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Transfer function phase prediction for a flat plate

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Abstract [139] This paper reports an investigation into the phase accumulation characteristics in the transfer function of a flat plate for bending waves. The paper starts with an overview of previous research on the phase characteristics in an ideal vibrational wave field i.e. either a completely direct field or a completely diffuse field. This initial research is extended by considering the phase accumulation characteristics of a non-ideal vibrational wave field i.e. a wave field between a direct field and a diffuse field. Various factors likely to influence the phase accumulation are discussed. Based on these factors and a pole and zero model of the direct field phase and the diffuse field phase, two mathematical models are presented for predicting the transfer function phase of a flat plate supporting a non-ideal wave field. Computer simulation results are given which compare the characteristics of the different models.

1 INTRODUCTION

Transfer mobility can be used in analysing dynamic system characteristics. Often the phase of the transfer function is discarded because only the amplitude of the vibration is of interest. Yet the phase of the transfer function may also reveal information about the structure. For example, the phase accumulation in a diffuse field for a finite flat plate greatly exceeds the phase accumulation in a direct field. This phenomenon implies that the phase accumulation properties may be used to investigate the characteristics of different wave fields.

The phase accumulation in an ideal wave field, i.e. either a completely direct field or a completely diffuse field, has been extensively investigated by Lyon and his co-investigators [1-8] during last two decades. However, the phase in a non-ideal wave field, a wave field between a direct field and a diffuse field, received only limited attention. In this paper, the phase characteristics in a non-ideal wave field of flat plates for bending waves are investigated. The paper starts with the discussion of the influence of poles and zeros of the transfer function on the accumulated phase characteristics. Then, the factors, which are related to the phase accumulation such as the damping of the structure, the structure size and the source-receiver separation distance, are discussed. It is found that the phase accumulation in a non-ideal wave field can be expressed generally by the number of the non-minimum phase zeros of the transfer function multiplied by -2π plus the contribution from the propagation phase, -kr. Based upon this general approach, a mathematical model for predicting the phase accumulation of flat plates in a non-ideal wave field is presented. Computer simulation results are given in Section 3 to study the model characteristics.

2 THEORY

2.1 Existing Theoretical Models

In a completely direct field, for example, within an "infinite" plate, the phase relationship of a transfer function, $\phi_{dir}(\omega)$, between source and receiver locations is given by the propagant phase

$$\phi_{dir}(\omega) = -kr \tag{1}$$

where k is the wavenumber and r is the distance between source and receiver locations.

However, Lyon noted in reference [1] that in real structures the measured phase accumulation greatly exceed the propagation phase given by equation (1). Lyon attributed this effect to reverberant wave behaviour in the structure. Through the study of the polynomial modal expansion of a transfer function, Lyon found that, as the frequency, ω, increases, the accumulated phase will undergo a phase change of $+\pi$ if passing a zero and a phase change of $-\pi$ if passing a pole of the transfer function. Thus, if N_p poles and N_z zeros have been passed up to a frequency ω, the phase accumulation in the transfer function of a reverberant environment, $\phi_r(\omega)$, will be approximately [1]

$$\phi_r(\omega) = -(N_p - N_z)\pi \pm \pi/2 \tag{2}$$

where the term $\pi/2$ arises from the possibility of a pole or zero near $\omega = 0$. It can be seen from equation (2) that the problem of estimating the phase accumulation in a transfer function, therefore, becomes a problem of estimating the number of poles and zeros that occur within the frequency interval of interest.

Based upon the analysis of poles and zeros in the complex frequency plane, Lyon [1,2] found that the phase accumulation in a transfer function for a finite two-dimensional system was shown to be half the number of poles of the system multiplied by $-\pi$ radians. Thus,

$$\phi_r(\omega) = -N_p \pi/2 \tag{3}$$

In reference [2] Lyon termed $\phi_r(\omega)$ the reverberant phase limit.

In reference [3] Tohyama and Lyon proposed a relationship between the reverberant phase, $\phi_r(\omega)$, and the occurrence of double-zeros in the transfer function. Double-zeros are either both located on the pole line or occur as a pair of complex conjugate zeros located symmetrically with respect to each other at equal distances from the pole line. If a conjugate zero is located below the real frequency axis, it is called a Non-Minimum Phase zero (NMP zero). It was shown that a NMP zero has the same effect upon phase accumulation as that of a pole. Thus, the accumulated phase will undergo a phase change of $-\pi$ if passing a NMP zero. They further suggest that the accumulated phase of a transfer function for a finite system can be expressed by using the number of NMP zeros, N_z^+. Hence, the accumulated reverberant phase can be expressed as:

$$\phi_r = -2\pi N_z^+ \tag{4}$$

It was noted in reference [3] that the number of NMP zeros, N_z^+, is related to the damping condition of the structure. As damping in the system increases the number of NMP zeros will decrease. Hence, the reverberant phase, $\phi_r(\omega)$, will also decrease.

In references [5] and [6] Tohyama, Lyon and Koike investigated the accumulated phase in terms of the group delay

$$\tau_g(\omega) = \frac{d\phi}{d\omega} \tag{5}$$

which is the first derivative of the accumulated phase, ϕ, with respect to frequency, ω. It was shown that the group delay is given by

$$\tau_g(\omega) = -\frac{\pi}{2} n(\omega)[1 - \frac{2}{\pi}\tan^{-1}(\frac{2}{\pi} M(\omega))] \tag{6}$$

where $n(\omega)$ is the modal density of the system and M is the modal overlap defined by

$$M(\omega) = \frac{\pi}{2}\omega\eta n(\omega) \tag{7}$$

and η is the damping loss factor. Equations (6) and (7) indicate that as the damping loss factor, η, increases the accumulated phase, $\phi(\omega)$, decreases. Since the predicted phase results from equation (6) are independent of the source-receiver separation distance, r, it is only true in a diffuse field. Therefore, the phase predicted from equation (6) is termed as Tohyama's diffuse field phase, $\phi_{diff}(\omega)$, and

$$\phi_{diff}(\omega) = \int \tau_g(\omega) d\omega \tag{8}$$

Tohyama's diffuse field phase is deduced from the prediction of the number of NMP zeros of the transfer function, i.e. equation (4). However, this model can not explain the formation of the phase accumulation in a direct field, i.e. propagant phase, -kr. The wave field in real structures is neither a completely direct field nor a completely diffuse field. It lies between of these two extremes. As the damping increases from very little to very high damping, the wave field will gradually turn from a diffuse field into a direct field. With the increase of damping the number of NMP zeros decreases because some of original NMP zeros may move up above to the real axis, then become non-NMP zeros [5]. Consequently, the phase accumulation will decrease by equation (4). When the damping is large enough to stop any reflections from the boundary, the wave field becomes a direct field, and the phase should be the propagation phase, -kr. The number of NMP zeros in this condition, however, will be expected to decrease to zero, as the damping is so large that all the NMP zeros should have moved up above the real axis, and changed to become non-NMP zeros. By equation (4), the phase in this case should be zero. This is contrary to the outcome of the direct field phase.

2.2 Phase Models for a "Non-ideal" Wave Field

The existence of simple propagant phase behaviour in a direct field implies that the number of zeros in any finite frequency range is systematically less than the number of poles by an amount just sufficient to generate the propagant phase. This was already established for the case of a one-dimensional acoustics pipe [2]. However, it is difficult to establish for two-dimensional systems as it is unable to write down an explicit closed form for the transfer functions. Taking this assumption into consideration, however, the phase accumulation in a non-ideal wave field can be generally assumed as the phase accumulation due to the NMP zeros plus the contribution of propagant phase, -kr, i.e.

$$\phi(\omega) = -2\pi N_z^+ - \mu_2 kr \tag{9}$$

where μ_2 is the contribution coefficient of the propagant phase and $0 \le \mu_2 \le 1$. When the wave field is a completely direct field, $\mu_2 = 1$. When the wave field tends to be close to a diffuse field, μ_2 is zero.

To calculate the phase accumulation using equation (9), the number of NMP zeros needs to be estimated. The phase accumulation in a non-ideal wave field is influenced by several factors: (i) frequencies range (ii) damping of the structure (iii) the structure size (iv) the source-receiver separation distance. Since the phase is accumulating as a function of frequencies, ω, it is natural that the phase accumulation is related to the studied frequency range. The wavelength at low frequencies is relatively large, hence, the waves at low frequencies are reflected more from the boundary so that the wave field is more diffused. With the frequency increasing, the wavelength becomes shorter, and there are fewer reflections from the boundary, thus the wave field tends to be controlled by the direct field. The effect of damping, η, on the phase accumulation has been discussed in reference [5] for diffuse fields and in reference [9] for direct fields. Generally, when the damping is small, the phase accumulation tends to be the Lyon's reverberant phase limit, $-\pi N_P / 2$. As the damping increases, the phase accumulation will decrease due to the decreasing number of NMP zeros. When the damping is large enough, the reflected waves in the structure will become very weak compared to the outgoing waves and the reflective waves can be ignored. The accumulated phase will then show propagant phase characteristics. The structure size can also influence the phase accumulation in a transfer function. When the structure size is very large, for instance, "infinite" size, the outgoing waves will never meet the boundaries hence there are no reflective waves and the phase will show a propagant phase. As the size of the structure becomes smaller, there will be increasingly reflected waves generated within the system, and the wave field will gradually turn into a diffuse field where the phase accumulation will show a rapid increase. The structure mean size can be expressed by using the mean free path, R, of the propagant wave, i.e. the average distance traversed between reflections, and for a flat plate [10],

$$R = \frac{\pi A_p}{P} \tag{10}$$

where A_p is the area of the plate, and P is the perimeter of the plate.

The source-receiver separation distance, r, is also related to the phase accumulation in a transfer function. When the distance is zero, it is the driving point mobility and the phase will not be accumulated (a value within $\pm\pi$) as the poles and zeros alternatively occur and the phase changes by poles and zeros are cancelled by each other [2]. As the distance increases, assuming the distance remains very small, the outgoing wave is still very strong and the reflected wave is relatively weak, in this case the phase show more or less a propagation phase. When the distance continues to increase, the receiver move far away from the source excitation point, the outgoing waves will become weaker, and the reflective waves start to dominate the wave field. At this time, the phase tend toward the diffuse field phase and has very weak relation to the distance, r.

From the discussion above, it can be seen that, the phase accumulation in a non-ideal wave field should be a function of frequency, structure damping, the structure size and the source-receiver separation distance. With the increasing damping, increasing structure size and decreasing distance, the phase will gradually turn from a diffuse field phase into a direct field propagant phase.

It has been established that equation (9) can be used to predict the accumulated phase in a non-ideal wave field, and the phase lies between the propagant phase curve given by equation (1) and the diffuse field phase given by equation (6) [11]. Actually, equation (9) can be conceived to be a combined contribution from both the direct wave field (propagant phase contribution) and the diffuse wave field (NMP zeros contribution). For the phase accumulation in real structures, the phase contribution from the direct field and the diffuse field is related to their energy density distribution. The energy density at any point within the structure is a combination of the energy density of the direct field and that of the diffuse field. If the diffuse field energy is the primary part, the diffuse field phase then plays a more important role. Conversely, if the direct field dominates the wave field, the phase will show more characteristics of a direct field propagant phase. Based upon the above analysis, the phase accumulation is predicted as

$$\phi(\omega) = \mu_1\phi_{diff} + \mu_2\phi_{dir} \tag{11}$$

where ϕ_{diff} is Tohyama's diffuse field phase. μ_1 is the contribution coefficient of diffuse field phase, $0 \le \mu_1 \le 1$ and μ_2 is the contribution coefficient of direct field propagant phase, ϕ_{dir}. The coefficients μ_1 and μ_2 are expressed by using the ratio of the energy density contribution from the direct field and the diffuse field at that point.

For a flat plate, the ratio between the energy density of the direct field, E_{dir}, and that of the diffuse field, E_{diff}, at the receiver point of distance, r, is [10]:

$$\frac{E_{diff}}{E_{dir}} = \frac{2\pi r c_g}{\omega\eta A_p} e^{-\omega\eta(R-r)/c_g} \tag{12}$$

and μ_1 and μ_2 are estimated as

$$\mu_1 = \frac{E_{diff}}{E_{diff}+E_{dir}} = \frac{1}{1+\dfrac{\omega\eta A_p}{2\pi r c_g} e^{\omega\eta(R-r)/c_g}} \tag{13}$$

$$\mu_2 = \frac{E_{dir}}{E_{diff}+E_{dir}} = \frac{1}{1+\dfrac{2\pi r c_g}{\omega\eta A_p} e^{-\omega\eta(R-r)/c_g}} \tag{14}$$

where c_g is the group velocity. r is the distance between the source and receiver and A_p is the area of the plate.

If the coefficients μ_1 and μ_2 are used to express the energy-weighted group delay, τ, instead of the phase contributions directly (equation (11)), the group delay of a non-ideal wave field can be then written

$$\tau = \frac{d\phi}{d\omega} = \mu_1\tau_g + \mu_2\tau_p \tag{15}$$

and the accumulated phase is

$$\phi(\omega) = \int(\mu_1\tau_g + \mu_2\tau_p)d\omega \tag{16}$$

where τ_g is the group delay for a diffuse field expressed by equation (6), τ_p is the group delay for a direct field, and

$$\tau_p = \frac{d\phi_{dir}}{d\omega} = \frac{d(-kr)}{d\omega} = -\frac{r}{c_g} \tag{17}$$

3 COMPUTER SIMULATION RESULTS

To investigate the characteristics of the mathematical model, both equation (11) and equation (16) for predicting the phase accumulation in a non-ideal wave field are simulated using a rectangular steel plate. The plate's dimensions and material properties were given in Table 1. The bending waves within the plate are of interest in this paper.

Young' s modulus, E [N/m^2]	Density, ρ [kg/m^3]	Poisson' s ratio, υ	Length, l [m]	Width, w [m]	Thickness, h [m]
210*10^9	7870	0.3	0.8	0.5	0.002

Table 1. Dimensions and material properties of the rectangular steel plate.

To calculate the group delay of the diffuse field, τ_g, the modal density of the poles, $n(\omega)$, is theoretically obtained for the plate by assuming simply supported boundary condition [10]. Thus,

$$n(\omega) = \frac{A_p}{4\pi\kappa c_l} \tag{18}$$

where $c_l = \sqrt{E/\rho(1-\nu^2)}$ is the longitudinal wave speed in the material and $\kappa = h/2\sqrt{3}$ is the radius of gyration of the cross-section of the plate.

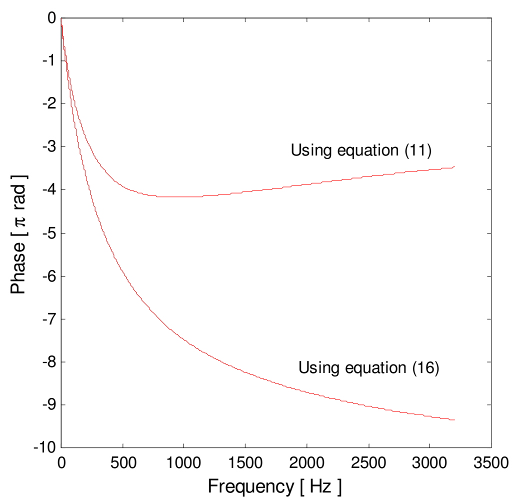

Figure 1. Comparison of phase predictions using equation (11) and using equation (16) at the damping of η =4e-2 and at the distance of r=2.8 cm.

Figure 1 shows the comparison of phase predictions by equation (11) and by equation (16). The distance between the source and the receiver is 2.8 cm and the damping loss factor, η, is set to $4*10^{-2}$. The test frequency range is from 0 Hz to 3200 Hz. In equation (11), the direct field contribution coefficient, μ_1, and the diffuse field contribution coefficient, μ_2, are used to directly express the respective phase contribution percentage from the diffuse field phase, ϕ_{diff}, and from the direct field propagant phase, ϕ_{dir}. These two terms are then added together to predict the phase accumulation in a non-ideal wave field. It can be seen in Figure 1 that this phase prediction from equation (11) has a disadvantage: the phase accumulation increases at low frequencies, then reaches a phase maximum, and turns to decrease at high frequencies. In reality, the phase accumulation should generally have a mono-increasing trend with increasing frequencies. Therefore, equation (11) is not proper to be used to predict the phase accumulation. However, when equation (16) is employed to predict the phase by expressing the group delay contribution from the direct field and the diffuse field rather than the phase contributions directly, this problem disappears. The reason behind this is that the coefficients $\mu_1(\omega)$ and $\mu_2(\omega)$ are functions of specific frequency, ω, while the accumulated phase ϕ_{diff} and ϕ_{dir} are functions of a frequency band $[\omega_1 \ \ \omega_2]$. It is not appropriate to express the phase contribution for a frequency band $[\omega_1 \ \ \omega_2]$ by using the coefficients μ_1 and μ_2 at specific frequency, ω_2. Therefore, equation (16) is a better approach to predict the phase accumulation in a non-ideal wave field.

Figure 2 displays the comparison of phase accumulation predicted by equation (16) and Tohyama's diffuse field phase, ϕ_{diff}, at the same distance r=8.5 cm. Different lines in Figure 2 show the phase predictions for different damping loss factor, η. Dot lines show the phases from Tohyama's model and solid lines display the phases from equation (16). The frequency range is from 0 to 3200 Hz. It can be seen that the phase accumulation in a non-ideal wave field predicted by equation (16) is very close to the phase from Tohyama's model when the damping is small ($\eta = 2*10^{-4}$). With increasing damping, the phase from equation (16) becomes less than Tohyama's diffuse field phase and will have much less phase accumulation when the damping is considerable ($\eta = 2.7*10^{-2}$). This phenomenon implies that the phase in a non-ideal wave field is less diffused and the direct field phase has more contribution on high damping conditions.

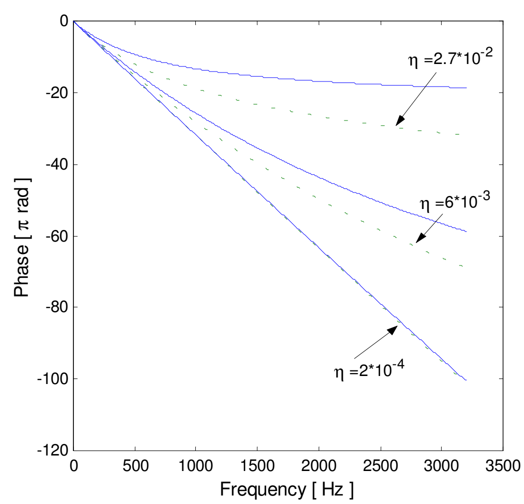

Figure 2. Comparison of phase prediction using equation (16) and Tohyama's diffuse field phase for different damping loss factor, η, and at the same distance of r=8.5 cm.

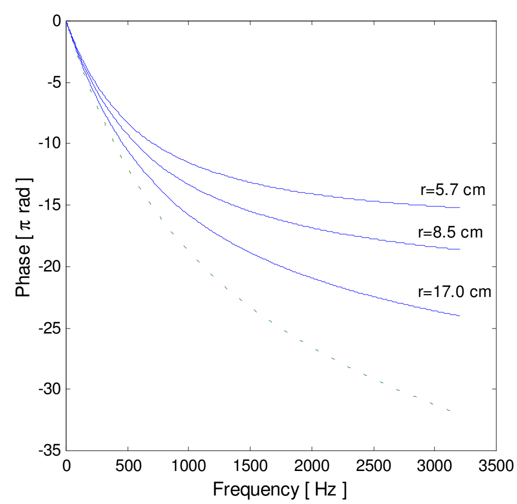

Figure 3. Comparison of phase predictions using equation (16) and Tohyama's diffuse field phase for different distances and at the same damping of $\eta = 2.7*10^{-2}$.

Figure 3 shows the comparison of phase predictions between equation (16) and Tohyama's model at different distances. The damping loss factor, η, is assumed $2.7*10^{-2}$. The diffuse field phase from Tohyama's model remains the same for all the distances as it is independent of distances. However, the phases for the non-ideal wave field expressed by equation (16) change with the distances. The phase accumulates much less than the diffuse field phase when the receiver is close to the source and the accumulated phase increases quickly as the distance goes up. This indicates that the direct field phase has more contribution to the phase in a non-ideal wave field at small distances and the diffuse field phase has gradually increasing contribution to the accumulated phase with increasing distances. In Figure 3, it can also be seen that all the phase accumulation lines are close to the diffuse field phase at low frequencies but gradually deviate away from the diffuse field phase with increasing frequencies. This implies the phase in a non-ideal wave field is more diffused at low frequencies than that at high frequencies.

4 CONCLUSIONS

In this paper, the phase accumulation in the transfer function of flat plates in a non-ideal wave field was investigated. The phase accumulation can generally be expressed by the number of NMP zeros multiplied by -2π plus the contribution from the direct field propagation phase, -kr. Based upon this approach, a mathematical model was presented to predict the phase accumulation by using the energy density distribution of the structure. Compared to Lyon's reverberant phase limit and Tohyama's diffuse field phase model, this model takes the distance into account, thus it can be used to predict the phase in a non-ideal wave field.

Computer simulation results show that equation (16) is a better approach to predict the phase than equation (11). It was also shown that the predicted phase from equation (16) is a combination of the direct field phase contribution and the diffuse field phase contribution. The diffuse field phase has less contribution at small distances or at high damping conditions and will gradually increase the contribution on the predicted phase with increasing distance and decreasing damping. For the future research, experiments on the plates need to be carried out to compare the predicted results from this model to real measurement data.

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