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Stability of Internet-Based Control Systems with Uncertainties and Multiple Time-Varying Delays

Zhi-Hong Guan, Hao Zhang, Shuang-Hua Yang, and Hua O. Wang

Abstract—In this paper, based on remote control and local control strategy, a class of hybrid multi-rate control models with uncertainties and multiple time-varying delays is formulated and their robust stability properties are investigated. By Lyapunov-Krasovskii functions and apply it to a descriptor model transformation, some new criteria of robust stability for such Internet-based control systems are established. Numerical example and simulation are given to illustrate the effectiveness of the theoretical results.

I. INTRODUCTION

The Internet is playing an important role in information retrieval, exchange, and applications. Internet based control, a new type of control systems, is characterized as globally remote monitoring and adjustment of plants over the Internet. In recent years, Internet based control systems have gained considerable attention in science and engineering [1], [3], [4], [7], [10], [12], [13], [14], since they provide a new and convenient unified framework for system control and practical applications. Examples include intelligent home environments, windmill and solar power stations, small-scale hydroelectric power stations, and other highly geographically distributed devices, as well as tele-manufacturing, tele-surgery, and tele-control of spacecrafts.

Internet-based control is an interesting and challenging topic. One of the major challenges in Internet based control systems is how to deal with the Internet transmission delay, specially, time-varying delay. The existing approaches of overcoming network transmission delay mainly focus on designing a model based time delay compensator or a state observer to reduce the effect of the transmission delay. Being distinct from the existing approaches, literatures [11], [12] have been investigating the overcoming of the Internet time delay from the control system architecture angle, including introducing a tolerant time to the fixed sampling interval to potentially maximize the possibility of succeeding the transmission on time. Most recently, a dual-rate control scheme for Internet based control systems has been proposed in literature [11]. A two-level hierarchy was used in the dual-rate control scheme. At the lower level a local controller

is implemented to control the plant at a higher frequency to stabilize the plant and guarantee the plant being under control even the network communication is lost for a long time. At the higher level a remote controller is employed to remotely regulate the desirable reference at a lower frequency to reduce the communication load and increase the possibility of receiving data over the Internet on time. A typical dual-rate control scheme is demonstrated in a process control rig [12], [13] and has shown a great potential to overcome Internet time delay and bring this new generation of control systems into industries. However, since the time delay is variable and the uncertainty of the process parameters is unavoidable, a dual-rate Internet based control system may be unstable for certain control intervals. A sufficient stability condition of dual-rate Internet based control systems with uncertainties and time-varying delay is urgently required to guide the system design process. The interest in the stability of networked control systems have grown in recent years due to its theoretical and practical significance [1], [2], [3], [5], [6], [15], but to our knowledge there are few reports dealing with such kind of Internet-based control systems. This motivates the present stability investigation of multi-rate Internet-based control systems with time-varying delay and uncertainties. We found that such multi-rate Internet-based control systems can be rewritten as a time-delay system through our modeling process, and there are many results in the fields [4], [8], [9], [17], [25], [27]. These criteria can be classified into two categories according to their dependence on the size of the delays, namely, delay-independent stability criteria [18], [19], [20], and delay-dependent stability criteria [21], [22], [23], [24]. The delay-independent stability criteria guarantee the stability of the system irrespective of the size of the time-delay. On the other hand, the delay-dependent stability criteria are concerned with the size of delay and provide an upper bound of time-delay size. In general, the delay-dependent stability criteria are less conservative than delay-independent ones when the size of the time-delay is small.

In this paper, based on remote control and local control strategy, a new class of multi-rate Internet-based network model is presented, and some new delay-dependent robust stability criteria for such system with multiple time-varying delays in terms of LMIs are proposed. Example and simulation are given to illustrate the effectiveness of the theoretical results.

II. PROBLEM FORMULATION

Let $N = \{1, 2, \dots\}$ and $N_+ = \{0, 1, 2, \dots\}$ denote the sets of positive integer and nonnegative integer, respectively.

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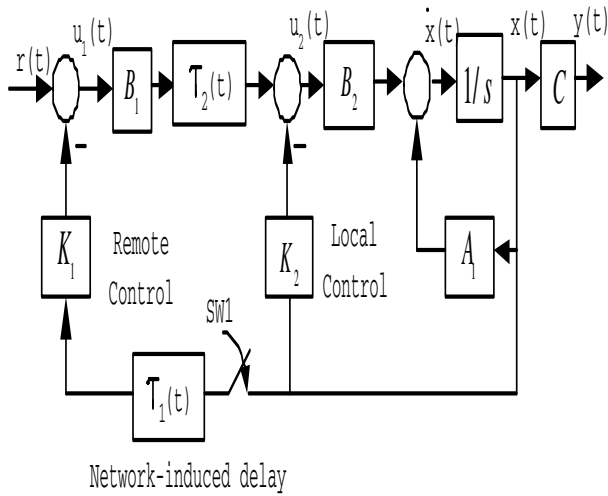


Fig. 1. Multi-rate network control loop with time delays

For the vector $x = \text{col}(x_1, \dots, x_n) \in R^n$ and the matrix $A = (a_{ij})_{n \times n} \in R^{n \times n}$. The identity matrix of order m is denoted as I_m (or simply I if no confusion arises) and $*$ denotes transpose of the corresponding sub-matrix.

A typical multi-rate control structure with remote controller and local controller can be shown as Fig. 1. The control architecture gives a continuous dynamical system, where plant is in circle with broken line, $x(t) \in R^n$ is the system state, $y(t)$ is the output, and $r(t)$ is the input, $u_1(t)$ and $u_2(t)$ are the output of remote control and local control, respectively. A_1, B_1, B_2 and C are parameter matrices of the mode with appropriate dimensions, K_1 and K_2 are control gain matrices, and $\tau_1(t)$ and $\tau_2(t)$ are time delays caused by communication delay in systems.

For the system given by Fig. 1, it is assumed that, the sampling interval of remote controller is the m multiple of local controller, with m being positive integer, and the switching device SW1 closes only at the instant time $t = im, i \in N_+$, and otherwise, it switches off. Correspondingly, remote controller $u_1(t)$ updates its state at $t = im, i \in N_+$ only, and otherwise, it keeps invariable. Also, it is assumed that the benchmark of continuous systems is the same as local controller. In this case, the system can be described by the following continuous system with time delays

$$\begin{cases} \dot{x}(t) = A_1 x(t) + B_2 u_2(t), \\ u_2(t) = B_1 u_1(t - \tau_2(t)) - K_2 x(t), \\ y(t) = C x(t), \end{cases} \quad (1)$$

where remote controller $u_1(t - \tau_2(t))$ is given by

$$\begin{cases} u_1(t - \tau_2(t)) = r(t - \tau_2(t)) \\ \quad - K_1 x(t - \tau_1(t) - \tau_2(t)), \\ \quad t = im, \\ u_1(t - \tau_2(t)) = r(im - \tau_2(t)) \\ \quad - K_1 x(im - \tau_1(t) - \tau_2(t)), \\ \quad t \in (im, im + m), \end{cases} \quad (2)$$

with $t > 0$ and $i \in N_+$. Moreover, it follows from (1) and (2) that, for $t = im$,

$$\begin{cases} \dot{x}(t) = (A_1 - B_2 K_2) x(t) \\ \quad - B_2 B_1 K_1 x(t - \tau_1(t) - \tau_2(t)) \\ \quad + B_2 B_1 r(t - \tau_2(t)), \\ y(t) = C x(t), \end{cases} \quad (3)$$

and for $t \in (im, im + m)$,

$$\begin{cases} \dot{x}(t) = (A_1 - B_2 K_2) x(t) \\ \quad - B_2 B_1 K_1 x(im - \tau_1(t) - \tau_2(t)) \\ \quad + B_2 B_1 r(im - \tau_2(t)), \\ y(t) = C x(t). \end{cases} \quad (4)$$

For the stability analysis, one can let $r(t) = 0$, and then the system (3) and (4) become

$$\begin{cases} \dot{x}(t) = (A_1 - B_2 K_2) x(t) \\ \quad - B_2 B_1 K_1 x(t - \tau(t)), t = im, \\ \dot{x}(t) = (A_1 - B_2 K_2) x(t) \\ \quad - B_2 B_1 K_1 x(im - \tau(t)), \\ \quad t \in (im, im + m), \end{cases} \quad (5)$$

where

$$\tau(t) = \tau_1(t) + \tau_2(t) > 0, t > 0,$$

$i \in N_+$ and $m > 0$ is a positive integer.

Obviously, if define

$$A = A_1 - B_2 K_2,$$

$$B = -B_2 B_1 K_1,$$

then the controlled system (5) is

$$\begin{cases} \dot{x}(t) = A x(t) + B x(t - \tau(t)), \quad t = im, \\ \dot{x}(t) = A x(t) + B x(im - \tau(t)), \\ \quad t \in (im, im + m), \end{cases} \quad (6)$$

where A_1, B_1, B_2, C are matrices with appropriate dimensions, K_1 and K_2 are control gain matrices.

$$\tau(t) = \tau_1(t) + \tau_2(t) > 0,$$

and $m > 0$ are integers, $t \geq 0, i = 0, 1, 2, \dots$.

Furthermore, note that, as $t = im + s$ with $s \in [0, m)$,

$$im - \tau(t) = t - (\tau(t) + s).$$

Equation (6) can be rewritten as

$$\dot{x}(t) = A x(t) + B x(t - h(t)), \quad (7)$$

with

$$0 \leq \tau(t) \leq h(t) < \tau(t) + m.$$

Accordingly, for the case of uncertainties, (7) becomes

$$\dot{x}(t) = (A + \Delta A(t)) x(t) + (B + \Delta B(t)) x(t - h(t)), \quad (8)$$

with

$$0 \leq \tau(t) \leq h(t) < \tau(t) + m,$$

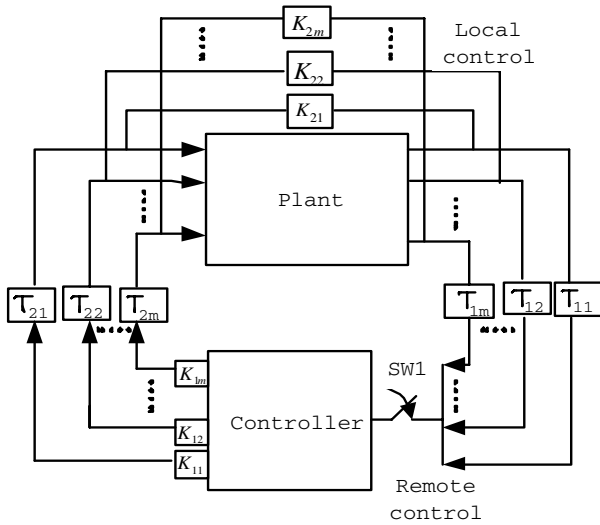


Fig. 2. Internet-Based control system with multiple delays.

and $\Delta A(t)$ and $\Delta B(t)$ being time-varying structured uncertainties.

To generalize results for the Internet-based systems with multiple time-delays case, without loss of generality, consider the following Internet-based system with uncertainties and multiple network-induced time-delays. As shown in Fig.2.

$$\dot{x}(t) = (A + \Delta A(t))x(t) + \sum_{i=1}^m (B_i + \Delta B_i(t))x(t - h_i(t)). \quad (9)$$

System (9) can be written as

$$\dot{x}(t) = \sum_{i=0}^m (A_i + \Delta A_i(t))x(t - \tau_i(t)), \quad (10)$$

where

$$A_i = A, \Delta A_i(t) = \Delta A(t), i = 0,$$

$$A_i = B_i, \Delta A_i(t) = \Delta B_i(t), i = 1, \dots, m,$$

and $\tau_0(t) \equiv 0, \tau_i(t)$ is continuous function and satisfy

$$0 \leq \tau_i(t) \leq \tau_i,$$

$$\bar{\tau} = \max_{1 \leq i \leq m} \{\tau_i\},$$

$$\dot{\tau}_i(t) \leq \mu_i \leq \mu < 1, i = 1, 2, \dots, m,$$

and

$$A_i(t) = A_i + \Delta A_i(t), A_i \in R^{n \times n}$$

are constant matrices, $\Delta A_i(t)$ are the structure uncertainties, satisfying

$$\Delta A_i(t) = E_i F_i(t) H_i,$$

where E_i, H_i are known constant real matrices with appropriate dimensions, $F_i(t)$ are unknown matrices satisfy

$$F_i^\top(t) F_i(t) \leq I, \forall t.$$

In what follows, the global asymptotic stability of the hybrid model (6) is first studied, and then, the example of the controlled systems (9) and (10) are investigated.

III. STABILITY ANALYSIS

Consider a system with multiple time-varying delays and uncertainties

$$\begin{cases} \dot{x}(t) = \sum_{i=0}^m A_i(t)x(t - \tau_i(t)), & t \geq 0, \\ x(t) = \phi(t), & t \in [-\bar{\tau}, 0], \end{cases} \quad (11)$$

Lemma 1:[16],[26] If A, P, D, F, E are matrices with appropriate dimensions and $F^\top F \leq I$, let $P > 0$, for any scalar $\varepsilon > 0$, then

$$DFE + E^\top F^\top D^\top \leq \varepsilon^{-1}DD^\top + \varepsilon E^\top E, \quad (12)$$

$$2x^\top y \leq x^\top P^{-1}x + y^\top Py. \quad (13)$$

If there is a number $\varepsilon > 0$ such that

$$P - \varepsilon DD^\top > 0,$$

then

$$[A + DFE]^\top P^{-1}[A + DFE] \leq \begin{matrix} A^\top R^{-1}A \\ + \varepsilon^{-1}E^\top E, \end{matrix} \quad (14)$$

where

$$R := P - \varepsilon DD^\top.$$

Next, stability properties of uncertain systems with time-varying delay described in (11) are investigated. The first equation in (11) can be recast as:

$$\begin{cases} \dot{x}(t) = y(t), \\ y(t) = \sum_{i=0}^m (A_i + \Delta A_i(t))x(t - \tau_i(t)), \end{cases} \quad (15)$$

with the identical initial conditions as expressed in (11). It is noted that (15) is completely equivalent to (11).

Theorem 1: Given scalars $0 \leq \tau_i \leq \bar{\tau}, \mu_i < 1, i = 1, \dots, m$, then for all time-varying delays satisfying $0 \leq \tau_i(t) \leq \tau_i$ and $\dot{\tau}_i(t) \leq \mu_i < 1$, system (11) is asymptotically stable, if there exist matrices $P > 0, P_1, P_2, Q_i > 0, S_i > 0, i = 1, 2, \dots, m$, scalars $\alpha_i > 0, i = 0, 1, \dots, m, \beta_i > 0, i = 1, \dots, m$, such that the following LMI holds:

$$\Lambda = \begin{bmatrix} \Theta_1 + \sum_{i=0}^m S_i & \Theta_2 & M_{11} & M_{21} & M_{31} & 0 \\ * & \Theta_3 & M_{12} & M_{22} & M_{32} & 0 \\ * & * & -\phi_1 & 0 & 0 & 0 \\ * & * & * & -\phi_2 & 0 & 0 \\ * & * & * & * & -\phi_3 & 0 \\ * & * & * & * & * & -\phi_4 \end{bmatrix} < 0, \quad (16)$$

where

$$\begin{aligned} \Theta_1 &= P_1^\top \sum_{i=0}^m A_i + \sum_{i=0}^m A_i^\top P_1 + \sum_{i=0}^m \alpha_i H_i^\top H_i, \\ \Theta_2 &= P - P_1^\top + \sum_{i=0}^m A_i^\top P_2, \\ \Theta_3 &= -P_2 - P_2^\top + \sum_{i=1}^m \tau_i Q_i, \\ M_{11} &= [P_1^\top E_0, P_1^\top E_1, \dots, P_1^\top E_m], \\ M_{12} &= [P_2^\top E_0, P_2^\top E_1, \dots, P_2^\top E_m], \\ M_{21} &= [\tau_1 P_1^\top A_1, \tau_2 P_1^\top A_2, \dots, \tau_m P_1^\top A_m], \\ M_{22} &= [\tau_1 P_2^\top A_1, \tau_2 P_2^\top A_2, \dots, \tau_m P_2^\top A_m], \\ M_{31} &= [\tau_1 P_1^\top E_1, \tau_2 P_1^\top E_2, \dots, \tau_m P_1^\top E_m], \\ M_{32} &= [\tau_1 P_2^\top E_1, \tau_2 P_2^\top E_2, \dots, \tau_m P_2^\top E_m], \\ \phi_1 &= \text{diag}\{\alpha_0 I, \dots, \alpha_m I\}, \\ \phi_2 &= \text{diag}\{\tau_1(Q_1 - \beta_1 H_1^\top H_1), \dots, \tau_m(Q_m - \beta_m H_m^\top H_m)\}, \\ \phi_3 &= \text{diag}\{\tau_1 \beta_1, \dots, \tau_m \beta_m\}, \\ \phi_4 &= \text{diag}\{(1 - \mu_1)S_1, \dots, (1 - \mu_m)S_m\}. \end{aligned}$$

Proof. From (15) we can obtain

$$\dot{x}(t) = y(t). \tag{17}$$

For $i \in \{1, 2, \dots, m\}$,

$$\begin{aligned} x(t - \tau_i(t)) &= x(t) - \int_{t-\tau_i(t)}^t \dot{x}(s) ds \\ &= x(t) - \int_{t-\tau_i(t)}^t y(s) ds, \end{aligned} \tag{18}$$

substituting (18) into (15) we can obtain

$$0 = -y(t) + \sum_{i=0}^m A_i(t)x(t) - \sum_{i=1}^m A_i(t) \int_{t-\tau_i(t)}^t y(s) ds. \tag{19}$$

It is easy to see that system (11) is equivalent to the descriptor model transformation (17) and (19). The Lyapunov-Krasovskii functions are constructed as follows:

$$V = V_1 + V_2 + V_3$$

where

$$\begin{aligned} V_1 &= x^\top(t)Px(t), \\ V_2 &= \sum_{i=1}^m \int_{-\tau_i}^0 d\theta \int_{t+\theta}^t y^\top(s)Q_i y(s) ds, \\ V_3 &= \sum_{i=1}^m \int_{t-\tau_i(t)}^t x^\top(s)S_i x(s) ds. \end{aligned}$$

Now we consider the derivative of V . For the first term V_1 , we have

$$\begin{aligned} \dot{V}_1 &= 2x^\top(t)P\dot{x}(t) = 2x^\top(t)Py(t) \\ &= 2\eta^\top(t)G^\top \begin{bmatrix} y^\top(t) & 0 \end{bmatrix}^\top, \end{aligned}$$

where

$$\eta^\top(t) = \begin{bmatrix} x^\top(t) & y^\top(t) \end{bmatrix},$$

$$G = \begin{bmatrix} P & 0 \\ P_1 & P_2 \end{bmatrix}.$$

Consider (19),

$$\begin{aligned} &2\eta^\top(t)G^\top \begin{bmatrix} y(t) \\ 0 \end{bmatrix} \\ &= 2\eta^\top(t)G^\top \left\{ \begin{bmatrix} y(t) \\ \sum_{i=0}^m A_i(t)x(t) - y(t) \end{bmatrix} \right. \\ &\quad \left. - \sum_{i=1}^m \begin{bmatrix} 0 \\ A_i(t) \end{bmatrix} \int_{t-\tau_i(t)}^t y(s) ds \right\}. \end{aligned} \tag{20}$$

From (12) in Lemma 1, for any scalars

$$\alpha_i > 0, i \in \{0, 1, 2, \dots, m\},$$

we have

$$\begin{aligned} &2\eta^\top(t)G^\top \begin{bmatrix} 0 & \Delta A_i^\top(t) \end{bmatrix}^\top x(t) \\ &= 2\eta^\top(t)G^\top \begin{bmatrix} 0 & E_i^\top \end{bmatrix}^\top F_i(t)H_i x(t) \\ &\leq \alpha_i^{-1} \eta^\top(t)G^\top \begin{bmatrix} 0 & E_i^\top \end{bmatrix}^\top \\ &\quad \times \begin{bmatrix} 0 & E_i^\top \end{bmatrix} G\eta(t) \\ &\quad + \alpha_i x^\top(t)H_i^\top H_i x(t), \end{aligned} \tag{21}$$

then

$$\begin{aligned} &2\eta^\top(t)G^\top \begin{bmatrix} y(t) \\ \sum_{i=0}^m A_i(t)x(t) - y(t) \end{bmatrix} \\ &\leq 2\eta^\top(t)G^\top \begin{bmatrix} 0 & \sum_{i=0}^m A_i^\top \end{bmatrix}^\top x(t) \\ &\quad + \sum_{i=0}^m \{ \alpha_i^{-1} \eta^\top(t)G^\top \begin{bmatrix} 0 & E_i^\top \end{bmatrix}^\top \\ &\quad \times \begin{bmatrix} 0 & E_i^\top \end{bmatrix} G\eta(t) \\ &\quad + \alpha_i x^\top(t)H_i^\top H_i x(t) \} \\ &\quad + 2\eta^\top(t)G^\top \begin{bmatrix} I & -I \end{bmatrix}^\top y(t). \end{aligned} \tag{22}$$

According to (13) in Lemma 1, for $Q_i > 0$ we get

$$\begin{aligned} &-2 \sum_{i=1}^m \eta^\top(t)G^\top \\ &\quad \times \begin{bmatrix} 0 & A_i^\top(t) \end{bmatrix}^\top \int_{t-\tau_i(t)}^t y(s) ds \\ &= -2 \sum_{i=1}^m \int_{t-\tau_i(t)}^t \eta^\top(t)G^\top \begin{bmatrix} 0 & A_i^\top(t) \end{bmatrix}^\top y(s) ds \\ &\leq \sum_{i=1}^m \left\{ \int_{t-\tau_i(t)}^t \eta^\top(t)G^\top \begin{bmatrix} 0 \\ A_i(t) \end{bmatrix} Q_i^{-1} \right. \\ &\quad \times \begin{bmatrix} 0 & A_i^\top(t) \end{bmatrix} G\eta(t) ds \\ &\quad \left. + \int_{t-\tau_i(t)}^t y^\top(s)Q_i y(s) ds \right\} \\ &\leq \sum_{i=1}^m \left\{ \tau_i \eta^\top(t)G^\top \begin{bmatrix} 0 \\ A_i(t) \end{bmatrix} Q_i^{-1} \right. \\ &\quad \times \begin{bmatrix} 0 & A_i^\top(t) \end{bmatrix} G\eta(t) \\ &\quad \left. + \int_{t-\tau_i}^t y^\top(s)Q_i y(s) ds \right\}. \end{aligned} \tag{23}$$

For any scalars $\beta_i, i = 1, 2, \dots, m$ satisfying

$$Q_i - \beta_i H_i^\top H_i > 0.$$

From (14) in Lemma 1

$$\begin{aligned}
& G^\top \begin{bmatrix} 0 \\ A_i(t) \end{bmatrix} Q_i^{-1} \begin{bmatrix} 0 & A_i^\top(t) \end{bmatrix} G \\
&= G^\top \left(\begin{bmatrix} 0 \\ A_i \end{bmatrix} + \begin{bmatrix} 0 \\ E_i \end{bmatrix} F_i(t) H_i \right) Q_i^{-1} \\
&\quad \times \left(\begin{bmatrix} 0 \\ A_i \end{bmatrix} + \begin{bmatrix} 0 \\ E_i \end{bmatrix} F_i(t) H_i \right)^\top G \\
&\leq G^\top \begin{bmatrix} 0 \\ A_i \end{bmatrix} (Q_i - \beta_i H_i^\top H_i)^{-1} \begin{bmatrix} 0 & A_i^\top \end{bmatrix} G \\
&\quad + \beta_i^{-1} G^\top \begin{bmatrix} 0 \\ E_i \end{bmatrix} \begin{bmatrix} 0 & E_i^\top \end{bmatrix} G.
\end{aligned} \tag{24}$$

The derivative for V_2 is

$$\dot{V}_2 = \sum_{i=1}^m y^\top(t) \tau_i Q_i y(t) - \sum_{i=1}^m \int_{t-\tau_i}^t y^\top(s) Q_i y(s) ds. \tag{25}$$

The derivative for V_3 is

$$\begin{aligned}
\dot{V}_3 &= \sum_{i=1}^m (x^\top(t) S_i x(t) \\
&\quad - (1 - \mu_i) x^\top(t - \tau_i(t)) S_i x(t - \tau_i(t))).
\end{aligned} \tag{26}$$

From (20)-(26) we obtain

$$\begin{aligned}
\dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \\
&\leq 2\eta^\top(t) G^\top \begin{bmatrix} 0 & \sum_{i=0}^m A_i^\top \end{bmatrix}^\top x(t) \\
&\quad + 2\eta^\top(t) G^\top \begin{bmatrix} I & -I \end{bmatrix}^\top y(t) \\
&\quad + \sum_{i=0}^m \{ \alpha_i^{-1} \eta^\top(t) G^\top \begin{bmatrix} 0 \\ E_i \end{bmatrix} \\
&\quad \times \begin{bmatrix} 0 & E_i^\top \end{bmatrix} G \eta(t) \\
&\quad + \alpha_i x^\top(t) H_i^\top H_i x(t) \} + \sum_{i=1}^m y^\top(t) \tau_i Q_i y(t) \\
&\quad + \sum_{i=1}^m \tau_i \eta^\top(t) \{ G^\top \begin{bmatrix} 0 \\ A_i \end{bmatrix} \\
&\quad \times (Q_i - \beta_i H_i^\top H_i)^{-1} \\
&\quad \times \begin{bmatrix} 0 & A_i^\top \end{bmatrix} G + \beta_i^{-1} G^\top \begin{bmatrix} 0 \\ E_i \end{bmatrix} \\
&\quad \times \begin{bmatrix} 0 & E_i^\top \end{bmatrix} G \} \eta(t) \\
&\quad + \sum_{i=1}^m (x^\top(t) S_i x(t) \\
&\quad - (1 - \mu_i) x^\top(t - \tau_i(t)) S_i x(t - \tau_i(t))) \\
&= \xi^\top(t) \Lambda \xi(t),
\end{aligned} \tag{27}$$

where

$$\xi^\top(t) = \begin{bmatrix} \eta^\top(t) & x^\top(t - \tau_1(t)) & \dots & x^\top(t - \tau_m(t)) \end{bmatrix},$$

and Λ is defined in (16). This complete the proof.

Remark: For a time-invariant delay system, according to the procedure of the proof of Theorem 1, it is clear that setting $\mu_i = 0$ in (16) the condition for time-invariant delay system can be obtained.

In the case of we want to obtain a stability condition for (11), which is derivative-independent, or in the case of $\tau(t)$ is not differentiable, we can construct a Lyapunov-Krasovskii functional which is similar to that in Theorem 1 but without V_3 . According to the proof of Theorem 1, the following Theorem is followed:

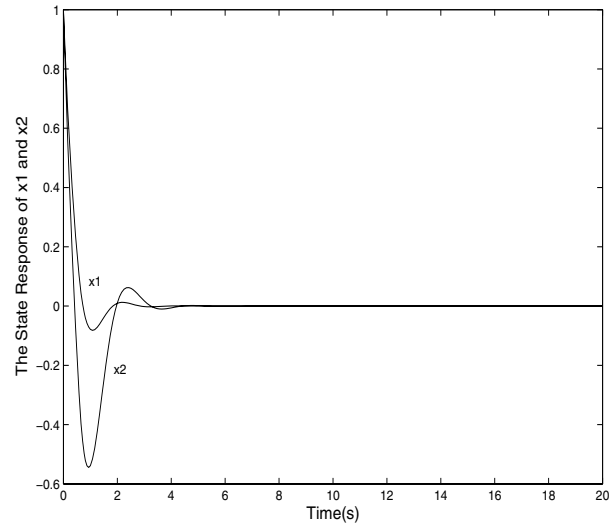


Fig. 3. States of example 1 with $\tau = 0.6$.

Theorem 2: Given scalars $0 \leq \tau_i \leq \bar{\tau}, i = 1, \dots, N$, for all time-varying delays $0 \leq \tau_i(t) \leq \tau_i$, system (11) is asymptotically stable, if there exist matrices $P > 0, P_1, P_2, Q_i > 0, i = 1, 2, \dots, m$, scalars $\alpha_i > 0, i = 0, 1, \dots, m, \beta_i > 0, i = 1, \dots, m$, such that the following LMI holds:

$$\begin{bmatrix} \Theta_1 & \Theta_2 & M_{11} & M_{21} & M_{31} \\ * & \Theta_3 & M_{12} & M_{22} & M_{32} \\ * & * & -\phi_1 & 0 & 0 \\ * & * & * & -\phi_2 & 0 \\ * & * & * & * & -\phi_3 \end{bmatrix} < 0, \tag{28}$$

where $\Theta_1, \Theta_2, \Theta_3, M_{11}, M_{21}, M_{31}, M_{12}, M_{22}, M_{32}$ are defined in Theorem 1.

IV. NUMERICAL EXAMPLE

In this section, an example is given to demonstrate that the stability results presented in this paper is effective and is an improvement over existing results.

Example 1. Consider system (11) as

$$\begin{aligned}
\dot{x}(t) &= \left\{ \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} \sqrt{0.3} & 0 \\ 0 & \sqrt{0.2} \end{bmatrix} \right. \\
&\quad \times \left. \begin{bmatrix} \cos t & 0 \\ 0 & \sin t \end{bmatrix} \begin{bmatrix} \sqrt{0.3} & 0 \\ 0 & \sqrt{0.2} \end{bmatrix} \right\} x(t) \\
&\quad + \left\{ \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} + \begin{bmatrix} \sqrt{0.3} & 0 \\ 0 & \sqrt{0.2} \end{bmatrix} \right. \\
&\quad \times \left. \begin{bmatrix} \cos t & 0 \\ 0 & \sin t \end{bmatrix} \begin{bmatrix} \sqrt{0.3} & 0 \\ 0 & \sqrt{0.2} \end{bmatrix} \right\} \\
&\quad \times x(t - \tau \sin t).
\end{aligned}$$

Using the LMI toolbox in Matlab and applying Theorem 2 to the system, then the maximal admissible time delay for stability is $\tau = 0.6164$, the delay bound for guaranteeing asymptotic stability of the system given in [17] is $\tau \leq 0.2588$, and the state response of example 1 with $\tau = 0.6$ is shown in Fig.3.

V. CONCLUSIONS

In this paper, based on remote control and local control strategy, a class of hybrid multi-rate control models with uncertainties and multiple time-varying delay is formulated and their stability properties are investigated. Using the Lyapunov-Krasovskii function approach on an equivalent descriptor model transformation, a new method of determining delay-dependent stability criteria, stability conditions based on LMIs are obtained for such systems, which are less conservative. Numerical example is given to illustrate our theoretical result.

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