$S = a H^m \tag{5.3}$$

With

$$a = \psi(t) \tag{5.4}$$

and

$$m = f(t) \tag{5.5}$$

These time functions are resulting from sedimentation processes.

5.3.1 Determination of Coefficients a & m - Roseries

Roseries reservoir was completed in 1966. During the course of operation, the dam was surveyed in 1976, 1981, 1985 and 1992. The results of the surveys are given in Table (5.2), (Gismalla, 1993).

Table (5.2) Storage capacity, S, in (million m^3) and depth (m) - Roseries

Reduced Level (m)	Depth H (m)	1966	1976	1981	1985	1992
465	2	454	66	34	24	23
467	4	638	152	91	80	62
470	7	992	443	349	341	236
475	12	1821	1271	1156	1088	934
480	17	3024	2474	2359	2020	1888
481	18	3329	2779	2664	2227	2106

A relation between S & H for each survey has been fitted, Table (5.3) and Figures (5.7), (5.8), (5.9) and (5.10). It should be noticed that H used does not represent the stage with mean sea level used as a reference. Instead level 463 m is used (Figure 5.6). Therefore, H represents the reservoir depth which is a more meaningful representation.

Table (5.3) Variation of a & m with time- Roseries

Year	Years of operation(t)	a	m	R^2
1976	10	16.333	1.7524	0.99
1981	15	6.8101	2.0507	0.99
1985	19	5.1237	2.1171	0.99
1992	26	4.1763	2.1404	0.99

Both "a" and "m" vary with time. While "a" is decreasing with time, "m" is increasing. Fitting trends for "a" and "m", using Software Excel, it has been found here that "a" is better expressed in a power form while "m" is better represented logarithmically, equations (5.6) and (5.7) and Figures (5.11) and (5.12).

$$a = 395.47\, t^{-1.4399}, \quad R^2 = 0.93 \tag{5.6}$$

$$m = 0.4101 \ln(t) + 0.8655\, , R^2 = 0.85 \tag{5.7}$$

Where t is time in years during which reservoir has been in operation.

Using equations (5.6) and (5.7), coefficients "a" and "m" can be predicted for any number of years in which the reservoir has been in operation. These values can be substituted in equation (5.3) to relate reservoir storage, S, with varying reservoir depth, H. With the possibility of predicting "a" and "m" equation (5.3) becomes a useful tool in estimation of change of storage with both time and reservoir depth. Table (5.4) shows the predicted storage capacity with varying time and depth.

Table (5.4) Variation of contents, in million m^3, with time and depth - Roseries

	Depth (m)	2	4	7	12	17	18
year	Time year						
1976	10	50.35	176.53	486.03	1289.16	2421.41	2685.3
1981	15	31.51	123.99	374.66	1086.92	2163.29	2421.97
1985	19	23.98	100.91	321.91	984.00	2025.70	2280.52
1992	26	16.69	76.77	263.21	862.32	1856.55	2105.52

5.3.2 Verification of the Developed Model

To verify the model, then for any year, t, and its preceding year, t-1, equations (5.6) and (5.7) are used to obtain coefficients "a" and "m". Using equation (5.3) the storage capacities S_t and S_{t-1} in years t and t-1 can be obtained. Then the amount of deposits in year t is obtained by deducting S_{t-1} from S_t. For the year 1993, i.e. t=27, H = 18 m is taken and equations (5.6), (5.7) and (5.3) are used to estimate the storage capacity. In 1993 Roseries storage capacity was equal to 2211.287 million m^3. Similarly for the preceding year, the storage capacity was calculated and found to be equal to 2232.731 million m^3. This means that the amount of sediment deposited in 1993 was equal to 21.444 million m^3. The amount of sediment entering the reservoir was 125 million m^3, Table (5.1). Dividing the amount of deposits by the total volume of the sediment entering, the trapping efficiency is obtained and is equal to 17.2 %

Alternatively the reservoir trapping efficiency in 1993 is calculated as shown in Table (5.5) using collected sediment samples upstream and downstream the reservoir and the entering and leaving discharges. The trapping efficiency calculated is 18.9 %. The two trapping efficiencies found differs by 1.7 %. This finding verifies the developed model.

Table(5.5) Trapping efficiency using sediment sampling - Roseries - 1993

(1) period ending on	(2) discharge entering in million m^3	(3) sediment concentration entering-ppm	(4) discharge leaving in million m^3	(5) sediment concentration leaving-(ppm)	(6)=2*3 Sediment load entering in (tons)	(7)=4*5 Sediment load leaving in (tons)
30-JUN	1258	1956	1311.2	1171	2460648	1535415
10-JUL	1663.7	3387	1702.92	2032	5634952	3460333
20-JUL	2172.9	3897	2249.84	3193	8467791	7183739
31-JUL	4560.6	3941	4528.64	3490	17973325	15804954
10-AUQ	5546.5	3687	5264.9	3303	20449946	17389965
20-AUQ	4486.6	3433	4589.75	3090	15402498	14182328
31-AUQ	5086.2	2948	5243.55	2366	14994118	12406239
10-SEP	5480.67	3590	5325.95	2595	19675593	13820832
20-SEP	3888.87	2324	3904.02	2177	9037726	8499044
30-SEP	3292.57	1734	2357.24	1711	5709311	4033232
10-OCT	3595	1165	3317.66	833	4187097	2763611
20-OCT	2190	609	1658.61	312	1334586	517486.3
total					125E+61	102E+6

Trapping Efficiency = Sediment Deposited / Sediment Input = [Σ(6) - Σ(7)] / Σ(6)

= 18.9 %

5.3.3 Determination of Coefficients a and m for Sennar

For Sennar Reservoir only two surveys were made Table(5.6), (MOI, 1968; MOI, 1986). For the two survey results, fitting between reservoir content, S, and reservoir depth, H, has been done, Table (5.7)

Table (5.6) Variation of storage volume, million m^3, with depth - Sennar

Reduced Level (m)	Reservoir Depth -m	1925	1986
417.2	7.2	330	116
418	8	411	145
418.5	8.5	471	166
419	9	537	190
419.5	9.5	605	217
420	10	678	245
420.5	10.5	752	278
421	11	825	315
421.7	11.7	930	370

Reference level for Sennar is taken at 410 m.

Table(5.7) Results of determination of coefficient of equation (5.3) - Sennar

	a	m	R^2
1925	4.623	2.1662	0.99
1986	0.99	2.3993	0.99

It is not possible here to fit a trend for the variation of "a" and "m" with time, since only two points of data are available. But generally as in Roseries "a" decreases while "m" increases with time. However the relation derived from 1986 can be used to relate storage to depth in the optimization model, since:

1) Being the downstream reservoir, Sennar is less vulnerable to sedimentation.

2) Being in operation from the twenties, it is expected that the reservoir is approaching

a stable state.

5.4 COMMENTS

The Blue Nile high flood and high sediment load occur concurrently over the period from July to September. During this period water levels, in the reservoirs under study, are kept at a minimum level which guarantees the diversion of water for irrigation and generating some power on one hand and guarantees the safety of the dam and minimises sediment deposits in the reservoir on the other. Sediment sluicing was adopted in the design and operation of both reservoirs. According to Mahmood (1987), Roseries trapping efficiency could have risen from 44 % to 83 % if the reservoir was operated at its maximum level during high sediment flow and the sluicing seemed to have saved 3.6 million m^3 of deposits annually during the period 1966 - 1981.

During sluicing the river flow far exceeds the requirements and the level of operation facilitates the diversion for irrigation. That is to say, with the current operation sluicing policy, conditions necessary to satisfy irrigation, which is the primary purpose, are fully met. However keeping reservoir level low, will reduce generated power, especially during flood where large volumes of water passing increases downstream water level significantly. So increasing reservoir level during sluicing will increase sediment deposits and power generation at the same time. However, the primary objective of the operation policy during this time of the year is to minimise the sediment deposits by effectively sluicing it and not to maximise any other purpose. The current operation policy is obviously the best in fulfilling this objective, since it keeps the operation levels at the dead storage levels. Therefore, in this study, reservoirs will be operated in the same way during July to August while different scenarios of operation policies will be studied in the remaining part of the year.

Almost all the incoming sediment enters the reservoir system in July and August. This means that the change in the storage - stage relationship defined in equation (5.3) takes place during this period of the year and remains almost the same for the whole year before another sediment wave occurs. This justifies the determination of the coefficients of equation (5.3) on annual basis.

Trends defining the variation of "a" and "m" with time are expected to change, if sluicing is carried out under varying water levels. But since the level is kept constant, the trends are expected to be the same.

5.5 CONCLUSION

To model the reservoir sedimentation process the relationship between the storage, S, and the water level, H, is fitted. In the relation $S = aH^m$, both coefficients "a" and "m" vary with time. Coefficient "a" decreases with time in a power form while "m" increases logarithmically. Since "a" and "m" vary with time, their values can be predicted and hence the relation between S and H. Data from different surveys are used for this purpose. The fitted model is verified using sediment samples and discharges measured upstream and downstream the reservoir. This finding verifies the first part of hypothesis 2. These fitted models will be used in formulating the optimization problem Chapter IX and in investigating the effect of sedimentation on optimization results in Chapter XI.

Figure (5.1) Blue Nile discharge and sediment hydrographs, June 93-June 94

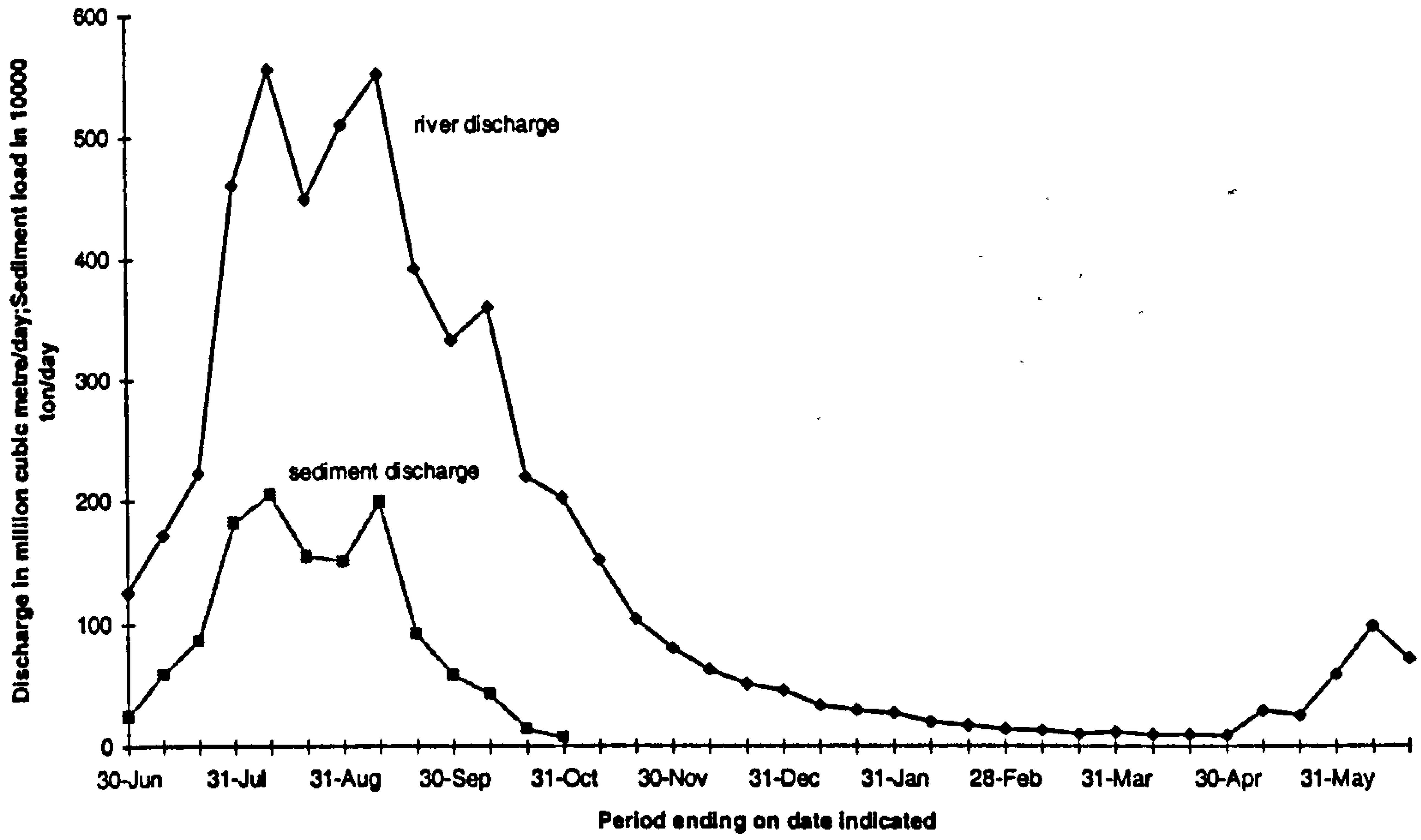


Figure (5.2) Sediment discharge versus flow discharge - Blue Nile

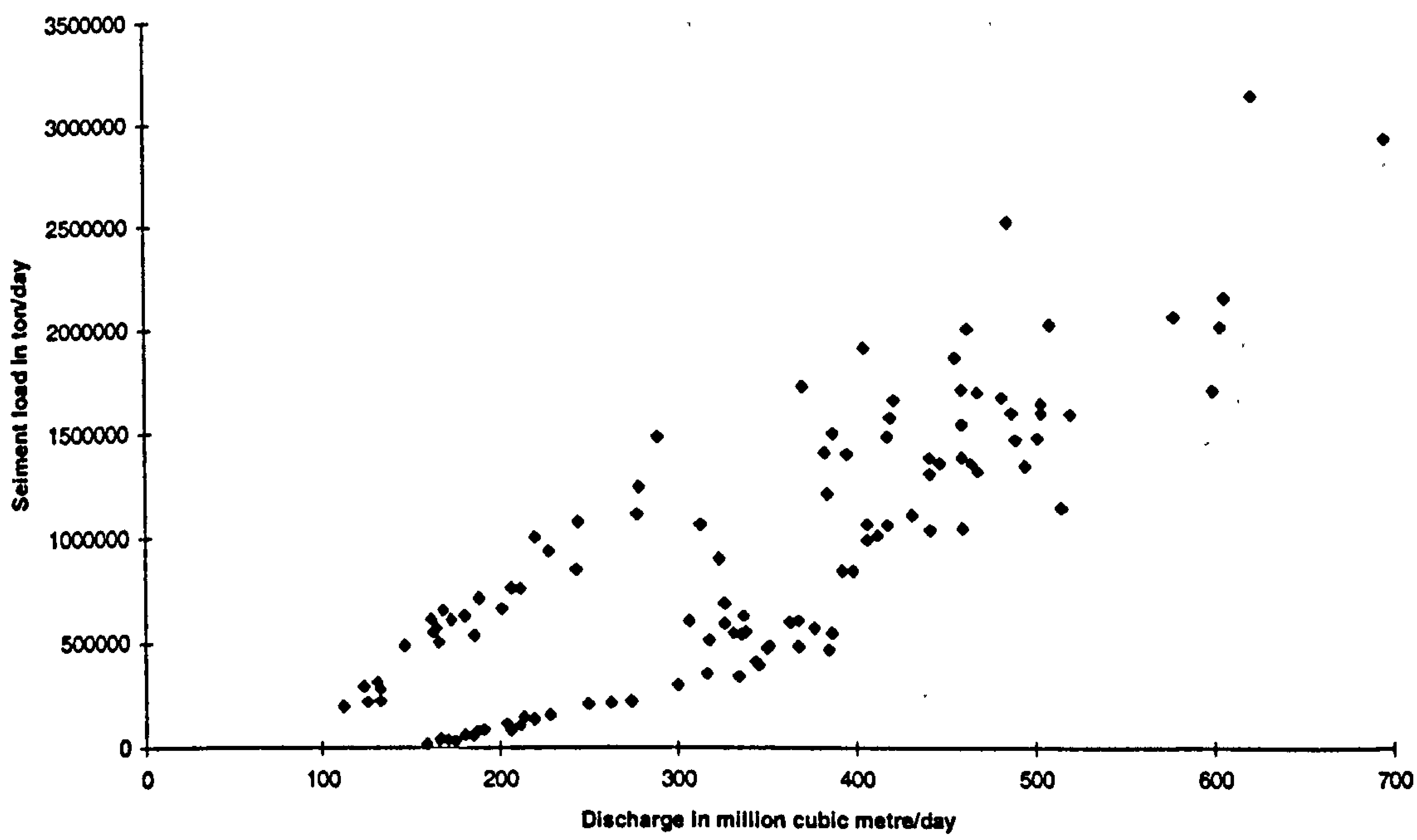


Figure (5.3) Sediment discharge versus flow discharge during rising and peak flood - Blue Nile

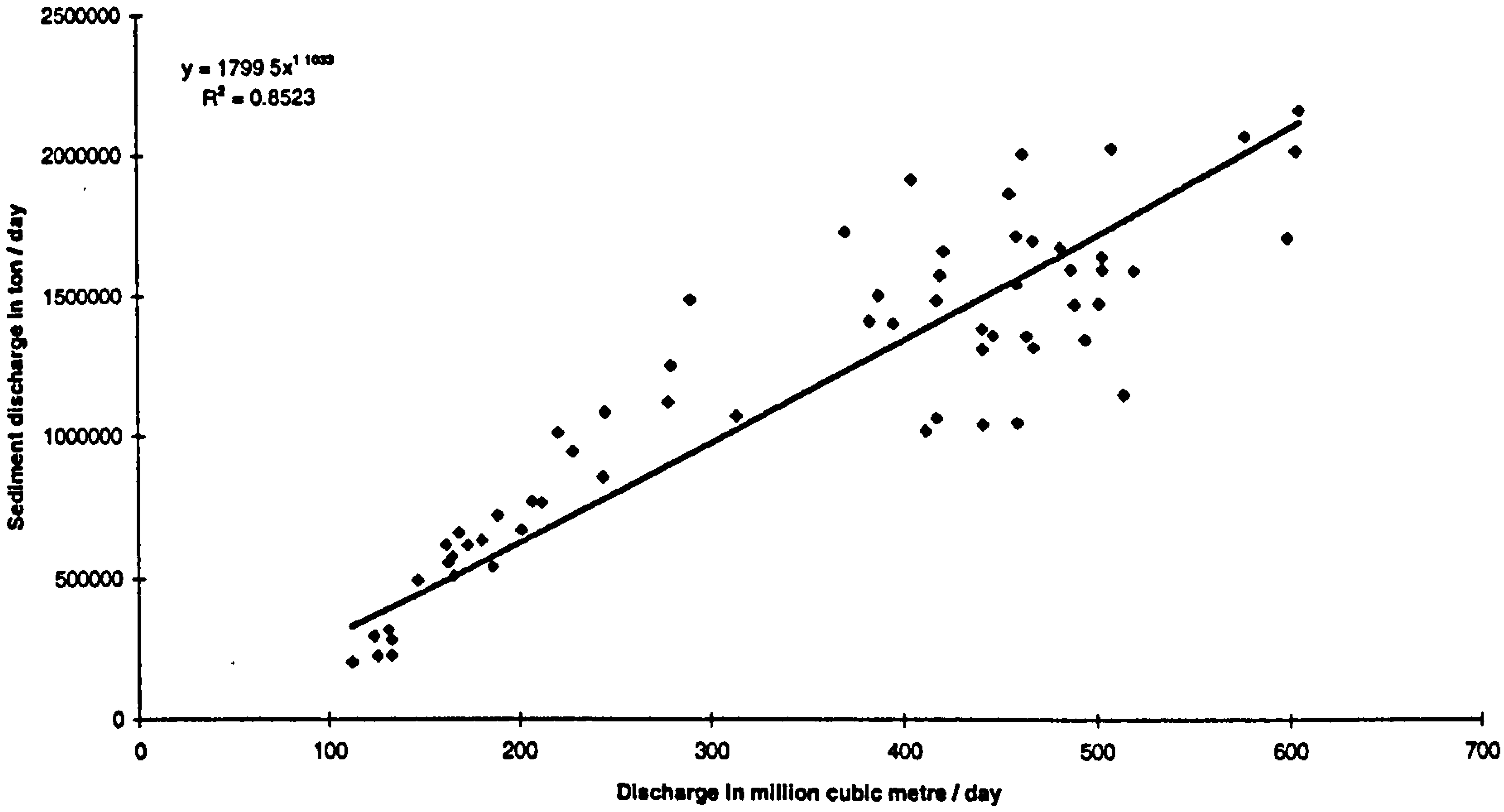


Figure (5.4) Sediment discharge versus flow discharge - falling flood - Blue Nile

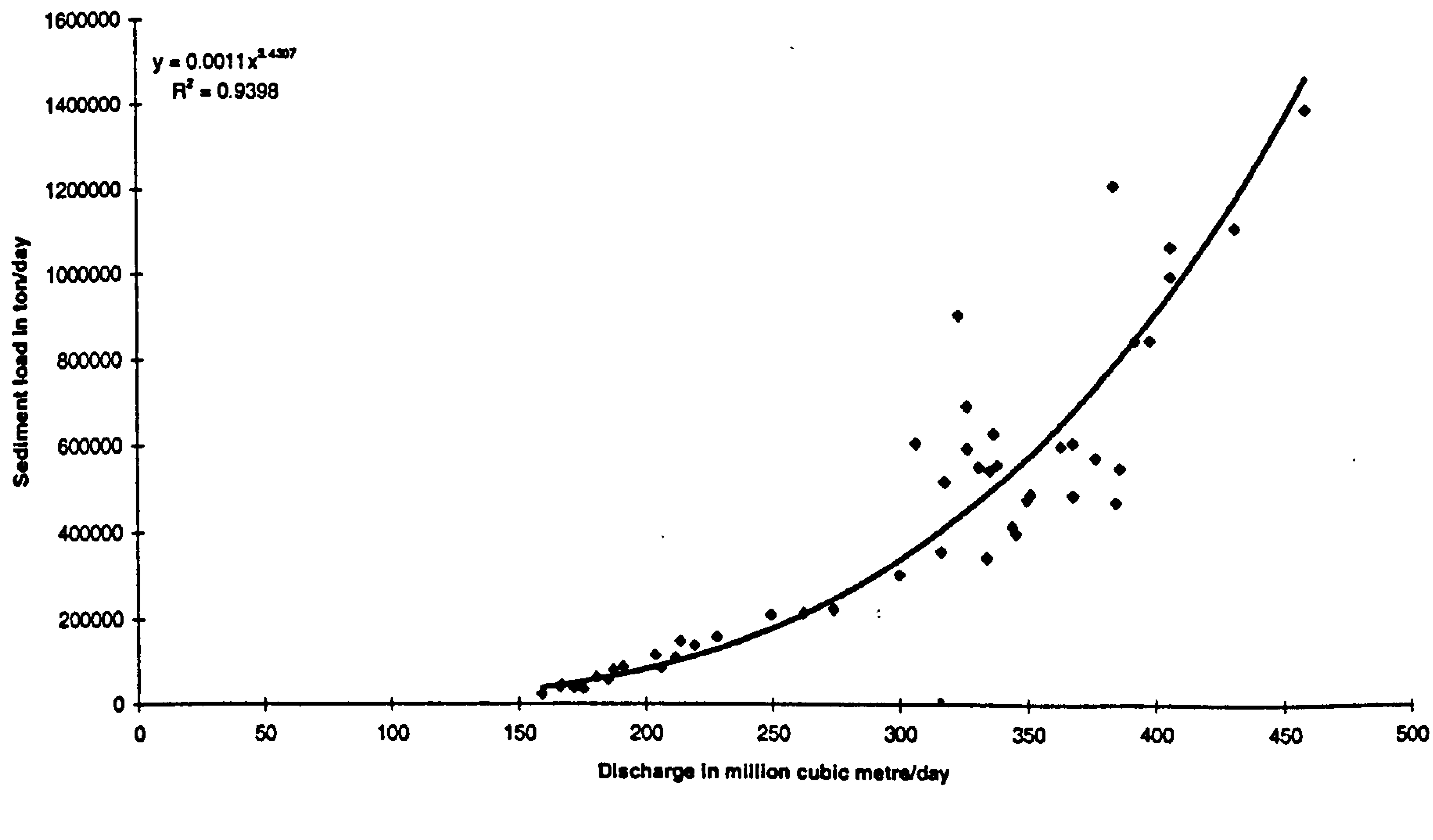


Figure (5.5) Sediment load during rising and falling flood - Blue Nile

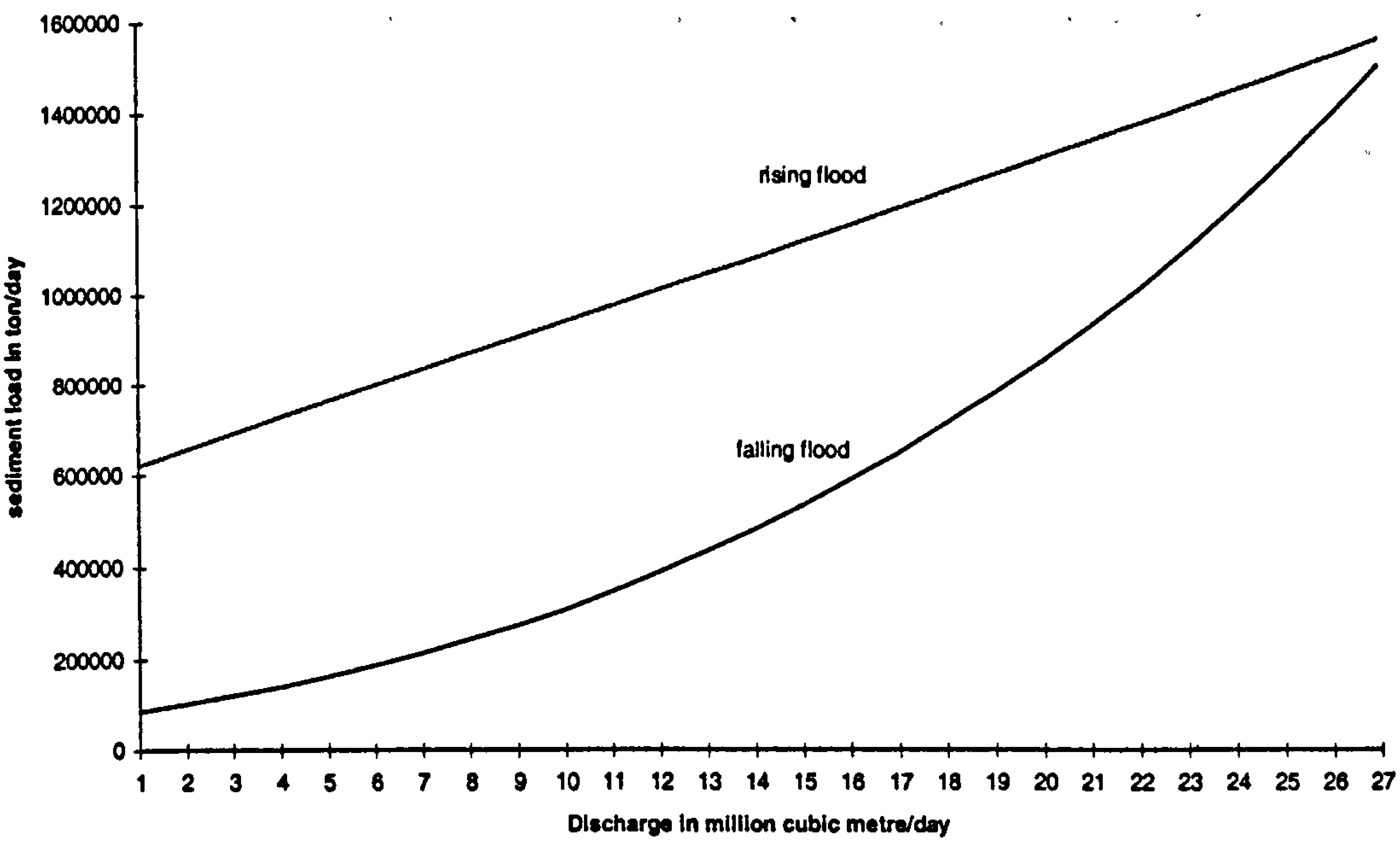


Figure (5.6) Sediment deposits - Roseries - 1992

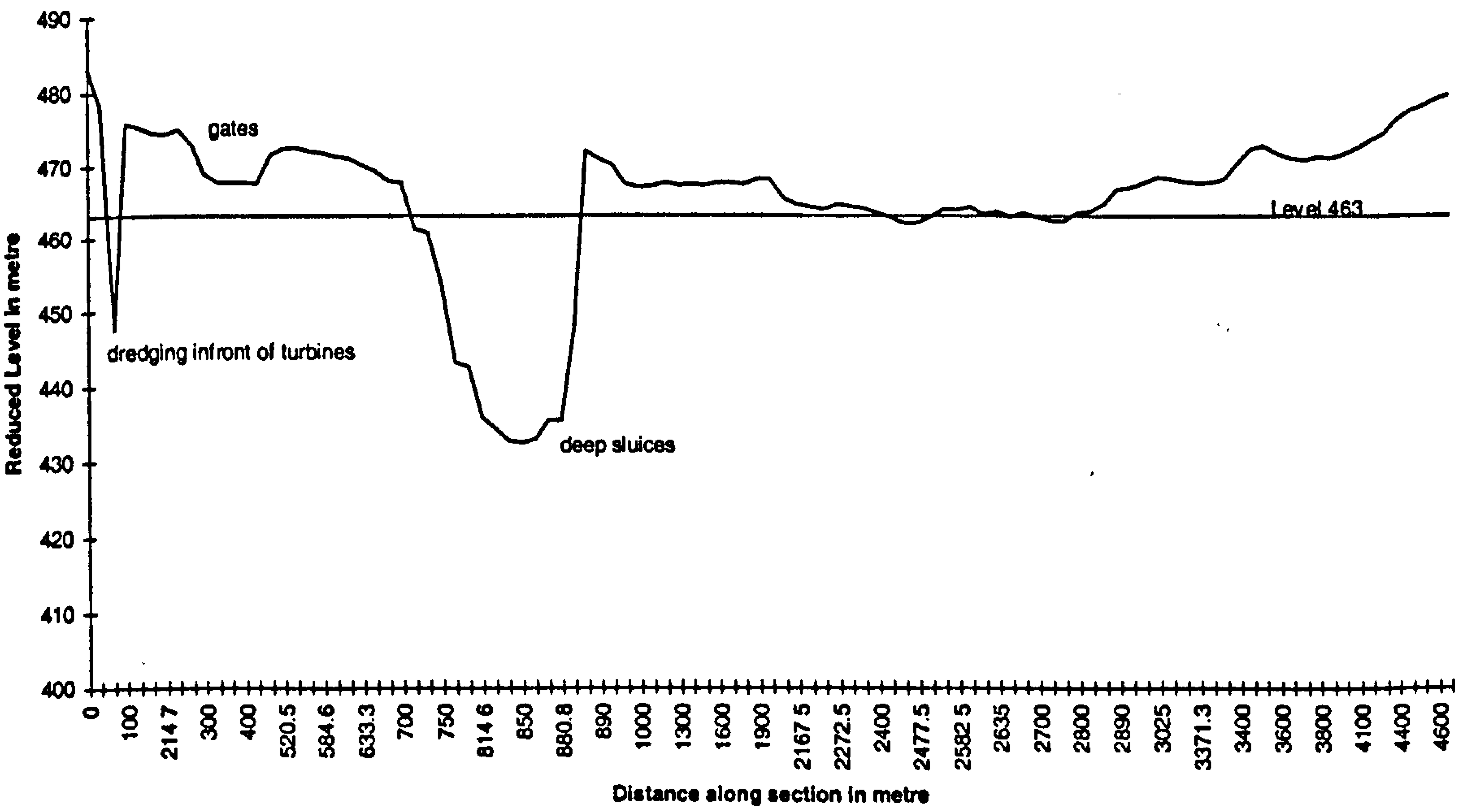


Figure (5.7) Variation of reservoir content with depth - Roseries - 1976

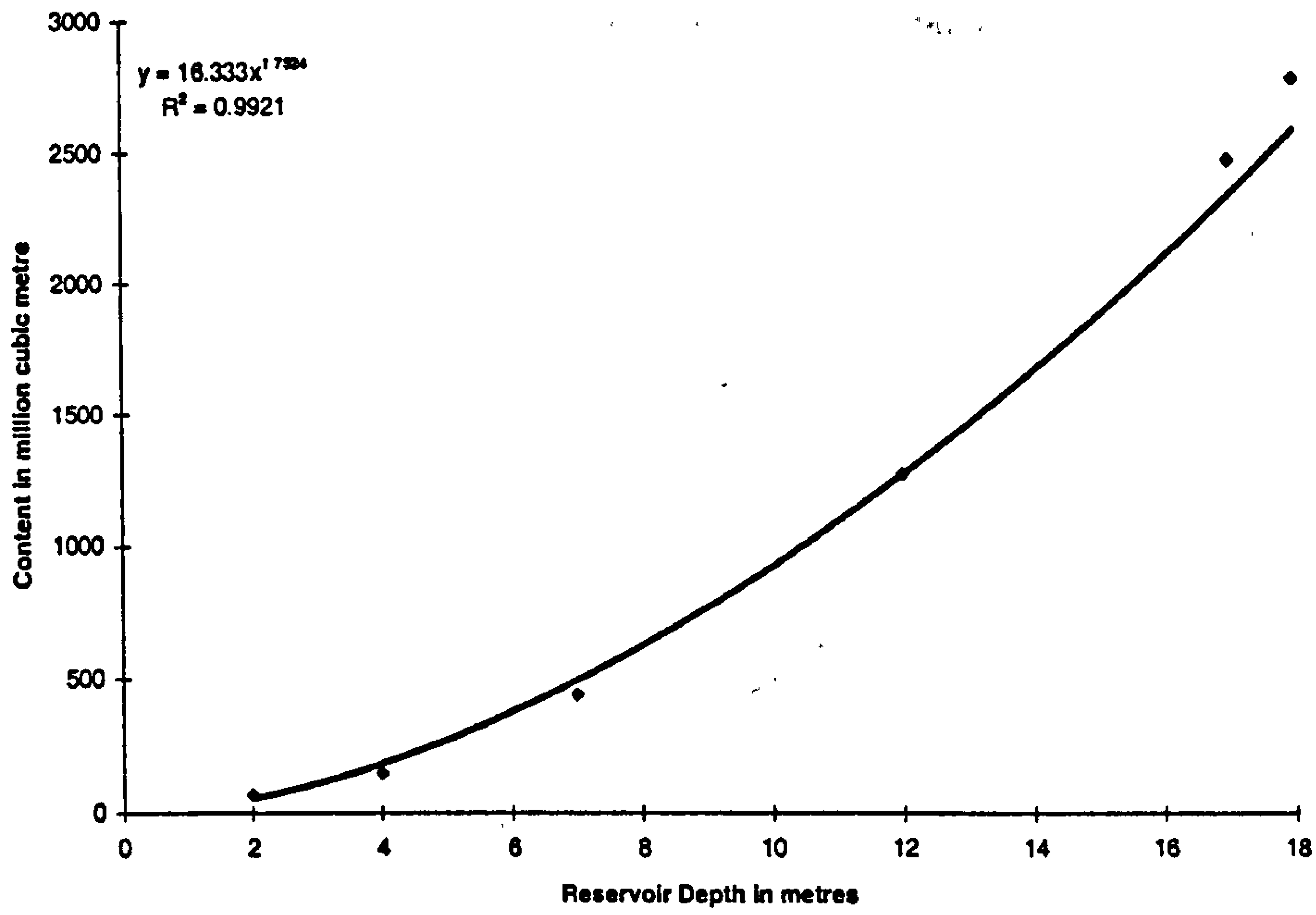


Figure (5.8) Variation of reservoir content with depth - Roseries - 1981

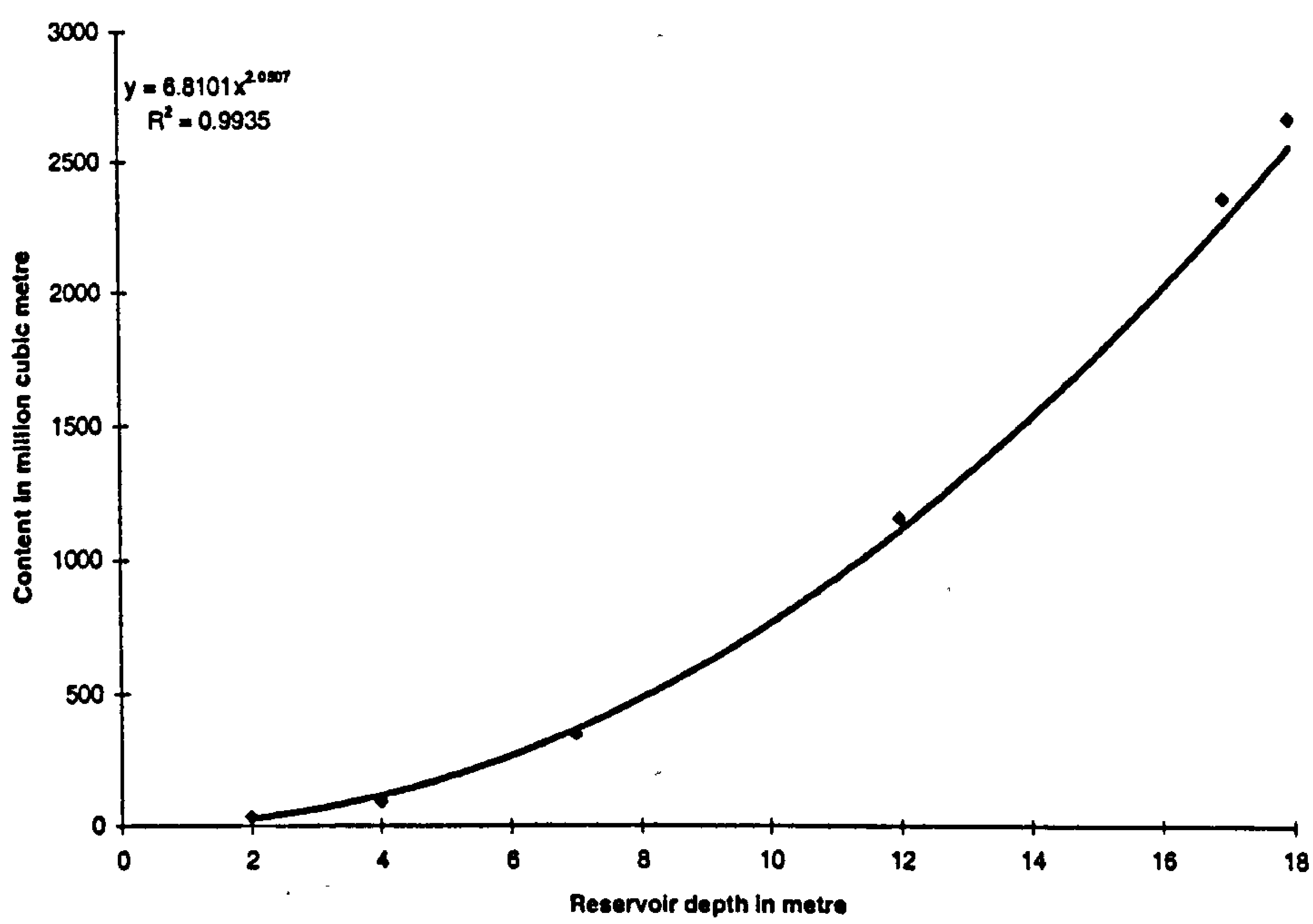


Figure (5.9) Variation of reservoir content with depth - Roseries - 1985

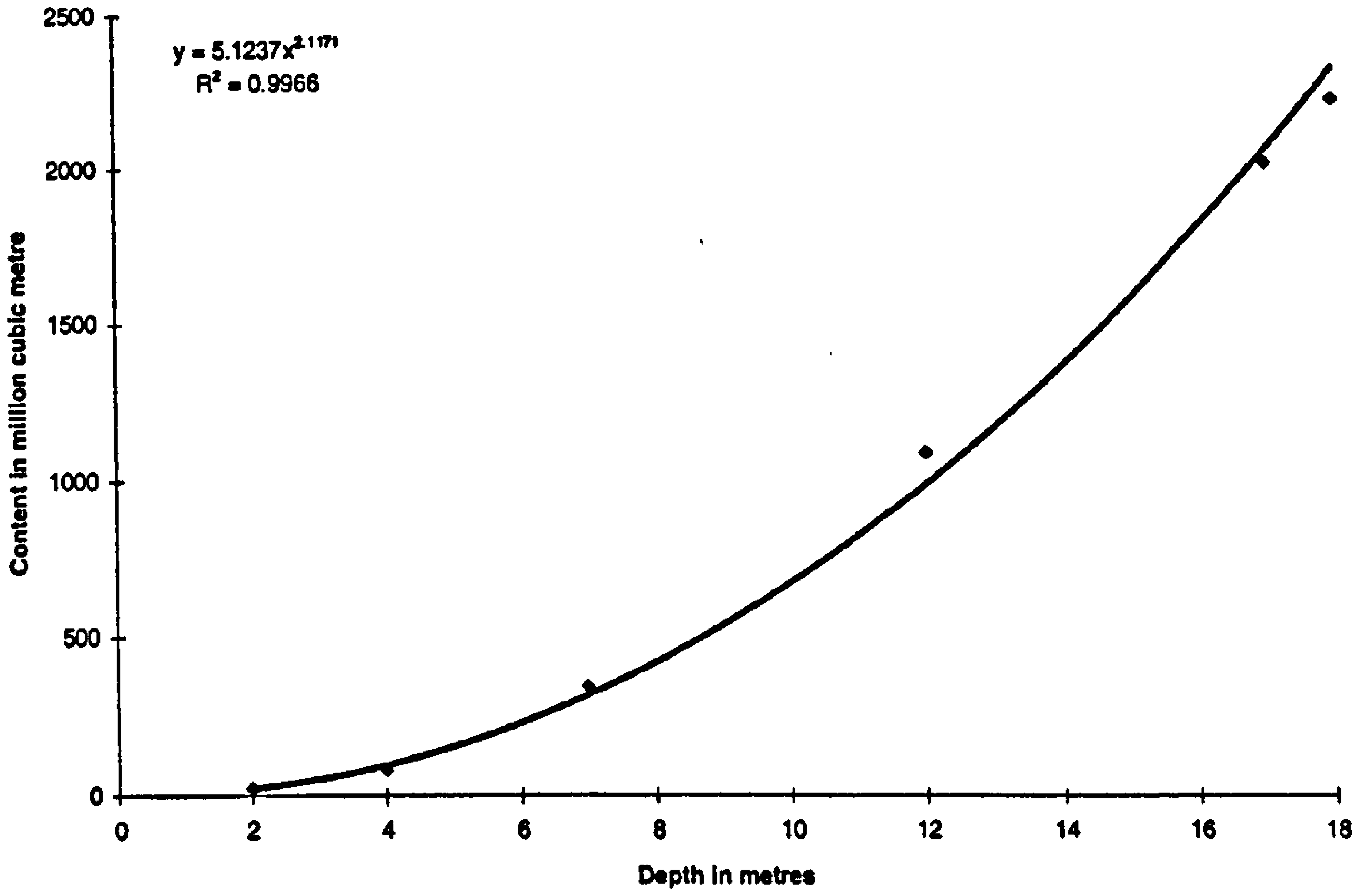


Figure (5.10) Variation of content with depth - Roseries - 1992

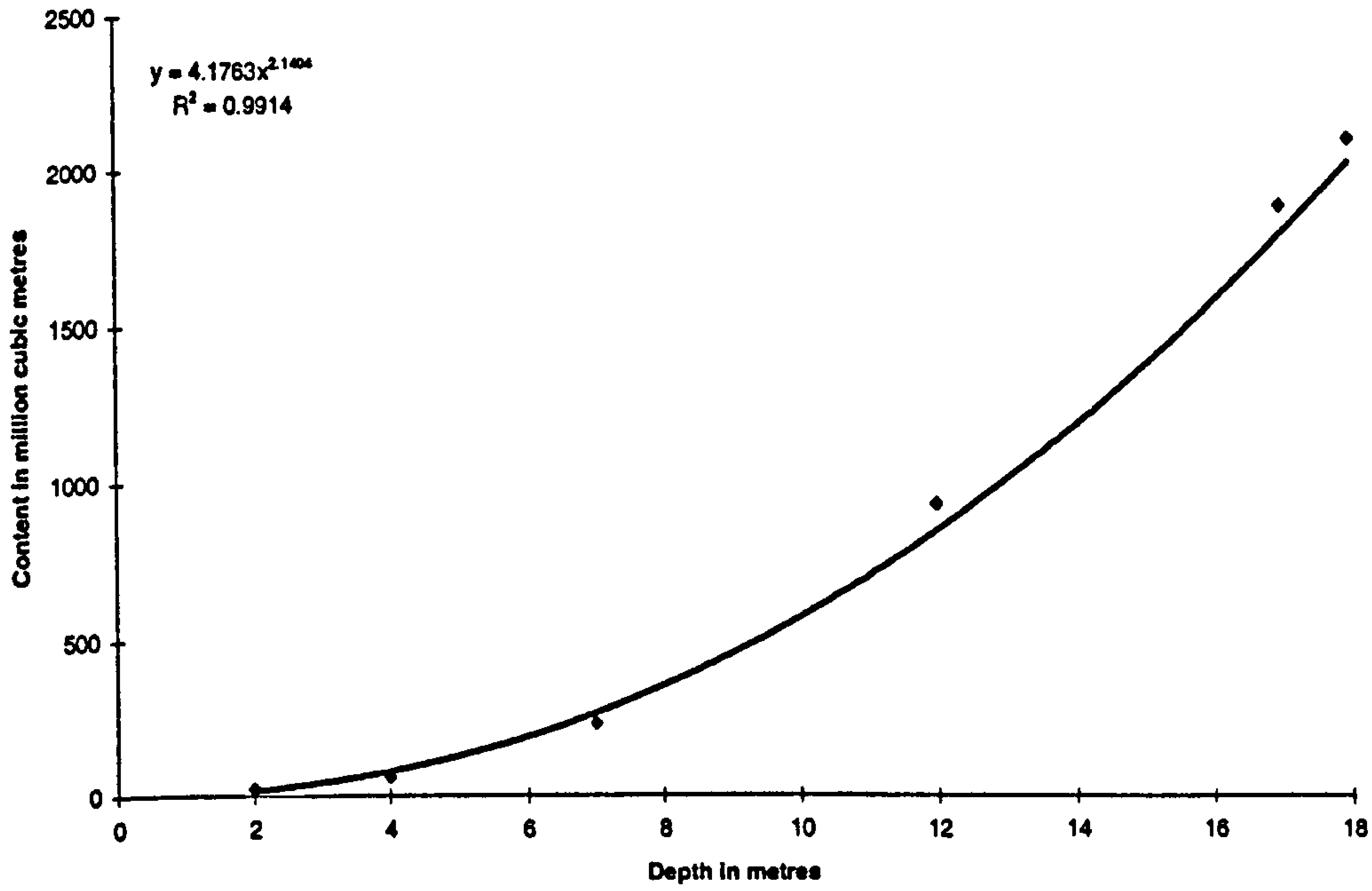


Figure (5.11) Variation of coefficient " a" with time - Roseries

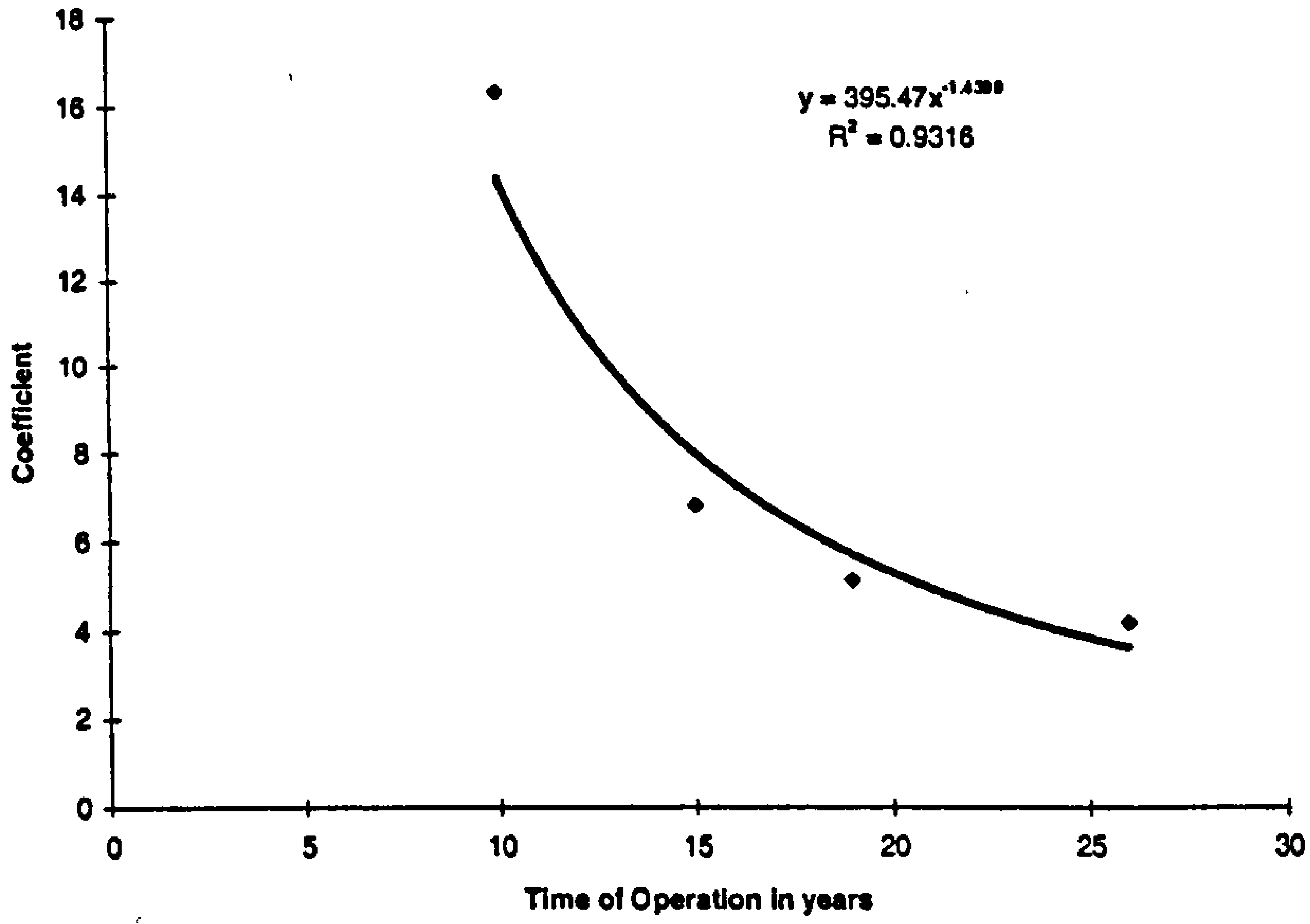
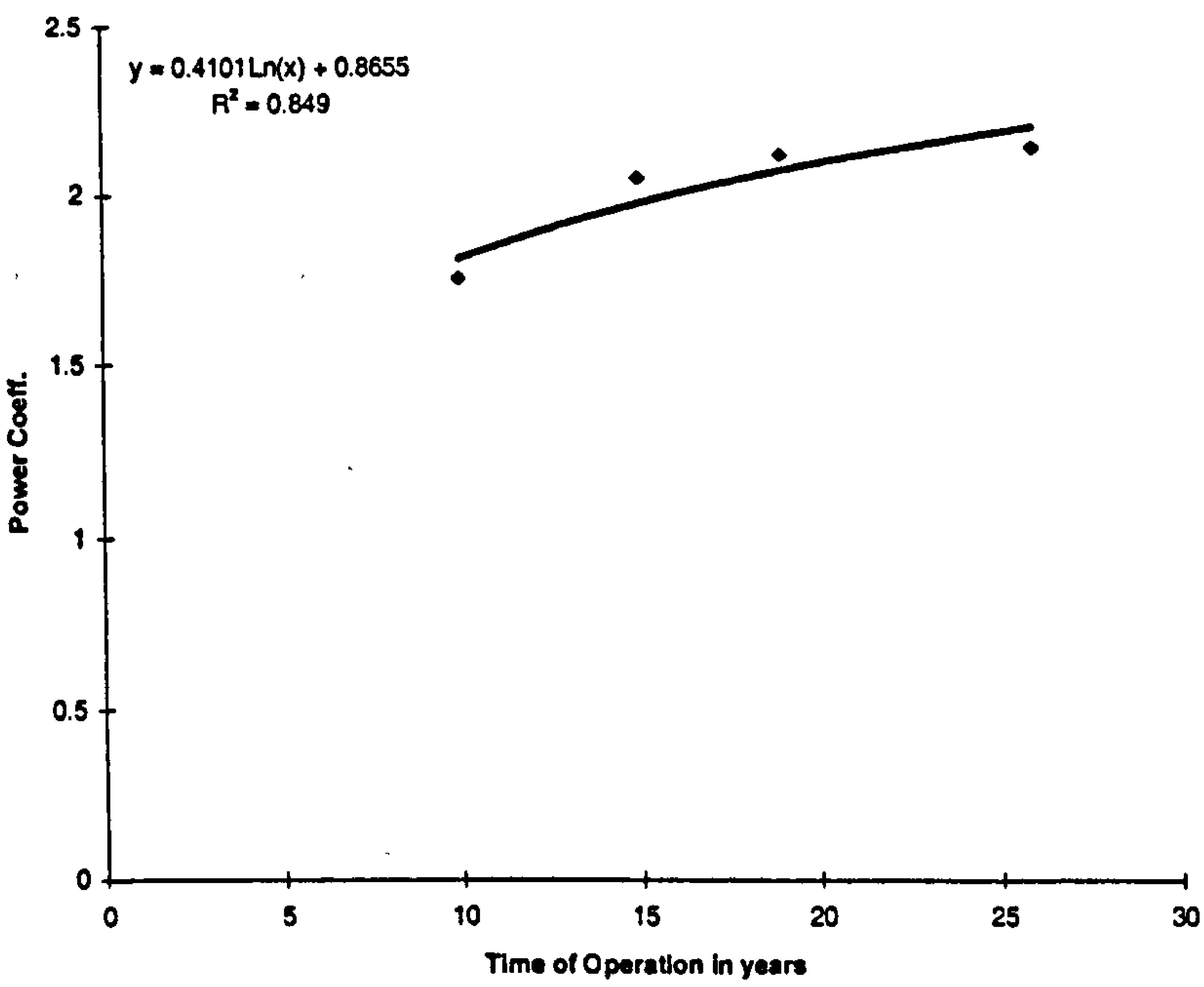


Figure (5.12) Variation of power coefficient "m" with time - Roseries



CHAPTER VI

EVAPORATION MODELLING

Summary ~ Here the reservoir evaporation losses are modelled using Penman equation so as to be incorporated in the optimization model later in Chapter IX. The developed model is then verified using water balance method.

6.1 INTRODUCTION

The word evaporation is used to describe water loss from water or bare-soil surfaces. It is a process in which moisture is vaporised and moved up in the atmosphere. To cause evaporation solar radiation provides energy that causes vaporisation while wind moves vapour upwards (Ayoade, 1988). Many methods can be applied to estimate evaporation losses from open water bodies, i.e. canals and reservoirs. These methods can estimate evaporation from water balance, use evaporation pan approach, mass transfer approach or Penman approach. It has been found that the latter approach is among the methods that give good evaporation losses estimates (Winter et al., 1995). Therefore it is applied here to estimate the evaporation losses from the Blue Nile system, which includes two in series reservoirs and a network of irrigation canals. Three stations that have been chosen for this purpose are Damazin, Singa and Wad Medani (Figure 4.2). The first two stations are located nearby Roseries and Sennar reservoirs respectively while the third station lies in the heart of the irrigated area and the evaporation estimated using data from this station can, thus, represent evaporation losses from irrigation canals.

Although Penman method is well theoretically based and widely recognised, it assumes that the evaporating body has no heat capacity and isolated. In other words the body doesn't absorb or transfer heat to the surroundings (Sibbons, 1962). Therefore, if the heat capacity of a large water body is neglected, then the evaporation might have been underestimated. For this reason and to validate the use of Penman equation, the evaporation losses from Roseries reservoir have been calculated from water balance and the results obtained from the two methods are then compared.

6.2 PENMAN EQUATION

Penman equation is formed of two terms: the energy or radiation and the wind (Penman, 1948). The equation has the following form (McCulloch, 1965):

$$E_o = \frac{\Delta}{\Delta+\gamma}\{R_a(1-r)(a+b*n/N)\} - \frac{\Delta}{\Delta+\gamma}\{\sigma T_a^4(0.56-0.092*(e_a)^{0.5})(0.10+0.9\,n/N)\} + \frac{\Delta}{\Delta+\gamma}\{0.35(1+U/100)(e_s-e_a)\} \quad (6.1)$$

The first term is the incoming radiation, the second is the outgoing radiation and the third is the aerodynamic.

E_o is evaporation in [mm / day].

Δ slope of the saturation vapour pressure curve for water at mean air temperature.

γ constant of the wet and dry bulb psychorometer equation.

$\frac{\Delta}{\Delta+\gamma}$ is a weighting factor to relate solar radiation to evaporation.

R_a the theoretical radiation, extraterrestrial radiation, that would be received at the ground surface in absence of atmosphere [mm/day].

r is the albedo (reflection coefficient) of the evaporating surface. r is 0.05 for open water surface and 0.25 for grass surfaces (Ayoade, 1988).

a,b are constants. 0.25 and 0.5 can be used for a and b respectively (FAO, 1984).

n actual hours of bright sunshine.

N possible hours of bright sunshine.

σ Boltzman constant = $4.903*10^{-9}$ [MJ/ m^2,$^0K^4$,day] = $2.0177*10^{-9}$ [mm/day,$^0K^4$], (FAO and WMO Training Manual, nd).

T_a Average temperature in Kelvin, ^{0}K.

e_a actual vapour pressure at the mean air temperature [mbar].

e_s saturation vapour pressure of water at the mean air temperature [mbar].

U wind speed at 2 metre level above the ground [mile/day]

6.3 PENMAN EQUATION PARAMETERS DETERMINATION

To calculate E_0 then Δ, γ, R_a, N, e_s and e_a have to be calculated first.

6.3.1 Determination of e_s and e_a

The saturation vapour pressure of water at mean air temperature, e_s, can be determined from equation (6.2) while the actual vapour pressure of water at mean air temperature, e_a, can be determined from equation (6.3) (FAO and WMO, nd).

$$e_s = 6.108 * e^{\frac{17.27T}{T+237.27}} \quad (6.2)$$

$$e_a = e_s \frac{RH}{100} \quad (6.3)$$

Where:

e_s and e_a are in mbar.

RH is relative humidity in % .

T average air temperature in ^{0}c.

Knowing average temperature and relative humidity RH, e_s and e_a can be calculated.

6.3.2 Determination of R_a and N

Determination of R_a and N goes as follows (FAO and WMO, nd):

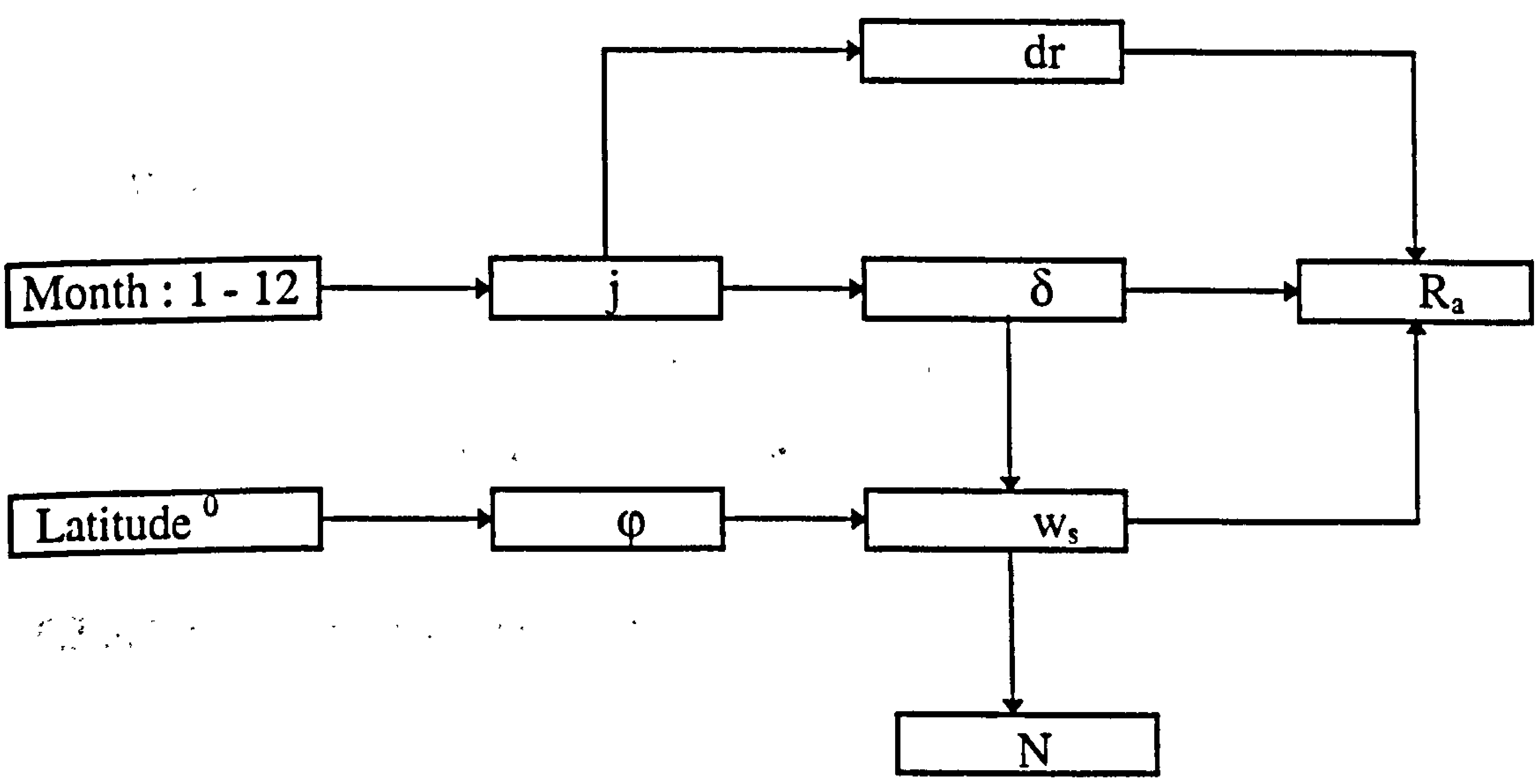


Figure (6.1) Flowchart showing R_a and N calculation steps

$$R_a = 37.586\, d_r [w_s \sin(\varphi) \sin(\delta) + \sin(\varphi) \sin(\delta) \sin(w_s)] \quad (6.4)$$

$$N = \frac{24}{\pi} w_s \quad (6.5)$$

Where :

R_a extra terrestrial radiation in [MJ / m^2, day] , to convert R_a in mm/day, the right hand side of equation (6.4) should be divided by 2.45.

d_r relative distance earth - sun.

w_s sunset hour angle.

φ Latitude [rad]

δ solar declination [rad]

With :

$$d_r = 1 + 0.033 \cos((2\pi/365)*J) \quad (6.6)$$

$$J = \text{Integer}(30.42M - 15.23) \quad (6.7)$$

(J Julian day number and M month number = 1 for January and 12 for December)

$$\delta = 0.4093 \sin((2\pi/365)*J - 1.405) \quad (6.8)$$

$$w_s = \text{acos}(-\tan(\varphi) \tan(\delta)) \quad (6.9)$$

6.3.3 Determination of Weighting Factor $\frac{\Delta}{\Delta + \gamma}$

6.3.3.1 Determination of slope vapour pressure curve Δ

$$\Delta = \frac{4098\, e_s}{(T+237.3)^2} \quad (6.10)$$

Where :

Δ = Slope vapour pressure curve [kpa / ^{0}C]

T = average air temperature [^{0}C].

e_s = saturation vapour pressure [kpa].

6.3.3.2 Psychometric Constant (γ)

$$\gamma = \frac{C_p \,.\, P}{E\lambda} * 10^{-3} = 0.016286 \frac{P}{\lambda} \quad (6.11)$$

Where :

C_p = specific heat of moist air = 1.013 [kj /kg ,^{0}C]

P = atmospheric pressure [kpa]

E = ratio molecular weight water vapour / dry air = 0.062

λ = Latent heat of vaporisation.

6.3.3.3 Latent Heat of Vaporisation (λ)

$$\lambda = 2.501 - (2.361*10^{-3})*T \tag{6.12}$$

Where:

λ = Latent heat of vaporisation [MJ/kg]

T = mean air temperature [^{0}C]

6.3.3.4 Atmospheric Pressure (P) :

$$P = 101.3\left[\frac{293 - 0.0065\,Z}{293}\right]^{5.256} \tag{6.13}$$

P is in [kpa]

Z altitude in metres.

6.3.4 Steps for Calculating E_0

To calculate E_o , the following steps are followed :

a) Knowing T_{max} and T_{min}, find the mean temperature, T.

b) Substitute T in equation (6.2) to find e_s.

c) Substitute the known relative humidity, RH, and e_s, calculated in (b), in equation (6.3), to find e_a .

d) From T and equation (6.12), calculate λ .

e) From altitude Z and equation (6.13), find P.

f) From T and e_s and equation (6.10), calculate Δ .

g) Substitute P and λ in equation (6.11) to find γ .

h) From the month number, M, and equation (6.7), find Julian day number J.

j) Substitute J in equation (6.8) to find δ.

k) Substitute δ and latitude φ, in radians, in equation (6.9), to find w_s.

l) Substitute, J, found in step h, in equation (6.6), to find d_r.

m) Substitute for w_s in equation (6.5) to find N.

n) Substitute for d_r, w_s, φ and δ, found in the above steps, in equation (6.4) to find R_a in [MJ/m², day]. Divide by 2.45 to obtain R_a in mm/ day

o) Substitute the following in equation (6.1) to obtain E_0

1. The obtained results from steps b, c, f, g, m and n.
2. Values for $r = 0.05$, $a = 0.25$, $b = 0.5$ and $\sigma = 2.0177 * 10^{-9}$.
3. The known, n, in hours, the known T_a in Kelvin and the known wind speed in mile/day.

6.4 DATA FROM SELECTED STATIONS

The following tables show the data for the three selected locations, required for calculating, E_0. The source of the data is the FAO climwat database, (FAO, 1993). The stations are Wad Medani, Singa and Damazin (Figure 4.2). The results obtained from the first station will represent evaporation losses from irrigation canals, while the results from the second and third stations will represent the losses from Sennar and Roseries reservoirs respectively.

Table (6.1) Wad Medani station data, altitude = 408 m, latitude = 14.24 °N.

Month	T_{max} °C	T_{min} °C	R.H %	Wind speed km/day	actual bright sunshine-hours	rain (mm)
January	33.5	14	35	216	10.3	0
February	35	14.8	27	242	10.7	0
March	38.3	18.1	21	216	10.4	0
April	40.2	21	19	190	10.6	1
May	41.3	23.8	28	216	10.1	15
June	39.6	24.5	39	268	9.3	28
July	35.7	22.7	57	268	7.7	94
August	33.2	21.8	71	242	7.6	105
Sept-	35.2	21.7	65	190	9.2	44
October	37.7	21.5	48	138	9.9	18
Novemb	36.5	18	37	190	10.4	1
December	33.7	14.5	38	216	10.5	0

Table (6.2) Singa station data, altitude = 430 m, latitude = 13.09 °N.

Month	T_{max} °C	T_{min} °C	R.H %	Wind speed km/day	actual bright sunshine-hours	rain (mm)
January	34.8	16.5	44	164	10.4	0
February	36.3	17.5	38	164	10.6	0
March	39.5	19.8	34	164	10.9	0
April	41.1	22.7	32	190	11.1	4
May	40.2	24.3	39	190	10.7	25
June	37.5	23.5	50	164	10.0	64
July	33.7	22	63	164	8.5	120
August	32	21.5	71	164	8.1	135
Sept-	33.5	20.3	74	164	9.1	81
October	37.1	21.5	60	138	9.9	53
Novemb	37.5	20	45	138	10.4	1
December	35.1	17.5	44	138	10.4	0

Table (6.3) Damazin station data, altitude = 470 m, latitude = 11.47 °N.

Month	T_{max} °C	T_{min} °C	RH %	Wind speed km/day	actual bright sunshine- hours	rain (mm)
January	35.5	16.6	33	138	9.8	0
February	37	18	27	138	10.1	0
March	39.5	21.6	25	138	9.7	1
April	40.1	23.5	29	138	9.5	19
May	38.5	24.8	41	164	8.2	34
June	35	22.7	58	190	6.9	96
July	31.6	21.5	73	164	5.3	110
August	30.7	21	80	138	5.6	120
Sept-	32.5	20.8	77	112	6.9	117
October	34.8	20.5	66	112	7.9	107
Novemb	36.5	18.5	46	112	9.5	3
December	35.8	16.2	36	138	10.3	0

6.5 EVAPORATION LOSSES ESTIMATION

6.5.1 Evaporation rate, E_0

Using the data in Tables (6.1) to (6.3) and following the steps described in Section (6.3.4) above, the evaporation rate, E_0, is calculated. Tables (6.4), (6.5) and (6.6) show the results for Wad Medani, Singa and Damazin respectively.

Table (6.4) Eo calculation results for Wad Medani

Month	T C	es mbar	ea mbar	Δ kpa/C	P Kpa	λ MJ/Kg	γ kpa/c	$\frac{\Delta}{\Delta+\gamma}$	M	J	δ rad	Ws rad	dr	N hour	Ra mm/day	Eo mm/day
Jan	23.75	29.39	10.29	1.768	96.57	2.445	0.643	0.733	1	15	-0.373	1.471	1.032	11.25	12.13	7.4
Feb	24.9	31.49	8.5	1.877	96.57	2.442	0.644	0.745	2	46	-0.236	1.51	1.023	11.54	13.41	8.7
Mar	28.2	38.24	8.03	2.223	96.57	2.434	0.646	0.775	3	76	-0.04	1.561	1.009	11.93	14.75	9.7
Aprl	30.6	43.91	8.34	2.507	96.57	2.429	0.648	0.795	4	106	0.166	1.613	0.992	12.33	15.53	10.1
May	32.55	49.05	13.73	2.76	96.57	2.424	0.649	0.81	5	137	0.333	1.659	0.977	12.68	15.67	10.8
June	32.05	47.68	18.6	2.693	96.57	2.425	0.648	0.806	6	167	0.407	1.68	0.968	12.84	15.57	10.9
Jul	29.2	40.52	23.1	2.338	96.57	2.432	0.647	0.783	7	198	0.372	1.67	0.968	12.76	15.56	8.9
Aug	27.5	36.71	26.07	2.146	96.57	2.436	0.646	0.769	8	228	0.239	1.633	0.977	12.48	15.51	7.7
Sept	28.45	38.8	25.22	2.251	96.57	2.434	0.646	0.777	9	259	0.037	1.58	0.992	12.08	14.95	8.3
Oct	29.6	41.47	19.9	2.385	96.57	2.431	0.647	0.787	10	289	-0.169	1.527	1.008	11.68	13.79	8.2
Nov	27.25	36.18	13.39	2.118	96.57	2.437	0.645	0.766	11	319	-0.331	1.484	1.023	11.34	12.47	7.9
Dec	24.1	30.02	11.41	1.8	96.57	2.444	0.644	0.737	12	350	-0.407	1.461	1.032	11.17	11.75	7.2

Table (6.5) Eo calculation results for Singa

Month	T C	es mbar	ea mbar	Δ kpa/C	P Kpa	λ MJ/Kg	γ kpa/c	$\frac{\Delta}{\Delta+\gamma}$	M	J	δ rad	Ws rad	dr	N hour	Ra mm/day	Eo mm/day
Jan	25.65	32.92	14.49	1.951	96.32	2.44	0.643	0.752	1	15	-0.373	1.48	1.032	11.31	12.37	7.18
Feb	26.9	35.44	13.47	2.081	96.32	2.437	0.644	0.764	2	46	-0.236	1.515	1.023	11.58	13.59	8.12
Mar	29.65	41.59	14.14	2.391	96.32	2.431	0.645	0.788	3	76	-0.04	1.562	1.009	11.94	14.84	9.37
Aprl	31.9	47.28	15.13	2.674	96.32	2.426	0.647	0.805	4	106	0.166	1.61	0.992	12.3	15.52	10.55
May	32.25	48.22	18.81	2.72	96.32	2.425	0.647	0.808	5	137	0.333	1.651	0.977	12.62	15.58	10.43
June	30.5	43.66	21.83	2.495	96.32	2.429	0.646	0.794	6	167	0.407	1.671	0.968	12.77	15.44	9.24
Jul	27.85	37.47	23.61	2.184	96.32	2.435	0.644	0.772	7	198	0.372	1.662	0.968	12.7	15.45	7.86
Aug	26.75	35.13	24.95	2.065	96.32	2.438	0.643	0.762	8	228	0.239	1.627	0.977	12.44	15.46	7.31
Sept	26.9	35.44	26.23	2.081	96.32	2.437	0.644	0.764	9	259	0.037	1.579	0.992	12.07	15.01	7.51
Oct	29.3	40.76	24.45	2.35	96.32	2.432	0.645	0.785	10	289	-0.169	1.531	1.008	11.7	13.94	8.1
Nov	28.75	39.48	17.77	2.286	96.32	2.433	0.645	0.78	11	319	-0.331	1.491	1.023	11.4	12.69	7.7
Dec	26.3	34.22	15.05	2.018	96.32	2.439	0.643	0.758	12	350	-0.407	1.47	1.032	11.24	12	6.87

Table (6.6) Eo calculation results for Damazin (Roseries)

Month	T C	es mbar	ea mbar	Δ kpa/C	P Kpa	λ MJ/Kg	γ kpa/c	Δ / Δ+γ	M	J	δ rad	Ws rad	dr	N hour	Ra mm/day	Eo mm/day
Jan	26.05	33.71	11.13	1.992	95.87	2.439	0.64	0.757	1	15	-0.373	1.491	1.032	11.4	12.69	7.1
Feb	27.5	36.71	9.91	2.146	95.87	2.436	0.641	0.77	2	46	-0.236	1.522	1.023	11.63	13.83	7.9
Mar	30.55	43.79	10.95	2.501	95.87	2.429	0.643	0.796	3	76	-0.04	1.563	1.009	11.94	14.96	8.9
Aprl	31.8	47.01	13.63	2.66	95.87	2.426	0.644	0.805	4	106	0.166	1.605	0.992	12.27	15.5	9.4
May	31.65	46.62	19.11	2.641	95.87	2.426	0.644	0.804	5	137	0.333	1.641	0.977	12.54	15.44	9.1
June	28.85	39.71	23.03	2.297	95.87	2.433	0.642	0.782	6	167	0.407	1.658	0.968	12.68	15.25	7.8
Jul	26.55	34.72	25.35	2.044	95.87	2.438	0.64	0.761	7	198	0.372	1.65	0.968	12.61	15.29	6.1
Aug	25.85	33.32	26.65	1.972	95.87	2.44	0.64	0.755	8	228	0.239	1.62	0.977	12.38	15.39	5.8
Sept	26.65	34.93	26.89	2.054	95.87	2.438	0.641	0.762	9	259	0.037	1.578	0.992	12.06	15.07	6.4
Oct	27.65	37.06	24.44	2.162	95.87	2.436	0.641	0.771	10	289	-0.169	1.536	1.008	11.74	14.14	6.9
Nov	27.5	36.71	16.89	2.146	95.87	2.436	0.641	0.77	11	319	-0.331	1.501	1.023	11.47	12.99	7.1
Dec	26	33.61	12.1	1.987	95.87	2.44	0.64	0.756	12	350	-0.407	1.483	1.032	11.34	12.34	7

6.5.2 Amount of Losses Using Penman Method

Losses from reservoirs can be estimated as the difference between evaporation from and rainfall on those reservoirs in accordance with the following relationship:

Volume of Losses = (Evaporation rate, E_0, - Rainfall) * Water surface area, A. (6.14)

Water surface area varies with the water level of stored water in a reservoir. Variation of water surface area with water level, (MOI, 1968), for both Roseries and Sennar reservoirs can be approximated, using software excel, by the following relations:

Roseries Reservoir

$$A = 0.4809\, H^2 - 441.5\, H + 101404,\ R^2 = 0.99 \qquad (6.15)$$

Sennar Reservoir

$$A = -2.1943\, H^2 + 1855.3\, H - 391978,\ R^2 = 0.99 \qquad (6.16)$$

Where :

A is the area in squared kilometre.

H is the stage in metres.

Substituting A, E_0 and rainfall in relation (6.14), losses can be estimated in volume. Knowing the daily water levels, during the period July 1993 - June 1994, the daily water surface area is obtained from relation (6.15). This water surface area, together with rainfall (Table 6.3) and E_0 (Table 6.6) are substituted in relation (6.14) to estimate daily evaporation losses using Penman approach (Appendix A). The monthly losses are found by summing up the daily losses, Table (6.8).

6.6 ESTIMATION OF LOSSES FROM WATER BALANCE

Losses can be estimated as a residual from water balance. They can be included in mass balance equation as follows:

Monthly inflow = monthly outflow + monthly losses +

final contents - initial contents (6.17)

The monthly inflows and outflows, Table (6.7), are obtained from Roseries reservoir resident engineer operation book. The contents at the beginning and at the end of the month, Table (6.7), are obtained by substituting the water levels at the beginning and the end of the month, from appendix A, in the storage-water level relationship, equation (6.18), derived for year 1993 using models (5.3), (5.6) and (5.7).

$$S = 3.261\ H^{2.232} \qquad (6.18)$$

The monthly inflows, monthly outflows, contents at the beginning of the month and at its end are used in relation (6.17) to obtain the monthly losses, Table (6.7). The results of the two methods are compared, Table (6.8). The difference between the two methods lies in the range ± 6 percent.

Table (6.7) Calculation of losses from water balance - Roseries

(Figures are in million m^3)

Month	Inflow	Outflow	Initial Content	Final Content	Losses
Jul - 93	8397.2	8481.4	183.25	89.94	9.045
Aug - 93	15119.3	15098.2	89.94	104.14	6.847
Sept - 93	12662.1	11587.2	104.14	1169.72	9.361
Oct - 93	7724	6917.2	1169.72	1947.68	28.791
Nov - 93	3318.5	3133.1	1947.68	2066.17	66.94
Dec - 93	1564.2	1518	2066.17	2045.73	66.648
Jan - 94	890.0	952.5	2045.73	1913.13	70.140
Feb - 94	502.9	787.2	1913.13	1566.44	62.444
Mar - 94	341.6	741.1	1566.44	1103.41	63.446
April - 94	275.2	818.2	1103.41	512.68	47.714
May - 94	1119.7	1338.7	512.68	257.43	36.281
June - 94	2927.4	3049.3	257.43	117.38	18.183
Total	54842	54422.1			485.838

Table (6.8) Evaporation losses, Roseries, in million m^3, using different methods

Month	Water Balance Method	Penman Method	% difference
Jul - 93	9.045	8.781	-2.92
Aug - 93	6.847	7.005	2.31
Sept - 93	9.361	9.674	3.35
Oct - 93	28.791	28.892	0.35
Nov - 93	66.94	64.569	-3.54
Dec - 93	66.648	65.917	-1.1
Jan - 94	70.140	65.671	-6.37
Feb - 94	62.444	61.814	-1.01
Mar - 94	63.446	66.727	5.17
April - 94	47.714	50.447	5.73
May - 94	36.281	35.588	-1.91
June - 94	18.183	17.966	-1.19

6.7 CONCLUSIONS

Penman original equation has been used to estimate monthly evaporation rate, E_0, in the Blue Nile system. Further the monthly losses for Roseries reservoir have been estimated as a product of water surface area and the term (E_0 - rainfall). Alternatively, the monthly losses have been estimated for the same reservoir, as a residual from water balance. The difference between the methods lies in the range ± 6 percent. This difference is acceptable, if the error in measuring inflowing and outflowing discharges is taken into account. Therefore Penman approach can be used in modelling the evaporation losses from the Blue Nile system. This finding verifies the first part of hypothesis 4. The evaporation models fitted here will be incorporated in the optimization model to be developed in Chapter IX.

CHAPTER VII

BLUE NILE FLOW MODELLING

Summary ~ In this Chapter the Blue Nile flow is modelled. The model is used in generating samples that will be used as inputs to the optimization model in Chapter IX.

7.1 INTRODUCTION

Recent applications, as will be done in the following chapters, mixed implicit optimization, regression analysis and simulation to derive and test operation rules (Section 2.5). The Blue Nile flow is characterised by a period of low flow (Section 4.2.2.b). Recession and low flow analysis are basically used in separation of base flow from flood flow and in low flow forecast (Martin, 1973). Therefore these models are not suitable for representing the flow implicitly. Alternatively time series analysis approach will be used to model the flow implicitly. A statistical model will be fixed and used in generating samples with equal probabilities of occurrence. This would allow testing the operation policies, to be derived later, against a range of flow sequences that could occur. The widely used models for hydrologic time series modelling are autoregressive, AR(p), and autoregressive moving average, ARMA(p,q) models (Salas et al., 1997). They have the following model forms (Box and Jenkins 1970),

AR(p): $$Z_t = \sum_{j=1}^{p} \phi_j Z_{t-j} + \varepsilon_t \qquad (7.1)$$

ARMA(p,q): $$Z_t = \sum_{j=1}^{p} \phi_j Z_{t-j} + \varepsilon_t - \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} \qquad (7.2)$$

Where ϕ_j the autoregressive parameter, Z_t standardised time series; ε_t residuals, (white noise); p autoregressive model order; q the moving average model order and θ_j is the moving average parameter.

7.2 DESCRIPTION OF THE MODELLING PROCEDURE

This section summarises the modelling process described by Salas et al., (1997).

7.2.1 Preliminary Analysis

The aim of the preliminary analysis is to have a general idea about the types of models that may be selected. They follow the following steps:

step(1a) check the original time series $X_{v,\tau}$ for normality, v is for years and τ is for months. If the series is normal take $y_{v,\tau} = X_{v,\tau}$ and go to step(1c).

step(1b) if the original series, $X_{v,\tau}$, is not normal, make a suitable transformation to bring it to normality and denote the transformed series by $y_{v,\tau}$.

step(1c) Plot the time series $y_{v,\tau}$ and observe its main characteristics.

step(1d) Find the estimates of the mean, $\ddot{y}_\tau$, the standard deviation, S_τ, and the periodic correlation coefficients, $r_{(k,\tau)}(y)$, as follows:

$$\ddot{y}_\tau = \frac{1}{N}\sum_{v=1}^{N} y_{v,\tau}, \tau = 1,\ldots\ldots\ldots\ldots, w \tag{7.3}$$

$$S_\tau = \sqrt{\frac{1}{N-1}\sum_{v=1}^{N}\left(y_{v,\tau} - \ddot{y}_\tau\right)^2} \tau = 1,\ldots\ldots\ldots, w \tag{7.4}$$

Where N is the number of years and w is the number of intervals during the year. These estimates are used instead of Fourier series fit when $w \leq 12$. Therefore they are suitable for monthly intervals.

$$r_k = \frac{1}{N}\sum_{v=1}^{N}\frac{(y_{v,\tau} - \ddot{y}_\tau)(y_{v,\tau-k} - \ddot{y}_{\tau-k})}{S_\tau - S_{\tau-k}} \tag{7.5}$$

When τ-k < 1, N is replaced by N-1, $y_{v,\tau-k}$ by $y_{v-1,w+\tau-k}$ and $y_{\tau-k}$ is replaced by $y_{w+\tau-k}$.

step(1e) Plot mean $\ddot{y}_\tau$ and S_τ, τ = 1, ...,12.

step(1f) Plot estimates $r_{k,\tau}$ for k = 1,2,3 and τ = 1, ...,12. If $r_{k,\tau}$ coefficients do not vary significantly from month to month, the AR or ARMA models with constant coefficients may be selected.

7.2.2 Estimation of parameters

The parameters of the likely suitable models are estimated as follows:

step(2a) From the samples the monthly mean, $\ddot{y}_\tau$, and standard deviation, S_τ, are determined. These are the estimates of the population mean, μ_τ, and the standard deviation, σ_τ, respectively.

step(2b) Standardise the series $y_{v,\tau}$, using the relation

$$Z_{v,\tau} = \frac{y_{v,\tau} - \mu_\tau}{\sigma_\tau} = \frac{y_{v,\tau} - \ddot{y}_\tau}{S_\tau}, v = 1, \ldots\ldots\ldots, N; \tau = 1, \ldots\ldots\ldots w \quad (7.6)$$

This series, $Z_{v,\tau}$, has approximately zero mean and variance 1.

step(2c)

(i) If a model with constant coefficients is selected from step(1f), the series $Z_{v,\tau}$ can be represented by a series Z_t with $t = (v-1)w+\tau$.

(ii) The sample correlogram $r_k(z)$, k=1, 2, 3, ..., of the series Z_t is determined by the following equation:

$$r_k(z_t) = \frac{\sum_{t=1}^{N-k} (Z_t - \ddot{Z}_t)(Z_{t+k} - \ddot{Z}_{t+k})}{\sqrt{\sum_{t=1}^{N-k} (Z_t - \ddot{Z}_t)^2 (Z_{t+k} - \ddot{Z}_{t+k})^2}} \quad (7.7)$$

Where

$\ddot{Z}_t$ is the mean of the first N-k values Z_1, ..., Z_{N-k}.

$\ddot{Z}_{t+k}$ is the mean of the last N-k values, Z_{k+1}, ..., Z_N.

In general $-1 \leq r_k(z) \leq +1$.

(iii) Parameter Estimation of AutoRegressive, AR, Models:

(a) Method of Moments:

AR(1):

Autoregressive Parameter ϕ_1:

$$\phi_1 = r_1 \quad (7.8)$$

Autocorrelation Function, r_k:

$$r_k = \phi_1^k \quad (7.9)$$

Estimate of the Residuals Variance, σ_ε^2:

$$\sigma_\varepsilon^2 = \frac{N\sigma^2(1-\phi_1^2)}{N-1} \quad (7.10)$$

σ^2, the variance of the series Z_t, is approximately 1.

AR(2):

Autoregressive Parameters, ϕ_1 and ϕ_2:

$$\phi_1 = \frac{r_1(1-r_2)}{1-r_1^2} \quad (7.11)$$

$$\phi_2 = \frac{r_2 - r_1^2}{1 - r_1^2} \tag{7.12}$$

Autocorrelation Function, r_k:

$$r_k = \phi_1 r_{k-1} + \phi_2 r_{k-2} \quad k \geq 2 \tag{7.13}$$

Estimate of the Residual Variance, σ_ε^2 :

$$\sigma_\varepsilon^2 = \frac{N\sigma^2(1+\phi_2)[(1-\phi_2)^2 - \phi_1^2]}{(N-2)(1-\phi_2)} \tag{7.14}$$

(b)Likelihood Method:

First the coefficients D_{ij} are calculated according to the following relation:

$$D_{ij} = D_{ji} = \frac{N}{N+2-i-j} \sum_{L=0}^{N+1-(i+j)} Z_{i+L} Z_{j+L} \tag{7.15}$$

AR(1):

Autoregressive Parameter ϕ_1:

$$\phi_1 = \frac{D_{12}}{D_{22}} \tag{7.16}$$

Residuals Variance, σ_ε^2

$$\sigma_\varepsilon^2 = \frac{1}{N-1}(D_{11} - \phi_1 D_{12}) \tag{7.17}$$

AR(2):

Autoregressive Parameter ϕ_1,ϕ_2:

$$\phi_1 = \frac{D_{12}D_{33} - D_{13}D_{23}}{D_{22}D_{33} - D_{23}^2} \tag{7.18}$$

$$\phi_2 = \frac{D_{13}D_{22} - D_{12}D_{23}}{D_{22}D_{33} - D_{23}^2} \tag{7.19}$$

Residuals Variance, σ_ε^2:

$$\sigma_\varepsilon^2 = \frac{1}{N-2}(D_{11} - \phi_1 D_{12} - \phi_2 D_{13}) \tag{7.20}$$

For large samples, methods of moments and likelihood give approximately the same estimation of autoregressive parameters.

(c) Stationary Conditions:

The following conditions are to be met by the estimated parameters.

For AR(1) model, the conditions are :

$$-1 < \phi_1 < 1 \tag{7.21}$$

For AR(2) model, the conditions are:

$$\phi_1 + \phi_2 < 1$$
$$\phi_2 - \phi_1 < 1 \qquad (7.22)$$
$$-1 < \phi_2 < 1$$

(iv) Parameter Estimation for ARMA(1,1) Model:

To obtain ARMA(1,1) general equation, substitute for p = q = 1 in relation (7.2).

$$Z_t = \phi_1 Z_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1} \qquad (7.23)$$

The parameters to be estimated are the autoregressive parameter, ϕ_1, and the moving average parameter, θ_1.

Initial Estimates of the Parameters:

ϕ_1:

$$\phi_1 = C_2 / C_1 \qquad (7.24)$$

Where C_1 and C_2 are found for k = 1 and 2 in the relation

$$C_k = \frac{1}{N}\sum_{t=1}^{N-k}(Z_t - \ddot{Z})(Z_{t+k} - \ddot{Z}), 0 \leq k < N \qquad (7.25)$$

C_k is the lag-k autocovariance.

θ_1:

Calculate C'_o, C'_1

$$C'_o = C_o + \phi_1^2 C_1 - 2\phi_1 C_1 \qquad (7.26)$$

$$C'_1 = C_1 + \phi_1^2 C_1 - \phi_1(C_2 + C_o) \qquad (7.27)$$

Substitute C'_o, C'_1 in (7.28) and (7.29)

$$\sigma_\varepsilon^2 = C'_o / (1 + \theta_1^2) \qquad (7.28)$$

$$\theta_1 = - C'_1 / \sigma_\varepsilon^2 \qquad (7.29)$$

Equations (7.28) and (7.29) are solved simultaneously for σ_ε^2 and θ_1.

Maximum Likelihood Estimate for ϕ_1, θ_1:

The maximum likelihood estimate corresponds to the minimum sum of squares of errors,

$$S(\phi_1, \theta_1) = \sum_{t=1}^{N} \varepsilon_t^2 \qquad (7.30)$$

Where $\varepsilon_1 = 0$.

$\varepsilon_2 = Z_2 - \phi_1 Z_1$.

$\varepsilon_3 = Z_3 - \phi_1 Z_2 + \theta_1 \varepsilon_2$

. . .

$$\varepsilon_N = Z_N - \phi_1 Z_{N-1} + \theta_1 \varepsilon_{N-1} \qquad (7.31)$$

In the neighbourhood of the initially estimated ϕ_1 and θ_1, calculate S using (7.30), for different values of ϕ_1 and θ_1. The values of ϕ_1 and θ_1 that give minimum value of S are the optimal estimation of the two parameters.

Residuals Variance, σ_ε^2:

$$\sigma_\varepsilon^{\,2} = \frac{1}{N} S(\phi,\theta) = \frac{1}{N} \sum_{t=1}^{N} \varepsilon_t^{\,2} \tag{7.32}$$

Stationarity Conditions:

$$-1 < \phi_1 < 1, -1 < \theta_1 < 1 \tag{7.33}$$

In typical hydrologic models,

$$0 < \phi_1 < 1, 0 < \theta_1 < 1 \text{ and } \phi_1 > \theta_1 \tag{7.34}$$

ARMA(1,1) Autocorrelation Coefficients:

$$r_1 = \frac{(1-\phi_1\theta_1)(\phi_1-\theta_1)}{1+\theta_1^{\,2}-2\phi_1\theta_1} \tag{7.35}$$

$$r_k = \phi_1\, r_{k-1},\ k \geq 2 \tag{7.36}$$

7.2.3 Goodness of Fit of Selected Models

The goodness of fit is determined by testing residuals for independence and normality. The following steps are followed:

step(3a): First the residuals, ε_t, are calculated. For,

AR(1) : $\varepsilon_2 = Z_2 - \phi_1 Z_1, \ldots\ldots\ldots\ldots, \varepsilon_N = Z_N - \phi_1 Z_{N-1},$ (7.37)

AR(2): $\varepsilon_3 = Z_3 - \phi_1 Z_2 - \phi_2 Z_1, \ldots\ldots, \varepsilon_N = Z_N - \phi_1 Z_{N-1} - \phi_2 Z_{N-2},$ (7.38)

ARMA(1,1): $\varepsilon_1=0, \varepsilon_2=Z_2-\phi_1 Z_1, \varepsilon_3=Z_3-\phi_1 Z_2+\theta_1\varepsilon_2, \ldots\ldots, \varepsilon_N=Z_N-\phi_1 Z_{N-1}+\theta_1 Z_{N-2}.$ (7.39)

step(3b): Test of residuals for independence

If the residuals are independent, then the model is accepted. Any of the following two tests can be applied to decide whether the residuals are correlated or not.

(1) Porte Manteau Test:

The hypothesis to be tested is that the residuals are independent. To carry out the test, the steps described below are to be followed:

(i) Calculate $r_k(\varepsilon)$ of the residuals ε_t for lags k = 1 to L, equation (7.7). Where ;

$$L = N/10 + p + q \tag{7.40}$$

(ii) Calculate Statistic Q

$$Q = N\sum_{k=1}^{L} r_k(\varepsilon)^2 \qquad (7.41)$$

(iii) Find, from χ^2 Tables, chi-square value with (L-p-q) degree of freedom.

(iv) If $Q < \chi^2$ value, then the hypothesis and the model are accepted.

(2) Anderson Test of Correlogram:

(i) Calculate correlogram $r_k(\varepsilon)$ of residuals (ε_t) from equation (7.7).

(ii) Anderson (1941) gave the limits for $r_k(95\%)$ as:

$$r_k(95\%) = \frac{-1 \pm 1.96(N-k-1)^{1/2}}{N-k} \qquad (7.42)$$

(iii) Plot $r_k(\varepsilon)$ and $r_k(95\%)$. If, for example, k=1,.........., 40, then the maximum number of points allowed out of the limit is (1-0.95)*40 = 2. If the number of points out of the limit are ≤ 2, then it can be concluded that the residuals of the model are uncorrelated.

step(3c): Test of Residuals for Normality:

Many tests can be conducted to test the hypothesis that a time series is normal. If its found that the series of the residuals is not normal, another trial to bring it to normality should be made by trying another transformation of the original time series. Here three tests of normality are described:

(i) The first test is done by plotting the residuals series on a normal probability paper. If the plot is approximately a straight line, then the hypothesis of normality is accepted.

(ii) χ^2, chi-square, test:

(a) Let X_t, t=1,.....N, be a time series with mean $\ddot{X}$ and standard deviation σ and sample size N. In this test it is assumed that this series is fitted with a normal distribution and χ^2 test is used to test the goodness of fit.

(b) The series is arranged in ascending order of magnitude.

(c) The series is divided into k class intervals, each with 1/k probability.

(d) From Normal probability tables, obtain the values $u_1, u_2, \ldots\ldots, u_{k-1}$ corresponding to commulative probabilities 1/k, 2/k, ..., (k-1)/k.

(e) The values in the X range which determine the class intervals will be, $X_1' = \ddot{X} + \sigma u_1$, $X_2' = \ddot{X} + \sigma u_2, \ldots\ldots, X'_{k-1} = \ddot{X} + \sigma u_{k-1}$.

(f) The absolute frequency of the ordered sample series, which falls within the class interval i-th, is N_i, i = 1,..............,k.

(g) The expected number of points which falls in each interval would be N/k.

h) Find, $\chi^2 = \sum \frac{(N_i - \frac{N}{k})^2}{(\frac{N}{k})}$ (7.43)

The above relation has a χ^2 distribution with (k-2) degree of freedom.

(i) From χ^2 tables, obtain $\chi^2_{1-\alpha}(k-2)$, α is the probability level.

(j) If $\chi^2 < \chi^2_{1-\alpha}(k-2)$, then the hypothesis of normality of the time series X_t is accepted.

(iii) Skewness test of normality:

For a normal distribution, the skewness coefficient is zero. The skewness coefficient, γ, for variable X_t, t = 1,..........., N, can be estimated by:

$$\gamma = \frac{\frac{1}{N}\sum_{t=1}^{N}(X_t - \ddot{X})^3}{\sqrt[3/2]{\frac{1}{N}\sum_{t=1}^{N}(X_t - \ddot{X})^2}} \qquad (7.44)$$

γ is normally distributed with mean zero and variance 6/N, (Snedecor and Cochran, 1967). Then the (1-α) probability limits on γ may be defined by

$$[\, - u_{1-\alpha/2}\sqrt{(6/N)}\, ,\; u_{1-\alpha/2}\sqrt{(6/N)}] \qquad (7.45)$$

Where, $u_{1-\alpha/2}$ is the 1-α/2 quantile of the standard normal distribution.

If γ falls within the above range, the hypothesis of normality is accepted. This test is sufficiently accurate for N > 150.

7.2.4 Selection among Competent Models

If more than one model pass the test of goodness of fit described in Section (7.2.3), Akaike test, proposed by Akaike (1974), is performed to select among competing models. For ARMA(p,q) models, Akaike Information Criterion is:

$$AIC(p,q) = N \ln(\sigma_\varepsilon^2) + 2(p+q) \qquad (7.46)$$

Where N is the sample size and σ_ε^2 is the estimate of the residuals variance.

According to this criterion, the model with minimum AIC is the one to be selected.

7.2.5 Model Use for Samples Generation

Different models are used for generation of synthetic samples as follows:

AR Models:

The general form of AR models is,

$$Z_t = \phi_1 Z_{t-1} + \dots\dots\dots + \phi_p Z_{t-p} + \varepsilon_t \tag{7.47}$$

ε_t is normal with mean zero and variance σ_ε^2.

The standard normal variable is introduced,

$$\xi_t = (\varepsilon_t - 0)/\sigma_\varepsilon \tag{7.48}$$

obtain ε_t from (7.48) and substitute it in (7.47) to get,

$$Z_t = \phi_1 Z_{t-1} + \dots\dots\dots + \phi_p Z_{t-p} + \sigma_\varepsilon \xi_t \tag{7.49}$$

ξ_t is independent, normal variable with mean 0 and variance 1.

A series of ξ is to be generated and substituted in (7.49) to generate a time series of Z_t.

To generate ξ, Box and Muller (1958) proposed these equations:

$$\xi_1 = [\ln(1/u_1)]^{1/2}\cos(2\pi u_2) \tag{7.50}$$

$$\xi_2 = [\ln(1/u_1)]^{1/2}\sin(2\pi u_2) \tag{7.51}$$

ξ_1, ξ_2 are standard normal random numbers.

u_1 u_2 are random numbers of the uniform (0,1) distribution.

Normality and independence tests have to be carried out for newly or previously generated random numbers to avoid the failure of the test by the generated time series. Take ξ_1 from the generated numbers and find Z_1 from (7.49), assuming Z_0, Z_{-1},........, Z_{-p+1} are zeros. Take ξ_2, find Z_2, using the derived Z_1 and taking Z_0, Z_{-1}, ..., Z_{-p+1} as zeros. Repeat until a series $Z_1, Z_2, \dots\dots\dots, Z_{N'}$ is generated.

$$N' = N_g + N_w \tag{7.52}$$

N_w is the warm-up length and N_g is the desired generated length. N_w is necessary to remove the effect of taking $Z_0 = Z_{-1} = \dots = Z_{-p+1} = 0$ and it may be 50 (Fiering and Jackson, 1971). The first N_w values are deleted and the last N_g values, Z_{Nw+1}, ... Z_{Nw+Ng}, are taken as Z_1, Z_2, ..., Z_{Ng}.

The generated standardised series, $Z_{v,\tau}$, can be substituted in equation (7.6) together with the monthly mean, μ_τ, and monthly standard deviation, σ_τ, to get the transformed series $y_{v,\tau}$. X_t, the generated time series is found by finding the inverse of $y_{v,\tau}$.

ARMA(1,1):

ε_t obtained from (7.48) is substituted in the general form of ARMA(1,1) model, equation (7.23), to have the general form of ARMA(1,1) in terms of ξ_t, i.e.

$$Z_t = \phi_1 Z_{t-1} + \xi_t \sigma_\varepsilon - \theta_1 \varepsilon_{t-1} \quad (7.53)$$

For generation the same procedure described above can be followed by taking Z_0 & ε_0 equal to zeros.

Alternatively, from the time series of Z_t and ε_t, take the last point of each and substitute their values in equation (7.53) for Z_{t-1} & ε_{t-1}. Then use the generated standard random numbers to generate Z_t. Then the generated Z_t and ε_t are used with another generated number to find Z_{t+1}. The process is repeated until a Z_t series of the required length is generated.

7.2.6 Preservation of Statistics

The generated samples should preserve some statistical properties. In the case of the monthly analysis carried out here, the generated time series has to preserve the mean and standard deviation of the original time series.

7.3 MODELLING APPLICATION TO THE BLUE NILE

To model the flow of the Blue Nile, the calculations follow the same steps previously described. The time series used in the analysis is 30 years long, extending from July 1962 to June 1992. Table (7.1) shows the monthly discharges, their averages, standard deviations and skewness coefficients.

7.3.1 Preliminary Analysis

Step(1a):

For the original time series, $X_{v,\tau}$, the monthly skewness coefficients are calculated. The average monthly skewness coefficient found is equal to 0.584, Table (7.1). The original time series is not normal, since the skewness coefficient is not close to zero.

Step(1b):

A power transformation is used to bring the time series, $X_{v,\tau}$, to normality. The best transformation is found by taking $y_{v,\tau} = X_{v,\tau}^{0.225}$. The transformed series, $y_{v,\tau}$, has 0 average monthly skewness coefficient, Table (7.2). It can be concluded that the original time series is brought to normality .

step (1d):

The mean $\ddot{y}_\tau$, the standard deviation, S_τ, and the periodic correlation coefficient, $r_{(1,\tau)}(y)$ are calculated and shown at the bottom of Table (7.2).

step (1f):

The estimates of $r_{(1,\tau)}(y)$, Table (7.2), do not vary significantly from month to month. Therefore, models with constant parameters can be selected.

7.3.2 Estimation of Parameters

step (2a):

Since w = 12, the sample mean, $\ddot{y}_\tau$ and standard deviation, S_τ,shown in Table (7.2), are taken as estimates of the population mean, μ_τ, and standard deviation, σ_τ, respectively.

step (2b):

The series $y_{v,\tau}$ is standardised using relation (7.6).Table (7.3), shows the standardised series $Z_{v,\tau}$.

Step (2c):

(i) Since models with constant parameters have been selected from step (1f), $Z_{v,\tau}$ is represented by Z_t with $t = (v-1)w + \tau$. Z_t mean is $\ddot{Z}$ = 5.84E-8 ≈ 0 and its standard deviation = 0.985 $\approx$ 1.

(ii) The correlogram of time series Z_t, $r_k(z)$, k = 1, 2, ..., 20 is determined using relation (7.7) and results are shown in Table (7.4). $r_k(z)$ lies within the range -1 to 1.

(iii)Parameter Estimation of AR models:

<u>a.</u> Method of Moments:

AR(1):

Autoregressive Coefficient ϕ_1:

$\phi_1 = r_1 = 0.6276$, r_1 from Table (7.4) and relation (7.8) is used.

Residual Variance σ_ε^2 :

$\sigma_\varepsilon^2 = 0.6078$ by substituting for N = 360 , $\phi_1 =$ 0.6276 and $\sigma^2 = 1$ in equation (7.10).

Table (7.1) Blue Nile monthly flows, $X_{v,\tau}$, in million m^3, and their statistics

yr.	july	aug.	sept.	oct.	nov.	dec.	jan.	feb.	mar.	apr	may	jun
62-	5359	15066	14220	8942	2558	1464	881	482	401	415	559	1703
63-	6497	17463	12980	4857	2624	2296	943	565	340	397	1246	1516
64-	8855	16904	14570	10596	3720	1953	986	571	448	494	443	1662
65-	4765	13407	9400	7047	3065	1742	752	492	424	321	277	1203
66-	6346	13249	11620	4108	2285	1403	691	387	351	323	551	1934
67-	6672	14949	13030	9727	3020	2028	887	575	340	250	498	1393
68-	8945	16195	10510	5980	2021	1262	686	475	748	375	296	1624
69-	7278	19620	10600	4036	1760	971	586	317	327	240	702	1776
70-	5884	17397	12110	6836	2475	1131	680	363	237	161	262	1039
71-	6678	16561	11550	5641	2741	1280	747	413	282	293	414	1698
72-	5002	9723	6920	3219	1672	907	521	274	178	171	492	1256
73-	5416	17551	12080	6507	2491	1254	783	440	380	241	703	1757
74-	8822	16859	12120	5806	2232	1313	656	480	320	207	788	2106
75-	7498	17343	17930	7229	2709	1484	918	513	434	282	311	1406
76-	5592	15285	9360	3938	2509	1265	708	439	343	254	618	1484

Table (7.1) Blue Nile monthly flows, $X_{v,\tau}$, in million m^3, and their statistics - continued

yr.	july	aug.	sept.	oct.	nov.	dec.	jan.	feb.	mar.	apr	may	jun
77-	10004	15804	11920	6410	3813	1502	794	449	357	271	471	1755
78-	7483	12649	10890	7771	2456	1323	834	507	324	246	507	1564
79-	5612	12327	8470	4374	1850	1004	584	357	271	324	768	1679
80-	7687	15125	9080	4675	1803	1006	569	315	268	252	458	1309
81-	6233	13940	11690	5283	1855	978	618	344	324	218	491	1076
82-	4398	11027	7730	5307	1821	945	504	311	242	249	323	1039
83-	4011	14740	10220	5740	2110	1020	558	310	192	125	426	1099
84-	6306	9732	7280	2476	965	570	321	197	151	230	295	1842
85-	6091	15442	12910	4583	1736	979	542	315	293	297	737	1410
86-	6693	10210	8715	3794	1389	740	419	251	354	309	1456	1430
87-	4731	10160	6810	4220	1932	939	513	367	396	221	813	2473
88-	12593	19911	14940	10121	3187	1506	775	429	370	423	274	2117
89-	6345	12181	10760	4840	1682	1115	847	453	353	273	440	1160
90-	4712	12789	9792	5100	1661	869	512	287	262	334	291	804
91-92	8462	14642	11174	4733	2077	1206	683	444	323	253	637	1561
av	6699	14575	11046	5797	2274	1249	683	404	334	282	552	1529
sd	1857	2862	2531	2024	655	385	164	99	107	81	275	368
sc	1.223	-0.245	0.46	0.916	0.57	0.94	-.072	-.072	1.79	0.64	1.73	.352

Table (7.2) Transformed time series , $y_{v,\tau}$, and its statistics

yr	july	aug.	sept.	oct	nov	dec	jan	feb	march	april	may	jun
62-	6.903	8.711	8.598	7.746	5.845	5.155	4.599	4.015	3.852	3.882	4.151	5.334
63-	7.209	9.005	8.423	6.752	5.879	5.705	4.669	4.161	3.712	3.844	4.972	5.196
64-	7.729	8.939	8.645	8.047	6.359	5.501	4.717	4.171	3.949	4.037	3.939	5.304
65-	6.723	8.485	7.833	7.342	6.088	5.361	4.438	4.034	3.901	3.664	3.545	4.932
66-	7.171	8.462	8.216	6.502	5.698	5.106	4.354	3.822	3.738	3.669	4.138	5.488
67-	7.252	8.695	8.431	7.894	6.067	5.547	4.606	4.178	3.712	3.464	4.045	5.098
68-	7.747	8.853	8.033	7.076	5.543	4.986	4.347	4.002	4.432	3.795	3.598	5.277
69-	7.395	9.244	8.048	6.476	5.373	4.700	4.195	3.654	3.679	3.432	4.369	5.384
70-	7.05	8.997	8.293	7.292	5.802	4.864	4.338	3.767	3.422	3.137	3.5	4.772
71-	7.253	8.898	8.205	6.983	5.936	5.002	4.431	3.878	3.559	3.59	3.88	5.33
72-	6.797	7.893	7.312	6.155	5.312	4.629	4.086	3.536	3.209	3.18	4.034	4.98
73-	6.92	9.015	8.288	7.211	5.810	4.979	4.478	3.933	3.806	3.435	4.371	5.371
74-	7.722	8.934	8.294	7.029	5.668	5.030	4.303	4.011	3.661	3.32	4.485	5.595
75-	7.445	8.991	9.058	7.384	5.921	5.171	4.641	4.072	3.921	3.559	3.638	5.109
76-	6.97	8.739	7.826	6.441	5.82	4.989	4.378	3.931	3.719	3.476	4.246	5.171

Table (7.2) Transformed time series, $y_{v,\tau}$, and its statistics - continued

yr	july	aug.	sept.	oct	nov	dec	jan	feb	march	april	may	jun
77-	7.944	8.805	8.263	7.187	6.394	5.185	4.492	3.951	3.753	3.527	3.994	5.37
78-	7.442	8.375	8.097	7.505	5.792	5.039	4.542	4.061	3.672	3.451	4.061	5.232
79-	6.975	8.326	7.652	6.595	5.434	4.736	4.192	3.753	3.527	3.672	4.459	5.317
80-	7.487	8.718	7.773	6.694	5.403	4.738	4.168	3.649	3.518	3.47	3.969	5.027
81-	7.142	8.56	8.227	6.881	5.437	4.708	4.246	3.722	3.672	3.359	4.032	4.81
82-	6.603	8.12	7.496	6.888	5.415	4.672	4.056	3.638	3.438	3.461	3.669	4.772
83-	6.467	8.668	7.982	7.011	5.597	4.753	4.149	3.635	3.264	2.963	3.905	4.833
84-	7.161	7.705	7.396	5.802	4.694	4.169	3.664	3.283	3.092	3.399	3.595	5.429
85-	7.105	8.759	8.413	6.664	5.357	4.709	4.122	3.649	3.59	3.601	4.418	5.112
86-	7.257	7.981	7.701	6.387	5.095	4.422	3.98	3.467	3.746	3.633	5.149	5.128
87-	6.712	7.972	7.285	6.542	5.487	4.665	4.072	3.776	3.841	3.369	4.516	5.801
88-	8.366	9.275	8.694	7.965	6.141	5.188	4.468	3.911	3.783	3.899	3.536	5.601
89-	7.17	8.304	8.075	6.747	5.319	4.849	4.558	3.959	3.743	3.533	3.933	4.892
90-	6.706	8.395	7.906	6.827	5.304	4.584	4.07	3.573	3.5	3.697	3.584	4.505
91-92	7.65	8.655	8.144	6.713	5.577	4.935	4.343	3.941	3.669	3.473	4.275	5.23
AV	7.216	8.616	8.087	6.958	5.652	4.936	4.32	3.838	3.669	3.533	4.067	5.18
S.D	0.427	0.399	0.420	0.533	0.371	0.335	0.246	0.224	0.250	0.231	0.412	0.286
S.C	.5993	-.573	-.065	.2931	-.121	.2419	-.5571	-.4745	.2698	-.1057	.7366	-.1939
r_1	.4535	.4853	.7563	.6885	.7694	.842	.8741	.9091	.6473	.5771	.0507	.3308

average skewness coefficient (S.C.) = 0.004

Table (7.3) Standardised time series - Z_t

year	july	august	sept.	oct.	nov.	dec.	Jan.	Feb.	March	April	May	June
62-	-0.7326	0.2373	1.2161	1.4796	0.5194	0.6552	1.1326	0.7931	0.7305	1.5106	0.2047	0.5369
63-	-0.0164	0.9747	0.8004	-0.3866	0.6100	2.2958	1.4213	1.4469	0.1693	1.3439	2.1966	0.0553
64-	1.2024	0.8101	1.3283	2.0458	1.9042	1.6866	1.6130	1.4912	1.1192	2.1823	-0.3091	0.4349
65-	-1.1545	-0.3282	-0.6034	0.7208	1.1735	1.2694	0.4773	0.8763	0.9249	0.5674	-1.2681	-0.8657
66-	-0.1055	-0.3849	0.3074	-0.8555	0.1245	0.5084	0.1366	-0.0726	0.2760	0.5896	0.1721	1.0782
67-	0.0850	0.1991	0.8177	1.7576	1.1190	1.8265	1.1612	1.5206	0.1693	-0.2998	-0.0539	-0.2873
68-	1.2437	0.5952	-0.1293	0.2208	-0.294	0.1493	0.1077	0.7341	3.0489	1.1319	-1.1387	0.3387
69-	0.4206	1.5739	-0.0926	-0.9039	-0.752	-0.7036	-0.5088	-0.8232	0.0398	-0.4368	0.7347	0.7138
70-	-0.3888	0.9555	0.4899	0.6268	0.4029	-0.2134	0.0727	-0.3171	-0.9877	-1.7127	-1.3752	-1.4250
71-	0.0884	0.7070	0.2808	0.0475	0.7662	0.1968	0.4502	0.1794	-0.4420	0.2451	-0.4537	0.5246
72-	-0.9816	-1.811	-1.8447	-1.5073	-0.918	-0.9173	-0.9548	-1.3506	-1.8409	-1.5274	-0.0807	-0.6976
73-	-0.6940	1.0002	0.4789	0.4757	0.4256	0.1280	0.6423	0.4284	0.5452	-0.4229	0.7381	0.6683
74-	1.1872	0.7967	0.4936	0.1329	0.0436	0.2826	-0.0695	0.7763	-0.0317	-0.9230	1.0142	1.4496
75-	0.5371	0.9397	2.3116	0.8001	0.7240	0.7022	1.3067	1.0468	1.0069	0.1120	-1.0410	-0.25
76-	-0.5770	0.3083	-0.6212	-0.9710	0.4510	0.1572	0.2338	0.4194	0.1987	-0.2462	0.4349	-0.0317

Table (7.3) Standardised time series - Z_t - continued

year	july	august	sept.	oct.	nov.	dec.	Jan.	Feb.	March	April	May	June
77-	1.7064	0.4735	0.4198	0.4300	1.9997	0.7442	0.6996	0.5088	0.3331	-0.0253	-0.1763	0.6635
78-	0.5293	-0.6049	0.0240	1.0275	0.3759	0.3083	0.9030	0.9987	0.0093	-0.3541	-0.0143	0.1831
79-	-0.5638	-0.7263	-1.0352	-0.6819	-0.588	-0.5976	-0.5219	-0.3802	-0.5688	0.6006	0.9514	0.4775
80-	0.6351	0.2565	-0.7481	-0.4951	-0.673	-0.5913	-0.6216	-0.8464	-0.6041	-0.2729	-0.2372	-0.5350
81-	-0.1733	-0.1409	0.3338	-0.1445	-0.579	-0.6809	-0.3032	-0.5197	0.0093	-0.7546	-0.0851	-1.2932
82-	-1.4361	-1.2430	-1.4059	-0.1313	-0.640	-0.7891	-1.0785	-0.8933	-0.9233	-0.3132	-0.9654	-1.4250
83-	-1.7534	0.1302	-0.2493	0.0990	-0.148	-0.5472	-0.6960	-0.9051	-1.6206	-2.4641	-0.3930	-1.2129
84-	-0.1294	-2.2839	-1.6450	-2.170	-2.583	-2.2892	-2.6725	-2.4825	-2.3069	-0.5783	-1.1453	0.8690
85-	-0.2598	0.3588	0.7760	-0.5512	-0.796	-0.6776	-0.8062	-0.8464	-0.3191	0.2926	0.8515	-0.2386
86-	0.0970	-1.592	-0.9180	-1.0720	-1.503	-1.536	-1.7507	-1.6597	0.3046	0.4321	2.6271	-0.1819
87-	-1.1799	-1.6145	-1.9073	-0.7814	-0.445	-0.8090	-1.0126	-0.2755	0.6871	-0.7099	1.0910	2.1695
88-	2.6958	1.6508	1.4447	1.8906	1.3182	0.7535	0.6002	0.3284	0.4543	1.5830	-1.2891	1.4725
89-	-0.1061	-0.7821	-0.0280	-0.3966	-0.899	-0.2599	0.9675	0.5441	0.2951	-1.9E-5	-0.3237	-1.0063
90-	-1.1941	-0.5528	-0.4312	-0.2466	-0.939	-1.0498	-1.0199	-1.1848	-0.6755	0.7097	-1.1721	-2.3603
91-92	1.0184	0.0976	0.1360	-0.4602	-0.202	-0.0020	0.0902	0.4643	-0.0009	-0.2595	0.5053	0.1752

Autocorrelation function r_k:

The autocorrelation function is calculated for k = 1 to 20, using relation (7.9) and results are shown in Table (7.4).

AR(2):

Autoregressive Coefficient ϕ_1, ϕ_2:

Substitute for $r_1(Z_t) = 0.6276$ and $r_2(Z_t) = 0.5043$ in relations (7.11) and (7.12) to find ϕ_1 and ϕ_2 respectively. $\phi_1 = 0.5132$ and $\phi_2 = 0.1822$.

Residual Variance σ_ε^2:

$\sigma_\varepsilon^2 = 0.5893$ by substituting for N = 360, $\phi_1 = 0.5132$, $\phi_2 = 0.1822$, and $\sigma^2 = 1$ in relation (7.14).

Autocorrelation function r_k:

The autocorrelation function, r_k, is determined by substituting $\phi_1 = 0.5132$, $\phi_2 = 0.1822$ and r_{k-1} and r_{k-2} from Table (7.4) in relation (7.13). The results of r_k for k = 2 to 20 are shown in Table (7.4).

Table (7.4) Correlogram, r_k, of fitted models

k	Z_t	AR(1)	AR(2)	ARMA(1,1)
1	0.627	0.627	0.627	0.627
2	0.504	0.394	0.504	0.507
3	0.407	0.247	0.373	0.409
4	0.355	0.155	0.283	0.331
5	0.307	0.097	0.213	0.267
6	0.234	0.061	0.161	0.216
7	0.201	0.038	0.122	0.175
8	0.188	0.024	0.092	0.141
9	0.181	0.015	0.069	0.114
10	0.182	0.009	0.052	0.092
11	0.217	0.006	0.039	0.074
12	0.179	0.004	0.030	0.060
13	0.211	0.002	0.022	0.049
14	0.187	0.001	0.017	0.039
15	0.199	0.001	0.013	0.032
16	0.149	0.001	0.010	0.026
17	0.140	0.000	0.007	0.021
18	0.180	0.000	0.005	0.017
19	0.180	0.000	0.004	0.014
20	0.170	0.000	0.003	0.011
21	0.150	0.000	0.002	0.009
22	0.194	0.000	0.002	0.007
23	0.152	0.000	0.001	0.006
24	0.116	0.000	0.001	0.005
25	0.108	0.000	0.001	0.004
26	0.054	0.000	0.001	0.003
27	0.045	0.000	0.000	0.002
28	0.045	0.000	0.000	0.002
29	0.040	0.000	0.000	0.002

b. Maximum Likelihood Method:

Using relation (7.15), the D_{ij} coefficients necessary for estimating parameters of AR models are calculated. The results are as follows:

$D_{11} = 348.0001$, $D_{12} = 218.8462$, $D_{13} = 176.2635$,

$D_{22} = 349.4046$, $D_{23} = 220.2476$, $D_{33} = 351.3106$.

AR(1):

Autoregressive Coefficient, ϕ_1:

$\phi_1 = 0.6263$, using relation (7.16).

Residual Variance σ_ε^2:

$\sigma_\varepsilon^2 = 0.5876$ using relation (7.17).

AR(2):

Autoregressive Coefficient, ϕ_1 ϕ_2:

$\phi_1 = 0.5127$ and $\phi_2 = 0.1803$, using relations (7.18) and (7.19) respectively..

Residual Variance σ_ε^2:

$\sigma_\varepsilon^2 = 0.5699$ using relation (7.20).

As expected, for large samples, i.e. N = 360, the methods of moment and likelihood give almost the same results when estimating ϕ_1 and ϕ_2 (Table 7.5). However results obtained from the method of moment will be used for further analysis.

Table (7.5) Comparison between likelihood and method of moment estimation

	method of moment	likelihood
AR(1)		
ϕ_1	0.6276	0.6263
σ_ε^2	0.6078	0.5876
AR(2)		
ϕ_1	0.5132	0.5127
ϕ_2	0.1822	0.1803
σ_ε^2	0.5893	0.5699

c. Conditions to be met by parameters:

The stationary conditions for AR(1) and AR(2) are satisfied, since the estimated parameters satisfy the relations (7.21) and (7.22).

AR(1): $-1 < (\phi_1 = 0.6276) < 1$

AR(2): $(\phi_1 + \phi_2) = 0.6954 < 1$

$(\phi_2 - \phi_1) = -0.331 < 1$

$-1 < \phi_2 = 0.1822 < 1$

(iv) Parameter Estimation for ARMA(1,1) Model:

Initial Estimation of the model parameters:

For initial estimation lag 0,1 and 2 autocovariance i.e. C_0 C_1 and C_2 are estimated using relation (7.25) when k = 0, 1 and 2.

$C_0 = 0.9667$, $C_1 = 0.6062$, $C_2 = 0.4869$.

Initial Estimate of the Autoregressive Parameter, ϕ_1:

According to relation (7.24) ,

$\phi_1 = C_2 / C_1 = 0.4869/0.6062 = 0.8032$.

Initial Estimate of the Moving Average Parameter, θ_1:

To estimate θ_1 the following steps are followed:

1. Substitute C_0, ϕ_1 and C_1 in relation (7.26) to find $C_0' = 0.6165$.
2. Substitute C_0, ϕ_1, C_1 and C_2 in relation (7.27) to find $C_1' = -0.1703$.
3. Substitute C_0' and C_1' in relations (7.28) and (7.29) and solve for θ_1. θ_1, found, is equal to 0.3.

Thus the initial estimates of the model parameters are: $\phi_1 = 0.80$ and $\theta_1 = 0.30$.

Maximum Likelihood Estimation of Parameters:

Maximum likelihood method gives an estimate of ϕ_1 and θ_1 which give the minimum value of the sum of squares of errors, equation (7.30).

In the vicinity of initially estimated ϕ_1 and θ_1, the errors are calculated according to relation (7.31). The sum of squares are shown in Tables (7.6), (7.7) and (7.8).

Table (7.6) Sum of squares - first trial

$\phi_1 =$	0.7	0.8	0.9
$\theta_1 =$			
0.2	205.295	205.0743	214.27
0.3	209.0712	<u>202.9881</u>	208.3497
0.4	219.4164	205.3679	205.7538

Table (7.7) Sum of squares - second trial

ϕ_1 = / θ_1 =	0.77	0.78	0.79	0.8	0.81	0.82	0.83
0.27	3.3022	3.1554	3.1163	3.1847	3.3606	3.6441	4.0352
0.28	3.358	4.1545	3.0607	3.0766	3.2023	3.4378	3.7831
0.29	3.4608	3.1987	3.0487	3.0108	3.085	3.2712	3.5695
0.30	3.6114	3.2892	3.0814	2.9881	3.0093	3.1449	3.3949
0.31	3.8111	3.4269	3.1597	3.0093	2.976	3.0595	3.2599
0.32	4.0611	3.613	3.2845	3.0755	2.9859	3.0159	3.1653
0.33	4.3628	3.849	3.4571	3.1875	3.0401	3.0148	3.117
0.34	4.7178	4.1358	3.6787	3.3466	3.1395	3.0573	3.1001
0.35	5.1276	4.4752	3.9507	3.554	3.2852	3.1443	3.1313
0.36	5.5941	4.8689	4.2746	3.8112	3.4786	3.277	3.2063

Note: Sum of squares = figures in table + 200

Table (7.8) Sum of squares - third trial

ϕ_1 = / θ_1 =	0.807	0.808	0.809	0.810	0.811	0.812
0.304	2.9788	2.9817	2.9857	2.9908	2.9971	3.0046
0.305	2.9769	2.9792	2.9827	2.9873	2.993	2.9958
0.306	2.9754	2.9771	2.9801	2.9841	2.9894	2.992
0.307	2.9743	2.9755	2.9779	2.9815	2.9862	2.9887
0.308	2.9737	2.9743	2.9762	2.9792	2.9834	2.9887
0.309	2.9735	2.9736	2.9749	2.9774	2.981	2.9858
0.310	2.9737	2.9733	2.974	2.976	2.9791	2.9833
0.311	2.9744	2.9734	2.9736	2.975	2.9775	2.9813
0.312	2.9755	2.974	2.9736	2.9745	2.9765	2.9797
0.313	2.977	2.975	2.9741	2.9744	2.9758	2.9785
0.314	2.979	2.9764	2.975	2.9747	2.9756	2.9777
0.315	2.9814	2.9783	2.9763	2.9755	2.9758	2.9774
0.316	2.9843	2.9806	2.978	2.9767	2.9765	2.9775
0.317	2.9876	2.9833	2.9802	2.9783	2.9776	2.9781

Note Sum of squares = figures in table + 200

The first trial, Table (7.6), shows that the parameters ϕ_1= 0.8 and θ_1= 0.3 give minimum value of sum squares of errors, 202.9881. For more precision, as in Table (7.7), points are chosen around the point (0.8,0.3). The second trial shows that ϕ_1= 0.81 and θ_1= 0.31 give minimum sum of squares, 202.976. Again points are chosen around ϕ_1= 0.81 and θ_1= 0.31 and it has been found that ϕ_1= 0.808 and θ_1= 0.31 give the minimum estimate of the sum of squared errors, 202.9733, Table (7.8). No significant reduction in the sum of squares, when fourth decimals are added to ϕ_1 and θ_1. Therefore 0.808 and 0.31 are taken as the maximum likelihood estimation of parameters ϕ_1 and θ_1 respectively.

Estimate of the Residual Variance, σ_ε^2:

Using relation (7.32), N=360 and $\sum\varepsilon_t^2 = 202.9733$, it has been found that $\sigma_\varepsilon^2 = 0.5638$.

Autocorrelation Function, r_k:

Substitute for $\phi_1 = 0.808$ and $\theta_1 = 0.31$ in relation (7.35) to find r_1. r_1 found and ϕ_1 are substituted in relation (7.36) to find r_2. r_2 is substituted in relation (7.36) to find r_3. The method is repeated to calculate r_k for k = 2 to 20. Results are shown in Table (7.4).

Conditions to be met by the Model:

The estimated parameters satisfy conditions (7.34).

$0 < \phi_1 (= 0.808) < 1$,

$0 < \theta_1 (= 0.31) < 1$

$\phi_1 (= 0.808) > \theta_1 (= 0.31)$

7.3.3 Goodness of Fit

Goodness of fit of a model is determined by testing the residuals for independence and normality. If a certain model does not pass the independence test, then that model is rejected. If the model does not pass the normality test, then another transformation of the original data should be tried.

step (3a):

For AR(1), AR(2) and ARMA(1,1) models, the residuals ε_t can be calculated from time series, Z_t, using relations (7.37), (7.38) and (7.39) respectively. Residuals statistics are summarised in Table (7.9), below.

Table (7.9) Residuals statistics

Model	AR(1)	AR(2)	ARMA(1,1)
Mean	0.00234	0.000926	0.003573
Stand. Dev.	0.7659	0.752	0.7519
Skewness	0.293	0.2856	0.2964

Step (3b):

Porte Manteau Test of Independence:

(i) The autocorrelation function $r_k(\varepsilon)$ of the residuals ε_t are calculated for k = 1, ..., L using relation (7.7), Table (7.10). L is calculated as in relation (7.40). L = 37 for AR(1) and 38 for AR(2) and ARMA(1,1).

Table (7.10) Residuals correlogram, $r_k(\varepsilon)$

k	AR(1)	AR(2)	ARMA(1,1)	k	AR(1)	AR(2)	ARMA(1,1)
1	-.1168	-.0126	-.0012	20	.046	.0182	.0232
2	.0901	-.0454	-.0116	21	-.0352	-.036	-.0344
3	.0509	-.0011	-.0275	22	.1564	.1465	.1424
4	.0751	.043	.0181	23	.0209	.0286	.0287
5	.0972	.0687	.0464	24	.0023	-.0047	-.002
6	.013	-.0141	-.03	25	.0768	.0598	.053
7	.0262	-.0084	-.0226	26	-.0391	-.0458	-.0526
8	.0346	.011	-.0066	27	.0052	-.0153	-.0236
9	.0402	.0114	.0032	28	.0165	-.0016	-.0111
10	.0018	-.0002	-.0105	29	-.034	-.0364	-.0455
11	.1302	.1031	.0984	30	.0628	.0401	.0347
12	-.0373	-.0291	-.034	31	-.033	-.0293	-.0336
13	.1134	.0794	.0829	32	.0672	.0504	.0497
14	.0093	.0226	.0155	33	.0339	.0347	.0351
15	.1107	.091	.0843	34	.0402	.0406	.0439
16	-.0126	-.0333	-.0385	35	.0848	.0828	.0841
17	-.0142	-.0409	-.0429	36	.0273	.0302	.0278
18	.0874	.0752	.0661	37			.0203
19	.0593	.0648	.0557	38			-.0481

(ii) Calculation of Statistic Q: For each model Q is calculated using relation (7.41). Table (7.11) shows the results.

Table (7.11) Statistic Q calculation results

Model	N	$\Sigma(r_k(\varepsilon))^2$	Q
AR(1)	360	0.1556	56.002
AR(2)	360	0.09531	34.312
ARMA(1,1)	360	0.0905	32.57

(iii) From chi- square tables , it is found for:

AR(1): $\chi_{1-\alpha}^2(L-p-q) = \chi_{1-0.05}^2(37-1-0) = \chi_{.95}^2 36 = 51 \quad < Q = 56.002$

AR(2): $\chi_{1-\alpha}^2(L-p-q) = \chi_{1-0.05}^2(38-2-0) = \chi_{.95}^2 36 = 51 \quad > Q = 34.312$

AR(1,1): $\chi_{1-\alpha}^2 (L-p-q) = \chi_{1-0.05}^2(38-1-1) = \chi_{.95}^2 36 = 51 \quad > Q = 32.57$

α is the level of significance = 0.05 and (L - p - q) is the degree of freedom.

AR(2) and ARMA(1,1) models are accepted since Q is less than the tabulated χ^2, while AR(1) model is rejected since Q is greater than tabulated χ^2.

This result is also clear from Table (7.4) and Figure (7.1), where the correlograms of AR(2) and ARMA(1,1) are more closer to the correlogram of the original series Z_t.

STEP (3C):

Test of Residuals for Normality:

Since AR(1) is rejected in step (3b), the test of normality will be carried out for AR(2) and ARMA(1,1) models.

χ^2 Test of Normality:

The calculations follow section 3c-(ii) and the results of the test are shown in Table (7.12) for AR(2) and in Table (7.13) for ARMA(1,1).

For AR(2) χ^2 = 25.444, the sum of the last column of Table (7.12).

From χ^2 tables , $\chi_{1-\alpha}^2(k-2) = \chi_{.95}^2(18) = 28.9$.

χ^2 (= 25.444) < $\chi_{.95}^2(18)$ (= 28.9), therefore the hypothesis of normality of the residuals of AR(2) is accepted.

For ARMA(1,1) χ^2 = 23.6667, the sum of the last column of Table (7.13).

From χ^2 tables, $\chi_{1-\alpha}^2(k-2) = \chi_{.95}^2(18) = 28.9$.

χ^2 (= 23.6667) < $\chi_{.95}^2(18)$ (= 28.9), therefore the hypothesis of normality of the residuals of ARMA(1,1) is accepted.

Skewness Test of Normality:

The skewness coefficients, γ, for AR(2) and ARMA(1,1) models are calculated using relation (7.44). γ for AR(2) is 0.2856 while it is 0.2964 for ARMA(1,1), Table (7.9).

The allowable range for γ as defined in relation (7.45), is (-0.3003,0.3003). This is obtained by having N = 360 and taking α = 0.02 and consequently $u_{0.99}$ = 2.326.

For both models skewness coefficients fall within the allowable range and hence the hypothesis of normality is accepted.

Table (7.12) χ^2 Test of normality - AR(2)

Interval k	Cumulative Probability	u -normal distribution	Class limit	N_i	$\frac{(N_i-N/k)^2}{N/k}$
1	0.05	-1.645	-1.23661	15	0.5
2	0.1	-1.2816	-0.96322	17	0.055556
3	0.15	-1.0367	-0.77898	12	2
4	0.2	-0.8415	-0.63213	18	0
5	0.25	-0.675	-0.50688	18	0
6	0.3	-0.525	-0.39403	20	0.222222
7	0.35	-0.385	-0.28871	25	2.722222
8	0.4	-0.2533	-0.18963	23	1.388889
9	0.45	-0.125	-0.09311	25	2.722222
10	0.5	0	0.000926	15	0.5
11	0.55	0.125	0.094964	27	4.5
12	0.6	0.2533	0.191484	20	0.222222
13	0.65	0.385	0.290562	20	0.222222
14	0.7	0.525	0.395884	13	1.388889
15	0.75	0.675	0.508729	13	1.388889
16	0.8	0.8415	0.633986	11	2.722222
17	0.85	1.0367	0.780835	18	0
18	0.9	1.2816	0.965074	12	2
19	0.95	1.645	1.23846	14	0.888889
20	1			24	2

Table (7.13) χ^2 Test of normality - AR(1,1)

Interval k	Cumulative Probability	u -normal distribution	Class limit	N_i	$\frac{(N_i-N/k)^2}{N/k}$
1	0.05	-1.645	-1.2333	15	0.5
2	0.1	-1.2816	-0.96006	13	1.388889
3	0.15	-1.0367	-0.77592	16	0.222222
4	0.2	-0.8415	-0.62915	19	0.055556
5	0.25	-0.675	-0.50396	17	0.055556
6	0.3	-0.525	-0.39117	20	0.222222
7	0.35	-0.385	-0.28591	28	5.555556
8	0.4	-0.2533	-0.18688	24	2
9	0.45	-0.125	-0.09041	18	0
10	0.5	0	0.003573	20	0.222222
11	0.55	0.125	0.097561	24	2
12	0.6	0.2533	0.194029	21	0.5
13	0.65	0.385	0.293055	19	0.055556
14	0.7	0.525	0.398321	15	0.5
15	0.75	0.675	0.511106	12	2
16	0.8	0.8415	0.636297	12	2
17	0.85	1.0367	0.783068	17	0.055556
18	0.9	1.2816	0.967208	11	2.722222
19	0.95	1.645	1.240449	14	0.888889
20	1			25	2.722222

7.3.4 Selection among Competent Models

Since two models have passed the goodness of fit test, Akaike Information Criterion, AIC, is to be performed to select one of them. Relation (7.46) is used to calculate AIC(p,q) and the result is shown in Table (7.14).

Table(7.14) Selection among competent models

Model	N	p	q	σ_ε^2	AIC
AR(1)	360	1	0	0.6078	-177.25
AR(2)	360	2	0	0.5893	-186.375
ARMA(1,1)	360	1	1	0.5638	-202.3

ARMA(1,1) gives minimum value of AIC, thus it is the one to be selected. The already rejected AR(1) gives the highest AIC value. It can be seen from Table (7.4) and Figure (7.1), that the correlogram of the selected model is the closest to the correlogram of the original time series Z_t.

7.3.5 Generation of Synthetic Time Series

The selected model is used to generate samples. The general form of ARMA(1,1), relation (7.53), is used for this purpose using the following already obtained values: the last value of the series Z_t is taken as Z_{t-1} = 0.175195, Table(7.3), the last value in the residual series ε_{t-1} = -0.04627, ϕ_1 = 0.808, θ_1 = 0.31 and σ_ε = 0.7509 as estimated before. A previously generated series of standard normal random numbers, ξ, is used. The sample size is 360, Table (7.15). The sample has a mean of 0.00423 ($\approx$ 0), a standard deviation of 1.0389($\approx$ 1.0) and a skewness coefficient of 0.03.

Test of ξ for Normality:

The skewness test of normality for ξ is carried out. Since N= 360 and α = 0.02, then as found in step 3c in Section 7.3.3, the skewness coefficient of a normally distributed variable should fall within the range (-0.3003, 0.3003). The standard normal number, ξ, skewness coefficient is 0.03. Therefore ξ is normally distributed.

Test of ξ for Independence:

For independence Port Manteau test is done. The autocorrelation function $r_k(\xi)$ is calculated using equation (7.7), Table (7.16). For N = 360 and $\sum (r_k(\xi))^2$ = 0.08575, Q = 30.87. Q is less than $\chi^2_{0.95}(36)$ = 51. Therefore ξs chosen are independent.

The independent normal numbers are used to generate the time series, Z_t, Table (7.17). Figure (7.2) shows the actual and generated standardised time series Z_t .

Table (7.15) Standard normal random number series, ξ.

ξ(1-40)	ξ(41-80)	ξ(81-120)	ξ(121-160)	ξ(161-200)	ξ(201-240)	ξ(241-280)	ξ(281-320)	ξ(321-360)
-0.319	-1.125	2.272	0.662	-1.843	-1.914	0.049	-1.258	0.359
-1.749	2.633	-1.928	0.603	-1.149	-0.562	-1.101	-0.44	0.242
0.358	0.516	-1.513	-1.533	-0.347	0.339	-0.16	-0.813	0.545
-0.802	0.81	0.862	-0.295	-0.219	-1.405	0.716	-0.307	-1.552
0.109	-0.09	2.152	0.317	0.277	1.155	1.244	1.185	-0.026
0.391	0.023	0.447	0.497	-1.662	-0.538	-0.519	-0.854	0.971
0.773	0.627	-1.297	-1.317	0.588	1.028	-0.06	-1.771	-2.213
-0.815	0.1	0.287	0.093	1.311	-0.128	-0.656	2.759	1.039
-1.043	0.735	-0.278	-0.192	1.009	-1.426	-2.005	0.194	-0.735
-0.915	0.171	-0.056	-0.203	-0.792	0.507	-0.954	-0.047	1.214
0.14	2.538	-1.014	-0.489	1.403	0.573	-1.508	-0.518	-0.359
1.641	0.255	0.913	-1.21	-0.76	1.058	2.014	1.905	0.9
0.604	0.371	0.522	-0.035	-1.134	0.23	0.325	1.524	-1.5
-0.366	1.17	1.671	-0.032	0.427	1.763	-0.335	2.16	1.481
-0.349	0.438	2.19	0.483	1.24	0.239	0.17	-2.638	-0.074
0.25	1.319	1.392	0.741	-0.581	-1.072	-2.213	0.196	0.051
2.367	-0.444	-1.035	-0.385	-2.355	-0.537	-0.325	-0.049	0.955
-0.9	0.698	-1.352	0.496	1.397	-0.306	-1.119	-0.92	-1.206
-1.309	1.137	0.749	-0.061	-0.737	0.299	1.065	0.061	-2.228
0.004	2.129	-0.304	0.503	-0.718	0.066	-0.267	0.414	-1.72
1.006	0.994	-2.448	1.225	-0.204	-0.638	-1.227	1.256	-0.097
-0.724	0.998	1.015	-0.816	0.846	-1.154	0.76	-1.166	1.107
0.515	0.087	-0.538	0.811	1.311	-1.514	-1.855	-0.612	0.546
0.188	-1.356	0.304	-0.785	0.342	-1.714	-1.097	-0.942	-0.001
0.859	-0.475	0.267	1.354	-0.911	0.359	0.631	0.118	-0.05
-0.84	0.1	0.195	-1.438	0.348	0.592	-0.596	-0.253	-0.222
0.757	1.161	-0.61	0.276	-0.768	-2.995	-0.372	-0.926	-0.228
-0.192	-0.144	0.487	-0.959	0.938	1.286	-1.063	2.201	1.299
0.496	-2.014	0.509	-0.626	-0.395	-0.934	1.222	0.439	-0.162
1.085	0.325	0.932	1.505	-1.982	0.345	-0.125	-0.058	0.219
-1.163	-1.528	-0.858	0.304	0.161	0.198	1.087	2.009	0.092
1.342	0.718	0.73	1.239	0.978	-0.047	-0.811	0.398	0.775
-0.543	-0.514	-1.394	0.363	-0.937	-0.409	0.057	-0.366	1.418
-0.028	0.634	-0.36	-0.508	-0.732	-1.257	-0.473	1.091	-0.03
-0.669	0.561	-0.33	0.066	-0.352	-0.94	-0.522	-0.644	0.714
-0.436	0.806	0.31	3.085	1.667	-0.999	0.009	-0.145	0.1
0.172	1.014	0.561	0.716	-0.139	0.712	0.308	0.917	-0.715
-0.138	-1.437	-0.35	-0.093	-0.829	-0.017	-0.312	-0.606	0.471
-0.131	0.952	-2.147	-0.851	-1.954	-0.458	0.245	-1.112	2.0801
2.53	0.194	0.797	-1.617	1.485	0.731	0.235	-1.725	1.31

Table (7.16) Correlogram standard random numbers, $r_k(\xi)$

k	$r_k(\xi)$	k	$r_k(\xi)$	k	$r_k(\xi)$	k	$r_k(\xi)$
1	-.0374	10	-.1007	19	.0284	28	-.0111
2	-.0083	11	.022	20	.0016	29	.0482
3	-.0606	12	.0475	21	-.0348	30	.0121
4	.0612	13	-.0374	22	-.0264	31	-.0107
5	.0481	14	.1116	23	.05	32	.0636
6	-.055	15	.0452	24	-.0481	33	.048
7	.0613	16	-.0162	25	.0406	34	.0579
8	.0054	17	.0563	26	.038	35	-.0106
9	.0277	18	.0996	27	-.0349	36	.0219

Table (7.17) Generated standard time series, Z_t.

Z(1-40)	Z(41-80)	Z(81-120)	Z(121-160)	Z(161-200)	Z(201-240)	Z(241-280)	Z(281-320)	Z(321-360)
-0.0836	-0.0331	2.0888	0.0886	-1.5288	-1.6469	-0.2128	-0.9480	-0.5297
-1.3066	2.2122	-0.2888	0.3703	-1.669	-1.3072	-1.0101	-0.8035	-0.3299
-0.3798	1.5620	-0.9206	-0.9923	-1.3417	-0.6708	-0.6800	-1.1573	0.0864
-0.9924	1.7502	0.2556	-0.6664	-1.1677	-1.6759	0.0254	-0.9764	-1.2224
-0.5333	1.1580	1.6217	-0.2318	-0.6846	-0.1598	0.7880	0.1723	-0.6460
-0.1627	0.9739	1.1451	0.1121	-1.8655	-0.8020	-0.0426	-0.7778	0.2132
0.3579	1.2524	-0.1527	-1.0140	-0.6790	0.2491	0.0413	-1.7595	-1.7154
-0.5027	0.9411	0.3940	-0.4429	0.2989	-0.1341	-0.4452	1.0622	-0.0908
-0.9996	1.2890	0.0428	-0.5237	0.6940	-1.1493	-1.7125	0.3617	-0.8671
-1.2520	0.9988	0.0573	-0.5309	-0.2688	-0.2160	-1.6333	0.2118	0.3820
-0.6935	2.6729	-0.7021	-0.7489	1.0206	0.1377	-2.2230	-0.2069	-0.2435
0.6393	1.7604	0.3543	-1.4000	-0.0726	0.7723	0.0614	1.3838	0.5626
0.5881	1.6416	0.4657	-0.8757	-0.7332	0.5504	-0.1751	1.8190	-0.8812
0.0598	2.1186	1.5095	-0.7234	-0.0079	1.715	-0.4687	2.7369	0.7492
-0.1286	1.7684	2.4751	-0.2144	0.8253	1.1548	-0.1731	-0.2721	0.2050
0.1651	2.3173	2.5353	0.2707	-0.0580	0.0725	-1.8411	0.5413	0.2212
1.8525	1.2320	0.9474	-0.2428	-1.6799	-0.0951	-1.2165	0.3550	0.8839
0.2706	1.6229	-0.0088	0.2658	0.2397	-0.1816	-1.7475	-0.3926	-0.4136
-0.5552	2.0025	0.8700	0.0535	-0.6849	0.1490	-0.3519	-0.0573	-1.7264
-0.1409	2.9520	0.3004	0.4351	-0.9209	0.1004	-0.7327	0.2504	-2.1678
0.6406	2.6360	-1.5247	1.1543	-0.7302	-0.4133	-1.4512	1.049	-1.4241
-0.2602	2.6479	0.1000	0.0349	0.0927	-1.0520	-0.3163	-0.3202	-0.2969
0.3450	1.9725	-0.5594	0.8271	0.8624	-1.7182	-1.8253	-0.4469	-0.0876
0.3000	0.5554	-0.0985	-0.1100	0.6485	-2.3229	-1.8668	-0.9259	-0.1986
0.8437	0.4077	0.0501	1.1106	-0.2397	-1.2084	-0.7792	-0.4403	-0.1978
-0.1490	0.5151	0.1248	-0.4976	0.2797	-0.6154	-1.2240	-0.5732	-0.3149
0.6435	1.2647	-0.4026	0.1399	-0.4317	-2.8839	-1.1296	-1.0996	-0.3739
0.1996	0.6435	0.1824	-0.6713	0.5343	-0.6674	-1.6243	0.9798	0.7263
0.5784	-0.9588	0.4162	-0.7892	-0.0832	-1.5399	-0.1474	0.6090	0.1629
1.1666	-0.0619	0.9176	0.6381	-1.4635	-0.7678	-0.4974	0.3463	0.3337
-0.1832	-1.2730	-0.1198	0.3935	-0.6003	-0.5520	0.4434	1.8018	0.2878
1.1303	-0.1338	0.6511	1.1775	0.2118	-0.5274	-0.5037	1.2871	0.7930
0.1932	-0.6612	-0.6906	0.9356	-0.7601	-0.7223	-0.1754	0.6725	1.5251
0.2615	0.0615	-0.5038	0.2900	-0.9457	-1.4323	-0.5102	1.4478	0.8797
-0.2845	0.3233	-0.5711	0.4022	-0.858	-1.5705	-0.6941	0.4323	1.2539
-0.4016	0.7359	-0.1518	2.626	0.6404	-1.8003	-0.4326	0.3903	0.9220
-0.0938	1.1683	0.2264	1.9413	0.0250	-0.6875	-0.1203	1.0377	0.1849
-0.2195	-0.371	-0.2105	1.332	-0.5699	-0.7340	-0.4032	0.1700	0.6694
-0.2436	0.7495	-1.7007	0.4590	-1.7347	-0.9330	-0.0692	-0.5566	1.9932
1.7333	0.5297	-0.2760	-0.6452	0.1682	-0.0984	0.0635	-1.4861	2.1099

The relation, $y_{v,\tau} = \mu_\tau + \sigma_\tau Z_{v,\tau}$, is used to find the generated transformed series, $y_{v,\tau}$. μ_τ, i.e. $\bar{y}_\tau$, and σ_τ, i.e. S_τ, are taken as found in Table (7.2) and $Z_{v,\tau}$ are taken from Table (7.17). The results are shown in Table (7.18).

Table (7.18) Generated transformed time series, $y_{v,\tau}$

	july	auq.	sept	oct.	nov.	dec
1	7.1801	8.0946	7.9274	6.4294	5.4543	4.8814
2	7.4667	8.6398	8.0330	7.0458	6.3396	5.0263
3	7.5758	8.5565	8.3575	7.0642	5.8668	5.3265
4	7.1757	8.5284	7.9847	7.8811	5.6399	5.6765
5	7.7658	9.0145	9.2103	7.8955	6.2613	5.6452
6	8.3407	9.6725	8.9160	7.2537	5.8035	5.1083
7	6.9335	8.6405	8.2229	7.3498	6.0857	4.8116
8	7.9078	9.0728	8.0229	7.1677	5.6681	4.9550
9	7.6201	8.6124	8.4527	7.1179	5.0864	4.9693
10	7.3934	8.9207	8.0367	7.3046	5.3959	4.7672
11	7.2535	8.7637	7.6700	6.6030	5.5662	4.9734
12	6.8420	8.3273	7.9969	7.1021	5.5621	5.0249
13	7.6897	8.4174	8.1458	6.6004	5.3593	5.1495
14	8.0443	9.1475	8.2799	6.6143	5.0849	4.3770
15	7.5119	8.5087	8.5159	6.9192	5.3801	4.9332
16	6.9041	8.6529	8.4495	7.3032	5.5632	5.0295
17	6.8914	8.2386	7.7264	7.2989	5.6615	4.7450
18	7.1475	8.2959	8.1917	6.8865	5.2257	4.8635
19	7.1752	8.5435	8.1496	7.0113	5.4988	4.5836
20	6.5585	8.3096	7.8550	6.6770	5.3841	4.4563
21	7.1249	8.2129	7.8013	6.9714	5.9446	4.9216
22	7.1410	8.4289	8.0143	5.9774	5.2008	4.3508
23	6.8832	8.1275	7.6123	6.0928	5.5975	4.7693
24	7.1643	8.4550	8.0580	6.9917	5.3004	4.6668
25	7.3701	8.7005	8.0001	7.6949	6.3272	4.8522
26	7.6634	8.4881	7.8992	6.4648	5.4888	4.7439
27	7.5027	9.1936	8.2687	7.1658	6.0372	4.9928
28	6.9401	8.7010	7.3661	6.9095	5.3304	5.0638
29	7.5930	8.4509	7.3615	5.8034	5.1237	4.8365
30	7.2852	8.7491	8.2080	7.3802	6.2181	5.2304
av.	7.3349	8.6175	8.0911	6.9659	5.6152	4.9577
s.d	0.3952	0.3502	0.3898	0.4993	0.3722	0.3466

Table (7.18)- continued.

	jan	feb	march	april	may	june
1	4.4083	3.7254	3.4193	3.2436	3.7812	5.3629
2	4.1840	3.8062	3.8297	3.4728	4.2089	5.2659
3	4.2754	4.0904	3.7178	3.5934	3.9496	5.0652
4	4.7040	4.2289	3.9592	3.7580	4.5826	5.4492
5	4.7547	4.3557	3.9777	3.9080	4.8916	6.0245
6	4.6310	3.9816	3.4295	3.5186	3.5425	5.1418
7	4.5045	3.9561	4.1921	3.4662	3.6876	5.2532
8	4.1480	3.9169	3.7859	3.8818	5.0863	5.9053
9	4.1830	3.8157	3.6820	3.5618	3.9010	5.2322
10	4.1801	3.8038	3.7261	3.4843	3.3663	5.1011
11	4.0714	3.7387	3.5384	3.4103	3.7584	4.7796
12	4.3335	3.9350	3.9582	3.5410	4.4075	5.1486
13	4.4170	4.1009	3.9035	3.6000	4.2325	5.9312
14	3.9909	3.5767	3.4981	3.1019	3.7872	5.2656
15	4.5231	3.8248	3.2491	3.5884	3.7847	4.9166
16	4.2144	3.9571	3.6486	3.1947	3.8196	5.2406
17	3.8944	3.8753	3.2574	3.2309	3.7905	4.7007
18	4.3542	4.0103	3.8071	3.9293	4.5425	5.2008
19	3.8984	3.3185	3.3671	3.3907	2.8790	4.9891
20	3.9347	3.4354	3.4974	3.3633	3.6825	5.1519
21	4.3305	3.7382	3.2409	3.1555	3.1483	5.1963
22	4.2340	3.6740	3.3063	3.4599	3.3150	4.6461
23	4.4293	3.7251	3.6255	3.4150	3.7809	5.0563
24	4.0362	3.6195	3.7125	3.3532	3.3421	5.4839
25	4.2536	3.9587	3.7582	3.4422	4.0432	5.2517
26	4.0504	4.0567	3.8218	3.6130	4.8089	5.5482
27	4.1837	3.5056	3.5369	3.4567	4.1024	4.8304
28	4.2606	3.9635	3.4489	3.7061	4.1513	5.2433
29	4.2989	3.7933	3.6199	3.4602	3.9128	5.3878
30	4.6282	4.0438	3.7157	3.6877	4.8878	5.7836
av.	4.2770	3.8511	3.6410	3.4996	3.9725	5.2518
s.d	0.2307	0.2274	0.2411	0.2095	0.5370	0.3408

To find the original generated time series, $X_{v,\tau}$, the relation $y_{v,\tau} = X_{v,\tau}^{0.225}$ is rewritten as $X_{v,\tau} = y_{v,\tau}^{1/0.225}$ and values of $y_{v,\tau}$ from Table (7.18) are used to obtain the generated series $X_{v,\tau}$, Table (7.19). Figure (7.3) shows the actual and generated time series.

Table(7.19) generated series $Xv_{,\tau}$ in million m^3

yr	July	aug.	sept	oct.	nov	dec.	jan.	feb.	mar	aprl	may	june
1	6383	10875	9912	3907	1881	1149	730	346	236	187	369	1745
2	7596	14529	10512	5869	3670	1308	579	380	391	253	594	1609
3	8102	13917	12535	5938	2601	1693	637	524	342	294	448	1354
4	6366	13714	10234	9656	2183	2246	974	607	453	359	868	1873
5	9045	17546	19305	9735	3473	2192	1022	692	462	427	1159	2926
6	12423	23998	16710	6679	2478	1406	909	464	239	268	276	1447
7	5465	14534	11662	7081	3061	1077	804	451	584	251	330	1592
8	9803	18057	10453	6334	2232	1228	557	432	371	415	1379	2678
9	8314	14326	13182	6141	1379	1244	578	384	328	283	424	1564
10	7270	17267	10533	6890	1793	1034	577	379	346	257	220	1397
11	6678	15478	8559	4398	2059	1248	513	351	275	233	359	1046
12	5151	12334	10304	6080	2052	1306	677	441	452	276	730	1456
13	8657	12939	11184	4391	1740	1457	737	530	425	297	609	2730
14	10577	1826	12026	4432	1377	707	469	288	261	153	372	1608
15	7802	13574	13626	5415	1770	1204	819	388	188	293	371	1186
16	5363	14628	13159	6884	2054	1312	598	452	315	175	386	1575
17	5319	11761	8843	6866	2220	1013	421	412	190	184	373	971
18	6255	12130	11467	5302	1555	1130	691	480	381	438	834	1522

Table(7.19) generated series $Xv_{,\tau}$ in million m^3 - continued

yr	July	aug.	sept.	oct.	nov	dec.	jan.	feb.	mar	aprl	may	june
19	6363	13823	11207	5743	1950	868	423	207	220	227	110	1266
20	4268	12218	9516	4622	1776	766	441	241	261	219	328	1460
21	6168	11599	9229	5599	2758	1191	675	351	186	165	164	1518
22	6230	13018	10403	2826	1522	689	610	325	203	249	206	922
23	5291	11073	8277	3077	2111	1036	746	345	306	235	369	1343
24	6321	13198	10658	5672	1656	941	493	304	340	216	213	1927
25	7169	14988	10322	8683	3639	2572	623	453	359	243	497	1590
26	8527	13429	9756	4004	1934	1012	501	505	387	302	1075	2029
27	7760	19150	11954	6327	2954	1270	579	264	274	248	530	1096
28	5488	14992	7151	5381	1698	1352	628	455	245	338	559	1579
29	8184	13169	7132	2478	1425	1102	653	374	304	249	430	1781
30	6809	15364	11568	7213	3368	1561	907	498	342	330	1155	2441
Av	7172	14545	11046	5787	2212	1277	652	411	322	269	525	1641
SD	1782	2812	2504	1755	682	430	159	106	95	73	323	499
SC	.996	1.521	1.424	.339	.908	1.505	.679	.438	.638	.783	1.217	1.148

7.3.6 Statistics Preservation

The means and standard deviations of the generated and original time series are compared, Table (7.20) and Figure (7.4). The generated sample preserves the mean and standard deviation of the original series during high and low flow periods. Therefore time series analysis can be used to implicitly model flows with periods of low flows. Unlike what was found by Hurst (1952) for the Main Nile, the clustering of low flow in successive years and droughts have little effects on the Blue Nile System operation. The system is operated on annual basis. In the early flood season, that follows the low flow period, the reservoirs are emptied and operated at low levels to pass sediments. It is always guaranteed to fill the reservoirs, during the falling flood, since they have very low storage capacities compared to the river recorded flow. Therefore drought effect is not carried from one year to the next.

Table (7.20) Comparison of original & transformed generated series statistics

Month	Mean Original	Mean Generated	Standard deviation original	Standard deviation generated
July	7.216	7.335	0.427	0.395
August	8.616	8.618	0.399	0.35
September	8.087	8.091	0.42	0.39
October	6.958	6.966	0.533	0.499
November	5.652	5.615	0.371	0.372
December	4.936	4.958	0.335	0.347
January	4.320	4.277	0.246	0.231
February	3.838	3.851	0.224	0.227
March	3.669	3.641	0.250	0.241
April	3.533	3.50	0.231	0.209
May	4.067	3.972	0.412	0.537
June	5.180	5.252	0.286	0.341

7.4 CONCLUSION

The Blue Nile flow is modelled using ARMA(1,1) model. The fitted model has been used to generate a flow sequence that preserves the mean and standard deviation of the original sample, during high and low flow periods. This shows the suitability of time series analysis approach to implicitly modelling the Blue Nile flow where the low flow clusters, due to the operation of the system on annual basis, and droughts, due to the small reservoir capacities, are expected to have little effects on system operation. This finding verifies partially hypothesis 3. The generated samples are used as inputs to the optimization model in Chapter IX and the output is used in deriving operation rules in Chapter X.

Figure (7.1) Models correlogram compared to standardised series

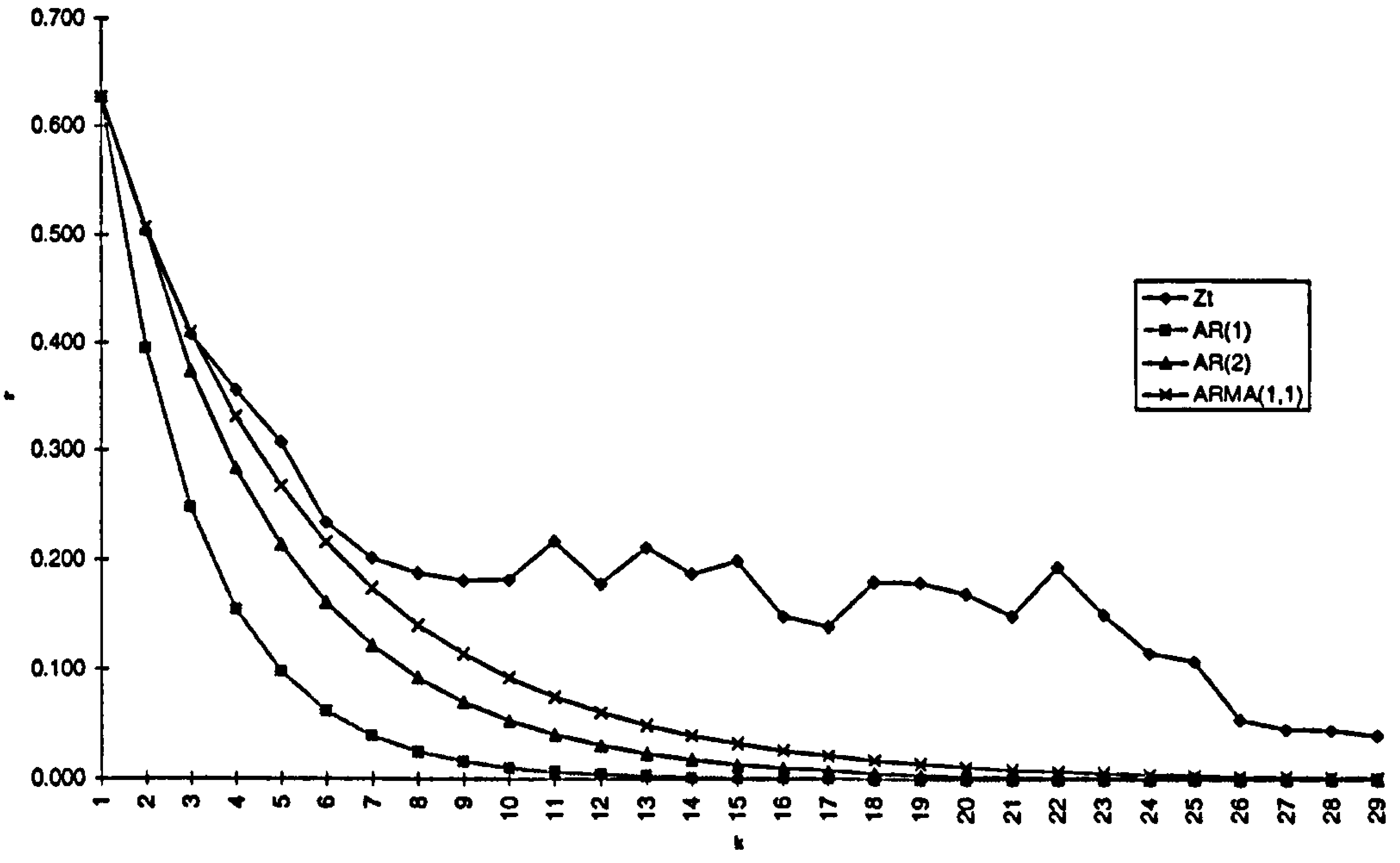


Figure (7.2) Generation of Zt series , Blue Nile

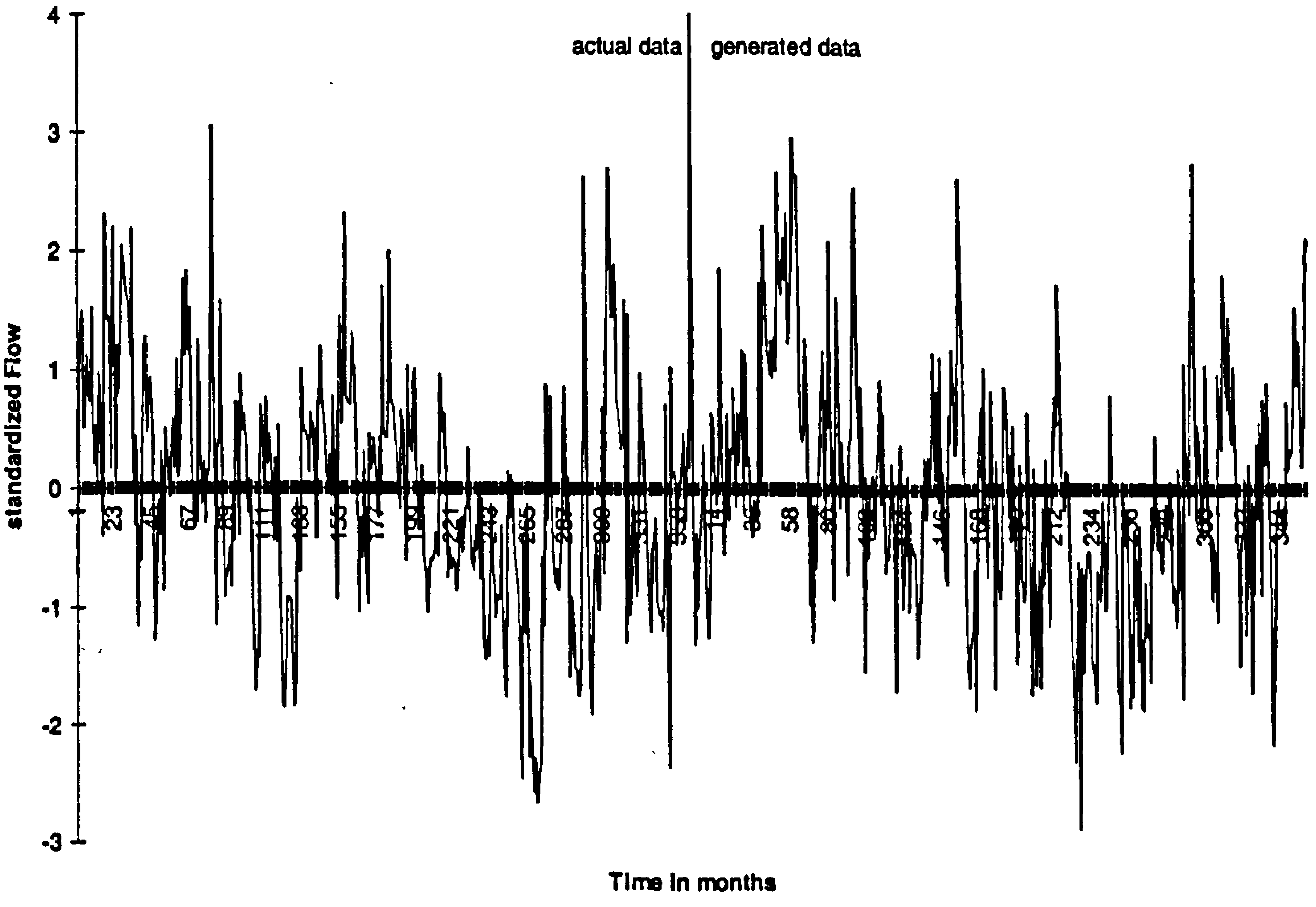


Figure (7.3) Actual and syntheticly generated monthly flows, Blue Nile

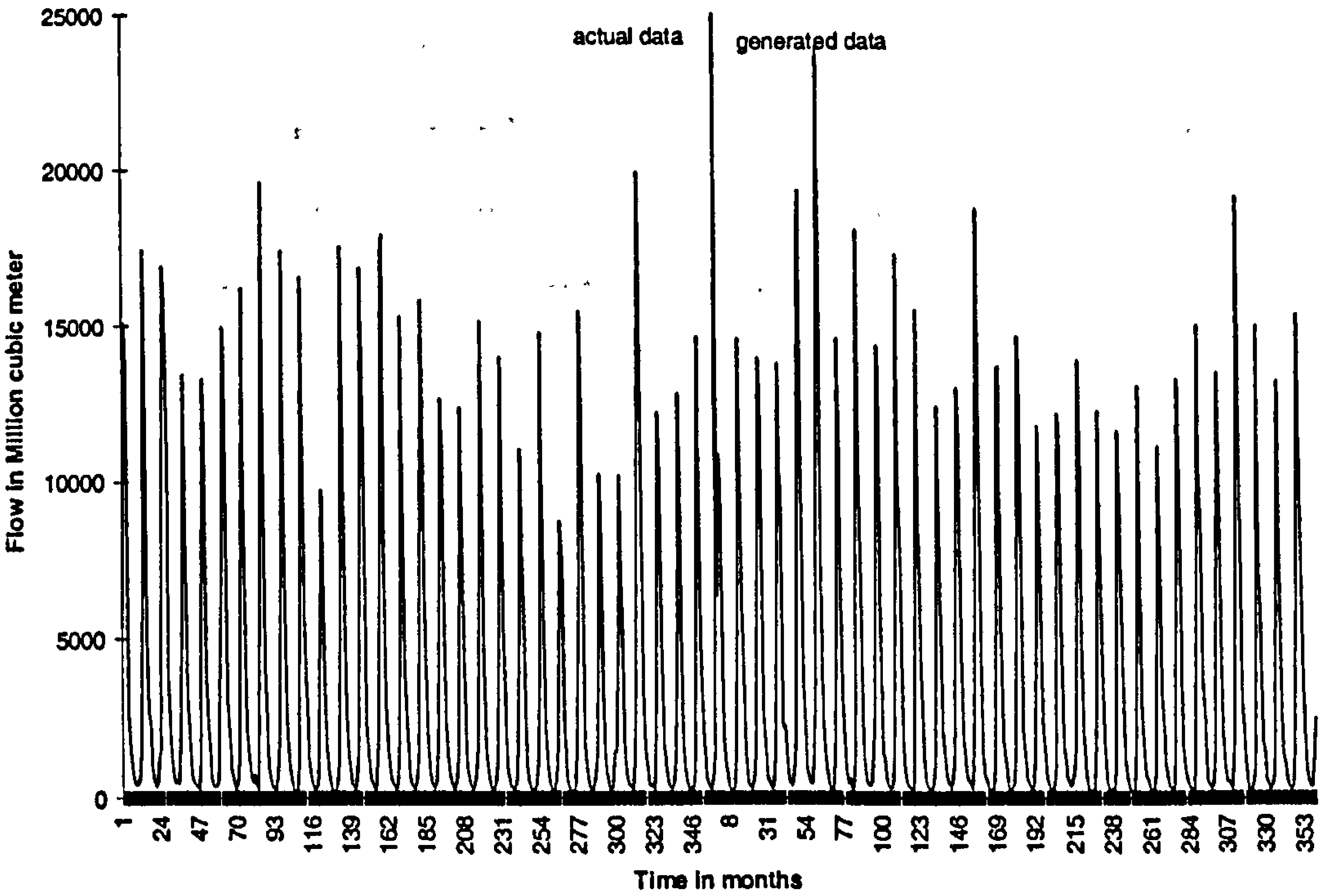
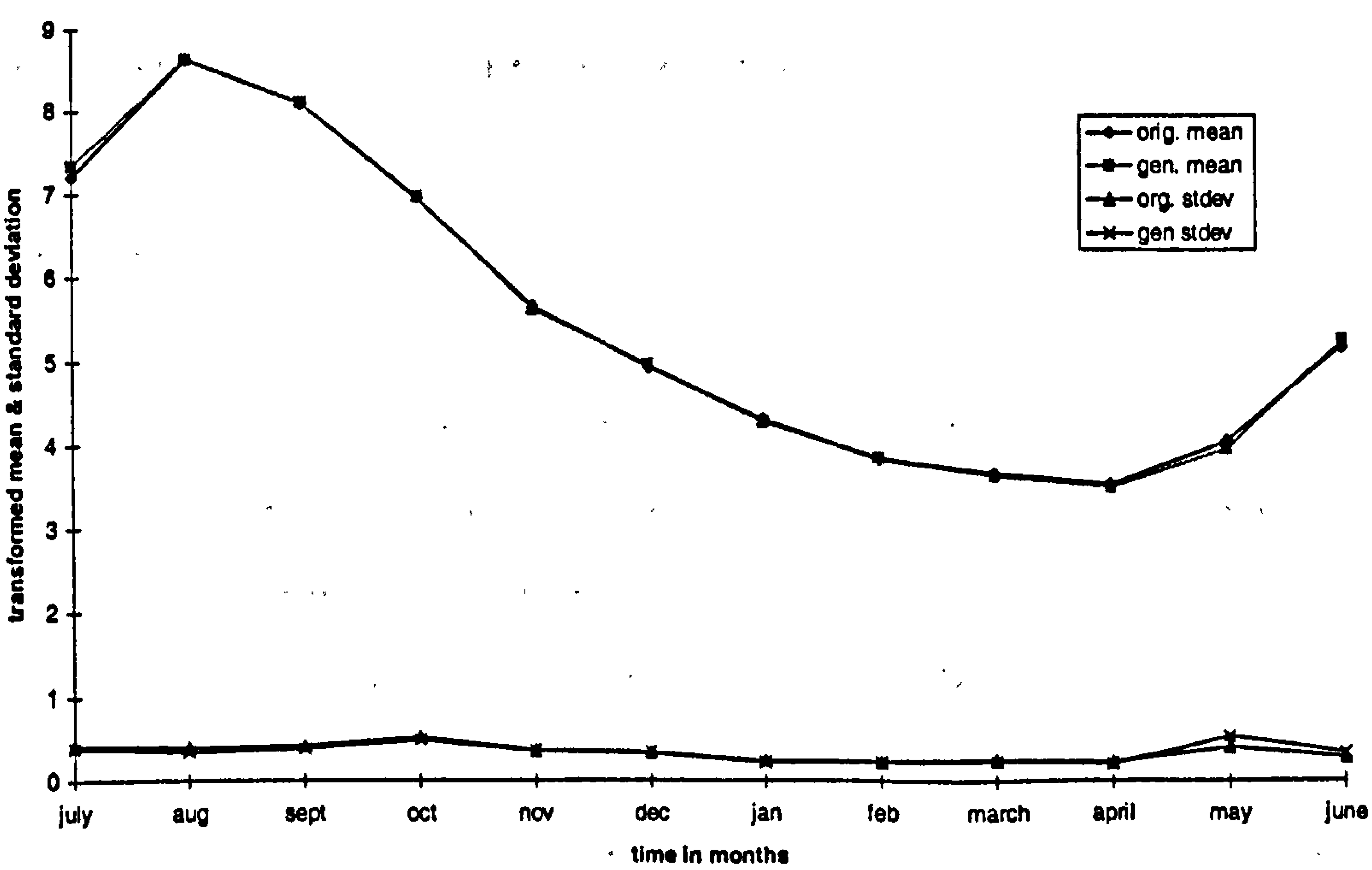


Figure (7.4) Comparison of statistics of the transformed actual & generated time series



CHAPTER VIII

IRRIGATION REQUIREMENTS

Summary ~ In this chapter, actual irrigation requirements to be supplied from the reservoir system are estimated. The efficiency of water use is investigated and the possible requirements, resulting from using water more efficiently, are estimated. Both requirements are to be used as inputs to the optimization model in Chapter IX.

8.1 INTRODUCTION

The total area fed by the Blue Nile System, is about 2685383 feddans, (1128312 ha). The area of the Gezira Scheme represents 77.5 % of this area (2081692 feddan, 874661 ha), (Sir Alexander Gib and Partners, 1978). This scheme is the oldest and other schemes follow it in their design and operation. Being the largest and the pilot scheme, data from the Gezira Scheme would be used in estimating the crop water requirements, irrigation efficiencies and hence the irrigation requirements that need to be supplied by the reservoir system.

8.2 GEZIRA IRRIGATION SCHEME

8.2.1 Canalisation

The Gezira irrigation system comprises two main canals taking water from Sennar dam reservoir. Branch and major canals take off the main canals to supply water to minor canals which in turn supply water to field ditches canals called Abu Ishreen, (Abu XX). The latter supplies water to a standard area of 90 feddans, (38 ha), called a number, through smaller field canals called Abu Sitta (Abu VI). Figure (8.1) shows the layout of the system while Table (8.1) shows the characteristics of different canals.

Table (8.1) Gezira scheme canals characteristics

Canal	Number	Total Length (Km)	Surface Width (m)
Main	2	261	40
Branch	11	651	40
Major	107	1652	10 to 21
Minor	1498	8119	10
Abu XX	29000	40000 -1.5 km each	4 to 2.5
Abu VI	350000	100000-0.3 " "	1.4

Source : (Ministry of Irrigation and Water Resources, May 1990)

8.2.2 System Operation

The system applied in the Gezira and other irrigation schemes is the night storage system. Under this system the minor canals are enlarged to store water during the night. Abu xxs stay closed at the time water is stored and the release is made during day time only (12 hours). The irrigation interval is two weeks. The application period is one week. In the first three to four days water is supplied to half of Abu VIs, while it is supplied to the remaining Abu VIs in the last days of the week.

The method used in estimating crop water requirements is an empirical one. According to this method all crop requirements are estimated at 30 m^3/ fd per day including the field losses up-to the head of Abu XX (Plusquellec, 1990). This is equivalent to 420 m^3/ fd per fortnight, i.e. 100 mm application depth or 7.14 mm / day.

8.3 CROPWAT

Cropwat is a software developed by FAO (FAO, 1992). The program will be used here to calculate evapotranspiration, ET_0. The program uses Penman-Monteith method for calculating ET_0 and USDA method, among others, to calculate effective rainfall. Both methods are described below.

8.3.1 Penman - Monteith

The method used for calculating crop water requirements in Gezira does not take account of neither the crop type nor its stage of growth. Therefore in this study the internationally accepted and more precise approach of Penman-Monteith will be used. This method calculates reference crop evapotranspiration, ETo, as follows (FAO, 1993):

$$ET_0 = \frac{0.408\,\Delta\,(R_n - G) + \gamma(900/(T+273))U_2(e_s - e_a)}{\Delta + \gamma\,(1 + 0.34\,U_2)} \qquad (8.1)$$

Where

ET_0 reference crop evapotranspiration [mmd^{-1}].

R_n net radiation at crop surface [$MJm^{-2}d^{-1}$].

G soil heat flux [MJ $m^{-2}d^{-1}$]

T average temperature [^0C]

U_2 wind speed measured at 2 metre height [ms^{-1}]

(e_s-e_a) vapour pressure deficit [kpa]

Δ slope vapour pressure curve [kpa $^0C^{-1}$]

γ psychometric constant [kpa $^0C^{-1}$]

900 conversion factor.

The net radiation is determined as follows :

$$R_n = R_{ns} - R_{nl} \qquad (8.2)$$

$$R_{ns} = 0.77(0.25 + 0.5\ n/N)\ R_a \qquad (8.3)$$

$$R_{nl} = 2.45E\text{-}9(0.9n/N + 0.1)(0.34\text{-}0.14\sqrt{e_a})(T_{kx}^4 + T_{kn}^4) \qquad (8.4)$$

$$G = 0.14(T_{monthn} - T_{monthn\text{-}1}) \qquad (8.5)$$

Where

R_n net radiation [$MJm^{-2}d^{-1}$]

R_{ns} net short-wave radiation [$MJm^{-2}d^{-1}$]

R_{nl} net long-wave radiation[$MJm^{-2}d^{-1}$]

R_a extraterrestrial radiation[$MJm^{-2}d^{-1}$]

n/N relative sunshine fraction

T_{kx} maximum temperature [K]

T_{kn} minimum temperature [K]

e_a actual vapour pressure [kpa].

T_{monthn} & $T_{monthn-1}$: mean temperature in months n & n-1 respectively [^0C].

0.25 , 0.5 Angstorm coefficients.

Finding e_a, e_s, R_a, N, Δ and γ is described in Chapter VI.

8.3.2 Effective Rainfall

To calculate the effective rainfall, which is the part of rain that actually used for evapotranspiration of the crop, the USDA Soil Conservation Services method is used. This method is as follows (FAO, 1993):

$$P_{eff} = P_{tot} (125 - 0.2 P_{tot})/125, \text{ for } P_{tot} \angle 250 \text{ mm}$$

$$P_{eff} = 125 + 0.1 P_{tot}, \text{ for } P_{tot} \geq 250 \text{ mm} \qquad (8.6)$$

Effective rainfall for the selected canals are shown in Table (8.5).

8.4 SELECTED CANALS

Three major canals that offtake from the main canal have been selected. These are: Zananda, Gamusia and Kab Elgidad. They are located at the head, the middle and the tail of the Gezira scheme respectively. In each major, three minor canals located at the head, middle and tail of each major are selected (Figure 8.2). These are, from head to tail of the majors are: Gymaillia, Toman and Wad Numan for Zananda; Hamza, Umuud and Fadlein for Gamusia; Eltuweir, Elmardi and Beibash for Kabelgidad. The position of the canal will be described later as H-H for head minor canal in a head major or H-T for a tail minor in a head major. The following tables show data available on these canals, collected by the Hydraulic Research Station, Wad Medani, Sudan.

Table (8.2) Location, length and command area of selected canals

Minor canal	Position	Command area -fed	Canal length-km
Gymailya	H-H	1591	4.6
Tuman	H-M	1839	6.0
Wad Numan	H-T	2719	6.7
Hamza	M-H	2672	8.6
Um uud	M-M	2415	8.5
Fadlein	M-T	1653	5.7
Eltuweir	T-H	1397	4.1
Elmardi	T-M	1135	3.4
Beibash	T-T	893	3.3

Table (8.3) Cropping pattern - 1988/89

Minor canal	Cotton-fed	Groundnut fed	Sorghum fed	Wheat fed	cropping intensity
Gymailya	390	-	426	410	0.77
Tuman	391	-	543	299	0.67
WadNuman	653	-	714	496	0.68
Hamza	428	332	663	307	0.65
Um uud	278	223	446	435	0.57
Fadlein	397	175	350	355	0.77
Eltuweir	283	122	245	231	0.63
Elmardi	298	101	203	179	0.69
Beibash	161	97	195	145	0.67
the scheme	422085	135160	487220	367764	0.67

Table (8.4) Actual and recommended sowing dates - 1988/89

Minor canal	Cott. act	Cott rec	G.N act.	G.N rec.	Sorg. act.	Sorg rec.	Wheat act.	Wheat rec
Gymailya	20/7	6/7	-	-	15/6	15/6	10/11	20/11
Tuman	20/7	6/7	-	-	22/6	15/6	10/11	20/11
WadNuman	15/7	6/7	-	-	24/6	15/6	10/11	20/11
Hamza	3/8	6/7	25/6	1/6	1/7	15/6	15/11	20/11
Um uud	3/8	6/7	25/6	1/6	1/7	15/6	15/11	20/11
Fadlein	3/8	6/7	25/6	1/6	1/7	15/6	15/11	20/11
Eltuweir	23/8	25/6	20/6	1/6	10/7	15/6	20/11	20/11
Elmardi	1/9	25/6	20/6	1/6	10/7	15/6	20/11	20/11
Bebash	23/8	25/6	20/6	1/6	10/7	15/6	20/11	20/11

The recommended sowing dates are given by Plusquellec (1990).

Table (8.5) Rainfall in mm per month

Month	Gymalia	Tuman	Wad Numan	Hamza Umuud Fadlein	Eltuweir	Elmardi	Beibash
Jun 88	0	0	0	0	0	0	0
Jul 88	109/90	93/79.2	91/77.8	94/79.9	21/20.3	18/17.5	24/23.1
Aug 88	105/87.4	87/74.9	106/88	96/81.3	64/57.4	82/71.2	97/81.9
Sept 88	53/48.5	80/69.8	42/39.2	5751.8	20/19.4	20/19.4	20/19.4
Oct 88	20/19.4	21/20.3	12/11.8	8/7.9	0	0	0
Nov 88	0	0	0	0	0	0	0
Dec 88	0	0	0	0	0	0	0
Jan 89	0	0	0	0	0	0	0
Feb 89	0	0	0	0	0	0	0
Mar 89	0	0	0	0	0	0	0
April 89	0	0	0	0	0	0	0
May 89	0	0	0	0	0	0	0

The second figure represents the effective rainfall.

Table (8.6) Actual measured releases to minor canals - in 10^3 m^3

Month	Gym.	Tum.	Numan	hamza	umud	fadlen	tuwer	mardi	bbsh
Jun 88	-	-	-	-	-	-	-	-	-
Jul 88	304.8	457.9	892.4	805.6	528.5	484.1	239.3	304.5	380.7
aug88	196.2	381.5	403.6	939.8	620.1	658.7	247.7	373.7	684.3
sep88	356.0	433.2	833.0	882.8	606.2	515.2	421.8	414.8	557.0
oct88	377.7	443.2	880.5	899.2	607.0	603.1	615.1	598.8	509.9
nov88	797.3	698.8	1225.5	837.6	791.6	762.4	599.0	620.1	417.3
dec88	588.8	523.6	942.2	613.4	561.0	569.7	462.5	780.7	482.2
jan89	615.0	513.4	961.1	655.7	646.6	625.6	496.1	832.5	627.8
feb89	315.6	224.2	516.4	491.1	489.3	416.4	436.3	520.5	256.2
mar 89	25.6	13.3	164.0	130.0	128.1	63.4	431.6	90.4	55.8
apr 89	-	-	-	-	-	-	-	-	-
may89	-	-	-	-	-	-	-	-	-

Table (8.7) Meteorological data - Wad Medani

Month	T_{max} ^0C	T_{min} ^0C	RH %	Wind speed km / day	sun bright hours
Jan	33.5	14	35	216	10.3
Feb	35	14.8	27	242	10.7
Mar	38.3	18.1	21	216	10.4
April	40.2	21	19	190	10.6
May	41.3	23.8	28	216	10.1
June	39.6	24.5	39	268	9.3
July	35.7	22.7	57	268	7.7
August	33.2	21.8	71	242	7.6
Sept	35.2	21.7	65	190	9.2
Oct.	37.7	21.5	48	138	9.9
Nov.	36.5	18.0	37	190	10.4
Dec.	33.7	14.5	38	216	10.5

The source of the Meteorological data is the FAO climwat database. Wad Medani station has been chosen for this purpose, since it is located at the centre of the Gezira.

8.5 REFERENCE CROP - EVAPOTRANSPIRATION, ET_0

Using FAO Cropwat Software and data from Table (8.7), ET_0 for Wad Medani is calculated. Results are shown in Table (8.8). Software Cropwat uses Penman - Monteith, described above, to calculate ET_0

Table (8.8) Reference Crop Evapotranspiration, ET_0, Wad Medani in mm/day

Month	jan	feb	mar	apr	may	jun	jul	aug	sep	oct	nov	dec
ET_0	6.2	7.3	7.9	8.0	8.5	8.6	6.9	5.7	6.0	5.9	6.3	6.1

8.6 CROP FILES

For further use of Cropwat to calculate crop evapotranspiration, crop files for main crops have to be prepared. Part of the crop files such as, the length of the growing period (LGP) and root depth, D, have been obtained from local conditions, while other parts like crop factor, K_c, depletion factor, P, and yield response factor, K_y, are estimated from FAO prepared tables, as follows :

8.6.1 Crop Factor, K_c

The crop factor, K_c, which relates crop evapotranspiration to reference crop evapotranspiration, has been found from Table (18), FAO (1986). To find K_c, recommended sowing dates for different crops have been used and for each crop stage, wind speed and minimum humidity have been found. Then by interpolation, Table (18), K_c values are found. The following tables show results obtained for the main crops grown in the Blue Nile System.

Table (8.9) K_c for ELS Cotton

Stage	Initial	Development	Mid season	Late season
Length	25/6-4/8	5/8-29/9	30/9-8/11	9/11-15/1
Wind speed m/s	3.1	2.5	1.6	2.4
Min. Humidity %	39	46.5	27	19
K_c	0.45	0.75	1.22	0.75

Table (8.10) K_c for MS Cotton

Stage	Initial	Development	Mid season	Late season
Length	6/7-5/8	6/8-24/9	25/9-4/11	5/11-6/1
Wind speed m/s	3.1	2.5	1.6	2.35
Min. Humidity %	39	46.5	27	19
K_c	0.45	0.75	1.22	0.75

Table (8.11) K_c for Sorghum

Stage	Initial	Development	Mid season	Late season
Length	15/6-5/7	6/7-30/7	31/7-3/9	4/9-1/10
Wind speed m/s	3.1	3.1	2.8	2.2
Min. Humidity %	23	39	51	42
K_c	0.35	0.75	1.08	0.65

Table (8.12) K_c for Ground Nut

Stage	Initial	Development	Mid season	Late season
Length	1/6-20/6	21/6-20/7	21/7-4/9	5/9-15/10
Wind speed m/s	3.1	3.1	3.0	1.9
Min. Humidity %	23	31	45	35
K_c	0.45	0.75	1.03	0.69

Table (8.13) K_c for Wheat

Stage	Initial	Development	Mid season	Late season
Length	20/11-10/12	11/12-5/1	6/1-3/2	4/2-5/3
Wind speed m/s	2.35	2.5	2.5	2.8
Min. Humidity %	19	19	18	13
K_c	0.35	0.75	1.12	0.45

late season K_c values are the average of late season and harvest stage of Table (18), FAO Paper 33.

8.6.2 Depletion Factors

The allowable depletion factor, P, is the soil moisture level at which first drought stress occurs affecting evapotranspiration and production. The value of P depends on the crop, the magnitude of the maximum crop evapotranspiration and the soil. The values of P for different crops have been found using Tables (19) and (20), FAO (1986). To use these tables, ET_m, is found first by multiplying K_c for each crop, from tables above, by ET_0 calculated for each stage from Table (8.8). Table (8.14) shows ET_m values while the depletion factor, P, calculated are shown in crop file tables.

Table (8.14) Maximum crop evapotranspiration, ET_m in mm/day

Stage	Initial	Development	Mid season	Late season
ELS Cotton	3.15	4.4	7.3	4.64
MS Cotton	3.02	4.4	7.83	4.64
Ground Nut	3.87	5.59	6.45	4.11
Sorghum	2.86	5.18	6.22	3.9
Wheat	2.17	4.59	7.07	3.31

8.6.3 Yield Response Factor

The yield response factor relates the relative decrease in yield to relative evapotranspiration deficit, such that (FAO, 1986):

1-actual yield/maximum yield = Ky (1 - actual evapotranspiration / maximum evapotranspiration)

K_y values have been found from Table (24), FAO (1986), and shown in the crop files below.

Table (8.15) ELS Cotton crop file

Stage	Initial	Development	Mid season	Late season	Total
LGP (day)	40	55	40	65	200
Kc	0.45	0.75	1.22	0.75	
D (m)	0.3	-	0.7	0.7	
P	0.8	0.65	0.49	0.64	
Ky	0.2	0.2	0.5	0.25	0.85

Table (8.16) MS Cotton crop file

Stage	Initial	Development	Mid season	Late season	Total
LGP (day)	30	50	40	60	180
Kc	0.45	0.75	1.22	0.75	
D (m)	0.3	-	0.65	0.65	
P	0.8	0.65	0.46	0.64	
Ky	0.2	0.2	0.5	0.25	0.85

Table (8.17) Ground Nut crop file

Stage	Initial	Development	Mid season	Late season	Total
LGP (day)	20	30	45	40	135
Kc	0.45	0.75	1.03	0.7	
D (m)	0.2	-	0.4	0.4	
P	0.61	0.47	0.445	0.58	
Ky	0.2	0.2	0.8	0.6	0.7

Table (8.18) Sorghum crop file

Stage	Initial	Development	Mid season	Late season	Total
LGP (day)	20	25	35	25	105
Kc	0.35	0.75	1.08	0.65	
D (m)	0.3	-	0.6	0.6	
P	0.808	0.585	0.54	0.71	
Ky	0.2	0.2	0.65	0.55	1.15

Table (8.19) Wheat crop file

Stage	Initial	Development	Mid season	Late season	Total
LGP (day)	20	25	30	30	105
Kc	0.35	0.75	1.12	0.45	
D (m)	0.2	-	0.5	0.5	
P	0.78	0.54	0.42	0.67	
Ky	0.2	0.2	0.65	0.55	1.15

8.7 IRRIGATION EFFICIENCIES

As was described earlier, the water is conveyed in the Gezira Scheme to field blocks through a canal system that comprises main, major and minor canals. Then the water is passed through the field blocks by a field canal called Abu XX and made available to plants through smaller canals, called Abu VI. In other words, it can be said that the conveyance system is formed of the main, major and minor canals while the distribution system is formed of a network of Abu xxs and Abu vis. The irrigation efficiencies for such a system can be defined as (Bos and Nugteren, 1990):

$$\text{Conveyance efficiency } Ec = \frac{Vd}{Vc} \tag{8.7}$$

Where:

Vc = Volume of water diverted or pumped from the river (m^3)

Vd = Volume of water delivered to the distribution system (m^3).

In Gezira, water delivered to the distribution system, Vd, and water diverted to scheme, Vc, can be estimated from measurements made at the head of minor canals and estimated losses in minor, major and main canals as follows:

Water delivered to the distribution system, Vd =

Water measured at minor head - losses in minor (8.8)

Water diverted to scheme, Vc = Water measured at head of minor +

major canal losses + main canal losses (8.9)

$$\text{Distribution efficiency, } Ed = \frac{V_f}{V_d} \tag{8.10}$$

Where:

V_f = Volume of water furnished to the fields (m^3)

The estimate of water furnished to fields, V_f, can be made as follows :

$V_f = V_d$ - estimated losses from field canals (8.11)

$$\text{Application efficiency} = \frac{V_m}{V_f} \quad (8.12)$$

Where :

V_m = Volume of irrigation water needed, and made available for transpiration by crop to avoid undesirable water stress in plants throughout the growing cycle (m^3).

$V_m = ET_{crop} - P_e$

Where:

ET_{crop} is the crop water requirements (m^3).

P_e is the effective precipitation (m^3).

8.7.1 Estimation of Canals' Losses

Canal losses include the following transit and management losses:

1) Seepage through bed and banks of canals.

2) Spillage over banks.

3) Leakage through cracks and holes in the bed and banks.

4) Leakage through structures (gates and escapes).

5) Evaporation from canals.

In the Gezira system the seepage losses are very small due to the presence of the impermeable clayey soil (Plusquellec, 1990). Leakage through the cracks and holes is also very small, since cracks and holes are filled up by the large amount of sediment deposited in the irrigation canals. Yousif and Hussein (1994) estimated that about 4 million tonnes of sediment enters the Gezira canalisation system annually and most of that amount is deposited in canals. The leakage losses through the gates are high. To take account of this the flows were measured by using currentmeters and not gate settings, so more accurate discharges were obtained. The spillage over banks and escape losses are very small. Thanks to the large sizes of minor canals which enable them to store any excess water. On the other hand the enlargement of the minor canals is expected to lead to significant increase in evaporation losses. Therefore, only evaporation losses are going to be considered when estimating canal losses.

To estimate evaporation rate, E_0, Penman original equation is used on data from Wad Medani station. E_0 values are shown in Table (8.20). The details of calculations are shown in Chapter VI.

Table (8.20) Penman, E_0, in mm/day, Wad Medani Station

mnth	jan	feb	mar	apr	may	jun	jul	aug	sep	oct	nov	dec
E_0	7.4	8.7	9.7	10.1	10.8	10.9	8.9	7.7	8.3	8.2	7.9	7.2

The monthly amount of water that evaporates from a canal or a group of canals can be found using the following equation:

Evaporation losses (m^3) = E_0(mm/day)*Canal length(m)*Canal width(m)*Time (days)*number of canals/1000 (8.13)

For the canal widths, the following values are taken in calculations: 40 m for main and branch canals, 20 m for major canals, 8.5 m for minors, 1.4 m for AbuVI and an average of 3.25 m for Abu XX, Table (8.1).

According to the operation policy of the Gezira Scheme, (Plusquellec, 1990), the main, branch and major canals flow continuously, while the minor canals store water during the night to be released during the day. This means that the water level in minors fluctuate and hence the water depth, between top and lower values. It can be assumed that minor canals are operated continuously at the average depth which has a corresponding water width of 8.5 m. Minor canals have 10 m top width, 7 m bed width and a 1:2 side slope (Sir Alexander Gibb and Partners, 1978).

According to the operation policy of the Gezira scheme, the irrigation interval is two weeks. Every Abuxx remains open in one week and closed during the second. Since Abuxx is open for half a day, then the actual application of water in Abuxx is only one week in a month. Abuxx supplies water to ten AbuVI. Five of them remain open in half of the week, while the other five are open in the second half. Therefore it can be assumed that five Abu VI are always open when Abuxx is open. Abuxx supplies water to a standard area of 90 feddan. Therefore the number of Abuxxs is found by dividing the cropped area by 90. For estimating the number of AbuVIs, in operation, the number of Abuxxs is multiplied by 5, as explained above. The lengths of Abuxx and Abu VI are standard. These are 1.5 km and 0.3 km respectively (Figure 8.2).

8.7.1.1 Losses in Selected Minor Canals

To find the losses from the nine selected canals, equation (8.13) is used. E_0 is substituted from Table (8.20). The length of each canal is taken from Table (8.2). The width is 8.5 m as explained earlier. The number of days in which each canal has been in operation, is counted from the day on which first crop is sown, Table (8.4), to the date on which water has been stopped from the last crop grown in the command area of the canal. The results are shown in Table(8.21) below.

Table (8.21) Minor canal losses, m^3

canal	gym.	tuman	numn	hamza	umud	fadlen	tuwer	mardi	bebsh
june	6393	4447	4345	4781	4725	3169	3799	3150	3057
july	10440	14071	15713	19518	19291	12936	9305	7716	7489
august	9333	12174	13594	17449	17246	11565	8319	6898	6696
sept	9736	12699	14181	18202	17990	12064	8678	7196	6984
oct	9939	12964	14477	18582	18366	12315	8859	7346	7130
nov	9267	12087	13497	17325	17123	11483	8259	6849	6648
dec	8727	11383	12711	16316	16126	10814	7779	6450	6261
jan	8970	11699	13064	16769	16574	11114	7995	6630	6435
feb	9525	9761	10900	17807	17600	11802	8489	7040	6833
mar							4395	5607	2721
apr									
may									

8.7.1.2 Losses in Field Canals Belonging to Selected Canals

Equation (8.13), following the same procedure used in Section (8.7.1.1), has been applied to estimate Abuxx and AbuVI losses separately and then the results are summed up to find losses from field canals. The results are shown in Table (8.22).

Table (8.22) Field canal losses, m^3

canal	gym.	tuman	numn	hamza	umud	fadlen	tuwer	mardi	bebsh
june	1426	2889	1064	1254	855	684	-	1292	608
july	2406	4475	5537	5292	3608	2887	1304	1636	1540
august	2746	4546	6368	6525	4290	4250	1966	1415	1598
sept	4190	3364	6643	7019	4602	4559	3173	2957	2197
oct	1995	2541	4265	7092	4654	4654	2848	2970	2171
nov	3100	2994	4835	7315	3375	3568	1681	1873	1185
dec	3503	3051	4982	7551	3114	3308	2257	2063	1323
jan	2961	1470	4555	7760	3200	3400	2320	2120	1360
feb	1024	100	1836	2172	2260	1904	2464	2251	1444
mar						338	924	1285	440
apr									
may									

8.7.2 Water Delivered to the Distribution System, V_d

Water delivered to the distribution system of selected canals, V_d, can be estimated according to equation (8.8). This can be achieved by subtracting Table (8.21) from Table(8.6). Water delivered to the distribution system for the whole scheme is calculated by summing up the values found for the nine selected canals and dividing and multiplying the resultant by the command area of the canals and the whole scheme area respectively. Table (8.23) shows the results.

8.7.3 Water Furnished to Fields, V_f

Water furnished to fields can be estimated according to equation (8.11) i.e. [Table (8.23) - Table(8.22)]. Table (8.24) shows the results.

8.7.4 Volume of Irrigation Water Needed, V_m

The reference crop evapotranspiration found in Table (8.8) was fed to Software Cropwat, together with sowing dates from Table (8.4) and crop files from Tables (8.15) to Table (8.19), to calculate the crop maximum requirements, ET_m, for the four main crops and the nine selected canals. ET_m average over the irrigation interval is found. The irrigation interval in Gezira is 14 days (Plusquellec, 1990).

After heavy rain or irrigation, the actual crop evapotranspiration, ET_a, is equal to ET_m. Then after a certain period ET_a becomes less than ET_m (FAO, 1986). To calculate ET_a the following steps are followed:

1) Calculate ET_m as described above.

2) From Tables (8.15) to (8.19) find the average root depth, D, in metres, over the irrigation intervals.

3) Find the available soil moisture in the root depth, $D.S_a$, by multiplying D from 2 above by S_a. S_a value recommended by the Hydraulic Research Station, Sudan, for the Gezira soils is 120 mm/m.

4) From Tables (19) and (20) of FAO (1986), calculate the depletion factor, P, during each irrigation interval.

5) Calculate t', time during which $ET_m = ET_a$. According to FAO (1986),

$$t' = PS_a D / ET_m \tag{8.14}$$

6) If the length of the irrigation interval, $t < t'$, then $ET_a = ET_m$.

7) Otherwise if $t \geq t'$, then according to Rijtema and Aboukhaled (1975):

$$ET_a = S_a .D \left[1 - (1- P)\, e^{\frac{-ETm \cdot t}{(1-p)SaD} + \frac{P}{1-P}}\right] \tag{8.15}$$

Appendix (B) shows the results of calculating ET_a for different crops and canals. For each canal, ET_a for each crop is multiplied by the corresponding area of that crop to obtain ET_{crop} for that crop. For different crops ET_{crop} values are summed up to obtain ET_{crop} for different canals. Table (8.25) shows the results. The effective rainfall from Table (8.5) is multiplied by the crop areas to obtain the volume of effective rain water in m^3 (Table 8.26). Table (8.26) is subtracted from Table (8.25), to obtain irrigation water needed, V_m, and results are shown in Table (8.27).

Table (8.23) Water delivered to the distribution system, V_d (m^3)

canal	gym.	tuman	numan	hamza	umud	fadlen	tuwer	mardi	bebsh	scheme
june										
july	294360	443829	876688	786082	509209	471164	230981	296784	373211	5.51E8
august	186867	369326	390006	922351	602854	647135	239022	366802	677604	5.59E8
sept	346264	420501	818820	864598	588210	503136	412941	407604	550016	6.32E8
oct	367761	430236	866023	880618	588634	590784	606841	591454	502770	6.98E8
nov	788033	686713	1212003	820275	774477	750917	591222	613251	410652	8.56E8
dec	580073	512217	929489	597084	544874	558886	454505	774250	475939	6.99E8
jan	606030	501701	948036	638931	630025	614486	487611	825870	621365	7.56E8
feb	306075	214439	505500	473293	471700	404598	431905	513460	249367	4.6E8
mar	25600	13300	164000	130000	128100	63400	431600	84793	53079	1.41E8
apr										
may										

Table (8.24) Water furnished to fields, V_f, m^3

canal	gym.	tuman	numan	hamza	umud	fadlen	tuwer	mardi	bebsh	Scheme
june										
july	289549	434879	865613	775498	501993	465391	227388	293512	370132	5.44E+8
august	181375	360233	377269	909300	594274	638636	235450	363971	674409	5.58E+8
sept	337884	413773	805533	850559	579005	494018	406777	401690	545622	6.22E+8
oct	363771	425154	857493	866434	579326	581476	600545	585514	498429	6.9E+8
nov	781834	680725	1202332	805646	767727	743782	587379	609504	408283	8.48E+8
dec	573067	506116	919525	581983	538647	552270	450207	770124	473293	6.91E+8
jan	600108	498761	938926	623410	623626	607685	483465	821630	618645	7.49E+8
feb	304027	214238	501829	468948	467179	400790	422883	508957	246479	4.55E+8
mar							425358	82222	52200	0.72E+8
apr										
may										

Table (8.25) Crop requirements, ET_{crop} - depending on ET_a , of selected canals (m^3)

canal	gym.	tuman	numan	hamza	umud	fadlen	tuwer	mardi	bebsh	total
june	71721	51314	52479	14223	9553	7497	8711	7283	6926	229707
july	243134	263326	402836	313439	211106	165397	85497	71096	66740	1822571
aug.	387084	449550	652186	625854	418494	380920	194711	140176	134939	3383914
sept	433787	500301	744003	689974	460589	433030	278909	259171	201148	4000912
oct	254585	320030	538352	576168	381813	405990	282620	248257	221363	3229178
nov	311861	299427	523075	349407	255016	314586	193054	181510	157899	2585835
dec	413714	365148	604021	384728	334129	381974	266141	258395	169086	3177336
jan	365197	305257	442526	419553	394013	425387	311949	296374	238972	3199228
feb	175127	116614	184238	171321	221686	191827	279117	266438	207622	1813990
mar							105533	149658	62286	317477
apr										
may										

Table (8.26) Effective rain, Pe (m^3)

canal	gym.	tuman	numan	hamza	umud	fadlen	tuwer	mardi	bebsh	total
june										
july	214033	226775	377851	333902	224839	176180	24494	17555	22161	1617790
augt	302107	293818	505243	485898	323704	314826	112444	90908	119749	2548697
sept	162237	273811	217803	299601	206247	200590	56292	47469	35641	1499691
oct	33512	48271	43777	39882	26192	25859				217493
nov										
dec										
jan										
feb										
mar										
apr										
may										

Table (8.27) Irrigation requirements, V_m, depending on ET_a, in (m^3)

canal	gym.	tuman	numan	hamza	umud	fadlen	tuwer	mardi	bebsh	total
june	71721	51314	52479	14223	9553	7497	8711	7283	6926	229707
july	29100	36552	24985	0	0	0	61003	53541	44579	249760
aug.	84977	155732	146943	139956	94790	66094	82266	49268	15189	835215
sept	271550	226490	526200	390372	254342	232439	222617	211703	165507	2501220
oct	221073	271759	494574	536285	355622	380131	282620	248257	221363	3011684
nov	311861	299427	523075	349407	255016	314586	193054	181510	157899	2585835
dec	413714	365148	604021	384728	334129	381974	266141	258395	169086	3177336
jan	365197	305257	442526	419553	394013	425387	311949	296374	238972	3199228
feb	175127	116614	184238	171321	221686	191827	279117	266438	207622	1813990
mar							105533	149658	62286	317477
apr										
may										

8.7.5 Application, E_a, and Distribution, E_d, Efficiencies

Application and distribution efficiencies for each minor canal can be estimated using relations (8.12) and (8.10) respectively and the total water delivered to the distribution system, V_d (V_d is obtained by summing up Table 8.23), water furnished to fields, V_f (V_f is obtained by summing up Table 8.24) and irrigation water needed and made available to crops, V_m (V_m is obtained by summing up Table 8.27 starting from July). Table (8.28) shows the results.

Table (8.28) Application and distribution efficiencies

Canal	Canal Posi-tion	water delivered to distribution system, V_d -m^3	water furnished to fields, V_f, m^3	irrigation requirement V_m m^3	application efficiency %	distribution efficiency %
Gymailya	H - H	3501063	3431616	1872600	54.6	98.02
Tuman	H - M	3592261	3533879	1776978	50.3	98.4
W.Numan	H - T	6710565	6468520	2946562	45.6	96.4
Hamza	M - H	6113233	5881778	2391621	40.7	96.2
Um uud	M - M	4838083	4651775	1909597	41.1	96.1
Fadlein	M - T	4604505	4484046	1992439	44.4	97.4
Eltuweir	T - H	3886628	3839452	1804300	47.0	98.8
Elmardi	T - M	4474268	4437125	1715144	38.7	99.2
Beibash	T - T	3914003	3887489	1282503	33	99.3
Average		-	-	-	44	97.76

8.7.6 Conveyance Efficiency, E_c

The conveyance efficiency for the whole scheme can be estimated using equation (8.7). The water delivered to the distribution system is found, as shown in Table (8.23), by dividing the total amount of water supplied to the selected canals by their total command area, 16314 feddans, and multiplying the resultant by the area of the scheme, 2.1 million feddans. The total amount of water delivered to the distribution system, V_d, is $5.352*10^9$ m^3. To estimate the amount of water diverted to the scheme from the reservoir system, V_c, the losses from main, major and minor canals have to be estimated and added to the water delivered to the distribution system. As shown in Table (8.29), the total losses from the conveying system is $3.15*10^8$ m^3. Therefore the total amount of water diverted to the scheme is $5.667*10^9$ m^3 and the conveyance efficiency is 94.4 %. The losses from main, major and minor canals have been found from equation (8.13), by using E_0 from Table (8.20) and widths and total lengths of canals from Table (8.1).

Table (8.29) Losses from main, major and minor canals(m^3)

Canal	main & branch	major	minor	total
june				
july	10064832	9115736	19040273	38220841
august	8707776	7886648	16473045	33067469
sept.	9083520	8226960	17756659	35067139
oct.	9273216	8398768	17542723	35214707
nov.	8645760	7830480	16900916	33377156
dec.	8142336	7374528	15403367	30920231
jan.	8368512	7579376	15831238	31779126
feb.	8886528	8048544	18612402	35547474
march	10969536	9935128	20751758	41656422
total	82142016	74396168	158312381	314850565

8.7.7 Overall efficiency, Ep

The overall efficiency of the scheme is the product of the application efficiency, distribution efficiency and conveyance efficiency. i.e. Ep = 0.44*0.978*0.944 = 0.41.

8.8 COMMENTS ON WATER USE IN GEZIRA

8.8.1 Conveyance and Distribution Efficiencies

The values obtained for the conveyance and distribution efficiencies are 0.944 and 0.978 respectively. These values, compared to other projects, seem to be too high. However a figure of 0.93 for the combined conveyance and distribution efficiency was given by some previous estimations (Plusquellec, 1990). Combining the distribution and conveyance efficiencies obtained here, a figure of 0.924 is obtained. This is very close to the previous observed figure. According to Plusquellec (1990), these high values are attributed to the impermeable clayey soils, the low level of escapage in the system and the important role of the minor canals which act as storage reservoirs.

8.8.2 Water Use and Application Efficiencies Differences from Head to Tail

The cropping intensity, Table (8.30), does not vary much from the head to the tail of the Gezira scheme. This indicates that there is no water shortage at the tail of the system. It is expected that the area and consequently the cropping intensity would be reduced, if water shortage is experienced. The use of water varies drastically from the head to the tail of the scheme. In the head canal, Gymaillya, the unit area, feddan, receives 2801 m^3 of water, while the unit area in the canal located at the tail, Beibash, receives 6501 m^3. This is attributed to the fact, that being at the head of the scheme, the system is more reliable. Therefore, water is applied more efficiently. Going down the scheme, the system becomes less reliable, therefore farmers tend to overirrigate to face any possible water crisis. The application efficiency drops from 54.6 % at the head to only 33 % at the tail, Table (8.30). If the reliability of the system is restored, then the application efficiency could be increased and consequently a large amount of water could be saved. There is a possibility to raise the application efficiency up-to the observed value of 54.6 %. If this is achieved, the overall efficiency will be 51 %.

Table (8.30) water applied to plant in different parts of Gezira scheme

canal	Canal Position	cropping intensity	cropped area feddan	water furnished to fields - m^3	water furnished to fields m^3/ feddan	Application efficiency %
Gymailya	H - H	0.77	1225	3431616	2801.32	54.6
Tuman	H - M	0.67	1232	3533879	2868.41	50.3
WadNuman	H - T	0.68	1860	6468520	3477.7	45.6
Hamza	M - H	0.65	1737	5881778	3386.17	40.7
Um uud	M - M	0.57	1377	4651775	3378.2	41.1
Fadlein	M - T	0.77	1273	4484046	3522.42	44.4
Eltuweir	T - H	0.63	880	3839452	4363.01	47.0
Elmardi	T - M	0.69	783	4437125	5666.83	38.7
Beibash	T -T	0.67	598	3887489	6500.82	33

8.9 REQUIREMENT FOR THE WHOLE BLUE NILE SYSTEM

To find the requirements for the whole Blue Nile System, the Gezira Scheme requirements have to be increased by 29 %. As was discussed earlier the Gezira Scheme represents 77.5 % of the Blue Nile System irrigated area and other schemes in the system follow it in their design and operation. To calculate the requirement first the

Gezira requirements have to be calculated. For this purpose the reference crop evapotranspiration from Table (8.8) is fed to Cropwat, together with sowing dates from Table (8.14), rainfall from Table (8.5) and crop files from Tables (8.15) to (8.19) to calculate ET_m. Results are shown in Appendix (C). ET_m is multiplied by the corresponding area of each crop, to find the irrigation requirements that should be made available to crops, V_m', for each selected canal. The results are shown in Table (8.31). The requirements for the selected canals are summed up, divided by canal areas and multiplied by the area of the scheme to obtain V_m' for the whole scheme (Table 8.31). Irrigation water that need to be made available to crops, V_m', is divided by the project overall efficiency, 0.41, to find the amount of water that need to be diverted from the reservoir system to meet the irrigation requirements, Table (8.32). This amount is multiplied by 0.29 to find the requirement of the Blue Nile remaining irrigation development (Table 8.32). These calculations have been repeated with the possibly achieved overall efficiency of 0.51 and results are shown in Table (8.33). In April and May an average monthly amount of 60 million m^3 is released to satisfy the domestic requirements (Sennar Dam Resident Engineer Operation Book).

It is noticed that the requirements of the Gezira Scheme, during some periods, exceed the carrying capacity of its main canals. The capacities of the two canals are 186 and 168 m^3/ sec i.e. 30.5 Million m^3/day (Plusquellec, 1990). This problem was caused by the implementation, in mid seventies, of the intensification and diversification policies and also observed by Sir Alexander Gibb and Partners (1978) and Hydraulic Research Wallingford (1991). Therefore the requirements in Tables (8.32) and (8.33), that exceed the canals capacities are replaced by the maximum canal capacities to obtain the requirements that can actually be satisfied from the reservoir system, Tables (8.34) and (8.35). The requirements that can be supplied to Gezira have been added to the requirements of the rest of the Blue Nile System to obtain the whole Blue Nile requirements, Tables (8.34) and (8.35). For 96 % of the irrigated area in the Blue Nile system, water is withdrawn from upstream Sennar, while water is withdrawn for only 4 % of the irrigated area from downstream Sennar. Therefore the figures shown in Tables (8.34) and (8.35) have to be divided in these proportions when they are used as inputs to the optimization model.

Table (8.31) Irrigation requirement, V_m' - depending on ET_m - in m^3

canal	gym.	tuman	numan	hamza	umud	fadlen	tuwer	mardi	bebsh	scheme
june	69758	44745	45972	35016	16803	13186	17678	14908	13933	35012743
july	89839	72477	87345	74557	50151	39323	83009	71753	62450	81212357
augt	200228	261339	317653	381093	256525	202662	153336	103264	85871	252552353
sept	363009	381814	753522	691276	462681	411907	332007	297458	245850	507110481
oct	334036	372418	672284	754877	499697	538864	345196	298084	263392	525044795
nov	449366	422654	696744	518963	373187	466198	283050	245674	161653	465656914
dec	584814	508705	827045	552345	488904	551136	421589	368095	213918	581387587
jan	535135	432996	644445	587819	596211	609162	481582	446934	278408	593763221
feb	220933	161119	246151	234888	311757	265333	369351	389974	234710	313341522
mar							111481	173499	32918	40921037
apr										
may										

Table (8.32) Blue Nile actual requirements to be diverted from reservoirs (million m^3)

month	Gezira	Rest of the system	Total
june	85	24.8	109.8
july	198	57.4	255.4
aug.	616	178.6	794.6
sept.	1237	358.7	1595.7
oct	1281	371.4	1652.4
nov.	1136	329.4	1465.4
dec.	1418	411.2	1829.2
jan.	1448	420	1868
feb.	764	221.6	985.6
march	100	28.9	128.9
april	60	17.4	77.4
may	60	17.4	77.4
total	8403	2436.8	10839.8

Table (8.33) Blue Nile requirements to be diverted, with possible improvement (million m^3)

	Gezira	Rest of the System	Total
jun	69	19.9	88.9
jul	159	46.2	205.2
aug	495	143.6	638.6
sept	994	288.4	1282.4
oct	1029	298.6	1327.6
nov	913	264.8	1177.8
dec	1140	330.6	1470.6
jan	1164	337.6	1501.6
feb	614	178.2	792.2
mar	80	23.3	103.3
apr	60	17.4	77.4
may	60	17.4	77.4
total	6777	1966	8743

Table (8.34) Possible delivery of the Blue Nile

actual requirements (million m^3)

	Gezira	Rest of the System	Total
jun	85	24.8	109.8
jul	198	57.4	255.4
aug	616	178.6	794.6
sept	918	358.7	1276.7
oct	949	371.4	1320.4
nov	918	329.4	1247.4
dec	949	411.2	1360.2
jan	949	420	1369
feb	764	221.6	985.6
mar	100	28.9	128.9
apr	60	17.4	77.4
may	60	17.4	77.4
total	6566	2436.8	9002.8

Table (8.35) Possible delivery of Blue Nile requirements,

with possible improvement (million m^3)

	Gezira	Rest of the System	Total
jun	69	19.9	88.9
jul	159	46.2	205.2
aug	495	143.6	638.6
sept	918	288.4	1206.4
oct	949	298.6	1247.6
nov	913	264.8	1177.8
dec	949	330.6	1279.6
jan	949	337.6	1286.6
feb	614	178.2	792.2
mar	80	23.3	103.3
apr	60	17.4	77.4
may	60	17.4	77.4
total	6215	1966	8181

8.10 CONCLUSIONS

For reservoir modelling purposes, irrigation requirements have to be calculated. For the Blue Nile System, these requirements have been estimated by estimating the requirements of the Gezira irrigation scheme and increase that by an amount of 29 %. Gezira Scheme area represents 77.5 % of the area irrigated from the reservoir system and other schemes follow it in their design and operation. The crop requirements for the Gezira have been calculated out of data collected from nine selected canals. The data on these canals, combined with data about scheme canalisation, have been used to estimate application, distribution, conveyance and overall efficiencies. The average values of these efficiencies are 44, 97.8, 94.4 and 41 % respectively.

The analysis reveals that more water is supplied to canals, going from the head to the tail of the scheme, as the system is getting less reliable. Therefore, there is a scope for improvement and saving the over-supplied water if the reliability of the system is restored. This can raise the application efficiency to 54.6 % and the overall efficiency to 51 %. This shows that inappropriate water supply is practised in the Blue Nile System and this justifies the first part of hypothesis 1. However the actual requirements and the requirements resulting from improved application of water will be inputted to the optimization model to investigate the consequent effect on reservoir system operation, Chapter XI, and to justify the second part of hypothesis 1.

Figure (8.1) Gezira irrigation field layout

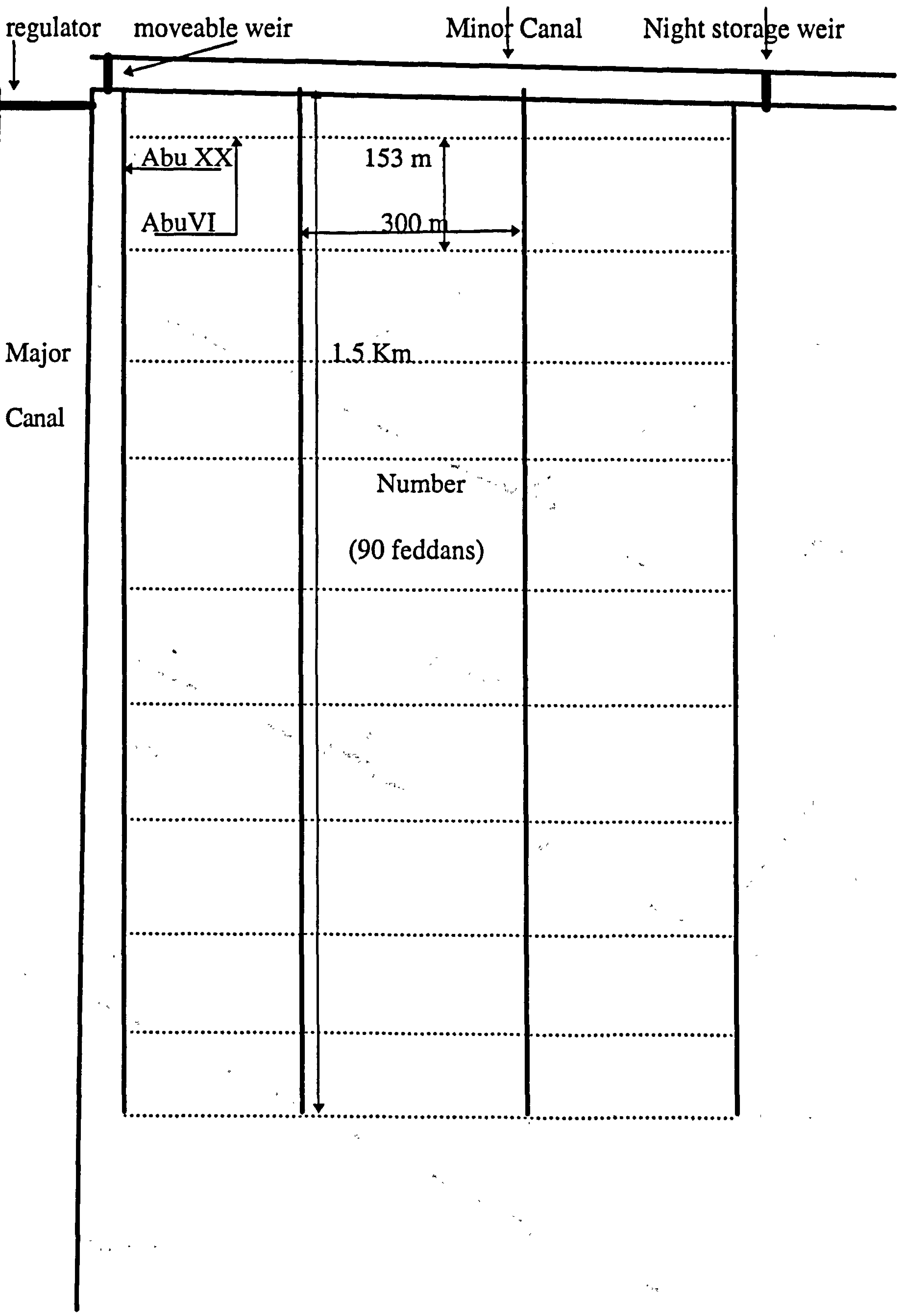
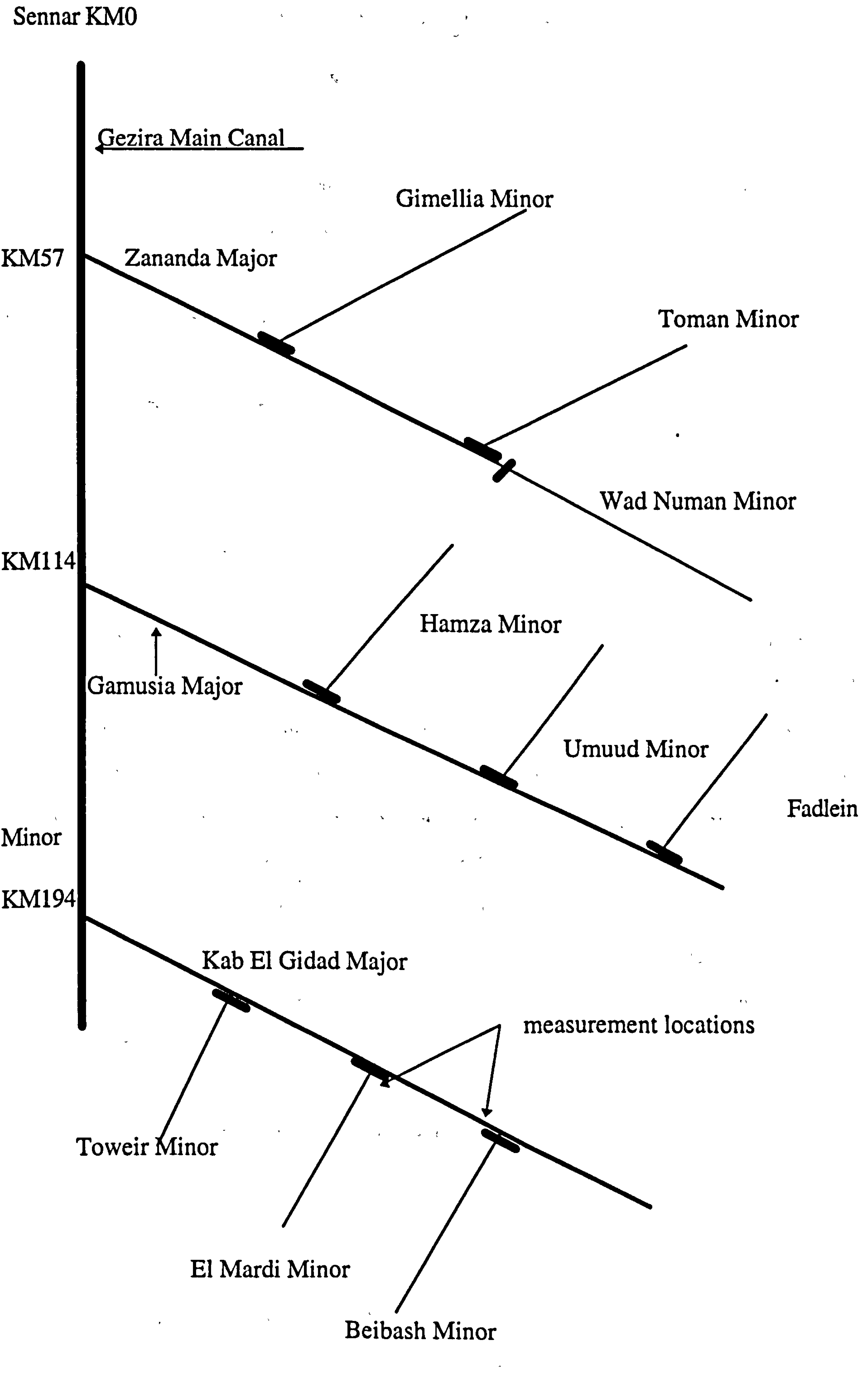


Figure (8.2) Selected canals and measurements locations



CHAPTER IX

OPTIMIZATION PROBLEM FORMULATION AND SOLUTION

Summary ~ In this chapter an optimization problem for the Blue Nile Reservoir System is formulated and solved. In formulating the problem use will be made of the sedimentaion model developed in Chapter V, evaporation models fitted in Chapter VI, flow model fitted in Chapter VII and demands estimated in Chapter VIII.

9.1 INTRODUCTION

The objective of the developed algorithm is to maximise the revenues of the power generated from two reservoirs in series (Figure 9.3), on condition that certain irrigation and downstream requirements be satisfied. The algorithm will represent the system without simplification i.e. no linearisation, decomposition or aggregation techniques, usually used to overcome nonlinearity and dimensionality problems, are to be applied. The algorithm uses synthetically generated inflows and deterministic irrigation requirements as inputs, incorporates non-linear hydropower and evaporation functions and is linked to a sedimentation model that predicts the change in reservoir's storage capacity. To solve the problem, the most suitable non-linear optimization techniques are to be applied. These are, as reached in Chapter III, are the Lagrangian and Conjugate Gradient methods. A general purpose software package, Lancelot, is used. To formulate the algorithm, relations for reservoirs evaporation, head difference across reservoirs, storages and power production have to be built first.

9.2 UPSTREAM WATER LEVELS

9.2.1 Roseries

The storage - upstream water level relationship and its variation with time can be modelled, as found in Chapter V, with the following set of equations:

$$S_{av} = a(H_{us} - 463)^m \quad (9.1)$$

$$a = 395.47\ (t)^{-1.4399} \quad (9.2)$$

$$m = 0.4101\ln(t) + 0.8655 \quad (9.3)$$

Where

H_{us} is the average upstream water level in (m)

S_{av} is the average storage in million m^3

a & m are constants.

t is the number of years in which the reservoir had been in operation,

For 1988, t = 22 , a = 4.61 and m = 2.13, thus:

$$H_{US} = 463 + 0.49\ S_{av}^{0.47} \quad (9.4)$$

9.2.2 Sennar

As was shown in Chapter V, the storage - upstream water level for Sennar is not affected by sedimentation and can be represented, for all years, by:

$$H_{US} = 410 + 1.004\ S_{av}^{0.417} \quad (9.5)$$

9.3 DOWNSTREAM WATER LEVELS

From the readings of both releases and water levels downstream Sennar and Roseries, the following relations for the downstream water levels are obtained. The source of the data are the operation books of residents engineers of Sennar and Roseries Reservoirs.

9.3.1 Roseries

$$H_{ds} = 0.00032X + 0.00032Y + 444.21, \quad R^2 = 0.858 \quad (9.6)$$

9.3.2 Sennar

$$H_{ds} = 0.00032X + 0.00032Y + 404.12, R^2 = 0.849 \quad (9.7)$$

Where

H_{ds} is downstream level in (m)

X is the release through turbines in million m^3/month

Y is the release through other gates in million m^3/month

9.4 HEAD DIFFERENCE (H)

The head difference is simply the difference between upstream and downstream levels.

9.4.1 Roseries (in 1988)

$$H=18.79-0.00032X-0.00032Y+0.49\,S_{av}^{0.47} \quad (9.8)$$

9.4.2 Sennar (all years)

$$H = 5.88 - 0.00032\,X - 0.00032Y + S_{av}^{0.417} \quad (9.9)$$

9.5 EVAPORATION

The reservoir evaporation losses is taken as the product of reservoir area and the resultant of subtraction of rainfall from evaporation rate, E_0.

$$L = 0.03A[E_0 - \text{rainfall}] \quad (9.10)$$

Where

L is the monthly losses in million m^3

A is the area in squared kilometre

E_0 is evaporation rate in mm/day

rainfall in mm/day

From evaporation modelling results, Chapter VI, it has been found that [E_0 - rainfall] for Sennar and Roseries are as shown in Table (9.1).

Table (9.1) [E_0 - rainfall] in mm/day

Month	Roseries	Sennar	Month	Roseries	Sennar
Sept.	2.5	4.81	Mar.	8.9	9.37
Oct.	3.44	6.39	April	8.8	10.42
Nov.	7.1	7.67	May	8.0	9.62
Dec.	7.0	6.87	June	4.6	7.11
Jan.	7.1	7.18	July	2.6	3.99
Feb.	7.9	8.12	Aug.	1.9	2.96

In Chapter VI, the following relations for reservoir's areas have been found.

Roseries:

$$A = 0.4809H_{us}^2 - 441.5H_{us} + 101404 \tag{9.11}$$

Sennar:

$$A = -2.1943H_{us}^2 + 1855.3H_{us} - 391978 \tag{9.12}$$

Where

A is the reservoir's area in squared km.

H_{us} is the upstream water level in (m).

Substituting for H_{us} from equations (9.4) and (9.5), these relations become

Roseries (in 1988):

$$A = 79.55 + 1.869\, S_{av}^{0.47} + 0.1155\, S_{av}^{0.94} \tag{9.13}$$

Sennar:

$$A = 56\, S_{av}^{0.417} - 2.1943\, S_{av}^{0.834} - 167 \tag{9.14}$$

Where

S_{av} is average storage and equal to $0.5[S_{i,j} + S_{i,j+1}]$, and

$S_{i,j}$ is the storage of reservoir i at the beginning of month j .

$S_{i,j+1}$ is the storage of reservoir i at the beginning of month j+1 .

For this problem, i will be taken as 1 for Roseries and 2 for Sennar and j = 1 for September, 2 for October,............., and 12 for June.

Substituting for S_{av}, the above relations become:

Roseries:

$$A = 79.55 + 1.349(S_{1,J} + S_{1,j+1})^{0.47} + 0.06(S_{1,j+1}+S_{1,j+1})^{0.94} \quad (9.15)$$

Sennar:

$$A = 41.94(S_{2,J} + S_{2,j+1})^{0.417} - 1.231(S_{2,j+1}+S_{2,j+1})^{0.834} - 167 \quad (9.16)$$

Substituting for these relations and [E_0 - rainfall] from Table (9.1) in relation (9.10), the following relations for reservoir losses are obtained:

Roseries:

$$L_{1,1} = 5.966 + 0.101(S_{1,1}+S_{1,2})^{0.47} + 0.005(S_{1,1}+S_{1,2})^{0.94} \quad (9.17)$$

$$L_{1,2} = 8.21 + 0.139(S_{1,2}+S_{1,3})^{0.47} + 0.006(S_{1,2}+S_{1,3})^{0.94} \quad (9.18)$$

$$L_{1,3} = 16.948 + 0.287(S_{1,3}+S_{1,4})^{0.47} + 0.013(S_{1,3}+S_{1,4})^{0.94} \quad (9.19)$$

$$L_{1,4} = 16.709 + 0.283(S_{1,4}+S_{1,5})^{0.47} + 0.013(S_{1,4}+S_{1,5})^{0.94} \quad (9.20)$$

$$L_{1,5} = 16.948 + 0.287(S_{1,5}+S_{1,6})^{0.47} + 0.013(S_{1,5}+S_{1,6})^{0.94} \quad (9.21)$$

$$L_{1,6} = 18.857 + 0.319(S_{1,6}+S_{1,7})^{0.47} + 0.014(S_{1,6}+S_{1,7})^{0.94} \quad (9.22)$$

$$L_{1,7} = 21.244 + 0.36(S_{1,7}+S_{1,8})^{0.47} + 0.016(S_{1,7}+S_{1,8})^{0.94} \quad (9.23)$$

$$L_{1,8} = 21.006 + 0.356(S_{1,8}+S_{1,9})^{0.47} + 0.016(S_{1,8}+S_{1,9})^{0.94} \quad (9.24)$$

$$L_{1,9} = 19.096 + 0.324(S_{1,9}+S_{1,10})^{0.47} + 0.015(S_{1,9}+S_{1,10})^{0.94} \quad (9.25)$$

$$L_{1,10} = 10.98 + 0.186(S_{1,10}+S_{1,11})^{0.47} + 0.008(S_{1,10}+S_{1,11})^{0.94} \quad (9.26)$$

$$L_{1,11} = 6.206 + 0.106(S_{1,11}+S_{1,12})^{0.47} + 0.005(S_{1,11}+S_{1,12})^{0.94} \quad (9.27)$$

$$L_{1,12} = 4.535 + 0.077(S_{1,12}+S_{1,13})^{0.47} + 0.003(S_{1,12}+S_{1,13})^{0.94} \quad (9.28)$$

Sennar:

$$L_{2,1} = 6.049(S_{2,1}+S_{2,2})^{0.417} - 0.177(S_{2,1}+S_{2,2})^{0.834} - 24.1 \quad (9.29)$$

$$L_{2,2} = 8.040\ (S_{2,2}+S_{2,3})^{0.417} - 0.236(S_{2,2}+S_{2,3})^{0.834} - 32.02 \quad (9.30)$$

$$L_{2,3} = 9.65(S_{2,3}+S_{2,4})^{0.417} - 0.284(S_{2,3}+S_{2,4})^{0.834} - 38.43 \quad (9.31)$$

$$L_{2,4} = 8.644(S_{2,4}+S_{2,5})^{0.417} - 0.254(S_{2,4}+S_{2,5})^{0.834} - 34.42 \quad (9.32)$$

$$L_{2,5} = 9.03(S_{2,5}+S_{2,6})^{0.417} - 0.265(S_{2,5}+S_{2,6})^{0.834} - 35.97 \quad (9.33)$$

$$L_{2,6} = 10.217(S_{2,6}+S_{2,7})^{0.417} - 0.30(S_{2,6}+S_{2,7})^{0.834} - 40.68 \quad (9.34)$$

$$L_{2,7} = 11.789(S_{2,7}+S_{2,8})^{0.417} - 0.346(S_{2,7}+S_{2,8})^{0.834} - 46.94 \quad (9.35)$$

$$L_{2,8} = 13.11(S_{2,8}+S_{2,9})^{0.417} - 0.385(S_{2,8}+S_{2,9})^{0.834} - 52.2 \quad (9.36)$$

$$L_{2,9} = 12.10(S_{2,9}+S_{2,10})^{0.417} - 0.355(S_{2,9}+S_{2,10})^{0.834} - 48.2 \quad (9.37)$$

$$L_{2,10} = 8.945(S_{2,10}+S_{2,11})^{0.417} - 0.263(S_{2,10}+S_{2,11})^{0.834} - 35.62 \quad (9.38)$$

$$L_{2,11} = 5.02(S_{2,11}+S_{2,12})^{0.417} - 0.147(S_{2,11}+S_{2,12})^{0.834} - 19.99 \quad (9.39)$$

$$L_{2,12} = 3.724(S_{2,12}+S_{2,13})^{0.417} - 0.110(S_{2,12}+S_{2,13})^{0.834} - 14.83 \quad (9.40)$$

9.6 POWER PRICES

Power prices vary with time. There are different prices for the period September to February and the period March to August. Higher prices are charged during the period March to August. During this dry period shortage in power is expected. Therefore, the price is increased, with the intention to decrease demand. Also the prices vary from sector to sector. There are five sectors, namely, domestic, commercial, public, industrial and agricultural. Table (9.2) shows the consumption of power and the prices for each sector. The source is the Sudanese National Electric Corporation (NEC).

Table (9.2) Power prices in Sudanese dinnar / KWh

Sector	Sale-Gwh	%	Price Sept-Feb	Price Mar -Aug
domestic	825	57.9	14	14
commercial	75	5.3	7	12
public	87.5	6.1	15	15
industry	275	19.3	7	12
agriculture	162.5	11.4	7	12

However for the purpose of this model average prices for the sectors are to be used.

Average price for Sept to Feb. = 0.579*14+0.053*7+0.061*15+0.193*7+0.114*7

= 11.54 dinnars / kwh

Average price for Mar. to Aug.= 0.579*14+0.053*12+0.061*15+0.193*12+0.114*12

= 13.34 dinnars / kwh

9.7 HYDROELECTRIC POWER PRODUCTION FUNCTION

The power production during any period at any site is dependent on the installed capacity, the flow through turbines, the average productive storage head, the number of hours in the period, the plant factor and a constant to convert the product of flow, head and plant efficiency into watt-hour (Loucks et al., 1982). Therefore, the total kilowatt-hours of energy produced in period t, with C_p the efficiency of conversion of potential energy to electrical energy, is:

$$KWH_t = \frac{C_p\, 9.81\, q_t\, H_t\, (\text{seconds in period } t)}{3.6 * 10^3} \quad (9.41)$$

Where :

KWH_t is the hydropower in kwh

q_t is the average flow rate in m^3/sec

H_t is the average productive head in (m)

C_p is the overall efficiency Coefficient

For both reservoirs, average C_p is 0.88 (Chapter IV - Table 4.1).

To get the monthly power production, HP, $C_p = 0.88$ and the number of seconds in a month are substituted in equation (9.41), to obtain:

$$HP = 6215.616\, Hq_t \quad (9.42)$$

Taking, X as the discharge in million m^3/month, the relation becomes (1 million m^3 / month = 0.3858 m^3 / sec);

$$HP = 2398HX \quad (9.43)$$

Where

HP is the monthly power generated in KWh

X is the release for power generation in million m^3 /month

H is the average head difference in (m).

When substituting for H from equations (9.8) and (9.9) for Roseries and Sennar respectively, the power functions become:

Roseries:

$$HP = 45058.42X + 1175X\, S_{av}^{0.47} - 0.767X^2 - 0.767XY \qquad (9.44)$$

Sennar:

$$HP = 14100X + 2407.6\, X\, S_{av}^{0.417} - 0.767\, X^2 - 0.767\, XY \qquad (9.45)$$

Take HP(1,i), X(1,i) and Y(1,i) respectively as the power generated, releases through turbines, releases through other gates for reservoir 1, Roseries, in month i and similarly HP(2,i), X(2,i) and Y(2,i) for Sennar. Substitute these symbols for HP, X and Y in the above relations. S(1,i), S(1,i+1), S(2,i) and S(2,i+1) are taken as the storages in Roseries and Sennar at the beginning and at the end of month i respectively. Then the average storage, S_{av}, which is equal to 0.5[S(1,i)+S(1,i+1)] for Roseries and 0.5[S(2,i)+S(2,i+1)] for Sennar, is also substituted for in equations (9.44) & (9.45) respectively. As a result, the relations above become:

Roseries:

$$HP(1,i) = 45058.42X_{(1,i)} + 848.31X_{(1,i)}[S_{(1,i)} + S_{(1,i+1)}]^{0.47} - 0.767X_{(1,i)}^2 - 0.767X_{(1,i)}\, Y_{(1,i)} \qquad (9.46)$$

Sennar:

$$HP(2,i) = 14100X_{(2,i)} + 1803.2X_{(2,i)}[S_{(2,i)} + S_{(2,i+1)}]^{0.417} - 0.767X_{(2,i)}^2 - 0.767X_{(2,i)}\, Y_{(2,i)} \qquad (9.47)$$

The total power produced by the two plants in month i is the summation of equations (9.46) and (9.47):

$$\begin{aligned} HP(i) = {} & 45058.42X_{(1,i)} + 848.31X_{(1,i)}[S_{(1,i)} + S_{(1,i+1)}]^{0.47} - 0.767X_{(1,i)}^2 - 0.767X_{(1,i)}\, Y_{(1,i)} \\ & + 14100X_{(2,i)} + 1803.2X_{(2,i)}[S_{(2,i)} + S_{(2,i+1)}]^{0.417} - 0.767X_{(2,i)}^2 - 0.767X_{(2,i)}\, Y_{(2,i)} \end{aligned} \qquad (9.48)$$

Where

HP(i) is power generated in month i in KWh.

$S_{(1,i)}$ and $S_{(2,i)}$ are storages in Roseries and Sennar, respectively, at the beginning of month i, in million m^3.

$S_{(1,i+1)}$ and $S_{(2,i+1)}$ are storages in Roseries and Sennar, respectively, at the end of month i, in million m^3.

$X_{(1,i)}$ and $X_{(2,,i)}$ are hydropower releases in Roseries and Sennar respectively, in million m^3/month.

$Y_{(1,i)}$ and $Y_{(2,,i)}$ are releases through other gates in Roseries and Sennar respectively, in million m^3/month.

9.8 OBJECTIVE FUNCTION

The objective of this model is to maximise power revenues on conditions that certain irrigation and downstream requirements be satisfied. The maximisation of revenues will be reflected in the objective function while other conditions will be dealt with as constraints. For a reservoir operation problem, decision variables included in the objective function are typically release rates and end of period storage (Yeh, 1985). Therefore the objective function would be a function of releases through turbines, releases through other gates and end of period storages for both reservoirs.

The monthly revenues is the product of power generated in that month, HP(i), and the price, C(i), charged in that month. The objective function to be maximised, F, is the sum of the monthly revenues, i.e.

$$F = 10^{-6}\sum_{i=1}^{12} C(i)HP(i) \qquad (9.49)$$

Where

C(i) is the power price in month i in Sudanese dinnars / kwh

HP(i) is the power generated in kwh

F is the revenues from power generated in million Sudanese dinnars. In practice there will be distribution losses and not all the power generated will reach the consumer.

Similarly there will be distribution costs. These however do not affect the optimization problem, which can be expressed as maximising F in equation (9.49).

Substituting for C(i), from Section (9.6), and HP(i), from equation (9.48), the objective function would be:

$$F = \sum_{i=1}^{12} a(1,i)X_{(1,i)}+b(1,i)X_{(1,i)}[S_{(1,i)}+S_{(1,i+1)}]^{0.47}+ c(1,i)X_{(1,i)}^{2} + d(1,i)X_{(1,i)}\, Y_{(1,i)}$$

$$+a(2,i)X_{(2,i)}+b(2,i)X_{(2,i)}[S_{(2,i)}+S_{(2,i+1)}]^{0.417}+ c(2,i)X_{(2,i)}^{2} + d(2,i)X_{(2,i)}\, Y_{(2,i)} \qquad (9.50)$$

Where :

a(1,1) ,..........................., a(1,6) = 0.52

a(1,7) ,..........................., a(1,12) = 0.601

a(2,1) ,..........................., a(2,6) = 0.163

a(2,7) ,..........................., a(2,12) = 0.188

b(1,1) ,..........................., b(1,6) = 0.0098

b(1,7) ,..........................., b(1,12) = 0.0113

b(2,1) ,..........................., b(2,6) = 0.021

b(2,7) ,..........................., b(2,12) = 0.024

c(1,1) ,..........................., c(1,6) = $-8.85*10^{-6}$

c(1,7) ,..........................., c(1,12) = $-1.02*10^{-5}$

c(2,1) ,..........................., c(2,6) = $-8.85*10^{-6}$

c(2,7) ,..........................., c(2,12) = $-1.02*10^{-5}$

d(1,1) ,..........................., d(1,6) = $-8.85*10^{-6}$

d(1,7) ,..........................., d(1,12) = $-1.02*10^{-5}$

d(2,1) ,..........................., d(2,6) = $-8.85*10^{-6}$

d(2,7) ,..........................., d(2,12) = $-1.02*10^{-5}$

9.9 CONSTRAINTS

In a reservoir operation problem, constraints typically include storage capacities and other physical characteristics of the reservoir/stream system, diversion or stream flow requirements for various purposes and mass balance (Yeh, 1985). Therefore, the optimization problem has to satisfy the following constraints:

1) Continuity equations, mass balance, for Roseries.

2) Continuity equations, mass balance, for Sennar.

3) Irrigation requirements and minimum downstream flow.

4) Bounds on releases and storages imposed by maximum gate capacities and maximum reservoirs storage capacities.

9.9.1 Continuity Equations for Roseries

The continuity equation for Roseries is

$$S_{(1,i+1)} = S_{(1,i)} - X_{(1,i)} - Y_{(1,i)} - L_{1,i} + q_i$$

Where

$S_{(1,i+1)}$, $S_{(1,i)}$, $X_{(1,i)}$, $Y_{(1,i)}$, $L_{1,i}$ are as defined before

q_i is the river flow in month i, in million m^3. Flows generated in Chapter VII would be used as input.

Substituting for $L_{1,i}$, from equations (9.17) to (9.28), and rearranging the equations by putting the constants on the right hand side, the following results are obtained:

$S_{(1,2)} - S_{(1,1)} + X_{(1,1)} + Y_{(1,1)} + 0.101(S_{1,1}+S_{1,2})^{0.47} + 0.005(S_{1,1}+S_{1,2})^{0.94} = q_1 - 5.966 = e1$ (cons1)

$S_{(1,3)} - S_{(1,2)} + X_{(1,2)} + Y_{(1,2)} + 0.139(S_{1,2}+S_{1,3})^{0.47} + 0.006(S_{1,2}+S_{1,3})^{0.94} = q_2 - 8.21 = e2$ (cons2)

$S_{(1,4)} - S_{(1,3)} + X_{(1,3)} + Y_{(1,3)} + 0.287(S_{1,3}+S_{1,4})^{0.47} + 0.013(S_{1,3}+S_{1,4})^{0.94} = q_3 - 16.948 = e3$ (cons3)

$S_{(1,5)} - S_{(1,4)} + X_{(1,4)} + Y_{(1,4)} + 0.283(S_{1,4}+S_{1,5})^{0.47} + 0.013(S_{1,4}+S_{1,5})^{0.94} = q_4 - 16.709 = e4$ (cons4)

$S_{(1,6)} - S_{(1,5)} + X_{(1,5)} + Y_{(1,5)} + 0.287(S_{1,5}+S_{1,6})^{0.47} + 0.013(S_{1,5}+S_{1,6})^{0.94} = q_5 - 16.948 = e5$ (cons5)

$S_{(1,7)} - S_{(1,6)} + X_{(1,6)} + Y_{(1,6)} + 0.319(S_{1,6}+S_{1,7})^{0.47} + 0.014(S_{1,6}+S_{1,7})^{0.94} = q_6 - 18.857 = e6$ (cons6)

$S_{(1,8)} - S_{(1,7)} + X_{(1,7)} + Y_{(1,7)} + 0.36(S_{1,7}+S_{1,8})^{0.47} + 0.016(S_{1,7}+S_{1,8})^{0.94} = q_7 - 21.244 = e7$ (cons7)

$S_{(1,9)} - S_{(1,8)} + X_{(1,8)} + Y_{(1,8)} + 0.356(S_{1,8}+S_{1,9})^{0.47} + 0.016(S_{1,8}+S_{1,9})^{0.94} = q_8 - 21.006 = e8$ (cons8)

$S_{(1,10)} - S_{(1,9)} + X_{(1,9)} + Y_{(1,9)} + 0.324(S_{1,9}+S_{1,10})^{0.47} + 0.015(S_{1,9}+S_{1,10})^{0.94} = q9 - 19.096 = e9$ (cons9)

$S_{(1,11)} - S_{(1,10)} + X_{(1,10)} + Y_{(1,10)} + 0.186(S_{1,10}+S_{1,11})^{0.47} + 0.008(S_{1,10}+S_{1,11})^{0.94} = q_{10} - 10.98 = e10$ (cons10)

$S_{(1,12)} - S_{(1,11)} + X_{(1,11)} + Y_{(1,11)} + 0.106(S_{1,11}+S_{1,12})^{0.47} + 0.005(S_{1,11}+S_{1,12})^{0.94} = q_{11} - 6.206 = e11$ (cons11)

$S_{(1,13)} - S_{(1,12)} + X_{(1,12)} + Y_{(1,12)} + 0.077(S_{1,12}+S_{1,13})^{0.47} + 0.003(S_{1,12}+S_{1,13})^{0.94} = q_{12} - 4.535 = e12$ (cons12)

e1 to e12 are constants introduced for use in writing the SIF, standard input file, later.

9.9.2 Continuity Equations for Sennar

The continuity equation for Sennar is

$S_{(2,i+1)} = S_{(2,i)} - X_{(2,i)} - Y_{(2,i)} + X_{(1,i)} + Y_{(1,i)} - L_{2,i} - rui$

Where

$S_{(2,i+1)}$, $S_{(2,i)}$, $X_{(2,i)}$, $Y_{(2,i)}$, $L_{2,i}$ are as defined before

Neglecting the transmission losses, the outflow from Roseries, $(X_{(1,i)} + Y_{(1,i)})$, is taken as the inflow to Sennar.

rui is the irrigation requirement upstream Sennar in month i. Their estimations are given in Chapter VIII.

Substituting for $L_{2,i}$, from equations (9.29) to (9.40) and rearranging and introducing constants h1 to h12, the following continuity equations for Sennar are obtained;

$S_{(2,2)} - S_{(2,1)} + X_{(2,1)} + Y_{(2,1)} - X_{(1,1)} - Y_{(1,1)} + 6.049(S_{2,1}+S_{2,2})^{0.417} - 0.177(S_{2,1}+S_{2,2})^{0.834}$
$= 24.1 - ru1 = h1$ (cons13)

$S_{(2,3)} - S_{(2,2)} + X_{(2,2)} + Y_{(2,2)} - X_{(1,2)} - Y_{(1,2)} + 8.04 (S_{2,2}+S_{2,3})^{0.417} - 0.236(S_{2,2}+S_{2,3})^{0.834}$
$= 32.02 - ru2 = h2$ (cons14)

$S_{(2,4)} - S_{(2,3)} + X_{(2,3)} + Y_{(2,3)} - X_{(1,3)} - Y_{(1,3)} + 9.65(S_{2,3}+S_{2,4})^{0.417} - 0.284(S_{2,3}+S_{2,4})^{0.834}$
$= 38.43 - ru3 = h3$ (cons15)

$S_{(2,5)} - S_{(2,4)} + X_{(2,4)} + Y_{(2,4)} - X_{(1,4)} - Y_{(1,4)} + 8.644(S_{2,4}+S_{2,5})^{0.417} - 0.254(S_{2,4}+S_{2,5})^{0.834}$
$= 34.42 - ru4 = h4$ (cons16)

$S_{(2,6)} - S_{(2,5)} + X_{(2,5)} + Y_{(2,5)} - X_{(1,5)} - Y_{(1,5)} + 9.03(S_{2,5}+S_{2,6})^{0.417} - 0.265(S_{2,5}+S_{2,6})^{0.834}$
$= 35.97 - ru5 = h5$ (cons17)

$S_{(2,7)} - S_{(2,6)} + X_{(2,6)} + Y_{(2,6)} - X_{(1,6)} - Y_{(1,6)} + 10.217(S_{2,6}+S_{2,7})^{0.417} - 0.30(S_{2,6}+S_{2,7})^{0.834}$
$= 40.68 - ru6 = h6$ (cons18)

$S_{(2,8)} - S_{(2,7)} + X_{(2,7)} + Y_{(2,7)} - X_{(1,7)} - Y_{(1,7)} + 11.789(S_{2,7}+S_{2,8})^{0.417} - 0.346(S_{2,7}+S_{2,8})^{0.834}$
$= 46.94 - ru7 = h7$ (cons19)

$S_{(2,9)} - S_{(2,8)} + X_{(2,8)} + Y_{(2,8)} - X_{(1,8)} - Y_{(1,8)} + 13.11(S_{2,8}+S_{2,9})^{0.417} - 0.385(S_{2,8}+S_{2,9})^{0.834}$
$= 52.2 - ru8 = h8$ (cons20)

$S_{(2,10)} - S_{(2,9)} + X_{(2,9)} + Y_{(2,9)} - X_{(1,9)} - Y_{(1,9)} + 12.10(S_{2,9}+S_{2,10})^{0.417} - 0.355(S_{2,9}+S_{2,10})^{0.834}$
$= 48.2 - ru9 = h9$ (cons21)

$S_{(2,11)} - S_{(2,10)} + X_{(2,10)} + Y_{(2,10)} - X_{(1,10)} - Y_{(1,10)} + 8.945(S_{2,10}+S_{2,11})^{0.417} - 0.263(S_{2,10}+S_{2,11})^{0.834}$
$= 35.62 - ru10 = h10$ (cons22)

$S_{(2,12)} - S_{(2,11)} + X_{(2,11)} + Y_{(2,11)} - X_{(1,11)} - Y_{(1,11)} + 5.02 (S_{2,11}+S_{2,12})^{0.417} - 0.147(S_{2,11}+S_{2,12})^{0.834}$
$= 19.99 - ru11 = h11$ (cons23)

$S_{(2,13)} - S_{(2,12)} + X_{(2,12)} + Y_{(2,12)} - X_{(1,12)} - Y_{(1,12)} + 3.724(S_{2,12}+S_{2,13})^{0.417} - 0.110(S_{2,12}+S_{2,13})^{0.834}$
$= 14.83 - ru12 = h12$ (cons24)

9.9.3 Requirements Downstream Sennar

Part of the irrigation requirements, rd1 to rd12, to be satisfied are withdrawn from locations lying downstream Sennar. According to the regulations for the operation of the reservoirs (MOI, 1968), the releases from Sennar should satisfy these requirements as well as a further downstream requirements, ds1,........................,ds12. Values for rd1 to rd12 were derived in Chapter VIII, irrigation requirements estimation, while the further downstream requirements should not be lower than 105 million m^3/month (MOI 1968). These conditions can be transferred in the following constraints.

$X_{2,1} + Y_{2,1} \geq f1(= rd1+ ds1)$ (cons25)

$X_{2,2} + Y_{2,2} \geq f2(= rd2+ ds2)$ (cons26)

$X_{2,3} + Y_{2,3} \geq f3(= rd3+ ds3)$ (cons27)

$X_{2,4} + Y_{2,4} \geq f4(= rd4+ ds4)$ (cons28)

$X_{2,5} + Y_{2,5} \geq f5(= rd5+ ds5)$ (cons29)

$X_{2,6} + Y_{2,6} \geq f6(= rd6+ ds6)$ (cons30)

$X_{2,7} + Y_{2,7} \geq f7(= rd7+ ds7)$ (cons31)

$X_{2,8} + Y_{2,8} \geq f8(= rd8+ ds8)$ (cons32)

$X_{2,9} + Y_{2,9} \geq f9(= rd9+ ds9)$ (cons33)

$X_{2,10} + Y_{2,10} \geq f10(= rd10+ ds10)$ (cons34)

$X_{2,11} + Y_{2,11} \geq f11(= rd11+ ds11)$ (cons35)

$X_{2,12} + Y_{2,12} \geq f12(= rd12+ ds12)$ (cons36)

9.9.4 Requirements between the Reservoirs

The constraints in Section (9.9.3) guarantee that the requirements downstream Sennar are met. According to the regulations of reservoirs operations (MOI, 1968), the irrigation requirements upstream Sennar should also be satisfied. That is to say, releases from Roseries minus the change in storage in Sennar should at least cover the upstream requirements, ru1 to ru12, as well as the releases from Sennar required to satisfy the downstream requirements. These conditions can be expressed in the following set of constraints.

$X_{1,1} + Y_{1,1} - X_{2,1} - Y_{2,1} + S_{2,1} - S_{2,2} \geq ru1$ (cons37)

$X_{1,2} + Y_{1,2} - X_{2,2} - Y_{2,2} + S_{2,2} - S_{2,3} \geq ru2$ (cons38)

$X_{1,3} + Y_{1,3} - X_{2,3} - Y_{2,3} + S_{2,3} - S_{2,4} \geq ru3$ (cons39)

$X_{1,4} + Y_{1,4} - X_{2,4} - Y_{2,4} + S_{2,4} - S_{2,5} \geq ru4$ (cons40)

$X_{1,5} + Y_{1,5} - X_{2,5} - Y_{2,5} + S_{2,5} - S_{2,6} \geq ru5$ (cons41)

$X_{1,6} + Y_{1,6} - X_{2,6} - Y_{2,6} + S_{2,6} - S_{2,7} \geq ru6$ (cons42)

$X_{1,7} + Y_{1,7} - X_{2,7} - Y_{2,7} + S_{2,7} - S_{2,8} \geq ru7$ (cons43)

$X_{1,8} + Y_{1,8} - X_{2,8} - Y_{2,8} + S_{2,8} - S_{2,9} \geq ru8$ (cons44)

$X_{1,9} + Y_{1,9} - X_{2,9} - Y_{2,9} + S_{2,9} - S_{2,10} \geq ru9$ (cons45)

$X_{1,10} + Y_{1,10} - X_{2,10} - Y_{2,10} + S_{2,10} - S_{2,11} \geq ru10$ (cons46)

$X_{1,11} + Y_{1,11} - X_{2,11} - Y_{2,11} + S_{2,11} - S_{2,12} \geq ru11$ (cons47)

$X_{1,12} + Y_{1,12} - X_{2,12} - Y_{2,12} + S_{2,12} - S_{2,13} \geq ru12$ (cons48)

9.9.5 Bounds on Releases

The releases from the reservoirs should not exceed the maximum capacity of the gates. The maximum discharges that can be passed through the gates of the reservoirs are as follows (MOI, 1968):

Maximum capacity of Roseries power house is 2014 million m^3 / month.

Maximum capacity of Roseries other gates is 17250 million m^3 / month.

Maximum capacity of Sennar power house is 330 million m^3 / month.

Maximum capacity of Sennar other gates is 28500 million m^3 / month.

These conditions can be expressed in the following simple bounds

$0 \leq X_{1,i} \leq 2014$ for i = 1,................,12 (cons 49)

$0 \leq Y_{1,i} \leq 17250$ for i = 1,................,12 (cons 50)

$0 \leq X_{2,i} \leq 330$ for i = 1,................,12 (cons 51)

$0 \leq Y_{2,i} \leq 28500$ for i = 1,................,12 (cons 52)

9.9.6 Bounds on Storages

The water levels in reservoirs should not exceed the maximum storage levels and should not go below minimum levels that guarantee the diversion of water in irrigation canals (MOI, 1968). The minimum operation level for Sennar is 417.2 m and the maximum is 421.7 m. For Roseries, these levels are 467.0 m and 481.0 m respectively. Using storage-upstream levels relationships, derived in Section (9.2), the equivalent maximum and minimum storages can be obtained. These are 2175 million m^3, 88.3 million m^3 for Roseries and 362.5 million m^3 and 113 million m^3 for Sennar. Due to the high sediment load in July and August, the reservoirs are operated at the minimum levels. These conditions can be expressed in the following bounds:

$88.3 \leq S_{1,i} \leq 2175$ for i = 2,................,10 (cons 53)

$S_{1,i} = 88.3$ for i = 1, 11, 12, 13 (cons 54)

$113 \leq S_{2,i} \leq 362.5$ for i = 2,................,10 (cons 55)

$S_{2,i} = 113$ for i = 1, 11, 12, 13 (cons 56)

Note that $S_{1,13}$ and $S_{2,13}$ are equal to $S_{1,1}$ and $S_{2,1}$ respectively.

9.10 PROBLEM SOLUTION

A non-linear optimization problem is formulated. The objective is to maximise the power revenues, i.e. function F defined by equation (9.50), subject to constraints 1 to 56. The aim is to find the releases and storages that maximise the benefits and satisfies the constraints. To solve the problem the most efficient non-linear optimization techniques discussed in Chapter III will be used. These are Augmented Lagrangian and Conjugate Gradient methods. A general purpose software package, named Lancelot will be used. This package is designed for solving large scale non-linear problems (Conn et al., 1996). The features of the package, how it works, how it would be used to solve the problem and the problem solution will be discussed in detail hereafter.

9.10.1 General Features and Structure of Lancelot

Lancelot package solves the general non-linear programming problem of the form:

$$\min_{x \in R^n} \quad f(x) \tag{9.51}$$

Subject to the constraints

$$C(x_i) = 0 \quad i = 1, \ldots, m \tag{9.52}$$

and to the simple bounds

$l_i \leq x_i \leq u_i \quad i = 1, \ldots, n$ (9.53)

The functions f and C are assumed to be smooth. The package is designed to solve problems with large n and/or m values. The algorithms are designed to achieve convergence to minimisers from all starting points. The inequality constraints are transformed automatically by Lancelot into equality constraints by adding slack or surplus variables. Any maximisation problem can be transferred into a minimisation one, as shown in Chapter III - Section (3.1.1), and solved by Lancelot.

9.10.2 Algorithmic Structure of the Package

Conn et al., (1996) summarised the structure of Lancelot algorithms in Figure (9.1) below.

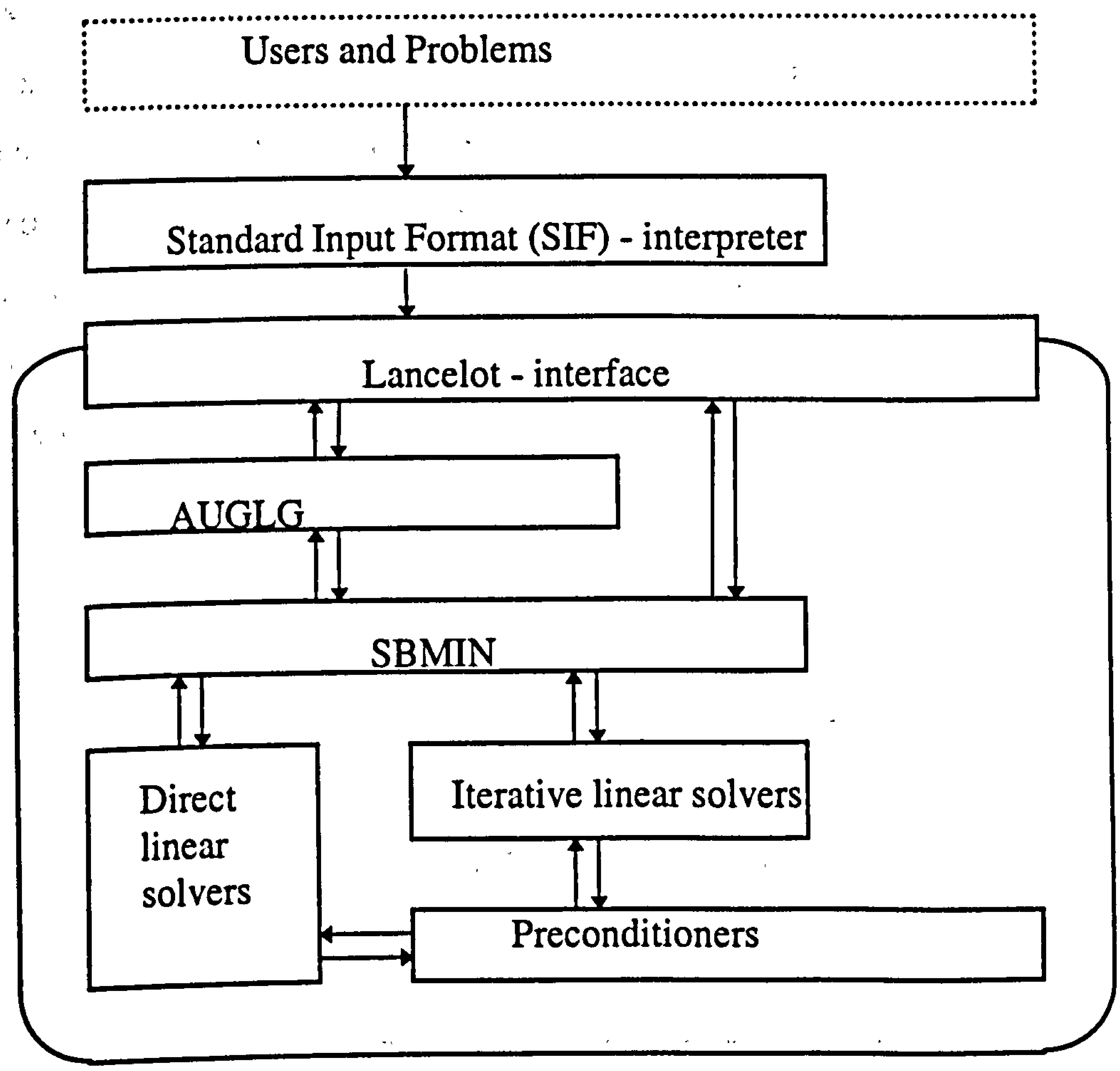


Figure (9.1) Structure of Lancelot package

After the problem is formulated, it is expressed in a Standard Input Format (SIF). The SIF file for the optimization problem formulated in Sections (9.8) and (9.9) is written and shown in Section (9.10.6). To know about the techniques used in writing the SIF file the reader is referred to Conn et al., (1992).

For the optimization problem at hand, Lancelot uses an augmented Lagrangian approach. This approach was discussed in detail in Chapter III - Section (3.4.3). The augmented Lagrangian method proceeds by solving a sequence of non-linear optimization problems with simple constraints. Conn et al., (1996) called these iterations of the augmented Lagrangian algorithms " major iterations". The equality and the transformed inequality constraints are included in the augmented Langrangian function and only the simple bounds are left. To solve the problem with only simple bounds, a specialised algorithm, SBMIN, can be applied (Conn et al., 1996). In SBMIN, a quadratic problem with simple bounds (BQP) is approximately solved at every SBMIN iteration. These are called "minor iterations".

Solving the BQP involves the solution, approximately, of a linear system of equations. This can be achieved by applying either direct or iterative linear solvers. The latter requires preconditioning, which in turn might call specialised versions of the direct solvers (Figure 9.1). The iterative technique used with the package is the conjugate gradient method. Iterations at this level are called cg-iterations.

The three nested iteration levels are shown in Figure (9.2) below.

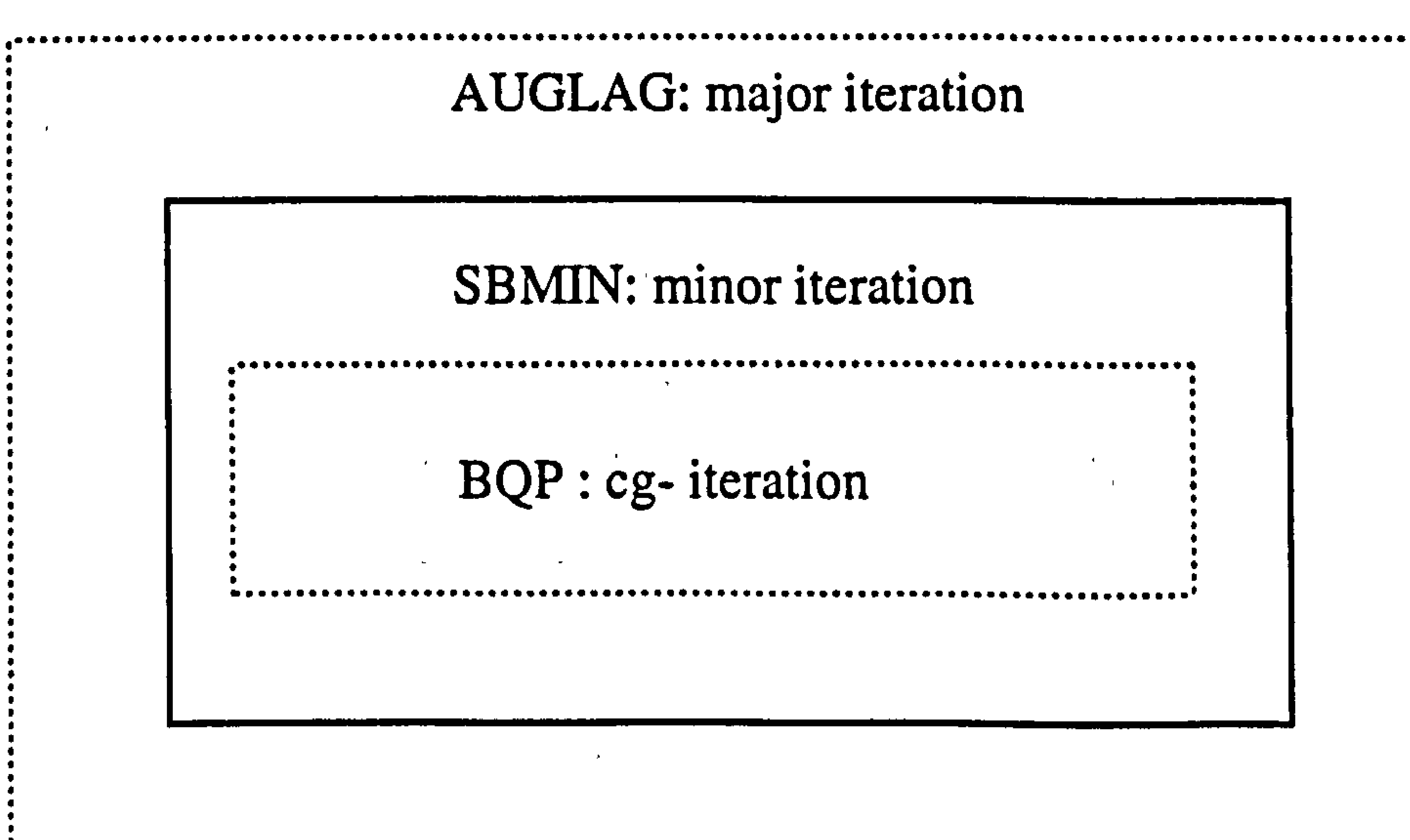


Figure (9.2) The nested iteration levels within Lancelot

9.10.3 Outline of SBMIN

SBMIN is a method for solving the bound-constrained minimisation problem defined by (9.51) and the simple constraints (9.53). f is assumed to be twice - continuously differentiable. The set of points that satisfy (9.53) are known as the feasible box and any point lying in it, is feasible.

SBMIN is an iterative method. A solution $x^{(k)}$, which satisfies the simple bounds (9.53) is obtained at the k^{th} iteration. To improve this solution a (k+1)st iteration is carried and a solution $x^{(k+1)}$ is obtained.

In the (k+1)st iteration, a quadratic model for the non-linear objective function f(x) is built (In this case f(x) is the constructed Lagrangian function). The quadratic model is of the form :

$$m^{(k)}(x) = f(x^{(k)}) + g(x^{(k)})^T(x - x^{(k)}) + (1/2)(x - x^{(k)})^T B^{(k)}(x - x^{(k)}) \quad (9.54)$$

where :

g(x) is the first partial derivative $\nabla_x f(x)$

G(x) is the Hessian matrix $\nabla_{xx} f(x)$

$B^{(k)}$ is a symmetric approximation of the hessian matrix $G(x^{(k)})$. This approximation can be achieved by using Rank one Update Formula (Sr1) as will be used later or the Broydon-Fletcher-Goldfarb-Shanno Formula (BFGS). These methods are described in detail in Chapter III - Section (3.2.4).

A region in which the values of $m^{(k)}(x)$ and f(x) generally agree is called the trust region and defined as

$$\| x - x^{(k)} \| \leq \Delta^{(k)} \quad (\Delta^{(k)} \text{ is a scalar})$$

The (k+1)st iteration proceeds in the following stages:

1) Test of convergence for true objective function f(x)

The first necessary conditions for a feasible point x^* to solve the problem is that the projected gradient at x^* be zero. The projected gradient of f(x) into the feasible box is:

$$x - p(x - g(x), l, u)$$

where the projection operator is defined as

$$p(x,l,u)_i = \begin{cases} l_i & \text{if } x_i < l_i \\ u_i & \text{if } x_i > u_i \\ x_i & \text{otherwise} \end{cases}$$

The iteration can be stopped when the projected gradient is getting small i.e.

$$||x^{(k)} - p(x^{(k)} - g(x^{(k)}),l,u)|| \leq \varepsilon_g$$

Where ε_g is a small convergence tolerance.

2) The generalised Cauchy point of the quadratic model is found. This point lies within the intersection of the feasible box and the trust region.

3) Obtain a new point that further reduces the quadratic model within the intersection of the feasible box and the trust region.

4) Test whether there is a general agreement between the values of the model and the true objective function at the new point. If so, accept the new point as the next iterate. Otherwise, keep the existing iterate as the next iterate and adjust the trust region.

9.10.3.1 The Generalised Cauchy Point (GCP)

To minimise the quadratic function at the (k+1)st iteration within the intersection of the feasible box and the trust region, a point called generalised Cauchy point has to be found. This point is obtained through the minimisation of the model along the path defined by its negative gradient. This point is useful since (Conn et al., 1996):

1) it is guaranteed that the algorithm converges to a point at which the projected gradient is zero if the value of the quadratic model is not larger than its value at the Cauchy point.

2) The variables which are equal to their lower or upper bounds at the generalised Cauchy point are expected to have the same values at the solution of the problem. It is not necessary to calculate the generalised Cauchy point exactly (Conn et al. 1996).

9.10.3.2 Beyond the Generalised Cauchy Point

If the Cauchy point is found, then it is guaranteed that SBMIN will converge. This convergence is achieved by further reducing the model. To reduce the quadratic model, the points which are on their bounds at the generalised Cauchy point are kept fixed and only the values of the free variables are changed. If $x^{(k,1)}$ is the obtained generalised Cauchy point, let $x^{(k,j)}$, j = 2,3,........ be distinct points such that:

* $x^{(k,j)}$ lies within the intersection of the feasible and trust regions.

* those variables which lie on their bounds at $x^{(k,1)}$ lie on the same bounds at $x^{(k,j)}$.

* $x^{(k,j+1)}$ is constructed from $x^{(k,j)}$ by:

(1) determining a non-zero search direction $d^{(k,j)}$ for which

$$\nabla_x m^{(k)}(x^{(k,j)})^T d^{(k,j)} < 0 \qquad (9.55)$$

(2) finding a step length $\alpha^{(k,j)} > 0$ which minimises $m^k(x^{(k,j)} + \alpha^{(k,j)} d^{(k,j)})$ within the intersection of the feasible box and the trust region; and

$$(3)\ x^{(k,j+1)} = x^{(k,j)} + \alpha^{(k,j)} d^{(k,j)} \qquad (9.56)$$

This process is stopped when the norm of the free gradient of the model at $x^{(k,j)}$ is sufficiently small.

The quadratic model is expressed as function of the free variables. Let $\zeta^{(k,j)}$ be the set of variables which are to be fixed because they are on their bounds at the generalised Cauchy point. Let e_i be the ith column of the n*n identity matrix I and $\dot{I}^{(k,j)}$ be the matrix made up of columns e_i, $i \notin \zeta^{(k,j)}$. Now define

$$\dot{g}^{(k,j)} \equiv \dot{I}^{(k,j)^T} g^{(k,j)} \quad \& \quad \dot{B}^{(k,j)} \equiv \dot{I}^{(k,j)^T} B^{(k,j)} \dot{I}^{(k,j)}$$

Then the quadratic model 9.54 at $(x^{(k,j)} + d)$, considered as a function of the free variables $\dot{d} \equiv \dot{I}^{(k,j)^T} d$, is (Conn et al., 1992):

$$\dot{m}^{(k,j)}(\dot{d}) = m^{(k)}(x^{(k,j)}) + \dot{g}^{(k,j)^T} \dot{d} + (1/2)\, \dot{d}^T \dot{B}^{(k,j)} \dot{d} \qquad (9.57)$$

($\dot{g}^{(k,j)}$ & $\dot{d}$ are the gradient and search direction of the free variable respectively).

The iteration used by Lancelot is the conjugate gradient method. This method is described in detail in Chapter III. The convergence of iterative methods can be accelerated by preconditioning. Preconditioning is a function factorisation to accelerate convergence (Conn et al., 1996). However, as discussed in Chapter III the conjugate gradient method minimises quadratic functions in a limited number of iterations. Therefore, this method without preconditioning will be used to solve the formulated reservoir system optimization problem.

9.10.3.3 Accepting the New Point

The point $x^{(k,j)}$ reduces the quadratic model, significantly. The ultimate purpose is to reduce the true objective function f(x). Therefore, it has to be decided whether this point reduces the true objective function as well. Let $r^{(k)}$ be the ratio of the actual reduction in the objective function to that predicted by the quadratic model,

$$r^{(k)} = \frac{f(x^{(k)}) - f(x^{(k,j)})}{m^{(k)}(x^{(k)}) - m^{(k)}(x^{(k,j)})}$$

Let $0 < u < 1$. Then the update $x^{(k+1)}$ is chosen according to the following

$$x^{(k+1)} = \begin{cases} x^{(k,j)} & \text{if } r^{(k)} > u \\ x^{(k)} & \text{otherwise} \end{cases}$$

9.10.4 A General Description of AUGLG

In the AUGLG, the augmented Lagrangian function is constructed. The augmented Lagrangian function is described in Chapter III. The AUGLG makes repeated use of the SBMIN. At the start of each iteration, Lagrange multipliers and penalty parameters have to be given.

9.10.5 Lancelot Specification File

To select specific algorithmic options to be used in solving the optimization problem, Lancelot specification file has to be written. For more information on the specification language and file layout, the reader is referred to Conn et al., (1992).

The main features of the optimization problem, at hand, that need to be reflected in the specification file and other specifications needed to do the calculations are:

a) a maximiser is sought.

b) the function first derivatives are evaluated using finite difference.

c) the function second derivatives are approximated according to the symmetric - rank - one, SR1, formula.

d) conjugate gradient method without preconditioner is used to minimise the quadratic model.

f) exact Cauchy point is to be obtained.

g) specifications about the trust region radius, results printing levels, maximum number of iterations and the penalty parameter have to be included in the specification file.

h) the origin is taken as the starting point. Therefore, all the variables and the Lagrange multipliers are equal to zero when the optimization is started.

i) the starting penalty parameter is taken as 0.1.

j) Both "h" and "i" above will not be stated explicitly in the specification file, since the starting values given for the variables, Lagrange multipliers and penalty parameter are the default values and will be taken automatically by the software.

Taking these considerations into account, the following specification file is prepared. The file is inputted to Lancelot under the name "SPEC.SPC".

9.10.5.1 Specification File Content

```
BEGIN
  maximizer-sought
  check-derivatives
  ignore-derivative-bugs
  finite-difference-gradients
  sr1-approximate-second-derivatives-used
  cg-method-used
  exact-cauchy-point-required
  trust-region-radius  5.0D+0
  maximum-number-of-iterations  2000000
  print-level              1
  start-printing-at-iteration   0
END
```

9.10.6 Standard Input Format, SIF, File

For an optimization problem to be solved by Lancelot, the problem has to be prepared in an understandable manner to the software. This is achieved by writing the standard input format, SIF, file for that problem.

When specifying a problem in SIF, one or more combined files are written. For the problem at hand two files have been prepared and shown in Appendix (D). These are the standard data input file (SDIF) and the standard element input file (SEIF). The SIF file contains a number of ordered sections using five kinds of objects: keywords, codes, numbers, names and Fortran names.

* The keywords are titles of different sections.

* Names are required to be given to various parts of the problem specification, as variables, constraints.... In the SIF file prepared in Appendix (D) names Cons1, Cons2, Cons3,............., Cons48 are given to the problem constraints defined in Section 9.9.1 to Section 9.9.4. The objective function (equation 9.50) is divided into groups. The variables shown in the left-hand-side of the following equations are taken as names for objective function groups.

$obj(1,i) = a(1,i)\, X_{1,i}$ $i = 1$ to 12
$obj(2,i) = a(2,i)\, X_{2,i}$ $i = 1$ to 12
$obj(3,i) = c(1,i)\, (X_{1,i})^2$ $i = 1$ to 12
$obj(4,i) = c(2,i)\, (X_{2,i})^2$ $i = 1$ to 12
$obj(5,i) = d(1,i)\, X_{1,i}\, Y_{1,i}$ $i = 1$ to 12
$obj(6,i) = d(2,i)\, X_{2,i}\, Y_{2,i}$ $i = 1$ to 12
$obj(7,i) = b(1,i)\, X_{1,i}\, [S_{1,i} + S_{1,i+1}]^{0.47}$ $i = 1$ to 12
$obj(8,i) = b(2,i)\, X_{2,i}\, [S_{2,i} + S_{2,i+1}]^{0.417}$ $i = 1$ to 12

* Codes consist of one or two upper case letters. Their purpose is to specify various kinds of information on the problems. Preparing the SIF file for the problem at hand, the following codes are used:

IE used to associate a value to an integer.

IA used to add integer.

RE used to associate a value to a real parameter.

RA used to add real parameters (also code R+ can be used).

RS used to subtract real parameters (also code R- can be used).

X used to declare decision variables.

XN used to define the objective function (in SIF usually Z replaces X if the value is to be defined by that of an already defined parameter).

DO used to start a loop.

OD used to end a loop.

XE is used to specify equality constraints.

XG specifies greater or equal constraints.

XL specifies less or equal constraints.

Z to assign value to constraints constants.

XL to specify a variable lower bound.

XU to specify a variable upper bound.

EV used to define elemental variable.

IV used to define internal variable.

XT is used to assign the element type to a specific elemental variable.

ZV is used to assign the problem variable to elemental variable.

T is used to define the element type in the SEIF file.

R is used to relate the elemental variables to internal variables in the SEIF file.

F is used to specify the functional expression of the group in the SEIF file.

G is used to specify the gradient of the group (in the SEIF file).

H is used to define the function second derivative (Hessian) of the group function (in the SEIF file).

In the SIF file, Appendix (D), the following sections have been introduced:

NAME : defines the problem name. RESERV is the name given to the problem.

VARIABLES : starts section where the problem variables are given names.

GROUPS : start the section where the objective function and constraints are given names and where the linear contribution of each variable to these is specified.

CONSTANTS : starts the section where the constant terms of the groups are named and defined. These are the constant term in the right-hand-side of the constraints.

BOUNDS : starts the section where the bounds on variables are named and defined.

START POINT : starts the section where the proposed starting point for the problem is named and specified using the code XV.

ELEMENT TYPE : in this section, suitable element types with their elemental and internal variables are specified.

ELEMENT USES : in this section a type to each element function appearing in the problem is assigned. Then the relevant problem variables are assigned to the elemental variables of the corresponding element type.

GROUP USES : assign the elements to their groups, together with their associated weighting factors.

ENDATA : declares the end of the SDIF file for the current problem.

ELEMENTS : declares the start of the SEIF file.

INDIVIDUALS : here the particular linear combinations of the elemental variables that define the internal variables associated with the element type are stated. Then expressions for calculating the value and derivatives of the non-linear functions associated with the element type are specified.

ENDATA : declare the end of the SEIF file.

Full description of the techniques used in writing SIF files are described in Conn et. al. (1992). For the optimization problem defined by equation (9.50) and constraints cons1 to cons56, SIF file is prepared (Appendix D). The file is put under one directory with other Lancelot subroutines under the name "RESERV.SIF". In preparing the file, provisions are made to accommodate changes made in the optimization problem. The following items can easily be changed:

a) constants of the objective functions: these change due to changes in power prices.

b) inflows to reservoir systems: therefore the different generated flow sequences can easily be accommodated. The inflows shown in this file are the average river flows.

c) Irrigation requirements: the values shown in the SIF file stand for actual irrigation requirements. It is possible to consider different scenarios of irrigation demands.

d) Requirements downstream the reservoir system.

9.10.7 Problem Solution Results

Lancelot package was installed in a hp-UNIX system. The package can also be installed in a PC. The version installed was the double precision large one. The software has small, medium and large versions in single and double precision. For steps of programme installation, the reader is referred to Conn et al., (1992). The specification and the SIF files were put in the same directory as Lancelot. Then the program was run. Appendix (E) shows the exact output. However, the results can be summarised as follows:

a) The solution includes values for the objective function, decision variables (releases and storage volumes), penalty parameter, Lagrange multipliers and surplus variables.

b) Projected gradient norm of the final iteration is 6.1D-02. This shows that convergence is approximately obtained. This is clear in appendix (E.2) where, in the last iterations, the gradient has become small while the objective function remains unchanged.

c) Objective function value is 1.55983D+04 (15.5983 billion Sudanese dinnars).

d) The solution was obtained in about 4 minutes. Hence the processor is shared, this time may vary if an other run is made.

e) The variables to be optimized are the releases and storages for both Roseries and Sennar reservoirs. Optimum values of these variables are shown in Table (9.3) and in Figures (9.4) and (9.5) for Roseries and Sennar respectively.

f) It can be noticed from the solution, Figures (9.4) and (9.5), that all the releases are made through the power house gates and other gates are only used during flood or when the requirements exceed the power house gates capacity. This agrees with the general optimization objective that aims at maximising the power and the power revenue.

g) It can also be noticed from the solution, Figures (9.4) and (9.5), that the storage is kept at lower levels during July and August. This agrees with the operation policy aiming at sediment management at this period.

Table (9.3) Optimum solution using average flow and actual irrigation requirements

month	Roseries			Sennar		
	power release 10^6 m^3/month	other releases 10^6 m^3/month	storage at the beg. of month -10^6 m^3	power release 10^6 m^3/month	other releases 10^6 m^3/month	storage at the beg. of month-10^6 m^3
sept	2014	6928.4	88.3	330	7082.3	113
oct	2014	3741.87	2175	330	4064.97	362.5
nov	2014	132.122	2175	330	542.143	362.5
dec	1278.84	0	2175	160.3	0	362.5
jan	1662.12	0	2108.33	160.66	0	175.239
feb	842.02	0	1043.34	145.32	0	362.5
mar	555.859	0	568.706	182.659	0	113
apr	404.3	0	295.743	330	0	362.5
may	154.8	0	128.599	330	0	362.5
jun	2002.14	0	467.084	330	1516.6	113
july	2014	5149.94	88.3	330	6560.63	113
aug	2014	12525.2	88.3	330	13425.5	113

9.10.8 Comparison of model results to the current system operation

It is not possible, due to lack of data, to compare this average return with the average actual benefits obtained from the current system operation policy. However, for the year 1993/1994 both actual irrigation requirement and recorded power production are available. In year 1993/1994 the recorded power production was 1200 GWh which has a benefit of 14.8416 billion Sudanese dinnar. The software was run for that year. It has been found that the power and benefits have increased to 1407.573 GWh and 17.2421 billion Sudanese dinnars respectively. These are increments of 14.75 % in power production and 13.92 % in annual benefits.

9.11 CONCLUSIONS

A non-linear model has been formulated for two reservoirs in series. The objective is to maximise power revenues on conditions that irrigation and downstream requirements be satisfied. The formulated algorithm uses synthetically generated flows and deterministic irrigation requirements as inputs, incorporates non-linear power and evaporation functions and is linked to a sedimentation model that predicts the reservoir storage-level relationship. The model is then solved using one of the most efficient non-linear optimization techniques. A general purpose Software package, designed for large scale non-linear programming, named Lancelot is used. To solve the problem the augmented Lagrangian function is constructed and then the conjugate gradient method is used to maximise the function within the feasible box, defined by simple bounds.
The problem is solved in less than 4 minutes (this time may vary if an other processor is used). The solution increased the actual benefits in year 1993/1994 by 13.92 %. The problem is solved without any simplification, i.e. linearisation, decomposition or aggregation, usually used to alleviate the effects of nonlinearity and dimensionality associated with reservoir optimization. Therefore it can be concluded that non-linear programming can be applied successfully without simplifications to multipurpose multiple reservoir systems and this justifies hypothesis 5 and objectives 1 and 2.

Figure (9.3) Blue Nile Double Reservoir System

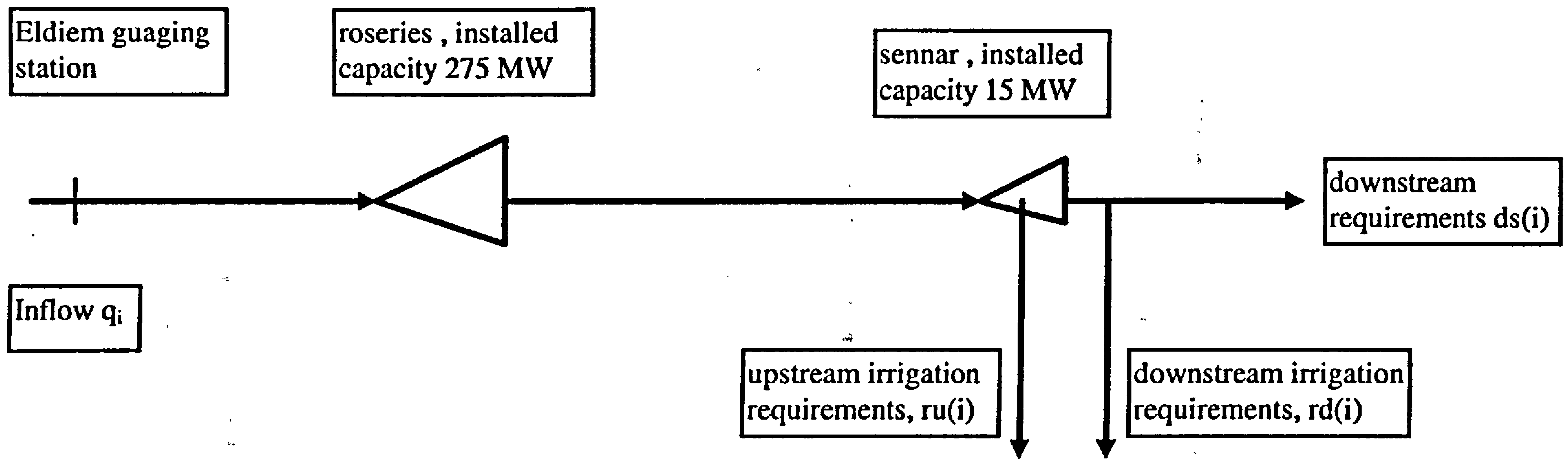


Figure (9.4) Optimization reults for average flow - Roseries

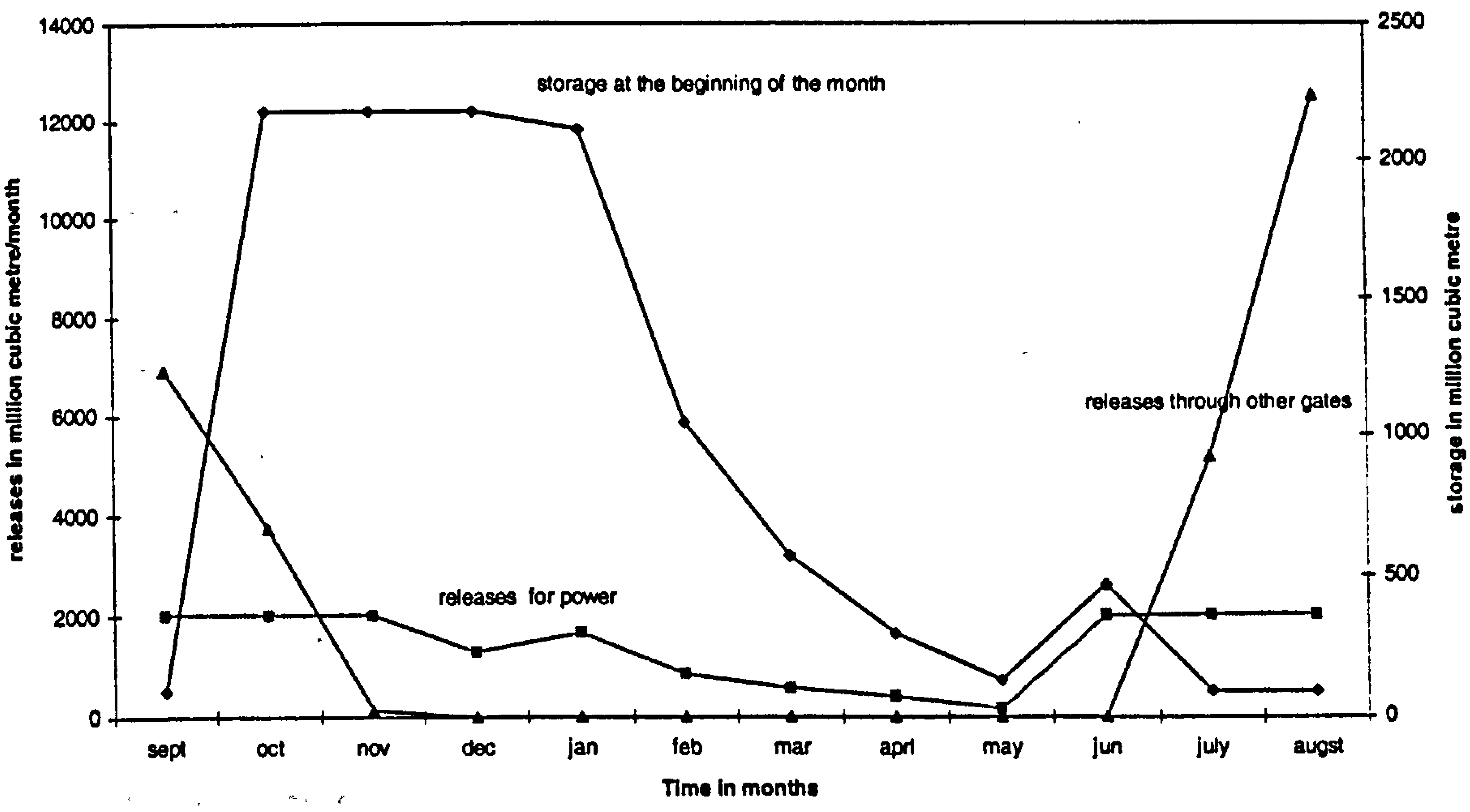
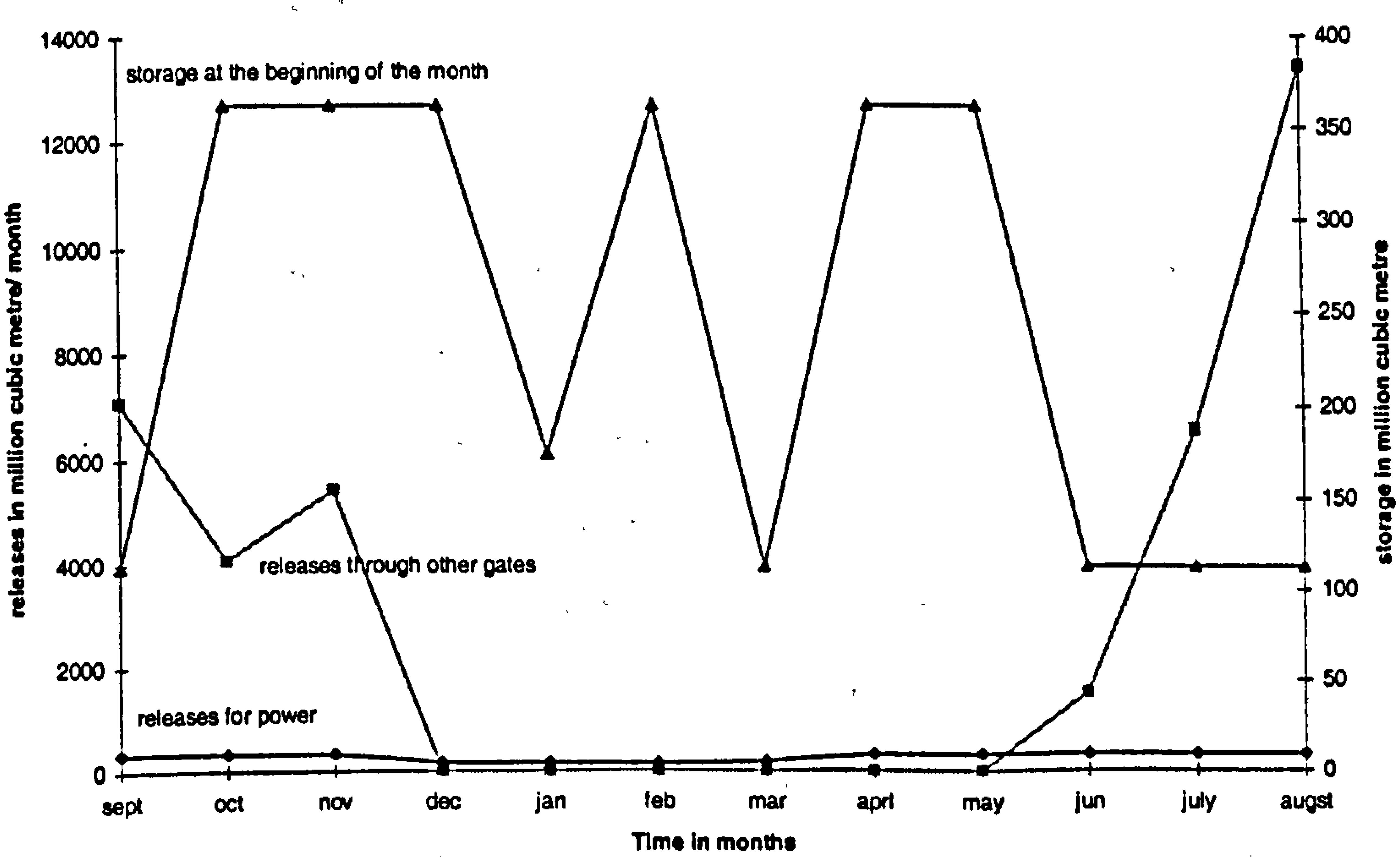


Figure (9.5) Optimization results for average flow - Sennar



CHAPTER X

DERIVATION OF MONTHLY OPERATION RULES

Summary ~ In Chapter VII synthetic samples were generated. These samples are used as inputs to the optimization model developed in Chapter IX. The ouput is used in this Chapter to derive operation rules. Releases are regressed against significant independent variables to derive these rules. Then the performance of these rules is judged both statistically and with using simulation techniques.

10.1 INTRODUCTION

There are three common applications of mathematical programming in water resources planning. These are concerned with water allocations, capacity expansion and reservoir operation. In these applications dynamic programming is widely used (Loucks, 1981). A trial would be made here to apply non-linear programming for reservoir operation.

10.2 AN APPROACH TO MONTHLY OPERATION RULES DERIVATION

Reservoirs are operated to achieve certain objectives. In the case at hand, (Chapter IX), it is aimed to maximise the annual hydropower benefits from a double reservoir system, on condition that certain minimum flows and irrigation requirements are met. To achieve these objectives, decisions have to be made on releases. Therefore decision rules or operation rules have to be developed by regressing the releases on storages and inflows. To develop these rules, the stochastic nature of inflows have to be included. To achieve this, inflow to reservoirs has to be modelled and used to generate flow sequences of equal probabilities of occurrences. The flow modelling and samples generation have been done in Chapter VII. These samples are inputted to the optimization model and the optimal releases are then regressed on important independent variables to derive the operation rules. The most famous work to derive monthly policies and widely referred to in literature was done by Bhasker and

Whitlatch (1980). They applied linear and non-linear regression models on optimal releases obtained from application of dynamic programming on a single reservoir. The purpose of optimization was to reduce the losses from a reservoir. The regression models they used, are of the following general forms:

Linear Model M1:

$$REL = B_0 + B_1(QFL) + B_2(STG) + B_3(QFL1) + \ldots\ldots + B_6(QFL4) \qquad (10.1)$$

Nonlinear Model M2:

$$REL = B_0 + B_1(SUM1) + B_2(SUM2) + B_3(SUM3) \qquad (10.2)$$

Nonlinear Model M3:

$$REL = B_0 + B_1(CRP) \qquad (10.3)$$

Where

REL release in month i
QFL inflow in month i
STG storage at the beginning of month i
QFL1, QFL2 ....QFL4 lagged inflows in month i-1, i-2, ... , i-4 respectively.
SUM1 = (QFL + STG)
SUM2 = $(QFL + STG)^2$
SUM3 = $(QFL + STG)^3$
CRP = (QFL * STG)

It can be noticed from the results obtained by Bhasker and Whitlach (1980), that the application of the above linear and non-linear regression models on dynamic programming, sometimes, yield very poor results. For the linear model, R^2 values range from 0.05 to 0.355 for the months of September - November and for the best non-linear model R^2 values range from 0.048 to 0.226 for the same period. Therefore, following the same approach, an attempt is to be made here to apply these linear and non-linear regression models to the output of a non-linear optimization model.

Earlier, in Chapter VII, a model for the Blue Nile flow was developed and used to generate flow sequences. Each sequence is used as an input for the optimization model, defined by equations (9.50) and cons1 to cons56. Then the model is solved, as described in Chapter IX. Appendix (F) shows the inputs to the model and the results obtained from the model solution. In Appendix (F), Tables (f.1) and (f.2) show the input to the model, while Tables (f.3) - (f.14) show the results obtained from model solutions for the upstream reservoir, Roseries, and Tables (f.15) - (f.26) show the results obtained for the downstream reservoir, Sennar. Also these tables include the independent variables required for regression analysis; QFL, QFL1, QFL2, QFL3, QFL4, SUM1, SUM2, SUM3 and CRP.

10.3 REGRESSION MODELS FORMS

To decide on the important variables on which the releases can be regressed, simple correlation between the release and these variables has to be done. The equation for the correlation coefficient (Haan, 1977) is:

$$\rho_{x,y} = \frac{COV(X,Y)}{\sigma_x * \sigma_y} \qquad (10.4)$$

Where :

$-1 \leq \rho_x \leq 1$,

The correlation is positive if large values of one set are associated with large values of another set. It is negative when small values of one set are associated with large values of the other. While there is no relation between the two sets, if the correlation is 0.

$$COV(X,Y) = (1/n) \sum_{i=1}^{n} (X_i - \mu_x)(Y_i - \mu_y) \qquad (10.5)$$

σ_x, σ_y standard deviation of array x and array y respectively.

μ_x, μ_y mean of array x and array y respectively.

Using Software Excel, the correlation analysis have been done and the results are shown in Tables (10.1) and (10.2) for Roseries and Sennar respectively.

Table (10.1) Simple correlation coefficients of optimal monthly releases with independent variables - Roseries

month	QFL	QFL1	QFL2	QFL3	QFL4	STG	SUM	CRP
sept.	1	0.368	0.358	0.349	0.281	0	1	1
october	0.898	0.49	0.495	0.128	0.298	-	0.898	0.898
november	1	0.562	0.335	0.357	0.049	-	1	1
december	0.945	0.469	0.690	0.302	0.207	-	0.945	0.945
january	0.460	-0.26	-0.153	-0.001	0.099	0.235	0.416	0.444
february	0.708	0.747	0.539	0.241	0.459	0.725	0.778	0.804
march	0.564	0.685	0.465	0.549	0.346	0.755	0.818	0.779
april	-0.041	0.330	0.265	0.100	0.301	0.502	0.408	0.345
may	0.892	0.708	0.379	0.680	0.509	0.821	0.941	0.919
june	0.94	0.765	0.620	0.511	0.718	-0.032	1	0.273
july	1	0.516	0.440	0.405	0.041	0	1	1
august	1	0.457	0.186	0.329	0.278	0	1	1

Table (10.2) Simple correlation coefficients of optimal monthly releases with independent variables - Sennar

month	QFL	QFL1	QFL2	QFL3	QFL4	STG	SUM	CRP
sept.	1	0.368	0.358	0.335	0.431	-	1	1
october	0.901	0.595	0.499	0.042	0.297	-	0.901	0.9
november	0.999	0.468	0.328	0.333	0.061	-	0.999	0.996
december	0.982	0.475	0.712	0.275	0.167	0.087	0.983	0.981
january	0.217	0.514	0.301	0.548	0.40	0.621	0.998	0.720
february	1	0.218	0.465	0.241	0.455	-	1	1
march	0.939	0.483	-0.025	0.446	0.319	-	0.939	0.939
april	0.322	0.684	0.24	0.087	0.210	0.447	0.488	0.516
may	0.96	-0.26	0.458	0.731	0.100	-0.443	0.979	0.871
june	0.993	0.793	-0.114	0.675	0.648	0.827	1	0.916
july	1	0.447	0.468	-0.20	0.062	-	1	1
august	1	0.457	0.126	0.373	-0.050	-	1	1

Blanks are noticed in these tables, when the independent variable, storage, is constant. When a variable has constant values, its standard deviation is zero. Therefore no value is obtained for the correlation coefficient as the denominator in equation (10.4) is divided by 0. Since the independent variable, storage, is kept constant and does not change with the dependent variable, release, no correlation between the two variables is expected.

From the correlation results, it is clear that there is a correlation between the release and all independent variables of the regression models (10.1) to (10.3), except the storage in some months. The reason for this no correlation, is that the storages at the beginning of these months are maintained constant (either the reservoirs are full or kept at a level to minimise sedimentation). However the inclusion of this variable in the derived models, require that the reservoirs be operated at or close to these constant levels. This may limit the applications of the non-linear models, since the storage is a significant variable of them.

10.4 REGRESSION ANALYSIS RESULTS

Using Software Excel and the data in Appendix (E), Tables (e.3) to (e.26), regression analysis have been carried out to find the constants of regression models defined in equation (10.1) to (10.3) as well as the following reduced forms of these models:

a) Reduced linear model, M1, that includes storage, STG, and the current monthly inflow, QFL, as independent variables.

b) Reduced linear model, M1, that includes storage, STG, and the preceding monthly inflow, QFL1, as independent variables.

c) Reduced non-linear model, M2, that includes SUM1 as the independent variable.

d) Reduced non-linear model, M2, that includes SUM2 as the independent variable.

e) Reduced non-linear model, M2, that includes SUM3 as the independent variable.

The results of the regression for these models are shown in Tables (10.3) to (10.6).

Table (10.3) Results of the regression for the complete and reduced linear model, M1, Roseries

	Complete Linear Model M1							M1 with STG & QFL			M1 with STG & QFL1		
month	B_0	B_1	B_2	B_3	B_4	B_5	B_6	B_0	B_1	B_2	B_0	B_2	B_3
sept.	-4760.54	1	30.09	7.71E-16	6.3E-15	1.18E-14	-1.5E-14	-5494.32	1	38.4	-2335.53	57.143	0.4287
octob	18845.07	0.885	-8.397	-0.092	-0.004	0.161	0.093	1213.196	0.819	0	2515.7	0	0.3258
nov.	-3090.72	1	1.391	6.32E-8	-9.4E-8	6.51E-9	-5.5E-9	-65.88	1	0	950.032	0	1
dec.	1638.66	0.784	-0.554	-0.014	0.002	-0.007	0.003	402.258	0.766	0	957.77	0	0.2187
jan.	1085.15	0.302	0.279	-0.172	-0.013	0.013	-0.005	1426.067	0.1904	0.0295	1147.43	0.3056	-0.1095
feb.	553.45	0.277	0.079	0.196	0.001	0.0002	-0.0034	581.647	0.306	0.1507	583.155	0.1217	0.2374
march	435.04	0.508	0.6005	-0.155	-0.237	-0.0924	-0.003	256.531	0.43	0.32	297.762	0.2859	0.2888
april	691.75	0.048	0.805	-0.029	-0.4414	-0.4152	-0.0943	322.36	-0.298	0.282	229.658	0.2223	0.0727
may	-148.752	1.24	1.136	-1.202	-0.483	-0.238	0.3597	-355.219	1	1.0688	-430.964	1.6782	1.9058
june	-101.916	1	0.991	-4.2E-5	5.01E-5	-0.0001	0.00024	-101.872	1	0.9913	1242.066	0.045	1.115
july	704.977	1	-8.075	-2.3E-14	5.17E-14	-1.3E-13	-5.5E-14	668.907	1	-7.667	5134.26	-3.6667	1.3328
aug.	1984.87	1	-22.54	1.95E-13	-1.3E-14	2.21E-13	-7.7E-13	2012.486	1	-22.857	9771.016	-8.6667	0.7476

Table (10.4) Results of the regression for the nonlinear model, M3, and complete and Reduced nonlinear model, M2, Roseries

	Complete Nonlinear Model M2				M2 with SUM1		M2 with SUM2		M2 with SUM3		M3 Model	
month	B_0	B_1	B_2	B_3	B_0	B_1	B_0	B_2	B_0	B_3	B_0	B_1
sept.	-2191.9	1	-4.7E-16	1.2E-20	-2191.9	1	3982.652	3.8E-5	6144.323	1.72E-9	-2103.6	0.0113
october	33364.02	-11.64	0.0015	-5.42E-8	-568.67	0.819	2646.275	4.968E-5	3810.502	3.688E-9	1213.196	0.00038
nov.	-2240.79	1	1.6E-8	-1.2E-12	-2240.88	1	71.988	0.0001	835.489	1.45E-8	-65.88	0.00046
dec.	42470.56	-31.433	0.0078	-6.2E-7	-1263.82	0.766	189.479	9.92E-5	670.939	1.69E-8	402.258	0.000352
jan.	-6053.82	7.922	-0.0027	3.15E-7	1323.85	0.108	1475.326	1.9E-5	1526.29	4.41E-9	1504.203	8.03E-5
feb.	2904.745	-3.893	0.0023	-4.3E-7	586.952	0.193	733.864	6.11E-5	783.505	2.47E-8	736.507	0.000291
march	-613.921	2.615	-0.0017	3.61E-7	273.514	0.343	446.058	0.00016	503.455	8.5E-8	431.812	0.000761
april	-320.628	2.452	-0.0028	1.02E-6	224.268	0.165	287.555	8.38E-5	305.355	4.84E-8	283.404	0.000412
may	517.482	-1.703	0.0022	-5.2E-7	-360.206	1.025	55.204	0.00047	189.008	2.39E-7	143.296	0.00198
june	-97.565	0.997	-2.4E-16	9.16E-10	-104.769	1	926.239	0.00023	1285.159	6.58E-8	1668.294	0.00052
july	-96.36	1	1.23E-14	-5.4E-19	-96.36	1	3466.915	6.8E-5	4671.956	5.93E-9	-8.06	0.01133
august	-94.1	1	-6.4E-15	1.45E-19	-94.1	1	7262.76	3.32E-5	9756.039	1.43E-9	-5.8	0.01133

Table (10.5) Results of the regression for the complete and reduced linear model, M1, Sennar

	Complete Linear Model M1							M1 with STG & QFL			M1 with STG & QFL1		
month	B_0	B_1	B_2	B_3	B_4	B_5	B_6	B_0	B_1	B_2	B_0	B_2	B_3
sept.	-1530.1	1	0	-2.62E-15	-2.6E-14	-4.8E-14	5.86E-14	-1530.1	1	0	1182.57	0	0.429
october	-1798.18	0.775	0	0.186	0.132	-0.332	0.238	-1437.9	0.986	0	729.104	0	0.433
nov.	-1197.49	0.979	0	-0.0028	0.0019	-0.0045	0.0068	-1204.05	0.974	0	-221.366	0	0.2012
dec.	-789.554	0.751	0	0.0125	0.0101	-0.0032	-0.0074	-880.19	0.783	0	-131.243	0	0.176
jan.	-1597.63	0.956	0.975	-0.0056	0.0019	-0.0007	0.0007	-1599.87	0.958	0.961	57.652	0.4182	0.0173
feb.	-696.7	1	0	-1.1E-13	-2.6E-14	-1.7E-16	-1.4E-15	-696.7	1	0	-233.2	0	0.254
march	127.989	0.812	0	0.1693	-0.3046	-0.015	-0.0055	-251.356	0.812	0	-370.054	0	0.682
april	247.112	0.2374	0.384	0.3428	0.1093	-0.2363	-0.0528	-58.64	0.2597	0.7723	15.981	0.1895	0.3621
may	398.338	0.899	1.677	-0.2651	-0.1297	-0.870	0.0498	-182.084	0.7509	1.0496	975.496	-2.359	0.7254
june	-254.976	1.0001	0.873	0.00032	-0.0004	-0.0014	0.0016	-254.444	1	0.875	1156.698	2.828	0.2938
july	-273.31	1	0	-6.3E-14	1.31E-14	-9.5E-14	2.02E-13	-273.31	1	0	4441.682	0	1.2407
august	-783.685	1	0	-2.7E-7	-4.9E-7	5.38E-8	-8.1E-6	-783.69	1	0	8228.08	0	0.7476

Table (10.6) Results of the regression for the nonlinear model, M3, and complete and reduced nonlinear model, M2, Sennar

	Complete Nonlinear Model M2				M2 with SUM1		M2 with SUM2		M2 with SUM3		M3 Model	
month	B_0	B_1	B_2	B_3	B_0	B_1	B_0	B_2	B_0	B_3	B_0	B_1
sept.	-1643.1	1	3.93E-16	-1.2E-20	-1643.1	1	3445.43	4.48E-5	5230.996	2.31E-9	-1530.1	0.00885
october	-2296.8	1.513	-0.00012	7.545E-9	-1795.249	0.986	1425.023	7.082E-5	2565.84	6.183E-9	-1437.9	0.00272
nov.	-16650.3	18.192	-0.00612	6.92E-7	-825.842	0.7979	269.707	0.000135	619.688	2.92E-8	-536.6	0.0022
dec.	5247.977	-7.313	0.00325	-4.2E-7	-1160.5	0.7816	-356.921	0.000181	-100.421	5.43E-8	-861.6	0.00213
jan.	281137.2	-440.725	0.2298	-4E-5	-1603.02	0.96	-682.242	0.00025	-375.974	8.66E-8	40.643	0.00037
feb.	-165.719	-0.00474	-5.5E-5	2.27E-7	-1059.2	1	-414.057	0.000386	-200.461	1.98E-7	-696.7	0.00276
march	3207.061	-15.778	0.0252	-1.3E-5	-343.161	0.812	-83.318	0.000614	5.1963	5.96E-7	-251.356	0.00719
april	19667.81	-91.743	0.1426	-7.3E-5	68.788	0.3379	174.234	0.000264	209.047	2.7E-7	183.555	0.001
may	879.516	-2.0132	0.0021	-4.5E-7	-70.335	0.723	237.862	0.000311	324.598	1.48E-7	207.215	0.00246
june	-538.425	1.4186	-0.0002	2.92E-8	-234.058	0.979	827.337	0.000211	1189.366	5.63E-8	1210.422	0.00154
july	-386.31	1	1.01E-14	-4.5E-19	-386.31	1	3185.481	6.78E-5	4393.306	5.91E-9	-273.31	0.00885
august	-896.271	0.9999	5.33E-9	-1.1E-13	-896.69	1	6469.751	3.32E-5	8966.187	1.43E-9	-783.69	0.00885

10.5 CHOICE OF THE BEST REGRESSION MODEL

To determine the best model among the fitted ones, values of the coefficient of determination, R^2, have to be found. R^2 is calculated, as part of the regression analysis, according to the following equation (Haan, 1977):

$$R^2 = (B-A)/B \tag{10.6}$$

Where

$A = \Sigma (y_i - y_i')^2$
$B = \Sigma (y_i - \mu)^2$
y_i denotes the observed y-values
y_i' denotes the estimated y-values
μ is the mean of observed y-value

This coefficient compares estimated and actual y-values. In this case these are the releases. It ranges from 0 to 1. If it is 1, then there is a perfect correlation and there is no difference between the estimated and actual y-values. At the other extreme, if the coefficient of determination is 0, then the regression equation is not helpful in predicting y-values. Tables (10.7) and (10.8) show the R^2 values for the different models for Roseries and Sennar respectively.

Examining these results according to the above criterion, it is clear that the complete linear model M1 produces the best results and the complete non-linear model M2 is the second best model. M1 complete model slightly improves the results obtained from model M1 with storage and current inflow QFL. This indicates that, for the linear models, the current period flow is the most significant variable. When examining R^2 results, it can be noticed that some of its values are equal to 1. These values are obtained in models where storage and current inflows are the significant variables and the reservoirs are operated with constant storages. When reservoirs are operated at constant levels, then there would be high correlation between inflows and outflows (releases), which resulted in this high R^2 values. Excluding these special cases, R^2 values for the best linear model, M1 complete, range from 0.512 to 0.908 for Roseries and 0.565 to 0.998 for Sennar. For the best non-linear model M2 complete, R^2 values range from 0.235 to 0.985, Roseries, and 0.345 to 0.998, Sennar. Comparison of these values to R^2 obtained by Bhasker and Whitlach (1980), shown earlier in Section (10.2), indicates that the result of application of these regression models to non-linear optimization output give better results than their application to dynamic programming output. However the two superior models will be subjected to further testing. Recent applications use simulation to test regression models derived from the optimization results (Karamouz and Vasiliadis, 1992).

Table (10.7) Coefficient of determination R^2, for different models - Roseries

Month	M1 Complete	M1 with QFL & Storage	M1 with QFL1 & Storage	M2 Complete	M2 with sum1 only	M2 with sum2 only	M2 with sum3 only	M3
sept.	1	1	0.135	1	1	0.968	0.891	1
october	0.825	0.806	0.240	0.93	0.806	0.855	0.860	0.806
nov.	1	1	0.220	1	1	0.996	0.985	1
dec.	0.895	0.893	0.220	0.985	0.893	0.917	0.934	0.893
jan.	0.776	0.216	0.362	0.235	0.173	0.161	0.149	0.197
feb.	0.684	0.619	0.615	0.713	0.606	0.643	0.666	0.646
march	0.726	0.674	0.59	0.811	0.669	0.574	0.473	0.606
april	0.512	0.303	0.256	0.378	0.167	0.095	0.051	0.119
may	0.908	0.886	0.711	0.945	0.886	0.933	0.901	0.845
june	1	1	0.586	1	1	0.981	0.934	0.075
july	1	1	0.267	1	1	0.992	0.969	1
august	1	1	0.209	1	1	0.992	0.971	1
Range	0.512-0.908	0.216-0.893	0.135-0.711	0.235-0.985	0.167-0.893	0.095-0.992	0.051-0.985	0.075-0.893

Table (10.8) Coefficient of determination R^2, for different models - Sennar

Month	M1 Complete	M1 with QFL & Storage	M1 with QFL1 & Storage	M2 Complete	M2 with sum1 only	M2 with sum2 only	M2 with sum3 only	M3
sept.	1	1	0.135	1	1	0.955	0.856	1
october	0.877	0.802	0.354	0.817	0.812	0.811	0.778	0.812
nov.	0.998	0.998	0.219	0.345	0.295	0.277	0.260	0.295
dec.	0.969	0.963	0.225	0.998	0.966	0.983	0.992	0.962
jan.	0.998	0.997	0.39	1	0.997	0.996	0.995	0.519
feb.	1	1	0.05	1	1	1	1	1
march	0.91	0.876	0.233	0.971	0.882	0.921	0.941	0.882
april	0.565	0.322	0.477	0.419	0.238	0.249	0.256	0.266
may	0.976	0.963	0.211	0.996	0.958	0.986	0.951	0.759
june	1	1	0.70	1	1	0.979	0.932	0.840
july	1	1	0.20	1	1	0.992	0.969	1
august	1	1	0.209	1	1	0.992	0.972	1
Range	0.565-0.998	0.322-0.998	0.05-0.7	0.345-0.998	0.238-0.997	0.249-0.992	0.256-0.992	0.266-0.962

10.6 RESERVOIR SIMULATION

Here simulation is going to be used to test the derived operation rules. When carrying out the simulation, the fitted operation rules will be used to operate the reservoirs. Knowing the inflows and storages, the releases can be calculated using the fitted operation rules.

As explained earlier in Chapter II - Section (2.3.1) of the literature review, the two basic equations used in reservoir simulation are:

a) Mass balance equation

Inflow = Outflow + spill + losses + Ds

b) Reservoir state equation

Ste = Stb + Ds

Combining the two equations by substituting the change in storage, Ds, from one equation into the other, the following equation is obtained:

$$(Ste+ spill) = Stb + Inflow - Outflow - losses \qquad (10.7)$$

Where :

Stb is the reservoir storage at the beginning of the month. This term is known when simulation is started. The same simples used in optimization will be used here. i.e. $S_{1,1}$, $S_{1,2}$, ... , $S_{1,12}$ for storage in Roseries at the beginning of September, October, ..., August, and $S_{2,1}$, $S_{2,2}$, ... , $S_{2,12}$ for storage in Sennar at the beginning of September, October,............, August.

Inflow is the inflow to the reservoir during the month. For the upstream reservoir, Roseries, this would be the river flow i.e. q_1, q_2 , q_3,...........q_{12} for September, October, ..., August respectively. In this simulation, average river flows will be used. For the downstream reservoir, Sennar, the inflow would be the releases from the upstream reservoir.

Outflow or releases can be calculated using the developed operation rules which are function of the known inflows and storage. The storage at the beginning of the simulation is known. Therefore, this term is known since it is a function of known variables. The releases found using the developed operation rules represent the total releases, i.e. releases for hydropower, irrigation and spill.

Losses: These are defined by equations (9.17) to (9.28) for Roseries and equations (9.29) to (9.40) for Sennar. These terms are not known since they are function of the unknown end of period storage Ste.

Substituting for these terms in equation (10.7), the following monthly relations, starting with September, are obtained.

Roseries:

$S_{(1,2)}$ +spill = $S_{(1,1)}$ -[$X_{(1,1)}$ + $Y_{(1,1)}$] - [$0.101(S_{1,1}+S_{1,2})^{0.47} + 0.005(S_{1,1}+S_{1,2})^{0.94}$ +5.966]+q_1..........(10.8)

$S_{(1,3)}$ + spill = $S_{(1,2)}$ - [$X_{(1,2)}$ + $Y_{(1,2)}$]- [$0.139(S_{1,2}+S_{1,3})^{0.47}$ + $0.006(S_{1,2}+S_{1,3})^{0.94}$ +8.21] + q_2 ....(10.9)

$S_{(1,4)}$ + spill = $S_{(1,3)}$ - [$X_{(1,3)}$ + $Y_{(1,3)}$]- [$0.287(S_{1,3}+S_{1,4})^{0.47}$ + $0.013(S_{1,3}+S_{1,4})^{0.94}$ +16.948]+q_3 ....(10.10)

$S_{(1,5)}$ + spill = $S_{(1,4)}$ -[$X_{(1,4)}$ + $Y_{(1,4)}$]-[$0.283(S_{1,4}+S_{1,5})^{0.47}$ + $0.013(S_{1,4}+S_{1,5})^{0.94}$ +16.709]+q_4 ....(10.11)

$S_{(1,6)}$ +spill = $S_{(1,5)}$ -[$X_{(1,5)}$ + $Y_{(1,5)}$]-[$0.287(S_{1,5}+S_{1,6})^{0.47}$ + $0.013(S_{1,5}+S_{1,6})^{0.94}$ + 16.948]+ q_5 ....(10.12)

$S_{(1,7)}$ +spill= $S_{(1,6)}$ -[$X_{(1,6)}$ + $Y_{(1,6)}$]-[$0.319(S_{1,6}+S_{1,7})^{0.47}$ + $0.014(S_{1,6}+S_{1,7})^{0.94}$ +18.857]+q_6 ....(10.13)

$S_{(1,8)}$ +spill=$S_{(1,7)}$ - [$X_{(1,7)}$ + $Y_{(1,7)}$]- [$0.36(S_{1,7}+S_{1,8})^{0.47}$ + $0.016(S_{1,7}+S_{1,8})^{0.94}$ +21.244]+q_7 ....(10.14)

$S_{(1,9)}$ +spill = $S_{(1,8)}$ - [$X_{(1,8)}$ + $Y_{(1,8)}$]-[$0.356(S_{1,8}+S_{1,9})^{0.47}$ + $0.016(S_{1,8}+S_{1,9})^{0.94}$ +21.006]+q_8 ....(10.15)

$S_{(1,10)}$ + spill= $S_{(1,9)}$ -[$X_{(1,9)}$ + $Y_{(1,9)}$]-[$0.324(S_{1,9}+S_{1,10})^{0.47}$ + $0.015(S_{1,9}+S_{1,10})^{0.94}$ +19.096]+ q_9 ....(10.16)

$S_{(1,11)}$ +spill =$S_{(1,10)}$ -[$X_{(1,10)}$ +$Y_{(1,10)}$]-[$0.186(S_{1,10}+S_{1,11})^{0.47}$ + $0.008(S_{1,10}+S_{1,11})^{0.94}$ +10.98]+q_{10} ....(10.17)

$S_{(1,12)}$+spill=$S_{(1,11)}$ -[$X_{(1,11)}$ +$Y_{(1,11)}$]-[$0.106(S_{1,11}+S_{1,12})^{0.47}$ + $0.005(S_{1,11}+S_{1,12})^{0.94}$ +6.206]+q_{11}. ....(10.18)

$S_{(1,13)}$+spill=$S_{(1,12)}$+[$X_{(1,12)}$+$Y_{(1,12)}$]- [$0.077(S_{1,12}+S_{1,13})^{0.47}$ + $0.003(S_{1,12}+S_{1,13})^{0.94}$ +4.535] + q_{12} ....(10.19)

Sennar:

$S_{(2,2)}$ +spill=$S_{(2,1)}$ -[$X_{(2,1)}$ + $Y_{(2,1)}$]+[$X_{(1,1)}$ +$Y_{(1,1)}$]-[$6.049(S_{2,1}+S_{2,2})^{0.417} - 0.177(S_{2,1}+S_{2,2})^{0.834}$ - 24.1] - ru1 ....(10.20)

$S_{(2,3)}$ +spill = $S_{(2,2)}$ -[$X_{(2,2)}$ + $Y_{(2,2)}$]+[$X_{(1,2)}$ + $Y_{(1,2)}$]-[$8.04\,(S_{2,2}+S_{2,3})^{0.417} - 0.236(S_{2,2}+S_{2,3})^{0.834}$ - 32.02] - ru2 ....(10.21)

$S_{(2,4)}$ +spill = $S_{(2,3)}$ -[$X_{(2,3)}$ + $Y_{(2,3)}$]+[$X_{(1,3)}$ + $Y_{(1,3)}$]-[$9.65(S_{2,3}+S_{2,4})^{0.417} - 0.284(S_{2,3}+S_{2,4})^{0.834}$ -38.43] -ru3 ....(10.22)

$S_{(2,5)}$ +spill = $S_{(2,4)}$ -[$X_{(2,4)}$ + $Y_{(2,4)}$]+[$X_{(1,4)}$ + $Y_{(1,4)}$]-[$8.644(S_{2,4}+S_{2,5})^{0.417} - 0.254(S_{2,4}+S_{2,5})^{0.834}$ 34.42] -ru4 ....(10.23)

$S_{(2,6)}$ +spill =$S_{(2,5)}$ -[$X_{(2,5)}$ + $Y_{(2,5)}$]+[$X_{(1,5)}$ + $Y_{(1,5)}$]-[$9.03(S_{2,5}+S_{2,6})^{0.417} - 0.265(S_{2,5}+S_{2,6})^{0.834}$ - 35.97]- ru5 ....(10.24)

$S_{(2,7)}$ + spill = $S_{(2,6)}$ -[$X_{(2,6)}$ + $Y_{(2,6)}$]+[$X_{(1,6)}$ + $Y_{(1,6)}$]-[$10.217(S_{2,6}+S_{2,7})^{0.417}$ - $0.30(S_{2,6}+S_{2,7})^{0.834}$ - 40.68] - ru6 ....(10.25)

$S_{(2,8)}$ +spill=$S_{(2,7)}$ -[$X_{(2,7)}$ + $Y_{(2,7)}$]+[$X_{(1,7)}$ + $Y_{(1,7)}$]-[$11.789(S_{2,7}+S_{2,8})^{0.417}$ - $0.346(S_{2,7}+S_{2,8})^{0.834}$ - 46.94] - ru7..(10.26)

$S_{(2,9)}$ +spill = $S_{(2,8)}$ -[$X_{(2,8)}$ + $Y_{(2,8)}$]+[$X_{(1,8)}$ + $Y_{(1,8)}$] -[$13.11(S_{2,8}+S_{2,9})^{0.417}$ - $0.385(S_{2,8}+S_{2,9})^{0.834}$ -52.2] - ru8 ....(10.27)

$S_{(2,10)}$ +spill = $S_{(2,9)}$ -[$X_{(2,9)}$ + $Y_{(2,9)}$]+[$X_{(1,9)}$ + $Y_{(1,9)}$]-[$12.10(S_{2,9}+S_{2,10})^{0.417}$ - $0.355(S_{2,9}+S_{2,10})^{0.834}$ -48.2]- ru9 ....(10.28)

$S_{(2,11)}$ +spill= $S_{(2,10)}$ -[$X_{(2,10)}$ + $Y_{(2,10)}$]+[$X_{(1,10)}$ +$Y_{(1,10)}$]-[$8.945(S_{2,10}+S_{2,11})^{0.417}$ - $0.263(S_{2,10}+S_{2,11})^{0.834}$ - 35.62] -ru10..............................(10.29)

$S_{(2,12)}$ +spill= $S_{(2,11)}$ -[$X_{(2,11)}$ + $Y_{(2,11)}$]+[$X_{(1,11)}$ + $Y_{(1,11)}$]-[5.02 $(S_{2,11}+S_{2,12})^{0.417}$ - $0.147(S_{2,11}+S_{2,12})^{0.834}$ - 19.99] -ru11..............................(10.30)

$S_{(2,13)}$ +spill= $S_{(2,12)}$ -[$X_{(2,12)}$ + $Y_{(2,12)}$]+[$X_{(1,12)}$ + $Y_{(1,12)}$]-[$3.724(S_{2,12}+S_{2,13})^{0.417}$ - $0.110(S_{2,12}+S_{2,13})^{0.834}$ - 14.83] -ru12..............................(10.31)

In these equations the only not known terms are (Ste + spill) and Ste. For example, in equation (10.8) these terms are ($S_{1,2}$ + spill) and $S_{1,2}$. If the reservoir is not full, the spill is 0 and the term (Ste + spill) = Ste. On the other hand, if the reservoir is full, it is expected that the spill might not be 0. The releases derived from the regression models, and used in simulation, represent the total releases from reservoirs including releases for hydropower and irrigation as well as spill. The spill included in the term (Ste + spill) is an additional spill resulting from the application of the regression models and is expected to be very small. Therefore, it can be assumed that (Ste + spill) ≈ Ste. Then the simulation equation is solved iteratively to find the term (Ste + spill). If the value obtained is less or equal to the reservoir storage capacity, then end of period storage is equal to the obtained value and the spill is equal to 0. If the obtained value is greater than the reservoir storage capacity, then the end of period storage would be equal to the reservoir storage capacity and the extra would be spilled. This additional release affects the power generated and hence the annual revenues. Therefore it is added to the releases found using operation rules to obtain the total releases. To minimise losses, water is released through the power house first, and then through other gates if the capacity of the power house is reached (MOI, 1968). The releases through the power house will be referred to as defined in Chapter IX as, $X_{1,1}$, $X_{1,2}$, ... , $X_{1,12}$ for Roseries and $X_{2,1}$, $X_{2,2}$, ... , $X_{2,12}$ for Sennar. The releases through other gates will be referred to as $Y_{1,1}$, $Y_{1,2}$, ... , $Y_{1,12}$ for Roseries and $Y_{2,1}$, $Y_{2,2}$,

......., $Y_{2,12}$ for Sennar. The end of period storage is then taken as the storage at the beginning of the next period and a similar process is repeated. The simulation steps used to produce Table (10.9), for example, can be summarised as follows:

1) The storage at the beginning of the first month is known. $S_{1,1} = 88.3$.

2) Column (2) shows the known average inflows, $q_1, q_2, ..., q_{12}$.

3) Knowing the inflow and initial storage, the outflow, $(X_{1,1} + Y_{1,1})$, in the first month is found using the operation rules, [Column 3]

4) q_1 and $(X_{1,1} + Y_{1,1})$ are substituted in equation (10.8) and the equation is solved iteratively to obtain the term ($S_{1,2}$ + spill), [Column 11].

5) If ($S_{1,2}$ + spill) is less than or equal to the storage capacity of the reservoir, 2175 million m^3, then the spill, shown in [column 4], is equal to 0 and the storage at the end of the month is equal to the value resulting from the solution of equation (10.8). The end of period storage should not be less than the minimum storage required for flow diversion i.e. 88.3 and 113 million m^3 for Roseries and Sennar respectively (MOI, 1968).

6) If ($S_{1,2}$ + spill) is greater than the storage capacity of the reservoir, the end of period storage, $S_{1,2}$ shown in [Column 10] is equal to the maximum capacity of the reservoir and the difference between the two values, [Column 11 - Column 10] is equal to the spill, [Column 4]. End of month storage for other months is found in a similar way except for the months of June, July and August. The end of period storage for these months is kept at a minimum and constant level to pass sediment (MOI, 1968). For Roseries, the storage at the minimum level is 88.3 million m^3 while it is 113 million m^3 at Sennar. During these months, the spill is calculated according to the following rearranged mass balance equation:

Spill = Inflow - Outflow - change in storage - losses

7) Since the initial and end of period storage are known, equation (9.17) is used to calculate losses [Column 8].

8) Column (9) shows the change in storage and is simply equal to the difference between the end of period storage and initial storage.

9) The release in column (3) is added to the spill in column (4) to obtain the total release, column (5).

10) The total release in column (5) is then divided between the release through the power house, column (6), and release through other gates, column(7). If the total release in column (5) is less than or equal to the maximum capacity of the power house, 2014 million m^3, then the release through the power house is equal to the total release and the release through other gates is equal to zero. Otherwise, the discharge through the power house will be equal to the maximum capacity of the power house, i.e. 2014 million m^3 and the excess, [Column 5 - 2014] will be released through other gates [Column 7].

11) The end of period storage of the first month is taken as the initial storage for the second month and steps 2 to 10 are repeated to produce the second row of Table (10.9).

The simulation is done first for the upstream reservoir, Roseries, using the average inflow to test the performance of the complete linear model, M1, and the complete non-linear model M2. Simulation results are shown in Tables (10.9) and (10.10).
The releases from the upstream reservoir, Roseries, using the linear and non-linear models are used as inputs to the downstream reservoir, Sennar. Then the simulation for Sennar has been carried out for the following combinations:

1) Using inflow resulting from applying full linear model, M1, to Roseries, linear model M1 is used in simulating the flow in Sennar. Table (10.11) shows the results.
2) Using inflow resulting from applying full non-linear model, M2, to Roseries, linear model M1 is used in simulating the flow in Sennar. Table (10.12) shows the results.
3) 1 and 2 above have been repeated with M1 model for Sennar, replaced by M2 model.
The last combination didn't work well, due to the fact that releases from Sennar, derived from M2 model, in some months, are highly affected by the shift of the simulated storages from the constant optimum storages used in deriving these rules.

Table (10.9) Simulation results for Roseries using full linear model M1

Month (1)	Inflow (2)	release (3)	spill (4)	total release (5)	X1,i release hydropower (6)	Y1,i other releases (7)	losses (8)	change in storage (9)	end of period storage Ste (10)	Ste+spill (11)
							initial storage = 88.3			
sept.	11046	8942.407	0.0	8942.407	2014	6928.41	16.896	2086.7	2175	2175
oct.	5787	5935.737	0.0	5935.737	2014	3921.737	30.377	-179.1	1995.9	1995.9
nov.	2212	1897.576	70.43	1968.01	1968.01	0.0	64.264	179.1	2175	2245.43
dec.	1277	1381.797	0.0	1381.797	1381.8	0.0	63.918	-168.7	2006.3	2006.3
jan.	652	1613.413	0.0	1613.413	1613.413	0.0	53.414	-1014.8	991.5	991.5
feb.	411	855.461	0.0	855.461	855.46	0.0	42.276	-486.74	504.76	504.76
march	322	558.865	0.0	558.865	558.865	0.0	37.173	-274.03	230.73	230.73
april	269	308.515	0.0	308.515	308.515	0.0	31.259	-70.77	159.96	159.96
may	525	341.805	0.0	341.805	341.805	0.0	29.861	153.34	313.3	313.3
june	1641	1849.592	0.07	1849.66	1849.66	0.0	16.336	-225	88.3	88.37
july	7172	7163.954	0.0	7163.954	2014	5149.954	8.059	0	88.3	88.3
august	14545	14539.23	0.0	14539.23	2014	12525.23	5.8	0	88.3	88.3

All releases and storage are in million m^3

Table (10.10) Simulation results for Roseries using full nonlinear model M2

Month	Inflow	release	spill	total release	X1,i release for hydropower	Y1,i other release	losses	change in storage	end of period storage Ste	(Ste+spill)
							initial storage = 88.3			
sept.	11046	8942.40	0.0	8942.40	2014	6928.4	16.896	2086.7	2175	2175
oct.	5787	5573.303	181.8	5755.103	2014	3741.1	31.128	0.0	2175	2356.8
nov.	2212	2146.417	0.0	2146.417	2014	132.42	65.875	-0.29	2174.71	2174.71
dec.	1277	1347.62	0.0	1347.62	1347.62	0.0	64.221	-134.61	2040.1	2040.1
jan.	652	1633.468	0.0	1633.468	1633.47	0.0	53.86	-1035.3	1004.8	1004.8
feb.	411	827.522	0.0	827.522	827.52	0.0	42.899	-459.4	545.4	545.4
march	322	611.134	0.0	611.134	611.13	0.0	37.607	-326.7	218.7	218.7
april	269	299.09	0.0	299.09	299.09	0.0	31.01	-61.1	157.6	157.6
may	525	224.02	0.0	224.02	224.02	0.0	31.558	269.9	427.5	427.5
june	1641	1963.188	-0.3	1962.888	1962.89	0.0	17.319	-339.2	88.3	88.
july	7172	7163.94	0.0	7163.94	2014	5149.94	8.059	0	88.3	88.3
august	14545	14539.2	0.0	14539.2	2014	12525.2	5.8	0	88.3	88.3

All releases and storage are in million m^3

Table (10.11) Simulation results for Sennar using full linear model M1 and inputs from Roseries using M2 model

Month	Inflow	release	spill	total release	X1,i hydropower release	Y1,i other releases	irrigation requirement ru(i)	losses	change in storage	end of period storage - Ste	Ste+spill
								initial storage = 113			
sept.	8942.40	7412.304	29.8	7442.08	330	7112.08	1225.63	24.729	249.5	362.5	392.28
october	5755.103	4341.328	109.6	4450.96	330	4120.96	1267.6	35.962	0	362.5	472.13
nov.	2146.417	789.943	115.5	905.42	330	575.42	1197.5	42.985	0	362.5	477.98
dec.	1347.62	172.135	0.0	172.13	172.13	0	1305.8	36.772	-167.09	195.41	195.41
jan.	1633.468	152.673	0.0	152.67	152.67	0	1314.2	37.761	128.83	324.24	324.24
feb.	827.522	130.822	0.0	130.82	130.82	0	946.2	40.49	-211.24	113	37.08
march	611.134	234.755	0.0	234.76	234.76	0	123.74	46.55	206.09	319.09	319.09
april	299.09	283.512	0.0	283.51	283.51	0	74.3	54.936	-113.66	205.43	205.43
may	224.02	147.135	0.0	147.14	147.14	0	74.3	44.978	-42.39	163.04	163.04
june	1962.888	1851.03	27.5	1878.49	330	1548.49	105.41	29.033	-50.04	113	
july	7163.94	6890.63	13.5	6904.14	330	6574.14	245.18	14.625	0	113	
august	14539.2	13755.51	10.1	13765.62	330	13435.62	762.82	10.761	0	113	

All releases and storage are in million m^3

Table (10.12) Simulation results for Sennar using full linear model M1 and inputs from Roseries using M1 model

Month	Inflow	release	spill	total release	X1,i hydropower release	Y1,i other releases	irrigation requirement ru(i)	losses	change in storage	end of period storage - Ste	Ste + spill
								initial storage	= 113		
sept.	8942.407	7412.307	29.8	7442.09	330	7112.09	1225.63	24.729	249.5	362.5	392.3
october	5935.737	4454.497	177.0	4631.53	330	4301.53	1267.6	35.962	0	362.5	539.53
nov.	1968.006	614.773	112.2	727.01	330	397.01	1197.5	42.985	0	362.5	474.74
dec.	1381.797	197.4	0.0	197.4	197.4	0.0	1305.8	36.911	-158.31	204.19	204.19
jan.	1613.413	141.404	0.0	141.4	141.4	0.0	1314.2	37.931	119.88	324.07	324.07
feb.	855.461	158.761	0.0	158.76	158.76	0.0	946.2	40.483	-211.07	113	36.8
march	558.865	203.620	0.0	203.62	203.62	0.0	123.74	45.755	185.75	298.75	298.75
april	308.515	266.005	0.0	266.01	266.01	0.0	74.3	54.546	-86.34	212.41	212.41
may	341.805	243.706	0.0	243.71	243.71	0.0	74.3	46.546	-22.75	189.66	189.66
june	1849.662	1761.246	29.3	1790.51	330	1460.51	105.41	30.402	-76.66	113	
july	7163.954	6890.644	13.5	6904.15	330	6574.15	245.18	14.625	0	113	
august	14539.23	13755.54	10.1	13765.65	330	13435.65	762.82	10.761	0	113	

All releases and storage are in million m^3

10.7 PERFORMANCE OF REGRESSION MODELS USING SIMULATION

The monthly storages and releases obtained from simulation, are substituted in the objective function, equation (9.50), to calculate the annual hydropower revenue and consequently assess the usefulness of the developed operation rules (Table 10.13). From these results, it is clear that the performance of both the linear model M1 and the non-linear model M2 are good when applied to the relatively large upstream reservoir, Roseries. The non-linear model is slightly better than the linear as they reduce the optimum annual revenues by 0.2 % and 1.2 % respectively.

On the other hand, the non-linear model, M2, is not successful when applied on the small downstream reservoir, Sennar, while the linear model gives better results. These results are highly affected by the model used in upstream reservoir. The reduction in Sennar annual revenues when operated using M1 model are 3.8 % and 6.9 % if linear model M1 and non-linear model M2 are, respectively, used to operate the upstream reservoir. However, for the whole system the application of M1 complete model reduces the annual hydropower revenues by only 1.4 %, while applying the non-linear model M2 to the upper reservoir and the linear model M1 to the downstream reservoir reduces the hydropower revenues by only 0.8 %. This shows that for more than one reservoir, a combination of different operation rules may yield better results.

This minimal reduction in power revenue is not obtained at the expense of the irrigation requirements. Shortage in supplying the upstream requirements occurred only once in February, when it was necessary to store water in Sennar to maintain the level required to divert these requirements. To estimate this shortage the mass balance equation for Sennar is used after being rearranged as follows:

Shortage = irrigation requirement - (Inflow - Outflow - change in storage - losses)

The shortage is estimated at 78.91 million m^3 when the inflow to Sennar is taken as the result of the application of the linear model M1 on Roseries and 78.75 million m^3 when the non-linear model M2 is applied. These figures represents 8 % of the total monthly requirement estimated at 946.2 million m^3.

Table (10.13) Performance of derived operation policies

Policy	Annual benefits from Roseries in million Sudanese dinnars	Annual benefits from Sennar in million Sudanese dinnars	Annual benefits from the whole system in million Sudanese dinnars
Optimum policy (average)	14181	1403	15584
Complete linear model M1 used to operate both reservoirs (average flow)	14013.8 (98.8%)	1349.9 (96.2%)	15363.7 (98.6%)
Complete non-linear model M2 used in Roseries & linear model M1 used in Sennar-average flow	14153.3 (99.8%)	1306.0 (93.1%)	15459.3 (99.2%)

Figures in parentheses give the percentage of the benefit obtained from the policy applied to the optimum policy

10.8 PRACTICAL USE OF THE FITTED MODELS

From the results obtained above it is justified that, the complete nonlinear model (equation 10.2) can be used in the operation of Roseries reservoir, while the Linear model (equation 10.1) can be used in the operation of Sennar reservoir. Knowing the inflow and the storage at the beginning of each month, the optimum release can be obtained. Coefficients of equations (10.2) and (10.1) are substituted from Table (10.4) and Table (10.5) respectively to obtain the monthly operation rules for the two reservoirs. These rules are easier in use if they are presented graphically. Figures (10.1) to (10.9) show the monthly curves for Roseries. These curves have been produced using the complete non-linear model, M2 (equation 10.2). Also the equations of the curves are shown in these figures. For Sennar, the complete linear model, M1 (equation 10.1), is the best to produce the operation curves. In this model, the release is function of four lagging inflows. This represents a problem in drawing the curves. The complete linear model slightly improves the results obtained by the reduced linear model, which is function of the current inflow and the beginning of the month storage (Section 10.5). Therefore this reduced linear model is used in drawing the operation curves for Sennar, Figures (10.10) to (10.21). However for more accuracy, the complete linear model can be used directly.

10.9 CONCLUSIONS

The monthly river flows generated in Chapter VII are used as inputs to the optimization model developed in Chapter IX. Each time the model is solved and the results are used to derive suitable operation policy by regressing the releases on suitable independent variables. To choose these variables, simple correlation analysis is carried out. Then using Software Excel, the optimum releases are regressed on the significant variables. Using R^2 criterion, it has been found that the complete linear model M1 and the complete non-linear model M2 are better than the non-linear model M3, reduced forms of the linear model M1 and other reduced forms of the non-linear model M2. Using R^2 criterion, it has been found that the application of these regression models to non-linear optimization output give higher R^2 values than those obtained by Bhaskar and Whitlach (1980) from application of these models to the outcome of dynamic programming. The performance of the two superior models is tested using simulation. For the relatively large upstream reservoir, the performance of the two models is good. For the small downstream reservoir, the full linear model M1 performs better while M2 model didn't work well. For a system with more than one reservoir, a combination of different operation rules may yield better results. To be easy in use the models can be presented in a graphical form. Knowing the inflows and the beginning of the month storage the graphs or the equations can be used to decide the amount of releases.

Figure (10.1) Relation between releases, inflow & storages - Roseries- Septemer, Octber.

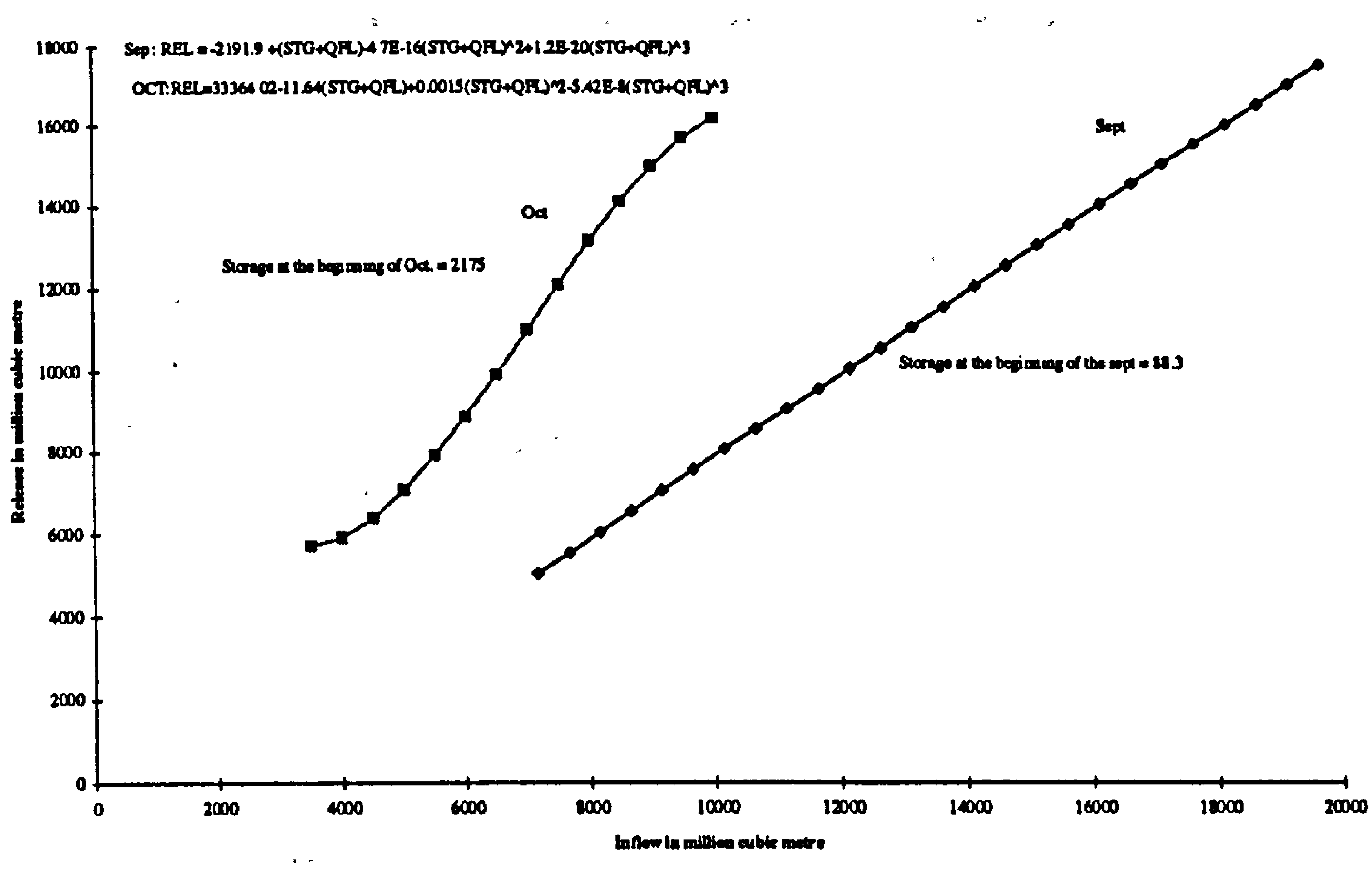


Figure (10.2) Relation between releases, inflows and storages - Roseries - Nov. & Dec.

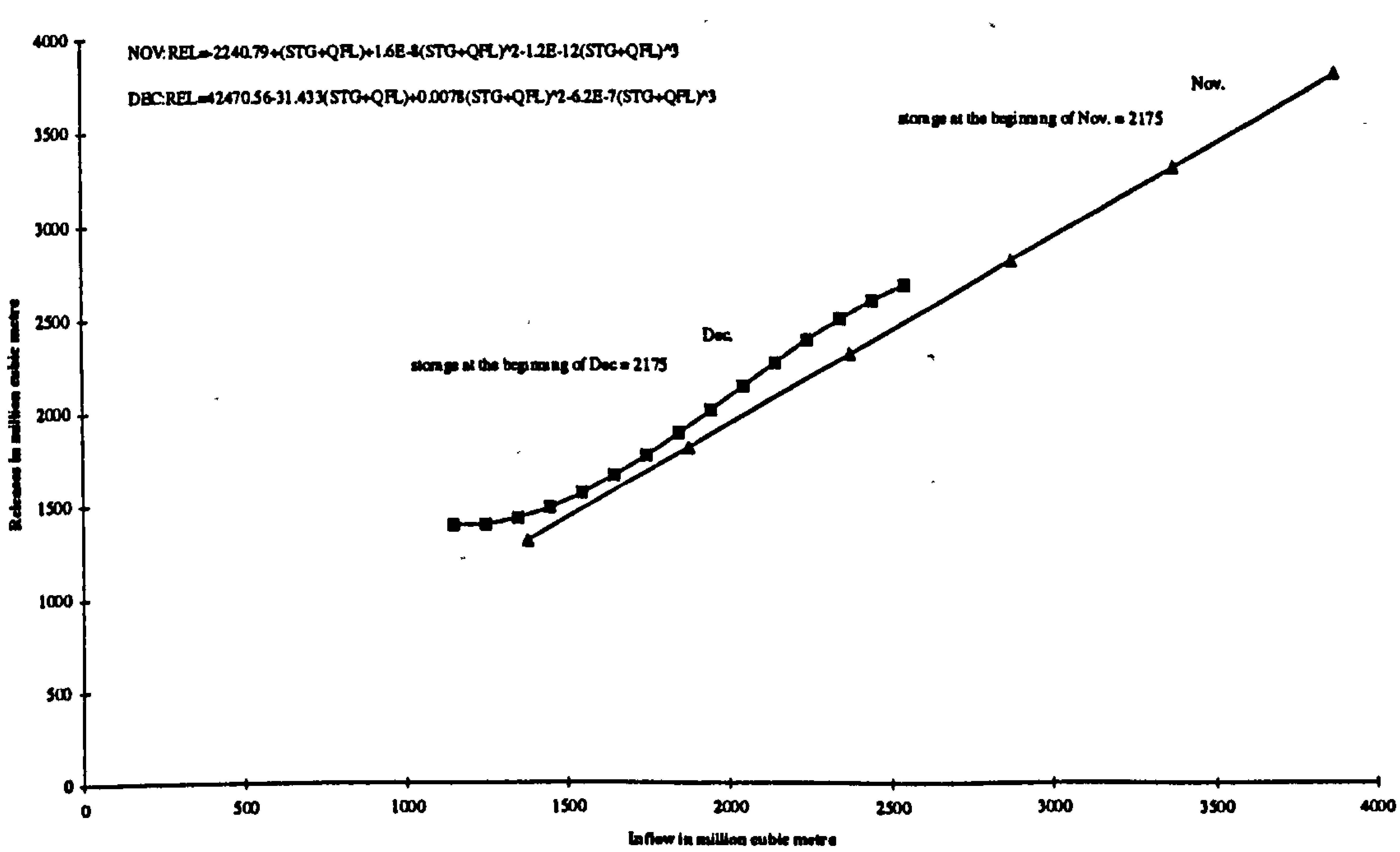


Figure (10.3) Relation between releases stotages and inflows - Roseries - January

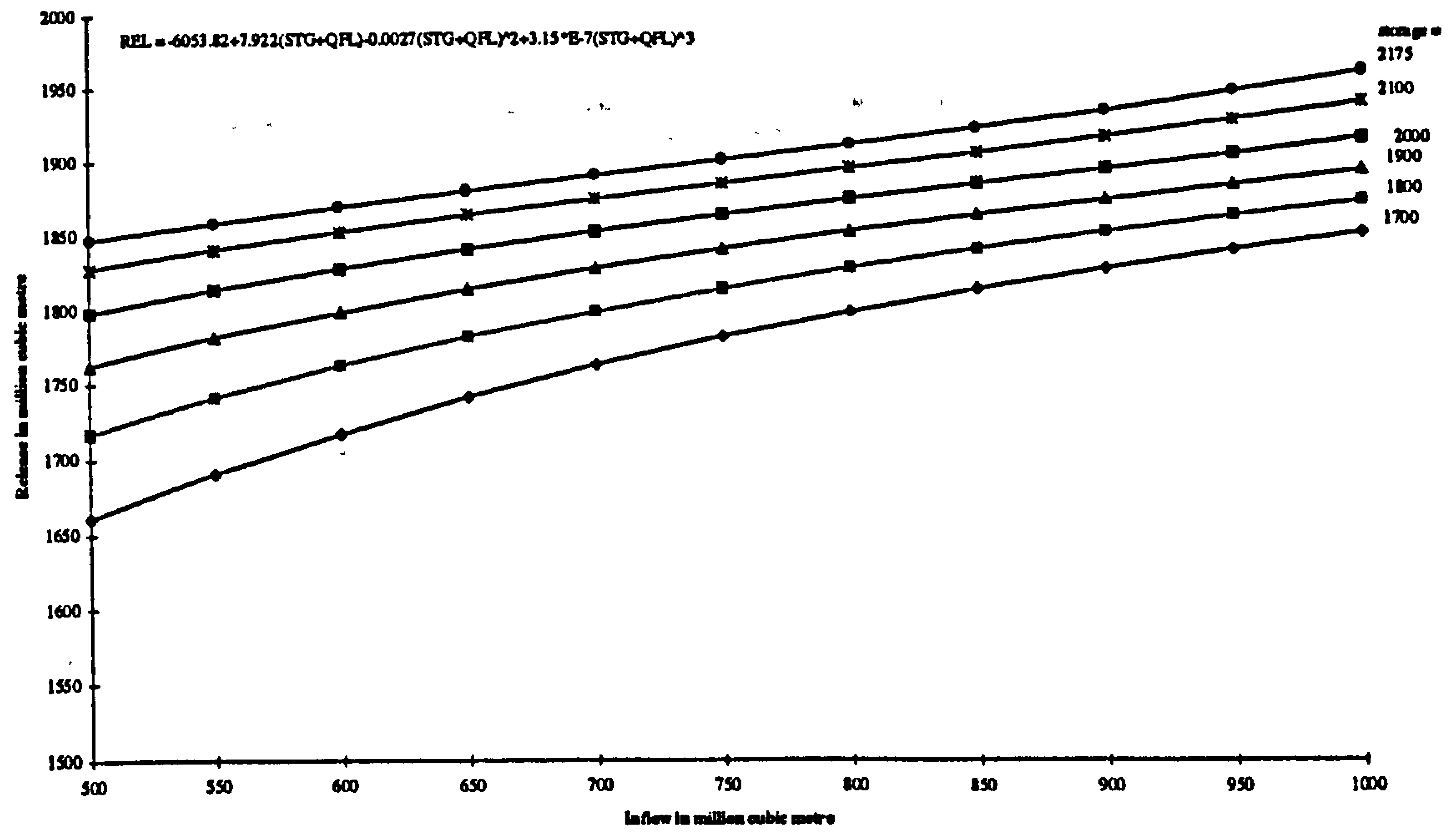


Figure (10.4) Relation between releases, inflows and storages - Roseries - February

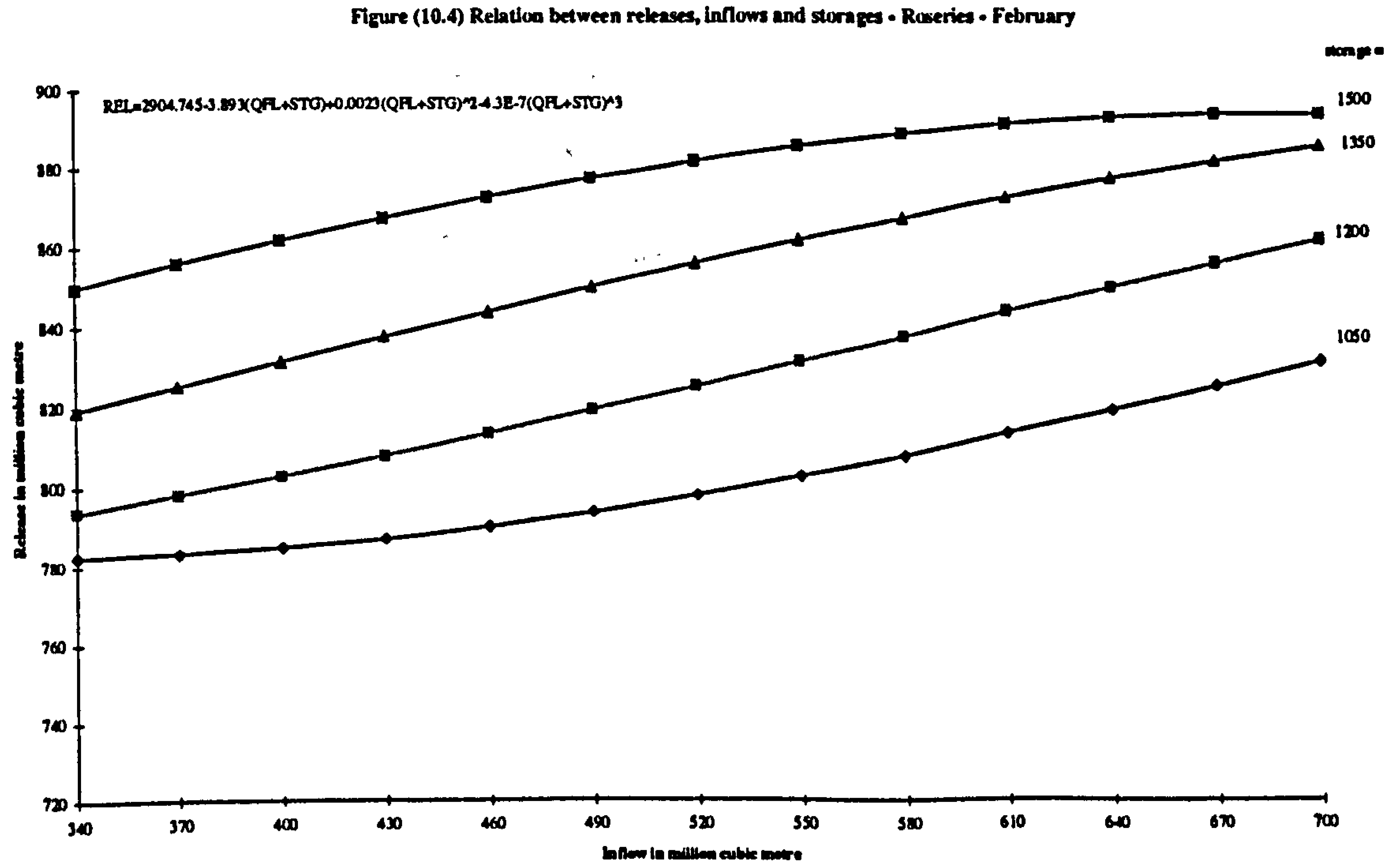


Figure (10.5) Relation between release, storage and inflow - Roseries - March

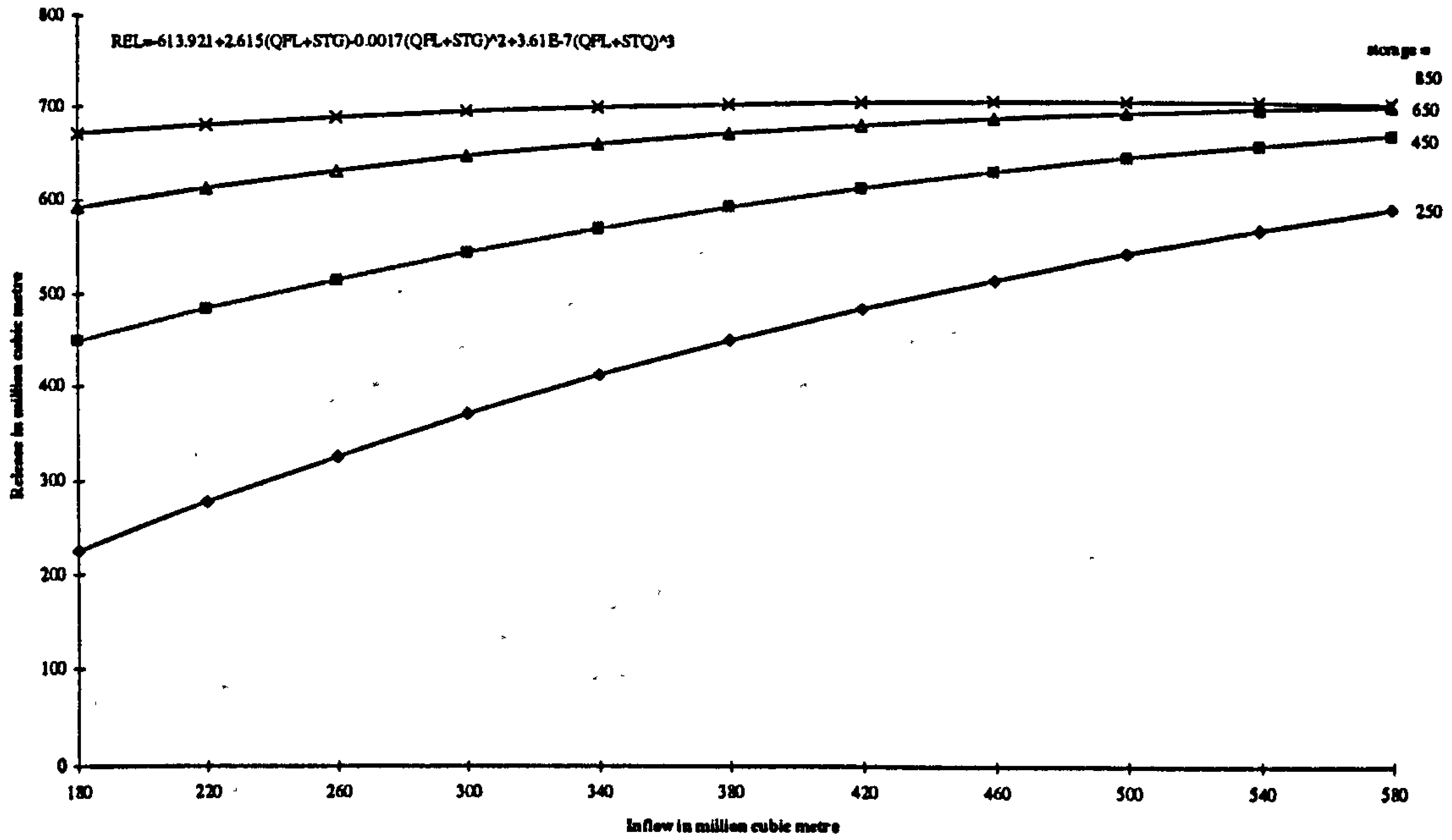


Figure (10.6) Relation between releases, storages and inflows - Roseries - April

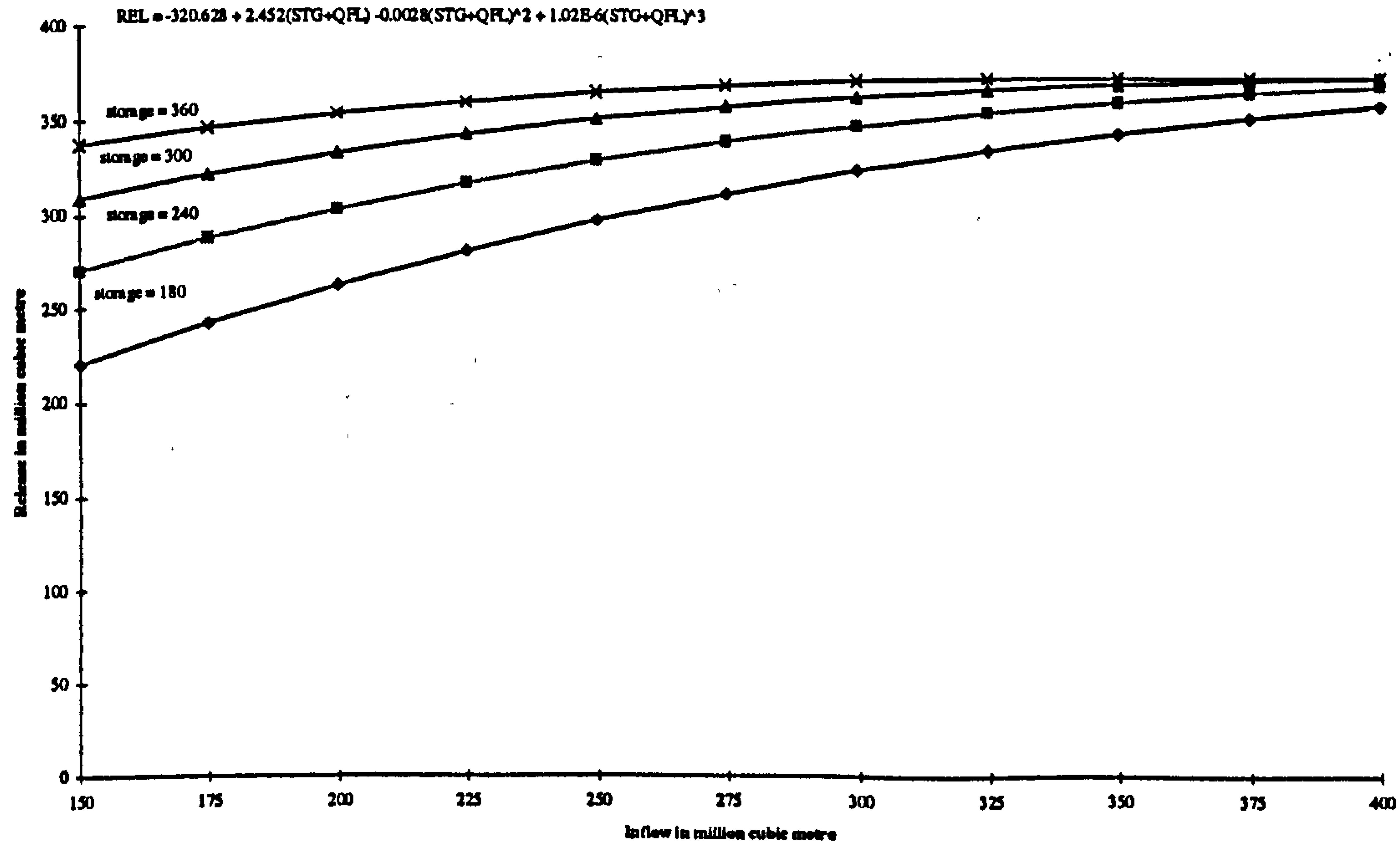


Figure (10.7) Relation between releases, inflows and storages - Roseries - May

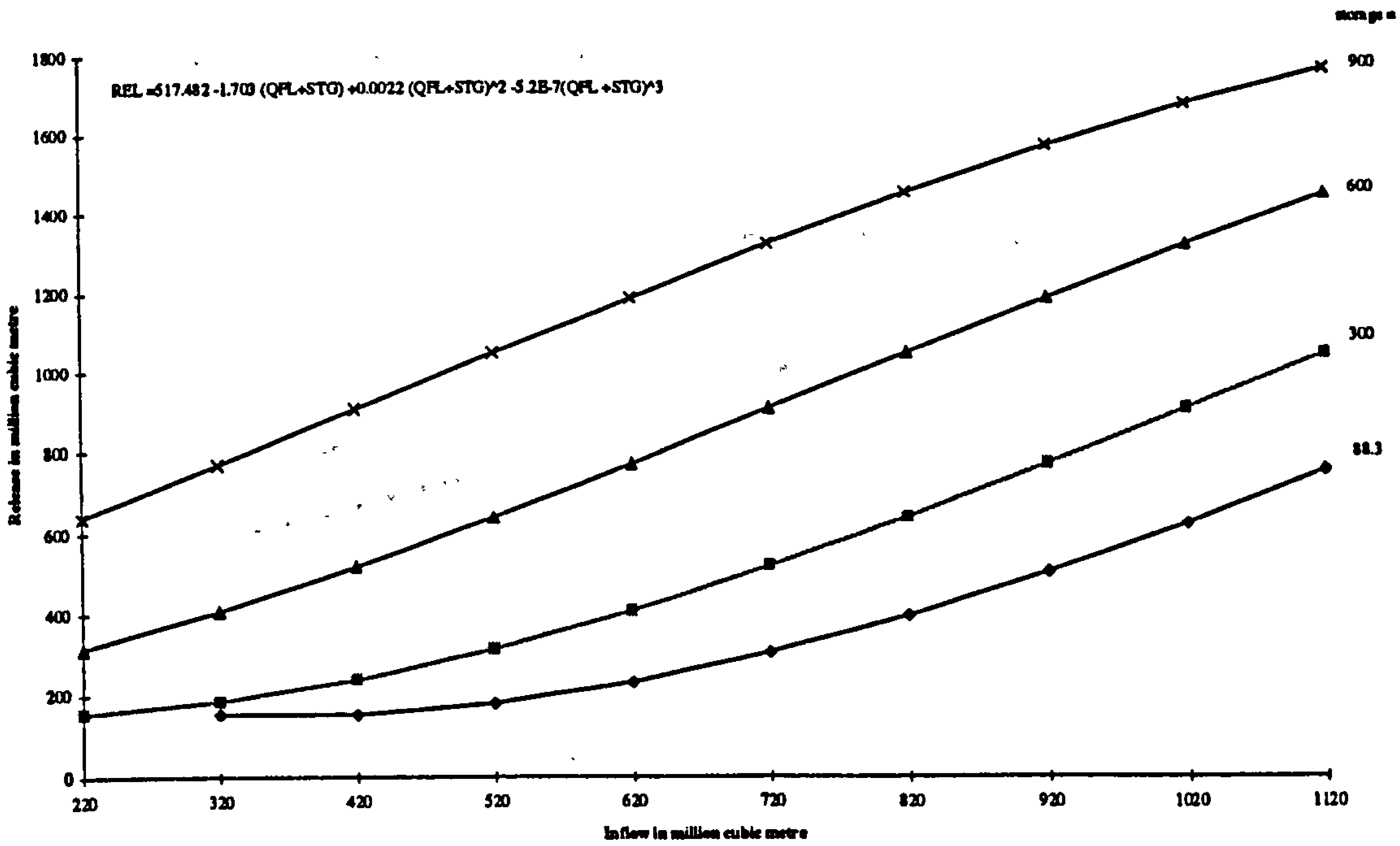


Figure (10.8) Relation between releases, inflows and storages - Roseries - June

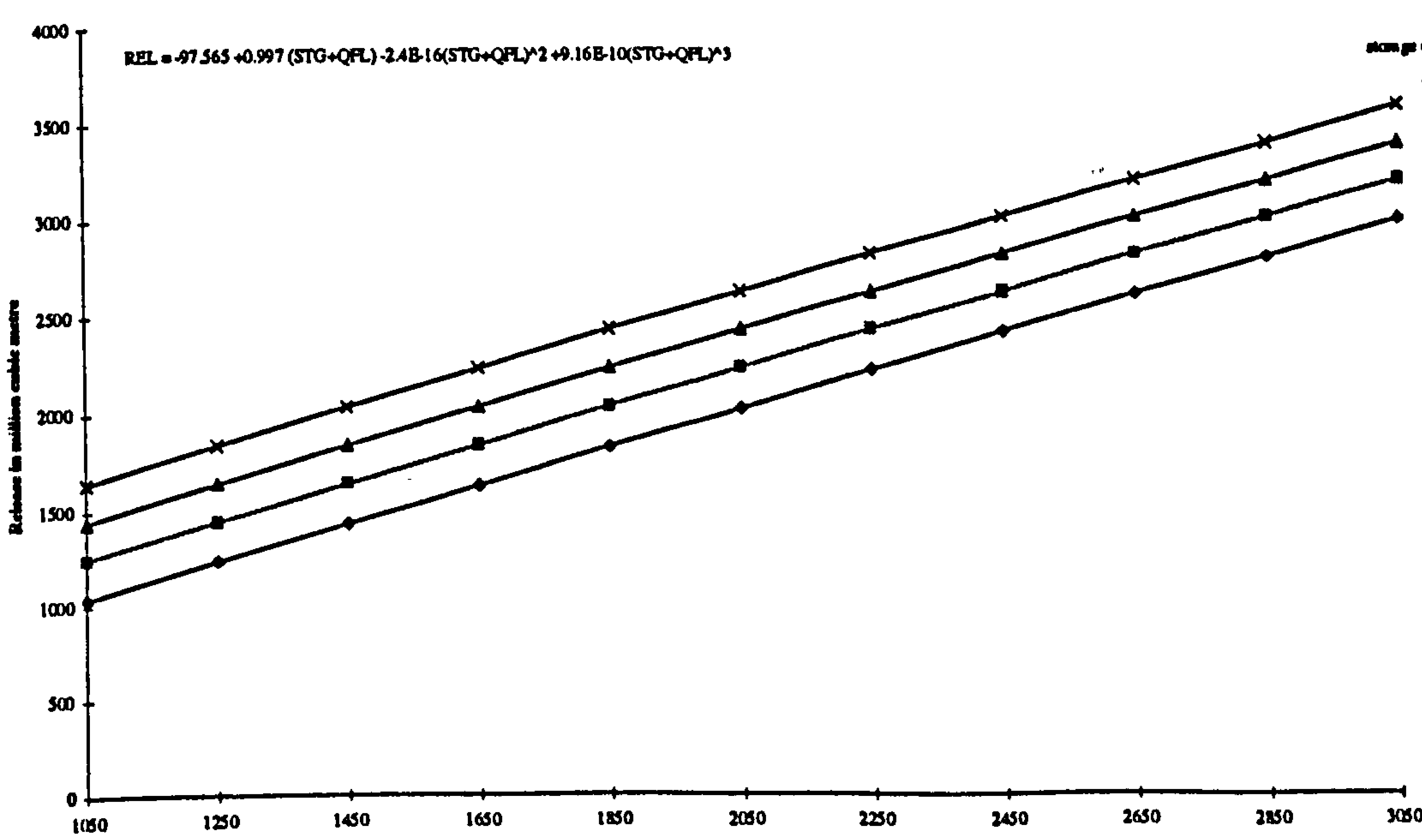


Figure (10.9) Relation between releases, inflows and storage - Roseries - July & August

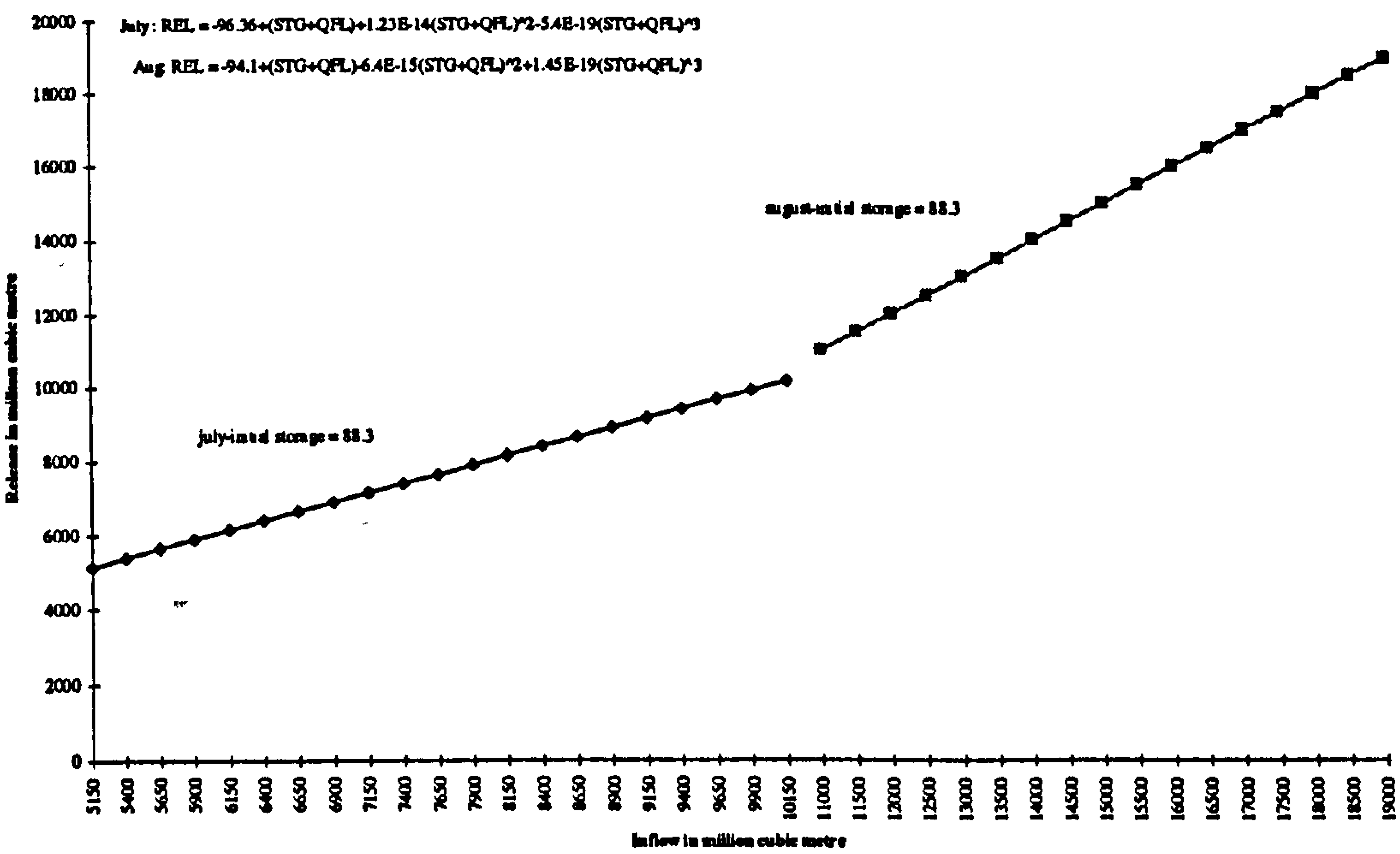


Figure (10.10) Relation between release, inflow and storage - Sennar - Sept

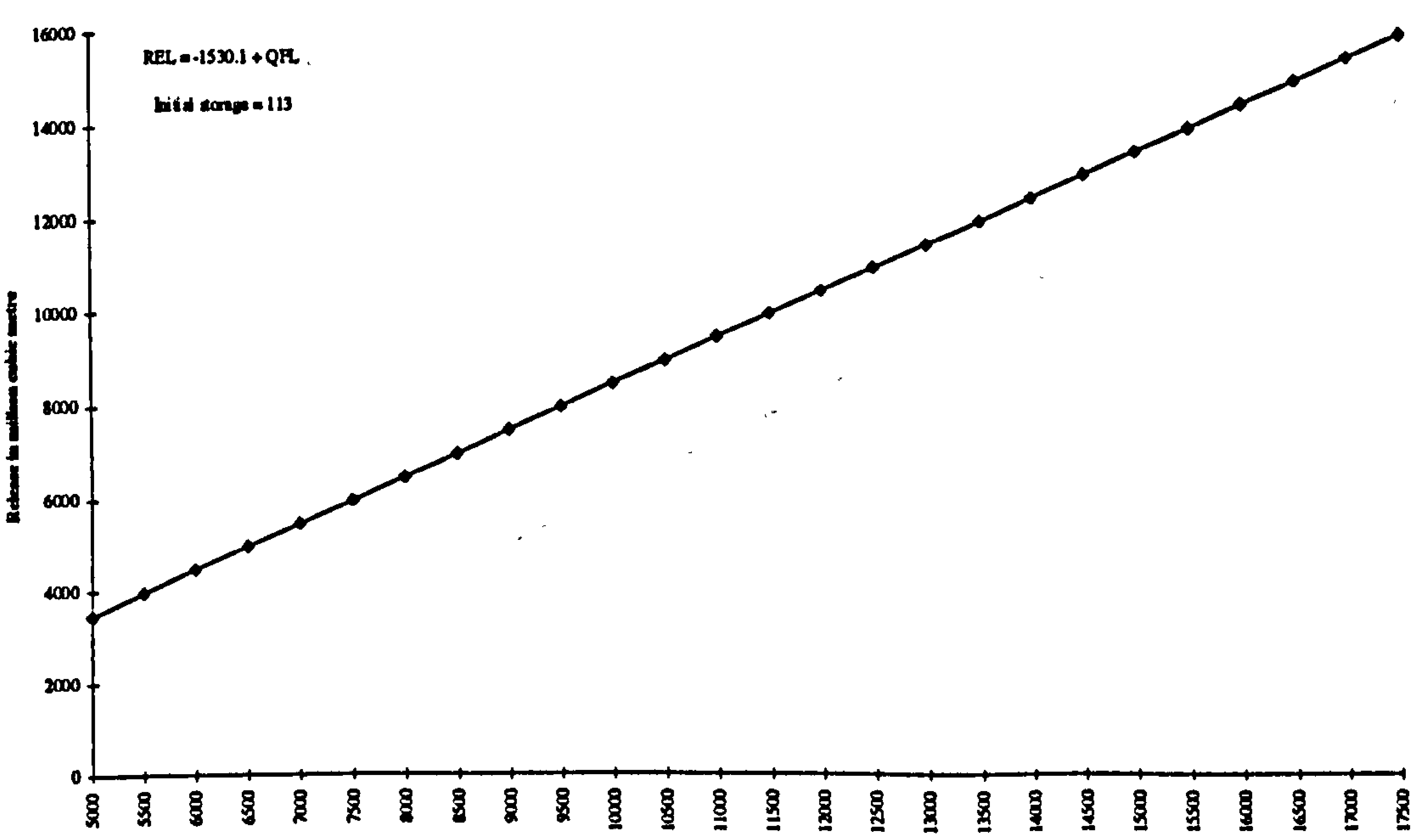


Figure (10.11) Relation between release, inflow and storage - Sennar - October

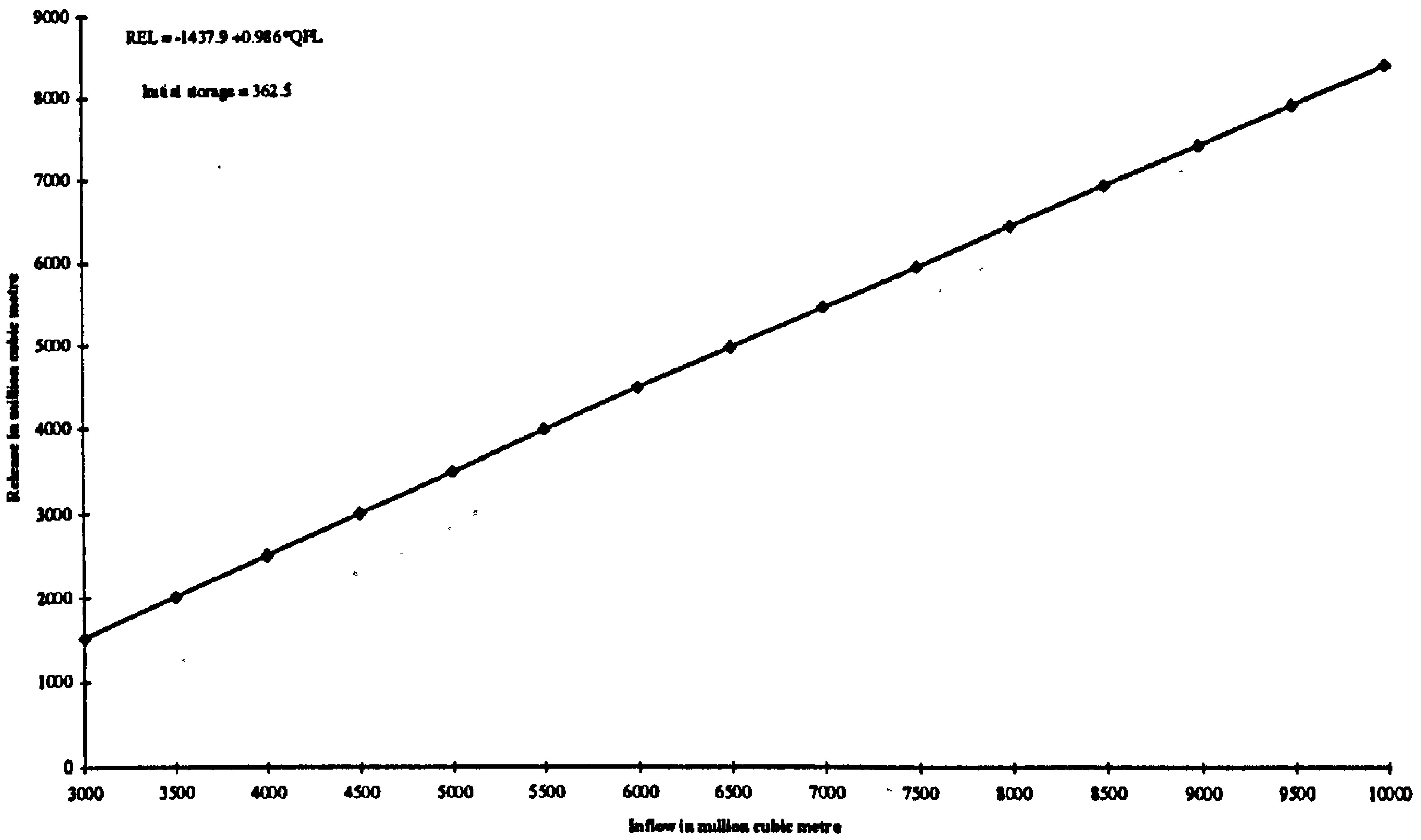


Figure (10.12) Relation between release, inflow and storage - Sennar - November

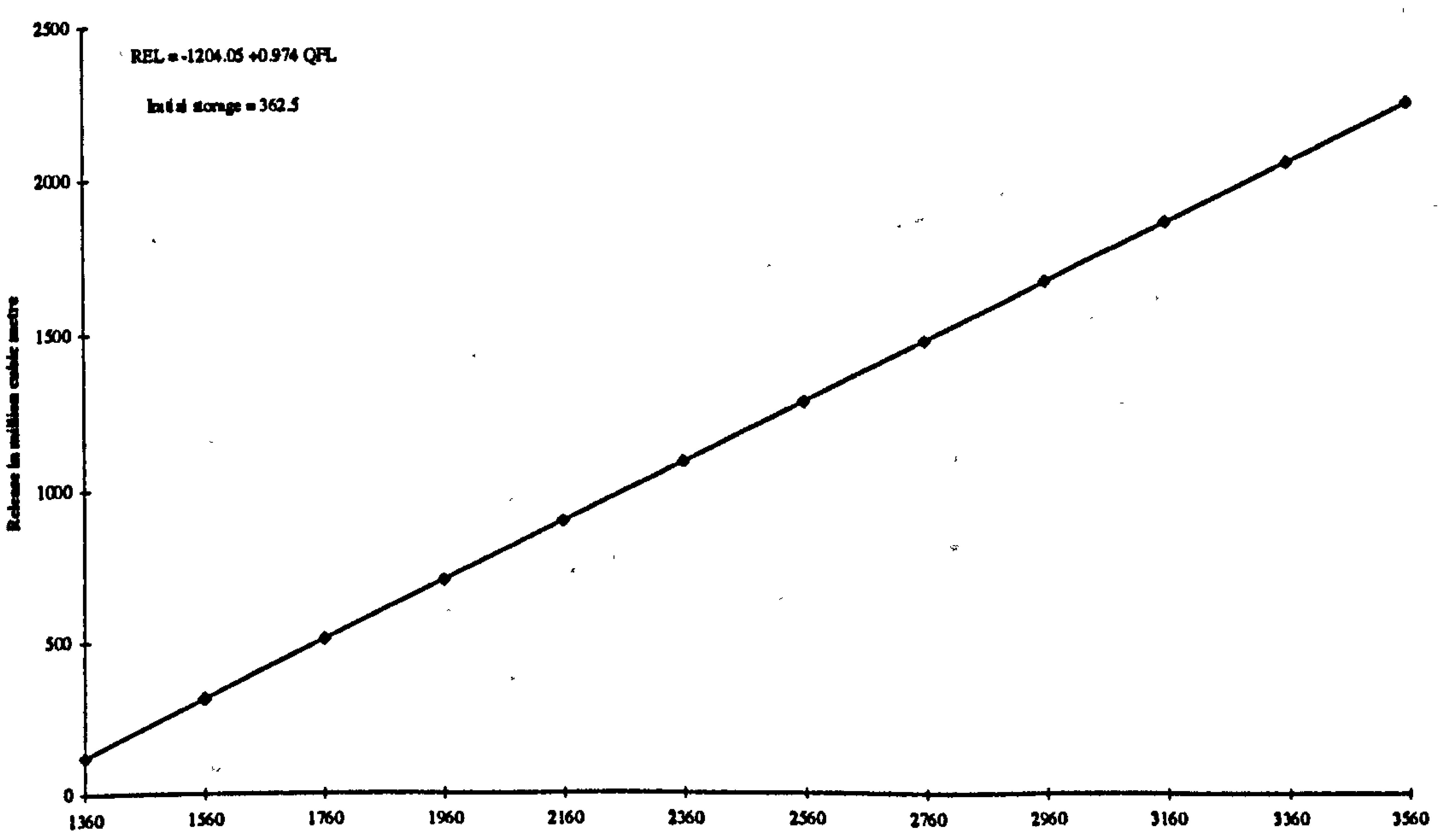


Figure (10.13) Relation between release, inflow and storage - Sennar - December

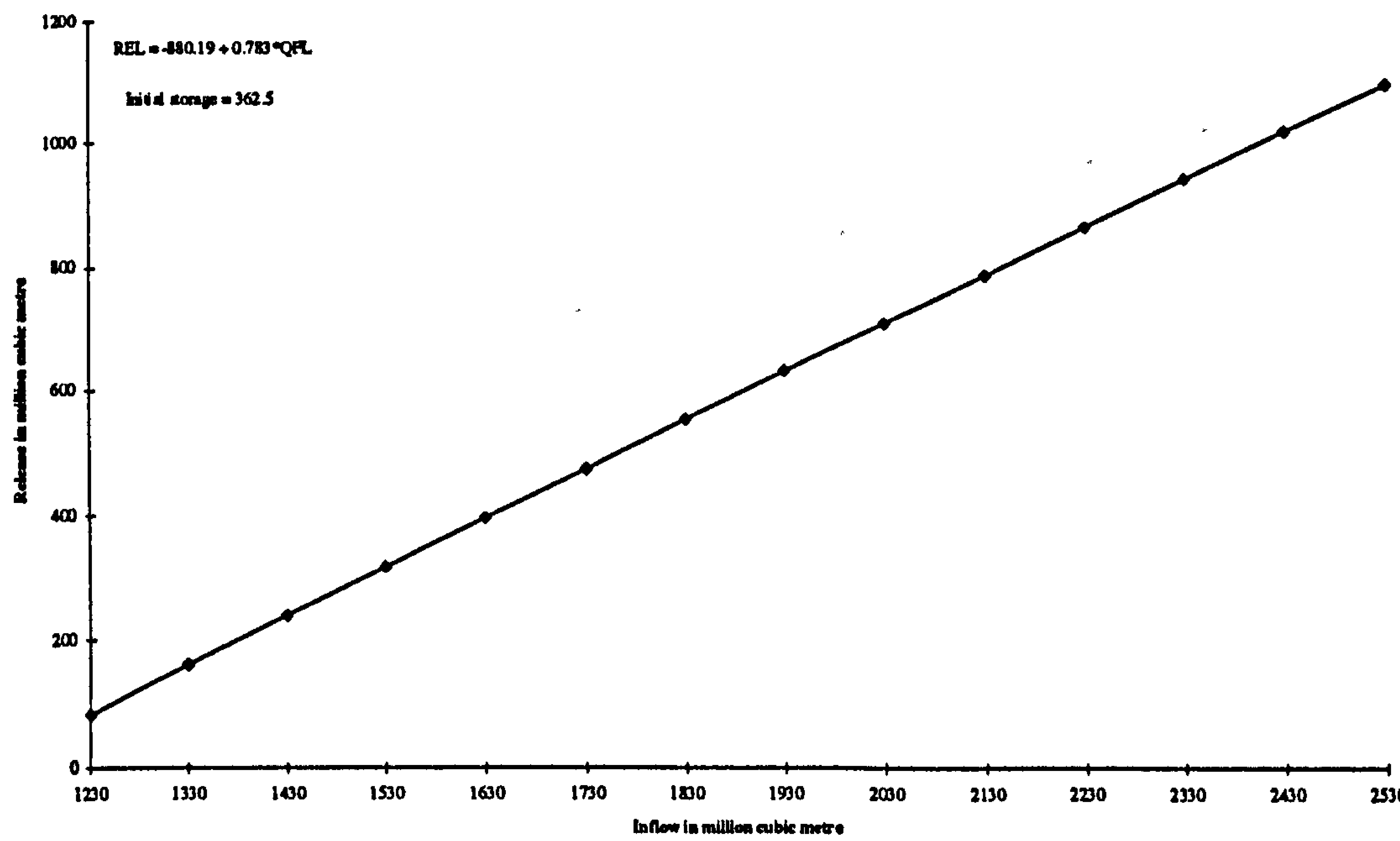


Figure (10.14) Relation between release, inflow and storage - Sennar - January

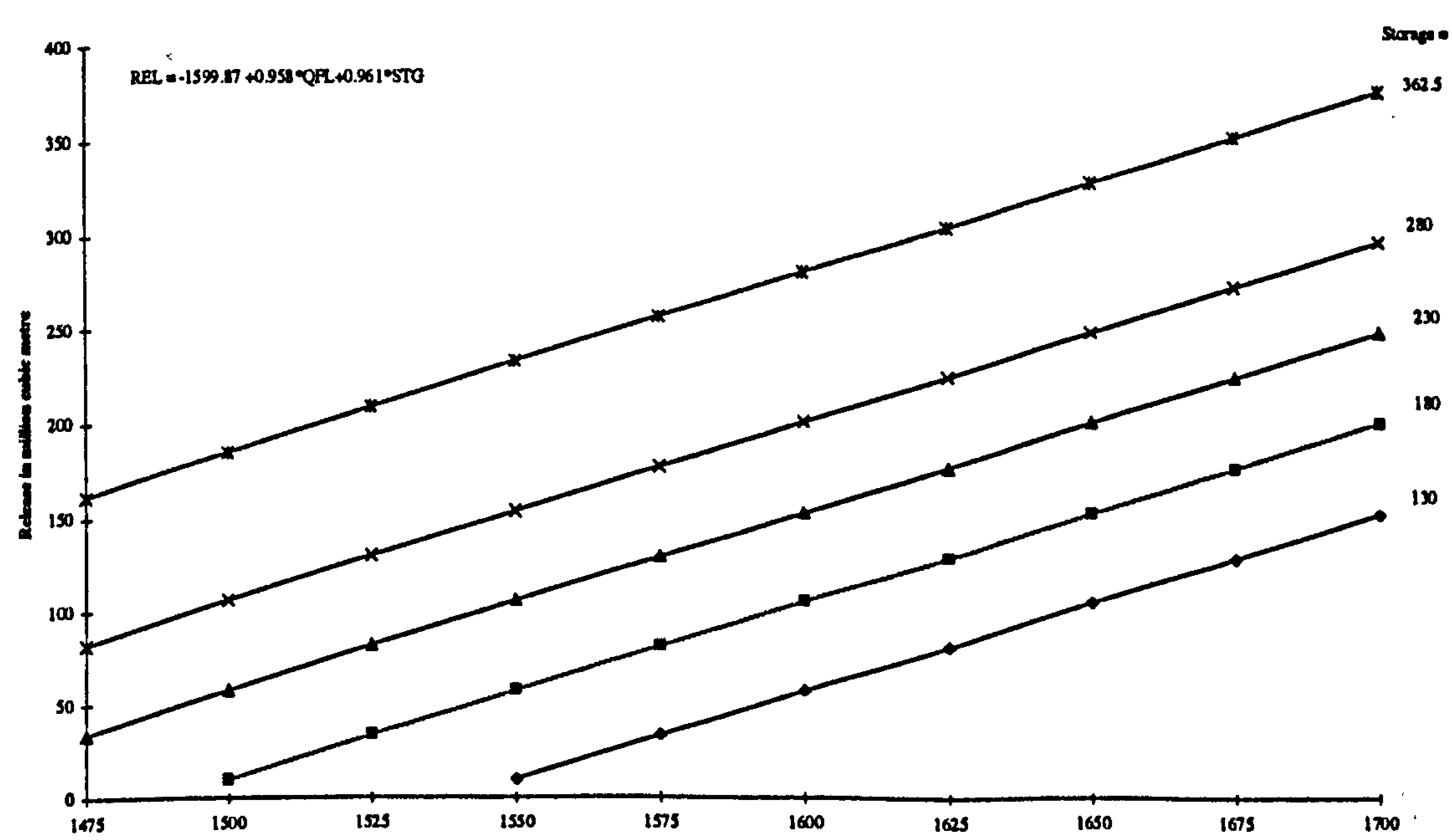


Figure (10.15) Relation between release, inflow and storage - Sennar - February

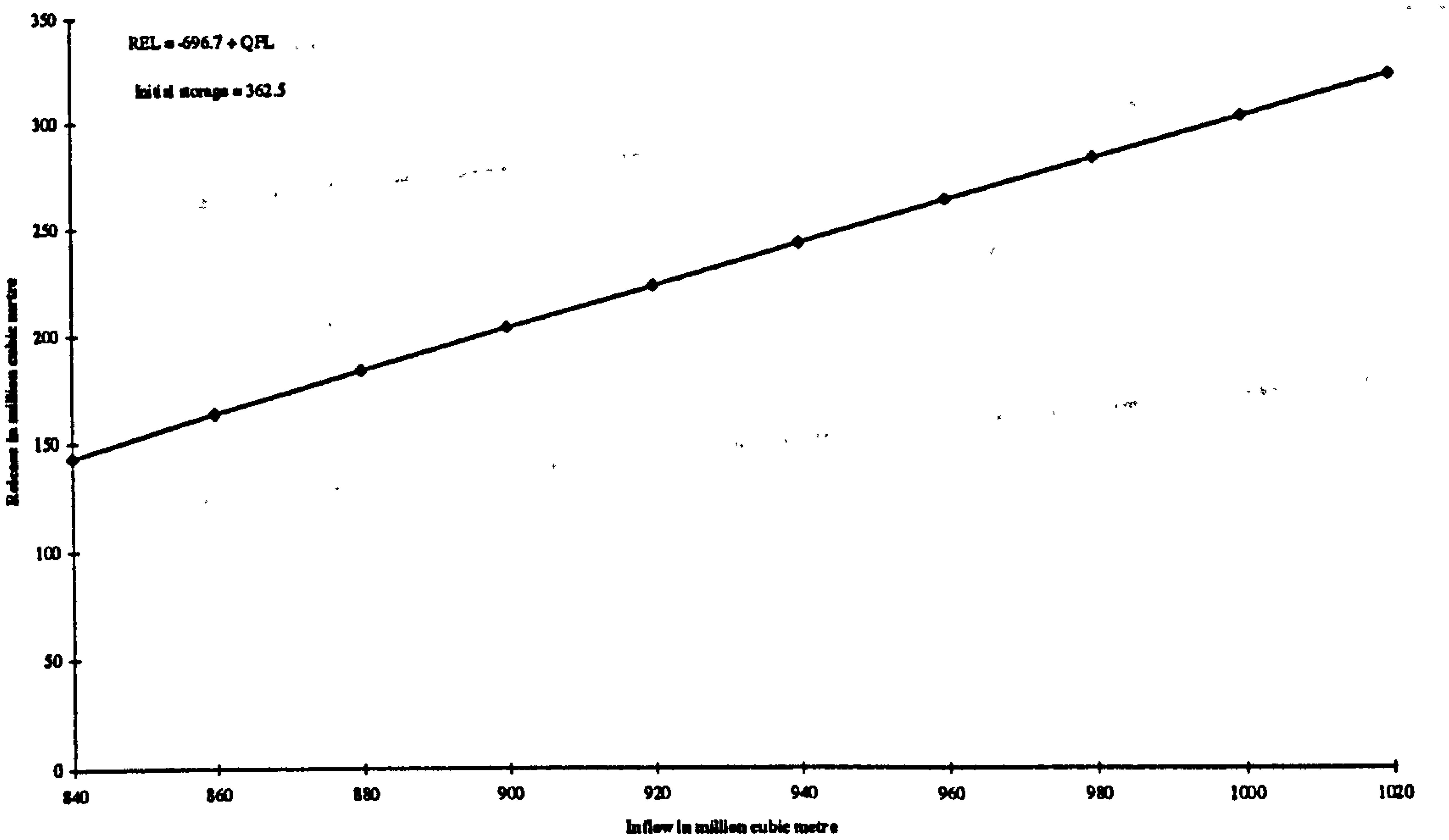


Figure (10.16) Relation between release, inflow and storage - Sennar - March

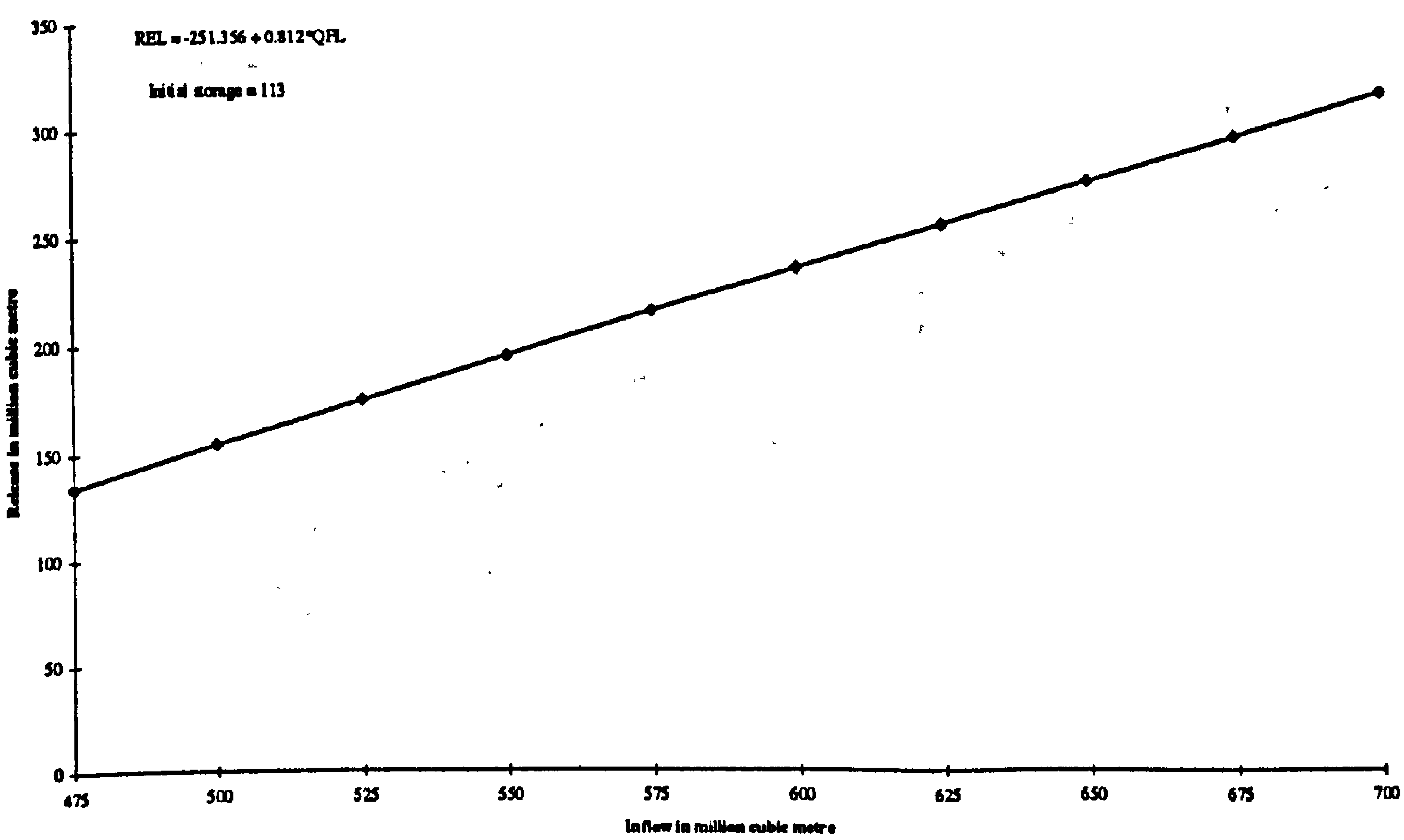


Figure (10.17) Relation between release, inflow and storage - Sennar - April

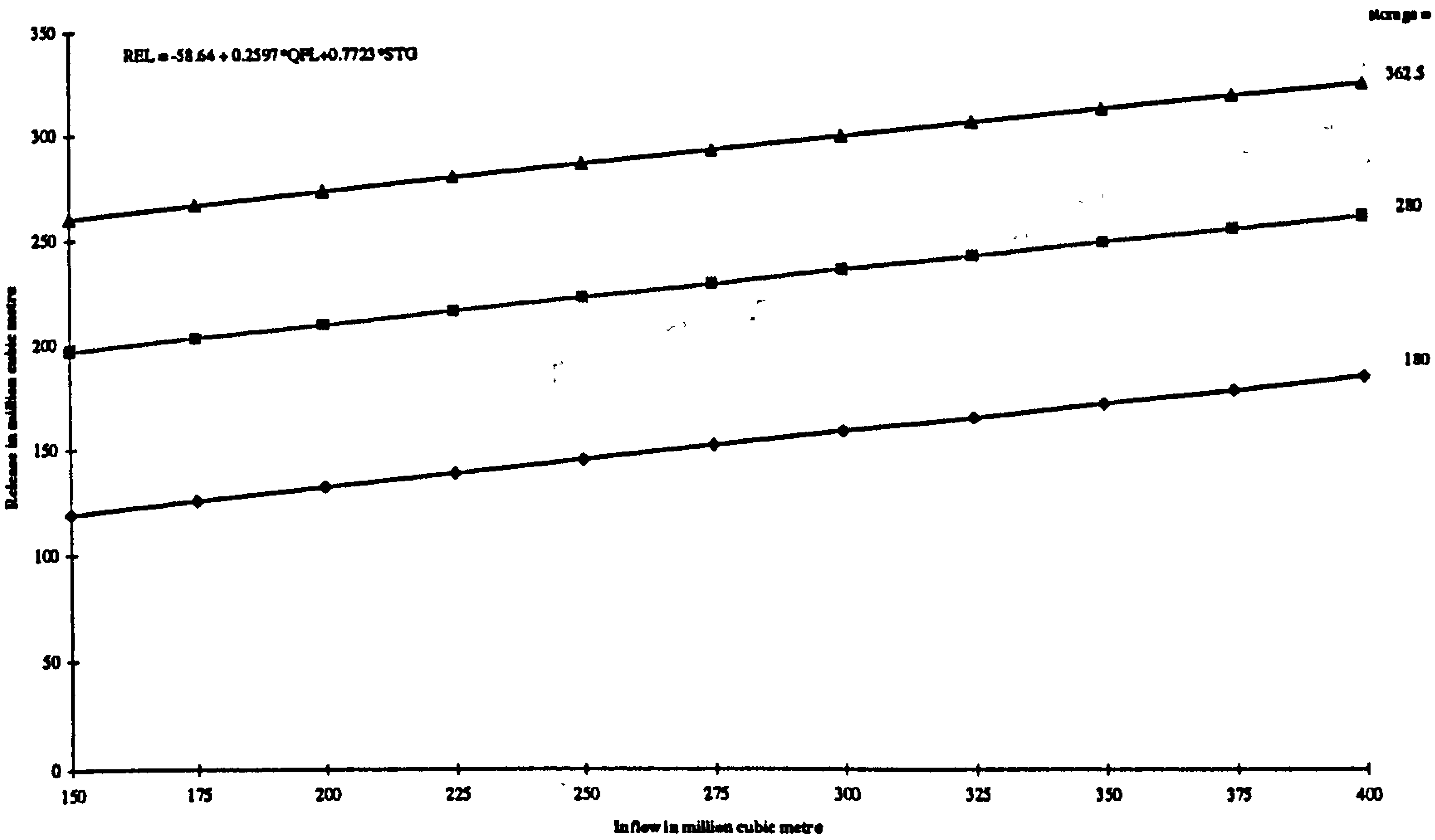


Figure (10.18) Relation between releases, inflows and storage - Sennar - May

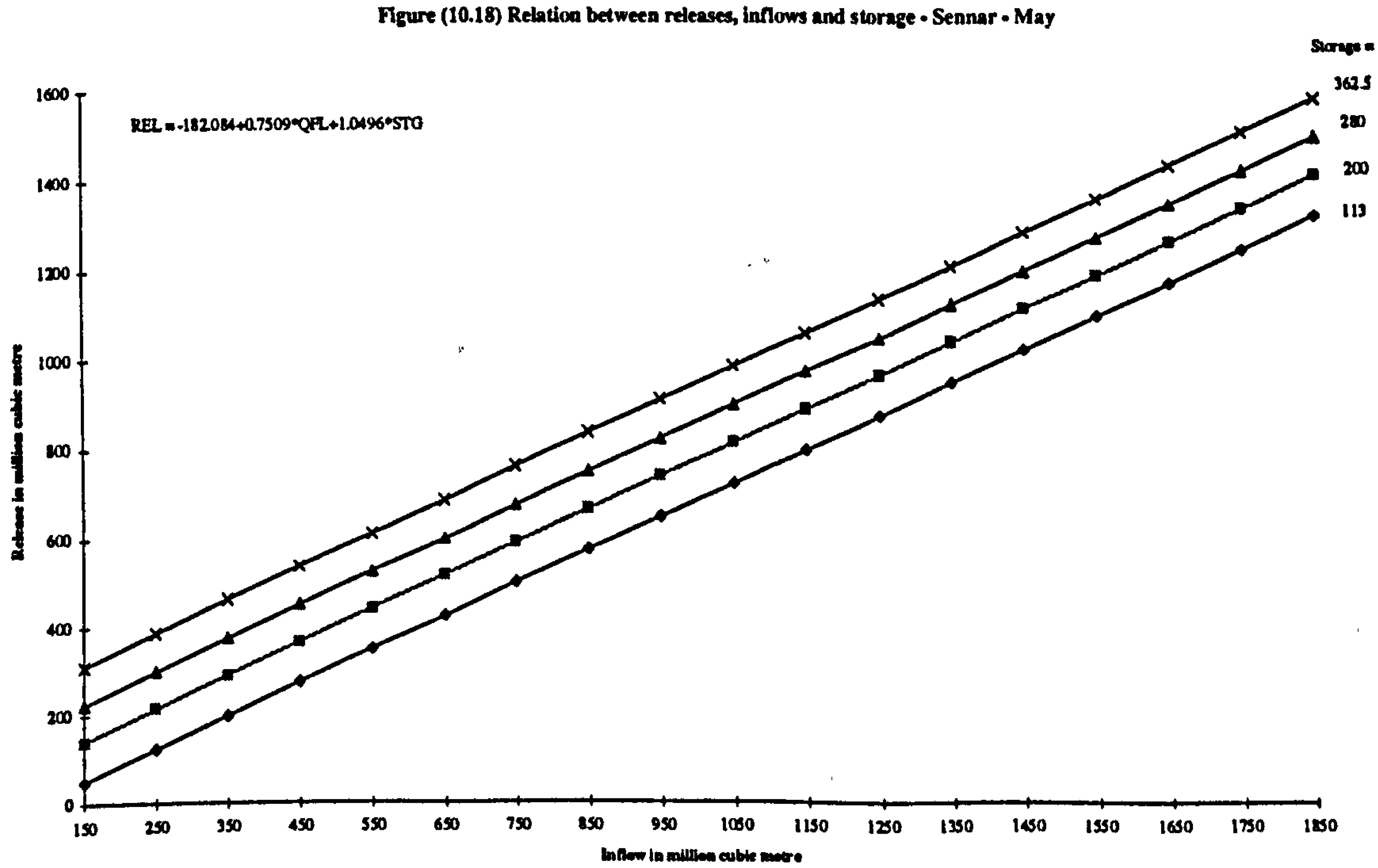


Figure (10.19) Relation between release, inflow and storage - Sennar - June

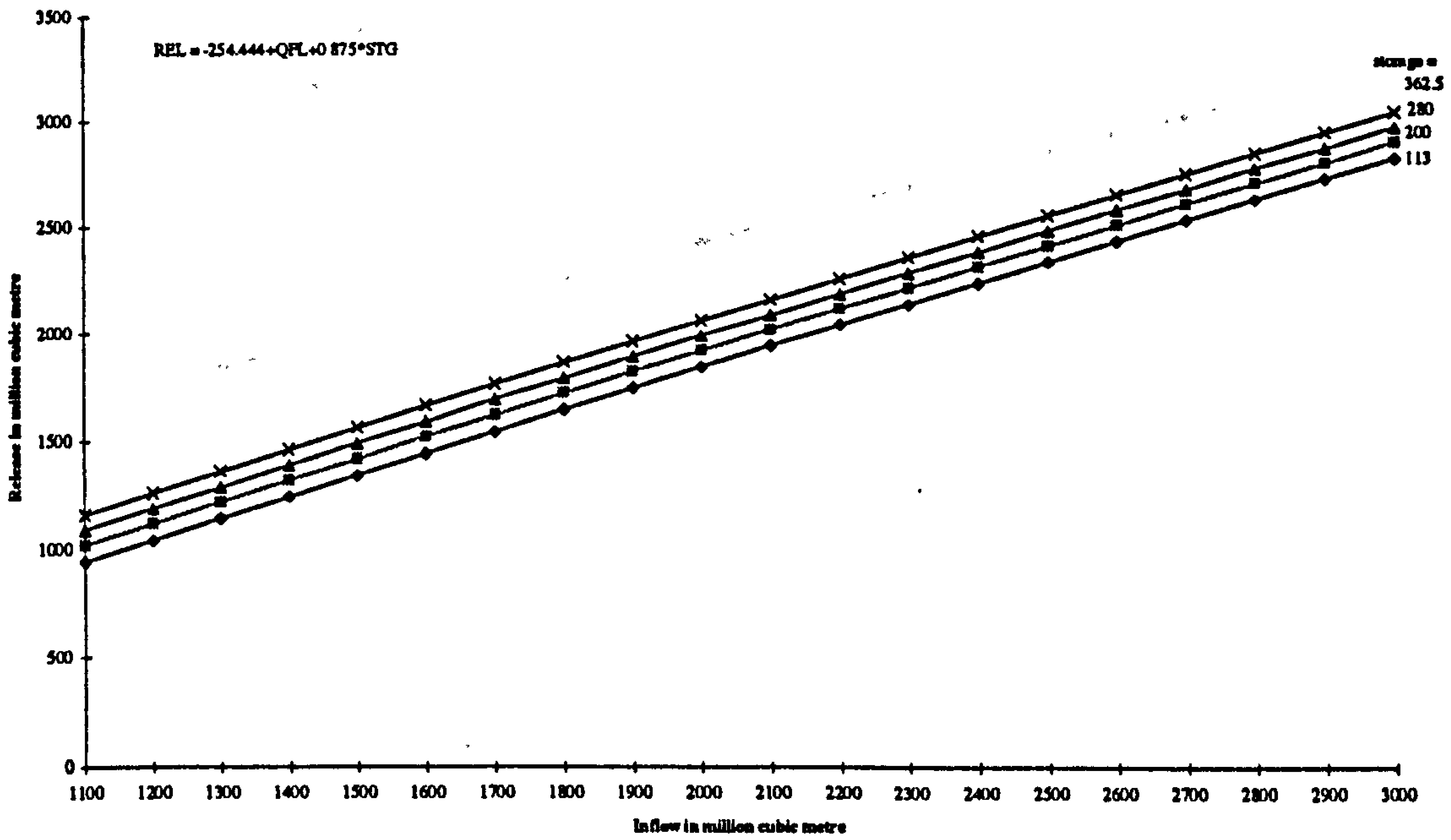


Figure (10.20) Relation between release, inflow and storage - Sennar - July

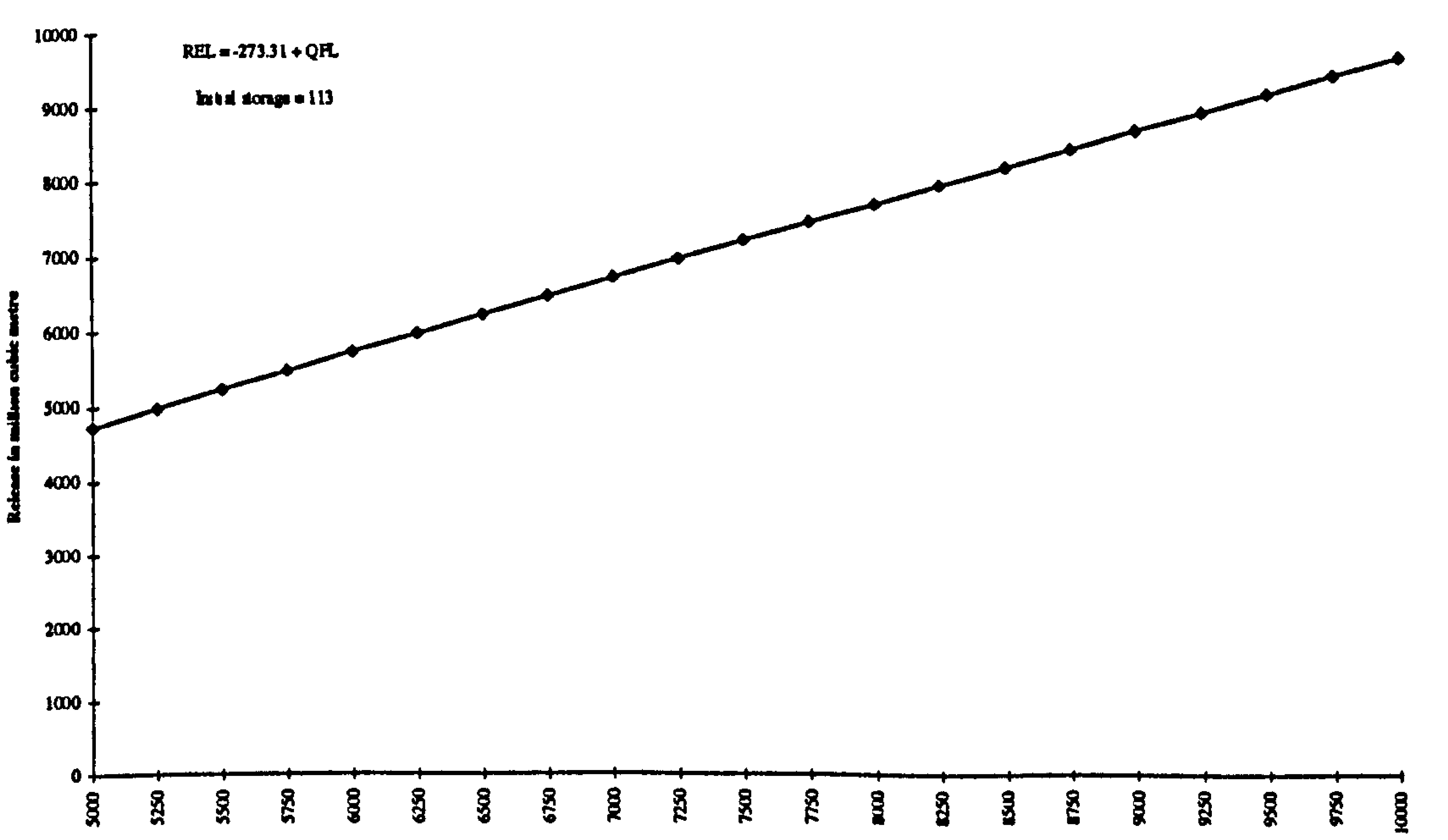
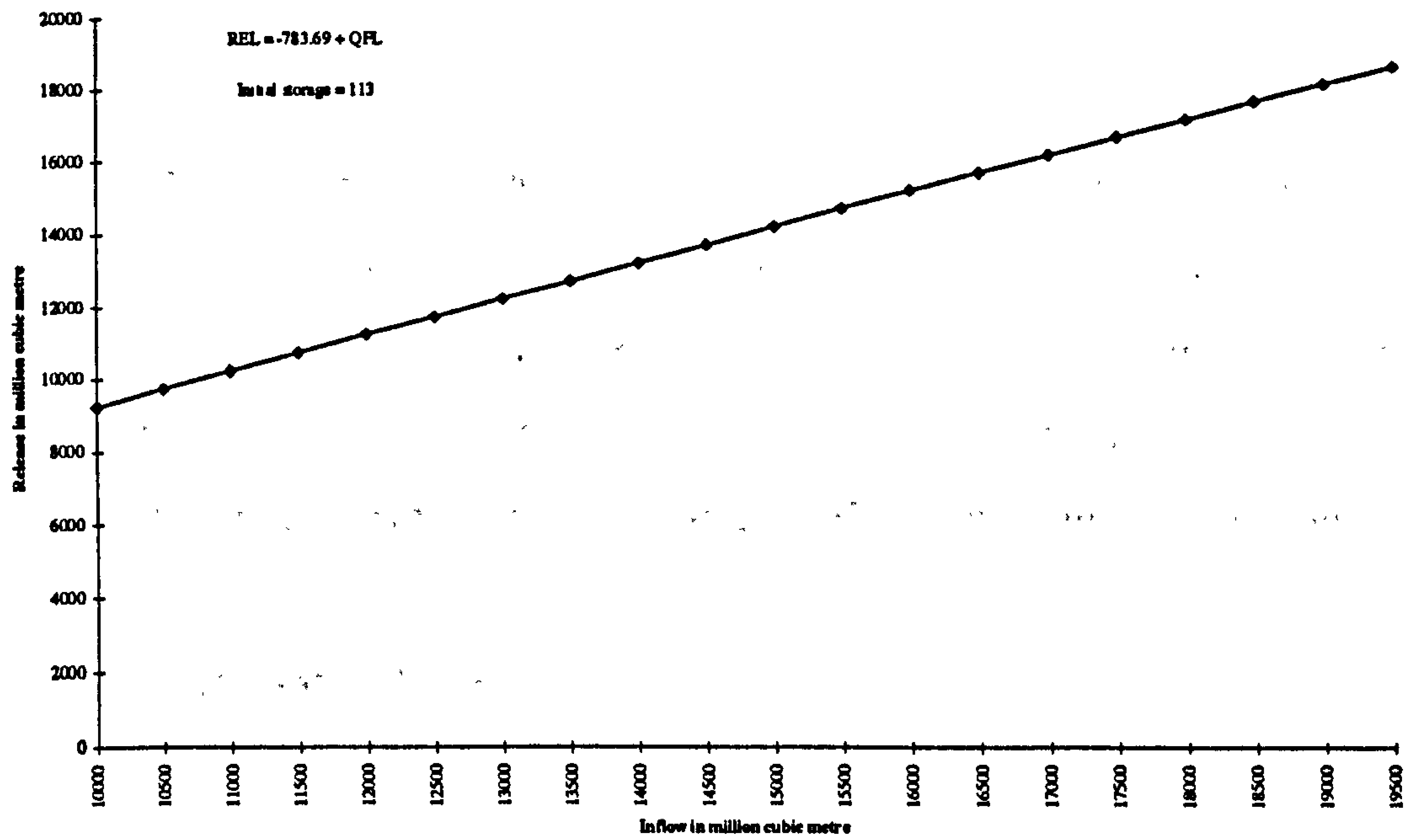


Figure (10.21) Relation between release, inflow and storage - Sennar - August



CHAPTER XI

EFFECTS OF SEDIMENT & WATER USE ON RESERVOIR OPERATION

Summary ~ More applications of the optimization model are to be considered in this Chapter. The sedimentation model developed in Chapter V is linked to the optimization model to study the effect of sedimentataion on optimum reservoir operation. Also the optimization model will be used in investigating the effect of efficient water use, investigated in Chapter VIII, on optimum reservoir operation.

11.1 INTRODUCTION

Reservoir optimum operation is affected by many issues. Among these are sedimentation and efficiency of water use in sectors served by the reservoir. An attempt will be made here to investigate the impact of these two issues on reservoir optimum operation. Firstly, the optimization model will be linked to a sedimentation model to assess the effect of reservoir sedimentation. Secondly, the results of the investigations of water use in irrigation, Chapter VIII, are inputted to the optimization model to evaluate the change in hydropower benefits due to inappropriate water use.

11.2 SEDIMENTATION EFFECTS

Sedimentation reduces reservoir capacity (both live and dead storage) and hence its capability to meet the objectives of operation. Here it is expected that the annual hydropower revenues may be reduced through the course of operation. Therefore, three years have been selected and the sedimentation is modelled each time as described in Chapter V. Each time the storage-water level relationship of the upstream reservoir, Roseries, is found. The storage - water level relationship for the downstream reservoir, Sennar, is kept constant, since it is less vulnerable to sedimentation. The optimization problem formulated in Chapter IX is for the year 1988 i.e. after 22 years of operation of the upstream reservoir, Roseries. For the other chosen years, 1978 and 1998 the optimization problem is reformulated and solved following similar steps to those described in Chapter IX.

11.2.1 Problems Reformulation

The optimization problem described in Chapter IX is for the year 1988. Optimization problems for years 1978 and 1998, due to the effect of sedimentation, will have the following changes:

11.2.1.1 Storage - Upstream Water Level Relationship-Roseries

1978

In 1978 Roseries Reservoir was in operation for 12 years. i.e. t = 12. Substituting for t = 12 in equations (9.2) and (9.3), values for a = 11.046 and m = 1.885 are obtained. Substituting for a & m in equation (9.1), the following storage - upstream water level is obtained:

$$H_{us} = 463 + 0.28\, S_{av}^{\;0.53} \qquad (9.4a)$$

1998

In 1998 Roseries Reservoir was in operation for 32 years. i.e. t = 32. Substituting for t = 32 in equations (9.2) and (9.3), values for a = 2.69 and m = 2.29 are obtained. Substituting for a & m in equation (9.1), the following storage - upstream water level is obtained:

$$H_{us} = 463 + 0.649\, S_{av}^{\;0.437} \qquad (9.4b)$$

Equations (9.4a) and (9.4b) replace equation (9.4) in the problem formulated for the year 1988. This replacement results in other changes in the formulated problems. These changes are described in the following sections.

11.2.1.2 Head Difference (H) - Roseries

<u>1978</u>

$$H = 18.79 - 0.00032\ X - 0.00032\ Y + 1.07\ S_{av}^{0.53} \qquad (9.8a)$$

<u>1998</u>

$$H = 18.79 - 0.00032\ X - 0.00032Y + 0.649S_{av}^{0.437} \qquad (9.8b)$$

11.2.1.3 Roseries Area-Upstream Water Level Relationship

<u>1978</u>

$$A = 79.55 + 1.07\ S_{av}^{0.53} + 0.038\ S_{av}^{1.06} \qquad (9.13a)$$

<u>1998</u>

$$A = 79.55 + 2.475\ S_{av}^{0.437} + 0.203\ S_{av}^{0.874} \qquad (9.13b)$$

11.2.1.4 Roseries Evaporation Relationships

<u>1978</u>

$$L1 = 5.966 + 0.056(S_{1,1}+S_{1,2})^{0.53} + 0.0014(S_{1,1}+S_{1,2})^{1.06} \qquad (9.17a)$$

$$L2 = 8.21 + 0.076(S_{1,2}+S_{1,3})^{0.53} + 0.0019(S_{1,2}+S_{1,3})^{1.06} \qquad (9.18a)$$

$$L3 = 16.948 + 0.158(S_{1,3}+S_{1,4})^{0.53} + 0.0038(S_{1,3}+S_{1,4})^{1.06} \qquad (9.19a)$$

$$L4 = 16.709 + 0.155(S_{1,4}+S_{1,5})^{0.53} + 0.0038(S_{1,4}+S_{1,5})^{1.06} \qquad (9.20a)$$

$$L5 = 16.948 + 0.158(S_{1,5}+S_{1,6})^{0.53} + 0.0038(S_{1,5}+S_{1,6})^{1.06} \qquad (9.21a)$$

$$L6 = 18.857 + 0.175(S_{1,6}+S_{1,7})^{0.53} + 0.0043(S_{1,6}+S_{1,7})^{1.06} \qquad (9.22a)$$

$$L7 = 21.244 + 0.198(S_{1,7}+S_{1,8})^{0.53} + 0.0048(S_{1,7}+S_{1,8})^{1.06} \qquad (9.23a)$$

$$L8 = 21.006 + 0.195(S_{1,8}+S_{1,9})^{0.53} + 0.0048(S_{1,8}+S_{1,9})^{1.06} \qquad (9.24a)$$

$$L9 = 19.096 + 0.178(S_{1,9}+S_{1,10})^{0.53} + 0.0043(S_{1,9}+S_{1,10})^{1.06} \quad (9.25a)$$

$$L10 = 10.98 + 0.102(S_{1,10}+S_{1,11})^{0.53} + 0.0025(S_{1,10}+S_{1,11})^{1.06} \quad (9.26a)$$

$$L11 = 6.206 + 0.058(S_{1,11}+S_{1,12})^{0.53} + 0.0014(S_{1,11}+S_{1,12})^{1.06} \quad (9.27a)$$

$$L12 = 4.535 + 0.042(S_{1,12}+S_{1,13})^{0.53} + 0.001(S_{1,12}+S_{1,13})^{1.06} \quad (9.28a)$$

<u>1998</u>

$$L1 = 5.966 + 0.137(S_{1,1}+S_{1,2})^{0.437} + 0.008(S_{1,1}+S_{1,2})^{0.874} \quad (9.17b)$$

$$L2 = 8.21 + 0.189(S_{1,2}+S_{1,3})^{0.437} + 0.011(S_{1,2}+S_{1,3})^{0.874} \quad (9.18b)$$

$$L3 = 16.948 + 0.389(S_{1,3}+S_{1,4})^{0.437} + 0.024(S_{1,3}+S_{1,4})^{0.874} \quad (9.19b)$$

$$L4 = 16.709 + 0.384(S_{1,4}+S_{1,5})^{0.437} + 0.023(S_{1,4}+S_{1,5})^{0.874} \quad (9.20b)$$

$$L5 = 16.948 + 0.389(S_{1,5}+S_{1,6})^{0.437} + 0.024(S_{1,5}+S_{1,6})^{0.874} \quad (9.21b)$$

$$L6 = 18.857 + 0.433(S_{1,6}+S_{1,7})^{0.874} + 0.026(S_{1,6}+S_{1,7})^{0.874} \quad (9.22b)$$

$$L7 = 21.244 + 0.488(S_{1,7}+S_{1,8})^{0.437} + 0.03(S_{1,7}+S_{1,8})^{0.874} \quad (9.23b)$$

$$L8 = 21.006 + 0.483(S_{1,8}+S_{1,9})^{0.437} + 0.029(S_{1,8}+S_{1,9})^{0.874} \quad (9.24b)$$

$$L9 = 19.096 + 0.439(S_{1,9}+S_{1,10})^{0.437} + 0.027(S_{1,9}+S_{1,10})^{0.874} \quad (9.25b)$$

$$L10 = 10.98 + 0.252(S_{1,10}+S_{1,11})^{0.437} + 0.015(S_{1,10}+S_{1,11})^{0.874} \quad (9.26b)$$

$$L11 = 6.206 + 0.143(S_{1,11}+S_{1,12})^{0.437} + 0.009(S_{1,11}+S_{1,12})^{0.874} \quad (9.27b)$$

$$L12 = 4.535 + 0.104(S_{1,12}+S_{1,13})^{0.437} + 0.006(S_{1,12}+S_{1,13})^{0.874} \quad (9.28b)$$

11.2.1.5 Hydroelectric Power Production Function

<u>1978</u>

For Roseries the hydropower production function is:

$$HP = 45058.42X + 671.44X\, S_{av}^{0.53} - 0.767X^2 - 0.767XY \quad (9.44a)$$

$$HP(1,i) = 45058.42X_{(1,i)} + 465.009X_{(1,i)}[S_{(1,i)}+S_{(1,i+1)}]^{0.53} - 0.767X_{(1,i)}^2 - 0.767X_{(1,i)}\, Y_{(1,i)} \quad (9.46a)$$

The power production function for Sennar remains unchanged, therefore the total power produced by the two plants in month i is:

$$HP(i) = 45058.42X_{(1,i)}+465.009X_{(1,i)}[S_{(1,i)}+S_{(1,i+1)}]^{0.53}- 0.767X_{(1,i)}^2 - 0.767X_{(1,i)} Y_{(1,i)} +14100X_{(2,i)}+1803.2X_{(2,i)}[S_{(2,i)}+S_{(2,i+1)}]^{0.417}- 0.767X_{(2,i)}^2 - 0.767X_{(2,i)} Y_{(2,i)} \quad (9.48a)$$

<u>1998</u>

For Roseries the hydropower production function is;

$$HP = 45058.42X +1556.3X S_{av}^{0.437} - 0.767X^2 - 0.767XY \quad (9.44b)$$

$$HP(1,i) =45058.42X_{(1,i)}+1149.6X_{(1,i)}[S_{(1,i)}+S_{(1,i+1)}]^{0.437}- 0.767X_{(1,i)}^2 - 0.767X_{(1,i)} Y_{(1,i)} \quad (9.46b)$$

The power production function for Sennar remains unchanged, therefore the total power produced by the two plants in month i is :

$$HP(i) = 45058.42X_{(1,i)}+1149.6X_{(1,i)}[S_{(1,i)}+S_{(1,i+1)}]^{0.437}- 0.767X_{(1,i)}^2 - 0.767X_{(1,i)} Y_{(1,i)} +14100X_{(2,i)}+1803.2X_{(2,i)}[S_{(2,i)}+S_{(2,i+1)}]^{0.417}- 0.767X_{(2,i)}^2 - 0.767X_{(2,i)} Y_{(2,i)} \quad (9.48a)$$

11.2.1.6 Objective Function

As before the objective function is obtained by multiplying the power produced by the power prices, Table (9.2).

<u>1978</u>:

$$F = \sum_{i=1}^{12} a(1,i)X_{(1,i)}+b(1,i)X_{(1,i)}[S_{(1,i)}+S_{(1,i+1)}]^{0.53}+ c(1,i)X_{(1,i)}^2 + d(1,i)X_{(1,i)} Y_{(1,i)} +a(2,i)X_{(2,i)}+b(2,i)X_{(2,i)}[S_{(2,i)}+S_{(2,i+1)}]^{0.417}+ c(2,i)X_{(2,i)}^2 + d(2,i)X_{(2,i)} Y_{(2,i)} \quad (9.50a)$$

Where :
b(1,1) ,........................., b(1,6) = 0.0053
b(1,7) ,........................., b(1,12) = 0.0062

Other constants have the same values as in Sect ion (9.8).

<u>1998</u>:

$$F = \sum_{i=1}^{12} a(1,i)X_{(1,i)}+b(1,i)X_{(1,i)}[S_{(1,i)}+S_{(1,i+1)}]^{0.437}+ c(1,i)X_{(1,i)}^2 + d(1,i)X_{(1,i)} Y_{(1,i)} +a(2,i)X_{(2,i)}+b(2,i)X_{(2,i)}[S_{(2,i)}+S_{(2,i+1)}]^{0.417}+ c(2,i)X_{(2,i)}^2 + d(2,i)X_{(2,i)} Y_{(2,i)} \quad (9.50b)$$

Where :
b(1,1) ,........................., b(1,6) = 0.0132
b(1,7) ,........................., b(1,12) = 0.0153

Other constants have the same values as in Section (9.8).

11.2.1.7 Constraints

a) Continuity Equations for Roseries

1978

$S_{(1,2)} - S_{(1,1)} + X_{(1,1)} + Y_{(1,1)} + 0.056(S_{1,1}+S_{1,2})^{0.53} + 0.0014(S_{1,1}+S_{1,2})^{1.06} = q_1 - 5.966 = e1$ cons1a

$S_{(1,3)} - S_{(1,2)} + X_{(1,2)} + Y_{(1,2)} + 0.076(S_{1,2}+S_{1,3})^{0.53} + 0.0019(S_{1,2}+S_{1,3})^{1.06} = q_2 - 8.21 = e2$ cons2a

$S_{(1,4)} - S_{(1,3)} + X_{(1,3)} + Y_{(1,3)} + 0.158(S_{1,3}+S_{1,4})^{0.53} + 0.0038(S_{1,3}+S_{1,4})^{1.06} = q_3 - 16.948 = e3$ cons3a

$S_{(1,5)} - S_{(1,4)} + X_{(1,4)} + Y_{(1,4)} + 0.155(S_{1,4}+S_{1,5})^{0.53} + 0.0038(S_{1,4}+S_{1,5})^{1.06} = q_4 - 16.709 = e4$ cons4a

$S_{(1,6)} - S_{(1,5)} + X_{(1,5)} + Y_{(1,5)} + 0.158(S_{1,5}+S_{1,6})^{0.53} + 0.0038(S_{1,5}+S_{1,6})^{1.06} = q_5 - 16.948 = e5$ cons5a

$S_{(1,7)} - S_{(1,6)} + X_{(1,6)} + Y_{(1,6)} + 0.175(S_{1,6}+S_{1,7})^{0.53} + 0.0043(S_{1,6}+S_{1,7})^{1.06} = q_6 - 18.857 = e6$ cons6a

$S_{(1,8)} - S_{(1,7)} + X_{(1,7)} + Y_{(1,7)} + 0.198(S_{1,7}+S_{1,8})^{0.53} + 0.0048(S_{1,7}+S_{1,8})^{1.06} = q_7 - 21.244 = e7$ cons7a

$S_{(1,9)} - S_{(1,8)} + X_{(1,8)} + Y_{(1,8)} + 0.195(S_{1,8}+S_{1,9})^{0.53} + 0.0048(S_{1,8}+S_{1,9})^{1.06} = q_8 - 21.006 = e8$ cons8a

$S_{(1,10)} - S_{(1,9)} + X_{(1,9)} + Y_{(1,9)} + 0.178(S_{1,9}+S_{1,10})^{0.53} + 0.0043(S_{1,9}+S_{1,10})^{1.06} = q_9 - 19.096 = e9$ cons9a

$S_{(1,11)} - S_{(1,10)} + X_{(1,10)} + Y_{(1,10)} + 0.102(S_{1,10}+S_{1,11})^{0.53} + 0.0025(S_{1,10}+S_{1,11})^{1.06} = q_{10} - 10.98 = e10$ cons10a

$S_{(1,12)} - S_{(1,11)} + X_{(1,11)} + Y_{(1,11)} + 0.058(S_{1,11}+S_{1,12})^{0.53} + 0.0014(S_{1,11}+S_{1,12})^{1.06} = q_{11} - 6.206 = e11$ cons11a

$S_{(1,13)} - S_{(1,12)} + X_{(1,12)} + Y_{(1,12)} + 0.042(S_{1,12}+S_{1,13})^{0.53} + 0.001(S_{1,12}+S_{1,13})^{1.06} = q_{12} - 4.535 = e12$ cons12a

1998

$S_{(1,2)} - S_{(1,1)} + X_{(1,1)} + Y_{(1,1)} + 0.137(S_{1,1}+S_{1,2})^{0.437} + 0.008(S_{1,1}+S_{1,2})^{0.874} = q_1 - 5.966 = e1$ cons1b

$S_{(1,3)} - S_{(1,2)} + X_{(1,2)} + Y_{(1,2)} + 0.189(S_{1,2}+S_{1,3})^{0.437} + 0.011(S_{1,2}+S_{1,3})^{0.874} = q_2 - 8.21 = e2$ cons2b

$S_{(1,4)} - S_{(1,3)} + X_{(1,3)} + Y_{(1,3)} + 0.389(S_{1,3}+S_{1,4})^{0.437} + 0.024(S_{1,3}+S_{1,4})^{0.874} = q_3 - 16.948 = e3$ cons3b

$S_{(1,5)} - S_{(1,4)} + X_{(1,4)} + Y_{(1,4)} + 0.384(S_{1,4}+S_{1,5})^{0.437} + 0.023(S_{1,4}+S_{1,5})^{0.874} = q_4 - 16.709 = e4$ cons4b

$S_{(1,6)} - S_{(1,5)} + X_{(1,5)} + Y_{(1,5)} + 0.389(S_{1,5}+S_{1,6})^{0.437} + 0.024(S_{1,5}+S_{1,6})^{0.874} = q_5 - 16.948 = e5$ cons5b

$S_{(1,7)} - S_{(1,6)} + X_{(1,6)} + Y_{(1,6)} + 0.433(S_{1,6}+S_{1,7})^{0.437} + 0.026(S_{1,6}+S_{1,7})^{0.874} = q_6 - 18.857 = e6$ cons6b

$S_{(1,8)} - S_{(1,7)} + X_{(1,7)} + Y_{(1,7)} + 0.488(S_{1,7}+S_{1,8})^{0.437} + 0.03(S_{1,7}+S_{1,8})^{0.874} = q_7 - 21.244 = e7$ cons7b

$S_{(1,9)} - S_{(1,8)} + X_{(1,8)} + Y_{(1,8)} + 0.483(S_{1,8}+S_{1,9})^{0.437} + 0.029(S_{1,8}+S_{1,9})^{0.874} = q_8 - 21.006 = e8$ cons8b

$S_{(1,10)} - S_{(1,9)} + X_{(1,9)} + Y_{(1,9)} + 0.439(S_{1,9}+S_{1,10})^{0.437} + 0.027(S_{1,9}+S_{1,10})^{0.874} = q_9 - 19.096 = e9$ cons9b

$S_{(1,11)} - S_{(1,10)} + X_{(1,10)} + Y_{(1,10)} + 0.252(S_{1,10}+S_{1,11})^{0.437} + 0.015(S_{1,10}+S_{1,11})^{0.874} = q_{10} - 10.98 = e10$ cons10b

$S_{(1,12)} - S_{(1,11)} + X_{(1,11)} + Y_{(1,11)} + 0.143(S_{1,11}+S_{1,12})^{0.437} + 0.009(S_{1,11}+S_{1,12})^{0.874} = q_{11} - 6.206 = e11$ cons11b

$S_{(1,13)} - S_{(1,12)} + X_{(1,12)} + Y_{(1,12)} + 0.104(S_{1,12}+S_{1,13})^{0.437} + 0.006(S_{1,12}+S_{1,13})^{0.874} = q_{12} - 4.535 = e12$ cons12b

b) Bounds on Storages

Roseries reservoir is operated between the minimum level of 467 m and the maximum level of 481 m (MOI, 1968). For 1988 these levels correspond to storages 88.3 and 2175 million m^3 respectively. For 1978, these levels are obtained, using relation (9.4a), if 150.3 and 2560.3 million m^3 are stored. For 1998, the minimum and maximum storages are 64.4 and 2016.2 million m^3 (using relation 9.4b).

<u>1978</u>

$150.3 \le S(1,i) \le 2560.3$, for i = 2,..............., 10		cons53a
$S(1,i) = 150.3$	for i = 1, 11, 12, 13	cons54a

<u>1998</u>

$64.4 \le S(1,i) \le 2016.2$, for i = 2,..............., 10		cons53b
$S(1,i) = 64.4$	for i = 1, 11, 12, 13	cons54b

11.2.2 <u>Reformulated Problems Solutions</u>

To solve the reformulated problems, their SIF files have to be written. These files are obtained by making the necessary changes in the SIF file for year 1988 shown in Appendix (D). Appendix (G) shows these changes made in 1988 SIF file to obtain 1978 and 1998 SIF files.

After these changes to the SIF file are made, Lancelot is run to obtain a solution for the optimum operation of the reservoir system during the chosen years. Inputs to the model are the average river inflow and actual estimated irrigation supplies. Tables (11.1) to (11.3) show the optimization results.

11.2.3 <u>Comments on Sedimentation Effects</u>

Sedimentation is expected to affect storages and operation levels. If the storages and operation levels are affected, the hydropower generated and the annual hydropower revenues will also be affected. Table (11.1) and Figure (11.1) compare the storages at Roseries reservoir for years 1978, 1988 and 1998. Table (11.2) shows the change in storages for Sennar. It can be seen, specially for the upstream reservoir which is more exposed to sedimentation, that the quantity of stored water decreases through the course of reservoir operation and this is expected to decrease the generated power and hence the annual revenues.

Sedimentation also affects water levels in reservoirs. Using storages obtained from the optimum solution, Table (11.1), and using equations (9.4) for 1988, equation (9.4a) for 1978 and equation (9.4b) for 1998, optimum operation levels for Roseries are obtained (Table 11.4 and Figure 11.2). The optimum storages for Sennar are substituted in equation (9.5) to obtain the optimum operation levels for this reservoir (Table 11.4). To evaluate the effect of sedimentation on power generation, the releases and storages obtained from the optimum solution for the three chosen years are substituted in equation (9.48) to obtain power generated in 1988, in equation (9.48a) to obtain the power generated for 1978 and in equation (9.48b) to obtain the power generated in 1998. Table (11.5) and Figure (11.3) show the reduction in generated power. This reduction in power reduces the annual revenues from the generated power. The maximum annual revenues, which are equal to the objective function value, are obtained directly from the model solution (Table 11.3 and Figure 11.4). The annual revenues in 1978 are estimated at 15.853 billions Sudanese Dinnars. This value has dropped by 1.7 % in 1988 and by about 3 % in 1998.

11.3 EFFECT OF WATER USE IN IRRIGATION SCHEMES ON RESERVOIR OPTIMUM OPERATION

From the investigation in Chapter VIII, irrigation requirements, it is found that water is inappropriately supplied in the Blue Nile System. An improvement in water application will result in a different demand sequence (Table 8.35). These requirements are divided into upstream and downstream requirements (Table 11.6). Having different demands to be supplied, is expected to affect the optimum reservoir operation. To estimate this effect, the optimization model is solved with the average inflow and the actual estimated irrigation requirements used as inputs. This is typical to the problem solved in Chapter IX. Then the actual estimated irrigation demand is replaced by the sequence resulting from the assumed improved water application, Table (11.6). Then the model

is solved again after the necessary changes in the SIF file are made. The changes in irrigation demand require changing the constants ru_1, ..., ru_{12} and rd_1, ..., rd_{12} in the SIF file shown in Appendix D. The results of the solution of the two problems are shown in Tables (11.7) and (11.8). Using these results and equation (9.48), the monthly power generated is calculated and shown in Tables (11.9) and Figure (11.5).
The improved water application increases the annual hydropower revenue from 15.5983 to 15.816 billion Sudanese Dinnars. This is an increase of 1.4 % which is attributed to the increase in the annual power generated from 1283.2 to 1296.4 Gwh. During the period September to February, the power generated is reduced by 2.7 % (from 847.9 to 824.9 Gwh). This is apparently due to the decrease in release caused by efficient water use. Using less water in September to February, resulted in higher reservoir storages in the period March to June. Therefore the power generated during the period March to August is increased by 8.3 % (From 435.2 to 471.5 Gwh).

11.4 CONCLUSION

Sedimentation affects reservoir storages and water levels. Using the sedimentation model developed in Chapter V, the storage-water level relationships for different years are predicted. By linking these results to the optimization model, effect of sedimentation on reservoirs' storage, reservoirs' operation levels and hydropower revenues have been evaluated. It has been found that the stored water is getting less and consequently reservoirs are operated at lower levels through the reservoirs course of operation. The revenues have also decreased due to sedimentation. These findings verify the second part of hypothesis 2.
The efficiency of water use investigated in Chapter VIII, resulted into two demand scenarios. The first represents the actual irrigation requirements while the second represents the irrigation requirements with improved water application. The two sequences are used as inputs to the optimization model. The better water application in irrigation increased the overall annual power production by 1.4 % and the production during the dry season by 8.3 %. This finding verifies the second part of hypothesis 1.
These findings illustrate that the developed non-linear model is a useful tool for investigating the effect of some reservoir issues on optimum reservoir operation.

Table (11.1) Roseries reservoir optimum releases and storages (million m^3)

month	1978			1988			1998		
	Power Release	Other Release	Storage at beginning of month	Power Release	Other Release	Storage at beginning of month	Power Release	Other Release	Storage at beginning of month
sept	2014	6606.24	150.3	2014	6928.4	88.3	2014	7064.02	64.4
oct	2014	3741.52	2560.3	2014	3741.87	2175	2014	3742.09	2016.2
nov	2014	133.96	2560.3	2014	132.122	2175	2014	132.411	2016.2
dec	1309.43	0	2560.3	1278.84	0	2175	1284.06	0	2016.2
jan	1631.53	0	2465.13	1662.12	0	2108.33	1656.9	0	1946
feb	842.02	0	1431.7	842.02	0	1043.34	842.02	0	886.636
mar	703.24	0	954.656	555.859	0	568.706	484.3	0	413.1
apr	404.3	0	531.611	404.3	0	295.743	387.088	0	213.101
may	315.132	0	361.713	154.8	0	128.599	172.012	0	64.4
jun	2014	0	540.096	2002.14	0	467.084	1945.16	0	386.317
july	2014	5150.01	150.3	2014	5149.94	88.3	2014	5149.97	64.4
aug	2014	12525.2	150.3	2014	12525.2	88.3	2014	12525.2	64.4

Table (11.2) Sennar reservoir optimum releases and storages (million m^3)

month	1978			1988			1998		
	Power Release	Other Release	Storage at beginning of month	Power Release	Other Release	Storage at beginning of month	Power Release	Other Release	Storage at beginning of month
sept	330	6760.13	113	330	7082.3	113	330	7217.91	113
oct	330	4064.62	362.5	330	4064.97	362.5	330	4065.19	362.5
nov	330	522.515	362.5	330	542.143	362.5	330	542.851	362.5
dec	160.3	0	362.5	160.3	0	362.5	160.3	0	362.5
jan	160.66	0	205.833	160.66	0	175.239	160.66	0	180.462
feb	145.32	0	362.5	145.32	0	362.5	145.32	0	362.5
mar	330	0	113	182.659	0	113	111.06	0	113
apr	330	0	362.5	330	0	362.5	330	0	362.5
may	330	0	362.5	330	0	362.5	330	0	345.288
jun	330	1667.31	273.332	330	1516.6	113	330	1459.62	113
july	330	6560.69	113	330	6560.63	113	330	6560.66	113
aug	330	13425.5	113	330	13425.5	113	330	13425.5	113

Table (11.3) Effect of sedimentation on revenues

Year	Revenue in billion Sudanese Dinnars
1978	15.853
1988	15.598
1998	15.388

1US$ = 245 Sudanese Dinnars - Bank of Sudan, 1999

Table (11.4) Effect of sedimentation on operation levels

	Operation level (m) -Roseries			Operation level (m) - Sennar		
month	78	88	98	78	88	98
sept	466.99	467.03	467.01	417.21	417.21	417.21
oct	480.93	481.15	481.04	421.72	421.72	421.72
nov	480.93	481.15	481.04	421.72	421.72	421.72
dec	480.93	481.15	481.04	421.72	421.72	421.72
jan	480.57	480.88	480.77	419.26	418.65	418.76
feb	476.18	475.85	475.60	421.72	421.72	421.72
mar	473.63	472.66	472.03	417.21	417.21	417.21
apr	470.79	470.10	469.76	421.72	421.72	421.72
may	469.35	467.80	467.01	421.72	421.72	421.49
jun	470.86	471.81	471.76	420.42	417.21	417.21
july	466.99	467.03	467.01	417.21	417.21	417.21
aug	466.99	467.03	467.01	417.21	417.21	417.21

Table (11.5) Effect of sedimentation on power production-Power in GWh.

month	78	88	98
sept	149.8775	151.9578	152.5069
oct	181.2615	182.3124	181.8152
nov	187.7308	188.7802	188.2827
dec	119.7367	117.8023	117.9623
jan	138.49	141.2003	140.2393
feb	67.00851	65.83444	65.0685
mar	59.35188	42.98126	35.10232
apr	38.82625	37.82884	36.27966
may	32.87118	21.95139	22.81464
jun	128.8574	130.1426	126.9524
july	107.5635	107.735	107.6428
aug	94.43325	94.60455	94.51243
Total	1306.008	1283.131	1269.179
reduction %		1.75	2.82

Table (11.6) Irrigation demand due to improved water application

month	requirements upstream Sennar, ru(i), in million m^3	requirements downstream Sennar, rd(i), in million m^3
sept	1158.144	48.256
oct	1197.696	49.904
nov	1130.688	47.112
dec	1228.416	51.184
jan	1235.136	51.464
feb	760.512	31.688
mar	99.168	4.132
apr	74.304	3.096
may	74.304	3.096
jun	85.344	3.556
july	196.992	8.208
auq	613.056	25.544

Table (11.7) Effect of efficiency of water use on reservoir operation-Roeries

month	Actual irrigation requirement			requirements with improved water application		
	Power Release	Other Release	Storage at beginning of month	Power Release	Other Release	Storage at beginning of month
sept	2014	6928.4	88.3	2014	6928.4	88.3
oct	2014	3741.87	2175	2014	3741.87	2175
nov.	2014	132.122	2175	2014	132.122	2175
dec.	1278.84	0	2175	1211.57	0	2175
jan.	1662.12	0	2108.33	1566.43	0	2175
feb.	842.02	0	1043.34	648.6	0	1203.57
mar.	555.859	0	568.706	678.67	0	916.693
apr.	404.3	0	295.743	404.3	0	513.04
may	154.8	0	128.599	350.225	0	339.152
june	2002.14	0	467.084	2014	0	479.045
july	2014	5149.94	88.3	2014	5149.94	88.3
aug.	2014	12525.2	88.3	2014	12525.2	88.3

All releases and storage are in million m^3

Table (11.8) Effect of efficiency of water use on reservoir operation-Sennar

month	Actual irrigation requirement			requirements with improved water application		
	Power Release	Other Release	Storage at beginning of month	Power Release	Other Release	Storage at beginning of month
sept	330	7082.3	113	330	7149.78	113
oct	330	4064.97	362.5	330	4134.88	362.5
nov.	330.0	542.143	362.5	330	582.454	362.5
dec.	160.3	0	362.5	157.08	0	362.5
jan.	160.66	0	175.239	157.36	0	188.566
feb.	145.32	0	362.5	137.59	0	362.5
mar.	182.659	0	113	330	0	113
apr.	330.0	0	362.5	330	0	362.5
may	330.0	0	362.5	330	0	362.5
june	330.0	1516.6	113	330	1718.51	308.425
july	330.0	6560.63	113	330	6608.81	113
aug.	330.0	13425.5	113	330	13575.3	113

All releases and storage are in million m^3

Table (11.9) Effect of efficiency of water use on power production-Power in GWh

month	Actual irrigation requirements	requirements with improved water application
sept	151.9578	151.9408
oct	182.3124	182.2947
nov.	188.7802	188.77
dec.	117.8343	112.3213
jan.	141.2002	135.3733
feb.	65.83438	54.2054
mar.	42.98126	60.08111
apr.	37.82884	40.11386
may	21.95139	36.18565
june	130.1426	132.8135
july	107.735	107.7228
aug.	94.60455	94.56664
Total	1283.163	1296.389
% Increase		1.02

Figure (11.1) Effect of sedimentation on reservoir optimum storages- Roseries

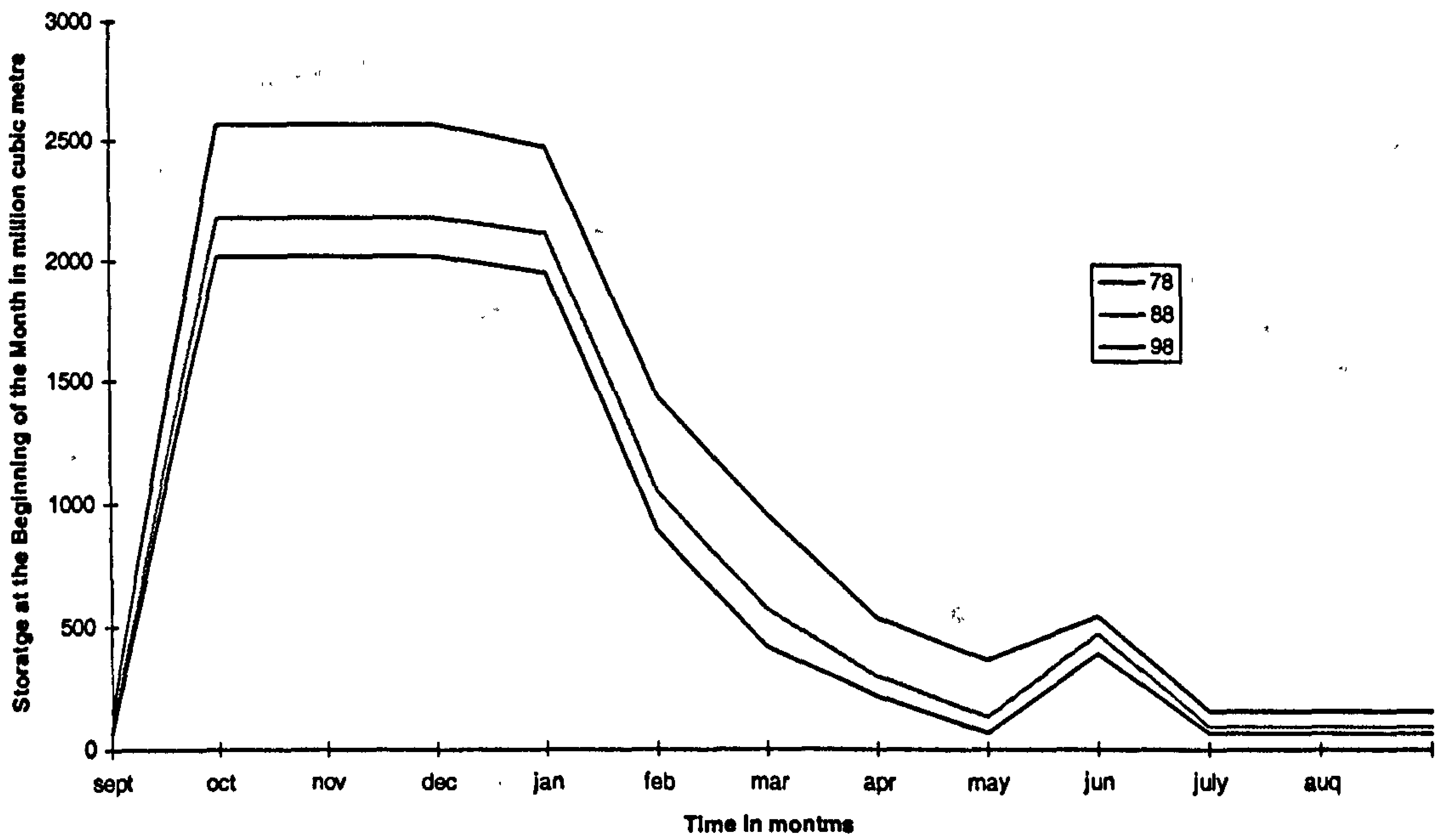


Figure (11.2) Effect of sedimentation on optimum operation levels - Roseries

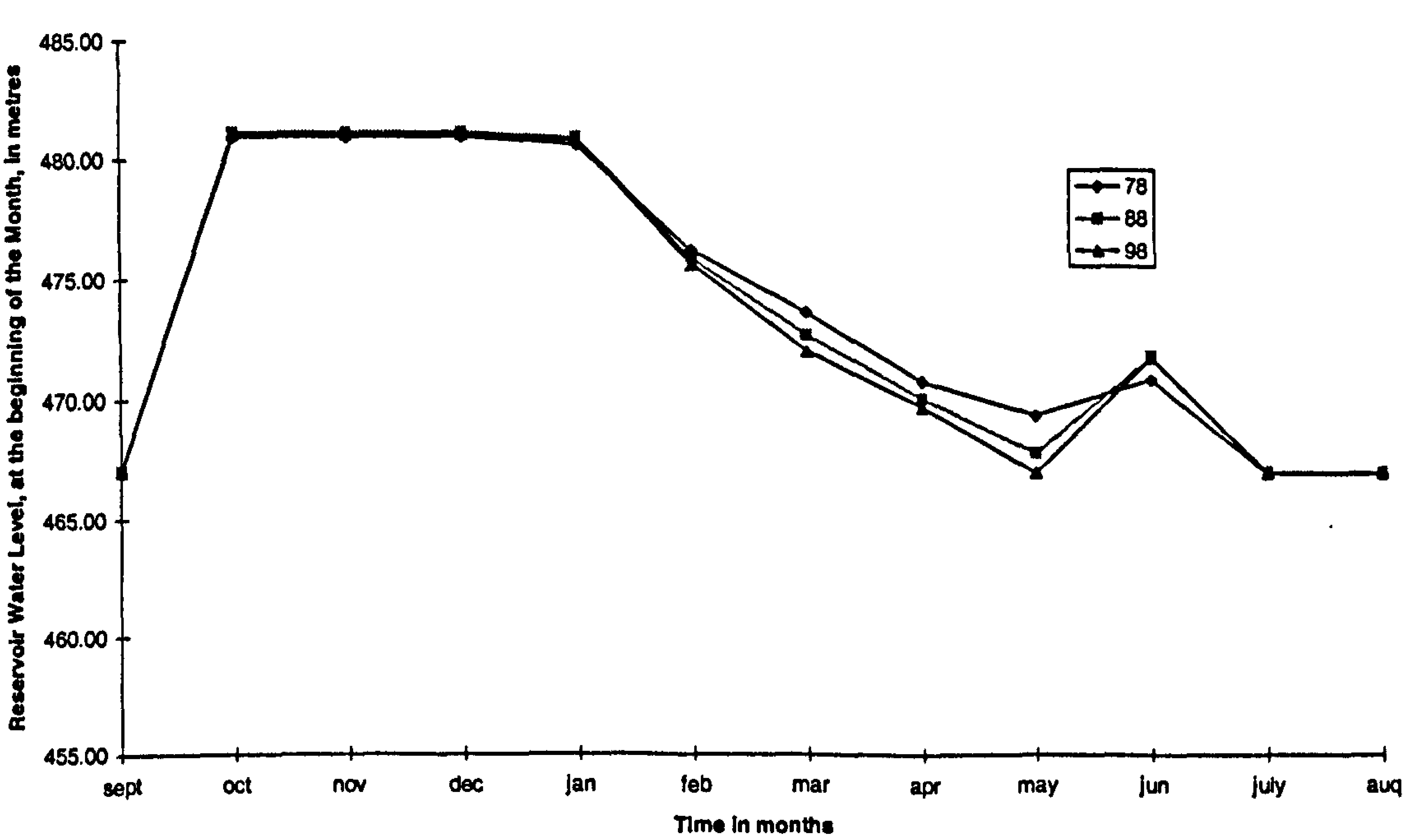


Figure (11.3) Effect of sedimentation on power production

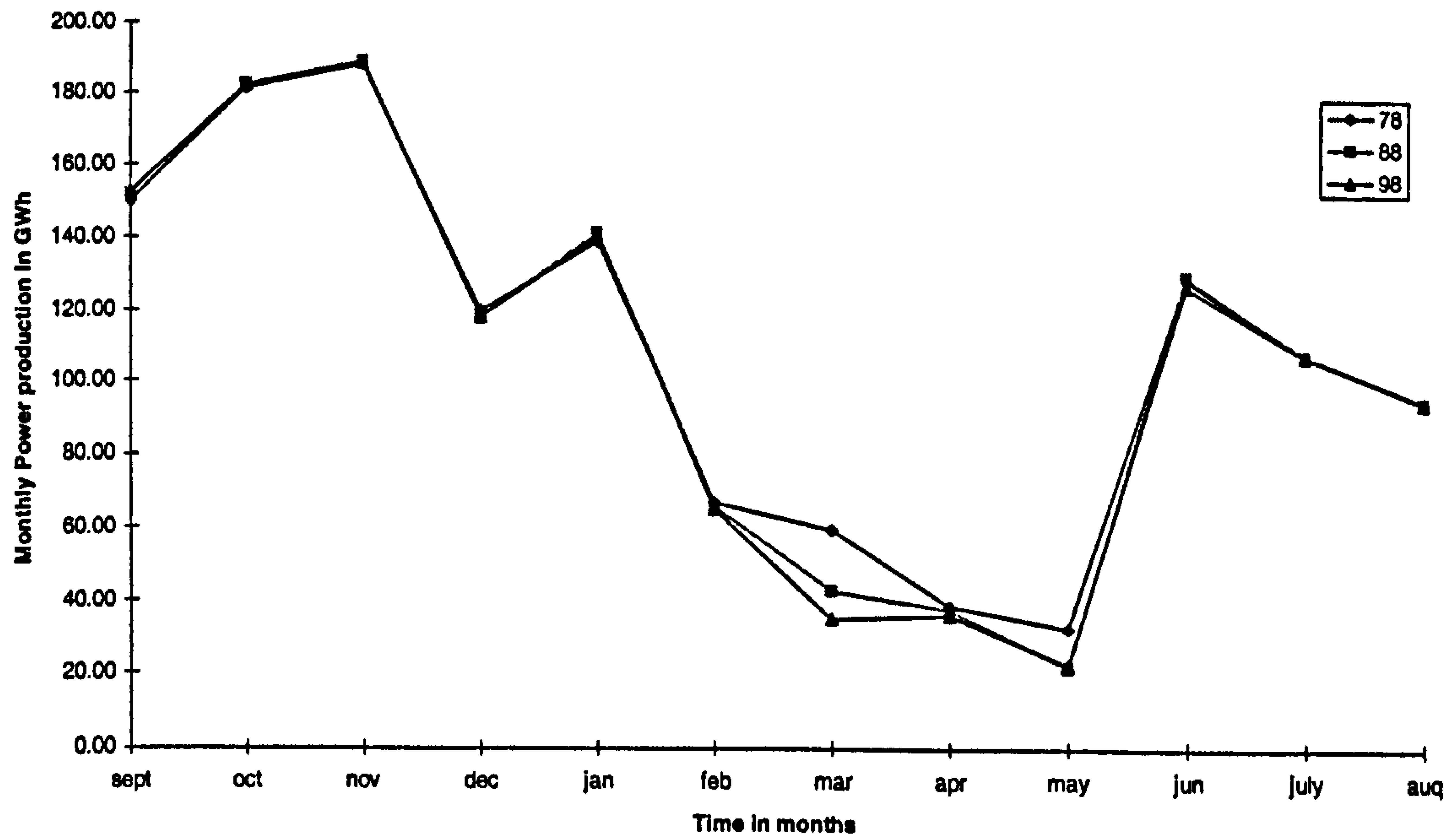


Figure (11.4) Effect of sedimentation on power revenues

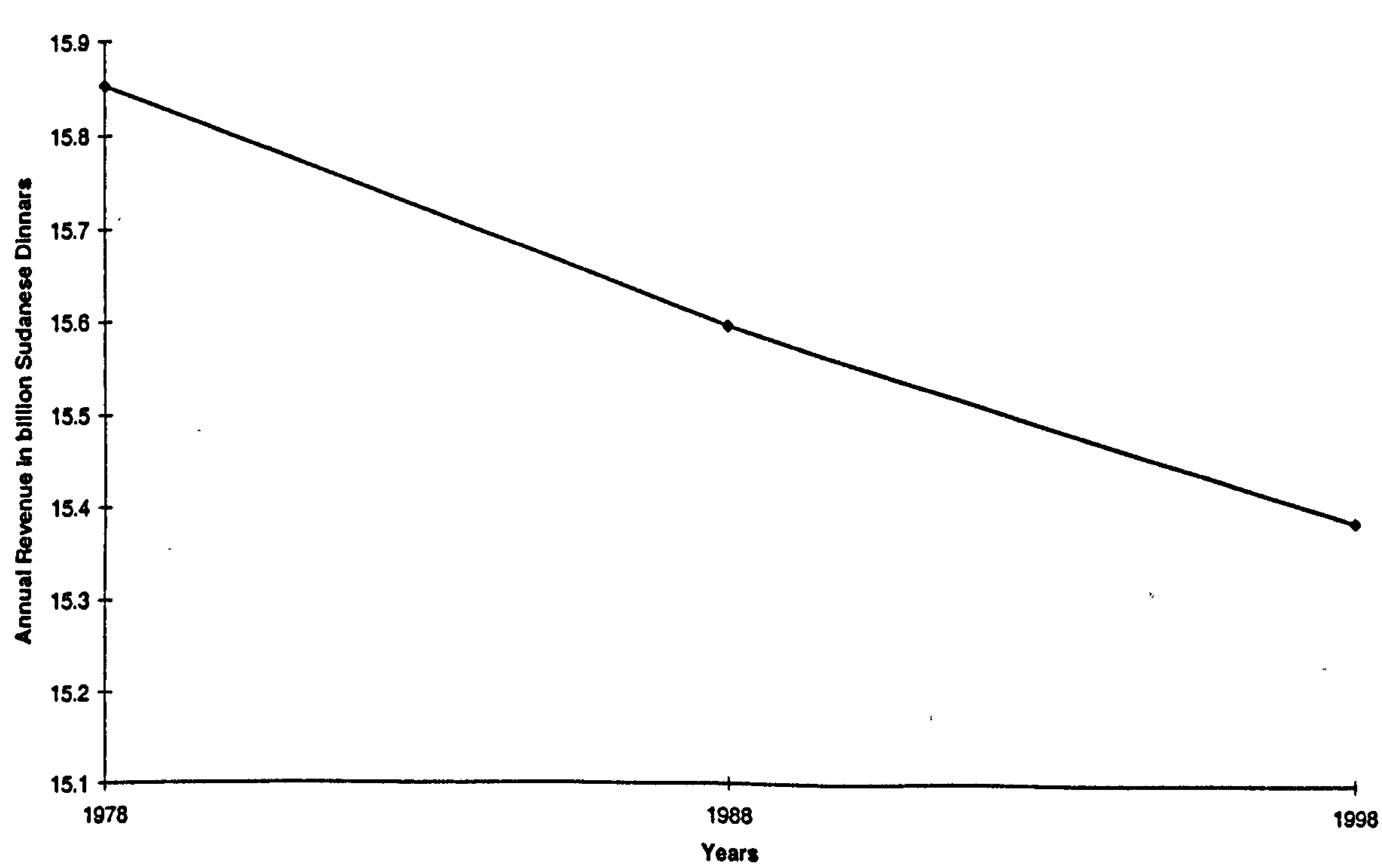


Figure (11.5) Effect of using irrigation water efficiently on power production

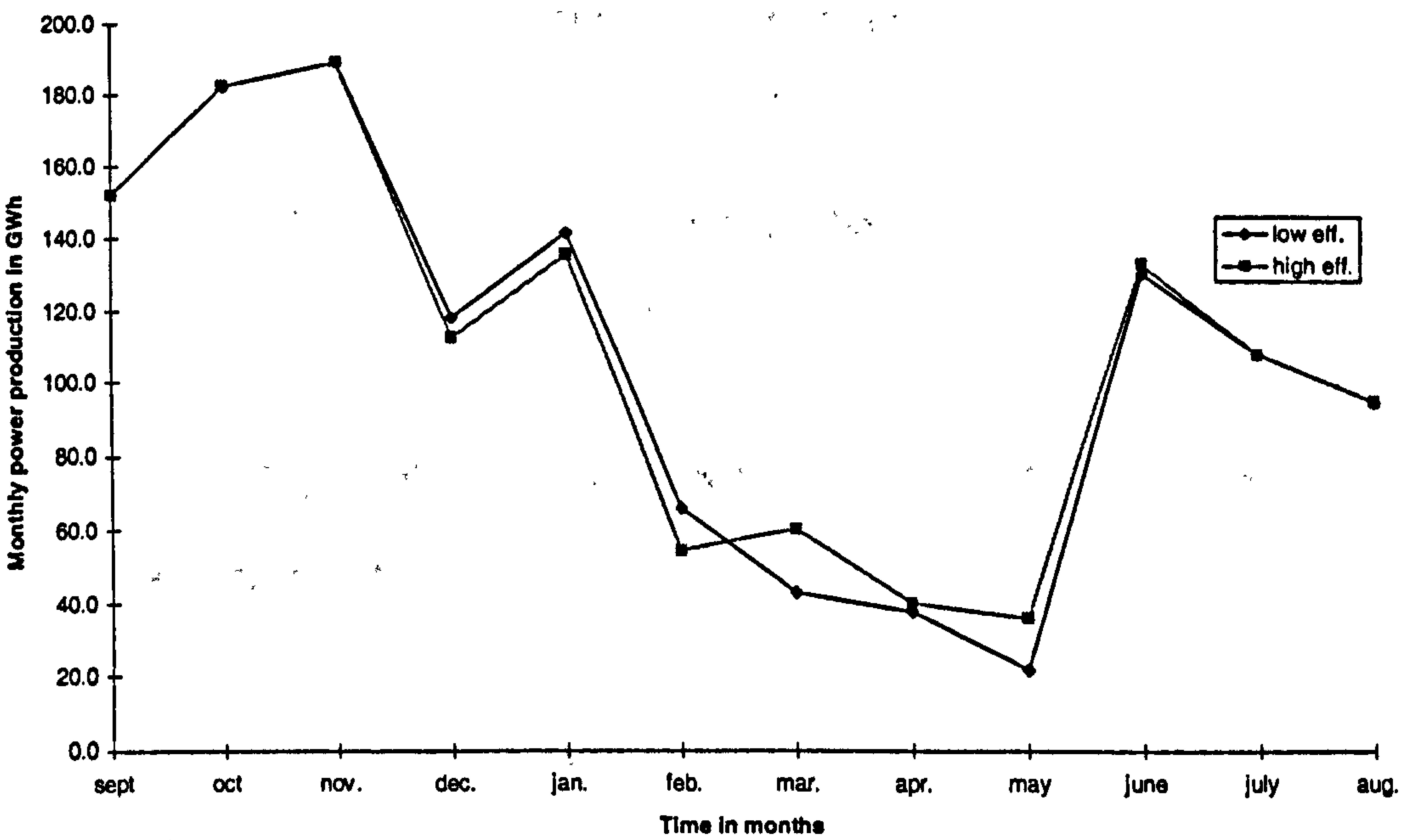
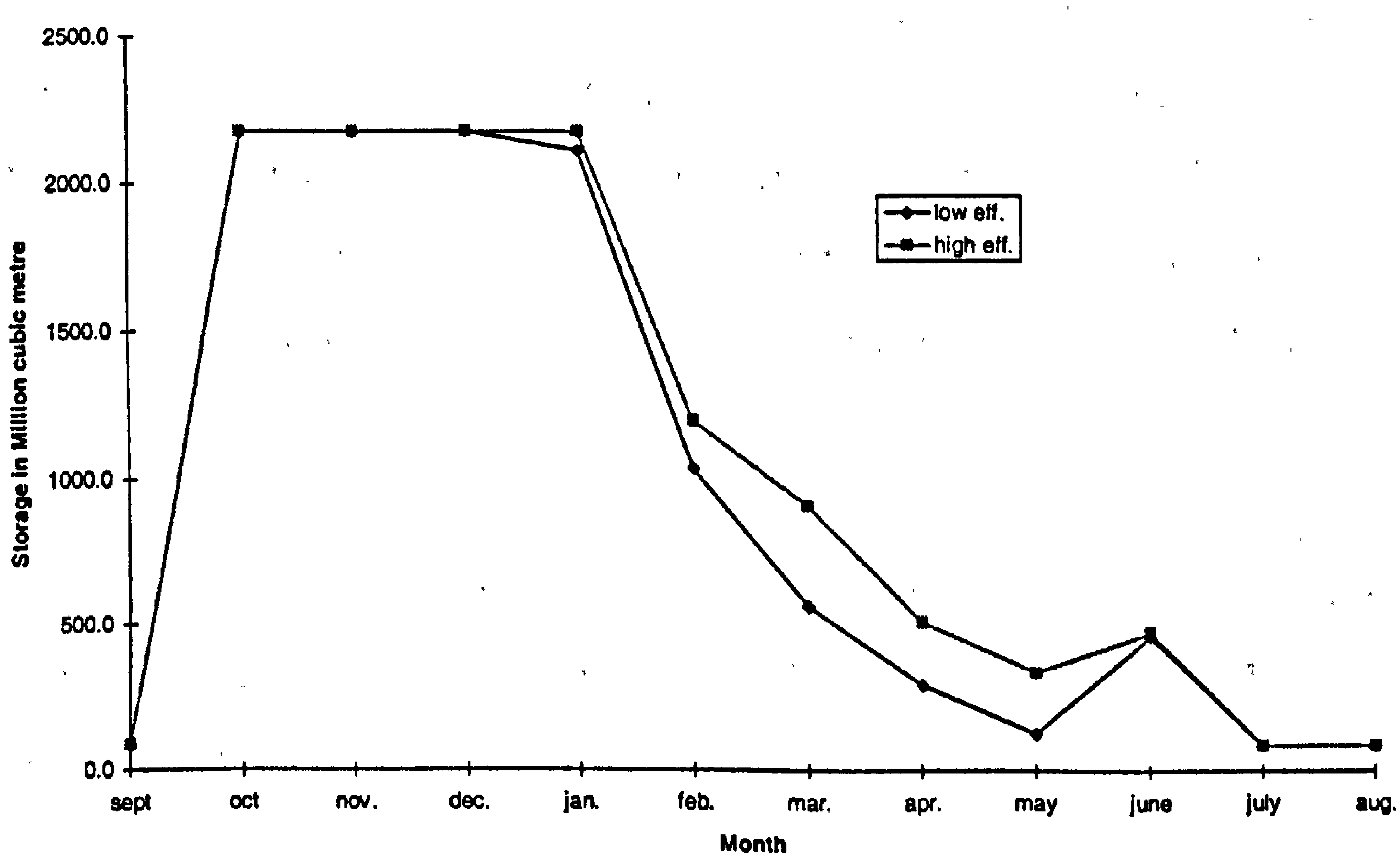


Figure (11.6) Effect of water use efficiently on storage - Roseries



CHAPTER XII

CONCLUSIONS

Summary ~ This Chapter describes the outcome of the research and recommendations for further research.

12.1 THE NEED FOR NON-LINEAR OPTIMIZATION TECHNIQUES APPLICATION

Optimization models determine values for a set of decision variables that maximise or minimise an objective function subject to constraints. For a reservoir operation problem the decision variables are release rates and end of period storage. Constraints include storage capacities, other physical characteristics of the reservoir/ river system, diversion or stream flow requirements and mass balance. In a system with multiple reservoirs and with decisions made monthly, weekly, daily or hourly, the number of the decision variables becomes very large. The objective function and constraints are represented by mathematical expressions which are functions of the decision variables. Modelling of evaporation and/or of power generation result in non-linear objective and/or constraint functions. Therefore a reservoir system operation problem can be described as large-scale non-linear.

The main three mathematical optimization techniques that have been applied to reservoir systems are: Linear Programming (LP), Dynamic Programming (DP) and the non-linear Programming (NLP). Although a reservoir optimization problem is non-linear, the application of NLP to reservoirs is very limited. LP and DP are the widely applied techniques. In LP the non-linear functions are linearised. As a result an approximate solution is obtained. To deal with dimensionality in DP, the state variables are discretized. The discretization improves the dynamic programming performance at the expense of accuracy. Therefore the approximate solution is obtained. If the reservoir optimization problem is to be closely modelled, then application of other techniques have to be sought. Since the reservoir optimization problem is identified as non-linear, then NLP is the most appealing technique.

12.2 STUDY HYPOTHESES AND OBJECTIVES

The output from optimum reservoir operation is affected by the variations in inflow, amount of sediment trapped, variation in demand due to efficiency of water use for one or more purposes, evaporation losses and the optimization techniques used to reach the solution. These considerations are reflected in the study hypotheses.

Application of non-linear programming in reservoir operation is problematic. The high dimensionality represents a problem in the application of NLP. Recent research in mathematical programming has addressed this problem. As a result software packages have been developed. Despite that the application of these techniques to reservoir systems remains limited. A trial is made here to apply the most suitable and efficient techniques to a major river system and this forms the basis for hypothesis (5) and objectives (1) and (2)

Due to the complexity of reservoir optimization problems, usually they are simplified by not considering all the issues involved at a time. However it is claimed here that most of the issues involved such as demand modelling, sedimentation effect, flow uncertainty, evaporation losses can be incorporated in or linked to the optimization model and this forms the basis for hypotheses (1), (2), (3), and (4).

Optimization models compute releases that maximise or minimise the objective function without tackling the details of the operating rules. No trials have been made to derive operation rules out of non-linear optimization results. General operation rules are needed more than computed releases corresponding to specified stream flow sequences. This forms the basis for hypothesis (6) and objectives (2) and (3).

12.3 HYPOTHESES VERIFICATION

This study is concerned with the application of non-linear optimization techniques to reservoir systems. The outputs from these applications are influenced by many factors. At the beginning of the research, hypotheses were made to reflect the effect of these issues on the outcome. Therefore these hypotheses have to be verified.

12.3.1 <u>Hypothesis 1</u>

Hypothesis 1 reads " *In a multipurpose reservoir system, where water is released for irrigation and hydropower generation, inappropriate water supply to irrigation schemes can be identified and reallocated to increase provisions for power generation*". This hypothesis proved more difficult to investigate through the Blue Nile case study than expected. The dependence of farmers on irrigation makes it necessary to ensure that reservoir releases satisfy the irrigation requirements. Therefore this was made a constraint in the optimization model, and the trade off between irrigation and power generation was not directly investigated.

Analysis of existing data however revealed oversupply of water to some parts of the Gezira irrigation scheme, indicated by low application efficiencies. The analysis, in Chapter VIII, reveals drastic change in water supplied to canals, going from the head to the tail of the Gezira scheme. The application efficiency drops from 54.6 % at the head of the scheme to only 33% at the tail. This is a direct result of the oversupply in water. In the head canal, the unit area, feddan, receives 2801 m^3 of water while it receives 6501 m^3 in the tail canal. This inappropriate water supply can be saved and used for power generation. The effect of reducing the oversupply on power production was investigated using the optimization model. It was found that this would reduce power generation in the September to February period due to lower releases, but the resulting storage allowed increased power generation in the March to August period, giving an overall increase in power and power revenues (Chapter XI). These findings verify hypothesis 1, while also show the need to take account of practical restrictions and complex interaction between irrigation releases and power generation.

12.3.2 Hypothesis 2

Hypothesis 2 reads "*Sedimentation effect on reservoir storage-water level relationship can be modelled. By linking this sedimentation model to the developed optimization model, effect of sedimentation on optimum reservoir operation can be investigated.*"
In Chapter V, the reservoir sedimentation process is modelled through fitting the relationship between the reservoir storage, S, and the water level, H. Data from different surveys are used for this purpose. The fitted model is then verified using sediment samples and discharges measured upstream and downstream the reservoir. The form of the storage-water level relationship is $S=aH^m$. It has been found that, both coefficients "a" and "m" vary with time. Coefficient "a" decreases with time in a power form while "m" increases logarithmically. The fitted model is used in formulating the optimization problem in Chapter IX. Also the fitted model is linked to the optimization model to investigate the effect of sedimentation on optimum reservoir output (Chapter XI). These findings verify hypothesis 2.

12.3.3 Hypothesis 3

Hypothesis 3 reads " *The stochastic nature of inflow can be implicitly incorporated in an optimization problem by synthetically generating inflows. (This approach does not consider the impact of droughts and low flow clusters on optimization, but in the Blue Nile System droughts do not affect the filling of reservoirs which have small capacities while low flow clusters have no effect due to the operation of the system on annual basis*").
Blue Nile monthly flow has been modelled (Chapter VII), using a 30 year flow record. The goodness of fit test, Akaike Information Criterion, AIC, is performed to select among the competent models that passed normality and independence tests. ARMA(1,1) model has given the best fitting. This model is used to generate synthetic samples. The generated samples preserve the original sample mean and standard deviation during high and low flow periods. Low flow clusters and drought effects are not built in this model. These issues have very little effect in operation of the Blue Nile system. Droughts do not affect the filling of reservoirs which have low capacities while

low flow clusters has no effect due to the operation of the system on annual basis. The generated samples have been used as inputs to the optimization model. In Chapter X different samples have been used to solve the optimization model repeatedly and the output is used to derive operation rules. These findings verify hypothesis 3.

12.3.4 Hypothesis 4

Hypothesis 4 reads "*Evaporation losses can be modelled and incorporated in an optimization problem*".

Evaporation losses from Roseries have been modelled using Penman equation which estimates the evaporation rate in mm/day. Multiplying this rate by the water surface area, the total evaporation losses are found. Thus a model that estimates the total evaporation losses has been fitted (Chapter VI). Alternatively, the monthly losses have been estimated from water balance using data collected by Roseries reservoir resident engineer. The two results compare well. Therefore Penman approach is used in modelling the evaporation losses from the Blue Nile system. Similarly a model is developed for Sennar. The models are used in formulating the optimization problem developed in Chapter IX. These findings verify hypothesis 4.

12.3.5 Hypothesis 5

Hypothesis 5 reads" *Non-linear programming, NLP, techniques can be applied to reservoir system real problems*".

In Chapter IX, the Augmented Lagrangian and Conjugate Gradient methods were used to solve the optimization problem, that maximises the revenues of the power generated from two reservoirs in series, on condition that certain irrigation and downstream requirements be satisfied. This finding verifies hypothesis 5.

12.3.6 Hypothesis 6

Hypothesis 6 reads "*Regression analysis can be used to derive operation rules out of the non-linear optimization results*".

Linear and non-linear regression models have been used to derive operation rules for multipurpose operation of two reservoirs in series, using optimization output (Chapter X). The derived rules have been tested statistically and using simulation. These findings verify hypothesis 6.

12.4 OBJECTIVES

A non-linear model has been formulated for the Blue Nile System which has two reservoirs in series (Chapter IX). The objective is to maximise power revenues on conditions that irrigation and downstream requirements be satisfied. The non-linear objective function is function of 72 decision variables which are the monthly releases for power, releases through other gates and end of month storage volumes for the two reservoirs. The problem is highly constrained to satisfy reservoir's mass balance equations and to satisfy minimum downstream flows and irrigation requirement. In total the problem has 24 non-linear and 24 linear constraints. The linear and non-linear constraints together with the objective function have been used to build the Augmented Lagrangian function. Maximum gates' capacities and minimum and maximum operation levels impose bounds on operation. These bounds constitute the feasible region in which the search for the optimum solution is conducted. The formulated algorithm uses synthetically generated flows and deterministic irrigation requirements as inputs, incorporates non-linear power and evaporation functions and is linked to a sedimentation model that predicts the reservoir storage-level relationship.

To solve the problem, the most suitable non-linear programming techniques have been used. These, as reached in Chapter III, are the Augmented Lagrangian Multiplier and Conjugate gradient methods. A general purpose FORTRAN Software package that use these methods, named Lancelot, is used. The package is designed for the solution of large scale non-linear optimization problems. The algorithms are designed to achieve convergence from all starting points. The package transforms inequality constraints into equality constraints by adding slack or surplus variables. Maximisation problems can be transferred into minimisation ones. The package is friendly used since the optimization problem is defined and the techniques to write the standard input format and the specification files are understood. To be used for solving the optimization problem, only the standard input format and specification file have been prepared and no changes in the package subroutines are required. The SIF file, for the optimization problem, has been prepared in a way that allows changes in the optimization problem. Changes in the objective function, inflows to the reservoir system and irrigation requirements can easily be made.

The problem is solved in few minutes, when average inflows are used as inputs. The problem is solved without any simplification, i.e. linearisation, decomposition, discretization or aggregation, usually used to alleviate the effects of nonlinearity and dimensionality associated with reservoir optimization. Therefore it can be concluded that non-linear programming can be applied successfully, without simplifications, to multipurpose multireservoir systems and this fulfils objectives 1 and 2.

In optimization there is a gap between theory and practice and techniques that practically use the optimization output have to be applied. Therefore, in reservoir operation, general operation rules are needed more than computed releases corresponding to specified flow sequences. To achieve this, the optimization model is solved repeatedly using different generated flow sequences. The optimum releases are then regressed linearly and nonlinearly on the important independent variables, flows and/or storage volumes, to derive operation rules. The derived rules have been tested successfully both statistically using R^2 criterion and simulation. Previous applications used the same regression models on dynamic programming output. Comparisons of R^2 values indicates that the application of these regression models on non-linear optimization output gives better results than their application on dynamic programming output. To be easily used in practice the rules are presented in a graphical form. Knowing the current flow and the storage volumes at the beginning of the month, these rules can be used to decide the optimal monthly releases without any need to run the software. These findings fulfil objective 3.

12.5 ACHIEVEMENTS AND RECOMMENDATIONS

12.5.1 Achievements

The achievements of this study can be summarised as follows:

* In this study the most suitable and efficient non-linear programming techniques for application to multipurpose reservoir systems have been identified.

* Submodels necessary for the optimization problem formulation have been developed. These include models for sedimentation, flow and estimation of evaporation losses.

* Development and solution of an optimization model using Software Lancelot. The software uses a combination of two methods, the Augmented Lagrangian Multiplier and the Conjugate Gradient, which have not been applied to reservoir systems before. The methods have been applied to a problem formulated for a major river system. The system has two in series reservoirs used for irrigation, low flow augmentation and hydropower generation. Optimization objectives were to maximise the power benefits on condition that other requirements are met first. The model increased the power benefits obtained from the current operation policy for year 1993/1994 by 13.92 %.
All the issues that affect the optimization results, such as sedimentation, evaporation and flow uncertainty are either directly incorporated in or linked to the optimization model. No decomposition, aggregation, linearisation or discretization have been used to simplify the problem. Most of the previously applied mathematical programming techniques used one or more of these techniques to simplify and solve reservoirs' optimization problems. Although the problem was not simplified, the solution was obtained in only few minutes. Obtaining the solution in this short time is a real achievement, as non-linear programming is generally criticised for being slow.

* In practice general operation rules are needed more than computed releases corresponding to specified stream flow sequences. Previous applications mixed implicit dynamic programming, linear regression analysis and simulation to derive and test

operation rules. In this study implicit non-linear programming, linear and non-linear regression analysis and simulation have been mixed to derive and test operation rules. No previous attempt had been made to derive operation rules using non-linear optimization output. Fitting linear and non-linear regression models to non-linear optimization output has given better results (higher R^2 values) than previous applications of these regression models to dynamic programming output. These previous applications derived operation rules for a single reservoir while in this study rules have been derived for a reservoir system.

* Development of an approach in which the developed sedimentation and optimization models have been linked to investigate the effect of reservoir sedimentation on reservoir optimum output.

* Development of an approach which is applicable in multipurpose reservoir systems, where priority is given for one purpose over the others. In this approach the efficiency of water use for the purpose with the highest priority and the effect of that on the output from other purposes are investigated.

*It is clearly shown, in this study, that the problematic non-linear programming techniques can be applied to multipurpose reservoir systems without any simplifications that may affect the accuracy. Also it has been shown that the developed model is a useful tool in investigating the effect of sedimentation and water use on reservoir optimum output and in deriving operation rules, when mixed with regression.

12.5.2 Recommendation for Further Research

For further and future research the following recommendations can be made:

1) In this research the monthly optimum releases and storage volumes have been determined. This is a typical example of the kind of studies known as strategic problems. It is recommended that the current research is extended to cover the tactical

problems. This type of studies looks at the short term; hourly, daily or weekly operation and uses the output of the strategic problem as boundary conditions.

2) The effect of sedimentation on reservoir optimum operation is modelled through fitting the storage-water level relationship. Alternatively sedimentation could have been modelled mathematically. Therefore it is recommended that the alternative approach may be adopted in future research.

3) In the considered case study the objective was to maximise the power benefits. It is recommended that other objectives, shown in Chapter II, be reflected in the objective function when non-linear optimization techniques are used in future research.

4) There are three common applications of mathematical programming, mainly dynamic programming, in water resources planning. These are concerned with water allocations, capacity expansion and reservoir operation. Here non-linear programming has been applied to a reservoir system operation problem. Application of the techniques used here to the other two previously mentioned areas is recommended.

5) The problem dealt with in this research is an operational one. In the considered system, priority is given to irrigation. Supplying irrigation requirements act as constraints to the maximisation of other objectives. Therefore they have been reflected in the constraints when the optimization problem is formulated. Since certain irrigation requirements have to be supplied, irrigation requirements have been inputted to the optimization model as constants. If priority is not given to irrigation, and it is required to maximise the return of the multipurpose reservoir system, then the irrigation releases will be considered as decision variables. Solving a problem with variable irrigation and hydropower requirements will provide the optimum hydropower releases as well as the optimum irrigation releases. The irrigation releases depend, among others, on crops, their areas and water prices. Thus reflecting these issues in an optimization problem will allow planning for crop types, areas and water prices. It is recommended to apply the techniques and the software used in this research to study these issues in a further research.

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APPENDIX A

This appendix shows the results of calculating Roseries daily evaporation losses using Penman approach for the period July 1993 - June 1994.

date	u/s Level m	area m^2	Losses million m^3	date	u/s level m	area m^2	Losses million m^3
01/07/93	469.08	1.21E+08	0.313338	01/01/94	480.92	3.02E+08	2.146457
02/07/93	469.88	1.29E+08	0.334234	02/01/94	480.92	3.02E+08	2.146457
03/07/93	469.33	1.23E+08	0.319696	03/01/94	480.92	3.02E+08	2.146457
04/07/93	469.22	1.22E+08	0.316879	04/01/94	480.91	3.02E+08	2.144963
05/07/93	468.34	1.14E+08	0.295435	05/01/94	480.91	3.02E+08	2.144963
06/07/93	467.75	1.09E+08	0.282142	06/01/94	480.9	3.02E+08	2.14347
07/07/93	467.63	1.08E+08	0.279545	07/01/94	480.9	3.02E+08	2.14347
08/07/93	467.62	1.07E+08	0.27933	08/01/94	480.92	3.02E+08	2.146457
09/07/93	467.6	1.07E+08	0.278901	09/01/94	480.91	3.02E+08	2.144963
10/07/93	467.26	1.05E+08	0.271763	10/01/94	480.9	3.02E+08	2.14347
11/07/93	467.7	1.08E+08	0.281055	11/01/94	480.89	3.02E+08	2.141977
12/07/93	467.48	1.06E+08	0.276349	12/01/94	480.87	3.01E+08	2.138993
13/07/93	467.48	1.06E+08	0.276349	13/01/94	480.85	3.01E+08	2.136013
14/07/93	467.43	1.06E+08	0.275296	14/01/94	480.81	3E+08	2.130059
15/07/93	467.64	1.08E+08	0.27976	15/01/94	480.79	3E+08	2.127087
16/07/93	467.18	1.04E+08	0.270126	16/01/94	480.75	2.99E+08	2.12115
17/07/93	467.56	1.07E+08	0.278046	17/01/94	480.72	2.98E+08	2.116704
18/07/93	467.66	1.08E+08	0.28019	18/01/94	480.7	2.98E+08	2.113744
19/07/93	467.7	1.08E+08	0.281055	19/01/94	480.67	2.97E+08	2.109309
20/07/93	467.32	1.05E+08	0.273002	20/01/94	480.65	2.97E+08	2.106355
21/07/93	467.4	1.06E+08	0.274667	21/01/94	480.6	2.96E+08	2.098984
22/07/93	467.6	1.07E+08	0.278901	22/01/94	480.6	2.96E+08	2.098984
23/07/93	467.52	1.07E+08	0.277195	23/01/94	480.57	2.95E+08	2.094569
24/07/93	467.4	1.06E+08	0.274667	24/01/94	480.56	2.95E+08	2.093099
25/07/93	467.57	1.07E+08	0.27826	25/01/94	480.54	2.94E+08	2.09016
26/07/93	467.3	1.05E+08	0.272588	26/01/94	480.53	2.94E+08	2.088692
27/07/93	467.5	1.06E+08	0.276772	27/01/94	480.53	2.94E+08	2.088692
28/07/93	467.45	1.06E+08	0.275716	28/01/94	480.51	2.94E+08	2.085758
29/07/93	467.6	1.07E+08	0.278901	29/01/94	480.49	2.93E+08	2.082826
30/07/93	467.4	1.06E+08	0.274667	30/01/94	480.46	2.93E+08	2.078434
31/07/93	467.48	1.06E+08	0.276349	31/01/94	480.46	2.93E+08	2.078434
01/08/93	467.42	1.06E+08	0.222185	01/02/94	480.39	2.91E+08	2.301247
02/08/93	467.5	1.06E+08	0.223546	02/02/94	480.39	2.91E+08	2.301247
03/08/93	467.63	1.08E+08	0.225786	03/02/94	480.38	2.91E+08	2.299625
04/08/93	467.93	1.1E+08	0.231085	04/02/94	480.35	2.9E+08	2.294763
05/08/93	467.75	1.09E+08	0.227884	05/02/94	480.3	2.89E+08	2.286675
06/08/93	467.42	1.06E+08	0.222185	06/02/94	480.28	2.89E+08	2.283445
07/08/93	467.23	1.04E+08	0.219004	07/02/94	480.2	2.87E+08	2.270555
08/08/93	467.73	1.08E+08	0.227532	08/02/94	480.15	2.86E+08	2.262524
09/08/93	467.52	1.07E+08	0.223889	09/02/94	480.07	2.85E+08	2.249713
10/08/93	467.32	1.05E+08	0.220502	10/02/94	480	2.83E+08	2.238544
11/08/93	467.53	1.07E+08	0.22406	11/02/94	479.9	2.81E+08	2.222652
12/08/93	467.4	1.06E+08	0.221847	12/02/94	479.89	2.81E+08	2.221067
13/08/93	467.7	1.08E+08	0.227006	13/02/94	479.83	2.8E+08	2.211574

14/08/93	467.72	1.08E+08	0.227357	14/02/94	479.8	2.79E+08	2.206837
15/08/93	467.48	1.06E+08	0.223205	15/02/94	479.76	2.79E+08	2.200532
16/08/93	467.65	1.08E+08	0.226134	16/02/94	479.73	2.78E+08	2.195811
17/08/93	467.72	1.08E+08	0.227357	17/02/94	479.71	2.78E+08	2.192668
18/08/93	467.67	1.08E+08	0.226482	18/02/94	479.69	2.77E+08	2.189527
19/08/93	467.76	1.09E+08	0.22806	19/02/94	479.68	2.77E+08	2.187958
20/08/93	467.68	1.08E+08	0.226657	20/02/94	479.64	2.76E+08	2.18169
21/08/93	467.9	1.1E+08	0.230547	21/02/94	479.56	2.75E+08	2.169189
22/08/93	467.66	1.08E+08	0.226308	22/02/94	479.48	2.73E+08	2.156738
23/08/93	467.7	1.08E+08	0.227006	23/02/94	479.44	2.72E+08	2.15053
24/08/93	467.76	1.09E+08	0.22806	24/02/94	479.34	2.7E+08	2.135064
25/08/93	467.6	1.07E+08	0.225266	25/02/94	479.25	2.69E+08	2.121209
26/08/93	467.7	1.08E+08	0.227006	26/02/94	479.15	2.67E+08	2.105888
27/08/93	467.7	1.08E+08	0.227006	27/02/94	479.07	2.65E+08	2.093685
28/08/93	467.72	1.08E+08	0.227357	28/02/94	479	2.64E+08	2.083048
29/08/93	467.84	1.09E+08	0.229476	01/03/94	478.9	2.62E+08	2.329677
30/08/93	467.72	1.08E+08	0.227357	02/03/94	478.83	2.6E+08	2.317795
31/08/93	467.75	1.09E+08	0.227884	03/03/94	478.75	2.59E+08	2.304267
01/09/93	467.72	1.08E+08	0.270663	04/03/94	478.69	2.58E+08	2.294157
02/09/93	467.86	1.09E+08	0.27361	05/03/94	478.67	2.57E+08	2.290794
03/09/93	467.75	1.09E+08	0.27129	06/03/94	478.55	2.55E+08	2.270686
04/09/93	467.8	1.09E+08	0.272341	07/03/94	478.46	2.53E+08	2.255687
05/09/93	467.53	1.07E+08	0.266738	08/03/94	478.42	2.53E+08	2.249042
06/09/93	467.7	1.08E+08	0.270245	09/03/94	478.35	2.51E+08	2.237448
07/09/93	468.2	1.12E+08	0.280963	10/03/94	478.26	2.5E+08	2.222602
08/09/93	467.6	1.07E+08	0.268174	11/03/94	478.16	2.48E+08	2.206189
09/09/93	467.48	1.06E+08	0.26572	12/03/94	478.1	2.47E+08	2.196382
10/09/93	467.48	1.06E+08	0.26572	13/03/94	478.02	2.45E+08	2.183353
11/09/93	467.47	1.06E+08	0.265517	14/03/94	477.96	2.44E+08	2.173618
12/09/93	467.58	1.07E+08	0.267763	15/03/94	477.9	2.43E+08	2.163914
13/09/93	467.69	1.08E+08	0.270037	16/03/94	477.82	2.42E+08	2.151022
14/09/93	467.52	1.07E+08	0.266534	17/03/94	477.76	2.41E+08	2.14139
15/09/93	467.67	1.08E+08	0.269621	18/03/94	477.7	2.4E+08	2.131788
16/09/93	467.7	1.08E+08	0.270245	19/03/94	477.61	2.38E+08	2.117443
17/09/93	467.53	1.07E+08	0.266738	20/03/94	477.54	2.37E+08	2.106334
18/09/93	467.56	1.07E+08	0.267352	21/03/94	477.45	2.35E+08	2.092113
19/09/93	467.5	1.06E+08	0.266127	22/03/94	477.37	2.34E+08	2.07953
20/09/93	468	1.11E+08	0.276604	23/03/94	477.33	2.33E+08	2.073259
21/09/93	469.14	1.21E+08	0.30274	24/03/94	477.21	2.31E+08	2.054528
22/09/93	470.4	1.34E+08	0.335263	25/03/94	477.15	2.3E+08	2.045209
23/09/93	471.42	1.46E+08	0.364388	26/03/94	477.07	2.28E+08	2.032831
24/09/93	472.32	1.57E+08	0.392163	27/03/94	477	2.27E+08	2.022045
25/09/93	473.08	1.67E+08	0.417134	28/03/94	476.99	2.27E+08	2.020508
26/09/93	473.93	1.79E+08	0.446708	29/03/94	476.86	2.25E+08	2.0006
27/09/93	474.6	1.88E+08	0.471244	30/03/94	476.78	2.23E+08	1.988421
28/09/93	475.22	1.98E+08	0.49491	31/03/94	476.69	2.22E+08	1.974785
29/09/93	475.8	2.07E+08	0.517886	01/04/94	476.59	2.2E+08	1.915677
30/09/93	476.34	2.16E+08	0.540004	02/04/94	476.47	2.18E+08	1.89811
01/10/93	476.95	2.26E+08	0.769533	03/04/94	476.33	2.16E+08	1.877767
02/10/93	477.37	2.34E+08	0.794427	04/04/94	476.25	2.15E+08	1.866217
03/10/93	477.77	2.41E+08	0.818672	05/04/94	476.16	2.13E+08	1.853286
04/10/93	478.12	2.47E+08	0.840315	06/04/94	476.08	2.12E+08	1.841849

05/10/93	478.16	2.48E+08	0.842814	07/04/94	476	2.1E+08	1.830466
06/10/93	478.1	2.47E+08	0.839067	08/04/94	475.92	2.09E+08	1.819136
07/10/93	478.07	2.46E+08	0.837198	09/04/94	475.84	2.08E+08	1.80786
08/10/93	478.05	2.46E+08	0.835954	10/04/94	475.74	2.06E+08	1.79384
09/10/93	478.1	2.47E+08	0.839067	11/04/94	475.63	2.04E+08	1.778515
10/10/93	478.44	2.53E+08	0.860453	12/04/94	475.46	2.02E+08	1.75503
11/10/93	478.79	2.6E+08	0.882863	13/04/94	475.37	2E+08	1.742694
12/10/93	479.08	2.65E+08	0.901735	14/04/94	475.22	1.98E+08	1.722286
13/10/93	479.28	2.69E+08	0.91491	15/04/94	475.05	1.95E+08	1.699384
14/10/93	479.57	2.75E+08	0.934247	16/04/94	474.86	1.92E+08	1.674073
15/10/93	479.75	2.78E+08	0.946387	17/04/94	474.7	1.9E+08	1.652994
16/10/93	479.93	2.82E+08	0.958633	18/04/94	474.58	1.88E+08	1.637325
17/10/93	480.09	2.85E+08	0.969607	19/04/94	474.49	1.87E+08	1.625652
18/10/93	480.21	2.88E+08	0.977893	20/04/94	474.32	1.84E+08	1.603788
19/10/93	480.32	2.9E+08	0.98553	21/04/94	474.16	1.82E+08	1.583432
20/10/93	480.4	2.92E+08	0.991109	22/04/94	474	1.8E+08	1.563289
21/10/93	480.48	2.93E+08	0.996708	23/04/94	473.87	1.78E+08	1.547081
22/10/93	480.54	2.94E+08	1.000922	24/04/94	473.73	1.76E+08	1.529784
23/10/93	480.6	2.96E+08	1.005147	25/04/94	473.59	1.74E+08	1.512652
24/10/93	480.67	2.97E+08	1.010092	26/04/94	473.46	1.72E+08	1.49689
25/10/93	480.74	2.99E+08	1.015052	27/04/94	473.33	1.7E+08	1.481269
26/10/93	480.79	3E+08	1.018605	28/04/94	473.21	1.69E+08	1.466976
27/10/93	480.74	2.99E+08	1.015052	29/04/94	473.02	1.66E+08	1.444591
28/10/93	480.8	3E+08	1.019317	30/04/94	472.85	1.64E+08	1.424818
29/10/93	480.84	3.01E+08	1.022166	01/05/94	472.64	1.61E+08	1.288025
30/10/93	480.86	3.01E+08	1.023593	02/05/94	472.47	1.59E+08	1.27034
31/10/93	480.88	3.01E+08	1.025021	03/05/94	472.11	1.54E+08	1.233625
01/11/93	480.53	2.94E+08	2.088692	04/05/94	472.17	1.55E+08	1.239675
02/11/93	480.54	2.94E+08	2.09016	05/05/94	472	1.53E+08	1.222605
03/11/93	480.92	3.02E+08	2.146457	06/05/94	471.94	1.52E+08	1.216633
04/11/93	480.96	3.03E+08	2.152441	07/05/94	471.89	1.51E+08	1.211678
05/11/93	480.97	3.03E+08	2.153938	08/05/94	471.78	1.5E+08	1.200845
06/11/93	480.98	3.04E+08	2.155436	09/05/94	471.74	1.5E+08	1.196928
07/11/93	480.99	3.04E+08	2.156935	10/05/94	471.68	1.49E+08	1.191077
08/11/93	480.99	3.04E+08	2.156935	11/05/94	471.59	1.48E+08	1.182351
09/11/93	481	3.04E+08	2.158435	12/05/94	471.5	1.47E+08	1.173688
10/11/93	481	3.04E+08	2.158435	13/05/94	471.4	1.46E+08	1.164136
11/11/93	480.99	3.04E+08	2.156935	14/05/94	471.4	1.46E+08	1.164136
12/11/93	481	3.04E+08	2.158435	15/05/94	471.38	1.45E+08	1.162234
13/11/93	480.99	3.04E+08	2.156935	16/05/94	471.31	1.44E+08	1.155604
14/11/93	481	3.04E+08	2.158435	17/05/94	471.22	1.43E+08	1.147135
15/11/93	481	3.04E+08	2.158435	18/05/94	471.14	1.42E+08	1.139659
16/11/93	480.99	3.04E+08	2.156935	19/05/94	471.04	1.41E+08	1.130384
17/11/93	480.98	3.04E+08	2.155436	20/05/94	470.98	1.41E+08	1.124855
18/11/93	481	3.04E+08	2.158435	21/05/94	470.8	1.39E+08	1.108437
19/11/93	480.99	3.04E+08	2.156935	22/05/94	470.68	1.37E+08	1.097629
20/11/93	481	3.04E+08	2.158435	23/05/94	470.65	1.37E+08	1.094945
21/11/93	481	3.04E+08	2.158435	24/05/94	470.56	1.36E+08	1.086933
22/11/93	481	3.04E+08	2.158435	25/05/94	470.38	1.34E+08	1.071095
23/11/93	480.99	3.04E+08	2.156935	26/05/94	470.28	1.33E+08	1.062405
24/11/93	480.97	3.03E+08	2.153938	27/05/94	470.15	1.31E+08	1.051222
25/11/93	481	3.04E+08	2.158435	28/05/94	470.13	1.31E+08	1.049513

26/11/93	481	3.04E+08	2.158435	29/05/94	470.13	1.31E+08	1.049513
27/11/93	480.99	3.04E+08	2.156935	30/05/94	470.13	1.31E+08	1.049513
28/11/93	481	3.04E+08	2.158435	31/05/94	470.15	1.31E+08	1.051222
29/11/93	480.99	3.04E+08	2.156935	01/06/94	470.08	1.31E+08	0.601021
30/11/93	481	3.04E+08	2.158435	02/06/94	470.1	1.31E+08	0.601999
01/12/93	481	3.04E+08	2.128034	03/06/94	469.98	1.3E+08	0.596157
02/12/93	480.99	3.04E+08	2.126556	04/06/94	470.09	1.31E+08	0.60151
03/12/93	481	3.04E+08	2.128034	05/06/94	470.08	1.31E+08	0.601021
04/12/93	481	3.04E+08	2.128034	06/06/94	470.15	1.31E+08	0.604453
05/12/93	481	3.04E+08	2.128034	07/06/94	470.34	1.33E+08	0.613876
06/12/93	480.99	3.04E+08	2.126556	08/06/94	470.68	1.37E+08	0.631137
07/12/93	480.98	3.04E+08	2.125078	09/06/94	470.86	1.39E+08	0.640482
08/12/93	481	3.04E+08	2.128034	10/06/94	470.8	1.39E+08	0.637351
09/12/93	481.01	3.04E+08	2.129513	11/06/94	470.98	1.41E+08	0.646792
10/12/93	481.01	3.04E+08	2.129513	12/06/94	471.02	1.41E+08	0.648909
11/12/93	481	3.04E+08	2.128034	13/06/94	470.95	1.4E+08	0.645208
12/12/93	480.99	3.04E+08	2.126556	14/06/94	470.87	1.39E+08	0.641005
13/12/93	481	3.04E+08	2.128034	15/06/94	470.8	1.39E+08	0.637351
14/12/93	481	3.04E+08	2.128034	16/06/94	470.65	1.37E+08	0.629593
15/12/93	481	3.04E+08	2.128034	17/06/94	470.59	1.36E+08	0.626518
16/12/93	481	3.04E+08	2.128034	18/06/94	470.59	1.36E+08	0.626518
17/12/93	481	3.04E+08	2.128034	19/06/94	470.53	1.36E+08	0.623459
18/12/93	480.98	3.04E+08	2.125078	20/06/94	470.39	1.34E+08	0.616382
19/12/93	480.99	3.04E+08	2.126556	21/06/94	470.23	1.32E+08	0.608401
20/12/93	480.99	3.04E+08	2.126556	22/06/94	470.02	1.3E+08	0.598097
21/12/93	480.99	3.04E+08	2.126556	23/06/94	469.86	1.28E+08	0.590378
22/12/93	481	3.04E+08	2.128034	24/06/94	469.79	1.28E+08	0.587036
23/12/93	481	3.04E+08	2.128034	25/06/94	469.07	1.2E+08	0.553923
24/12/93	481.01	3.04E+08	2.129513	26/06/94	468.4	1.14E+08	0.525171
25/12/93	480.99	3.04E+08	2.126556	27/06/94	468.2	1.12E+08	0.516972
26/12/93	480.99	3.04E+08	2.126556	28/06/94	468.06	1.11E+08	0.511339
27/12/93	480.98	3.04E+08	2.125078	29/06/94	467.66	1.08E+08	0.495722
28/12/93	480.97	3.03E+08	2.123601	30/06/94	467.98	1.1E+08	0.508159
29/12/93	480.95	3.03E+08	2.120649				
30/12/93	480.92	3.02E+08	2.116225				
31/12/93	480.92	3.02E+08	2.116225				

APPENDIX B

This appendix shows the results of calculating actual crop evapotranspiraataion, ET_a for different crops in the selected canal in Gezira. The calculations followed the method described in Chapter 8.7.4.

Table (B.1) Gymailia - Toman Minor - MS Cotton - (20/07//88 - 20/1/89)

period starting	Irrig Int. - days	ETm mm/day	D (m)	DSa mm/root depth	P	t' days	ETa mm/day
20/07/88	14	2.900	0.300	36	0.808	10.0	2.48
03/08/88	14	2.634	0.300	36	0.83	11.3	2.43
17/08/88	14	2.877	0.338	41	0.808	11.4	2.69
31/08/88	14	4.161	0.434	52	0.68	8.5	3.42
14/09/88	14	5.455	0.530	64	0.575	6.7	4.10
28/09/88	14	6.621	0.623	75	0.52	5.9	4.77
12/10/88	14	7.257	0.650	78	0.485	5.2	4.98
26/10/88	14	7.451	0.650	78	0.475	5.0	5.01
09/11/88	14	7.619	0.650	78	0.47	4.8	5.03
23/11/88	14	7.120	0.650	78	0.495	5.4	4.97
07/12/88	14	6.407	0.650	78	0.53	6.5	4.87
21/12/88	14	5.691	0.650	78	0.565	7.7	4.72
04/01/89	14	5.050	0.650	78	0.595	9.2	4.53
18/01/89	3	4.810	0.650	78	0.62	10.1	4.81

Table (B.2) Gymailia Sorghum - 15/6/88 - 1/10/88

period starting	Irrig Int. - days	ETm mm/day	D (m)	DSa mm/root depth	P	t' days	ETa mm/day
15/06/88	14	2.954	0.300	36	0.8	9.7	2.48
29/06/88	14	3.299	0.330	39.6	0.77	9.2	2.71
13/07/88	14	5.133	0.479	57	0.595	6.7	3.78
27/07/88	14	6.359	0.595	71	0.53	6.0	4.58
10/08/88	14	6.114	0.600	72	0.545	6.4	4.57
24/08/88	14	6.160	0.600	72	0.54	6.3	4.58
07/09/88	14	5.641	0.600	72	0.57	7.3	4.49
21/09/88	9	4.390	0.600	72	0.66	10.8	4.39

Table (B.3) Gymailia - Toman - Wad Numan - Wheat 10/11/88 - 25/2/89

period starting	Irrig Int. days	Etm mm/day	D (m)	DSa mm/ root depth	P	t' days	Eta mm/day
10/11/88	14	2.204	0.200	24	0.78	8.5	1.68
24/11/88	14	2.645	0.230	27.6	0.735	7.7	1.92
08/12/88	14	4.659	0.379	45	0.53	5.2	3.02
22/12/88	14	6.523	0.495	59	0.435	4.0	3.90
05/01/89	14	6.830	0.500	60	0.43	3.8	3.97
19/01/89	14	6.814	0.500	60	0.43	3.8	3.97
02/02/89	14	5.663	0.500	60	0.465	4.9	3.82
16/02/89	7	4.407	0.500	60	0.56	7.6	4.41

Table (B.4) Toman - Wad Numan - Sorghum - 22/6/88 - 7/10/88

period starting	Irrig Int. days	ETm mm/day	D (m)	Dsa mm/root depth	P	t' days	ETa mm/day
22/06/88	14	2.771	0.300	36	0.86	11.2	2.50
06/07/88	14	3.002	0.330	39.6	0.8	10.5	2.67
20/07/88	14	4.797	0.479	57	0.68	8.1	3.82
03/08/88	14	5.964	0.595	71	0.55	6.6	4.52
17/08/88	14	6.150	0.600	72	0.54	6.3	4.57
31/08/88	14	6.293	0.600	72	0.535	6.1	4.60
14/09/88	14	5.590	0.600	72	0.57	7.3	4.48
28/09/88	9	4.375	0.600	72	0.66	10.9	4.38

Table (B.5) Wad Numan-Cotton MS-15/7/88 - 15/1/89

period starting	Irrig Int. days	ETm mm/day	D (m)	DSa mm/root depth	P	t' days	ETa mm/day
15/07/88	14	3.007	0.300	36	0.8	9.6	2.49
29/07/88	14	2.766	0.300	36	0.785	10.2	2.43
12/08/88	14	2.997	0.338	41	0.8	10.8	2.72
26/08/88	14	4.037	0.434	52	0.7	9.0	3.41
09/09/88	14	5.454	0.530	64	0.575	6.7	4.10
23/09/88	14	6.639	0.623	75	0.52	5.9	4.77
07/10/88	14	7.146	0.650	78	0.495	5.4	4.98
21/10/88	14	7.394	0.650	78	0.47	5.0	4.99
04/11/88	14	7.540	0.650	78	0.475	4.9	5.02
18/11/88	14	7.174	0.650	78	0.49	5.3	4.98
02/12/88	14	6.364	0.650	78	0.53	6.5	4.86
16/12/88	14	5.721	0.650	78	0.565	7.7	4.73
30/12/88	14	5.056	0.650	78	0.595	9.2	4.53
13/01/89	3	4.580	0.650	78	0.64	10.9	4.58

Table (B.6) Hamza-Umuud-Fadlein Minors - Cotton MS-3/8/88 - 3/2/89

period starting	Irrig Int. days	ETm mm/day	D (m)	DSa mm/ root depth	P	t' days	ETa mm/day
03/08/88	14	2.634	0.300	36	0.83	11.3	2.43
17/08/88	14	2.563	0.300	36	0.83	11.7	2.41
31/08/88	14	3.122	0.338	41	0.79	10.3	2.74
14/09/88	14	4.260	0.434	52	0.67	8.2	3.43
28/09/88	14	5.464	0.530	64	0.575	6.7	4.10
12/10/88	14	6.738	0.623	75	0.515	5.7	4.78
26/10/88	14	7.391	0.650	78	0.47	5.0	4.98
09/11/88	14	7.653	0.650	78	0.465	4.7	5.03
23/11/88	14	7.493	0.650	78	0.475	4.9	5.01
07/12/88	14	7.007	0.650	78	0.5	5.6	4.96
21/12/88	14	6.319	0.650	78	0.535	6.6	4.86
04/01/89	14	5.675	0.650	78	0.565	7.8	4.72
18/01/89	14	5.314	0.650	78	0.585	8.6	4.62
01/02/89	3	5.090	0.650	78	0.595	9.1	5.09

Table (B.7) Hamza-Umuud-Fadlein - Sorghum - 1/7/88 - 15/10/88

period starting	Irrig Int. days	ETm mm/day	D (m)	DSa mm/root depth	P	t' days	ETa mm/day
01/07/88	14	2.556	0.300	36	0.83	11.7	2.40
15/07/88	14	2.877	0.330	39.6	0.808	11.1	2.64
29/07/88	14	4.486	0.479	57	0.65	8.3	3.70
12/08/88	14	5.836	0.595	71	0.56	6.9	4.51
26/08/88	14	6.293	0.600	72	0.535	6.1	4.60
09/09/88	14	6.386	0.600	72	0.53	6.0	4.61
23/09/88	14	5.480	0.600	72	0.58	7.6	4.46
07/10/88	9	4.302	0.600	72	0.67	11.2	4.30

Table (B.8) Hamza - Umuud - Fadlein; Ground nut - 25/6/88 - 10/11/89

period starting	Irrig Int. days	ETm mm/day	D (m)	Dsa mm/root depth	P	t' days	ETa mm/day
25/06/88	14	3.497	0.200	24	0.65	4.5	1.70
09/07/88	14	3.534	0.217	26.0	0.65	4.8	1.84
23/07/88	14	4.491	0.300	36	0.55	4.4	2.49
06/08/88	14	5.339	0.383	46	0.54	4.7	3.14
20/08/88	14	5.905	0.400	48	0.455	3.7	3.25
03/09/88	14	6.123	0.400	48	0.447	3.5	3.26
17/09/88	14	6.159	0.400	48	0.446	3.5	3.26
01/10/88	14	5.721	0.400	48	0.465	3.9	3.24
15/10/88	14	5.133	0.400	48	0.495	4.6	3.19
29/10/88	13	4.662	0.400	48	0.53	5.5	3.15

Table (B.9) Hamza - Umuud - Fadlein; Wheat 15/11/88 - 1/3/89

period starting	Irrig Int. days	ETm mm/day	D (m)	DSa mm/day	P	t' DAYS	ETa mm/day
15/11/88	14	2.193	0.200	24	0.78	8.5	1.68
29/11/88	14	2.763	0.230	27.6	0.72	7.2	1.92
13/12/88	14	4.829	0.379	45	0.52	4.9	3.04
27/12/88	14	6.505	0.495	59	0.438	4.0	3.90
10/01/89	14	6.950	0.500	60	0.425	3.7	3.98
24/01/89	14	7.135	0.500	60	0.42	3.5	4.00
07/02/89	14	5.891	0.500	60	0.455	4.6	3.85
21/02/89	7	4.210	0.500	60	0.58	8.3	4.21

Table (B.10) Tuweir - Beibash - Cotton ELS-23/8/88 - 13/3/89

period starting	Irrig Int. days	Etm mm/day	D (m)	DSa mm/root depth	P	t' days	ETa mm/day
23/08/88	14	2.609	0.3	36	0.83	11.5	2.42
06/09/88	14	2.686	0.3	36	0.82	11.0	2.44
20/09/88	14	2.749	0.302	36	0.82	10.8	2.46
04/10/88	14	3.305	0.368	44	0.77	10.3	2.94
18/10/88	14	4.373	0.468	56	0.66	8.5	3.63
01/11/88	14	5.826	0.568	68	0.56	6.6	4.36
15/11/88	14	6.951	0.666	80	0.5	5.8	5.03
29/11/88	14	7.471	0.700	84	0.475	5.3	5.27
13/12/88	14	7.457	0.700	84	0.475	5.4	5.27
27/12/88	14	7.429	0.700	84	0.48	5.4	5.27
10/01/89	14	7.149	0.700	84	0.49	5.8	5.23
24/01/89	14	7.121	0.700	84	0.495	5.8	5.23
07/02/89	14	6.996	0.700	84	0.5	6.0	5.21
21/02/89	14	6.437	0.700	84	0.53	6.9	5.11
07/03/89	7	6.044	0.700	84	0.55	7.6	6.04

Table (B.11) Tuweir - Mardi - Beibash - Sorghum - 10/7/88 - 25/10/88

period starting	Irrig Int. days	ETm mm/day	D (m)	DSa mm/root depth	P	t' days	ETa mm/day
10/07/88	14	2.388	0.300	36	0.845	12.7	2.34
24/07/88	14	2.579	0.330	39.6	0.83	12.7	2.53
07/08/88	14	4.004	0.479	57	0.7	10.0	3.61
21/08/88	14	5.899	0.595	71	0.555	6.7	4.51
04/09/88	14	6.425	0.600	72	0.53	5.9	4.62
18/09/88	14	6.410	0.600	72	0.53	6.0	4.62
02/10/88	14	5.525	0.600	72	0.575	7.5	4.47
16/10/88	9	4.385	0.600	72	0.66	10.8	4.39

Table (B.12) Tuweir - Mardi - Beibash - Ground nut - 20/6/88 - 5/11/89

period starting	Irrig Int. days	ETm mm/day	D (m)	DSa mm/root depth	P	t' days	ETa mm/day
20/06/88	14	3.609	0.200	24	0.64	4.3	1.70
04/07/88	14	3.560	0.217	26.0	0.64	4.7	1.84
18/07/88	14	4.587	0.300	36	0.54	4.2	2.49
01/08/88	14	5.670	0.383	46	0.465	3.8	3.12
15/08/88	14	5.850	0.400	48	0.455	3.7	3.24
29/08/88	14	6.051	0.400	48	0.45	3.6	3.26
12/09/88	14	6.123	0.400	48	0.448	3.5	3.26
26/09/88	14	5.763	0.400	48	0.46	3.8	3.24
10/10/88	14	5.056	0.400	48	0.498	4.7	3.18
24/10/88	13	4.566	0.400	48	0.54	5.7	3.15

Table (B.13) Tuweir - Mardi - Beibash - Wheat - 20/11/88 - 5/3/89

period starting	Irrig Int. days	Etm mm/day	D (m)	DSa mm/root depth	P	t' days	ETa mm/day
20/11/88	14	2.176	0.200	24	0.78	8.6	1.67
04/12/88	14	2.615	0.230	27.6	0.74	7.8	1.92
18/12/88	14	4.571	0.379	45	0.54	5.4	3.02
01/01/89	14	6.494	0.495	59	0.438	4.0	3.90
15/01/89	14	7.100	0.500	60	0.42	3.5	3.99
29/01/89	14	7.247	0.500	60	0.413	3.4	4.00
12/02/89	14	5.907	0.500	60	0.455	4.6	3.86
26/02/89	7	3.921	0.500	60	0.61	9.3	3.92

Table (B.14) Elmardi minor Cotton ELS-1/9/88 - 20/3/89

period starting	Irrig Int. days	Etm mm/day	D (m)	DSa mm/root depth	P	t' days	ETa mm/day
01/09/88	14	2.671	0.300	36	0.822	11.1	2.44
15/09/88	14	2.694	0.300	36	0.822	11.0	2.44
29/09/88	14	2.730	0.302	36	0.822	10.9	2.46
13/10/88	14	3.460	0.368	44	0.75	9.6	2.96
27/10/88	14	4.591	0.468	56	0.64	7.8	3.65
10/11/88	14	6.023	0.568	68	0.55	6.2	4.39
24/11/88	14	7.020	0.666	80	0.5	5.7	5.05
08/12/88	14	7.415	0.700	84	0.48	5.4	5.27
22/12/88	14	7.469	0.700	84	0.475	5.3	5.27
05/01/89	14	7.377	0.700	84	0.48	5.5	5.26
19/01/89	14	7.441	0.700	84	0.48	5.4	5.28
02/02/89	14	7.410	0.700	84	0.48	5.4	5.27
16/02/89	14	7.075	0.700	84	0.495	5.9	5.22
02/03/89	14	6.467	0.700	84	0.525	6.8	5.11
16/03/89	5	6.210	0.700	84	0.54	7.3	6.21

APPENDIX C

This appendix shows the results of crop evapotranspiration, ET_0, for different crops in the selected canals in Gezira. The results are the output of Software Cropwat. Reference crop evapotranspiration from Table(8.8), sowing dates from Table (8.14), rainfall from Table (8.5) and crop files from Tables (8.15) to (8.19) are fed to Cropwat to obtain ET_m in mm/day.

Table (C.1): ET_m in mm/day - Gymailya Minor - 1988

10 day period	MS - Cotton No rain 20/7 - 20/1	MS - Cotton rain 20/7 - 20/1	Sorghum No rain 15/6 - 1/10	Sorghum rain 15/6 - 1/10	Wheat No rain 10/11 - 25/2
Jun 1					
Jun 2			3.08	3.08	
Jun 3			2.86	2.06	
Jul 1			3.16	0.89	
Jul 2			4.43	1.03	
Jul 3	2.93	0.0	6.07	2.83	
Aug 1	2.72	0.0	6.52	3.44	
Aug 2	2.52	0.0	6.05	3.14	
Aug 3	3.02	0.54	6.19	3.71	
Sept 1	4.02	1.97	6.12	4.07	
Sept 2	5.01	3.39	5.45	3.83	
Sept 3	5.9	4.61	4.39	3.1	
Oct 1	6.78	5.81			
Oct 2	7.2	6.55			
Oct 3	7.36	6.93			
Nov 1	7.52	7.31			
Nov 2	7.69	7.69			2.21
Nov 3	7.36	7.36			2.18
Dec 1	6.8	6.8			3.11
Dec 2	6.25	6.25			4.95
Dec 3	5.8	5.8			6.40
Jan 1	5.29	5.29			6.83
Jan 2	4.81	4.81			6.83
Jan 3					6.92
Feb 1					6.22
Feb 2					4.92
Feb 3					3.38
Mar 1					
Mar 2					
Mar 3					

Table (C.2): ET_m in mm/day - Toman Minor - 1988/1989

10 day period	MS - Cotton No rain 20/7 - 20/1	MS - Cotton rain 20/7 - 20/1	Sorghum No rain 22/6 - 7/10	Sorghum rain 15/6 - 1/10	Wheat No rain 10/11 - 22/2
Jun 1					
Jun 2					
Jun 3			2.86	2.18	
Jul 1			2.61	0.65	
Jul 2			3.22	0.28	
Jul 3	2.93	0.13	4.74	1.95	
Aug 1	2.72	0.07	5.9	3.26	
Aug 2	2.52	0.02	6.05	3.55	
Aug 3	3.02	0.58	6.19	3.75	
Sept 1	4.02	1.57	6.37	3.92	
Sept 2	5.01	2.58	6.07	3.64	
Sept 3	5.9	4.06	5.11	3.27	
Oct 1	6.78	5.59	4.06	2.87	
Oct 2	7.2	6.62			
Oct 3	7.36	6.98			
Nov 1	7.52	7.33			
Nov 2	7.69	7.69			2.21
Nov 3	7.36	7.36			2.18
Dec 1	6.8	6.8			3.11
Dec 2	6.25	6.25			4.95
Dec 3	5.8	5.8			6.40
Jan 1	5.29	5.29			6.83
Jan 2	4.81	4.81			6.83
Jan 3					6.92
Feb 1					6.22
Feb 2					4.92
Feb 3					3.38
Mar 1					
Mar 2					
Mar 3					

Table (C.3): ET_m in mm/day - Wad Numan Minor - 1988/1989

10 day period	MS - Cotton No rain 15/7 - 15/1	MS - Cotton rain 15/7 - 15/1	Sorghum No rain 22/6 - 7/10	Sorghum rain 15/6 - 1/10	Wheat No rain 10/11 - 25/2
Jun 1					
Jun 2					
Jun 3			2.86	2.19	
Jul 1			2.61	0.75	
Jul 2	3.11	0.31	3.22	0.23	
Jul 3	2.93	0.09	4.74	1.52	
Aug 1	2.72	0.0	5.9	2.7	
Aug 2	2.74	0.0	6.05	2.91	
Aug 3	3.46	0.94	6.19	3.67	
Sept 1	4.47	2.62	6.37	4.52	
Sept 2	5.47	4.27	6.07	4.96	
Sept 3	6.36	5.43	5.11	4.38	
Oct 1	7.01	6.35	4.06	3.6	
Oct 2	7.2	6.81			
Oct 3	7.36	7.1			
Nov 1	7.52	7.39			
Nov 2	7.56	7.56			2.21
Nov 3	7.12	7.12			2.18
Dec 1	6.56	6.56			3.11
Dec 2	6.01	6.01			4.95
Dec 3	5.56	5.56			6.40
Jan 1	5.05	5.05			6.83
Jan 2	4.58	4.58			6.83
Jan 3					6.92
Feb 1					6.22
Feb 2					4.92
Feb 3					3.38
Mar 1					
Mar 2					
Mar 3					

Table (C.4): ET_m in mm/day - Hamza - Umuud - Fadlein - Minor - 1988/1989

10 day period	MS - Cotton No rain 15/7 - 15/1	MS - Cotton rain 15/7 - 15/1	Ground nut No rain 25/6 -10/11	Ground nut rain 25/6 -10/11	Sorghum No rain 1/7 - 15/10	Sorghum rain 1/7 - 15/10	Wheat No rain 15/11 - 1/3
Jun 1							
Jun 2							
Jun 3			3.68	2.99			
Jul 1			3.36	1.39	2.61	0.64	
Jul 2			3.44	0.48	2.42	0.0	
Jul 3			4.18	1.3	3.22	0.35	
Aug 1	2.72	0.0	5.05	2.25	4.75	1.96	
Aug 2	2.52	0.0	5.5	2.79	5.64	2.93	
Aug 3	2.58	0.2	5.91	3.52	6.19	3.81	
Sept 1	2.97	0.92	6.08	4.02	6.37	4.32	
Sept 2	3.81	2.08	6.18	4.45	6.48	4.75	
Sept 3	4.71	3.56	6.15	4.99	5.93	4.78	
Oct 1	5.59	5.15	5.87	5.42	4.88	4.43	
Oct 2	6.47	6.47	5.35	5.35	3.84	3.84	
Oct 3	7.22	7.22	4.97	4.97			
Nov 1	7.52	7.52	4.57	4.57			
Nov 2	7.69	7.69					2.21
Nov 3	7.6	7.6					2.18
Dec 1	7.35	7.35					2.63
Dec 2	6.87	6.87					4.01
Dec 3	6.43	6.43					5.92
Jan 1	5.91	5.91					6.83
Jan 2	5.44	5.44					6.83
Jan 3	5.28	5.28					7.28
Feb 1	5.09	5.09					6.99
Feb 2							5.73
Feb 3							4.21
Mar 1							
Mar 2							
Mar 3							

Table (C.5): ET_m in mm/day - Eltweir - Minor - 1988/1989

10 day period	ELS - Cotton No rain 23/8 - 13/3	ELS - Cotton rain 15/7 - 15/1	Ground nut No rain 20/6 -5/11	Ground nut rain 25/6 -10/11	Sorghum No rain 10/7 -25/10	Sorghum rain 10/7 -25/10	Wheat No rain 20/11 - 5/3
Jun 1							
Jun 2							
Jun 3			3.68	3.45			
Jul 1			3.36	2.91			
Jul 2			3.77	3.1	2.42	1.74	
Jul 3			4.81	3.72	2.27	1.19	
Aug 1			5.63	3.93	2.99	1.29	
Aug 2			5.77	3.55	4.41	2.2	
Aug 3	2.58	0.89	5.91	4.21	5.77	4.08	
Sept 1	2.66	1.55	6.08	4.98	6.37	5.27	
Sept 2	2.7	2.15	6.18	5.63	6.48	5.93	
Sept 3	2.69	2.32	6.02	5.66	6.44	6.08	
Oct 1	2.96	2.78	5.62	5.44	5.9	5.72	
Oct 2	3.65	3.65	5.10	5.10	4.85	4.85	
Oct 3	4.57	4.57	4.72	4.72	3.92	3.92	
Nov 1	5.54	5.54	4.32	4.32			
Nov 2	6.54	6.54					
Nov 3	7.26	7.26					2.18
Dec 1	7.52	7.52					2.16
Dec 2	7.44	7.44					3.07
Dec 3	7.48	7.48					4.98
Jan 1	7.4	7.4					6.36
Jan 2	7.13	7.13					6.83
Jan 3	7.13	7.13					7.28
Feb 1	7.11	7.11					7.38
Feb 2	6.95	6.95					6.55
Feb 3	6.6	6.6					5.05
Mar 1	6.22	6.22					3.47
Mar 2	5.81	5.81					
Mar 3							

Table (C.6): ET_m in mm/day - Elmardi - Minor - 1988/1989

10 day period	ELS - Cotton No rain 1/9 - 20/3	ELS - Cotton rain 1/9 - 20/3	Ground nut No rain 20/6 -5/11	Ground nut rain 25/6 -10/11	Sorghum No rain 10/7 -25/10	Sorghum rain 10/7 -25/10	Wheat No rain 20/11 - 5/3
Jun 1							
Jun 2							
Jun 3			3.68	3.48			
Jul 1			3.36	3.04			
Jul 2			3.77	3.29	2.42	1.93	
Jul 3			4.81	3.7	2.27	1.16	
Aug 1			5.63	3.62	2.99	0.98	
Aug 2			5.77	2.99	4.41	1.64	
Aug 3			5.91	3.84	5.77	3.71	
Sept 1	2.66	1.37	6.08	4.79	6.37	5.08	
Sept 2	2.7	2.15	6.18	5.63	6.48	5.93	
Sept 3	2.69	2.32	6.02	5.66	6.44	6.08	
Oct 1	2.67	2.49	5.62	5.44	5.9	5.72	
Oct 2	3.07	3.07	5.10	5.10	4.85	4.85	
Oct 3	3.98	3.98	4.72	4.72	3.92	3.92	
Nov 1	4.93	4.93	4.32	4.32			
Nov 2	5.92	5.92					
Nov 3	6.73	6.73					2.18
Dec 1	7.31	7.31					2.16
Dec 2	7.44	7.44					3.07
Dec 3	7.48	7.48					4.98
Jan 1	7.44	7.44					6.36
Jan 2	7.33	7.33					6.83
Jan 3	7.46	7.46					7.28
Feb 1	7.46	7.46					7.38
Feb 2	7.32	7.32					6.55
Feb 3	6.98	6.98					5.05
Mar 1	6.61	6.61					3.47
Mar 2	6.21	6.21					
Mar 3							

Table (C.7): ET_m in mm/day - Beibash - Minor - 1988/1989

10 day period	ELS - Cotton No rain 23/8 - 13/3	ELS - Cotton rain 23/8 - 13/3	Ground nut No rain 20/6 -5/11	Ground nut rain 25/6 -10/11	Sorghum No rain 10/7 -25/10	Sorghum rain 10/7 -25/10	Wheat No rain 20/11 - 5/3
Jun 1							
Jun 2							
Jun 3			3.68	3.42			
Jul 1			3.38	2.91			
Jul 2			3.77	3.1	2.42	1.75	
Jul 3			4.81	3.45	2.27	0.92	
Aug 1			5.63	3.25	2.99	0.62	
Aug 2			5.77	2.54	4.41	1.18	
Aug 3	2.58	0.21	5.91	3.54	5.77	3.40	
Sept 1	2.66	1.28	6.08	4.7	6.37	5.00	
Sept 2	2.7	2.25	6.18	5.73	6.48	6.03	
Sept 3	2.69	2.39	6.02	5.73	6.44	6.15	
Oct 1	2.96	2.81	5.62	5.47	5.9	5.75	
Oct 2	3.65	3.65	5.10	5.10	4.85	4.85	
Oct 3	4.57	4.57	4.72	4.72	3.92	3.92	
Nov 1	5.54	5.54	4.32	4.32			
Nov 2	6.54	6.54					
Nov 3	7.26	7.26					2.18
Dec 1	7.52	7.52					2.16
Dec 2	7.44	7.44					3.07
Dec 3	7.48	7.48					4.98
Jan 1	7.4	7.4					6.36
Jan 2	7.13	7.13					6.83
Jan 3	7.13	7.13					7.28
Feb 1	7.11	7.11					7.38
Feb 2	6.95	6.95					6.55
Feb 3	6.6	6.6					5.05
Mar 1	6.22	6.22					3.47
Mar 2	5.81	5.81					
Mar 3							

APPENDIX D

This appendix shows the text of the SIF File "RESERV.SIF" for the optimization problem formulated in Chapter IX.

```
NAME        RESERV
 IE 1          1
 IE 2          2
 IE 3          3
 IE 4          4
 IE 5          5
 IE 6          6
 IE 7          7
 IE 8          8
 IE P          11
 IE M          1
 IE N          12
 IA P+1    P     1
 IA M+1    M     1
 IA N+1    N     1
* CONSTANTS OF THE OBJECTIVE FUNCTION
 RE a1,1       0.52
 RE a1,2       0.52
 RE a1,3       0.52
 RE a1,4       0.52
 RE a1,5       0.52
 RE a1,6       0.52
 RE b1,1       0.0098
 RE b1,2       0.0098
 RE b1,3       0.0098
 RE b1,4       0.0098
 RE b1,5       0.0098
 RE b1,6       0.0098
 RE c1,1       -8.85D-6
 RE c1,2       -8.85D-6
 RE c1,3       -8.85D-6
 RE c1,4       -8.85D-6
 RE c1,5       -8.85D-6
 RE c1,6       -8.85D-6
 RE d1,1       -8.85D-6
 RE d1,2       -8.85D-6
 RE d1,3       -8.85D-6
 RE d1,4       -8.85D-6
 RE d1,5       -8.85D-6
 RE d1,6       -8.85D-6
 RE a2,1       0.163
 RE a2,2       0.163
 RE a2,3       0.163
 RE a2,4       0.163
```

RE a2,5	0.163
RE a2,6	0.163
RE b2,1	0.021
RE b2,2	0.021
RE b2,3	0.021
RE b2,4	0.021
RE b2,5	0.021
RE b2,6	0.021
RE c2,1	-8.85D-6
RE c2,2	-8.85D-6
RE c2,3	-8.85D-6
RE c2,4	-8.85D-6
RE c2,5	-8.85D-6
RE c2,6	-8.85D-6
RE d2,1	-8.85D-6
RE d2,2	-8.85D-6
RE d2,3	-8.85D-6
RE d2,4	-8.85D-6
RE d2,5	-8.85D-6
RE d2,6	-8.85D-6
RE a1,7	0.601
RE a1,8	0.601
RE a1,9	0.601
RE a1,10	0.601
RE a1,11	0.601
RE a1,12	0.601
RE b1,7	0.0113
RE b1,8	0.0113
RE b1,9	0.0113
RE b1,10	0.0113
RE b1,11	0.0113
RE b1,12	0.0113
RE c1,7	-1.02D-5
RE c1,8	-1.02D-5
RE c1,9	-1.02D-5
RE c1,10	-1.02D-5
RE c1,11	-1.02D-5
RE c1,12	-1.02D-5
RE d1,7	-1.02D-5
RE d1,8	-1.02D-5
RE d1,9	-1.02D-5
RE d1,10	-1.02D-5
RE d1,11	-1.02D-5
RE d1,12	-1.02D-5
RE a2,7	0.188
RE a2,8	0.188
RE a2,9	0.188
RE a2,10	0.188
RE a2,11	0.188

```
RE a2,12          0.188
RE b2,7           0.024
RE b2,8           0.024
RE b2,9           0.024
RE b2,10          0.024
RE b2,11          0.024
RE b2,12          0.024
RE c2,7           -1.02D-5
RE c2,8           -1.02D-5
RE c2,9           -1.02D-5
RE c2,10          -1.02D-5
RE c2,11          -1.02D-5
RE c2,12          -1.02D-5
RE d2,7           -1.02D-5
RE d2,8           -1.02D-5
RE d2,9           -1.02D-5
RE d2,10          -1.02D-5
RE d2,11          -1.02D-5
RE d2,12          -1.02D-5
*Roseires reservoir continuity equation
RE q1             11046.0
RE q2             5787.0
RE q3             2212.0
RE q4             1277.0
RE q5             652.0
RE q6             411.0
RE q7             322.0
RE q8             269.0
RE q9             525.0
RE q10            1641.0
RE q11            7172.0
RE q12            14545.0
RA e1      q1     -5.966
RA e2      q2     -8.21
RA e3      q3     -16.948
RA e4      q4     -16.709
RA e5      q5     -16.948
RA e6      q6     -18.857
RA e7      q7     -21.244
RA e8      q8     -21.006
RA e9      q9     -19.096
RA e10     q10    -10.98
RA e11     q11    -6.206
RA e12     q12    -4.535
* SENNAR reservoir continuity equation
RE ru1            1225.63
RE ru2            1267.6
RE ru3            1197.5
RE ru4            1305.8
```

RE ru5		1314.2	
RE ru6		946.2	
RE ru7		123.74	
RE ru8		74.3	
RE ru9		74.3	
RE ru10		105.41	
RE ru11		245.18	
RE ru12		762.82	
RS h1	ru1	24.1	
RS h2	ru2	32.02	
RS h3	ru3	38.43	
RS h4	ru4	34.42	
RS h5	ru5	35.97	
RS h6	ru6	40.68	
RS h7	ru7	46.94	
RS h8	ru8	52.2	
RS h9	ru9	48.2	
RS h10	ru10	35.62	
RS h11	ru11	19.99	
RS h12	ru12	14.83	
*requirements downstream			
RE ds1		105.9	
RE ds2		105.9	
RE ds3		105.9	
RE ds4		105.9	
RE ds5		105.9	
RE ds6		105.9	
RE ds7		105.9	
RE ds8		105.9	
RE ds9		105.9	
RE ds10		105.9	
RE ds11		105.9	
RE ds12		105.9	
RE rd1		51.07	
RE rd2		52.82	
RE rd3		49.90	
RE rd4		54.40	
RE rd5		54.76	
RE rd6		39.42	
RE rd7		5.160	
RE rd8		3.1	
RE rd9		3.1	
RE rd10		4.39	
RE rd11		10.22	
RE rd12		31.78	
R+ f1	rd1		ds1
R+ f2	rd2		ds2
R+ f3	rd3		ds3
R+ f4	rd4		ds4

```
R+ f5     rd5          ds5
R+ f6     rd6          ds6
R+ f7     rd7          ds7
R+ f8     rd8          ds8
R+ f9     rd9          ds9
R+ f10    rd10         ds10
R+ f11    rd11         ds11
R+ f12    rd12         ds12

VARIABLES
 DO i      1           M+1
 DO j      1           P+1
 X  X(i,j)
 X  Y(i,j)
 OD j
 OD i
 DO i      1           M+1
 DO j      1           N+1
 X  S(i,j)
 OD j
 OD i
GROUPS
* objective function
 DO i      1           P+1
 ZN Obj(1,i)  X(1,i)       a(1,i)
 OD i
 DO i      1           P+1
 ZN Obj(2,i)  X(2,i)       a(2,i)
 OD i
 DO i      1           P+1
 XN Obj(3,i)
 OD i
 DO i      1           P+1
 XN Obj(4,i)
 OD i
 DO i      1           P+1
 XN Obj(5,i)
 OD i
 DO i      1           P+1
 XN Obj(6,i)
 OD i
 DO i      1           P+1
 XN Obj(7,i)
 OD i
 DO i      1           P+1
 XN Obj(8,i)
 OD i
* continuity equation of roseries reservoir
*Cons1 - continuity equation roseries
```

```
XE   Cons1    S1,2     1.0      S1,1     -1.0
XE   Cons1    X1,1     1.0      Y1,1     1.0
*Cons2 - continuity equation roseries
XE   Cons2    S1,3     1.0      S1,2     -1.0
XE   Cons2    X1,2     1.0      Y1,2     1.0
*Cons3 - continuity equation roseries
XE   Cons3    S1,4     1.0      S1,3     -1.0
XE   Cons3    X1,3     1.0      Y1,3     1.0
*Cons4 - continuity equation roseries
XE   Cons4    S1,5     1.0      S1,4     -1.0
XE   Cons4    X1,4     1.0      Y1,4     1.0
*Cons5 - continuity equation roseries
XE   Cons5    S1,6     1.0      S1,5     -1.0
XE   Cons5    X1,5     1.0      Y1,5     1.0
*Cons6 - continuity equation roseries
XE   Cons6    S1,7     1.0      S1,6     -1.0
XE   Cons6    X1,6     1.0      Y1,6     1.0
*Cons7 - continuity equation roseries
XE   Cons7    S1,8     1.0      S1,7     -1.0
XE   Cons7    X1,7     1.0      Y1,7     1.0
*Cons8 - continuity equation roseries
XE   Cons8    S1,9     1.0      S1,8     -1.0
XE   Cons8    X1,8     1.0      Y1,8     1.0
*Cons9 - continuity equation roseries
XE   Cons9    S1,10    1.0      S1,9     -1.0
XE   Cons9    X1,9     1.0      Y1,9     1.0
*Cons10 - continuity equation roseries
XE   Cons10   S1,11    1.0      S1,10    -1.0
XE   Cons10   X1,10    1.0      Y1,10    1.0
*Cons11 - continuity equation roseries
XE   Cons11   S1,12    1.0      S1,11    -1.0
XE   Cons11   X1,11    1.0      Y1,11    1.0
*Cons12 - continuity equation roseries
XE   Cons12   S1,13    1.0      S1,12    -1.0
XE   Cons12   X1,12    1.0      Y1,12    1.0
 DO i     1                 P+1
 XE Cons(i)
 OD i
* continuity equation of sennar reservoir
*Cons13 - continuity equation sennar
XE   Cons13   S2,2     1.0      S2,1     -1.0
XE   Cons13   X1,1     -1.0     Y1,1     -1.0
XE   Cons13   X2,1     1.0      Y2,1     1.0
*Cons14 - continuity equation sennar
XE   Cons14   S2,3     1.0      S2,2     -1.0
XE   Cons14   X1,2     -1.0     Y1,2     -1.0
XE   Cons14   X2,2     1.0      Y2,2     1.0
*Cons15 - continuity equation sennar
XE   Cons15   S2,4     1.0      S2,3     -1.0
```

```
XE  Cons15  X1,3   -1.0  Y1,3   -1.0
XE  Cons15  X2,3   1.0   Y2,3   1.0
*Cons16 - continuity equation sennar
XE  Cons16  S2,5   1.0   S2,4   -1.0
XE  Cons16  X1,4   -1.0  Y1,4   -1.0
XE  Cons16  X2,4   1.0   Y2,4   1.0
*Cons17 - continuity equation sennar
XE  Cons17  S2,6   1.0   S2,5   -1.0
XE  Cons17  X1,5   -1.0  Y1,5   -1.0
XE  Cons17  X2,5   1.0   Y2,5   1.0
*Cons18 - continuity equation sennar
XE  Cons18  S2,7   1.0   S2,6   -1.0
XE  Cons18  X1,6   -1.0  Y1,6   -1.0
XE  Cons18  X2,6   1.0   Y2,6   1.0
*Cons19 - continuity equation sennar
XE  Cons19  S2,8   1.0   S2,7   -1.0
XE  Cons19  X1,7   -1.0  Y1,7   -1.0
XE  Cons19  X2,7   1.0   Y2,7   1.0
*Cons20 - continuity equation sennar
XE  Cons20  S2,9   1.0   S2,8   -1.0
XE  Cons20  X1,8   -1.0  Y1,8   -1.0
XE  Cons20  X2,8   1.0   Y2,8   1.0
*Cons21 - continuity equation sennar
XE  Cons21  S2,10  1.0   S2,9   -1.0
XE  Cons21  X1,9   -1.0  Y1,9   -1.0
XE  Cons21  X2,9   1.0   Y2,9   1.0
*Cons22 - continuity equation sennar
XE  Cons22  S2,11  1.0   S2,10  -1.0
XE  Cons22  X1,10  -1.0  Y1,10  -1.0
XE  Cons22  X2,10  1.0   Y2,10  1.0
*Cons23 - continuity equation sennar
XE  Cons23  S2,12  1.0   S2,11  -1.0
XE  Cons23  X1,11  -1.0  Y1,11  -1.0
XE  Cons23  X2,11  1.0   Y2,11  1.0
*Cons24 - continuity equation sennar
XE  Cons24  S2,13  1.0   S2,12  -1.0
XE  Cons24  X1,12  -1.0  Y1,12  -1.0
XE  Cons24  X2,12  1.0   Y2,12  1.0
XE  Cons13
XE  Cons14
XE  Cons15
XE  Cons16
XE  Cons17
XE  Cons18
XE  Cons19
XE  Cons20
XE  Cons21
XE  Cons22
XE  Cons23
```

```
XE   Cons24
*cons25 - Cons36 downstream requirements
XG   Cons25   X2,1    1.0    Y2,1    1.0
XG   Cons26   X2,2    1.0    Y2,2    1.0
XG   Cons27   X2,3    1.0    Y2,3    1.0
XG   Cons28   X2,4    1.0    Y2,4    1.0
XG   Cons29   X2,5    1.0    Y2,5    1.0
XG   Cons30   X2,6    1.0    Y2,6    1.0
XG   Cons31   X2,7    1.0    Y2,7    1.0
XG   Cons32   X2,8    1.0    Y2,8    1.0
XG   Cons33   X2,9    1.0    Y2,9    1.0
XG   Cons34   X2,10   1.0    Y2,10   1.0
XG   Cons35   X2,11   1.0    Y2,11   1.0
XG   Cons36   X2,12   1.0    Y2,12   1.0
*Cons37 - Cons48 relation between reservoirs
XG   Cons37   X1,1    1.0    Y1,1    1.0
XG   Cons37   X2,1   -1.0    Y2,1   -1.0
XG   Cons37   S2,1    1.0    S2,2   -1.0
XG   Cons38   X1,2    1.0    Y1,2    1.0
XG   Cons38   X2,2   -1.0    Y2,2   -1.0
XG   Cons38   S2,2    1.0    S2,3   -1.0
XG   Cons39   X1,3    1.0    Y1,3    1.0
XG   Cons39   X2,3   -1.0    Y2,3   -1.0
XG   Cons39   S2,3    1.0    S2,4   -1.0
XG   Cons40   X1,4    1.0    Y1,4    1.0
XG   Cons40   X2,4   -1.0    Y2,4   -1.0
XG   Cons40   S2,4    1.0    S2,5   -1.0
XG   Cons41   X1,5    1.0    Y1,5    1.0
XG   Cons41   X2,5   -1.0    Y2,5   -1.0
XG   Cons41   S2,5    1.0    S2,6   -1.0
XG   Cons42   X1,6    1.0    Y1,6    1.0
XG   Cons42   X2,6   -1.0    Y2,6   -1.0
XG   Cons42   S2,6    1.0    S2,7   -1.0
XG   Cons43   X1,7    1.0    Y1,7    1.0
XG   Cons43   X2,7   -1.0    Y2,7   -1.0
XG   Cons43   S2,7    1.0    S2,8   -1.0
XG   Cons44   X1,8    1.0    Y1,8    1.0
XG   Cons44   X2,8   -1.0    Y2,8   -1.0
XG   Cons44   S2,8    1.0    S2,9   -1.0
XG   Cons45   X1,9    1.0    Y1,9    1.0
XG   Cons45   X2,9   -1.0    Y2,9   -1.0
XG   Cons45   S2,9    1.0    S2,10  -1.0
XG   Cons46   X1,10   1.0    Y1,10   1.0
XG   Cons46   X2,10  -1.0    Y2,10  -1.0
XG   Cons46   S2,10   1.0    S2,11  -1.0
XG   Cons47   X1,11   1.0    Y1,11   1.0
XG   Cons47   X2,11  -1.0    Y2,11  -1.0
XG   Cons47   S2,11   1.0    S2,12  -1.0
XG   Cons48   X1,12   1.0    Y1,12   1.0
```

```
XG    Cons48   X2,12   -1.0     Y2,12     -1.0
XG    Cons48   S2,12   1.0      S2,13     -1.0
CONSTANTS
* for Roseries continuity equation
 Z RESERV   Cons1              e1
 Z RESERV   Cons2              e2
 Z RESERV   Cons3              e3
 Z RESERV   Cons4              e4
 Z RESERV   Cons5              e5
 Z RESERV   Cons6              e6
 Z RESERV   Cons7              e7
 Z RESERV   Cons8              e8
 Z RESERV   Cons9              e9
 Z RESERV   Cons10             e10
 Z RESERV   Cons11             e11
 Z RESERV   Cons12             e12
* for sennar continuity equation
 Z RESERV   Cons13             h1
 Z RESERV   Cons14             h2
 Z RESERV   Cons15             h3
 Z RESERV   Cons16             h4
 Z RESERV   Cons17             h5
 Z RESERV   Cons18             h6
 Z RESERV   Cons19             h7
 Z RESERV   Cons20             h8
 Z RESERV   Cons21             h9
 Z RESERV   Cons22             h10
 Z RESERV   Cons23             h11
 Z RESERV   Cons24             h12
* for requirements d\s sennar
 Z RESERV   Cons25             f1
 Z RESERV   Cons26             f2
 Z RESERV   Cons27             f3
 Z RESERV   Cons28             f4
 Z RESERV   Cons29             f5
 Z RESERV   Cons30             f6
 Z RESERV   Cons31             f7
 Z RESERV   Cons32             f8
 Z RESERV   Cons33             f9
 Z RESERV   Cons34             f10
 Z RESERV   Cons35             f11
 Z RESERV   Cons36             f12
* for relation between reservoirs
 Z RESERV   Cons37             ru1
 Z RESERV   Cons38             ru2
 Z RESERV   Cons39             ru3
 Z RESERV   Cons40             ru4
 Z RESERV   Cons41             ru5
 Z RESERV   Cons42             ru6
```

```
Z RESERV   Cons43          ru7
Z RESERV   Cons44          ru8
Z RESERV   Cons45          ru9
Z RESERV   Cons46          ru10
Z RESERV   Cons47          ru11
Z RESERV   Cons48          ru12
BOUNDS
DO i      1                P+1
XU RESERV    X(1,i)   2014.0
XU RESERV    Y(1,i)   17250.0
XU RESERV    X(2,i)   330.0
XU RESERV    Y(2,i)   28500.0
OD i
XU   RESERV   S1,2    2175
XU   RESERV   S2,2    362.5
XL   RESERV   S1,2    88.3
XL   RESERV   S2,2    113
XU   RESERV   S1,3    2175
XU   RESERV   S2,3    362.5
XL   RESERV   S1,3    88.3
XL   RESERV   S2,3    113
XU   RESERV   S1,4    2175
XU   RESERV   S2,4    362.5
XL   RESERV   S1,4    88.3
XL   RESERV   S2,4    113
XU   RESERV   S1,5    2175
XU   RESERV   S2,5    362.5
XL   RESERV   S1,5    88.3
XL   RESERV   S2,5    113
XU   RESERV   S1,6    2175
XU   RESERV   S2,6    362.5
XL   RESERV   S1,6    88.3
XL   RESERV   S2,6    113
XU   RESERV   S1,7    2175
XU   RESERV   S2,7    362.5
XL   RESERV   S1,7    88.3
XL   RESERV   S2,7    113
XU   RESERV   S1,8    2175
XU   RESERV   S2,8    362.5
XL   RESERV   S1,8    88.3
XL   RESERV   S2,8    113
XU   RESERV   S1,9    2175
XU   RESERV   S2,9    362.5
XL   RESERV   S1,9    88.3
XL   RESERV   S2,9    113
XU   RESERV   S1,10   2175
XU   RESERV   S2,10   362.5
XL   RESERV   S1,10   88.3
XL   RESERV   S2,10   113
```

```
 XL   RESERV   S1,13   88.3
 XU   RESERV   S1,13   88.3
 XL   RESERV   S2,13   113.0
 XU   RESERV   S2,13   113.0
 XL   RESERV   S1,1    88.3
 XU   RESERV   S1,1    88.3
 XL   RESERV   S2,1    113.0
 XU   RESERV   S2,1    113.0
 XL   RESERV   S1,11   88.3
 XU   RESERV   S1,11   88.3
 XL   RESERV   S2,11   113.0
 XU   RESERV   S2,11   113.0
 XL   RESERV   S1,12   88.3
 XU   RESERV   S1,12   88.3
 XL   RESERV   S2,12   113.0
 XU   RESERV   S2,12   113.0
*START POINT
ELEMENT TYPE
* FOR Obj(7,i)
 EV I3PR     V1                V2
 EV I3PR     V3
 IV I3PR     U1                U2
* For obj(8,i)
 EV I3XS     V1                V2
 EV I3XS     V3
 IV I3XS     U1                U2
* For Obj(3,i) & Obj(4,i)
 EV SQ       X
* For Obj(5,i) & Obj(6,i)
 EV 2PR      X                 Y
* continuity equation - roseries - Cons1 to Cons12
 EV I4SS     V1                V2
 IV I4SS     U
* continuity equation - roseries - Cons1 to Cons12
 EV I5SS     V1                V2
 IV I5SS     U
* continuity equation -sennar  - Cons13 to Cons24
 EV I6SS     V1                V2
 IV I6SS     U
* continuity equation -sennar - Cons13 to Cons24
 EV I7SS     V1                V2
 IV I7SS     U
ELEMENT USES
*For Obj(7,1)
 XT XSS71    I3PR
 ZV XSS71    V1                X1,1
 ZV XSS71    V2                S1,1
 ZV XSS71    V3                S1,2
*For Obj(7,2)
```

```
 XT XSS72    I3PR
 ZV XSS72    V1              X1,2
 ZV XSS72    V2              S1,2
 ZV XSS72    V3              S1,3
*For Obj(7,3)
 XT XSS73    I3PR
 ZV XSS73    V1              X1,3
 ZV XSS73    V2              S1,3
 ZV XSS73    V3              S1,4
*For Obj(7,4)
 XT XSS74    I3PR
 ZV XSS74    V1              X1,4
 ZV XSS74    V2              S1,4
 ZV XSS74    V3              S1,5
*For Obj(7,5)
 XT XSS75    I3PR
 ZV XSS75    V1              X1,5
 ZV XSS75    V2              S1,5
 ZV XSS75    V3              S1,6
*For Obj(7,6)
 XT XSS76    I3PR
 ZV XSS76    V1              X1,6
 ZV XSS76    V2              S1,6
 ZV XSS76    V3              S1,7
*For Obj(7,7)
 XT XSS77    I3PR
 ZV XSS77    V1              X1,7
 ZV XSS77    V2              S1,7
 ZV XSS77    V3              S1,8
*For Obj(7,8)
 XT XSS78    I3PR
 ZV XSS78    V1              X1,8
 ZV XSS78    V2              S1,8
 ZV XSS78    V3              S1,9
*For Obj(7,9)
 XT XSS79    I3PR
 ZV XSS79    V1              X1,9
 ZV XSS79    V2              S1,9
 ZV XSS79    V3              S1,10
*For Obj(7,10)
 XT XSS710   I3PR
 ZV XSS710   V1              X1,10
 ZV XSS710   V2              S1,10
 ZV XSS710   V3              S1,11
*For Obj(7,11)
 XT XSS711   I3PR
 ZV XSS711   V1              X1,11
 ZV XSS711   V2              S1,11
 ZV XSS711   V3              S1,12
```

```
*For Obj(7,12)
 XT XSS712   I3PR
 ZV XSS712   V1         X1,12
 ZV XSS712   V2         S1,12
 ZV XSS712   V3         S1,13
*For Obj(8,1)
 XT XSS81    I3XS
 ZV XSS81    V1         X2,1
 ZV XSS81    V2         S2,1
 ZV XSS81    V3         S2,2
*For Obj(8,2)
 XT XSS82    I3XS
 ZV XSS82    V1         X2,2
 ZV XSS82    V2         S2,2
 ZV XSS82    V3         S2,3
*For Obj(8,3)
 XT XSS83    I3XS
 ZV XSS83    V1         X2,3
 ZV XSS83    V2         S2,3
 ZV XSS83    V3         S2,4
*For Obj(8,4)
 XT XSS84    I3XS
 ZV XSS84    V1         X2,4
 ZV XSS84    V2         S2,4
 ZV XSS84    V3         S2,5
*For Obj(8,5)
 XT XSS85    I3XS
 ZV XSS85    V1         X2,5
 ZV XSS85    V2         S2,5
 ZV XSS85    V3         S2,6
*For Obj(8,6)
 XT XSS86    I3XS
 ZV XSS86    V1         X2,6
 ZV XSS86    V2         S2,6
 ZV XSS86    V3         S2,7
*For Obj(8,7)
 XT XSS87    I3XS
 ZV XSS87    V1         X2,7
 ZV XSS87    V2         S2,7
 ZV XSS87    V3         S2,8
*For Obj(8,8)
 XT XSS88    I3XS
 ZV XSS88    V1         X2,8
 ZV XSS88    V2         S2,8
 ZV XSS88    V3         S2,9
*For Obj(8,9)
 XT XSS89    I3XS
 ZV XSS89    V1         X2,9
 ZV XSS89    V2         S2,9
```

```
 ZV XSS89    V3                S2,10
*For Obj(8,10)
 XT XSS810   I3XS
 ZV XSS810   V1                X2,10
 ZV XSS810   V2                S2,10
 ZV XSS810   V3                S2,11
*For Obj(8,11)
 XT XSS811   I3XS
 ZV XSS811   V1                X2,11
 ZV XSS811   V2                S2,11
 ZV XSS811   V3                S2,12
*For Obj(8,12)
 XT XSS812   I3XS
 ZV XSS812   V1                X2,12
 ZV XSS812   V2                S2,12
 ZV XSS812   V3                S2,13
*For Obj(3,1)
 XT XSQ31    SQ
 ZV XSQ31    X                 X1,1
*For Obj(3,2)
 XT XSQ32    SQ
 ZV XSQ32    X                 X1,2
*For Obj(3,3)
 XT XSQ33    SQ
 ZV XSQ33    X                 X1,3
*For Obj(3,4)
 XT XSQ34    SQ
 ZV XSQ34    X                 X1,4
*For Obj(3,5)
 XT XSQ35    SQ
 ZV XSQ35    X                 X1,5
*For Obj(3,6)
 XT XSQ36    SQ
 ZV XSQ36    X                 X1,6
*For Obj(3,7)
 XT XSQ37    SQ
 ZV XSQ37    X                 X1,7
*For Obj(3,8)
 XT XSQ38    SQ
 ZV XSQ38    X                 X1,8
*For Obj(3,9)
 XT XSQ39    SQ
 ZV XSQ39    X                 X1,9
*For Obj(3,10)
 XT XSQ310   SQ
 ZV XSQ310   X                 X1,10
*For Obj(3,11)
 XT XSQ311   SQ
 ZV XSQ311   X                 X1,11
```

```
*For Obj(3,12)
 XT XSQ312   SQ
 ZV XSQ312   X          X1,12
*For Obj(4,1)
 XT XSQ41    SQ
 ZV XSQ41    X          X2,1
*For Obj(4,2)
 XT XSQ42    SQ
 ZV XSQ42    X          X2,2
*For Obj(4,3)
 XT XSQ43    SQ
 ZV XSQ43    X          X2,3
*For Obj(4,4)
 XT XSQ44    SQ
 ZV XSQ44    X          X2,4
*For Obj(4,5)
 XT XSQ45    SQ
 ZV XSQ45    X          X2,5
*For Obj(4,6)
 XT XSQ46    SQ
 ZV XSQ46    X          X2,6
*For Obj(4,7)
 XT XSQ47    SQ
 ZV XSQ47    X          X2,7
*For Obj(4,8)
 XT XSQ48    SQ
 ZV XSQ48    X          X2,8
*For Obj(4,9)
 XT XSQ49    SQ
 ZV XSQ49    X          X2,9
*For Obj(4,10)
 XT XSQ410   SQ
 ZV XSQ410   X          X2,10
*For Obj(4,11)
 XT XSQ411   SQ
 ZV XSQ411   X          X2,11
*For Obj(4,12)
 XT XSQ412   SQ
 ZV XSQ412   X          X2,12
*For Obj(5,1)
 XT XY51     2PR
 ZV XY51     X          X1,1
 ZV XY51     Y          Y1,1
*For Obj(5,2)
 XT XY52     2PR
 ZV XY52     X          X1,2
 ZV XY52     Y          Y1,2
*For Obj(5,3)
 XT XY53     2PR
```

```
 ZV XY53     X             X1,3
 ZV XY53     Y             Y1,3
*For Obj(5,4)
 XT XY54     2PR
 ZV XY54     X             X1,4
 ZV XY54     Y             Y1,4
*For Obj(5,5)
 XT XY55     2PR
 ZV XY55     X             X1,5
 ZV XY55     Y             Y1,5
*For Obj(5,6)
 XT XY56     2PR
 ZV XY56     X             X1,6
 ZV XY56     Y             Y1,6
*For Obj(5,7)
 XT XY57     2PR
 ZV XY57     X             X1,7
 ZV XY57     Y             Y1,7
*For Obj(5,8)
 XT XY58     2PR
 ZV XY58     X             X1,8
 ZV XY58     Y             Y1,8
*For Obj(5,9)
 XT XY59     2PR
 ZV XY59     X             X1,9
 ZV XY59     Y             Y1,9
*For Obj(5,10)
 XT XY510    2PR
 ZV XY510    X             X1,10
 ZV XY510    Y             Y1,10
*For Obj(5,11)
 XT XY511    2PR
 ZV XY511    X             X1,11
 ZV XY511    Y             Y1,11
*For Obj(5,12)
 XT XY512    2PR
 ZV XY512    X             X1,12
 ZV XY512    Y             Y1,12
*For Obj(6,1)
 XT XY61     2PR
 ZV XY61     X             X2,1
 ZV XY61     Y             Y2,1
*For Obj(6,2)
 XT XY62     2PR
 ZV XY62     X             X2,2
 ZV XY62     Y             Y2,2
*For Obj(6,3)
 XT XY63     2PR
 ZV XY63     X             X2,3
```

```
 ZV XY63     Y                    Y2,3
*For Obj(6,4)
 XT XY64    2PR
 ZV XY64     X                    X2,4
 ZV XY64     Y                    Y2,4
*For Obj(6,5)
 XT XY65    2PR
 ZV XY65     X                    X2,5
 ZV XY65     Y                    Y2,5
*For Obj(6,6)
 XT XY66    2PR
 ZV XY66     X                    X2,6
 ZV XY66     Y                    Y2,6
*For Obj(6,7)
 XT XY67    2PR
 ZV XY67     X                    X2,7
 ZV XY67     Y                    Y2,7
*For Obj(6,8)
 XT XY68    2PR
 ZV XY68     X                    X2,8
 ZV XY68     Y                    Y2,8
*For Obj(6,9)
 XT XY69    2PR
 ZV XY69     X                    X2,9
 ZV XY69     Y                    Y2,9
*For Obj(6,10)
 XT XY610    2PR
 ZV XY610    X                    X2,10
 ZV XY610    Y                    Y2,10
*For Obj(6,11)
 XT XY611    2PR
 ZV XY611    X                    X2,11
 ZV XY611    Y                    Y2,11
*For Obj(6,12)
 XT XY612    2PR
 ZV XY612    X                    X2,12
 ZV XY612    Y                    Y2,12
*For Cons1 - continuity  roseries
 XT SS12     I4SS
 ZV SS12     V1                  S1,1
 ZV SS12     V2                  S1,2
*For Cons2 - continuity  roseries
 XT SS22     I4SS
 ZV SS22     V1                  S1,2
 ZV SS22     V2                  S1,3
*For Cons3 - continuity  roseries
 XT SS32     I4SS
 ZV SS32     V1                  S1,3
 ZV SS32     V2                  S1,4
```

```
*For Cons4 - continuity  roseries
 XT SS42    I4SS
 ZV SS42    V1              S1,4
 ZV SS42    V2              S1,5
*For Cons5 - continuity  roseries
 XT SS52    I4SS
 ZV SS52    V1              S1,5
 ZV SS52    V2              S1,6
*For Cons6 - continuity  roseries
 XT SS62    I4SS
 ZV SS62    V1              S1,6
 ZV SS62    V2              S1,7
*For Cons7 - continuity  roseries
 XT SS72    I4SS
 ZV SS72    V1              S1,7
 ZV SS72    V2              S1,8
*For Cons8 - continuity  roseries
 XT SS82    I4SS
 ZV SS82    V1              S1,8
 ZV SS82    V2              S1,9
*For Cons9 - continuity  roseries
 XT SS92    I4SS
 ZV SS92    V1              S1,9
 ZV SS92    V2              S1,10
*For Cons10 - continuity  roseries
 XT SS102   I4SS
 ZV SS102   V1              S1,10
 ZV SS102   V2              S1,11
*For Cons11 - continuity  roseries
 XT SS112   I4SS
 ZV SS112   V1              S1,11
 ZV SS112   V2              S1,12
*For Cons12 - continuity  roseries
 XT SS122   I4SS
 ZV SS122   V1              S1,12
 ZV SS122   V2              S1,13
*For Cons1 - continuity  roseries
 XT SS13    I5SS
 ZV SS13    V1              S1,1
 ZV SS13    V2              S1,2
*For Cons2 - continuity  roseries
 XT SS23    I5SS
 ZV SS23    V1              S1,2
 ZV SS23    V2              S1,3
*For Cons3 - continuity  roseries
 XT SS33    I5SS
 ZV SS33    V1              S1,3
 ZV SS33    V2              S1,4
*For Cons4 - continuity  roseries
```

```
 XT SS43     I5SS
 ZV SS43     V1              S1,4
 ZV SS43     V2              S1,5
*For Cons5 - continuity  roseries
 XT SS53     I5SS
 ZV SS53     V1              S1,5
 ZV SS53     V2              S1,6
*For Cons6 - continuity  roseries
 XT SS63     I5SS
 ZV SS63     V1              S1,6
 ZV SS63     V2              S1,7
*For Cons7 - continuity  roseries
 XT SS73     I5SS
 ZV SS73     V1              S1,7
 ZV SS73     V2              S1,8
*For Cons8 - continuity  roseries
 XT SS83     I5SS
 ZV SS83     V1              S1,8
 ZV SS83     V2              S1,9
*For Cons9 - continuity  roseries
 XT SS93     I5SS
 ZV SS93     V1              S1,9
 ZV SS93     V2              S1,10
*For Cons10 - continuity  roseries
 XT SS103    I5SS
 ZV SS103    V1              S1,10
 ZV SS103    V2              S1,11
*For Cons11 - continuity  roseries
 XT SS113    I5SS
 ZV SS113    V1              S1,11
 ZV SS113    V2              S1,12
*For Cons12 - continuity  roseries
 XT SS123    I5SS
 ZV SS123    V1              S1,12
 ZV SS123    V2              S1,13
*For Cons13 - continuity sennar
 XT SSS12    I6SS
 ZV SSS12    V1              S2,1
 ZV SSS12    V2              S2,2
*For Cons14 - continuity sennar
 XT SSS22    I6SS
 ZV SSS22    V1              S2,2
 ZV SSS22    V2              S2,3
*For Cons15 - continuity  sennar
 XT SSS32    I6SS
 ZV SSS32    V1              S2,3
 ZV SSS32    V2              S2,4
*For Cons16 - continuity sennar
 XT SSS42    I6SS
```

ZV SSS42 V1 S2,4
ZV SSS42 V2 S2,5
*For Cons17 - continuity sennar
XT SSS52 I6SS
ZV SSS52 V1 S2,5
ZV SSS52 V2 S2,6
*For Cons18 - continuity sennar
XT SSS62 I6SS
ZV SSS62 V1 S2,6
ZV SSS62 V2 S2,7
*For Cons19 - continuity sennar
XT SSS72 I6SS
ZV SSS72 V1 S2,7
ZV SSS72 V2 S2,8
*For Cons20 - continuity sennar
XT SSS82 I6SS
ZV SSS82 V1 S2,8
ZV SSS82 V2 S2,9
*For Cons21 - continuity sennar
XT SSS92 I6SS
ZV SSS92 V1 S2,9
ZV SSS92 V2 S2,10
*For Cons22 - continuity sennar
XT SSS102 I6SS
ZV SSS102 V1 S2,10
ZV SSS102 V2 S2,11
*For Cons23 - continuity sennar
XT SSS112 I6SS
ZV SSS112 V1 S2,11
ZV SSS112 V2 S2,12
*For Cons24 - continuity sennar
XT SSS122 I6SS
ZV SSS122 V1 S2,12
ZV SSS122 V2 S2,13
*For Cons13 - continuity sennar
XT SSS13 I7SS
ZV SSS13 V1 S2,1
ZV SSS13 V2 S2,2
*For Con14 - continuity sennar
XT SSS23 I7SS
ZV SSS23 V1 S2,2
ZV SSS23 V2 S2,3
*For Cons15 - continuity sennar
XT SSS33 I7SS
ZV SSS33 V1 S2,3
ZV SSS33 V2 S2,4
*For Cons16 - continuity sennar
XT SSS43 I7SS
ZV SSS43 V1 S2,4

```
 ZV SSS43    V2              S2,5
*For Cons17 - continuity  sennar
 XT SSS53    I7SS
 ZV SSS53    V1              S2,5
 ZV SSS53    V2              S2,6
*For Cons18 - continuity  sennar
 XT SSS63    I7SS
 ZV SSS63    V1              S2,6
 ZV SSS63    V2              S2,7
*For Cons19 - continuity  sennar
 XT SSS73    I7SS
 ZV SSS73    V1              S2,7
 ZV SSS73    V2              S2,8
*For Cons20 - continuity sennar
 XT SSS83    I7SS
 ZV SSS83    V1              S2,8
 ZV SSS83    V2              S2,9
*For Cons21 - continuity  sennar
 XT SSS93    I7SS
 ZV SSS93    V1              S2,9
 ZV SSS93    V2              S2,10
*For Cons22 - continuity  sennar
 XT SSS103   I7SS
 ZV SSS103   V1              S2,10
 ZV SSS103   V2              S2,11
*For Cons23 - continuity  sennar
 XT SSS113   I7SS
 ZV SSS113   V1              S2,11
 ZV SSS113   V2              S2,12
*For Cons24 - continuity  sennar
 XT SSS123   I7SS
 ZV SSS123   V1              S2,12
 ZV SSS123   V2              S2,13
GROUP USES
*OBJ(7,1)
 ZE Obj7,1   XSS71            b1,1
*OBJ(7,2)
 ZE Obj7,2   XSS72            b1,2
*OBJ(7,3)
 ZE Obj7,3   XSS73            b1,3
*OBJ(7,4)
 ZE Obj7,4   XSS74            b1,4
*OBJ(7,5)
 ZE Obj7,5   XSS75            b1,5
*OBJ(7,6)
 ZE Obj7,6   XSS76            b1,6
*OBJ(7,7)
 ZE Obj7,7   XSS77            b1,7
*OBJ(7,8)
```

ZE Obj7,8	XSS78	b1,8
*OBJ(7,9)		
ZE Obj7,9	XSS79	b1,9
*OBJ(7,10)		
ZE Obj7,10	XSS710	b1,10
*OBJ(7,11)		
ZE Obj7,11	XSS711	b1,11
*OBJ(7,12)		
ZE Obj7,12	XSS712	b1,12
*OBJ(8,1)		
ZE Obj8,1	XSS81	b2,1
*OBJ(8,2)		
ZE Obj8,2	XSS82	b2,2
*OBJ(8,3)		
ZE Obj8,3	XSS83	b2,3
*OBJ(8,4)		
ZE Obj8,4	XSS84	b2,4
*OBJ(8,5)		
ZE Obj8,5	XSS85	b2,5
*OBJ(8,6)		
ZE Obj8,6	XSS86	b2,6
*OBJ(8,7)		
ZE Obj8,7	XSS87	b2,7
*OBJ(8,8)		
ZE Obj8,8	XSS88	b2,8
*OBJ(8,9)		
ZE Obj8,9	XSS89	b2,9
*OBJ(8,10)		
ZE Obj8,10	XSS810	b2,10
*OBJ(8,11)		
ZE Obj8,11	XSS811	b2,11
*OBJ(8,12)		
ZE Obj8,12	XSS812	b2,12
*OBJ(3,1-12)		
ZE Obj3,1	XSQ31	c1,1
ZE Obj3,2	XSQ32	c1,2
ZE Obj3,3	XSQ33	c1,3
ZE Obj3,4	XSQ34	c1,4
ZE Obj3,5	XSQ35	c1,5
ZE Obj3,6	XSQ36	c1,6
ZE Obj3,7	XSQ37	c1,7
ZE Obj3,8	XSQ38	c1,8
ZE Obj3,9	XSQ39	c1,9
ZE Obj3,10	XSQ310	c1,10
ZE Obj3,11	XSQ311	c1,11
ZE Obj3,12	XSQ312	c1,12
*OBJ(4,1-12)		
ZE Obj4,1	XSQ41	c2,1
ZE Obj4,2	XSQ42	c2,2

```
ZE Obj4,3   XSQ43        c2,3
ZE Obj4,4   XSQ44        c2,4
ZE Obj4,5   XSQ45        c2,5
ZE Obj4,6   XSQ46        c2,6
ZE Obj4,7   XSQ47        c2,7
ZE Obj4,8   XSQ48        c2,8
ZE Obj4,9   XSQ49        c2,9
ZE Obj4,10  XSQ410       c2,10
ZE Obj4,11  XSQ411       c2,11
ZE Obj4,12  XSQ412       c2,12
*OBJ(5,1-12)
ZE Obj5,1   XY51         d1,1
ZE Obj5,2   XY52         d1,2
ZE Obj5,3   XY53         d1,3
ZE Obj5,4   XY54         d1,4
ZE Obj5,5   XY55         d1,5
ZE Obj5,6   XY56         d1,6
ZE Obj5,7   XY57         d1,7
ZE Obj5,8   XY58         d1,8
ZE Obj5,9   XY59         d1,9
ZE Obj5,10  XY510        d1,10
ZE Obj5,11  XY511        d1,11
ZE Obj5,12  XY512        d1,12
*OBJ(6,1-12)
ZE Obj6,1   XY61         d2,1
ZE Obj6,2   XY62         d2,2
ZE Obj6,3   XY63         d2,3
ZE Obj6,4   XY64         d2,4
ZE Obj6,5   XY65         d2,5
ZE Obj6,6   XY66         d2,6
ZE Obj6,7   XY67         d2,7
ZE Obj6,8   XY68         d2,8
ZE Obj6,9   XY69         d2,9
ZE Obj6,10  XY610        d2,10
ZE Obj6,11  XY611        d2,11
ZE Obj6,12  XY612        d2,12
* Cons1 - continuity equation roseries
E Cons1    SS12    0.101     SS13    0.005
*Cons2 - continuity equation roseries
E Cons2    SS22    0.139     SS23    0.006
*Cons3 - continuity equation roseries
E Cons3    SS32    0.287     SS33    0.013
*Cons4 - continuity equation roseries
E Cons4    SS42    0.283     SS43    0.013
*Cons5 - continuity equation roseries
E Cons5    SS52    0.287     SS53    0.013
*Cons6 - continuity equation roseries
E Cons6    SS62    0.319     SS63    0.014
*Cons7 - continuity equation roseries
```

```
 E Cons7    SS72    0.360      SS73     0.016
*Cons8 - continuity equation roseries
 E Cons8    SS82    0.356      SS83     0.016
*Cons9 - continuity equation roseries
 E Cons9    SS92    0.324      SS93     0.015
*Cons10 - continuity equation roseries
 E Cons10  SS102    0.186      SS103    0.008
*Cons11 - continuity equation roseries
 E Cons11  SS112    0.106      SS113    0.005
*Cons12 - continuity equation roseries
 E Cons12  SS122    0.077      SS123    0.003
*  Cons13 - continuity equation sennar
 E Cons13  SSS12    6.049      SSS13    -0.177
*Cons14 - continuity equation sennar
 E Cons14  SSS22    8.040      SSS23    -0.236
*Cons15 - continuity equation sennar
 E Cons15  SSS32    9.650      SSS33    -0.284
*Cons16 - continuity equation sennar
 E Cons16  SSS42    8.644      SSS43    -0.254
*Cons17 - continuity equation sennar
 E Cons17  SSS52    9.03       SSS53    -0.265
*Cons18 - continuity equation sennar
 E Cons18  SSS62    10.217     SSS63    -0.30
*Cons19 - continuity equation sennar
 E Cons19   SSS72   11.789     SSS73    -0.346
*Cons20 - continuity equation sennar
 E Cons20   SSS82   13.11      SSS83    -0.385
*Cons21 - continuity equation sennar
 E Cons21   SSS92   12.1       SSS93    -0.355
*Cons22 - continuity equation sennar
 E Cons22   SSS102  8.945      SSS103   -0.263
*Cons23 - continuity equation sennar
 E Cons23   SSS112  5.02       SSS113   -0.147
*Cons24 - continuity equation sennar
 E Cons24   SSS122  3.724      SSS123   -0.11
ENDATA

ELEMENTS    RESERV
INDIVIDUALS
 T I3PR
 R U1      V1      1.0
 R U2      V2      1.0       V3      1.0
 F                U1*U2**0.47
 G U1             U2**0.47
 G U2             0.47*U1/U2**0.53
 H U1      U2     0.47/U2**0.53
 H U2      U2     -0.47*0.53*U1/U2**1.53
 T I3XS
 R U1      V1      1.0
```

```
R U2     V2     1.0        V3     1.0
F               U1*U2**0.417
G U1            U2**0.417
G U2            0.417*U1/U2**0.583
H U1     U2     0.417/U2**0.583
H U2     U2     -0.417*0.583*U1/U2**1.583
T SQ
F               X*X
G X             X+X
H X      X      2.0
T 2PR
F               X*Y
G X             Y
G Y             X
H X      Y      1.0
T I4SS
R U      V1     1.0        V2     1.0
F               U**0.47
G U             0.47/U**0.53
H U      U      -0.47*0.53/U**1.53
T I5SS
R U      V1     1.0        V2     1.0
F               U**0.94
G U             0.94/U**0.06
H U      U      -0.94*0.06/U**1.06
T I6SS
R U      V1     1.0        V2     1.0
F               U**0.417
G U             0.417/U**0.583
H U      U      -0.417*0.583/U**1.583
T I7SS
R U      V1     1.0        V2     1.0
F               U**0.834
G U             0.834/U**0.166
H U      U      -0.166*0.834/U**1.166
ENDATA
```

APPENDIX E

This appendix shows the exact output of Lancelot. These results are obtained when the average inflow is used as an input. The first file, section E.1, shows the problem solution while the second file, section E.2, shows the iteration carried out to reach the solution.

E.1 PROBLEM SOLUTION:

This file shows Lancelot solution for the problem defined in Chapter IX. In the solution values for releases, storage volumes, penalty parameter and objective function are given.

```
* Lancelot solution for problem name: RESERV
* penalty parameter value is  1.0000D-03

* variables

  SOLUTION  X1,1    2.01400D+03
  SOLUTION  Y1,1    6.92840D+03
  SOLUTION  X1,2    2.01400D+03
  SOLUTION  Y1,2    3.74187D+03
  SOLUTION  X1,3    2.01400D+03
  SOLUTION  Y1,3    1.32122D+02
  SOLUTION  X1,4    1.27884D+03
  SOLUTION  Y1,4     .00000D+00
  SOLUTION  X1,5    1.66212D+03
  SOLUTION  Y1,5     .00000D+00
  SOLUTION  X1,6    8.42020D+02
  SOLUTION  Y1,6     .00000D+00
  SOLUTION  X1,7    5.55859D+02
  SOLUTION  Y1,7     .00000D+00
  SOLUTION  X1,8    4.04300D+02
  SOLUTION  Y1,8     .00000D+00
  SOLUTION  X1,9    1.54800D+02
  SOLUTION  Y1,9     .00000D+00
  SOLUTION  X1,10   2.00214D+03
  SOLUTION  Y1,10    .00000D+00
  SOLUTION  X1,11   2.01400D+03
  SOLUTION  Y1,11   5.14994D+03
  SOLUTION  X1,12   2.01400D+03
  SOLUTION  Y1,12   1.25252D+04
  SOLUTION  X2,1    3.30000D+02
  SOLUTION  Y2,1    7.08230D+03
  SOLUTION  X2,2    3.30000D+02
  SOLUTION  Y2,2    4.06497D+03
  SOLUTION  X2,3    3.30000D+02
  SOLUTION  Y2,3    5.42143D+02
  SOLUTION  X2,4    1.60300D+02
  SOLUTION  Y2,4     .00000D+00
  SOLUTION  X2,5    1.60660D+02
  SOLUTION  Y2,5     .00000D+00
  SOLUTION  X2,6    1.45320D+02
  SOLUTION  Y2,6     .00000D+00
  SOLUTION  X2,7    1.82659D+02
```

SOLUTION	Y2,7	.00000D+00
SOLUTION	X2,8	3.30000D+02
SOLUTION	Y2,8	.00000D+00
SOLUTION	X2,9	3.30000D+02
SOLUTION	Y2,9	.00000D+00
SOLUTION	X2,10	3.30000D+02
SOLUTION	Y2,10	1.51660D+03
SOLUTION	X2,11	3.30000D+02
SOLUTION	Y2,11	6.56063D+03
SOLUTION	X2,12	3.30000D+02
SOLUTION	Y2,12	1.34255D+04
SOLUTION	S1,1	8.83000D+01
SOLUTION	S1,2	2.17500D+03
SOLUTION	S1,3	2.17500D+03
SOLUTION	S1,4	2.17500D+03
SOLUTION	S1,5	2.10833D+03
SOLUTION	S1,6	1.04334D+03
SOLUTION	S1,7	5.68706D+02
SOLUTION	S1,8	2.95743D+02
SOLUTION	S1,9	1.28599D+02
SOLUTION	S1,10	4.67084D+02
SOLUTION	S1,11	8.83000D+01
SOLUTION	S1,12	8.83000D+01
SOLUTION	S1,13	8.83000D+01
SOLUTION	S2,1	1.13000D+02
SOLUTION	S2,2	3.62500D+02
SOLUTION	S2,3	3.62500D+02
SOLUTION	S2,4	3.62500D+02
SOLUTION	S2,5	1.75239D+02
SOLUTION	S2,6	3.62500D+02
SOLUTION	S2,7	1.13000D+02
SOLUTION	S2,8	3.62500D+02
SOLUTION	S2,9	3.62500D+02
SOLUTION	S2,10	1.13000D+02
SOLUTION	S2,11	1.13000D+02
SOLUTION	S2,12	1.13000D+02
SOLUTION	S2,13	1.13000D+02
SOLUTION	2,1	.00000D+00
SOLUTION	2,2	.00000D+00
SOLUTION	2,3	.00000D+00
SOLUTION	2,4	3.26856D+02
SOLUTION	2,5	7.30426D+02
SOLUTION	2,6	3.28307D+02
SOLUTION	2,7	6.41318D+02
SOLUTION	2,8	3.28830D+02
SOLUTION	2,9	9.70337D+02
SOLUTION	2,10	.00000D+00
SOLUTION	2,11	.00000D+00
SOLUTION	2,12	.00000D+00
SOLUTION	2,13	.00000D+00
SOLUTION	Cons25	7.25533D+03
SOLUTION	Cons26	4.23625D+03
SOLUTION	Cons27	7.16343D+02
SOLUTION	Cons28	.00000D+00
SOLUTION	Cons29	.00000D+00
SOLUTION	Cons30	.00000D+00
SOLUTION	Cons31	7.15992D+01
SOLUTION	Cons32	2.21000D+02

SOLUTION Cons33 2.21000D+02
SOLUTION Cons34 1.73631D+03
SOLUTION Cons35 6.77451D+03
SOLUTION Cons36 1.36178D+04
SOLUTION Cons37 5.49779D+01
SOLUTION Cons38 9.32990D+01
SOLUTION Cons39 7.64789D+01
SOLUTION Cons40 .00000D+00
SOLUTION Cons41 .00000D+00
SOLUTION Cons42 .00000D+00
SOLUTION Cons43 .00000D+00
SOLUTION Cons44 .00000D+00
SOLUTION Cons45 .00000D+00
SOLUTION Cons46 5.01319D+01
SOLUTION Cons47 2.81346D+01
SOLUTION Cons48 2.08704D+01

* Lagrange multipliers

SOLUTION Cons1 -2.07452D-02
SOLUTION Cons2 -2.07467D-02
SOLUTION Cons3 -2.07443D-02
SOLUTION Cons4 1.77919D+00
SOLUTION Cons5 1.63346D+00
SOLUTION Cons6 1.48133D+00
SOLUTION Cons7 1.35892D+00
SOLUTION Cons8 1.23124D+00
SOLUTION Cons9 1.15477D+00
SOLUTION Cons10 7.77103D-01
SOLUTION Cons11 -2.39091D-02
SOLUTION Cons12 -2.39046D-02
SOLUTION Cons13 -2.92146D-03
SOLUTION Cons14 -2.92298D-03
SOLUTION Cons15 -2.92029D-03
SOLUTION Cons16 3.95591D-03
SOLUTION Cons17 -3.76338D-03
SOLUTION Cons18 3.26818D-03
SOLUTION Cons19 -2.83102D-03
SOLUTION Cons20 2.66809D-03
SOLUTION Cons21 -2.75409D-03
SOLUTION Cons22 -3.36624D-03
SOLUTION Cons23 -3.36617D-03
SOLUTION Cons24 -3.36160D-03
SOLUTION Cons25 3.67436D-07
SOLUTION Cons26 1.04146D-06
SOLUTION Cons27 -7.09974D-08
SOLUTION Cons28 -3.33611D-01
SOLUTION Cons29 -2.61693D-01
SOLUTION Cons30 -2.26003D-01
SOLUTION Cons31 -1.02636D-07
SOLUTION Cons32 -1.13118D-09
SOLUTION Cons33 .00000D+00
SOLUTION Cons34 1.27579D-07
SOLUTION Cons35 -4.63842D-09
SOLUTION Cons36 -1.89557D-06
SOLUTION Cons37 -2.83389D-07
SOLUTION Cons38 -5.77218D-07
SOLUTION Cons39 7.11140D-08

SOLUTION	Cons40	-7.78798D-01
SOLUTION	Cons41	-7.14593D-01
SOLUTION	Cons42	-6.57693D-01
SOLUTION	Cons43	-5.00855D-01
SOLUTION	Cons44	-4.41686D-01
SOLUTION	Cons45	-3.31992D-01
SOLUTION	Cons46	1.84741D-10
SOLUTION	Cons47	-4.00281D-08
SOLUTION	Cons48	9.52259D-07

XU SOLUTION 1.55983D+04

E.2 LANCELOT ITERATIONS:

This file shows the iterations carried out by Lancelot to reach the solution.

Problem name: RESERV

Double precision version will be formed.

The objective function uses 24 linear groups
The objective function uses 72 nonlinear groups

There are 24 linear inequality constraints
There are 24 nonlinear equality constraints

There are 13 variables bounded only from below
There are 66 variables bounded from below and above
There are 8 fixed variables
There are 24 slack variables

--*-*-* LANCELOT A -*- AUGLG Minimizer *-*-*-*-*

************* Problem RESERV *****************

*********** Checking group derivatives **********
*   *

*   *
*********** Derivatives checked O.K. ************

--*-*-*-*-* Maximizer sought *-*-*-*-*-*-*-*-*

********* Starting optimization **************

Penalty parameter 1.0000D-01 Required projected gradient norm = 1.0000D-01
Required constraint norm = 1.0000D-01

There are 111 variables
There are 144 groups
There are 120 nonlinear elements

Objective function value .00000000000000D+00
Constraint norm 1.45392004423733D+04

Iter #g.ev c.g.it f proj.g rho radius step cgend #free time

0	1	0	-2.24D+09	1.4D+05	-	-	-	-	44	.0
1	2	0	-2.23D+09	1.7D+04	5.2D-01	5.0D+00	5.0D+00	CONVR	44	.0
2	3	2	-2.23D+09	1.7D+04	1.0D+00	5.0D+00	5.0D+00	BOUND	44	.1
3	4	2	-2.21D+09	1.7D+04	1.0D+00	1.0D+01	1.0D+01	CONVR	45	.1
4	5	3	-2.19D+09	1.7D+04	1.0D+00	2.0D+01	2.0D+01	CONVR	53	.1
5	6	4	-2.13D+09	1.7D+04	1.0D+00	4.0D+01	4.0D+01	CONVR	56	.2
6	7	7	-2.03D+09	1.7D+04	1.0D+00	8.0D+01	8.0D+01	BOUND	62	.2
7	8	10	-1.86D+09	1.7D+04	1.0D+00	1.6D+02	1.6D+02	BOUND	67	.2
8	9	14	-1.58D+09	1.7D+04	4.4D-01	3.2D+02	3.2D+02	-CURV	84	.2
9	10	22	-1.36D+09	1.6D+04	3.9D-01	3.2D+02	3.2D+02	-CURV	85	.3
10	11	25	-1.18D+09	1.8D+04	1.0D+00	3.2D+02	3.2D+02	BOUND	85	.3
11	12	28	-8.99D+08	2.3D+04	1.0D+00	6.4D+02	6.4D+02	BOUND	78	.3
12	13	30	-6.59D+08	1.6D+04	1.0D+00	1.3D+03	1.3D+03	BOUND	73	.4
13	14	36	-3.60D+08	1.3D+04	1.0D+00	2.6D+03	2.6D+03	BOUND	75	.4
14	15	49	-8.46D+07	2.0D+04	4.6D-01	5.1D+03	5.1D+03	-CURV	72	.4
15	16	63	-1.20D+05	1.0D+03	1.0D+00	5.1D+03	5.0D+03	CONVR	79	.5
16	17	87	1.12D+04	2.9D+01	1.0D+00	1.0D+04	1.4D+02	CONVR	89	.5
17	18	585	1.41D+04	1.2D+02	7.2D-01	1.0D+04	9.3D+02	CONVR	76	1.3
18	19	687	1.51D+04	1.9D+00	1.0D+00	1.0D+04	6.3D+01	CONVR	66	1.4
19	20	1022	1.53D+04	7.7D+01	4.8D-01	1.0D+04	3.4D+02	CONVR	71	1.9
20	20	1139	1.53D+04	7.7D+01	-2.5D-01	1.0D+04	5.4D+02	CONVR	71	2.1
21	21	1156	1.55D+04	9.4D-01	1.0D+00	3.1D+02	5.1D+00	CONVR	66	2.1
22	21	1405	1.55D+04	9.4D-01	1.6D-02	3.1D+02	3.0D+02	CONVR	66	2.5
23	21	1502	1.55D+04	9.4D-01	-2.5D+00	1.6D+02	1.6D+02	-CURV	66	2.7
24	22	1573	1.55D+04	1.4D+00	8.6D-01	9.8D+00	9.8D+00	BOUND	59	2.8
25	23	1643	1.55D+04	7.9D-01	1.0D+00	2.0D+01	2.0D+01	BOUND	59	2.9
26	24	1690	1.55D+04	1.0D+01	1.0D+00	3.9D+01	3.9D+01	BOUND	73	3.0
27	25	1777	1.55D+04	7.3D-01	1.0D+00	7.8D+01	6.1D+00	CONVR	62	3.1
28	26	1938	1.56D+04	5.1D+00	9.4D-01	7.8D+01	7.8D+01	BOUND	56	3.4
29	27	2008	1.56D+04	1.5D-01	1.0D+00	1.6D+02	1.0D+01	CONVR	56	3.5
30	28	2171	1.56D+04	4.8D+00	9.0D-01	1.6D+02	1.6D+02	BOUND	63	3.7
31	29	2214	1.56D+04	1.3D-01	1.0D+00	3.1D+02	1.9D+00	CONVR	54	3.8
32	30	2357	1.56D+04	6.1D+00	1.0D+00	3.1D+02	2.3D+02	CONVR	58	4.0
33	31	2393	1.56D+04	6.1D-02	1.0D+00	4.5D+02	3.9D+00	CONVR	53	4.1

Iteration number 33 Merit function value = 1.55980709335D+04
No. derivative evaluations 31 Projected gradient norm = 6.08994549861D-02
C.G. iterations 2393 Trust region radius = 4.54287907195D+02
Number of updates skipped 361

There are 111 variables and 60 active bounds

Times for Cauchy, systems, products and updates .06 .59 2.52 .08

Exact Cauchy step computed

Conjugate gradients without preconditioner used

Infinity-norm trust region used

Finite-difference approximations to nonlinear-element gradients used

S.R.1 Approximation to second derivatives used

Objective function value 1.55988611755501D+04

Penalty parameter = 1.0000D-01
Projected gradient norm = 6.0899D-02 Required gradient norm = 1.0000D-01

Constraint norm = 1.7597D-01 Required constraint norm = 1.0000D-01

*********** Reducing mu ***************

Penalty parameter 1.0000D-02 Required projected gradient norm = 1.0000D-02
Required constraint norm = 7.9433D-02

Iter	#g.ev	c.g.it	f	proj.g	rho	radius	step	cgend	#free	time
33	31	2393	1.56D+04	9.2D+00	-	-	-	-	56	4.1
34	31	2408	1.56D+04	9.2D+00	-1.4D+00	5.0D+00	5.0D+00	-CURV	56	4.1
35	32	2414	1.56D+04	2.2D+01	7.8D-01	1.6D-01	1.6D-01	BOUND	68	4.2
36	33	2440	1.56D+04	4.9D+00	9.5D-01	3.1D-01	3.1D-01	BOUND	64	4.2
37	34	2494	1.56D+04	2.4D+00	9.8D-01	6.3D-01	6.1D-01	CONVR	62	4.3
38	35	2569	1.56D+04	7.6D-02	1.0D+00	1.2D+00	4.5D-01	CONVR	54	4.4
39	36	2693	1.56D+04	1.3D+00	1.0D+00	1.2D+00	1.2D+00	BOUND	58	4.6
40	37	2750	1.56D+04	1.8D-01	1.0D+00	2.4D+00	2.5D-01	CONVR	53	4.7
41	38	2803	1.56D+04	4.1D-01	1.0D+00	2.4D+00	2.4D+00	BOUND	53	4.8
42	39	2857	1.56D+04	2.6D-02	1.0D+00	4.9D+00	5.9D-01	CONVR	52	4.9
43	40	2947	1.56D+04	6.4D-01	1.0D+00	4.9D+00	4.9D+00	BOUND	53	5.0
44	41	2989	1.56D+04	1.8D-02	1.0D+00	9.8D+00	7.0D-01	CONVR	50	5.1
45	42	3036	1.56D+04	4.1D-01	1.0D+00	9.8D+00	9.8D+00	BOUND	57	5.2
46	43	3125	1.56D+04	1.3D-02	1.0D+00	2.0D+01	6.5D-01	CONVR	50	5.3
47	44	3173	1.56D+04	1.4D+00	1.0D+00	2.0D+01	2.0D+01	BOUND	54	5.4
48	45	3212	1.56D+04	6.7D-03	1.0D+00	3.9D+01	1.0D-01	CONVR	50	5.5

Iteration number 48 Merit function value = 1.55982845836D+04
No. derivative evaluations 45 Projected gradient norm = 6.70265462136D-03
C.G. iterations 3212 Trust region radius = 3.91875265317D+01
Number of updates skipped 655

There are 111 variables and 61 active bounds

Times for Cauchy, systems, products and updates .07 .84 3.23 .12

Exact Cauchy step computed

Conjugate gradients without preconditioner used

Infinity-norm trust region used

Finite-difference approximations to nonlinear-element gradients used

S.R.1 Approximation to second derivatives used

Objective function value 1.55983641335018D+04

Penalty parameter = 1.0000D-02
Projected gradient norm = 6.7027D-03 Required gradient norm = 1.0000D-02
Constraint norm = 1.7820D-02 Required constraint norm = 7.9433D-02

******** Updating multiplier estimates **********

Penalty parameter 1.0000D-02 Required projected gradient norm = 1.0000D-04
Required constraint norm = 1.2589D-03

Iter	#g.ev	c.g.it	f	proj.g	rho	radius	step	cgend	#free	time
48	45	3212	1.56D+04	9.9D-01	-	-	-	-	54	5.5
49	46	3270	1.56D+04	1.7D-01	9.1D-01	5.0D+00	2.2D-01	CONVR	56	5.6

50	47	3340	1.56D+04	4.7D-03	9.8D-01	5.0D+00	2.2D-01	CONVR	50	5.7
51	47	3388	1.56D+04	4.7D-03	8.6D-02	5.0D+00	5.0D+00	-CURV	50	5.8
52	48	3436	1.56D+04	4.6D-01	4.2D-01	2.5D+00	2.5D+00	-CURV	58	5.8
53	49	3477	1.56D+04	1.0D-02	1.0D+00	2.5D+00	1.1D-01	CONVR	50	5.9
54	50	3627	1.56D+04	1.3D-02	1.1D+00	2.5D+00	1.1D+00	MAXIT	50	6.1
55	51	3676	1.56D+04	1.4D-01	1.0D+00	2.5D+00	2.5D+00	BOUND	55	6.2
56	52	3726	1.56D+04	2.4D-03	1.0D+00	5.0D+00	1.2D-01	CONVR	50	6.3
57	53	3772	1.56D+04	3.0D-01	5.9D-01	5.0D+00	5.0D+00	-CURV	58	6.4
58	54	3813	1.56D+04	5.8D-03	1.0D+00	5.0D+00	1.2D-01	CONVR	50	6.4
59	55	3861	1.56D+04	2.1D-01	2.7D-01	5.0D+00	5.0D+00	-CURV	57	6.5
60	56	3898	1.56D+04	9.6D-04	1.0D+00	5.0D+00	1.2D-02	CONVR	50	6.6
61	56	3942	1.56D+04	9.6D-04	-2.9D+00	5.0D+00	5.0D+00	-CURV	50	6.7
62	57	3983	1.56D+04	3.7D-02	5.1D-01	1.6D-01	1.6D-01	BOUND	51	6.7
63	58	4013	1.56D+04	2.4D-03	1.0D+00	1.6D-01	4.6D-02	CONVR	50	6.8
64	59	4061	1.56D+04	3.4D-02	8.3D-01	1.6D-01	1.6D-01	-CURV	50	6.9
65	60	4107	1.56D+04	1.1D-03	9.9D-01	3.1D-01	7.2D-02	CONVR	50	6.9
66	61	4156	1.56D+04	1.9D-02	9.7D-01	3.1D-01	3.1D-01	BOUND	50	7.0
67	62	4202	1.56D+04	1.0D-03	1.0D+00	6.3D-01	7.1D-02	CONVR	50	7.1
68	63	4252	1.56D+04	1.5D-01	9.7D-01	6.3D-01	6.3D-01	-CURV	50	7.2
69	64	4278	1.56D+04	2.5D-03	1.0D+00	1.3D+00	2.3D-02	CONVR	50	7.2
70	64	4336	1.56D+04	2.5D-03	-8.6D-01	1.3D+00	1.3D+00	-CURV	50	7.3
71	65	4380	1.56D+04	3.4D-02	1.0D+00	3.9D-02	3.9D-02	BOUND	50	7.4
72	66	4425	1.56D+04	9.6D-03	9.8D-01	7.8D-02	7.8D-02	BOUND	50	7.5
73	67	4471	1.56D+04	2.9D-02	1.0D+00	1.6D-01	1.6D-01	BOUND	50	7.6
74	68	4499	1.56D+04	1.2D-03	1.0D+00	3.1D-01	8.1D-02	CONVR	50	7.6
75	69	4547	1.56D+04	2.6D-02	1.4D+00	3.1D-01	3.1D-01	BOUND	50	7.7
76	70	4581	1.56D+04	9.3D-04	1.0D+00	6.3D-01	7.2D-02	CONVR	50	7.8
77	71	4629	1.56D+04	1.5D-01	1.2D+00	6.3D-01	6.3D-01	BOUND	52	7.8
78	72	4657	1.56D+04	3.1D-03	1.0D+00	1.3D+00	5.5D-02	CONVR	50	7.9
79	73	4708	1.56D+04	3.7D-03	1.8D+00	1.3D+00	1.9D-01	CONVR	50	8.0
80	74	4756	1.56D+04	5.1D-02	1.1D+00	1.3D+00	1.3D+00	BOUND	52	8.1
81	75	4800	1.56D+04	6.2D-04	1.0D+00	2.5D+00	9.3D-02	CONVR	50	8.1
82	76	4850	1.56D+04	1.7D-03	2.0D+00	2.5D+00	8.8D-02	CONVR	49	8.2
83	77	4898	1.56D+04	3.1D-02	8.6D-01	2.5D+00	2.5D+00	-CURV	50	8.3
84	78	4945	1.56D+04	4.5D-04	1.0D+00	5.0D+00	1.1D-01	CONVR	50	8.4
85	78	4988	1.56D+04	4.5D-04	-4.3D+00	5.0D+00	5.0D+00	BOUND	50	8.4
86	79	5016	1.56D+04	9.1D-03	6.5D-01	3.1D-01	3.1D-01	BOUND	51	8.5
87	80	5066	1.56D+04	2.6D-02	9.9D-01	3.1D-01	3.1D-01	BOUND	50	8.6
88	81	5115	1.56D+04	2.3D-01	1.1D+00	6.3D-01	6.3D-01	BOUND	50	8.7
89	82	5134	1.56D+04	3.1D-03	1.0D+00	1.3D+00	1.6D-02	CONVR	50	8.7
90	83	5182	1.56D+04	1.3D-01	1.5D+00	1.3D+00	1.3D+00	BOUND	52	8.8
91	84	5213	1.56D+04	4.6D-03	1.0D+00	2.5D+00	1.0D-01	CONVR	50	8.8
92	85	5261	1.56D+04	2.4D-01	1.3D+00	2.5D+00	2.5D+00	BOUND	52	8.9
93	86	5298	1.56D+04	4.3D-03	1.0D+00	5.0D+00	3.0D-02	CONVR	50	9.0
94	86	5356	1.56D+04	4.3D-03	-4.6D-01	5.0D+00	5.0D+00	-CURV	50	9.1
95	87	5403	1.56D+04	2.1D-02	1.1D+00	1.6D-01	1.6D-01	BOUND	50	9.2
96	88	5435	1.56D+04	8.8D-04	9.9D-01	3.1D-01	4.9D-02	CONVR	50	9.2
97	89	5482	1.56D+04	3.0D-02	1.1D+00	3.1D-01	3.1D-01	BOUND	51	9.3
98	90	5517	1.56D+04	1.3D-03	1.0D+00	6.3D-01	4.4D-02	CONVR	50	9.4
99	91	5564	1.56D+04	6.1D-02	9.1D-01	6.3D-01	6.3D-01	-CURV	50	9.4

INFORM = 0 Number of iterations = 4621
Time(LANCELOT) = 201.93 Time(other) = 33.80

--*-*-*-*-* Maximizer sought *-*-*-*-*-*-*-*-*

************* Problem RESERV *****************

APPENDIX F

This appendix shows the inputs and the output of the optimization model as well as the independent variables used in regression analysis. Tables (f.1) and (f.2) show the input to the optimization model. Tables (f.3) to (f.14) show the optimization output for Roseries and Tables (f.15) to (f.26) show the optimization output for Sennar. Also these tables include the independent variables used in regression analysis: QFL, QFL1, QFL2, QFL3, QFL4, SUM1, SUM2, SUM3 and CmR0

Table (f.1) : Generated Inflows, in million m^3 used as Inputs to the Optimization Model

sequence	q1	q2	q3	q4	q5	q6	q7	q8	q9	q10	q11	q12
1	9912	3907	1881	1149	730	346	236	187	369	1745	6383	10875
2	10512	5869	3670	1308	579	380	391	253	594	1609	7596	14529
3	12535	5938	2601	1693	637	524	342	294	448	1354	8102	13917
4	10234	9656	2183	2246	974	607	453	359	868	1873	6366	13714
5	19305	9735	3473	2192	1022	692	462	427	1159	2926	9045	17546
6	11662	7081	3061	1077	804	451	584	251	330	1592	5465	14534
7	10453	6334	2232	1228	557	432	371	415	1379	2678	9803	18057
8	13182	6141	1379	1244	578	384	328	283	424	1564	8314	14326
9	10533	6890	1793	1034	577	379	346	257	220	1397	7270	17267
10	8559	4398	2059	1248	513	351	275	233	359	1046	6678	15478
11	10304	6080	2052	1306	677	441	452	276	730	1456	5151	12334
12	11184	4391	1740	1457	737	530	425	297	609	2730	8657	12939
13	13626	5415	1770	1204	819	388	188	293	371	1186	7802	13574
14	13159	6884	2054	1312	598	452	315	175	386	1575	5363	14628
15	11467	5302	1555	1130	691	480	381	438	834	1522	6255	12130
16	9229	5599	2758	1191	675	351	186	165	164	1518	6168	11599
17	8277	3077	2111	1036	746	345	306	235	369	1343	5291	11073
18	10322	8683	3639	2572	623	453	359	243	497	1590	7169	14988
19	9756	4004	1934	1012	501	505	387	302	1075	2029	8527	13429
20	11954	6327	2954	1270	579	264	274	248	530	1096	7760	19150
21	7151	5381	1698	1352	628	455	245	338	559	1579	5488	14992
22	7132	2478	1425	1102	653	374	304	249	430	1781	8184	13169
23	11568	7213	3368	1561	907	498	342	330	1155	2441	6809	15364

Table (f.2) : Requirements upstream Sennar, ru(i), and Downstream Sennar, rd(i) irrigation & ds(i), other requirements

mnth	1	2	3	4	5	6	7	8	9	10	11	12
ru(i)	1225.63	1267.6	1197.5	1305.8	1314.2	946.2	123.74	74.3	74.3	105.41	245.18	762.82
rd(i)	51.07	52.82	49.9	54.4	54.76	39.42	5.16	3.1	3.1	4.39	10.22	31.78
ds(i)	105.9	105.9	105.9	105.9	105.9	105.9	105.9	105.9	105.9	105.9	105.9	105.9

i represents the month and i = 1 is for september

Table (f.3) Optimization Results for Upstream Reservoir, Roseries, September

NO	X1,1	Y1,1	R1,1	S1,1	QFL	QFL1	QFL2	QFL3	QFL4	CRP	SUM1	SUM2	SUM3
1	2014	5794.4	7808.4	88.3	9912	10875	6383	1745	369	875229.6	10000.3	1E+08	1E+12
2	2014	6394.4	8408.4	88.3	10512	14529	7596	1609	594	928209.6	10600.3	1.12E+08	1.19E+12
3	2014	8417.4	10431.4	88.3	12535	13917	8102	1354	448	1106841	12623.3	1.59E+08	2.01E+12
4	2014	6116.4	8130.4	88.3	10234	13714	6366	1873	868	903662.2	10322.3	1.07E+08	1.1E+12
5	2014	15187.4	17201.4	88.3	19305	17546	9045	2926	1159	1704632	19393.3	3.76E+08	7.29E+12
6	2014	7544.4	9558.4	88.3	11662	14534	5465	1592	330	1029755	11750.3	1.38E+08	1.62E+12
7	2014	6335.4	8349.4	88.3	10453	18057	9803	2678	1379	922999.9	10541.3	1.11E+08	1.17E+12
8	2014	9064.4	11078.4	88.3	13182	14326	8314	1564	424	1163971	13270.3	1.76E+08	2.34E+12
9	2014	6415.4	8429.4	88.3	10533	17267	7270	1397	220	930063.9	10621.3	1.13E+08	1.2E+12
10	2014	4441.4	6455.4	88.3	8559	15478	6678	1046	359	755759.7	8647.3	74775797	6.47E+11
11	2014	6186.4	8200.4	88.3	10304	12334	5151	1456	730	909843.2	10392.3	1.08E+08	1.12E+12
12	2014	7066.4	9080.4	88.3	11184	12939	8657	2730	609	987547.2	11272.3	1.27E+08	1.43E+12
13	2014	9508.4	11522.4	88.3	13626	13574	7802	1186	371	1203176	13714.3	1.88E+08	2.58E+12
14	2014	9041.4	11055.4	88.3	13159	14628	5363	1575	386	1161940	13247.3	1.75E+08	2.32E+12
15	2014	7349.4	9363.4	88.3	11467	12130	6255	1522	834	1012536	11555.3	1.34E+08	1.54E+12
16	2014	5111.4	7125.4	88.3	9229	11599	6168	1518	164	814920.7	9317.3	86812079	8.09E+11
17	2014	4159.4	6173.4	88.3	8277	11073	5291	1343	369	730859.1	8365.3	69978244	5.85E+11
18	2014	6204.4	8218.4	88.3	10322	14988	7169	1590	497	911432.6	10410.3	1.08E+08	1.13E+12
19	2014	5638.4	7652.4	88.3	9756	13429	8527	2029	1075	861454.8	9844.3	96910242	9.54E+11
20	2014	7836.4	9850.4	88.3	11954	19150	7760	1096	530	1055538	12042.3	1.45E+08	1.75E+12
21	2014	3033.4	5047.4	88.3	7151	14992	5488	1579	559	631433.3	7239.3	52407464	3.79E+11
22	2014	3014.4	5028.4	88.3	7132	13169	8184	1781	430	629755.6	7220.3	52132732	3.76E+11
23	2014	7450.4	9464.4	88.3	11568	15364	6809	2441	1155	1021454	11656.3	1.36E+08	1.58E+12

Table (f.4) Optimization Results for Upstream Reservoir, Roseries, October

sequence	X1,2	Y1,2	R1,2	S1,2	QFL	QFL1	QFL2	QFL3	QFL4	CRP	SUM1	SUM2	SUM3
1	2014	1861.87	3875.87	2175	3907	9912	10875	6383	1745	8497725	6082	36990724	2.25E+11
2	2014	3823.87	5837.87	2175	5869	10512	14529	7596	1609	12765075	8044	64705936	5.2E+11
3	2014	3892.87	5906.87	2175	5938	12535	13917	8102	1354	12915150	8113	65820769	5.34E+11
4	2014	7610.87	9624.87	2175	9656	10234	13714	6366	1873	21001800	11831	1.4E+08	1.66E+12
5	2014	7689.87	9703.87	2175	9735	19305	17546	9045	2926	21173625	11910	1.42E+08	1.69E+12
6	2014	5035.87	7049.87	2175	7081	11662	14534	5465	1592	15401175	9256	85673536	7.93E+11
7	2014	4288.87	6302.87	2175	6334	10453	18057	9803	2678	13776450	8509	72403081	6.16E+11
8	2014	4095.87	6109.87	2175	6141	13182	14326	8314	1564	13356675	8316	69155856	5.75E+11
9	2014	4844.87	6858.87	2175	6890	10533	17267	7270	1397	14985750	9065	82174225	7.45E+11
10	2014	2352.87	4366.87	2175	4398	8559	15478	6678	1046	9565650	6573	43204329	2.84E+11
11	2014	4034.87	6048.87	2175	6080	10304	12334	5151	1456	13224000	8255	68145025	5.63E+11
12	2014	2345.87	4359.87	2175	4391	11184	12939	8657	2730	9550425	6566	43112356	2.83E+11
13	2014	3369.87	5383.87	2175	5415	13626	13574	7802	1186	11777625	7590	57608100	4.37E+11
14	2014	4838.87	6852.87	2175	6884	13159	14628	5363	1575	14972700	9059	82065481	7.43E+11
15	2014	3256.87	5270.87	2175	5302	11467	12130	6255	1522	11531850	7477	55905529	4.18E+11
16	2014	3553.87	5567.87	2175	5599	9229	11599	6168	1518	12177825	7774	60435076	4.7E+11
17	2014	1031.87	3045.87	2175	3077	8277	11073	5291	1343	6692475	5252	27583504	1.45E+11
18	2014	6637.87	8651.87	2175	8683	10322	14988	7169	1590	18885525	10858	1.18E+08	1.28E+12
19	2014	1958.87	3972.87	2175	4004	9756	13429	8527	2029	8708700	6179	38180041	2.36E+11
20	2014	4281.87	6295.87	2175	6327	11954	19150	7760	1096	13761225	8502	72284004	6.15E+11
21	2014	3335.87	5349.87	2175	5381	7151	14992	5488	1579	11703675	7556	57093136	4.31E+11
22	2014	4328.72	6342.72	2175	2478	7132	13169	8184	1781	5389650	4653	21650409	1.01E+11
23	2014	5167.87	7181.87	2175	7213	11568	15364	6809	2441	15688275	9388	88134544	8.27E+11

Table (f.5) Optimization Results for Upstream Reservoir, Roseries, November

sequence	X1,3	Y1,3	R1,3	S1,3	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	1815.12	0	1815.12	2175	1881	3907	9912	10875	6383	4091175	4056	16451136	6.67E+10
2	2014	1590.12	3604.12	2175	3670	5869	10512	14529	7596	7982250	5845	34164025	2E+11
3	2014	521.122	2535.122	2175	2601	5938	12535	13917	8102	5657175	4776	22810176	1.09E+11
4	2014	103.122	2117.122	2175	2183	9656	10234	13714	6366	4748025	4358	18992164	8.28E+10
5	2014	1393.12	3407.12	2175	3473	9735	19305	17546	9045	7553775	5648	31899904	1.8E+11
6	2014	981.122	2995.122	2175	3061	7081	11662	14534	5465	6657675	5236	27415696	1.44E+11
7	2014	152.122	2166.122	2175	2232	6334	10456	18057	9803	4854600	4407	19421649	8.56E+10
8	1313.12	0	1313.12	2175	1379	6141	13182	14326	8314	2999325	3554	12630916	4.49E+10
9	1727.12	0	1727.12	2175	1793	6890	10533	17267	7270	3899775	3968	15745024	6.25E+10
10	1993.12	0	1993.12	2175	2059	4398	8559	15478	6678	4478325	4234	17926756	7.59E+10
11	1986.12	0	1986.12	2175	2052	6080	10304	12334	5151	4463100	4227	17867529	7.55E+10
12	1674.12	0	1674.12	2175	1740	4391	11184	12939	8657	3784500	3915	15327225	6E+10
13	1704.12	0	1704.12	2175	1770	5415	13626	13574	7802	3849750	3945	15563025	6.14E+10
14	1988.12	0	1988.12	2175	2054	6884	13159	14628	5363	4467450	4229	17884441	7.56E+10
15	1489.12	0	1489.12	2175	1555	5302	11467	12130	6255	3382125	3730	13912900	5.19E+10
16	2014	678.122	2692.122	2175	2758	5599	9229	11599	6168	5998650	4933	24334489	1.2E+11
17	2014	31.1222	2045.122	2175	2111	3077	8277	11073	5291	4591425	4286	18369796	7.87E+10
18	2014	1559.12	3573.12	2175	3639	8683	10322	14988	7169	7914825	5814	33802596	1.97E+11
19	1868.12	0	1868.12	2175	1934	4004	9756	13429	8527	4206450	4109	16883881	6.94E+10
20	2014	874.122	2888.122	2175	2954	6327	11954	19150	7760	6424950	5129	26306641	1.35E+11
21	1632.12	0	1632.12	2175	1698	5381	7151	14992	5488	3693150	3873	15000129	5.81E+10
22	1359.12	0	1359.12	2175	1425	2478	7132	13169	8184	3099375	3600	12960000	4.67E+10
23	2014	1288.12	3302.12	2175	3368	7213	11568	15364	6809	7325400	5543	30724849	1.7E+11

Table (f.6) Optimization Results for Upstream Reservoir, Roseries, December

sequence	X1,4	Y1,4	R1,4	S1,4	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	1290.49	0	1290.49	2175	1149	1881	3907	9912	10875	2499075	3324	11048976	3.67E+10
2	1299.08	0	1299.08	2175	1309	3670	5869	10512	14529	2847075	3484	12138256	4.23E+10
3	1627.57	0	1627.57	2175	1693	2601	5938	12535	13917	3682275	3868	14961424	5.79E+10
4	2014	166.566	2180.566	2175	2246	2183	9656	10234	13714	4885050	4421	19545241	8.64E+10
5	2014	112.566	2126.566	2175	2192	3473	9735	19305	17546	4767600	4367	19070689	8.33E+10
6	1270.78	0	1270.78	2175	1077	3061	7081	11662	14534	2342475	3252	10575504	3.44E+10
7	1327.91	0	1327.91	2175	1228	2232	6334	10453	18057	2670900	3403	11580409	3.94E+10
8	1372.38	0	1372.38	2175	1244	1379	6141	13182	14326	2705700	3419	11689561	4E+10
9	1393.88	0	1393.88	2175	1034	1793	6890	10533	17267	2248950	3209	10297681	3.3E+10
10	1356.57	0	1356.57	2175	1248	2059	4398	8559	15478	2714400	3423	11716929	4.01E+10
11	1252.11	0	1252.11	2175	1306	2052	6080	10304	12334	2840550	3481	12117361	4.22E+10
12	1391.57	0	1391.57	2175	1457	1740	4391	11184	12939	3168975	3632	13191424	4.79E+10
13	1233.95	0	1233.95	2175	1204	1770	5415	13626	13574	2618700	3379	11417641	3.86E+10
14	1291.71	0	1291.71	2175	1312	2054	6884	13159	14628	2853600	3487	12159169	4.24E+10
15	1294.3	0	1294.3	2175	1130	1555	5302	11467	12130	2457750	3305	10923025	3.61E+10
16	1304.36	0	1304.36	2175	1191	2758	5599	9229	11599	2590425	3366	11329956	3.81E+10
17	1316.95	0	1316.95	2175	1036	2111	3077	8277	11073	2253300	3211	10310521	3.31E+10
18	2014	492.566	2506.566	2175	2572	3639	8683	10322	14988	5594100	4747	22534009	1.07E+11
19	1427.69	0	1427.69	2175	1012	1934	4004	9756	13429	2201100	3187	10156969	3.24E+10
20	1317.48	0	1317.48	2175	1270	2954	6327	11954	19150	2762250	3445	11868025	4.09E+10
21	1286.57	0	1286.57	2175	1352	1698	5381	7151	14992	2940600	3527	12439729	4.39E+10
22	1342.27	0	1342.27	2175	1102	1425	2478	7132	13169	2396850	3277	10738729	3.52E+10
23	1495.57	0	1495.57	2175	1561	3368	7213	11568	15364	3395175	3736	13957696	5.21E+10

Table (f.7) Optimization Results for Upstream Reservoir, Roseries, January

sequence	X1,5	Y1,5	R1,5	S1,5	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	1650.47	0	1650.47	1969.92	730	1149	1881	3907	9912	1438042	2699.92	7289568	1.97E+10
2	1641.88	0	1641.88	2118.99	579	1308	3670	5869	10512	1226895	2697.99	7279150	1.96E+10
3	1474.86	0	1474.86	2175	637	1693	2601	5938	12535	1385475	2812	7907344	2.22E+10
4	1664.2	0	1664.2	2175	974	2246	2183	9656	10234	2118450	3149	9916201	3.12E+10
5	1630.7	0	1630.7	2175	1022	2192	3473	9735	19305	2222850	3197	10220809	3.27E+10
7	1670.18	0	1670.18	1918.1	804	1077	3061	7081	11662	1542152	2722.1	7409828	2.02E+10
8	1613.05	0	1613.05	2011.13	557	1228	2232	6334	10453	1120199	2568.13	6595292	1.69E+10
9	1608.75	0	1608.75	1982.91	578	1244	1379	6141	13182	1146122	2560.91	6558260	1.68E+10
10	1547.08	0	1547.08	1753.49	577	1034	1793	6890	10533	1011764	2330.49	5431184	1.27E+10
11	1584.39	0	1584.39	2002.54	513	1248	2059	4398	8559	1027303	2515.54	6327941	1.59E+10
12	1688.85	0	1688.85	2163.56	677	1306	2052	6080	10304	1464730	2840.56	8068781	2.29E+10
13	1642.77	0	1642.77	2175	737	1457	1740	4391	11184	1602975	2912	8479744	2.47E+10
15	1707.01	0	1707.01	2080.46	819	1204	1770	5415	13626	1703897	2899.46	8406868	2.44E+10
16	1649.25	0	1649.25	2130.25	598	1312	2054	6884	13159	1273890	2728.25	7443348	2.03E+10
18	1646.66	0	1646.66	1947.31	691	1130	1555	5302	11467	1345591	2638.31	6960680	1.84E+10
21	1636.6	0	1636.6	1997.8	675	1191	2758	5599	9229	1348515	2672.8	7143860	1.91E+10
23	1624.01	0	1624.01	1831.71	746	1036	2111	3077	8277	1366456	2577.71	6644589	1.71E+10
25	1474.86	0	1474.86	2175	623	2572	3639	8683	10322	1355025	2798	7828804	2.19E+10
26	1513.27	0	1513.27	1698.18	501	1012	1934	4004	9756	850788.2	2199.18	4836393	1.06E+10
27	1623.48	0	1623.48	2063.09	579	1270	2954	6327	11954	1194529	2642.09	6980640	1.84E+10
28	1654.39	0	1654.39	2175	628	1352	1698	5381	7151	1365900	2803	7856809	2.2E+10
29	1598.69	0	1598.69	1872.03	653	1102	1425	2478	7132	1222436	2525.03	6375777	1.61E+10
30	1644.2	0	1644.2	2175	907	1561	3368	7213	11568	1972725	3082	9498724	2.93E+10

Table (f.8) Optimization Results for Upstream Reservoir, Roseries, February

sequence	X1,6	Y1,6	R1,6	S1,6	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	842.02	0	842.02	996.33	346	730	1149	1881	3907	344730.2	1342.33	1801850	2.42E+09
2	842.02	0	842.02	1001.53	380	579	1308	3670	5869	380581.4	1381.53	1908625	2.64E+09
3	842.02	0	842.02	1279.44	524	637	1693	2601	5938	670426.6	1803.44	3252396	5.87E+09
4	1026.7	0	1026.7	1445.56	607	974	2246	2183	9656	877454.9	2052.56	4213003	8.65E+09
5	1026.7	0	1026.7	1506.5	692	1022	2192	3473	9735	1042498	2198.5	4833402	1.06E+10
6	842.02	0	842.02	999.27	451	804	1077	3061	7081	450670.8	1450.27	2103283	3.05E+09
7	842.02	0	842.02	902.467	432	557	1228	2232	6334	389865.7	1334.467	1780802	2.38E+09
8	842.02	0	842.02	899.838	384	578	1244	1379	6141	345537.8	1283.838	1648240	2.12E+09
9	842.02	0	842.02	734.904	379	577	1034	1793	6890	278528.6	1113.904	1240782	1.38E+09
10	842.02	0	842.02	878.851	351	513	1248	2059	4398	308476.7	1229.851	1512533	1.86E+09
11	842.02	0	842.02	1095.83	441	677	1306	2052	6080	483261	1536.83	2361846	3.63E+09
12	1026.7	0	1026.7	1212.15	530	737	1457	1740	4391	642439.5	1742.15	3035087	5.29E+09
13	842.02	0	842.02	1136.97	388	819	1204	1770	5415	441144.4	1524.97	2325534	3.55E+09
14	842.02	0	842.02	1024.11	452	598	1312	2054	6884	462897.7	1476.11	2178901	3.22E+09
15	842.02	0	842.02	939.3	480	691	1130	1555	5302	450864	1419.3	2014412	2.86E+09
16	842.02	0	842.02	982.947	351	675	1191	2758	5599	345014.4	1333.947	1779415	2.37E+09
17	842.02	0	842.02	902.805	345	746	1036	2111	3077	311467.7	1247.805	1557017	1.94E+09
18	842.02	0	842.02	1265.57	453	623	2572	3639	8683	573303.2	1718.57	2953483	5.08E+09
19	842.02	0	842.02	638.896	505	501	1012	1934	4004	322642.5	1143.896	1308498	1.5E+09
20	842.02	0	842.02	964.911	264	579	1270	2954	6327	254736.5	1228.911	1510222	1.86E+09
21	842.02	0	842.02	1092.65	455	628	1352	1698	5381	497155.8	1547.65	2395221	3.71E+09
22	842.02	0	842.02	875.316	374	653	1102	1425	2478	327368.2	1249.316	1560790	1.95E+09
23	1026.7	0	1026.7	1379.17	498	907	1561	3368	7216	686826.7	1877.17	3523767	6.61E+09

Table (f.9) Optimization Results for Upstream Reservoir, Roseries, March

sequence	X1,7	Y1,7	R1,7	S1,7	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	484.3	0	484.3	458.522	236	346	730	1149	1881	108211.2	694.522	482360.8	3.35E+08
2	563.631	0	563.631	497.209	391	380	579	1308	3670	194408.7	888.209	788915.2	7.01E+08
3	703.24	0	703.24	911.374	342	524	637	1693	2601	311689.9	1253.374	1570946	1.97E+09
4	703.24	0	703.24	973.364	453	607	974	2246	2183	440933.9	1426.364	2034514	2.9E+09
5	703.24	0	703.24	1117.14	462	692	1022	2192	3473	516118.7	1579.14	2493683	3.94E+09
6	703.24	0	703.24	565.187	584	451	804	1077	3061	330069.2	1149.187	1320631	1.52E+09
7	700.437	0	700.437	451.833	371	432	557	1228	2232	167630	822.833	677054.1	5.57E+08
8	484.3	0	484.3	401.828	328	384	578	1244	1379	131799.6	729.828	532648.9	3.89E+08
9	301.656	0	301.656	235.945	346	379	577	1034	1793	81636.97	581.945	338660	1.97E+08
10	484.3	0	484.3	348.729	275	351	513	1248	2059	95900.48	623.729	389037.9	2.43E+08
11	703.24	0	703.24	649.682	452	441	677	1306	2052	293656.3	1101.682	1213703	1.34E+09
12	703.24	0	703.24	668.809	425	530	737	1457	1740	284243.8	1093.809	1196418	1.31E+09
13	556.809	0	556.809	637.496	188	388	819	1204	1770	119849.2	825.496	681443.6	5.63E+08
14	515.568	0	515.568	590.455	315	452	598	1312	2054	185993.3	905.455	819848.8	7.42E+08
15	703.24	0	703.24	535.256	381	480	691	1130	1555	203932.5	916.256	839525.1	7.69E+08
16	484.3	0	484.3	450.385	186	351	675	1191	2758	83771.61	636.385	404985.9	2.58E+08
17	484.3	0	484.3	366.186	306	345	746	1036	2111	112052.9	672.186	451834	3.04E+08
18	703.24	0	703.24	827.566	359	453	623	2572	3639	297096.2	1186.566	1407939	1.67E+09
19	534.539	0	534.539	266.765	387	505	501	1012	1934	103238.1	653.765	427408.7	2.79E+08
20	484.3	0	484.3	346.782	274	264	579	1270	2954	95018.27	620.782	385370.3	2.39E+08
21	621.36	0	621.36	660.417	245	455	628	1352	1698	161802.2	905.417	819779.9	7.42E+08
22	484.3	0	484.3	368.005	304	374	653	1102	1425	111873.5	672.005	451590.7	3.03E+08
23	703.24	0	703.24	800.549	342	498	907	1561	3368	273787.8	1142.549	1305418	1.49E+09

Table (f.10) Optimization Results for Upstream Reservoir, Roseries, April

sequence	X1,8	Y1,8	R1,8	S1,8	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	244.433	0	244.433	174.634	187	236	346	730	1149	32656.56	361.634	130779.1	47294187
2	404.3	0	404.3	286.672	253	391	380	579	1308	72528.02	539.672	291245.9	1.57E+08
3	404.3	0	404.3	503.351	294	342	524	637	1693	147985.2	797.351	635768.6	5.07E+08
4	404.3	0	404.3	673.289	359	453	607	974	2246	241710.8	1032.289	1065621	1.1E+09
5	331.432	0	331.432	822.319	427	462	692	1022	2192	351130.2	1249.319	1560798	1.95E+09
6	404.3	0	404.3	405.301	251	584	451	804	1077	101730.6	656.301	430731	2.83E+08
7	157.603	0	157.603	88.3	415	371	432	557	1228	36644.5	503.3	253310.9	1.27E+08
8	375.389	0	375.389	210.273	283	328	384	578	1244	59507.26	493.273	243318.3	1.2E+08
9	385.569	0	385.569	247.137	257	346	379	577	1034	63514.21	504.137	254154.1	1.28E+08
10	223.919	0	223.919	106.744	233	275	351	513	1248	24871.35	339.744	115426	39215286
11	404.3	0	404.3	357.274	276	452	441	677	1306	98607.62	633.274	401036	2.54E+08
12	357.92	0	357.92	349.243	297	425	530	737	1457	103725.2	646.243	417630	2.7E+08
13	404.3	0	404.3	229.545	293	188	388	819	1204	67256.69	522.545	273053.3	1.43E+08
14	404.3	0	404.3	349.679	175	315	452	598	1312	61193.83	524.679	275288.1	1.44E+08
15	275.802	0	275.802	176.211	438	381	480	691	1130	77180.42	614.211	377255.2	2.32E+08
16	166.485	0	166.485	117.537	165	186	351	675	1191	19393.61	282.537	79827.16	22554125
17	272.318	0	272.318	154.115	235	306	345	746	1036	36217.03	389.115	151410.5	58916090
18	404.3	0	404.3	438.547	243	359	453	623	2572	106566.9	681.547	464506.3	3.17E+08
19	238.06	0	238.06	88.3	302	387	505	501	1012	26666.6	390.3	152334.1	59455995
20	236.115	0	236.115	103.879	248	274	264	579	1270	25761.99	351.879	123818.8	43569246
21	404.3	0	404.3	244.363	338	245	455	628	1352	82594.69	582.363	339146.7	1.98E+08
22	286.114	0	286.114	153.907	249	304	374	653	1102	38322.84	402.907	162334.1	65405525
23	154.8	0	154.8	395.491	330	342	498	907	1561	130512	725.491	526337.2	3.82E+08

Table (f.11) Optimization Results for Upstream Reservoir, Roseries, May

sequence	X1,9	Y1,9	R1,9	S1,9	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	164.835	0	164.835	88.3	369	187	236	346	730	32582.7	457.3	209123.3	95632081
2	154.8	0	154.8	104.112	594	253	391	380	579	61842.53	698.112	487360.4	3.4E+08
3	154.8	0	154.8	354.38	448	294	342	524	637	158762.2	802.38	643813.7	5.17E+08
4	1171.65	0	1171.65	583.687	868	359	453	607	974	506640.3	1451.687	2107395	3.06E+09
5	1867.1	0	1867.1	867.858	1159	427	462	692	1022	1005847	2026.858	4108153	8.33E+09
6	154.8	0	154.8	216.906	330	251	584	451	804	71578.98	546.906	299106.2	1.64E+08
7	1462.9	0	1462.9	314.23	1379	415	371	432	557	433323.2	1693.23	2867028	4.85E+09
8	183.711	0	183.711	88.3	424	283	328	384	578	37439.2	512.3	262451.3	1.34E+08
9	154.8	0	154.8	88.3	220	257	346	379	577	19426	308.3	95048.89	29303573
10	277.06	0	277.06	88.3	359	233	275	351	513	31699.7	447.3	200077.3	89494572
11	224.016	0	224.016	194.996	730	276	452	441	677	142347.1	924.996	855617.6	7.91E+08
12	717.054	0	717.054	253.535	609	297	425	530	737	154402.8	862.535	743966.6	6.42E+08
13	154.8	0	154.8	88.3	371	293	188	388	819	32759.3	459.3	210956.5	96892316
14	154.8	0	154.8	88.3	386	175	315	452	598	34083.8	474.3	224960.5	1.07E+08
15	504.526	0	504.526	305.59	834	438	381	480	691	254862.1	1139.59	1298665	1.48E+09
16	134.568	0	134.568	88.3	164	165	186	351	675	14481.2	252.3	63655.29	16060230
17	224.321	0	224.321	88.3	369	235	306	345	746	32582.7	457.3	209123.3	95632081
18	173.555	0	173.555	241.254	497	243	359	453	623	119903.2	738.254	545019	4.02E+08
19	978.753	0	978.753	124.347	1075	302	387	505	501	133673	1199.347	1438433	1.73E+09
20	316.104	0	316.104	88.3	530	248	274	264	579	46799	618.3	382294.9	2.36E+08
21	154.8	0	154.8	146.796	559	338	245	455	628	1352	705.796	498148	3.52E+08
22	172.507	0	172.507	88.3	430	249	304	374	653	37969	518.3	268634.9	1.39E+08
23	1505.89	0	1505.89	531.018	1155	330	342	498	907	613325.8	1686.018	2842657	4.79E+09

Table (f.12) Optimization Results for Upstream Reservoir, Roseries, June

sequence	X1,10	Y1,10	R1,10	S1,10	QFL	QFL1	QFL2	QFL3	QFL4	CRP	SUM1	SUM2	SUM3
1	1905.35	0	1905.35	264.543	1745	369	187	236	346	461627.5	2009.543	4038263	8.12E+09
2	2014	0	2014	511.307	1609	594	253	391	380	822693	2120.307	4495702	9.53E+09
3	1857.61	0	1857.61	610.709	1354	448	294	342	524	826900	1964.709	3860081	7.58E+09
4	2014	0	2014	245.014	1873	868	359	453	607	458911.2	2118.014	4485983	9.5E+09
5	2014	931.746	2945.746	122.55	2926	1159	427	462	692	358581.3	3048.55	9293657	2.83E+10
6	1847.61	0	1847.61	360.66	1592	330	251	584	451	574170.7	1952.66	3812881	7.45E+09
7	2014	760.253	2774.253	199.838	2678	1379	415	371	432	535166.2	2877.838	8281952	2.38E+10
8	1759.56	0	1759.56	300.08	1564	424	283	328	384	469325.1	1864.08	3474794	6.48E+09
9	1422.14	0	1422.14	127.999	1397	220	257	346	379	178814.6	1524.999	2325622	3.55E+09
10	1087.4	0	1087.4	144.43	1046	359	233	275	351	151073.8	1190.43	1417124	1.69E+09
11	2014	0	2014	665.519	1456	730	276	452	441	968995.7	2121.519	4500843	9.55E+09
12	2014	730.513	2744.513	117.26	2730	609	297	425	530	320119.8	2847.26	8106890	2.31E+10
13	1358.08	0	1358.08	276.381	1186	371	293	188	388	327787.9	1462.381	2138558	3.13E+09
14	1761.7	0	1761.7	291.137	1575	386	175	315	452	458540.8	1866.137	3482467	6.5E+09
15	2014	0	2014	599.004	1522	834	438	381	480	911684.1	2121.004	4498658	9.54E+09
16	1508.43	0	1508.43	92.9143	1518	164	165	186	351	141043.9	1610.914	2595045	4.18E+09
17	1445.41	0	1445.41	206.057	1343	369	235	306	345	276734.6	1549.057	2399578	3.72E+09
18	2014	0	2014	530.461	1590	497	243	359	453	843433	2120.461	4496355	9.53E+09
19	2014	104.748	2118.748	193.269	2029	1075	302	387	505	392142.8	2222.269	4938480	1.1E+10
20	1265.83	0	1265.83	274.114	1096	530	248	274	264	300428.9	1370.114	1877212	2.57E+09
21	1990.91	0	1990.91	518.27	1579	559	338	245	455	818348.3	2097.27	4398541	9.22E+09
22	1993.34	0	1993.34	317.009	1781	430	249	304	374	564593	2098.009	4401642	9.23E+09
23	2014	471.15	2485.15	147.211	2441	1155	330	342	498	359342.1	2588.211	6698836	1.73E+10

Table (f.13) Optimization Results for Upstream Reservoir, Roseries, July

sequence	X1,11	Y1,11	R1,11	S1,11	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	2014	4360.94	6374.94	88.3	6383	1745	369	187	236	563618.9	6471.3	41877724	2.71E+11
2	2014	5573.94	7587.94	88.3	7596	1609	594	253	391	670726.8	7684.3	59048466	4.54E+11
3	2014	6079.94	8093.94	88.3	8102	1354	448	294	342	715406.6	8190.3	67081014	5.49E+11
4	2014	4343.94	6357.94	88.3	6366	1873	868	359	453	562117.8	6454.3	41657988	2.69E+11
5	2014	7022.94	9036.94	88.3	9045	2926	1159	427	462	798673.5	9133.3	83417169	7.62E+11
6	2014	3442.94	5456.94	88.3	5465	1592	330	251	584	482559.5	5553.3	30839141	1.71E+11
7	2014	7780.94	9794.94	88.3	9803	2678	1379	415	371	865604.9	9891.3	97837816	9.68E+11
8	2014	6291.94	8305.94	88.3	8314	1564	424	283	328	734126.2	8402.3	70598645	5.93E+11
9	2014	5247.94	7261.94	88.3	7270	1397	220	257	346	641941	7358.3	54144579	3.98E+11
10	2014	4655.94	6669.94	88.3	6678	1046	359	233	275	589667.4	6766.3	45782816	3.1E+11
11	2014	3128.94	5142.94	88.3	5151	1456	730	276	452	454833.3	5239.3	27450264	1.44E+11
12	2014	6634.94	8648.94	88.3	8657	2730	609	297	425	764413.1	8745.3	76480272	6.69E+11
13	2014	5779.94	7793.94	88.3	7802	1186	371	293	188	688916.6	7890.3	62256834	4.91E+11
14	2014	3340.94	5354.94	88.3	5363	1575	386	175	315	473552.9	5451.3	29716672	1.62E+11
15	2014	4232.94	6246.94	88.3	6255	1522	834	438	381	552316.5	6343.3	40237455	2.55E+11
16	2014	4145.94	6159.94	88.3	6168	1518	164	165	186	544634.4	6256.3	39141290	2.45E+11
17	2014	3268.94	5282.94	88.3	5291	1343	369	235	306	467195.3	5379.3	28936868	1.56E+11
18	2014	5146.94	7160.94	88.3	7169	1590	497	243	359	633022.7	7257.3	52668403	3.82E+11
19	2014	6504.94	8518.94	88.3	8527	2029	1075	302	387	752934.1	8615.3	74223394	6.39E+11
20	2014	5737.94	7751.94	88.3	7760	1096	530	248	274	685208	7848.3	61595813	4.83E+11
21	2014	3465.94	5479.94	88.3	5488	1579	559	338	245	484590.4	5576.3	31095122	1.73E+11
22	2014	6161.94	8175.94	88.3	8184	1781	430	249	304	722647.2	8272.3	68430947	5.66E+11
23	2014	4786.94	6800.94	88.3	6809	2441	1155	330	342	601234.7	6897.3	47572747	3.28E+11

Table (f.14) Optimization Results for Upstream Reservoir, Roseries, August

sequence	X1,12	Y1,12	R1,12	S1,12	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	2014	8855.2	10869.2	88.3	10875	6383	1745	369	187	960262.5	10963.3	1.2E+08	1.32E+12
2	2014	12509.2	14523.2	88.3	14529	7596	1609	594	253	1282911	14617.3	2.14E+08	3.12E+12
3	2014	11897.2	13911.2	88.3	13917	8102	1354	448	294	1228871	14005.3	1.96E+08	2.75E+12
4	2014	11694.2	13708.2	88.3	13714	6366	1873	868	359	1210946	13802.3	1.91E+08	2.63E+12
5	2014	15526.2	17540.2	88.3	17546	9045	2926	1159	427	1549312	17634.3	3.11E+08	5.48E+12
6	2014	12514.2	14528.2	88.3	14534	5465	1592	330	251	1283352	14622.3	2.14E+08	3.13E+12
7	2014	16037.2	18051.2	88.3	18057	9803	2678	1379	415	1594433	18145.3	3.29E+08	5.97E+12
8	2014	12306.2	14320.2	88.3	14326	8314	1564	424	283	1264986	14414.3	2.08E+08	2.99E+12
9	2014	15247.2	17261.2	88.3	17267	7270	1397	220	257	1524676	17355.3	3.01E+08	5.23E+12
10	2014	13458.2	15472.2	88.3	15478	6678	1046	359	233	1366707	15566.3	2.42E+08	3.77E+12
11	2014	10314.2	12328.2	88.3	12334	5151	1456	730	276	1089092	12422.3	1.54E+08	1.92E+12
12	2014	10919.2	12933.2	88.3	12939	8657	2730	609	297	1142514	13027.3	1.7E+08	2.21E+12
13	2014	11554.2	13568.2	88.3	13574	7802	1186	293	388	1198584	13662.3	1.87E+08	2.55E+12
14	2014	12608.2	14622.2	88.3	14628	5363	1575	386	175	1291652	14716.3	2.17E+08	3.19E+12
15	2014	10110.2	12124.2	88.3	12130	6255	1522	834	438	1071079	12218.3	1.49E+08	1.82E+12
16	2014	9579.2	11593.2	88.3	11599	6168	1518	164	165	1024192	11687.3	1.37E+08	1.6E+12
17	2014	9053.2	11067.2	88.3	11073	5291	1343	369	235	977745.9	11161.3	1.25E+08	1.39E+12
18	2014	12968.2	14982.2	88.3	14988	7169	1590	497	243	1323440	15076.3	2.27E+08	3.43E+12
19	2014	11409.2	13423.2	88.3	13429	8527	2029	1075	302	1185781	13517.3	1.83E+08	2.47E+12
20	2014	17130.2	19144.2	88.3	19150	7760	1096	530	248	1690945	19238.3	3.7E+08	7.12E+12
21	2014	12972.2	14986.2	88.3	14992	5488	1579	559	338	1323794	15080.3	2.27E+08	3.43E+12
22	2014	11149.2	13163.2	88.3	13169	8184	1781	430	249	1162823	13257.3	1.76E+08	2.33E+12
23	2014	13344.2	15358.2	88.3	15364	6809	2441	1155	330	1356641	15452.3	2.39E+08	3.69E+12

Table (f.15) Optimization Results for Downstream Reservoir, Sennar, September

sequence	X2,1	Y2,1	R2,1	S2,1	QFL	QFL1	QFL2	QFL3	QFL4	CRP	SUM1	SUM2	SUM3
1	330	5948.3	6278.3	113	7808.4	10869.2	6374.94	1905.35	164.835	882349.2	7921.4	62748578	4.97E+11
2	330	6548.3	6878.3	113	8408.4	14523.2	7587.94	2014	154.8	950149.2	8521.4	72614258	6.19E+11
3	330	8571.3	8901.3	113	10431.4	13911.2	8093.94	1857.61	154.8	1178748	10544.4	1.11E+08	1.17E+12
4	330	6270.3	6600.3	113	8130.4	13708.2	6357.94	2014	1171.65	918735.2	8243.4	67953644	5.6E+11
5	330	15341.3	15671.3	113	17201.4	17540.2	9036.94	2945.746	1867.1	1943758	17314.4	3E+08	5.19E+12
6	330	7698.3	8028.3	113	9558.4	14528.2	5456.94	1847.61	154.8	1080099	9671.4	93535978	9.05E+11
7	330	6489.3	6819.3	113	8349.4	18051.2	9794.94	2774.253	1462.9	943482.2	8462.4	71612214	6.06E+11
8	330	9218.3	9548.3	113	11078.4	14320.2	8305.94	1759.56	183.711	1251859	11191.4	1.25E+08	1.4E+12
9	330	6569.3	6899.3	113	8429.4	17261.2	7261.94	1422.14	154.8	952522.2	8542.4	72972598	6.23E+11
10	330	4595.3	4925.3	113	6455.4	15472.2	6669.94	1087.4	277.06	729460.2	6568.4	43143879	2.83E+11
11	330	6340.3	6670.3	113	8200.4	12328.2	5142.94	2014	224.016	926645.2	8313.4	69112620	5.75E+11
12	330	7220.3	7550.3	113	9080.4	12933.2	8648.94	2744.513	717.054	1026085	9193.4	84518604	7.77E+11
13	330	9662.3	9992.3	113	11522.4	13568.2	7793.94	1358.08	154.8	1302031	11635.4	1.35E+08	1.58E+12
14	330	9195.3	9525.3	113	11055.4	14622.2	5354.94	1761.7	154.8	1249260	11168.4	1.25E+08	1.39E+12
15	330	7503.3	7833.3	113	9363.4	12124.2	6246.94	2014	504.526	1058064	9476.4	89802157	8.51E+11
16	330	5265.3	5595.3	113	7125.4	11593.2	6159.94	1508.43	134.568	805170.2	7238.4	52394435	3.79E+11
17	330	4313.3	4643.3	113	6173.4	11067.2	5282.94	1445.41	224.321	697594.2	6286.4	39518825	2.48E+11
18	330	6358.3	6688.3	113	8218.4	14982.2	7160.94	2014	173.555	928679.2	8331.4	69412226	5.78E+11
19	330	5792.3	6122.3	113	7652.4	13423.2	8518.94	2118.748	978.753	864721.2	7765.4	60301437	4.68E+11
20	330	7990.3	8320.3	113	9850.4	19144.2	7751.94	1265.83	316.104	1113095	9963.4	99269340	9.89E+11
21	330	3187.3	3517.3	113	5047.4	14986.2	5479.94	1990.91	154.8	570356.2	5160.4	26629728	1.37E+11
22	330	3168.3	3498.3	113	5028.4	13163.2	8175.94	1993.34	172.507	568209.2	5141.4	26433994	1.36E+11
23	330	7604.3	7934.3	113	9464.4	15358.2	6800.94	2485.15	1505.89	1069477	9577.4	91726591	8.79E+11

Table (f.16) Optimization Results for Downstream Reservoir, Sennar, October

sequence	X2,2	Y2,2	R2,2	S2,2	QFL	QFL1	QFL2	QFL3	QFL4	CRP	SUM1	SUM2	SUM3
1	330	2184.97	2514.97	362.5	3875.87	7808.4	10869.2	6374.94	1905.35	1405003	4238.37	17963780	7.61E+10
2	330	4146.97	4476.97	362.5	5837.87	8408.4	14523.2	7587.94	2014	2116228	6200.37	38444588	2.38E+11
3	330	4215.97	4545.97	362.5	5906.87	10431.4	13911.2	8093.94	1857.61	2141240	6269.37	39305000	2.46E+11
4	330	7933.97	8263.97	362.5	9624.87	8130.4	13708.2	6357.94	2014	3489015	9987.37	99747560	9.96E+11
5	330	8012.97	8342.97	362.5	9703.87	17201.4	17540.2	9036.94	2945.746	3517653	10066.37	1.01E+08	1.02E+12
6	330	5358.97	5688.97	362.5	7049.87	9558.4	14528.2	5456.94	1847.61	2555578	7412.37	54943229	4.07E+11
7	330	4611.97	4941.97	362.5	6302.87	8349.4	18051.2	9794.94	2774.253	2284790	6665.37	44427157	2.96E+11
8	330	4491.83	4821.83	362.5	6109.87	11078.4	14320.2	8305.94	1759.56	2214828	6472.37	41891573	2.71E+11
9	330	5167.97	5497.97	362.5	6858.87	8429.4	17261.2	7261.94	1422.14	2486340	7221.37	52148185	3.77E+11
10	330	2575.97	2905.97	362.5	4366.87	6455.4	15472.2	6669.94	1087.4	1582990	4729.37	22366941	1.06E+11
11	330	4357.97	4687.97	362.5	6048.87	8200.4	12328.2	5142.94	2014	2192715	6411.37	41105665	2.64E+11
12	330	2668.97	2998.97	362.5	4359.87	9080.4	12933.2	8648.94	2744.513	1580453	4722.37	22300778	1.05E+11
13	330	3692.97	4022.97	362.5	5383.87	11522.4	13568.2	7793.94	1358.08	1951653	5746.37	33020768	1.9E+11
14	330	5161.97	5491.97	362.5	6852.87	11055.4	14622.2	5354.94	1761.7	2484165	7215.37	52061564	3.76E+11
15	330	3670.95	4000.95	362.5	5270.87	9363.4	12124.2	6246.94	2014	1910690	5633.37	31734858	1.79E+11
16	330	3876.97	4206.97	362.5	5567.87	7125.4	11593.2	6159.94	1508.43	2018353	5930.37	35169288	2.09E+11
17	330	1354.97	1684.97	362.5	3045.87	6173.4	11067.2	5282.94	1445.41	1104128	3408.37	11616986	3.96E+10
18	330	6960.97	7290.97	362.5	8651.87	8218.4	14982.2	7160.94	2014	3136303	9014.37	81258866	7.32E+11
19	330	2281.97	2611.97	362.5	3972.87	7652.4	13423.2	8518.94	2118.748	1440165	4335.37	18795433	8.15E+10
20	330	4604.97	4934.97	362.5	6295.87	9850.4	19144.2	7751.94	1265.83	2282253	6658.37	44333891	2.95E+11
21	330	3658.97	3988.97	362.5	5349.87	5047.4	14986.2	5479.94	1990.91	1939328	5712.37	32631171	1.86E+11
22	330	828.703	1158.703	362.5	6342.72	5028.4	13163.2	8175.94	1993.34	2299236	6705.22	44959975	3.01E+11
23	330	5490.97	5820.97	362.5	7181.87	9464.4	15358.2	6800.94	2485.15	2603428	7544.37	56917519	4.29E+11

Table (f.17) Optimization Results for Downstream Reservoir, Sennar, November

sequence	X2,3	Y2,3	R2,3	S2,3	QFL	QFL1	QFL2	QFL3	QFL4	CRP	SUM1	SUM2	SUM3
1	330	211.384	541.384	362.5	1815.12	3875.87	7808.4	10869.2	6374.94	657981	2177.62	4742029	1.03E+10
2	330	2000.14	2330.14	362.5	3604.12	5837.87	8408.4	14523.2	7587.94	1306494	3966.62	15734074	6.24E+10
3	330	931.11	1261.11	362.5	2535.122	5906.87	10431.4	13911.2	8093.94	918981.7	2897.622	8396213	2.43E+10
4	330	477.638	807.638	362.5	2117.122	9624.87	8130.4	13708.2	6357.94	767456.7	2479.622	6148525	1.52E+10
5	330	1767.64	2097.64	362.5	3407.12	9703.87	17201.4	17540.2	9036.94	1235081	3769.62	14210035	5.36E+10
6	330	1391.14	1721.14	362.5	2995.122	7049.87	9558.4	14528.2	5456.94	1085732	3357.622	11273625	3.79E+10
7	330	527.153	857.153	362.5	2166.122	6302.87	8349.4	18051.2	9794.94	785219.2	2528.622	6393929	1.62E+10
8	155.8	0	155.8	362.5	1313.12	6109.87	11078.4	14320.2	8305.94	476006	1675.62	2807702	4.7E+09
9	330	128.722	458.722	362.5	1727.12	6858.87	8429.4	17261.2	7261.94	626081	2089.62	4366512	9.12E+09
10	330	392.553	722.553	362.5	1993.12	4366.87	6455.4	15472.2	6669.94	722506	2355.62	5548946	1.31E+10
12	330	122.443	452.443	362.5	1674.12	4359.87	9080.4	12933.2	8648.94	606868.5	2036.62	4147821	8.45E+09
13	330	100.152	430.152	362.5	1704.12	5383.87	11522.4	13568.2	7793.94	617743.5	2066.62	4270918	8.83E+09
14	330	384.141	714.141	362.5	1988.12	6852.87	11055.4	14622.2	5354.94	720693.5	2350.62	5525414	1.3E+10
15	291.622	0	291.622	362.5	1489.12	5270.87	9363.4	12124.2	6246.94	539806	1851.62	3428497	6.35E+09
16	330	1088.55	1418.55	362.5	2692.122	5567.87	7125.4	11593.2	6159.94	975894.2	3054.622	9330716	2.85E+10
17	330	442.948	772.948	362.5	2045.122	3045.87	6173.4	11067.2	5282.94	741356.8	2407.622	5796645	1.4E+10
18	330	1933.64	2263.64	362.5	3573.12	8651.87	8218.4	14982.2	7160.94	1295256	3935.62	15489105	6.1E+10
20	330	1288.92	1618.92	362.5	2888.122	6295.87	9850.4	19144.2	7751.94	1046944	3250.622	10566543	3.43E+10
21	330	8.1419	338.1419	362.5	1632.12	5349.87	5047.4	14986.2	5479.94	591643.5	1994.62	3978509	7.94E+09
22	161.622	0	161.622	362.5	1359.12	6342.72	5028.4	13163.2	8175.94	492681	1721.62	2963975	5.1E+09
23	330	1750.38	2080.38	362.5	3302.12	7181.87	9464.4	15358.2	6800.94	1197019	3664.62	13429440	4.92E+10

Table (f.18) Optimization Results for Downstream Reservoir, Sennar, December

sequence	X2,4	Y2,4	R2,4	S2,4	QFL	QFL1	QFL2	QFL3	QFL4	CRP	SUM1	SUM2	SUM3
1	160.3	0	160.3	362.5	1290.49	1815.12	3875.87	7808.4	10869.2	467802.6	1652.99	2732376	4.52E+09
2	160.3	0	160.3	362.5	1299.08	3604.12	5837.87	8408.4	14523.2	470916.5	1661.58	2760848	4.59E+09
3	321.766	0	321.766	362.5	1627.57	2535.122	5906.87	10431.4	13911.2	589994.1	1990.07	3960379	7.88E+09
4	330	476.493	806.493	362.5	2180.566	2117.122	9624.87	8130.4	13708.2	790455.2	2543.066	6467185	1.64E+10
5	330	422.493	752.493	362.5	2126.566	3407.12	9703.87	17201.4	17540.2	770880.2	2489.066	6195450	1.54E+10
6	160.3	0	160.3	362.5	1270.78	2995.122	7049.87	9558.4	14528.2	460657.8	1633.28	2667604	4.36E+09
7	160.3	0	160.3	362.5	1327.91	2166.122	6302.87	8349.4	18051.2	481367.4	1690.41	2857486	4.83E+09
8	160.3	0	160.3	322.322	1372.38	1313.12	6109.87	11078.4	14320.2	442348.3	1694.702	2872015	4.87E+09
9	160.3	0	160.3	362.5	1393.88	1727.12	6858.87	8429.4	17261.2	505281.5	1756.38	3084871	5.42E+09
10	160.3	0	160.3	362.5	1356.57	1993.12	4366.87	6455.4	15472.2	491756.6	1719.07	2955202	5.08E+09
11	160.3	0	160.3	362.5	1252.11	1986.12	6048.87	8200.4	12328.2	453889.9	1614.61	2606965	4.21E+09
12	160.3	0	160.3	362.5	1391.57	1674.12	4359.87	9080.4	12933.2	504444.1	1754.07	3076762	5.4E+09
13	160.3	0	160.3	362.5	1233.95	1704.12	5383.87	11522.4	13568.2	447306.9	1596.45	2548653	4.07E+09
14	160.3	0	160.3	362.5	1291.71	1988.12	6852.87	11055.4	14622.2	468244.9	1654.21	2736411	4.53E+09
15	160.3	0	160.3	362.5	1294.3	1489.12	5270.87	9363.4	12124.2	469183.8	1656.8	2744986	4.55E+09
16	160.3	0	160.3	362.5	1304.36	2692.122	5567.87	7125.4	11593.2	472830.5	1666.86	2778422	4.63E+09
17	160.3	0	160.3	362.5	1316.95	2045.122	3045.87	6173.4	11067.2	477394.4	1679.45	2820552	4.74E+09
18	330	851.077	1181.077	362.5	2506.566	3573.12	8651.87	8218.4	14982.2	908630.2	2869.066	8231540	2.36E+10
19	160.3	0	160.3	362.5	1427.69	1868.12	3972.87	7652.4	13423.2	517537.6	1790.19	3204780	5.74E+09
20	160.3	0	160.3	362.5	1317.48	2888.122	6295.87	9850.4	19144.2	477586.5	1679.98	2822333	4.74E+09
21	160.3	0	160.3	362.5	1286.57	1632.12	5349.87	5047.4	14986.2	466381.6	1649.07	2719432	4.48E+09
22	160.3	0	160.3	362.5	1342.27	1359.12	6342.72	5028.4	13163.2	486572.9	1704.77	2906241	4.95E+09
23	189.766	0	189.766	362.5	1495.57	3302.12	7181.87	9464.4	15358.2	542144.1	1858.07	3452424	6.41E+09

Table (f.19) Optimization Results for Downstream Reservoir, Sennar, January

sequence	X2,5	Y2,5	R2,5	S2,5	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	160.66	0	160.66	186.887	1650.47	1290.49	1815.12	3875.87	7808.4	308451.4	1837.357	3375881	6.2E+09
2	160.66	0	160.66	195.48	1641.88	1299.08	3604.12	5837.87	8408.4	320954.7	1837.36	3375892	6.2E+09
3	160.66	0	160.66	362.5	1474.86	1627.57	2535.122	5906.87	10431.4	534636.8	1837.36	3375892	6.2E+09
4	330	0	330	362.5	1664.2	2180.566	2117.122	9624.87	8130.4	603272.5	2026.7	4107513	8.32E+09
5	316.496	0	316.496	362.5	1630.7	2126.566	3407.12	9703.87	17201.4	591128.8	1993.2	3972846	7.92E+09
6	160.66	0	160.66	167.18	1670.18	1270.78	2995.122	7049.87	9558.4	279220.7	1837.36	3375892	6.2E+09
7	160.66	0	160.66	224.309	1613.05	1327.91	2166.122	6302.87	8349.4	361821.6	1837.359	3375888	6.2E+09
8	160.66	0	160.66	228.606	1608.75	1372.38	1313.12	6109.87	11078.4	367769.9	1837.356	3375877	6.2E+09
9	160.66	0	160.66	290.284	1547.08	1393.88	1727.12	6858.87	8429.4	449092.6	1837.364	3375906	6.2E+09
10	160.66	0	160.66	252.973	1584.39	1356.57	1993.12	4366.87	6455.4	400807.9	1837.363	3375903	6.2E+09
11	160.66	0	160.66	148.51	1688.85	1252.11	1986.12	6048.87	8200.4	250811.1	1837.36	3375892	6.2E+09
12	254.039	0	254.039	287.966	1642.77	1391.57	1674.12	4359.87	9080.4	473061.9	1930.736	3727742	7.2E+09
13	160.66	0	160.66	130.35	1707.01	1233.95	1704.12	5383.87	11522.4	222508.8	1837.36	3375892	6.2E+09
14	160.66	0	160.66	188.113	1649.25	1291.71	1988.12	6852.87	11055.4	310245.4	1837.363	3375903	6.2E+09
15	160.66	0	160.66	190.705	1646.66	1294.3	1489.12	5270.87	9363.4	314026.3	1837.365	3375910	6.2E+09
16	160.66	0	160.66	200.762	1636.6	1304.36	2692.122	5567.87	7125.4	328567.1	1837.362	3375899	6.2E+09
17	160.66	0	160.66	213.352	1624.01	1316.95	2045.122	3045.87	6173.4	346485.8	1837.362	3375899	6.2E+09
18	160.66	0	160.66	362.5	1474.86	2506.566	3573.12	8651.87	8218.4	534636.8	1837.36	3375892	6.2E+09
19	160.66	0	160.66	324.091	1513.27	1427.69	1868.12	3972.87	7652.4	490437.2	1837.361	3375895	6.2E+09
20	160.66	0	160.66	213.883	1623.48	1317.48	2888.122	6295.87	9850.4	347234.8	1837.363	3375903	6.2E+09
21	160.66	0	160.66	182.966	1654.39	1286.57	1632.12	5349.87	5047.4	302697.1	1837.356	3375877	6.2E+09
22	160.66	0	160.66	238.665	1598.69	1342.27	1359.12	6342.72	5028.4	381551.3	1837.355	3375873	6.2E+09
23	330	0	330	362.5	1644.2	1495.57	3302.12	7181.87	9464.4	596022.5	2006.7	4026845	8.08E+09

Table (f.20) Optimization Results for Downstream Reservoir, Sennar, February

sequence	X2,6	Y2,6	R2,6	S2,6	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	145.32	0	145.32	362.5	842.02	1650.47	1290.49	1815.12	3875.87	305232.3	1204.52	1450868	1.75E+09
2	145.32	0	145.32	362.5	842.02	1641.88	1299.08	3604.12	5837.87	305232.3	1204.52	1450868	1.75E+09
3	145.32	0	145.32	362.5	842.02	1474.86	1627.57	2535.122	5906.87	305232.3	1204.52	1450868	1.75E+09
4	330	0	330	362.5	1026.7	1664.2	2180.566	2117.122	9624.87	372178.8	1389.2	1929877	2.68E+09
5	330	0	330	362.5	1026.7	1630.7	2126.566	3407.12	9703.87	372178.8	1389.2	1929877	2.68E+09
6	145.32	0	145.32	362.5	842.02	1670.18	1270.78	2995.122	7049.87	305232.3	1204.52	1450868	1.75E+09
7	145.32	0	145.32	362.5	842.02	1613.05	1327.91	2166.122	6302.87	305232.3	1204.52	1450868	1.75E+09
8	145.32	0	145.32	362.5	842.02	1608.75	1372.38	1313.12	6109.87	305232.3	1204.52	1450868	1.75E+09
9	145.32	0	145.32	362.5	842.02	1547.08	1393.88	1727.12	6858.87	305232.3	1204.52	1450868	1.75E+09
10	145.32	0	145.32	362.5	842.02	1584.39	1356.57	1993.12	4366.87	305232.3	1204.52	1450868	1.75E+09
11	145.32	0	145.32	362.5	842.02	1688.85	1252.11	1986.12	6048.87	305232.3	1204.52	1450868	1.75E+09
12	330	0	330	362.5	1026.7	1642.77	1391.57	1674.12	4359.87	372178.8	1389.2	1929877	2.68E+09
13	145.32	0	145.32	362.5	842.02	1707.01	1233.95	1704.12	5383.87	305232.3	1204.52	1450868	1.75E+09
14	145.32	0	145.32	362.5	842.02	1649.25	1291.71	1988.12	6852.87	305232.3	1204.52	1450868	1.75E+09
15	145.32	0	145.32	362.5	842.02	1646.66	1294.3	1489.12	5270.87	305232.3	1204.52	1450868	1.75E+09
16	145.32	0	145.32	362.5	842.02	1636.6	1304.36	2692.122	5567.87	305232.3	1204.52	1450868	1.75E+09
17	145.32	0	145.32	362.5	842.02	1624.01	1316.95	2045.122	3045.87	305232.3	1204.52	1450868	1.75E+09
18	145.32	0	145.32	362.5	842.02	1474.86	2506.566	3573.12	8651.87	305232.3	1204.52	1450868	1.75E+09
19	145.32	0	145.32	362.5	842.02	1513.27	1427.69	1868.12	3972.87	305232.3	1204.52	1450868	1.75E+09
20	145.32	0	145.32	362.5	842.02	1623.48	1317.48	2888.122	6295.87	305232.3	1204.52	1450868	1.75E+09
21	145.32	0	145.32	362.5	842.02	1654.39	1286.57	1632.12	5349.87	305232.3	1204.52	1450868	1.75E+09
22	145.32	0	145.32	362.5	842.02	1598.69	1342.27	1359.12	6342.72	305232.3	1204.52	1450868	1.75E+09
23	330	0	330	362.5	1026.7	1644.2	1495.57	3302.12	7181.87	372178.8	1389.2	1929877	2.68E+09

Table (f.21) Optimization Results for Downstream Reservoir, Sennar, March

sequence	X2,7	Y2,7	R2,7	S2,7	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	111.06	0	111.06	113	484.3	842.02	1650.47	1290.49	1815.12	54725.9	597.3	356767.3	2.13E+08
2	190.391	0	190.391	113	563.631	842.02	1641.88	1299.08	3604.12	63690.3	676.631	457829.5	3.1E+08
3	330	0	330	113	703.24	842.02	1474.86	1627.57	2535.122	79466.12	816.24	666247.7	5.44E+08
4	330	0	330	113	703.24	1026.7	1664.2	2180.566	2117.122	79466.12	816.24	666247.7	5.44E+08
5	330	0	330	113	703.24	1026.7	1630.7	2126.566	3407.12	79466.12	816.24	666247.7	5.44E+08
6	330	0	330	113	703.24	842.02	1670.18	1270.78	2995.122	79466.12	816.24	666247.7	5.44E+08
7	330	0	330	113	700.437	842.02	1613.05	1327.91	2166.122	79149.38	813.437	661679.8	5.38E+08
8	111.06	0	111.06	113	484.3	842.02	1608.75	1372.38	1313.12	54725.9	597.3	356767.3	2.13E+08
9	111.06	0	111.06	113	301.656	842.02	1547.08	1393.88	1727.12	34087.13	414.656	171939.6	71295786
10	111.06	0	111.06	113	484.3	842.02	1584.39	1356.57	1993.12	54725.9	597.3	356767.3	2.13E+08
11	330	0	330	113	703.24	842.02	1688.85	1252.11	1986.12	79466.12	816.24	666247.7	5.44E+08
12	330	0	330	113	703.24	1026.7	1642.77	1391.57	1674.12	79466.12	816.24	666247.7	5.44E+08
13	183.569	0	183.569	113	556.809	842.02	1707.01	1233.95	1704.12	62919.42	669.809	448644.1	3.01E+08
14	142.328	0	142.328	113	515.568	842.02	1649.25	1291.71	1988.12	58259.18	628.568	395097.7	2.48E+08
15	330	0	330	113	703.24	842.02	1646.66	1294.3	1489.12	79466.12	816.24	666247.7	5.44E+08
16	111.06	0	111.06	113	484.3	842.02	1636.6	1304.36	2692.122	54725.9	597.3	356767.3	2.13E+08
17	111.06	0	111.06	113	484.3	842.02	1624.01	1316.95	2045.122	54725.9	597.3	356767.3	2.13E+08
18	330	0	330	113	703.24	842.02	1474.86	2506.566	3573.12	79466.12	816.24	666247.7	5.44E+08
19	244.559	0	244.559	113	534.539	842.02	1513.27	1427.69	1868.12	60402.91	647.539	419306.8	2.72E+08
20	111.06	0	111.06	113	484.3	842.02	1623.48	1317.48	2888.122	54725.9	597.3	356767.3	2.13E+08
21	248.12	0	248.12	113	621.36	842.02	1654.39	1286.57	1632.12	70213.68	734.36	539284.6	3.96E+08
22	111.06	0	111.06	113	484.3	842.02	1598.69	1342.27	1359.12	54725.9	597.3	356767.3	2.13E+08
23	330	0	330	113	703.24	1026.7	1644.2	1495.57	3302.12	79466.12	816.24	666247.7	5.44E+08

Table (f.22) Optimization Results for Downstream Reservoir, Sennar, April

sequence	X2,8	Y2,8	R2,8	S2,8	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	180.168	0	180.168	362.5	244.433	484.3	842.02	1650.47	1290.49	88606.96	606.933	368367.7	2.24E+08
2	330	0	330	362.5	404.3	563.631	842.02	1641.88	1299.08	146558.8	766.8	587982.2	4.51E+08
3	330	0	330	362.5	404.3	703.24	842.02	1474.86	1627.57	146558.8	766.8	587982.2	4.51E+08
4	330	0	330	362.5	404.3	703.24	1026.7	1664.2	2180.566	146558.8	766.8	587982.2	4.51E+08
5	330	0	330	362.5	331.432	703.24	1026.7	1630.7	2126.566	120144.1	693.932	481541.6	3.34E+08
6	330	0	330	362.5	404.3	703.24	842.02	1670.18	1270.78	146558.8	766.8	587982.2	4.51E+08
7	330	0	330	359.697	157.603	700.437	842.02	1613.05	1327.91	56689.33	517.3	267599.3	1.38E+08
8	330	0	330	362.5	375.389	484.3	842.02	1608.75	1372.38	136078.5	737.889	544480.2	4.02E+08
9	128.625	0	128.625	179.856	385.569	301.656	842.02	1547.08	1393.88	69346.9	565.425	319705.4	1.81E+08
10	271.879	0	271.879	362.5	223.919	484.3	842.02	1584.39	1356.57	81170.64	586.419	343887.2	2.02E+08
11	330	0	330	362.5	404.3	703.24	842.02	1688.85	1252.11	146558.8	766.8	587982.2	4.51E+08
12	330	0	330	362.5	357.92	703.24	1026.7	1642.77	1391.57	129746	720.42	519005	3.74E+08
13	330	0	330	362.5	404.3	556.809	842.02	1707.01	1233.95	146558.8	766.8	587982.2	4.51E+08
14	330	0	330	362.5	404.3	515.568	842.02	1649.25	1291.71	146558.8	766.8	587982.2	4.51E+08
15	330	0	330	362.5	275.802	703.24	842.02	1646.66	1294.3	99978.23	638.302	407429.4	2.6E+08
16	109	0	109	362.5	166.485	484.3	842.02	1636.6	1304.36	60350.81	528.985	279825.1	1.48E+08
17	267.539	0	267.539	362.5	272.318	484.3	842.02	1624.01	1316.95	98715.28	634.818	402993.9	2.56E+08
18	330	0	330	362.5	404.3	703.24	842.02	1474.86	2506.566	146558.8	766.8	587982.2	4.51E+08
19	330	0	330	279.24	238.06	534.539	842.02	1513.27	1427.69	66475.87	517.3	267599.3	1.38E+08
20	323.11	0	323.11	362.5	236.115	484.3	842.02	1623.48	1317.48	85591.69	598.615	358339.9	2.15E+08
21	330	0	330	362.5	404.3	621.36	842.02	1654.39	1286.57	146558.8	766.8	587982.2	4.51E+08
22	229.521	0	229.521	362.5	286.114	484.3	842.02	1598.69	1342.27	103716.3	648.614	420700.1	2.73E+08
23	330	0	330	362.5	154.8	703.24	1026.7	1644.2	1495.57	56115	517.3	267599.3	1.38E+08

Table (f.23) Optimization Results for Downstream Reservoir, Sennar, May

sequence	X2,9	Y2,9	R2,9	S2,9	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	330	0	330	352.465	164.835	244.433	484.3	842.02	1650.47	58098.57	517.3	267599.3	1.38E+08
2	330	0	330	362.5	154.8	404.3	563.631	842.02	1641.88	56115	517.3	267599.3	1.38E+08
3	330	0	330	362.5	154.8	404.3	703.24	842.02	1474.86	56115	517.3	267599.3	1.38E+08
4	330	671.571	1001.571	362.5	1171.65	404.3	703.24	1026.7	1664.2	424723.1	1534.15	2353616	3.61E+09
5	330	1291.88	1621.88	289.632	1867.1	331.432	703.24	1026.7	1630.7	540771.9	2156.732	4651493	1E+10
6	330	0	330	362.5	154.8	404.3	703.24	842.02	1670.18	56115	517.3	267599.3	1.38E+08
7	330	808.63	1138.63	113	1462.9	157.603	700.437	842.02	1613.05	165307.7	1575.9	2483461	3.91E+09
8	330	0	330	333.589	183.711	375.389	484.3	842.02	1608.75	61283.97	517.3	267599.3	1.38E+08
9	330	0	330	362.5	154.8	385.569	301.656	842.02	1547.08	56115	517.3	267599.3	1.38E+08
10	330	0	330	240.24	277.06	223.919	484.3	842.02	1584.39	66560.89	517.3	267599.3	1.38E+08
11	330	0	330	362.5	224.016	404.3	703.24	842.02	1688.85	81205.8	586.516	344001	2.02E+08
12	330	169.217	499.217	316.12	717.054	357.92	703.24	1026.7	1642.77	226675.1	1033.174	1067449	1.1E+09
13	330	0	330	362.5	154.8	404.3	556.809	842.02	1707.01	56115	517.3	267599.3	1.38E+08
14	330	0	330	362.5	154.8	404.3	515.568	842.02	1649.25	56115	517.3	267599.3	1.38E+08
15	330	0	330	234.002	504.526	275.802	703.24	842.02	1646.66	118060.1	738.528	545423.6	4.03E+08
16	292.954	0	292.954	345.685	134.568	166.485	484.3	842.02	1636.6	46518.14	480.253	230642.9	1.11E+08
17	330	0	330	292.979	224.321	272.318	484.3	842.02	1624.01	65721.34	517.3	267599.3	1.38E+08
18	330	0	330	362.5	173.555	404.3	703.24	842.02	1474.86	62913.69	536.055	287355	1.54E+08
19	330	214.971	544.971	113	978.753	238.06	534.539	842.02	1513.27	110599.1	1091.753	1191925	1.3E+09
20	330	0	330	201.196	316.104	236.115	484.3	842.02	1623.48	63598.86	517.3	267599.3	1.38E+08
21	330	0	330	362.5	154.8	404.3	621.36	842.02	1654.39	56115	517.3	267599.3	1.38E+08
22	330	0	330	344.793	172.507	286.114	484.3	842.02	1598.69	59479.21	517.3	267599.3	1.38E+08
23	330	742.11	1072.11	113	1505.89	154.8	703.24	1026.7	1644.2	170165.6	1618.89	2620805	4.24E+09

Table (f.24) Optimization Results for Downstream Reservoir, Sennar, June

sequence	X2,10	Y2,10	R2,10	S2,10	QFL	QFL1	QFL2	QFL3	QFL4	CRP	SUM1	SUM2	SUM3
1	330	1419.81	1749.81	113	1905.35	164.835	244.433	484.3	842.02	215304.6	2018.35	4073737	8.22E+09
2	330	1528.46	1858.46	113	2014	154.8	404.3	563.631	842.02	227582	2127	4524129	9.62E+09
3	330	1372.07	1702.07	113	1857.61	154.8	404.3	703.24	842.02	209909.9	1970.61	3883304	7.65E+09
4	330	1746.77	2076.77	362.5	2014	1171.65	404.3	703.24	1026.7	730075	2376.5	5647752	1.34E+10
5	330	2678.52	3008.52	362.5	2945.746	1867.1	331.432	703.24	1026.7	1067833	3308.246	10944492	3.62E+10
6	330	1362.07	1692.07	113	1847.61	154.8	404.3	703.24	842.02	208779.9	1960.61	3843992	7.54E+09
7	330	2507.03	2837.03	362.5	2774.253	1462.9	157.603	700.437	842.02	1005667	3136.753	9839219	3.09E+10
8	330	1274.02	1604.02	113	1759.56	183.711	375.389	484.3	842.02	198830.3	1872.56	3506481	6.57E+09
9	330	936.596	1266.596	113	1422.14	154.8	385.569	301.656	842.02	160701.8	1535.14	2356655	3.62E+09
10	330	601.856	931.856	113	1087.4	277.06	223.919	484.3	842.02	122876.2	1200.4	1440960	1.73E+09
11	330	1587.57	1917.57	182.216	2014	224.016	404.3	703.24	842.02	366983	2196.216	4823365	1.06E+10
12	330	2477.29	2807.29	362.5	2744.513	717.054	357.92	703.24	1026.7	994886	3107.013	9653530	3E+10
13	330	872.535	1202.535	113	1358.08	154.8	404.3	556.809	842.02	153463	1471.08	2164076	3.18E+09
14	330	1276.16	1606.16	113	1761.7	154.8	404.3	515.568	842.02	199072.1	1874.7	3514500	6.59E+09
15	330	1721.45	2051.45	334.228	2014	504.526	275.802	703.24	842.02	673135.2	2348.228	5514175	1.29E+10
16	330	1022.89	1352.89	113	1508.43	134.568	166.485	484.3	842.02	170452.6	1621.43	2629035	4.26E+09
17	330	959.87	1289.87	113	1445.41	224.321	272.318	484.3	842.02	163331.3	1558.41	2428642	3.78E+09
18	330	1544.31	1874.31	131.755	2014	173.555	404.3	703.24	842.02	265354.6	2145.755	4604265	9.88E+09
19	330	1851.52	2181.52	362.5	2118.748	978.753	238.06	534.539	842.02	768046.2	2481.248	6156592	1.53E+10
20	330	780.289	1110.289	113	1265.83	316.104	236.115	484.3	842.02	143038.8	1378.83	1901172	2.62E+09
21	330	1505.36	1835.36	113	1990.91	154.8	404.3	621.36	842.02	224972.8	2103.91	4426437	9.31E+09
22	330	1507.8	1837.8	113	1993.34	172.507	286.114	484.3	842.02	225247.4	2106.34	4436668	9.35E+09
23	330	2217.92	2547.92	362.5	2485.15	1505.89	154.8	703.24	1026.7	900866.9	2847.65	8109111	2.31E+10

Table (f.25) Optimization Results for Downstream Reservoir, Sennar, July

sequence	X2,11	Y2,11	R2,11	S2,11	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	330	5771.63	6101.63	113	6374.94	1905.35	164.835	244.433	484.3	720368.2	6487.94	42093365	2.73E+11
2	330	6984.63	7314.63	113	7587.94	2014	154.8	404.3	563.631	857437.2	7700.94	59304477	4.57E+11
3	330	7490.63	7820.63	113	8093.94	1857.61	154.8	404.3	703.24	914615.2	8206.94	67353864	5.53E+11
4	330	5754.63	6084.63	113	6357.94	2014	1171.65	404.3	703.24	718447.2	6470.94	41873064	2.71E+11
5	330	8433.63	8763.63	113	9036.94	2945.746	1867.1	331.432	703.24	1021174	9149.94	83721402	7.66E+11
6	330	4853.63	5183.63	113	5456.94	1847.61	154.8	404.3	703.24	616634.2	5569.94	31024232	1.73E+11
7	330	9191.63	9521.63	113	9794.94	2774.253	1462.9	157.603	700.437	1106828	9907.94	98167275	9.73E+11
8	330	7702.63	8032.63	113	8305.94	1759.56	183.711	375.389	484.3	938571.2	8418.94	70878551	5.97E+11
9	330	6658.63	6988.63	113	7261.94	1422.14	154.8	385.569	301.656	820599.2	7374.94	54389740	4.01E+11
10	330	6066.63	6396.63	113	6669.94	1087.4	277.06	223.919	484.3	753703.2	6782.94	46008275	3.12E+11
11	330	4539.63	4869.63	113	5142.94	2014	224.016	404.3	703.24	581152.2	5255.94	27624905	1.45E+11
12	330	8045.63	8375.63	113	8648.94	2744.513	717.054	357.92	703.24	977330.2	8761.94	76771593	6.73E+11
13	330	7190.63	7520.63	113	7793.94	1358.08	154.8	404.3	556.809	880715.2	7906.94	62519700	4.94E+11
14	330	4751.63	5081.63	113	5354.94	1761.7	154.8	404.3	515.568	605108.2	5467.94	29898368	1.63E+11
15	330	5643.63	5973.63	113	6246.94	2014	504.526	275.802	703.24	705904.2	6359.94	40448837	2.57E+11
16	330	5556.63	5886.63	113	6159.94	1508.43	134.568	166.485	484.3	696073.2	6272.94	39349776	2.47E+11
17	330	4679.63	5009.63	113	5282.94	1445.41	224.321	272.318	484.3	596972.2	5395.94	29116168	1.57E+11
18	330	6557.63	6887.63	113	7160.94	2014	173.555	404.3	703.24	809186.2	7273.94	52910203	3.85E+11
19	330	7915.63	8245.63	113	8518.94	2118.748	978.753	238.06	534.539	962640.2	8631.94	74510388	6.43E+11
20	330	7148.63	7478.63	113	7751.94	1265.83	316.104	236.115	484.3	875969.2	7864.94	61857281	4.87E+11
21	330	4876.63	5206.63	113	5479.94	1990.91	154.8	404.3	621.36	619233.2	5592.94	31280978	1.75E+11
22	330	7572.63	7902.63	113	8175.94	1993.34	172.507	286.114	484.3	923881.2	8288.94	68706526	5.7E+11
23	330	6197.63	6527.63	113	6800.94	2485.15	1505.89	154.8	703.24	768506.2	6913.94	47802566	3.31E+11

Table (f.26) Optimization Results for Downstream Reservoir, Sennar, August

sequence	X2,12	Y2,12	R2,12	S2,12	QFL	QFL1	QFL2	QFL3	QFL4	CRP	sum1	sum2	sum3
1	330	9755.51	10085.51	113	10869.2	6374.94	1905.35	164.835	244.433	1228220	10982.2	1.21E+08	1.32E+12
2	330	13409.5	13739.5	113	14523.2	7587.94	2014	154.8	404.3	1641122	14636.2	2.14E+08	3.14E+12
3	330	12797.5	13127.5	113	13911.2	8093.94	1857.61	154.8	404.3	1571966	14024.2	1.97E+08	2.76E+12
4	330	12594.5	12924.5	113	13708.2	6357.94	2014	1171.65	404.3	1549027	13821.2	1.91E+08	2.64E+12
5	330	16426.5	16756.5	113	17540.2	9036.94	2945.746	1867.1	331.432	1982043	17653.2	3.12E+08	5.5E+12
6	330	13414.5	13744.5	113	14528.2	5456.94	1847.61	154.8	404.3	1641687	14641.2	2.14E+08	3.14E+12
7	330	16937.5	17267.5	113	18051.2	9794.94	2774.253	1462.9	157.603	2039786	18164.2	3.3E+08	5.99E+12
8	330	13206.5	13536.5	113	14320.2	8305.94	1759.56	183.711	375.389	1618183	14433.2	2.08E+08	3.01E+12
9	330	16147.5	16477.5	113	17261.2	7261.94	1422.14	154.8	385.569	1950516	17374.2	3.02E+08	5.24E+12
10	330	14358.5	14688.5	113	15472.2	6669.94	1087.4	277.06	223.919	1748359	15585.2	2.43E+08	3.79E+12
11	330	11214.5	11544.5	113	12328.2	5142.94	2014	224.016	404.3	1393087	12441.2	1.55E+08	1.93E+12
12	330	11819.5	12149.5	113	12933.2	8648.94	2744.513	717.054	357.92	1461452	13046.2	1.7E+08	2.22E+12
13	330	12454.5	12784.5	113	13568.2	7793.94	1358.08	154.8	404.3	1533207	13681.2	1.87E+08	2.56E+12
14	330	13508.5	13838.5	113	14622.2	5354.94	1761.7	154.8	404.3	1652309	14735.2	2.17E+08	3.2E+12
15	330	11010.5	11340.5	113	12124.2	6246.94	2014	504.526	275.802	1370035	12237.2	1.5E+08	1.83E+12
16	330	10479.5	10809.5	113	11593.2	6159.94	1508.43	134.568	166.485	1310032	11706.2	1.37E+08	1.6E+12
17	330	9953.51	10283.51	113	11067.2	5282.94	1445.41	224.321	272.318	1250594	11180.2	1.25E+08	1.4E+12
18	330	13868.5	14198.5	113	14982.2	7160.94	2014	173.555	404.3	1692989	15095.2	2.28E+08	3.44E+12
19	330	12309.5	12639.5	113	13423.2	8518.94	2118.748	978.753	238.06	1516822	13536.2	1.83E+08	2.48E+12
20	330	18030.5	18360.5	113	19144.2	7751.94	1265.83	316.104	236.115	2163295	19257.2	3.71E+08	7.14E+12
21	330	13872.5	14202.5	113	14986.2	5479.94	1990.91	154.8	404.3	1693441	15099.2	2.28E+08	3.44E+12
22	330	12049.5	12379.5	113	13163.2	8175.94	1993.34	172.507	286.114	1487442	13276.2	1.76E+08	2.34E+12
23	330	14244.5	14574.5	113	15358.2	6800.94	2485.15	1505.89	154.8	1735477	15471.2	2.39E+08	3.7E+12

Appendix G

To obtain 1978 and 1998 SIF files, changes in 1988 SIF file, shown in appendix D, are made. This appendix shows these changes.

a) Section "Constants of the Objective Function"

In section "Constants of the Objective Function" of the SIF file, values assigned to the constants $b_{1,1}$ ...., $b_{1,12}$ are changed and the lines defining these constants are rewritten as follows:

<u>**1978**</u>

RE $b_{1,1}$	0.0053
⋮	⋮
RE $b_{1,6}$	0.0053
RE $b_{1,7}$	0.0062
⋮	⋮
RE $b_{1,12}$	0.0062

<u>**1998**</u>

RE $b_{1,1}$	0.0132
⋮	⋮
RE $b_{1,6}$	0.0132
RE $b_{1,7}$	0.0153
⋮	⋮
RE $b_{1,12}$	0.0153

b) Changes in Bound Sections

The lower and upper bounds of storage for Roseries have to be changed. For 1978, the figure 2175 is to be replaced by 2560.3 and figure 88.3 is replaced by figure 150.3 wherever they appear in this section. For 1998, figures 2175 and 88.3 have to be replaced by figures 2016.3 and 64.4 respectively wherever they occur in this section.

c) Changes in Group Uses Section

In the group uses section, all the lines under the subtitles 'continuity equation roseries' have to be rewritten as follows:

<u>1978</u>

```
* Cons1 - continuity equation roseries
 E Cons1    SS12   0.056      SS13    0.0014
*Cons2 - continuity equation roseries
 E Cons2    SS22   0.076      SS23    0.0019
*Cons3 - continuity equation roseries
 E Cons3    SS32   0.158      SS33    0.0038
*Cons4 - continuity equation roseries
 E Cons4    SS42   0.155      SS43    0.0038
*Cons5 - continuity equation roseries
 E Cons5    SS52   0.158      SS53    0.0038
*Cons6 - continuity equation roseries
 E Cons6    SS62   0.175      SS63    0.0043
*Cons7 - continuity equation roseries
 E Cons7    SS72   0.198      SS73    0.0048
*Cons8 - continuity equation roseries
 E Cons8    SS82   0.195      SS83    0.0048
*Cons9 - continuity equation roseries
 E Cons9    SS92   0.178      SS93    0.0043
```

*Cons10 - continuity equation roseries

E Cons10 SS102 0.102 SS103 0.0025

*Cons11 - continuity equation roseries

E Cons11 SS112 0.058 SS113 0.0014

*Cons12 - continuity equation roseries

E Cons12 SS122 0.042 SS123 0.0010

<u>1998</u>

* Cons1 - continuity equation roseries

E Cons1 SS12 0.137 SS13 0.008

*Cons2 - continuity equation roseries

E Cons2 SS22 0.189 SS23 0.011

*Cons3 - continuity equation roseries

E Cons3 SS32 0.389 SS33 0.024

*Cons4 - continuity equation roseries

E Cons4 SS42 0.384 SS43 0.023

*Cons5 - continuity equation roseries

E Cons5 SS52 0.389 SS53 0.024

*Cons6 - continuity equation roseries

E Cons6 SS62 0.433 SS63 0.026

*Cons7 - continuity equation roseries

E Cons7 SS72 0.488 SS73 0.03

*Cons8 - continuity equation roseries

E Cons8 SS82 0.483 SS83 0.029

*Cons9 - continuity equation roseries

E Cons9 SS92 0.439 SS93 0.027

*Cons10 - continuity equation roseries

E Cons10 SS102 0.252 SS103 0.015

*Cons11 - continuity equation roseries

E Cons11 SS112 0.143 SS113 0.009

*Cons12 - continuity equation roseries

E Cons12 SS122 0.104 SS123 0.006

d) Changes in Element Section

The SEIF, Standard Element Input Format, for some elements have to be changed. These elements are I3PR, I4SS and I5SS:

<u>**1978**</u>

```
T I3PR
 R U1    V1   1.0
 R U2    V2   1.0      V3    1.0
 F            U1*U2**0.53
 G U1         U2**0.53
 G U2         0.53*U1/U2**0.47
 H U1    U2   0.53/U2**0.47
 H U2    U2   -0.47*0.53*U1/U2**1.47
 T I4SS
 R U     V1   1.0      V2    1.0
 F            U**0.53
 G U          0.53/U**0.47
 H U     U    -0.47*0.53/U**1.47
 T I5SS
 R U     V1   1.0      V2    1.0
 F            U**1.06
 G U          1.06*U**0.06
 H U     U    1.06*0.06/U**0.94
```

<u>**1998**</u>

```
T I3PR
R U1    V1   1.0
R U2    V2   1.0      V3    1.0
```

```
F                  U1*U2**0.437
G U1               U2**0.437
G U2               0.437*U1/U2**0.563
H U1      U2       0.437/U2**0.563
H U2      U2       -0.437*0.563*U1/U2**1.563
T I4SS
R U       V1       1.0        V2      1.0
F                  U**0.437
G U                0.437/U**0.563
H U       U        -0.437*0.563/U**1.563
T I5SS
R U       V1       1.0        V2      1.0
F                  U**0.874
G U                0.874/U**0.126
H U       U        -0.874*0.126/U**1.126
```