Switching and symmetry breaking behaviour of discrete breathers in Josephson ladders

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We investigate the roto-breathers recently observed in experiments on Josephson ladders subjected to a uniform transverse bias current. We describe the switching mechanism in which the number of rotating junctions increases. In the region close to switching we find that frequency locking, period doubling, quasi-periodic behaviour and symmetry breaking all occur. This suggests that a chaotic dynamic occurs in the switching process. Close to switching the induced flux increases sharply and clearly plays an important role in the switching mechanism. We also find three critical frequencies which are independent of the dissipation constant α*, provided that* α *is not too large.*

The recent discovery[1, 2] of discrete breathers in Josephson ladder arrays (Fig. 1) driven by a transverse dc bias current has shown not only that these localised excitations exist but that they exhibit remarkable behaviour, which is the subject of this paper.

The Josephson ladder has been studied theoretically for many years. In the absence of a driving current the interaction between vortices has been found to be exponential and this leads to the vortex density exhibiting a devil's staircase as the magnetic field is increased[3, 4]. Quantum fluctuations[5], meta-stable states[6] and inductance effects[7] have been studied as has the interaction of vortices with a transverse dc bias current[8, 9].

In the case of a Josephson ladder, a roto-breather is a stable group of vertical junctions rotating together ($\theta'_j - \theta_j \approx \omega t$, see Fig. 1). Until recently, the interest has focussed on the case of a sinusoidal bias current, where it has been shown that roto-breather solutions exist and that a discrete breather may even include a chaotic trajectory[10].

Roto-breathers should also be stable[11] in the experimentally easier case of a ladder array subjected to a uniform dc transverse driving current. The case of rotation occurring at a single vertical junction has been studied numerically[12, 13]. Two experimental groups[1, 2] have now independently confirmed the existence of such solutions and discovered further unpredicted behaviour. In an annular ladder[1] (Fig. 1a) and in a linear ladder[2] (Fig. 1b) breathers were observed with various numbers of "vertical" (i.e. radial) junctions rotating. However the most interesting features of the observations were the switching between breathers and their *symmetry breaking* behaviour. Once a breather has been initialised it is stable, but if the bias current is slowly decreased then, at some critical value of the current, the number of rotating junctions switches to a larger number and a new breather is formed. Furthermore, *the new breather may have a different centre of symmetry from the old breather despite the symmetry of the ladder itself and the symmetry of the driving currents about a single vertical junction*; this is particularly remarkable in the case of the annular ladder where translational invariance should be exact. The production of breathers with an even number of rotating junctions is, by itself, a demonstration of symmetry breaking. The symmetry breaking effect was not mentioned by the experimenters, presumably because it could be due to experimental imperfections. However we show that it should also arise in a perfect experiment, the apparent mechanism being the occurrence of chaotic dynamics in the switching region.

First we construct a new model, similar to but significantly different from those proposed previously[12, 13, 14, 10]. The current through a junction is determined by the RCSJ model

$$\frac{I}{I_c} = \frac{d^2\varphi}{dt^2} + \alpha\frac{d\varphi}{dt} + \sin\varphi \tag{1}$$

where I_c is the critical current and

$$\varphi = \Delta\theta - \frac{2\pi}{\Phi_0}\int \mathbf{A}.\mathrm{d}\mathbf{l} \tag{2}$$

where $\Delta\theta$ is the change in superconducting order parameter θ across the junction and $\mathbf{A}$ is the vector potential. The "vertical" (i.e. radial) junctions may differ in area from the "horizontal" junctions by an anisotropy parameter $\eta = I_{ch}/I_{cv} = C_h/C_v = R_v/R_h$ where I_{ch} (I_{cv}), C_h (C_v) and R_h (R_v) are, respectively, the critical current, capacitance and resistance of a horizontal (vertical) junction. From Fig. 1a we see that there are three unknowns per plaquette: θ_j, θ'_j and f_j, where f_j is the total flux threading the jth plaquette. To solve for these unknowns we construct three equations per plaquette as follows. The first two equations are obtained from current conservation at the top (inner) and bottom (outer) rails. The third equation is obtained by making the approximation that the induced flux $f_j - f_a$ (where f_a is the applied flux) is produced solely by the currents flowing around the immediate perimeter of the jth plaquette (Fig. 1c):

$$\frac{f_j - f_a}{\Phi_0} = \frac{\beta_L}{8\pi}\left(I^v_j + I^h_j - I^v_{j+1} - I'^h_j\right) \tag{3}$$

where β_L is an inductance parameter. This last equation makes the plausible assumption that the plaquettes are more or less square and that it is only the currents flowing around the plaquette that produce the induced field. This differs from, and for square plaquettes is more accurate than, the common assumption[13, 12] that the induced field is proportional to the loop (or "mesh") current circulating the plaquette.

We impose the initial conditions, mimicing the experiments, that at $t = 0$, $\theta_j = \theta'_j = 0$, $f_j = 0$ and $I_j = 0$ for all j. This means that $\theta_j + \theta'_j = 0$ for all time, i.e. there is a symmetry between the inner and outer rails. Using Landau gauge we then have the following pair of coupled differential equations for each plaquette:

$$\left\{\frac{d}{dt^2} + \alpha\frac{d}{dt}\right\}\left(-\theta^-_{j-1} + 4\theta^-_j - \theta^-_{j+1}\right) = 2I_j + \sin\theta^-_{j-1} \\ -4\sin\theta^-_j + \sin\theta^-_{j+1} + \frac{8\pi}{\beta_L}(f_{j-1} - f_j) \tag{4}$$

$$\left\{\frac{d}{dt^2} + \alpha\frac{d}{dt}\right\}\left(-2\pi\eta f_j + (1-\eta)(\theta^-_{j+1} - \theta^-_j)\right) = \sin\theta^-_j \\ -\sin\theta^-_{j+1} + \frac{8\pi}{\beta_L}(f_j - f_a) + 2\eta\sin\chi^-_j \tag{5}$$

where $\theta^-_j = \theta'_j - \theta_j$ and $\chi^-_j = \frac{1}{2}(\theta^-_{j+1} - \theta^-_j + 2\pi f_j)$.

We now focus on determining whether or not our model exhibits the interesting switching and symmetry breaking behaviour observed in the data of Binder *et al*[1]. All parameter values are chosen to mimic the experimental setup i.e. $\eta = 0.44$, $\beta_L = 2.7$, $\alpha = 0.07$ and $f_a = 0$. Let

$$I_j = \begin{cases} I_B + I_\Delta & \text{if } j = 0 \\ I_B & \text{otherwise} \end{cases} \tag{6}$$

where I_B is called the bias current. Again following the experiment (Fig. 1a), we slowly increase I_Δ while keeping $I_B = 0$ until rotation starts at site $j = 0$. I_Δ is then slowly decreased while at the same time increasing I_B to keep I_0 constant. When I_B has reached the desired value it is then held fixed while I_Δ is slowly reduced to zero. Finally I_B is slowly reduced while keeping $I_\Delta = 0$. Note that the dynamical equations, initial conditions and injected currents are exactly symmetrical about site $j = 0$. One would expect only solutions which are also symmetrical about $j = 0$. However, like the experiments, the simulations also produce breathers which are *not symmetrical* about $j = 0$. Fig. 2 shows how the number N_R of rotating junctions changes as the bias current I_B is slowly reduced from various starting values. In agreement with experiment, breathers with *even* N_R are commonly produced (these *cannot* be symmetrical about $j = 0$), and N_R switches to larger and larger values until eventually all junctions are rotating. While we have found that similar behaviour occurs in a previously published model[13, 12], our model gives considerably better agreement with the experimentally observed switching currents, thus indicating the importance of inductance effects in the switching mechanism. We believe the origin of the symmetry breaking is the occurrence of chaotic dynamics in the switching region.

The chaotic nature of the switching is further suggested by the fact that when the same computer code which produced Fig. 2 is performed on a different computer (different floating point processor) we see significant changes in the switching behaviour of nearly all trajectories although the overall pattern remains identical. The minute differences in the handling of floating point numbers are amplified in the chaotic regime to produce significantly altered trajectories.

Fig. 3 shows the voltage-current characteristics obtained from the same simulations used in producing Fig. 2. At high frequencies ($V = \langle d\theta^-/dt\rangle > 4.3$) most of the breathers show purely resistive behaviour (i.e. $V = \langle d\theta^-/dt\rangle = I_B R$), the value of the resistance R increasing with N_R according to an expression deduced from experiment[1]:

$$\alpha R \approx 1/(1+\eta/N_R) \qquad (7)$$

This is the relationship expected if the current through each rotating vertical junction is αV and the four horizontal junctions surrounding the rotating region each carry current $\frac{1}{2}\eta\alpha V$ (to satisfy Kirchoff's rule). For $N_R = 1$ this expression was derived in ref. [13]. We find that no such resistive behaviour occurs for $V < 4.3$.

A typical example of a resistive breather (far away from any switching region) is shown in Fig. 4. Although the breather is not symmetric about $j = 0$ it shows exact symmetry about the midpoint of the rotating region and also appears to be exactly periodic, the period being two revolutions of a vertical junction (we call this "period 2"). The non-rotating junctions oscillate, the amplitude decreasing exponentially with distance from the rotating region. Far away from switching the magnitude of the flux f_j is everywhere small (< 0.1) and is limited to the edges of the rotating region.

Fig. 3 shows that there are at least two critical frequencies, $\langle d\theta^-/dt\rangle = 6.5$ and $\langle d\theta^-/dt\rangle = 5.0$, at which breathers become unstable and switching occurs. As the current is reduced the frequency falls until it reaches the critical value, at which point the breather becomes unstable and switches to a larger breather with a larger resistance (Eq. (7)) and therefore a higher frequency. The process then repeats as the current continues to be ramped down. We identify the larger of these two critical frequencies with that observed experimentally[1]. Although, at fixed I_B, the frequency of rotation depends on the dissipation constant α, we find that these two critical frequencies are more or less independent of α, as is the critical frequency below which breathers cease to show constant resistance. This of course breaks down when the bias current required to achieve the upper critical frequency exceeds unity, i.e. when $\alpha > 0.15$.

Note also that in the above simulations: (i) no breathers are seen for $I_B > 0.85$, (ii) only the single site breather is seen for $0.68 < I_B < 0.85$ and (iii) all rotating junctions normally stop together when the current I_B is reduced below 0.17, although sometimes a new breather containing only a few rotating junctions is produced which is then destroyed when I_B falls below 0.14.

As well as resistive period 2 breathers we have found *frequency-locked* breathers. In fact the usual $N_R = 1$ resistive breather makes a transition to a frequency-locked breather as the bias current is reduced below $I_B = 0.699$. At this point the period doubles, and the breather becomes *asymmetric* and jumps to a higher rotation frequency $\langle d\theta^-/dt\rangle = 6.32$. At $I_B = 0.6895$ the period doubles again and at $I_B = 0.6892$ it becomes quasi-periodic (Fig. 5). Switching to multi-site breathers occurs at $I_B \approx 0.6891$. Such frequency locked breathers have not been reported experimentally but have probably been overlooked as they are only stable in narrow current intervals.

While the maximum flux f^{max} is normally small ($f^{max} < 0.1$), it sharply increase at switching to $f^{max} \sim 1$. It would appear that an important part of the switching mechanism is the instability caused by having a large flux concentrated in a small area. The importance of the flux in the switching process is further confirmed by the fact that when I_B has finally been reduced all the way to zero we find that the ladder may contain one or more *stable vortex-antivortex pairs.*

The occurrence of frequency locking, period doubling, quasi-periodic behaviour and symmetry breaking as switching is approached strongly suggests that the origin of the symmetry breaking is the occurrence of chaotic dynamics in the switching region. From the theory of chaotic dynamics it is known that two or more coupled Josephson junctions (or, equivalently, two or more coupled pendula) may exhibit three types of motion: oscillatory, rotational and chaotic. Chaotic motion arises for particular initial conditions. An analogous situation arises in non-linear coupled lattices which also display both breathers and chaotic dynamics[15]. In a large array of coupled Josephson junctions (or, analagously, an array of coupled pendula) we must also expect these three types of motion. While roto-breathers are the most characteristic stable solutions it appears that the switching between breathers occurs only in the chaotic regime. Of course, the chaotic dynamics has an influence on the selection of breather type. Any small difference in the initial conditions leads to a completely different breather. Thus, in this chaotic regime, any small experimental imperfections or small inaccuracies in a computer simulation may lead to different chaotic trajectories and hence different roto-breathers. Symmetry breaking arises if the perturbation itself breaks symmetry.

We conclude that our model exhibits most of the main features [16] of the roto-breathers recently observed in Josephson ladder arrays, sheds light on the switching mechanism and predicts further observable behaviour. The occurrence of frequency locking, period doubling, quasi-periodic behaviour and symmetry breaking in the switching region suggests that switching occurs in the chaotic regime. We find that the maximum flux increases sharply in the switching region and that the flux clearly plays an important role in the switching mechanism. We also find two critical switching frequencies and a critical frequency below which no resistive behaviour is observed. All three frequencies are approximately independent of the dissipation constant α (for $\alpha < 0.15$),

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[16] Ref. [1] finds that breathers with even N_R are most common in the annular ladder. In agreement with this we find that, when switching, breathers with $N_R = 2$ or $N_R = 4$ have a strong tendency to grow symmetrically thus maintaining even N_R. No such effect is seen for breathers with $N_R = 1$ or $N_R = 3$. This is strongly suggestive of a parity effect, i.e. a difference in behaviour according to whether N_R is even or odd.

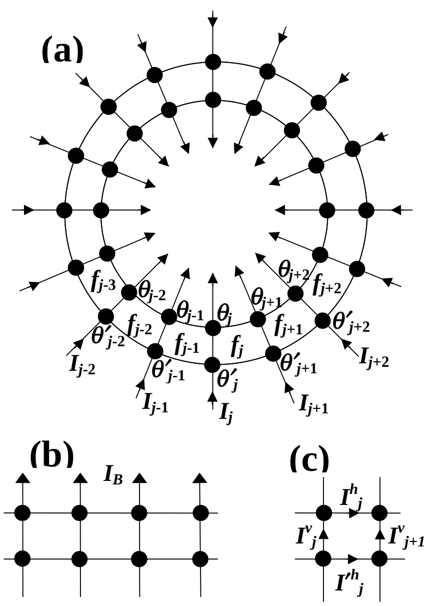

Figure 1: (a) An annular ladder subjected to a uniform transverse bias current I_B. Each circle represents a superconducting island. Each link between islands represents a Josephson junction. θ is the phase of the superconducting order parameter and f is the flux threading a plaquette. (b) A linear ladder. (c) Explanation of the notation used in Eq. (3).

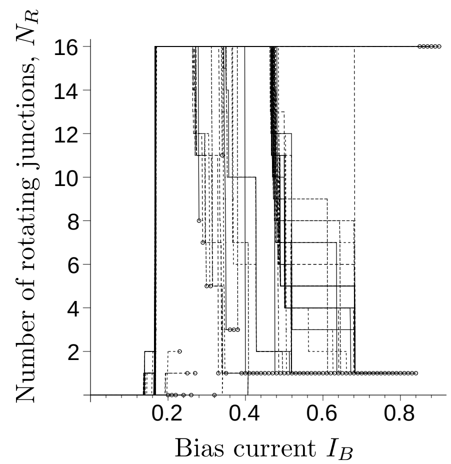

Figure 2: Result of smoothly ramping down the bias current. Each small circle marks the start of a new ramp.

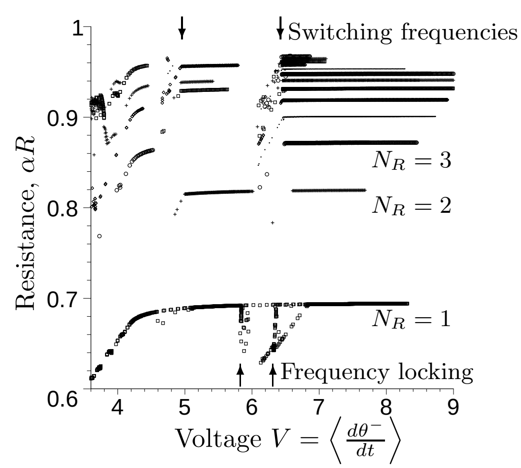

Figure 3: Resistance αR (where $R = V/I_B$) plotted against voltage $V = \langle d\theta^-/dt \rangle$ for the data shown in Fig. 2. Note the critical switching frequencies and frequency locked states.

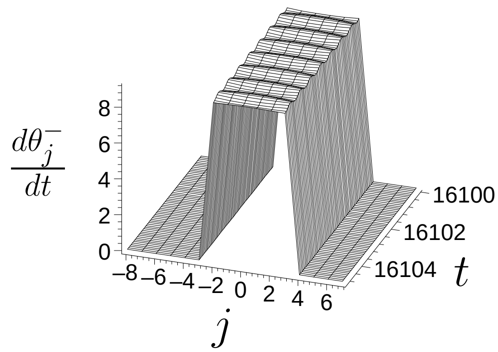

Figure 4: An example of a resistive breather far from switching. $d\theta_j^-/dt$ is plotted against t for all j. Note that six junctions ($-2 \leq j \leq 3$) are rotating and that the breather is *not symmetrical* about the injection site $j = 0$. The breather is periodic and exactly symmetrical about its centre. The peak flux (not shown) is small (~ 0.06).

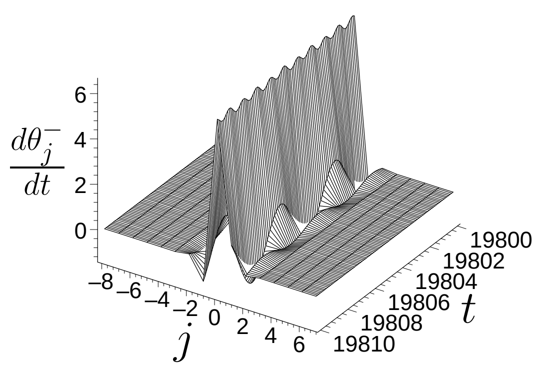

Figure 5: The quasi-periodic breather (approximately period 8) which occurs near $I_B = 0.6892$, just above the bias current at which switching to multi-site breathers occurs. It is frequency-locked at $\langle d\theta^-/dt \rangle = 6.32$. Note that $d\theta_j^-/dt$ is nearly anti-symmetric about $j = 0$. The maximum flux (not shown) is large (0.5) and increases still further at switching.