Modelling Requirements for the Design of Active Stability Control Strategies for a High Speed Bogie

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Abstract

The paper presents the findings of a study on active stability control and simulation for a railway bogie vehicle. For control design a plan-view partial railway vehicle model is described. This is a simplified model derived from research experience and appropriate modelling, and a frequency domain analysis illustrates the problems associated with system instability. A multi-body dynamics software, SIMPACK[1], is used to generate a detailed non-linear full vehicle model for simulation and control assessment. Model order reduction methods, both empirically and analytically based, are used to simplify the linear model generated from SIMPACK for further system analysis and control designs based upon the complex model. Comparisons between the simplified plan-view model and the exported reduced-order model are presented.

1 Introduction

Vehicle dynamicists have been aware of active suspensions for some time [1, 2], but so far they have only found substantial application in tilting trains, which can now be thought of an established suspension technology even though developments continue [3]. However there are two other major categories: active secondary suspensions for improved ride quality, and active primary suspensions for improved running stability and curving performance. Not only do these offer quite different possibilities for improved performance, but also they are significantly more challenging from a control design point of view. This arises from the fact that tilting is a relatively low frequency action and the current controllers

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[1]SIMPACK is being developed by INTEC GmbH in Germany.

are relatively simple, although choosing suitable parameters for the controllers is a difficult task. More complex tilting control strategies are possible [4], but as yet have not been used in practice, whereas for the other two categories mentioned above complex control structures and strategies are almost inevitable [5, 6, 7, 8].

Modelling of these systems in a manner which is suitable for control system design is a critical activity, but has not received much attention in the literature. This paper identifies the issues and discusses and compares different approaches that can be used for deriving appropriate models.

2 Modelling Overview

2.1 Modelling Requirements

For designing active suspension systems such as these, an important difference arises compared with passive suspensions. A conventional suspension is designed with as accurate a model as possible so that the computer simulation can predict the on-track performance effectively. The designer then adjusts the values of the suspension components based upon well-understood expectations for the particular vehicle configuration until the required performance is achieved. However for an active suspension it is important to distinguish between the design model and the simulation model: the former is a simplified model used for synthesis of the control strategy and algorithm, whereas the latter is a full-complexity model to test the system performance, i.e. as used for conventional suspensions. The importance of having an appropriately simplified design model is less profound when "classical" control design techniques are being used, although even here key insights arise with simplified models; the real issue arises when modern model-based design approaches are being used, either for the controller itself or for estimators to access difficult or impossible to measure variables, in which case the controller and/or estimator assumes a dynamic complexity equal to or greater than that of the design model. Since a good simulation model will usually have a few hundred or more states, a controller based upon such a model would be overly complex (perhaps impossible) to implement.

Engineering experience can often be used to provide a suitable abstraction. For example, there is a relatively weak coupling between the vertical and lateral motions of rail vehicles and, depending on the objectives, only selected degrees of freedom need to be included in the design model. Common simplifications are based around a vehicle model that is partitioned into side-view, plan-view and end-view models: the side-view model is concerned with the bounce and pitch degrees of freedom, and can be used for active vertical suspensions; the plan-view model deals with the lateral and yaw motions, and can be used for active lateral suspensions and active steering/stability control; the end-view model covers the bounce, lateral and roll motions, and can be used for the design of tilting controllers.

There are two requirements for system modelling: the first is for design models suitable for developing the controllers, and the second is for simulation models suitable to predict the complete behaviour of the vehicle and the control system. The design models tend to be simpler, (relatively) linear and aim to embody the fundamental dynamics only, whereas the simulation models are complex and non-linear. For the generation of the control system models, SIMULINK has been used, which also enables the controllers to be designed and analysed using specialist control techniques and functions; classical tools such as Nichols and Bode plots, and more sophisticated model-based approaches such as optimal and robust control.

2.2 Modelling Software

It is of course essential that such modelling software can support the integration of the controller into the mechanical system. This can either be achieved within a single package, but there is a strong argument for distinct but well-integrated software, i.e. one of the many MBS (Multi-Body Systems) dynamics packages plus a control design package such as MATLAB/SIMULINK. Ideally there should be a number of interface possibilities: controllers designed using the simplified design model need to be exported into the MBS package for simulation purposes; equally it is often valuable to be able to export a complex but linearised model from the MBS package for further controller evaluation using the targeted analytical tools provided for controller design; and finally running the two packages simultaneously in a co-simulation mode is also important because this avoids the need for conversion and export, although the data transfer process must be robust.

3 Design and simulation models

For the generation of the simulation models the non-linear MBS software package called SIMPACK has been used. This package is able to model, with high degrees of accuracy, particular features of railway vehicles such as: the non-linear wheel/rail contact geometry, non-linear characteristics of some suspension components, large displacements and rotations between bodies etc. Figure 1 shows the leading bogie of the vehicle model generated by SIMPACK.

The simulation models offer the best possible prediction of system performance, but as mentioned previously they generally are not suitable for model based control system design due to their complexity, and also because they may not be controllable and/or observable. The design model needs to be relatively simple and easy to validate. It provides insights into the system characteristics and enables control possibilities to be identified and investigated. The design models must also be viable for model based control system techniques. Two approaches for the derivation of design models have been investigated in this paper: the first is a physically-based method and the second uses model order reduction (analytical) techniques.

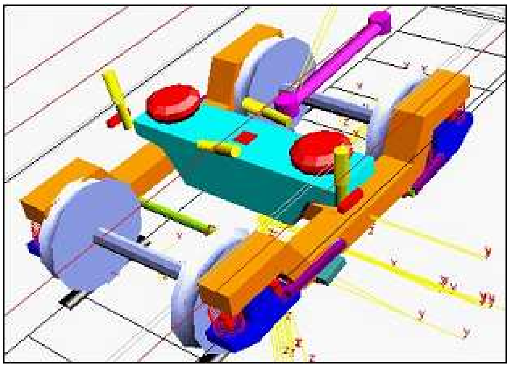

Figure 1: Simulation Model: Full SIMPACK Bogie

3.1 Physically-based methods

Figure 2 presents the plan-view half-vehicle design model used for this study - the parameters for the model are given in Appendix A and are representative of a modern high-speed railway vehicle. It includes seven degrees-of-freedom, i.e. lateral and yaw modes for each wheelset ($y_{\mathrm{w}_1}, \theta_{\mathrm{w}_1}, y_{\mathrm{w}_2}, \theta_{\mathrm{w}_2}$) and for the bogie frame (y_g, θ_g), and a lateral mode for the vehicle body (y_{v}). Active control is provided via the controlled torque for leading and trailing wheelsets ($T_{\mathrm{w}_1}, T_{\mathrm{w}_2}$). The equations of motion are [8, 9]:

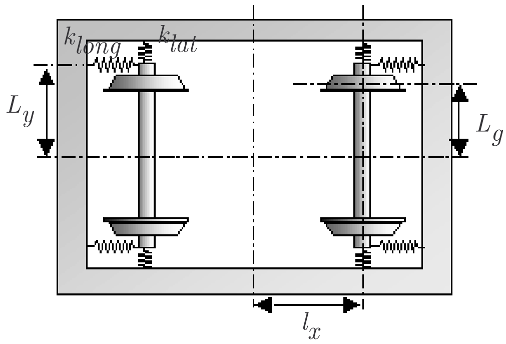

Figure 2: Design Model: Plan-view Bogie-SIMULINK

$$m_{\mathrm{w}}\ddot{y}_{\mathrm{w}_1} + \left(\frac{2f_{22}}{V_s} + C_s\right)\dot{y}_{\mathrm{w}_1} + K_s y_{\mathrm{w}_1} - 2f_{22}\theta_{\mathrm{w}_1} - C_s\dot{y}_g - K_s y_g \ldots$$
$$-C_s L_{\mathrm{v}}\dot{\theta}_g - K_s L_{\mathrm{v}}\theta_g = m_{\mathrm{w}}\left(\frac{V_s^2}{R_1} - g\theta_{c_1}\right) \quad (1)$$

$$I_{\mathrm{w}}\ddot{\theta}_{\mathrm{w}_1} + \frac{2f_{22}L_g^2}{V_s}\dot{\theta}_{\mathrm{w}_1} + \frac{2f_{11}\lambda L_g}{r_0}y_{\mathrm{w}_1} = \frac{2f_{11}L_g^2}{R_1} - \frac{2f_{11}\lambda L_g}{r_0}y_{t_1} + T_{\mathrm{w}_1} \tag{2}$$

$$\begin{aligned} m_{\mathrm{w}}\ddot{y}_{\mathrm{w}_2} + \left(\frac{2f_{22}}{V_s} + C_s\right)\dot{y}_{\mathrm{w}_2} + K_s y_{\mathrm{w}_2} - 2f_{22}\theta_{\mathrm{w}_2} - C_s\dot{y}_g - K_s y_g \ldots \\ +C_s L_{\mathrm{v}}\dot{\theta}_g + K_s L_{\mathrm{v}}\theta_g = m_{\mathrm{w}}\left(\frac{V_s^2}{R_2} - g\theta_{c_2}\right) \end{aligned} \tag{3}$$

$$I_{\mathrm{w}}\ddot{\theta}_{\mathrm{w}_2} + \frac{2f_{22}L_g^2}{V_s}\dot{\theta}_{\mathrm{w}_2} + \frac{2f_{11}\lambda L_g}{r_0}y_{\mathrm{w}_2} = \frac{2f_{11}L_g^2}{R_2} - \frac{2f_{11}\lambda L_g}{r_0}y_{t_2} + T_{\mathrm{w}_2} \tag{4}$$

$$\begin{aligned} m_g\ddot{y}_g + (2C_s + C_{sc})\,\dot{y}_g + (2K_s + K_{sc})\,y_g - C_s\dot{y}_{\mathrm{w}_1} - K_s y_{\mathrm{w}_1} - C_s\dot{y}_{\mathrm{w}_2} \ldots \\ -K_s y_{\mathrm{w}_2} - C_{sc}\dot{y}_{\mathrm{v}} - K_{sc}y_{\mathrm{v}} = \frac{m_g V_s^2}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{m_g g}{2}\left(\theta_{c_1} + \theta_{c_2}\right) \end{aligned} \tag{5}$$

$$\begin{aligned} I_g\ddot{\theta}_g + 2L_{\mathrm{v}}^2 C_s\dot{\theta}_g + 2L_{\mathrm{v}}^2 C_s\theta_g - L_{\mathrm{v}}C_s\dot{y}_{\mathrm{w}_1} + L_{\mathrm{v}}C_s\dot{y}_{\mathrm{w}_2} - L_{\mathrm{v}}K_s y_{\mathrm{w}_1} \ldots \\ +L_{\mathrm{v}}K_s y_{\mathrm{w}_2} = -\left(T_{\mathrm{w}_1} + T_{\mathrm{w}_2}\right) \end{aligned} \tag{6}$$

$$\begin{aligned} m_{\mathrm{v}}\ddot{y}_{\mathrm{v}} + C_{sc}\dot{y}_{\mathrm{v}} + K_{sc}y_{\mathrm{v}} - C_{sc}\dot{y}_g - K_{sc}y_g = \frac{m_{\mathrm{v}}V_s^2}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\ldots \\ -\frac{m_{\mathrm{v}}g}{2}\left(\theta_{c_1} + \theta_{c_2}\right) \end{aligned} \tag{7}$$

The linear model is therefore 14th order overall, and is a highly coupled complex MIMO (Multiple Input Multiple Output) system. It can be then represented in state space form by:

$$\frac{\mathbf{dX}}{\mathbf{dt}} = \mathbf{A}\cdot\mathbf{X} + \mathbf{B}\cdot\mathbf{u} + \boldsymbol{\Gamma}\cdot\mathbf{w} \tag{8}$$

$$\mathbf{X} = [\dot{y}_{\mathrm{w}_1}\;\; y_{\mathrm{w}_1}\;\; \dot{\theta}_{\mathrm{w}_1}\;\; \theta_{\mathrm{w}_1}\;\; \dot{y}_{\mathrm{w}_2}\;\; y_{\mathrm{w}_2}\;\; \dot{\theta}_{\mathrm{w}_2}\;\; \theta_{\mathrm{w}_2}\;\; \dot{y}_g\;\; y_g\;\; \dot{\theta}_g\;\; \theta_g\;\; \dot{y}_{\mathrm{v}}\;\; y_{\mathrm{v}}]^T \tag{9}$$

$$\mathbf{w} = [\frac{1}{R_1}\;\; \theta_{c_1}\;\; y_{t_1}\;\; \frac{1}{R_2}\;\; \theta_{c_2}\;\; y_{t_2}]^T, \quad \mathbf{u} = [T_{\mathrm{w}_1}\;\; T_{\mathrm{w}_2}]^T \tag{10}$$

The main dynamic modes for the wheelset and bogie lie in the range from 2-30Hz, and there is also a body mode at a little under 1Hz. The full list of eigenvalues is given in Table 1. Note that the eigenvalues are assessed without control, and with the actuators locked to provide a degree of stability.

MODES	Speed = 200km/h	
	Frequency (Hz)	Damping(%)
Body lateral	0.92	20.15
Kinematic 1st	5.99	0.80
Kinematic 2nd	13.68	87.06
Bogie yaw	14.71	26.75
Bogie lateral	26.69	99.98
Wheelset (high frequency)		
- 1st	65.83	31.67
- 2nd	72.84	22.20

Table 1: Plan-view (MATLAB/SIMULINK) vehicle model eigen-modes (actuator locked)

3.2 Model order reduction analytical approach

The linear model generated from SIMPACK [2] incorporates a large number of states (130). The exported model is complex and many aspects are rather redundant for representing the system properties of interest in this application. The model size also imposes difficulties both on rigorous system analysis and further control design (if desired) based upon the MBS-derived model (especially in the case of implementing model-based techniques, i.e. LQ Regulators or robust H-infinity controllers). *Model order reduction* [10] is a framework for simplifying the analysis and design procedures and thus the complexity of the final controller (i.e. practical implementation and computational issues in real time applications). The reduced order system to be used in the design must preserve the required Input-Output properties and also must be a good approximation of the full order equivalent for appropriate control design. Hence, the central problem addressed is: given an n^{th} high-order linear model $G(s)$, to derive a low order approximation $G_r(s)$ with a specified size k ($k << n$) such that the infinity-norm ($\infty - norm$) of their difference $||G - G_r||_\infty$ is sufficiently small.

The model reduction process for this paper was based upon the *Balanced Reduction* approach [11]. This is an SVD-based (Singular Value Decomposition) method in which the system is transformed in a form where the states that are difficult to control are also difficult to observe. The reduced model is then obtained simply by removing (truncating) the aforementioned states. The reduction objective is summarised as follows:

[2]SIMPACK is a software tool which can develop/analyse/simulate complex multi-body mechanical systems (`http://www.simpack.de`). SIMPACK comprises tools for exporting linearised equations of motion for mechanical and mechatronic systems by building a CAD style model. The analysis/simulation work for this project was based on SIMPACK and MATLAB.

Compute the k^{th} order reduced model $G_r(s) = C_r(sI - A_r)^{-1}B_r + D_r$ from an n^{th} order full model $G(s) = C(sI - A)^{-1}B + D$ such that

$$\|G - G_r\|_\infty \leq 2 \times \sum_{i=k+1}^{n} \sigma_i \tag{11}$$

where σ_i denotes the Hankel singular values of $G(j\omega)$, i.e. the square roots of the eigenvalues of their controllability P and observability Q grammians

$$\sigma_i := \sqrt{\lambda_i(PQ)} \tag{12}$$

where $\lambda_i(PQ)$ is the ith *largest eigenvalue of PQ and P, Q are the solutions of the following Lyapunov equalities*

$$PA^T + AP + BB^T = 0 \tag{13}$$

$$QA + A^TQ + C^TC = 0 \tag{14}$$

with the grammians equal and balanced, i.e. $P = Q = diag(\sigma_1, \ldots, \sigma_n)$, where $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n$.

Remarks: Balanced truncation preserves stability and provides the same error bound, Equation 11, as in the Hankel norm approximations [10]. In the case of unstable systems (as the case in this paper), the system $G(s)$ is first projected into its stable $G(s)_-$ and unstable $G(s)_+$ parts, i.e. $G(s) = [G(s)_- + G(s)_+]$ [12]. Model reduction is performed on $G(s)_-$ to obtain $G_r(s)_-$ which then combined with $G(s)_+$ gives the overall reduced order system $G_r(s)$. Note that the unstable part remains unchanged throughout the procedure.

Figure 3 presents the normalised Hankel singular values of the full model[3]. Moreover, Figure 4 depicts the relative degree of reduction $\frac{k}{n}$ versus a given error tolerance $\rho := \frac{\sigma_k}{\sigma_1}$ ($k :=$ order of reduced model, $n :=$ order of full model, σ_k, σ_1 are the k^{th} and *maximum* singular values respectively). This figure shows the extent to which the model size can be reduced for a given error tolerance, thus low curves (with gradual slopes) indicate easy approximations. Table 2 illustrates the concept from Figure 4. Note that both figures present results on the stable system part $G(s)_-$[4].

The reduced model sizes of orders 30, 20, 10 are good candidates, with order 20 being the most attractive choice as it relates to both acceptably low tolerance and relative reduction degree. Thus, the overall reduced order system becomes $G_r(s) = G_r(s)_- + G(s)_+$ 24^{th} order, which is rather manageable for subsequent control designs (comparison of all eigenvalues in Figure 9).

[3] A minimal realisation of the full order model is first obtained to remove the initial uncontrollable or unobservable modes. The maximum Hankel singular value of the system is normalised to 1.

[4] G(s)$_-$ after minimal realisation (minreal) has 93 states

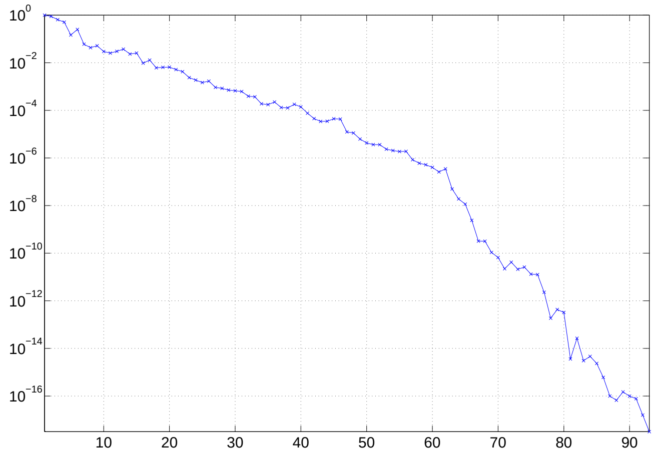

Figure 3: Normalised Hankel singular values of $G(s)_{-}$ (minreal full order model)

3.3 Control and model integration

The initial controller design and assessment used the models generated in SIMULINK, since these are relatively simple but embody the fundamental vehicle dynamics. For the final tuning and assessment of the controllers in realistic operating conditions before being implemented on the experimental vehicle, it was necessary to use the non-linear vehicle models in SIMPACK. Two methods were used to perform this task:

1. linearised, although complex, models were exported from SIMPACK into SIMULINK to enable the controller performance to be tested and simulated in SIMULINK using complex linear models,

order k	relative degree of reduction $\frac{k}{n}$	error tolerance $\frac{\sigma_k}{\sigma_1}$
93	1.0	3.1×10^{-18}
70	0.753	6.6×10^{-11}
50	0.538	4.3×10^{-6}
30	0.323	6.7×10^{-4}
20	0.215	6.5×10^{-3}
10	0.108	3.0×10^{-2}
2	0.0215	8.8×10^{-1}

Table 2: Characteristic values for reduced order degree, relative reduction degree and error tolerance for $G(s)_{-}$

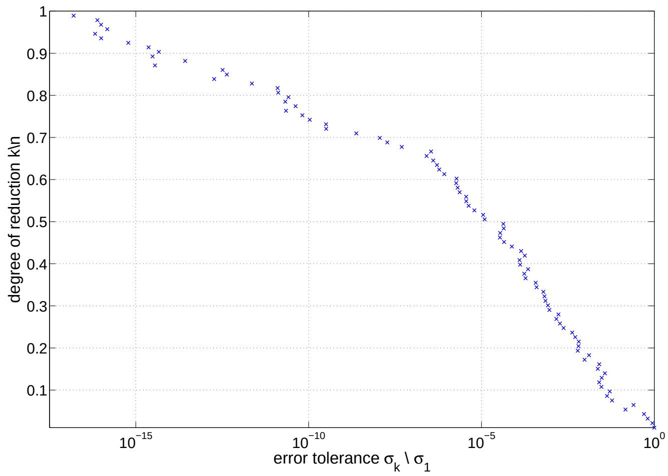

Figure 4: Relative degree of reduction $\frac{k}{n}$ vs error tolerance $\rho = \frac{\sigma_k}{\sigma_1}$ for $G(s)$

2. the controller was simulated in SIMULINK, while the non-linear vehicle dynamics were simulated in SIMPACK. In this case the two packages are linked using co-simulation [13] as shown in Figure 5. A summary of the two models is given in Table 3.

Comprehensive simulation studies have been performed of the various control options. These studies have included stability tests and straight track tests using recorded track data.

The controller as designed in SIMULINK has been translated into controller software written in C. This software was validated by first replacing the SIMULINK controller with the C-code in the simulations, and secondly by performing a frequency response analysis of the DSP (Digital Signal Processor) running the controller code and comparing it to the one taken of the SIMULINK controller.

4 Application of active bogie

The vehicle used as a case study in this paper has no secondary yaw dampers, which conventionally provide stability at high speeds. These dampers also have an unwanted effect of transmitting high frequency vibration to the vehicle body, and this has secondary effects on the body design (stiffness and consequently weight), the dampers themselves are also significantly heavy. Removing the secondary yaw dampers therefore offers significant advantages in terms of the vehicles weight and comfort, however once removed stability and consequently high speed operation are significantly compromised. An active stability system

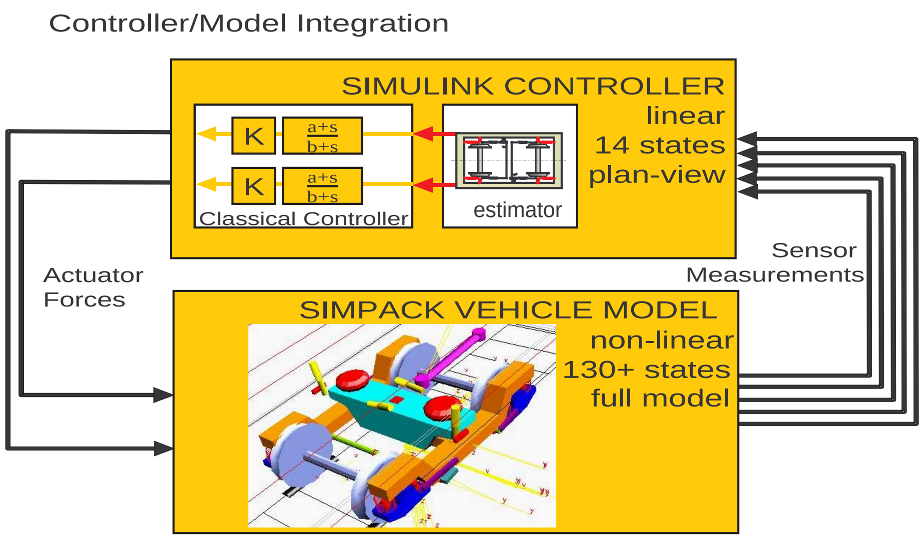

Figure 5: Control algorithm validation scheme

is used to provide vehicle stability at high speed without adding significant weight penalties.

4.1 Summary of problem

The models were validated against each other, by comparing eigenvalues and by the comparison of time histories in response to discrete inputs such as steps in the track. The models were also validated using experimental data obtained from the vehicle. This included a modal analysis of the bogie and also a stability test of the passive vehicle.

4.2 Analysis of the passive system (SIMULINK plan-view design model vs SIMPACK exported model)

The first stage in the validation process was to analyse the passive system and confirm that the design and simulation models were consistent with one another, and this was done prior to any controller design studies. The models were validated against each other, by comparing eigenvalues and by the comparison of time histories in response to discrete inputs such as steps in the track. Figure 6 shows the results for a critical speed analysis, the actuator was locked in position (this removed the effects of the actuator dynamics from the simulation) and the critical speed of the vehicle was calculated for each of the models. Lines are shown for both models corresponding to speeds where damping was zero and where there was only five percent damping remaining. The comparison was favourable and showed that the two models were consistent. Note that the

	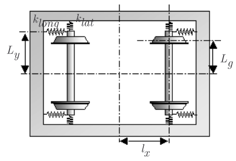	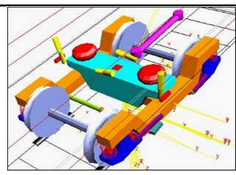
Package	MATLAB/SIMULINK	SIMPACK MBS
Format	Linear	Non-linear
Dynamics	Plan view only - single bogie	Full - complete vehicle
Model size	14	130 (linear exported)
Usage	Controller Development	Vehicle Simulation/Performance

Table 3: Control system design model (SIMULINK and vehicle simulation model (SIMPACK) (only the bogie is shown)

'design model' denotes the 14^{th} order SIMULINK plan-view model.

4.3 Model validation of the active (controlled) system

The active system was validated in a series of stages as the controller developed. The development stages were:

- simple controller only,
- controller with estimator,
- controller and estimator coded into C.

The validation was achieved by comparing the performance and results of the controller using different model approaches listed below:

- controller with SIMULINK plan-view versus controller with exported linear SIMPACK model,
- controller with SIMULINK plan-view model versus controller with SIMPACK nonlinear model,
- controller with SIMPACK nonlinear model versus controller with exported linear SIMPACK model,
- controller with exported linear SIMPACK (analytically) reduced model versus controller with exported linear SIMPACK model,

One used linearised vehicle models exported from SIMPACK into SIMULINK. The second used controllers in SIMULINK co-simulated with vehicle models in SIMPACK, this ensured that the various non-linearities modelled by SIMPACK would not compromise the controller functions.

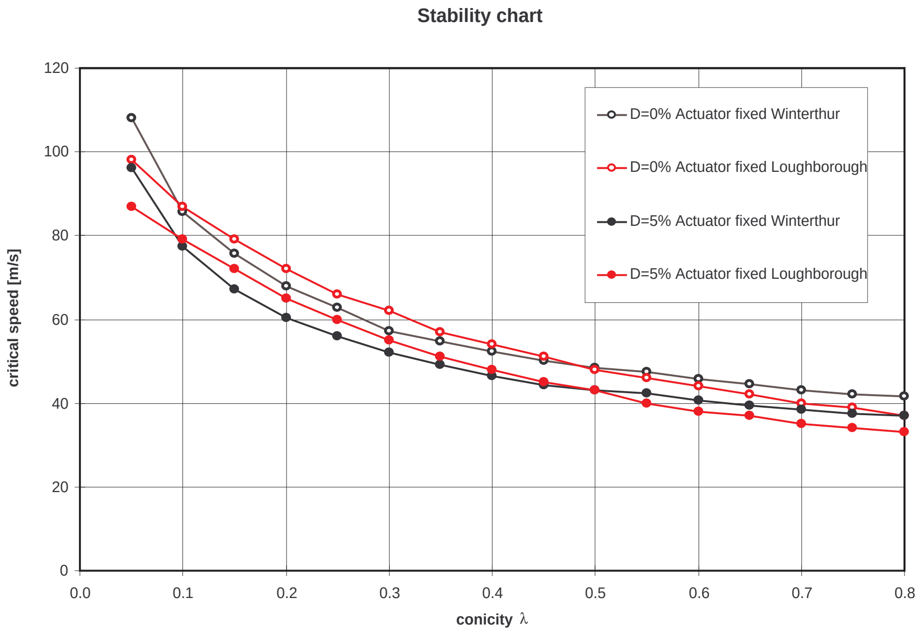

Figure 6: Validation of controller design model in SIMULINK with simulation models in SIMPACK ('D': remaining damping; 'Loughborough': SIMULINK model work at Loughborough, UK; 'Winterthur': SIMPACK nonlinear model work at Winterthur, SUI)

4.3.1 SIMULINK plan-view (design) model vs SIMPACK full non-linear model

Figure 7 shows the time history plots for the lateral velocity of the leading wheelset with the stability controller operating in SIMULINK only using the design model and for the stability controller operating under co-simulation with SIMPACK using the full simulation model. A track data file was used which represented typically good quality main line railtrack for a given forward vehicle speed of 65m/s.

4.3.2 SIMULINK plan-view (design) model vs SIMPACK exported full order linear

Figure 8 shows the time history plots for the lateral velocity of the leading wheelset with the stability controller operating in SIMULINK (design model) and for the stability controller operating with the exported SIMPACK model (speed 65m/s). The results in both Figures 7 and 8 are very similar (only minor differences can be seen).

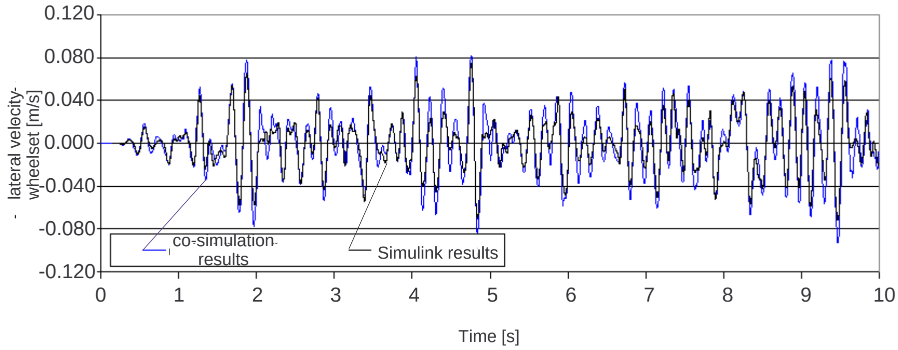

Figure 7: Design (SIMULINK plan-view) model vs SIMPACK co-simulation (stochastic simulation)

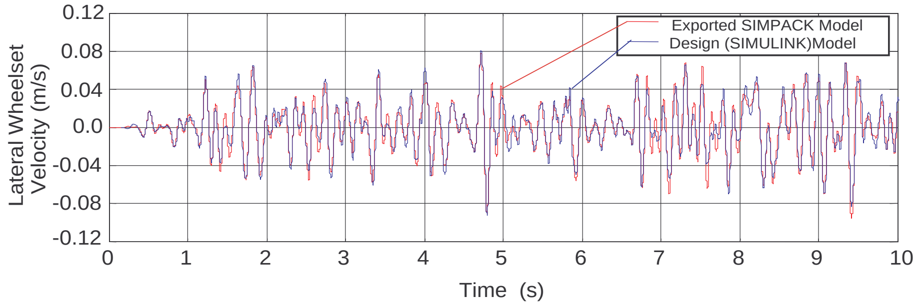

Figure 8: Design (SIMULINK plan-view) model vs SIMPACK exported model (stochastic simulation)

4.3.3 SIMPACK exported models - full order vs linear (analytically) reduced order

Comparisons have been made with a reduced order model having 24 states (discussed in Sections 3.2). The eigenvalue analysis in Figure 9 is interesting from a practical point-of-view but not straightforward to interpret the combination of the remaining poles. This is expected due to the model reduction being based upon the HSVs (Hankel Singular Values). It shows two groups of eigenvalues (identified by shaded ellipses A and B) which correspond to the unstable kinematic modes and the lateral suspension modes respectively; the reduced-order and SIMPACK models give pairs of eigenvalues for the full vehicle that are almost identical, with corresponding individual eigenvalues for the plan view half-vehicle model close by. Other groups such as C can also be identified relating to some of the higher frequency modes, but there are many eigenvalues from the SIMPACK model which have been correctly excluded from both the simplified models.

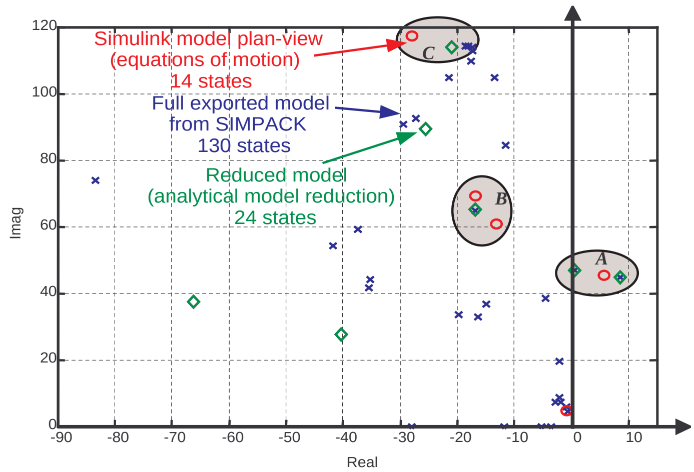

Figure 9: Eigenvalues comparison for the three vehicle models

The frequency domain analysis (singular value plot) in Figure 10 shows that the SIMPACK (130 states) and model reduction (24 states) responses are almost identical, whereas for the SIMULINK plan-view design model (14 states) there are noticeable differences. This suggests that refinement of the Kalman filter design model using the model-order reduction approach would yield valuable benefits.

Figure 11 presents the closed loop system response (with the same input-output relation as in Figure 10) using a baseline classical controller [8]. The models used to close the loop were: the full order, the 24^{th} analytically-reduced order $G_r(s)$ and the 14^{th} analytically-reduced order (for comparison purposes). Note that the four extra states come from the unstable system part as mentioned previously.

It can be seen from the figure that the 24^{th} order $G_r(s)$, within the closed loop, manages very well to sustain all important system characteristics within the desired base frequency range of interest $[0.1, 30]$Hz and is reliable up to around 1000rad/s (160Hz). The 14^{th} order model proves reliable within the window of interest, however its approximation deteriorates rapidly towards higher frequencies (unreliable information in such ranges). This also supports the choice of the 24^{th} order model as a reliable approximation for further analysis/design, compared to the original 130^{th} order exported SIMPACK linearised model.

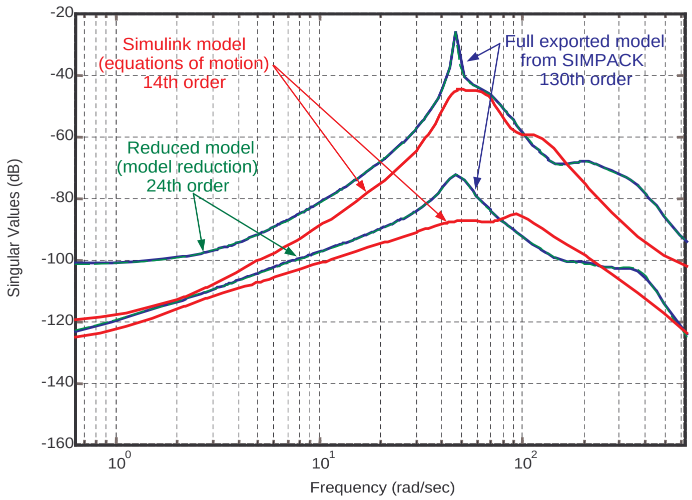

Figure 10: Frequency response comparison for the three (uncompensated open loop) models (Singular Value Plot - $u := [T_{w1}\ \ T_{w2}]^T$, $y := [\dot{y}_{w1}\ \ \dot{y}_{w2}]^T$)

5 Conclusion

The paper has discussed a number of issues related to creating models for designing controllers to be applied to complex dynamic systems. The particular system studied is for a railway vehicle, but the principles and techniques described are relevant to a variety of control system applications. The main contributions are: an identification of the need for both simulation and design models, the latter being a necessity for achieving a practical controller when using model-based control design concepts of all kinds; an exposition of the relative merits of producing an appropriately-simplified design model through physical understanding compared with using rigorous mathematical model reduction techniques; the use of a practical example both to give clarification of the ideas and to provide a numerical comparison.

The authors believe that the two model reduction techniques (mathematically based and physically based) should be used together, the former to provide important insights into the possibilities for achieving a reduced-order design model, in particular some guidance regarding the achievable size reduction for the design model, the latter to give significantly enhanced model accuracy than is possible with the former. Additional research is required to use the two techniques together in the most effective manner possible, but the paper should be of considerable interest to control engineers who have to provide practical solutions but may be put off by the potential complexity of normal model-based control techniques.

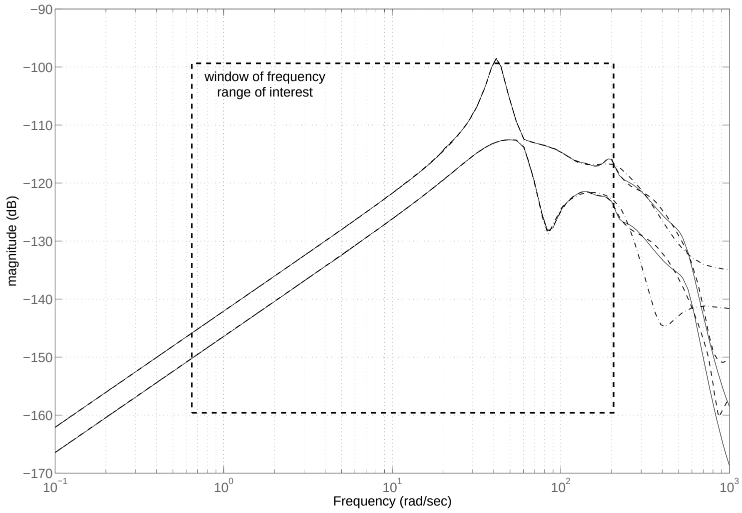

Figure 11: Singular value plot of compensated closed loop system (baseline classical controller) based upon: full $G(s)$ (solid), reduced $G_r(s)$ 24 states (dash), reduced $G_r(s)$ 14 states (dash-dot)

A Symbols and parameters used in the paper

Symbol/Parameter	Description/Value
$y_{w_1}, y_{w_2}, y_g, y_v$	lateral displacement of leading, trailing wheelset, bogie frame and vehicle body
$\theta_{w_1}, \theta_{w_2}, \theta_g, \theta_v$	yaw displ. of leading, trailing wheelset, bogie frame and vehicle body
V_s, g	forward vehicle speed and gravity ($9.81m/s^2$)
m_w, I_w	wheelset mass (1363kg) and yaw inertia (766kgm^2)
L_g, L_v	half wheelset gauge (0.75m) and half spacing of axles (1.225m)
r_0, λ	wheel radius (0.445m) and conicity (0.3)
m_g, I_g	bogie frame mass (3447kg) and yaw inertia (3200kgm^2)
K_s, C_s	lateral stiffness (471kN/m) and damping (12kNs/m) per wheelset
m_v	vehicle body mass (34460kg)
K_{sc}, C_{sc}	2ndary lateral stiffness (490kN/m) and damping per wheelset (40kNs/m)
f_{11}, f_{22}	longitudinal and lateral creepage coefficients (10MN each)
R_1, R_2	radius of curved track at leading and trailing wheelsets (3500m each)
$\theta_{c_1}, \theta_{c_2}$	Cant angle (track elevation) of the curved track at leading and trailing wheelsets (6^o each)
y_{t_1}, y_{t_2}	track lateral displacement (irregularities)
T_{w_1}, T_{w_2}	controlled torque for leading and trailing wheelsets

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