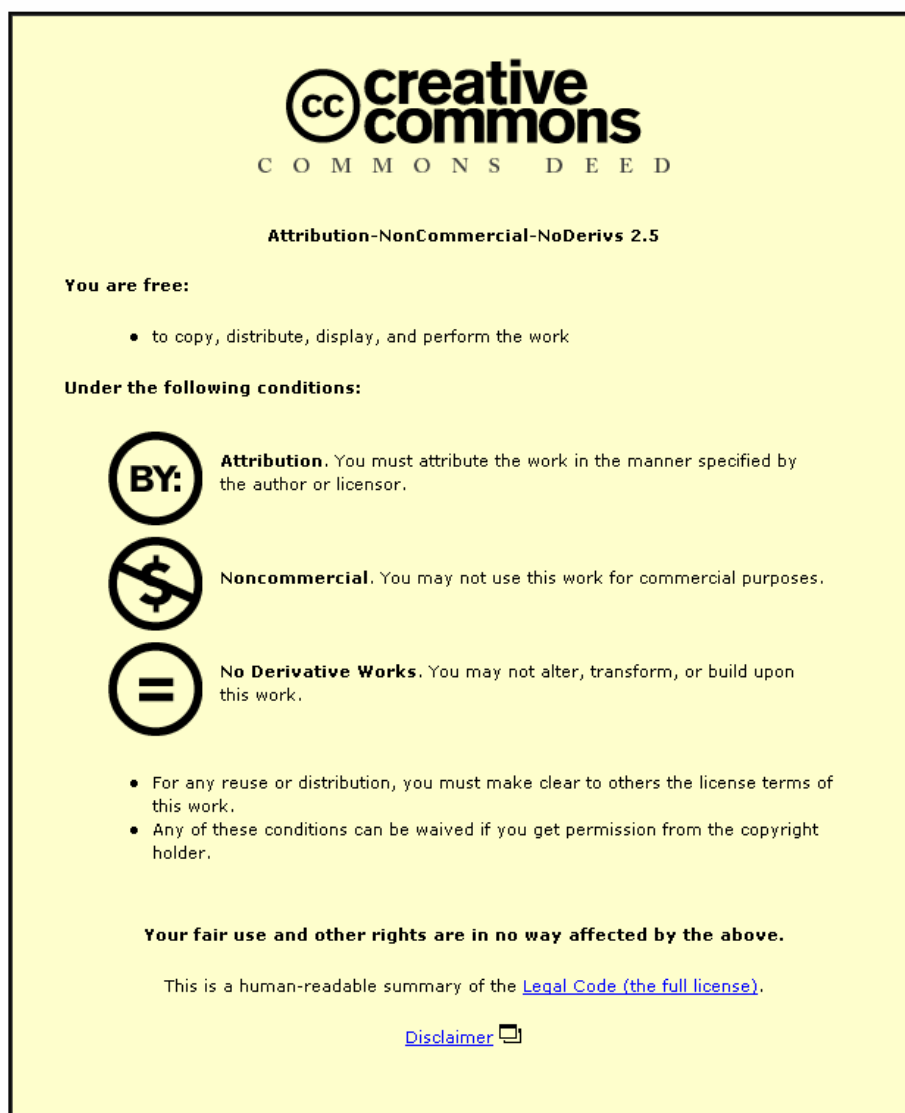




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Flexible vs Dedicated Technology Adoption in the Presence of a Public Firm*

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Abstract

We study firms' adoption of flexible versus dedicated technologies in the context of a mixed versus a private duopoly with product differentiation. The flexible technology allows a firm to become multiproduct or multimarket without bearing additional costs. We find that a configuration where both firms adopt flexible technologies is more likely to arise in equilibrium in the private duopoly. A similar result occurs when both firms use a dedicated technology in the case of either almost independent products or products that are close substitutes. Privatization of the public firm is socially beneficial only in limited circumstances.

Keywords: Flexible Technology, Privatization, Public Firm, Mixed Duopoly.

JEL Classification: L32, L33, L13, O33

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1 Introduction

In the recent past, many firms all over the world have substituted their traditional production processes by more flexible systems. One of the advantages of a flexible manufacturing system (FMS) over a dedicated equipment (DE) is that the former allows a firm to supply several products and consequently to participate in different markets (in other words, becoming a multiproduct or multimarket firm¹) without having to incur additional production costs.²

The following two examples serve as motivation for the analysis we present. First, consider the internet access, telephone and TV services. Traditionally the provision of these services required the use of different technologies and separate production processes for each one of them. At present though, cable technology can be used by firms in order to provide these three different services using the same production process, therefore enabling firms to be present in all three markets and to exploit economies of scope. In that sense, cable technology can be considered an example of FMS.³ Interestingly, the matter has raised public concern. In the UK regulators have encouraged cable companies to provide telephone services but did not allowed British Telecom to enter the television business (Waverman and Sirel, 1997). Similarly, Spanish Telefonica was not permitted to compete with cable operators for a certain period of time (Cantos-Sánchez et al., 2003).

The second example draws from the health care sector. There is evidence of economies of scope (Ozcan et al., 1992), which can be related to the use of FMS. There are several empirical studies stressing the fact that public (not for-profit) hospitals provide a wider range of services than private (for-profit) hospitals (Shortell and Morrison, 1986, 1987 and Schlesinger et al., 1997)

¹This represents an alternative interpretation of our model.

²Boyer and Moreaux (1997, 2002) report additional benefits of using FMS related to capacity flexibility in that FMS can increase the capacity of firms to adapt to fluctuations in demand.

³Dial-up internet access can also be provided using traditional telephone technology. In that sense, traditional telephone technology could also be seen as a flexible technology, since it can be used to service two markets: telephone and internet access services. However, cable technology also allows firms to provide TV services, which cannot be provided by using traditional telephone technology.

although public hospitals tend to provide more innovative services without competition and private hospitals are more likely to add these services when there is competition (Schlesinger, 1998). This body of observations suggest that not only the public or private character of firms but also the degree of competition among them seem to be key factors influencing the adoption of FMS (thus, the multiproduct/multimarket character of firms).

The study of the adoption of FMS by private firms was first introduced by Röller and Tombak (1990) and Kim, Röller and Tombak (1992), in the context of oligopolistic competition. Their findings indicate that the adoption of flexible technologies requires a sufficiently low adoption cost, sufficiently high product differentiation and large enough markets.⁴ Consumers benefit from the use of FMS, due to the increase in competition.⁵ In addition, Röller and Tombak (1993) validate these results by an empirical study.⁶ To the best of our knowledge, the issue of technology choice as exemplified by the adoption of FMS versus DE technologies has not been studied in the context of a mixed market where private (profit-maximising) firms co-exist with public (not-for-profit) ones. Such mixed markets are quite prevalent in transition economies but not exclusively so; telecommunications, health services and the postal sector in many countries are organized as a mixed market.

The aim of this paper is to provide an initial analysis into the choice of production flexibility by concentrating on a simple duopolistic market consisting of either a public and a private firm (mixed duopoly) or two private firms. In particular, we characterize the market conditions that would lead the public and private firms to adopt FMS as opposed to DE. A natural question to address in this context relates to the potential benefits of privatizing the public firm. This is of practical and policy relevance in the light

⁴Garcia-Gallego and Georgantzis (1996) use a similar model to assess multiproduct activity in relation to competition policy.

⁵See also Gupta (1998) for some corrections and reinterpretations of the results in Röller and Tombak (1990).

⁶Eaton and Schmitt (1994), in the context of horizontal product differentiation, point out that the adoption of FMS may correspond to pre-emptive strategies leading to higher levels of concentration.

of recent liberalization trends in many countries across the world. Interestingly, we find that privatization is socially beneficial only when both firms in the mixed duopoly adopt FMS *and* products are sufficiently differentiated.

The plan of the paper is as follows: first, we introduce the model (section 2) and then characterize the different equilibria (section 3). Next we analyze the behavior of firms in the mixed and private duopolies and consider social welfare and the question of privatization (section 4). Finally we summarize our main findings (section 5).

2 The Model

Our model keeps the main features from Röller and Tombak (1990) and Kim, Röller and Tombak (1992) but allows for decreasing returns to scale.⁷ Consider a duopoly competing in output and facing the choice between adopting a flexible manufacturing system (FMS) and a dedicated equipment (DE). The use of FMS allows participation in two existing markets, A and B . The use of the DE constraints firms to be active only in one of the markets. In the case of the mixed duopoly, one of the two firms, denoted by the subscript 2, is public (non-for-profit) and acts as social-welfare maximizer.⁸

The system of inverse demand functions is given by:

$$P^A = a - Q^A - \gamma Q^B \quad (1)$$

and

$$P^B = a - Q^B - \gamma Q^A \quad (2)$$

where P^A and P^B are the prices for products A and B respectively, Q^A and Q^B the total quantities in market A and market B respectively and $a > 0$

⁷This assumption is widely spread in the literature on mixed oligopoly, and is useful in order to avoid the case of natural monopolies which, considering the scope of our paper, is uninteresting.

⁸The assumption about social welfare maximization is in line with the majority of the literature on mixed oligopoly. An alternative, not pursued here, is provided by Matsumura (1998): partially privatized firms are assumed to combine the maximization of social welfare with the maximization of profits.

measures market potential. The parameter γ measures the substitutability of products A and B , $\gamma \in [0, 1)$, the higher γ the fiercer the competition between firms across markets.

The profit of each firm is given by:

$$\pi_{i,j} = P^A Q_{i,j}^A + P^B Q_{i,j}^B - F_k - C_i(Q_{i,j}^A + Q_{i,j}^B) \quad (3)$$

where i denotes the firm ($i = 1$ or 2) and j denotes the state of the industry according to the technologies used by the two firms. In particular,

- $j = 1$ if both firms are using FMS;
- $j = 2$ if firm 1 is using DE and firm 2 is using FMS;
- $j = 3$ if firm 1 is using FMS and firm 2 is using DE;
- $j = 4$ if both firms are using DE.

$Q_{i,j}^A$ and $Q_{i,j}^B$ are the quantities chosen by firm i in state j for markets A and B respectively. Without loss of generality, we assume that if only one firm is using DE, this firm competes only in market A while the other firm participates in both markets. If both firms use DE, they compete in different markets (without loss of generality, firm 1 in market A and firm 2 in market B). Thus, the use of FMS increases the degree of competition not only in the market where a firm is operating but also across markets (due to product substitutability).

F_k are the fixed costs of firms, which are related to the use of the available manufacturing technologies; $k = FMS$ or DE . The costs of using FMS are assumed higher than the costs of using a DE.⁹ For simplicity, we normalize the costs of the dedicated technology to $F_{DE} = 1$. The costs of the flexible technology are then $F_{FMS} = 1 + s$, where s captures the extent of the cost differential between the two manufacturing technologies. C_i are the costs of production, which are assumed to be quadratic and separable in output

$$C_i(Q_{i,j}^A + Q_{i,j}^B) = (Q_{i,j}^A)^2 + (Q_{i,j}^B)^2. \quad (4)$$

Total Surplus (TS) is the sum of consumers' surplus (CS) and producers'

⁹Developments costs are higher for FMS than DE; see Jaikumar (1986).

profits. Linear demand functions yield

$$CS = \frac{1}{2}((Q^A)^2 + (Q^B)^2). \quad (5)$$

Thus, TS is given by

$$TS = CS + \sum_{i=1}^2 \pi_{i,j}. \quad (6)$$

We consider two versions of a two-stage game: (i) a private duopoly and (ii) a mixed duopoly. In the first stage firms choose which technology to adopt and in the second stage they set quantity (Cournot). Decisions in each stage are taken simultaneously. Given technology choices made in stage one, it is straightforward to solve the output stage.¹⁰ We can then derive the relevant payoff functions ($\pi_{i,j}^*$) that firms use in solving the first stage. In other words, we use subgame perfection as our equilibrium concept. In the appendix,¹¹ we give the second-stage solutions for profits (and total surplus). We can then represent the technology choice stage using this simple matrix:

		<i>Firm 2</i>	
		<i>FMS</i>	<i>DE</i>
<i>Firm 1</i>	<i>FMS</i>	$\pi_{1,1}, \pi_{2,1}$	$\pi_{1,3}, \pi_{2,3}$
	<i>DE</i>	$\pi_{1,2}, \pi_{2,2}$	$\pi_{1,4}, \pi_{2,4}$

Table 1¹²

3 Equilibria Characterization

In this section we establish the conditions under which each of the combination of strategies in technology choice is a Nash equilibrium. We proceed by

¹⁰Second-order conditions are satisfied in all cases.

¹¹Second-order conditions are satisfied in all cases.

¹²In the private duopoly, Table 1 is symmetric since $\pi_{1,1} = \pi_{2,1}$, $\pi_{1,4} = \pi_{2,4}$, $\pi_{1,3} = \pi_{2,2}$ and $\pi_{1,2} = \pi_{2,3}$.

finding the critical value of the technology costs, s , above which investment in FMS becomes unprofitable. Using Table 1, we examine the conditions that guarantee one of the four possible pure-strategy equilibria in each of the regimes, private or mixed duopoly: (FMS, FMS) where both firms choose a flexible production technology and serve both markets, (DE, DE) where both firms choose a dedicated production process and serve different markets and (FMS, DE), (DE, FMS) where one firm chooses FMS and the other DE.

3.1 The (FMS,FMS) Equilibrium

Private Duopoly. From Table 1, it is clear that (FMS, FMS) is an equilibrium when (i) $\pi_{1,1}^* - \pi_{1,2}^* \geq 0$ for firm 1 and (ii) $\pi_{2,1}^* - \pi_{2,3}^* \geq 0$ for firm 2. Using the model outlined previously, these conditions are equivalent to:

$$\frac{a^2(3+2\gamma)}{(4+3\gamma)^2} - \frac{3a^2(2\gamma^2+\gamma-6)^2}{2(24-11\gamma^2)^2} - s \geq 0.$$

Let σ_1 denote the critical level in (the difference in) fixed costs s , that makes the above expression a strict equality. If s is lower than this critical value σ_1 then both firms will choose FMS as it improves their profits. From the above expression this critical value is,

$$\sigma_1 = \frac{a^2 f_1(\gamma)}{2(4+3\gamma)^2(24-11\gamma^2)^2} \quad (7)$$

where $f_1(\gamma) = 1728 + 288\gamma - 2172\gamma^2 - 324\gamma^3 + 867\gamma^4 + 88\gamma^5 - 108\gamma^6 > 0$. Note that the critical value is increasing in market size, $\partial\sigma_1/\partial a > 0$, while it is decreasing in product substitutability, $\partial\sigma_1/\partial\gamma < 0$. The larger market for either product makes firms wish to participate in flexible production in order to serve both markets. With a low degree of substitutability (small γ) firms' products are perceived as highly differentiated by consumers so that a firm that opts for a dedicated production process (DE) and thus serves only one market effectively loses out. Hence a larger market size and greater

product differentiation point towards the adoption of FMS by the firms.¹³

Mixed Duopoly. From Table 1, (FMS, FMS) is an equilibrium if (i) $\pi_{1,1}^* - \pi_{1,2}^* \geq 0$ for firm 1 and (ii) $\pi_{2,1}^* - \pi_{2,3}^* \geq 0$ for firm 2. The first condition yields

$$\frac{a^2(3+2\gamma)}{(5+2\gamma)^2} - \frac{3a^2}{50} - s \geq 0$$

which implies a corresponding critical value for s denoted,

$$\sigma_2 = \frac{a^2 f_2(\gamma)}{50(5+2\gamma)^2} \quad (8)$$

where $f_2(\gamma) = 75 + 40\gamma - 12\gamma^2 > 0$. The second condition is equivalent to

$$\frac{2a^2(8+5\gamma+\gamma^2)}{(1+\gamma)(5+2\gamma)^2} - \frac{2a^2(61-58\gamma-12\gamma^2+16\gamma^3)}{(15-8\gamma^2)^2} - s \geq 0$$

implying an associated critical value for s ,

$$\sigma_3 = \frac{2a^2 f_3(\gamma)}{(1+\gamma)(5+2\gamma)^2(8\gamma^2-15)^2} \quad (9)$$

where $f_3(\gamma) = 275 - 170\gamma - 249\gamma^2 + 88\gamma^3 + 72\gamma^4 - 16\gamma^5 > 0$. It is easy to establish that $\partial\sigma_2/\partial a > 0$, $\partial\sigma_3/\partial a > 0$, $\partial\sigma_2/\partial\gamma < 0$ and $\partial\sigma_3/\partial\gamma < 0$. A larger market (higher a) supports a larger critical difference in the fixed costs of the two different types of technology while increased product substitutability (higher γ) has the opposite effect. Taking the two conditions together implies that (FMS, FMS) is an equilibrium when $s < \sigma_2$ and $s < \sigma_3$ while it is not an equilibrium if $s > \sigma_2$ or $s > \sigma_3$. We then state the following Lemma¹⁴:

Lemma 1 : *In the mixed duopoly, (FMS, FMS) is an equilibrium if $s < \min\{\sigma_2, \sigma_3\}$. In particular, given market size a , there exists a critical value γ^* such that for $\gamma < \gamma^*$ (FMS, FMS) is an equilibrium if $s < \sigma_2$ and for*

¹³Röller and Tombak (1990, 1993) obtain a similar result for a different specification of the variable production costs.

¹⁴All proofs are included in the Appendix.

$\gamma > \gamma^*$ (FMS, FMS) is an equilibrium if $s < \sigma_3$. This critical value is $\gamma^* = 0.2432$.

This result implies that under low levels of competition the private firm is less likely to have a multiproduct profile than the public firm ($\sigma_2 < \sigma_3$ for $\gamma < \gamma^*$). On the other hand, the opposite happens for high degrees of competition ($\sigma_2 > \sigma_3$ for $\gamma > \gamma^*$).¹⁵ Having analyzed both the private and mixed duopoly cases we now proceed to a simple comparison of the two regimes. First, we consider the conditions for an (FMS, FMS) equilibrium to occur, i.e. we compare the three critical levels of fixed costs, σ_1, σ_2 and σ_3 (see expressions (7),(8) and (9)). The following proposition describes.

Proposition 1 *For given $\gamma \in [0, 1)$ and any $a > 0$ the critical value for the fixed technology costs s is lower in the mixed duopoly than in the private duopoly, that is $\min\{\sigma_2, \sigma_3\} < \sigma_1$. Hence from the necessary conditions for an (FMS, FMS) equilibrium, (7),(8) and (9):*

- (i) *if $s < \min\{\sigma_2, \sigma_3\}$ then (FMS, FMS) is an equilibrium in both the mixed and private duopolies;*
- (ii) *if $\min\{\sigma_2, \sigma_3\} < s < \sigma_1$ then (FMS, FMS) is an equilibrium in the private duopoly but not in the mixed duopoly;*
- (iii) *if $\sigma_1 < s$ then (FMS, FMS) is not an equilibrium.*

Proposition 1 implies that an equilibrium in (FMS, FMS) is more likely to arise in a private duopoly than in a mixed duopoly (i.e. it requires less demanding conditions of the technology costs and size of the market). Even if this result might seem surprising, the intuition behind it is clear. First, consider the case with relatively high substitutability between products. In such a case, the public firms is less inclined to invest in FMS since it is less profitable and also socially not meaningful: investing in FMS would imply bearing the higher technology costs in order to produce a new good which is perceived by consumers to be a very close substitute to the one already

¹⁵Note that this result is confirmed empirically by Schlesinger et al. (1997) and Schlesinger et al. (1998), in the context of competition among hospitals in the provision of several services.

produced by the public firm.¹⁶ Second, consider the case of relatively low substitutability. Here, the public firm produces more in each market to compensate for the low substitutability between products, therefore making it less profitable for the private firm to invest in technology adoption; in essence the public firm crowds out the private firm's investment.

Proposition 1 is illustrated in figure 1. The figure graphs σ_1 , σ_2 , σ_3 for given a .¹⁷ The area below σ_1 represents combinations of s and γ that guarantee an (FMS, FMS) equilibrium in the private duopoly and the area below the minimum of σ_2 and σ_3 represents equivalent combinations for the mixed duopoly. Therefore, the shadowed area represents parameter combinations that make (FMS, FMS) an equilibrium in the private but not the mixed duopoly. This indicates that, for given size of the market and product differentiation, lower values of the technology adoption costs correspond to an (FMS, FMS) equilibrium in the mixed duopoly.

[Insert figure 1 about here]

3.2 The (DE, DE) Equilibrium

Private Duopoly. In the case of the private duopoly the conditions for (DE, DE) to be an equilibrium (see Table 1) are (i) $\pi_{1,4}^* - \pi_{1,3}^* > 0$ and (ii) $\pi_{2,4}^* - \pi_{2,2}^* > 0$, for firms 1 and 2 respectively, implying

$$-\frac{3a^2}{2(3+\gamma)^2} + \frac{a^2(300 - 276\gamma - 85\gamma^2 + 122\gamma^3 - 21\gamma^4)}{2(24 - 11\gamma^2)^2} + s \geq 0.$$

Letting σ_4 denote the relevant critical value for s in this case, we obtain from the above expression,

$$\sigma_4 = \frac{a^2 f_4(\gamma)}{2(3+\gamma)^2(11\gamma^2 - 24)^2} \quad (10)$$

¹⁶In such a case, it would be more efficient to produce a higher quantity of the "old" good instead.

¹⁷We have set $a = 1$ in figure 1. The value of a does not affect the graphs qualitatively, since a is just a scaling parameter.

where $f_4(\gamma) = 972 - 684\gamma - 537\gamma^2 + 312\gamma^3 + 95\gamma^4 - 4\gamma^5 - 21\gamma^6 > 0$. If s is greater than this critical value, σ_4 , then (DE, DE) is an equilibrium. It is obvious that this critical value is increasing in market size, $\partial\sigma_4/\partial a > 0$, while it can be easily established that it is decreasing in the product differentiation parameter, $\partial\sigma_4/\partial\gamma < 0$. Consequently, (DE, DE) is an equilibrium for relatively smaller a and higher γ . The intuition behind this is clear, since the opposite to the FMS case holds: The smaller the market for either product makes firms less willing to participate in flexible technology adoption in order to serve both markets. With a high degree of substitutability (high γ) firms' products are perceived as close substitutes by consumers so that a firm that opts for a FMS is bearing a high fixed cost to produce two goods that are almost the same. Hence a smaller market size and lower product differentiation point towards the adoption of DE by the firms, given s .

Mixed Duopoly. From Table 1, the conditions ensuring that (DE, DE) is an equilibrium are (i) $\pi_{1,4}^* - \pi_{1,3}^* > 0$ (for the private firm) and (ii) $\pi_{2,4}^* - \pi_{2,2}^* > 0$ (for the public firm). The first condition can be written as

$$\frac{3a^2(2-\gamma)^2}{8(\gamma^2-3)^2} - \frac{a^2(51-48\gamma-14\gamma^2+16\gamma^3)}{(15-8\gamma^2)^2} + s \geq 0$$

implying that the associated critical value for s is

$$\sigma_5 = \frac{a^2 f_5(\gamma)}{8(\gamma^2-3)^2(8\gamma^2-15)^2} \quad (11)$$

where $f_5(\gamma) = 972 - 756\gamma - 1251\gamma^2 + 576\gamma^3 + 1032\gamma^4 - 384\gamma^5 - 304\gamma^6 + 128\gamma^7 > 0$. From the second condition we obtain

$$\frac{a^2(-57+60\gamma-4\gamma^2)}{100(-1+\gamma^2)} - \frac{a^2(17-14\gamma-\gamma^2+2\gamma^3)}{4(-3+\gamma^2)^2} + s \geq 0$$

with associated critical value

$$\sigma_6 = \frac{a^2 f_6(\gamma)}{50(\gamma^2-3)^2(1-\gamma^2)} \quad (12)$$

where $f_6(\gamma) = 44 - 95\gamma + 72\gamma^2 - 20\gamma^3 + 4\gamma^4 - 5\gamma^5 + 2\gamma^6 > 0$. Notice

that $\partial\sigma_5/\partial a > 0$ and $\partial\sigma_6/\partial a > 0$ while it is relatively easy to check that $\partial\sigma_5/\partial\gamma < 0$ and $\partial\sigma_6/\partial\gamma \begin{matrix} \leq \\ > \end{matrix} 0$ as $\gamma \begin{matrix} \leq \\ > \end{matrix} 0.6669$. Therefore a (DE, DE) equilibrium occurs when both $s > \sigma_5$ and $s > \sigma_6$. The following Lemma establishes that the latter inequality is sufficient for a (DE, DE) equilibrium; that is, the critical value in the mixed duopoly is the one corresponding to the public firm.

Lemma 2 : *In the mixed duopoly (DE, DE) is an equilibrium if $s > \sigma_6$ for all $\gamma \in [0, 1)$.*

In line with the discussion of the (FMS, FMS) equilibrium we now proceed in comparing the private and mixed duopolies in terms of the critical values for the difference in fixed costs as well as characterizing the (DE, DE) equilibrium.

Lemma 3 *Comparing the critical values for the private duopoly, σ_4 , and the mixed duopoly, σ_6 , we have: $\sigma_4 \geq \sigma_6$ for $\gamma_1 \leq \gamma \leq \gamma_2$ and $\sigma_4 < \sigma_6$ for $0 \leq \gamma < \gamma_1$ and $\gamma_2 < \gamma < 1$, where $\gamma_1 = 0.0056$ and $\gamma_2 = 0.6755$.*

We summarize the results obtained in this subsection in the following proposition:

Proposition 2(a) *For given $a > 0$ and $\gamma_1 \leq \gamma \leq \gamma_2$: (i) if $s > \sigma_4$ then (DE, DE) is an equilibrium in both the mixed and private duopolies, (ii) if $\sigma_4 > s > \sigma_6$ then (DE, DE) is an equilibrium in the mixed duopoly but not in the private duopoly, (iii) if $\sigma_6 > s$ then (DE, DE) is not an equilibrium;*

(b) For given $a > 0$, $0 < \gamma < \gamma_1$ and $\gamma_2 < \gamma < 1$: (i) if $s > \sigma_6$ then (DE, DE) is an equilibrium in both the mixed and the private duopolies, (ii) if $\sigma_6 > s > \sigma_4$ then (DE, DE) is an equilibrium in the private but not the mixed duopoly, (iii) if $\sigma_4 > s$ then (DE, DE) is not an equilibrium.

Figure 2 illustrates proposition 2.¹⁸ The white area above σ_4 and σ_6 represents combinations of the parameters s and γ such that a (DE, DE) equilibrium exists for both versions of duopoly. The dark shadowed area

¹⁸In Figure 2, $a = 1$.

represents combinations that guarantee a (DE, DE) equilibrium in the private duopoly but not in the mixed one. Finally, the light shadowed area represents parameter combinations that make (DE, DE) an equilibrium in the mixed duopoly only.

[Insert Figure 2 about here]

Figure 2 illustrates that the necessary conditions for a (DE, DE) equilibrium are more stringent in the case of the mixed duopoly for low and relatively high values of substitutability. For low values of substitutability, i.e. when products are perceived as highly differentiated by consumers, there is a strong incentive for the public firm to serve both markets and so increase the degree of competition. Thus, a (DE, DE) equilibrium is less likely in the mixed duopoly. For high values of substitutability, since the degree of competition across markets is already very high, either firm in the private duopoly is willing to adopt DE as a way of dampening down competition, provided that its counterpart behaves in the same way. Meanwhile, in the case of the mixed duopoly, if the private firm uses DE, the public firm has strong incentives to adopt FMS in order to increase the degree of competition. For intermediate values of product substitutability a (DE, DE) equilibrium is more prevalent in the mixed duopoly.

3.3 The (DE, FMS) and (FMS, DE) Equilibria

Private Duopoly. From Table 1, (DE, FMS) is an equilibrium when (i) $\pi_{1,1}^* - \pi_{1,2}^* \leq 0$ for firm 1 and (ii) $\pi_{2,4}^* - \pi_{2,3}^* > 0$ for firm 2. The two conditions taken together imply that if $\sigma_1 < s < \sigma_4$, (DE, FMS) is a Nash equilibrium in the case of a private duopoly. Given symmetry, it follows that (FMS, DE) is an equilibrium under the same conditions as (DE, FMS). Thus, if $\sigma_1 < s < \sigma_4$ there are two Nash Equilibria. We then state the following lemma.

Lemma 4 *In the private duopoly, (DE, FMS) and (FMS, DE) are Nash Equilibria if $\sigma_1 < s < \sigma_4$. In particular, given market size a , there exists a critical value γ^{**} such that if $\gamma > \gamma^{**}$ then $\sigma_4 > \sigma_1$ and therefore, (DE,*

FMS) and *(DE, FMS)* are Nash Equilibria. This critical value is $\gamma^{**} = 0.6442$.

It is interesting to note that only relatively high values of product substitutability guarantee the existence of asymmetric equilibria (in the sense that firms make differing technology choices).¹⁹ Intuitively, when there is high substitutability across markets, there are situations in which technology costs are high enough to make unprofitable the investment in FMS when the opponent is present in the two markets while they are not high enough to make the investment unprofitable when the counterpart is only present in one of the two markets. In such circumstances, the equilibrium outcome will be asymmetric.²⁰

Mixed Duopoly. We begin with the analysis of the *(DE, FMS)* equilibrium. In this case, from Table 1, the necessary conditions are (i) $\pi_{1,1}^* - \pi_{1,2}^* \leq 0$ and (ii) $\pi_{2,4}^* - \pi_{2,2}^* < 0$ implying that if $\sigma_2 < s < \sigma_6$, *(DE, FMS)* is a Nash Equilibrium in the mixed duopoly.

Lemma 5 *(DE, FMS) is a Nash equilibrium in the mixed duopoly only if $\sigma_2 < s < \sigma_6$. This is satisfied for values of the substitutability parameter $\gamma \leq 0.3133$ or $\gamma \geq 0.8172$. For $\gamma \in (0.3133, 0.8172)$, *(DE, FMS)* is not an equilibrium.*

Next we consider the case of the *(FMS, DE)* equilibrium. So that *(FMS, DE)* is an equilibrium it is required that (i) $\pi_{1,4}^* - \pi_{1,3}^* > 0$ and (ii) $\pi_{2,3}^* - \pi_{2,1}^* > 0$, implying that $\sigma_3 < s < \sigma_5$ must hold.

Lemma 6 *(FMS, DE) is a Nash equilibrium in the mixed duopoly if $\sigma_3 < s < \sigma_5$. In particular, given market size, a , there exists a critical value γ^{***}*

¹⁹This result is in contrast with Kim, Röller and Tombak (1992) where asymmetric equilibria in pure strategies do not exist.

²⁰Here the two firms are interested in being the one using FMS. Given that $\pi_{1,2}^* - \pi_{1,1}^* > 0$ and $\pi_{1,3}^* - \pi_{1,4}^* > 0$ must hold, and by definition $\pi_{1,4}^* > \pi_{1,2}^*$ ($\forall \gamma \neq 0$), then $\pi_{1,3}^* > \pi_{1,2}^*$. Given the symmetry of the game, the same applies to firm 2. Therefore, in the case of asymmetric equilibria the firm using FMS obtains higher profits than the one using DE. Therefore, given the multiplicity of equilibria firms might end up in the worst scenario possible unless some coordination mechanism is used.

such that for $\gamma > \gamma^{***}$ (FMS, DE) is an equilibrium. This critical value is $\gamma^{***} = 0.3133$.

Interestingly, we can show that given a set of market and technology conditions $(a, s$ and $\gamma)$, asymmetric equilibria never arise simultaneously in the private and in the mixed duopoly. In other words, the space of market and technology conditions required for an asymmetric equilibrium to arise in the private duopoly does not overlap with any of the two (one for (FMS, DE) , the other for (DE, FMS)) spaces of market and technology conditions required in the mixed duopoly.

Proposition 3 (i) For given $a > 0$ and $\gamma > \gamma^{**}$ if $\sigma_1 < s < \sigma_4$ (FMS, DE) and (DE, FMS) are equilibria in the private duopoly but not in the mixed duopoly; (ii) For given $a > 0$ and $\gamma \notin (0.313292, 0.817226)$ if $\sigma_2 < s < \sigma_6$ then (DE, FMS) is an equilibrium in the mixed duopoly but not in the private duopoly; (iii) For given $a > 0$ and $\gamma > \gamma^{***}$ if $\sigma_3 < s < \sigma_5$ then (FMS, DE) is an equilibrium in the mixed duopoly but not in the private duopoly.

4 Is Privatization Beneficial?

In this section, we examine social welfare across the two market arrangements. In doing so address the question of privatization of the public firm. Obviously, privatization is beneficial only if it leads to an increase in social welfare (total surplus).

Note that under the same market and technology conditions, the technology choice equilibrium outcomes of the mixed and the private duopoly might differ, as shown in Propositions 1 to 3. Therefore, in order to make a valid comparison across types of duopoly, we need to identify the equilibrium outcomes of the two duopolies *for given sets* of market and technology conditions. We proceed as follows: We start by considering one of the four possible equilibria in the mixed duopoly, say (FMS, FMS) . We know that this equilibrium requires a particular set of conditions related to the parameters of the model, s , a and γ (as established in Lemma 1). Then

we identify which would be the corresponding equilibrium outcome in the private duopoly under the same set of market and technology conditions. Having done this, we compare the equilibrium level of total surplus across the two regimes. We, then, repeat this procedure for the other three possible equilibria in the mixed duopoly (DE, FMS), (FMS, DE) and (DE, DE). We denote by subscripts M the mixed duopoly and by P the private duopoly, followed by 1, 2, 3 and 4 denoting the (FMS, FMS), (DE, FMS), (FMS, DE) and (DE, DE) equilibria respectively.

4.1 (FMS, FMS) Equilibrium in the mixed duopoly

Recall From Lemma 1 that (FMS, FMS) is an equilibrium in the mixed duopoly if $s < \min\{\sigma_2, \sigma_3\}$. The equivalent condition for the private duopoly is $s < \sigma_1$ while from Proposition 1 the critical value for the fixed technology costs s is lower in the mixed duopoly than in the private one, $\min\{\sigma_2, \sigma_3\} < \sigma_1$. So (FMS, FMS) is an equilibrium in both the mixed and private duopolies if $s < \min\{\sigma_2, \sigma_3\}$. A straightforward comparison of the total surplus in the two market regimes reveals that welfare is higher in the private duopoly except when products are nearly independent, as the following Lemma demonstrates.

Lemma 7 $TS_{P1} \geq TS_{M1}$ for $\gamma \geq 0.0223$ and $TS_{P1} < TS_{M1}$ for $\gamma < 0.0223$.

4.2 (DE, DE) Equilibrium in the mixed duopoly

As shown in lemma 3, the relevant condition for a (DE, DE) equilibrium in the mixed duopoly is $s > \sigma_6$ while the equivalent condition in the private duopoly requires $s > \sigma_4$. We then distinguish the following cases. *Case A*: $s > \sigma_6$ and $s > \sigma_4$. (DE, DE) is the outcome in both market arrangements. *Case B(i)*: $s > \sigma_6$, $s < \sigma_4$ and $s \geq \sigma_1$. (DE, DE) obtains in the mixed duopoly while either (DE, FMS) or (FMS, DE) occurs in the private duopoly; *Case B(ii)*: $s > \sigma_6$, $s < \sigma_4$ and $s < \sigma_1$ where (DE, DE) is the mixed duopoly

equilibrium and (FMS, FMS) is the private duopoly equilibrium. We next proceed to examine each of these cases in detail.

Case A. (DE, DE) is the equilibrium in both the mixed and private duopolies so we just need to compare TS_{P_4} and TS_{M_4} . This is done in the following Lemma.

Lemma 8 *For $a > 0$ and $\gamma \in [0, 1)$, when $s > \sigma_6$ and $s > \sigma_4$, $TS_{P_4} < TS_{M_4}$*

Case B(i). The mixed duopoly is characterized by a (DE, DE) equilibrium while the private duopoly equilibrium is either (DE, FMS) or (FMS, DE). Hence the relevant welfare comparison is between total surplus TS_{M_4} in the mixed duopoly and total surplus TS_{P_2} in the private duopoly - recall that the private duopoly equilibria are symmetric. The following lemma 9 illustrates.

Lemma 9 *For $a > 0$ and $\gamma \in (0.6442, 0.6755)$, when $s > \sigma_6$, $s < \sigma_4$ and $s \geq \sigma_1$, $TS_{P_2} < TS_{M_4}$.*

Case B(ii). In this case the mixed duopoly equilibrium is (DE, DE) while the private duopoly yields (FMS, FMS). In the following lemma, we compare total surpluses TS_{M_4} and TS_{P_1} .

Lemma 10 *For $a > 0$ and $\gamma \in (0.0536, 0.6736)$, when $s > \sigma_6$, $s < \sigma_4$ and $s < \sigma_1$, $TS_{P_1} < TS_{M_4}$.*

To sum up the results of this section, under the market and technology conditions that lead to an equilibrium with both firms choosing DE in the mixed duopoly, privatization will not be welfare enhancing.

4.3 (DE, FMS) Equilibrium in the mixed duopoly

Next we turn our attention to the (DE, FMS) equilibrium in the mixed duopoly. From Lemma 5, the relevant condition for a (DE, FMS) equilibrium is $\sigma_2 < s < \sigma_6$ and is satisfied when $\gamma \notin (0.3133, 0.8173)$. In this ranges of

values for γ , the corresponding equilibrium in the private duopoly would be either (DE, DE), if $s > \sigma_4$ (Case C) or (FMS, FMS) if $s < \sigma_1$ (Case D).²¹ We start by analyzing the first of these cases.

Case C: $\sigma_2 < s < \sigma_6$ and $s > \sigma_4$. (DE, FMS) is the outcome in the mixed duopoly while (DE, DE) obtains in the private duopoly. Comparing total surplus in the two market regimes yields the following Lemma.

Lemma 11 *For $a > 0$ and $\gamma \notin (0.0056, 0.8173)$ when $\sigma_2 < s < \sigma_6$ and $s > \sigma_4$, $TS_{P_4} < TS_{M_2}$.*

Case D: $\sigma_2 < s < \sigma_6$ and $s < \sigma_1$. (DE, FMS) is the outcome in the mixed duopoly and (FMS, FMS) in the private one. The relevant welfare comparison is between TS_{P_1} and TS_{M_2} .

Lemma 12 *For $a > 0$ and $\gamma \notin (0.3133, 0.8173)$, when $\sigma_2 < s < \sigma_6$ and $s < \sigma_1$, $TS_{P_1} < TS_{M_2}$.*

In both cases, privatization would not be beneficial. Therefore, under the market and technology conditions that lead to an equilibrium in the mixed duopoly with the private firm adopting DE and the public firm FMS, privatization is welfare reducing.

4.4 (FMS, DE) Equilibrium in the mixed duopoly

Finally, we consider the case of the (FMS, DE) equilibrium in the mixed duopoly. Lemma 6 requires that $\sigma_3 < s < \sigma_5$, which is guaranteed as long as $\gamma > \gamma^{***} = 0.3133$. In Proposition 3 we have shown that asymmetric equilibria do not arise in both types of duopoly for a given set of technology and market conditions. Moreover, it can be easily checked that $\sigma_4 > \sigma_5$ and thus, (DE, DE) is never an equilibrium in the private duopoly for values of s such that $\sigma_3 < s < \sigma_5$. On the contrary, the conditions for (FMS, FMS) to be an equilibrium in the private duopoly are compatible with $\sigma_3 < s < \sigma_5$, since

²¹In proposition 3 we have shown that asymmetric equilibria do not arise in both types of duopoly.

$\sigma_1 > \sigma_5$. Hence, whenever the equilibrium in the mixed duopoly is (FMS, DE) the counterpart in the private duopoly is (FMS, FMS). Therefore, the only comparison that is meaningful here is between TS_{P_1} and TS_{M_3} .

Lemma 13 *For $a > 0$ and $\gamma \in [0, 1)$, when $\sigma_3 < s < \sigma_5$ and $s < \sigma_1$, $TS_{P_1} < TS_{M_3}$.*

As a consequence, we can state that under the conditions that lead to an equilibrium in the mixed duopoly with the private firm using FMS and the public firm using DE, privatization would not lead to an increase in surplus.

The results we have obtained regarding welfare comparisons across the two market arrangements have some potential policy implications for the debate about the privatization of a public firm. As we have argued and shown, privatizing the public firm, i.e. switching from a mixed duopoly to a private one, would only enhance social welfare when the outcome in the mixed duopoly is (FMS, FMS), i.e. both firms are adopting flexibility in their production, *provided that products are not (almost) independent*. The private duopoly outcome would also be (FMS, FMS) but would result in higher levels of social welfare. In all other cases, privatization would result in a reduction in social welfare. The following Proposition summarizes.

Proposition 4: *Privatization is beneficial in that it increases social welfare when the equilibrium outcome in the mixed duopoly is (FMS, FMS) and $\gamma > 0.0223$. In the remaining cases privatization of the public firm is detrimental as it would reduce social welfare.*

The relative strength of the above proposition in terms of its policy implications is derived from the fact that it can be used even without knowing the exact values of a , γ and s . It seems quite plausible to assume that policy makers know accurately the strategic plans of public firms, in this case the FMS investment plan in technology choice and the closeness between the markets/goods. If the public firm does not have any intention of replacing DE with FMS, then privatizing it should not be considered. However, a word of caution is needed here. The results we obtain are based

on a simple duopoly model, with linear demand and quadratic costs. It would be interesting to examine the robustness of the model's predictions in a more general setting of an oligopoly with general demand and cost functions and also whether the results are sensitive to the mode of competition, i.e. quantity versus price. We leave this aside for future research.

5 Concluding Remarks

In this paper we have introduced a mixed duopoly in the context of a differentiated product, quantity-setting duopoly facing the decision of whether to adopt a flexible technology (and become a multiproduct or multimarket firm) or a dedicated technology. We have also the equivalent private duopoly so as to compare the outcomes of the two different market arrangements and provide some tentative policy guidelines on the privatization of a public firm.

Our main findings can be summarized as follows: An equilibrium with both firms choosing flexible technologies is more likely to arise in the case of the private duopoly. Further, an equilibrium involving the two firms using dedicated technologies is also more likely to arise in the private duopoly when products are very close substitutes or almost independent. Mixed (asymmetric) equilibria with one firm being flexible and the other dedicated, are less likely to be obtained in the private duopoly. In the case of a mixed duopoly, the public firm chooses a dedicated technology when products are very close substitutes, since it is not profitable for it to bear higher technology costs in order to produce almost the same good.

Privatization of the public firm is warranted, i.e., beneficial, when the market and technology conditions lead to an equilibrium outcome where both firms use flexible technologies and goods are not (almost) independent. The underlying conditions for this equilibrium to arise imply high potential profitability (low technology costs relative to the size of the market and/or the degree of substitutability between markets). In all remaining cases, privatizing the public firm would result in a reduction of social welfare. Thus, our results provide limited support for privatizing the public firm. However, this conclusion is qualified by the limitations of the model used.

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6 Appendix

6.1 Private duopoly: Equilibrium solutions

(FMS, FMS)	Firm 1	Firm 2
π	$\frac{a^2(3+2\gamma)}{(4+3\gamma)^2} - (1+s)$	$\frac{a^2(3+2\gamma)}{(4+3\gamma)^2} - (1+s)$
TS	$\frac{2a^2(5+2\gamma)}{(4+3\gamma)^2} - 2(1+s)$	

(DE, DE)	Firm 1	Firm 2
π	$\frac{3a^2}{2(3+\gamma)^2} - 1$	$\frac{3a^2}{2(3+\gamma)^2} - 1$
TS	$\frac{4a^2}{(3+\gamma)^2} - 2$	

(FMS, DE)	Firm 1	Firm 2
π	$\frac{a(300-\gamma(3-\gamma)(92+\gamma(59-21\gamma)))}{24-11\gamma^2} - (1+s)$	$\frac{3a^2(6-\gamma-2\gamma^2)^2}{2(24-11\gamma^2)^2} - 1$
TS	$\frac{a^2(2+\gamma)(154-\gamma(215-\gamma(81-4\gamma)))}{(24-11\gamma^2)^2} - (2+s)$	

(DE, FMS)	Firm 1	Firm 2
π	$\frac{3a^2(6-\gamma-2\gamma^2)^2}{2(24-11\gamma^2)^2} - 1$	$\frac{a(300-\gamma(3-\gamma)(92+\gamma(59-21\gamma)))}{24-11\gamma^2} - (1+s)$
TS	$\frac{a^2(2+\gamma)(154-\gamma(215-\gamma(81-4\gamma)))}{(24-11\gamma^2)^2} - (2+s)$	

6.2 Mixed duopoly: Equilibrium solutions

(FMS, FMS)	Private Firm	Public Firm
π	$\frac{a^2(3+2\gamma)^2}{(5+2\gamma)^2} - (1+s)$	$\frac{4a^2(1+3\gamma+\gamma^2)}{(1+\gamma)(5+2\gamma)^2} - (1+s)$
TS	$\frac{2a^2(8+\gamma(5+\gamma))}{(1+\gamma)(5+2\gamma)^2} - 2(1+s)$	

(DE, DE)	Private Firm	Public Firm
π	$\frac{3a^2(2-\gamma)^2}{8(3-\gamma^2)^2} - 1$	$\frac{a^2(9-2\gamma^2(5-2\gamma))}{8(3-\gamma^2)^2} - 1$
TS	$\frac{a^2(17-\gamma(14+\gamma(1-2\gamma)))}{4(3-\gamma^2)^2} - 2$	

(FMS, DE)	Private Firm	Public Firm
π	$\frac{a^2(51-48\gamma-14\gamma^2+16\gamma^3)}{(15-8\gamma^2)^2} - (1+s)$	$\frac{a^2(18+2\gamma(3-2\gamma(9-4\gamma)))}{(15-8\gamma^2)^2} - 1$
TS	$\frac{2a^2(61-2\gamma(29+2\gamma(3-4\gamma)))}{(15-8\gamma^2)^2} - (2+s)$	

(DE, FMS)	Private Firm	Public Firm
π	$\frac{3a^2}{50} - 1$	$\frac{a^2(41-\gamma^2(143-20\gamma(6-\gamma)))}{200(1-\gamma^2)^2} - (1 + s)$
TS		$\frac{a^2(57-4\gamma(15-\gamma))}{100(1-\gamma^2)} - (2 + s)$

6.3 Proofs

Proof of Lemma 1: Note that $\partial\sigma_2/\partial\gamma < 0$, and $\partial\sigma_3/\partial\gamma < 0$. Further, from (8) and (9), we obtain $\sigma_2|_{\gamma=0} = 0.06a^2$, $\sigma_2|_{\gamma \rightarrow 1} = 0.042a^2$, $\sigma_3|_{\gamma=0} = 0.0977a^2$, $\sigma_3|_{\gamma \rightarrow 1} = 0$ and $\sigma_3|_{\gamma=0} > \sigma_2|_{\gamma=0}$ while $\sigma_2|_{\gamma \rightarrow 1} > \sigma_3|_{\gamma \rightarrow 1} = 0$. Therefore σ_2 and σ_3 must cross. Setting (8) and (9) equal we obtain $\gamma^* = 0.2432$ where σ_2 and σ_3 cross. The result then follows immediately. QED

Proof of Proposition 1 Lemma 1 establishes that the relevant critical value for s in the mixed duopoly is $\min\{\sigma_2, \sigma_3\}$; in particular, for $\gamma < \gamma^*$ the relevant critical value is given by σ_2 and for $\gamma \geq \gamma^*$ it is given by σ_3 , $\gamma^* = 0.2432$. Thus, we need to show that $\sigma_2 < \sigma_1$ for $\gamma < \gamma^*$ and $\sigma_3 < \sigma_1$ for $\gamma \geq \gamma^*$. Note that $\partial\sigma_1/\partial\gamma < 0$, $\partial\sigma_2/\partial\gamma < 0$, $\partial\sigma_3/\partial\gamma < 0$. Further, from (7) and (8), we obtain $\sigma_1|_{\gamma=0} = 0.0937a^2$ and $\sigma_2|_{\gamma=0} = 0.06a^2$ respectively. $\sigma_1 = \sigma_2$ at $\gamma = 0.4593 > \gamma^*$ and $\sigma_2|_{\gamma=0} < \sigma_1|_{\gamma=0}$. Therefore, $\sigma_2 < \sigma_1$ when $\gamma < \gamma^*$. Similarly, from (7) and (9) we obtain $\sigma_1|_{\gamma \rightarrow 1} = 0.0221a^2$, and $\sigma_3|_{\gamma \rightarrow 1} = 0$ respectively. $\sigma_1 = \sigma_3$ at $\gamma = 0.0393 < \gamma^*$ and $\sigma_3|_{\gamma \rightarrow 1} < \sigma_1|_{\gamma \rightarrow 1}$. Therefore, $\sigma_3 < \sigma_1$ when $\gamma \geq \gamma^*$ and we have shown that $\min\{\sigma_2, \sigma_3\} < \sigma_1$. The rest of the proposition follows by considering the conditions for the (FMS, FMS) equilibrium, i.e. conditions (7), (8) and (9) QED

Proof of Lemma 2 From (11) and (12), $\sigma_6 - \sigma_5 = \frac{a^2 f_{5,6}(\gamma)}{200(\gamma^2 - 3)^2(\gamma^2 - 1)(8\gamma^2 - 15)^2}$. This is positive as $f_{5,6}(\gamma) < 0$, where $f_{5,6}(\gamma) = -15300 + 66600\gamma - 78135\gamma^2 - 39900\gamma^3 + 111331\gamma^4 - 14380\gamma^5 - 49792\gamma^6 + 13120\gamma^7 + 8496\gamma^8 - 1920\gamma^9 - 512\gamma^{10}$, and the denominator is negative as $\lim_{\gamma \rightarrow 1^-} < 0$. QED

Proof of Lemma 3 Note that $\sigma_4|_{\gamma=0} = 0.0937a^2$, $\sigma_6|_{\gamma=0} = 0.0978a^2$, $\sigma_4|_{\gamma=1} = 0.0246a^2$ and $\lim_{\gamma \rightarrow 1} \sigma_6 = \infty$. Therefore, $\sigma_4|_{\gamma=0} < \sigma_6|_{\gamma=0}$ and $\sigma_4|_{\gamma=1} < \lim_{\gamma \rightarrow 1} \sigma_6 = \infty$. σ_6 reaches its minimum at $\gamma = 0.6689$ while $\sigma_4|_{\gamma=0.6689} = 0.0393a^2$ and $\sigma_6|_{\gamma=0.6689} = 0.0388a^2$, meaning that $\sigma_4|_{\gamma=0.6689} > \sigma_6|_{\gamma=0.6689}$. Hence, σ_4 and σ_6 must cross twice: setting σ_4 and σ_6 equal, we find that they cross at $\gamma_1 = 0.0056$ and at $\gamma_2 = 0.6755$.

The rest of the lemma follows.QED

Proof of Proposition 2 Follows from Lemma 3 and the necessary conditions for equilibrium, i.e. conditions (7), (8) and (9). QED

Proof of Lemma 4 Using (7) and (10) we obtain $\sigma_1 - \sigma_4 = \frac{a^2 \gamma f_{1,4}(\gamma)}{2(3+\gamma)^2(4+3\gamma)^2(24-11\gamma^2)^2}$ where $f_{1,4}(\gamma) = 576 + 168\gamma - 1608\gamma^2 - 488\gamma^3 + 646\gamma^4 - 20\gamma^6 + 81\gamma^7 \geq 0$ for $\gamma \geq \gamma^{**} = 0.6442$. The rest of the lemma follows immediately. QED

Proof of Lemma 5 Note that $\sigma_6 |_{\gamma=0} = 0.1a^2$, $\sigma_2 |_{\gamma=0} = 0.06a^2$, $\sigma_6 |_{\gamma \rightarrow 1} = \infty$, and $\sigma_2 |_{\gamma \rightarrow 1} = 0.042a^2$. Further, $\partial\sigma_2/\partial\gamma < 0$ and $\partial\sigma_6/\partial\gamma \leq 0$ for $\gamma \leq 0.6669$. Setting σ_2 and σ_6 equal, we find that they cross at $\gamma = 0.3133$ and at $\gamma = 0.8172$. It is then obvious that $\sigma_2 < \sigma_6$ when $\gamma \leq 0.3133$ and when $\gamma \geq 0.8172$ and $\sigma_2 > \sigma_6$ when $\gamma \in (0.3133, 0.8172)$. The rest of the lemma follows from the equilibrium conditions. QED

Proof of Lemma 6 $\partial\sigma_3/\partial\gamma < 0$ and $\partial\sigma_5/\partial\gamma < 0$. Furthermore, $\sigma_3 |_{\gamma=0} = 0.0977a^2$, $\sigma_3 |_{\gamma \rightarrow 1} = 0$, $\sigma_5 |_{\gamma=0} = 0.06a^2$ and $\sigma_5 |_{\gamma \rightarrow 1} = 0.0083a^2$, so that $\sigma_3 |_{\gamma=0} > \sigma_5 |_{\gamma=0} = 0.06a^2$ while $\sigma_3 |_{\gamma \rightarrow 1} = 0 < \sigma_5 |_{\gamma \rightarrow 1}$. Therefore, σ_5 and σ_3 cross at a critical value of the parameter of substitutability, $\gamma^{***} = 0.3133$. Thus, if $\gamma \leq \gamma^{***}$, $\sigma_5 \leq \sigma_3$ and if $\gamma > \gamma^{***}$, $\sigma_5 > \sigma_3$. The rest of the lemma follows from equilibrium conditions (9) and (11).QED

Proof of Proposition 3 As shown in lemma 4, for (DE, FMS) or (FMS, DE) to be equilibria in the private duopoly $\sigma_1 < s < \sigma_4$ must hold; this can only happen for $\gamma > \gamma^{**} = 0.644205$. Recall that (DE, FMS) is an equilibrium in the mixed duopoly if $\sigma_2 < s < \sigma_6$. We know that $\partial\sigma_2/\partial\gamma < 0$ and $\partial\sigma_4/\partial\gamma < 0$ and that $\sigma_2 |_{\gamma=0} = 0.06a^2$, $\sigma_2 |_{\gamma \rightarrow 1} = 0.042a^2$, $\sigma_4 |_{\gamma=0} = 0.9375a^2$, $\sigma_4 |_{\gamma \rightarrow 1} = 0.02459a^2$. Therefore, $\sigma_2 |_{\gamma=0} < \sigma_4 |_{\gamma=0}$ while $\sigma_2 |_{\gamma \rightarrow 1} > \sigma_4 |_{\gamma \rightarrow 1}$. Thus, they must cross at a certain value of γ . Setting σ_2 and σ_4 equal, we know that $\sigma_2 \leq \sigma_4$ for $\gamma \leq 0.450595$. Therefore for $\gamma > \gamma^{**}$, $\sigma_2 > \sigma_4$, implying that $\sigma_1 < s < \sigma_4$ and $\sigma_2 < s < \sigma_6$ can not hold simultaneously. Furthermore, recall that (FMS, DE) is an equilibrium in the mixed duopoly if $\sigma_3 < s < \sigma_5$. We know that $\partial\sigma_1/\partial\gamma < 0$ and $\partial\sigma_5/\partial\gamma < 0$ and that $\sigma_1 |_{\gamma=0} = 0.09375a^2$, $\sigma_5 |_{\gamma=0} = 0.06a^2$, $\sigma_1 |_{\gamma \rightarrow 1} = 0.06a^2$ and $\sigma_5 |_{\gamma \rightarrow 1} = 0.009328a^2$. Thus, $\sigma_1 > \sigma_5$ for any γ and therefore $\sigma_1 < s < \sigma_4$

and $\sigma_3 < s < \sigma_5$. The rest of the proposition follows.QED

Proof of Lemma 7 $TS_{P_1} - TS_{M_1} = \frac{2a^2 f_{P_1 M_1}(\gamma)}{(1+\gamma)^2(5+2\gamma)^2(4+3\gamma)^2}$ where $f_{P_1, M_1}(\gamma) = -3 + 128\gamma + 277\gamma^2 + 209\gamma^3 + 67\gamma^4 + 8\gamma^5 \lesseqgtr 0$ for $\gamma \lesseqgtr 0.0223$. Hence, $TS_{P_1} \geq TS_{M_1}$ if $\gamma \geq 0.0223$ and $TS_{P_1} < TS_{M_1}$ if $\gamma < 0.0223$.QED

Proof of Lemma 8 $TS_{P_4} - TS_{M_4} = \frac{a^2(1-\gamma)^2 f_{P_4 M_4}(\gamma)}{4(3+\gamma)^2(3-\gamma^2)^2}$ where $f_{P_4, S_4}(\gamma) = (-9 + 6\gamma + \gamma^2 - 2\gamma^3) < 0$ for any γ . Hence $TS_{P_4} < TS_{M_4}$. QED

Proof of Lemma 9 From Lemma 4, $\sigma_1 < \sigma_4$ if and only if $\gamma > \gamma^{**} = 0.6442$. Further, from Lemma 3, $\sigma_6 - \sigma_4 < 0$ if and only if $0.0056 < \gamma < 0.6755$. Hence the relevant range for γ is $\gamma \in (0.6442, 0.6755)$. It can be checked that the difference $TS_{P_2} - TS_{M_4}$ is decreasing in s and $TS_{P_2} - TS_{M_4} |_{s=\sigma_1} = \frac{a^2 f_{P_2 M_4}(\gamma)}{4(4+3\gamma)^2(3-\gamma^2)^2(24-11\gamma^2)^2} > 0$ as $f_{P_2 M_4}(\gamma) = -10368 - 4032\gamma + 30600\gamma^2 + 9816\gamma^3 - 29466\gamma^4 - 6772\gamma^5 + 12203\gamma^6 + 1670\gamma^7 - 2041\gamma^8 - 110\gamma^9 + 72\gamma^{10} > 0$ for $\gamma \in (0.6442, 0.6755)$. Note also that in this region of γ , $\sigma_1 > \sigma_6$. Then, given that $s \geq \sigma_1$, it follows that $TS_{P_2} < TS_{M_4}$.QED

Proof of Lemma 10 From (7) and (12) $\sigma_1 - \sigma_6 = \frac{a^2 f_{1,6}(\gamma)}{50(4+3\gamma)^2(3-\gamma^2)^2(24-11\gamma^2)^2(1-\gamma^2)}$ and $sign(\sigma_1 - \sigma_6) = sign f_{1,6}(\gamma)$, where $f_{1,6}(\gamma) = -16704 + 332064\gamma - 343356\gamma^2 - 744420\gamma^3 + 706663\gamma^4 + 634292\gamma^5 - 531705\gamma^6 - 252133\gamma^7 + 180629\gamma^8 + 44928\gamma^9 - 24779\gamma^{10} - 2563\gamma^{11} + 522\gamma^{12}$. Note that $f_{1,6}(\gamma) > 0$ for $\gamma \in (0.0536, 0.6736)$, which is the relevant range for γ . It can be checked that the difference $TS_{P_1} - TS_{P_4}$ is decreasing in s and $TS_{P_1} - TS_{M_4} |_{s=\sigma_6} = \frac{a^2 f_{P_1 M_4}(\gamma)}{100(4+3\gamma)^2(3-\gamma^2)^2(1-\gamma^2)} < 0$ as $f_{P_1 M_4}(\gamma) = 616 - 856\gamma + 297\gamma^2 + 662\gamma^3 - 122\gamma^4 - 56\gamma^5 - 183\gamma^6 - 38\gamma^7 + 72\gamma^8 < 0$. Then, given that $s > \sigma_6$ it follows that, in the relevant region of γ , $TS_{P_1} - TS_{M_4} < 0$.QED.

Proof of Lemma 11 From Lemma 3, $\sigma_4 < \sigma_6$ if and only if $\gamma \notin (0.0056, 0.8173)$ and from Lemma 5, $\sigma_2 < \sigma_6$ if and only if $\gamma \notin (0.3133, 0.8173)$ so the relevant range for γ is $\gamma \notin (0.0056, 0.8173)$. It can be checked that the difference $TS_{P_4} - TS_{M_2}$ is increasing in s ; further

$TS_{P_2} - TS_{M_4} |_{s=\sigma_6} = \frac{a^2 f_{P_4 M_2}(\gamma)}{100(3+\gamma)^2(3-\gamma^2)^2(1-\gamma^2)^2} < 0$ as $f_{P_4 M_2}(\gamma) = -225 + 600\gamma - 4667\gamma^2 + 632\gamma^3 + 6381\gamma^4 - 414\gamma^5 - 2901\gamma^6 + 132\gamma^7 + 416\gamma^8 - 14\gamma^9 - 4\gamma^{10} < 0$. Then, given that $s < \sigma_6$ it follows that, in the relevant region of

γ , $TS_{P_2} < TS_{M_4}$. QED

Proof of Lemma 12 From Lemma 5, $\sigma_2 < \sigma_6$ if and only if $\gamma \notin (0.3133, 0.8173)$. Further, from the proof of Proposition 1, $\sigma_2 < \sigma_1$ if and only if $\gamma < 0.4593$. Therefore the relevant range for γ is $\gamma \notin (0.3133, 0.8173)$. It can be checked that the difference $TS_{P_1} - TS_{M_2}$ is decreasing in s . Then $TS_{P_1} - TS_{M_2} |_{s=\sigma_2} = \frac{a^2 f_{P_1 M_2}(\gamma)}{100(5+2\gamma)^2(4+3\gamma)^2(1-\gamma^2)^2} < 0$ as $f_{P_1 M_2}(\gamma) = -200 - 3320\gamma - 26319\gamma^2 - 22056\gamma^3 + 16020\gamma^4 + 23264\gamma^5 + 8538\gamma^6 + 1456\gamma^7 + 216\gamma^8 < 0$. Hence, given that $s > \sigma_2$ it follows that, in the relevant range for γ , $TS_{P_1} < TS_{M_2}$. QED

Proof of Lemma 13 It can be checked that the difference $TS_{P_1} - TS_{M_3}$ is decreasing in s . Then, $TS_{P_1} - TS_{M_3} |_{s=\sigma_3} = \frac{-a^2 f_{P_1 M_3}(\gamma)}{(1+\gamma)(5+2\gamma)^2(4+3\gamma)^2} < 0$ as $f_{P_1 M_3}(\gamma) = 3 - 3\gamma - 2\gamma^2 + \gamma^3 + \gamma^4 > 0$. Hence, given that $s > \sigma_3$ it follows that $TS_{P_1} < TS_{M_3}$. QED

Proof of Proposition 4 Follows from lemmata 7-13. QED

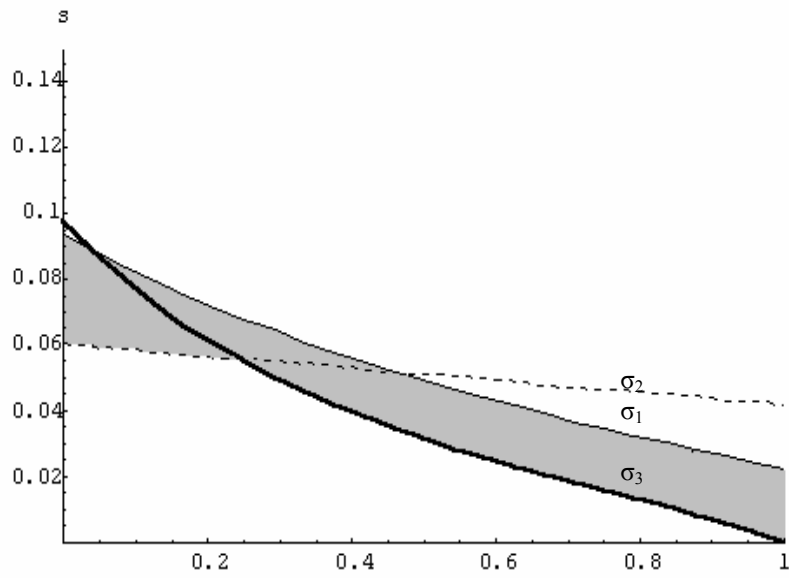


Figure 1: (FMS, FMS) Equilibrium

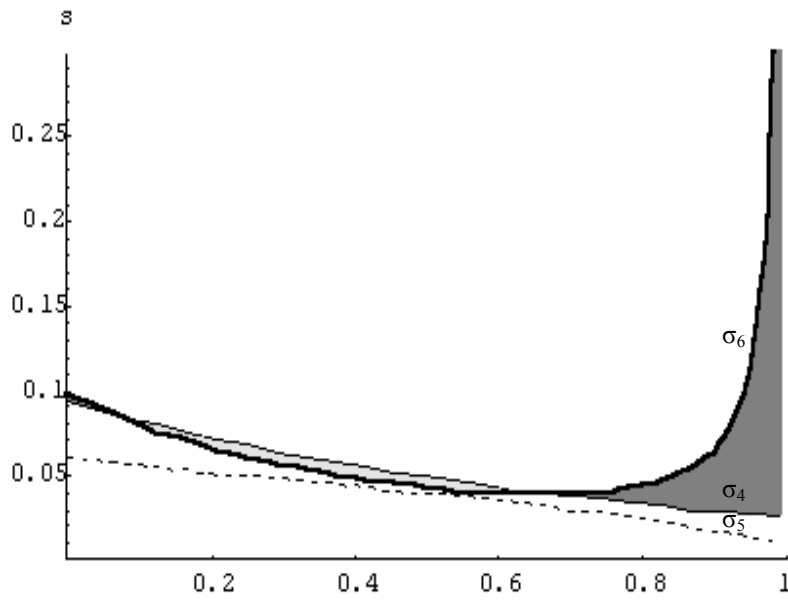


Figure 2: (DE, DE) Equilibrium