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ESTIMATING THE PERMANENT AND TRANSITORY COMPONENTS OF THE U.K. BUSINESS CYCLE

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ABSTRACT

We estimate a model that incorporates two key features of business cycles, comovement among economic variables and switching between regimes of boom and slump, to quarterly U.K. data for the last four decades. Common permanent and transitory factors, interpreted as composite indicators of coincident variables, and estimates of turning points from one regime to the other, are extracted from the data by using the Kalman filter and maximum likelihood estimation. Both comovement and regime switching are found to be important features of the U.K. business cycle. The components produce sensible representation of the cycles and the estimated turning points agree fairly well with independently determined chronologies.

Keywords: MARKOV CHAINS, REGIME SWITCHING, BUSINESS CYCLES

J.E.L. Classification: C5, E3, O4

1. INTRODUCTION

The two empirical regularities of business cycles highlighted by Burns and Mitchell (1946) - comovement among economic variables through the cycle and asymmetry in the evolution of the cycle - have undergone a resurgence of interest in recent years, prompted by the development of new time series techniques. Two of the most influential models of the business cycle are Stock and Watson's (1989, 1991, 1993, 1999) linear common factor model and Hamilton's (1989) regime switching model. Stock and Watson develop a linear dynamic factor model where business cycles are measured by comovements in various components of economic activity. Using several macroeconomic time series, they extract a single unobserved variable and interpret it as the "state of the economy". They then compare this variable with the U.S. Department of Commerce (DOC) composite index, and find that the similarity between the two is striking, especially over the business-cycle horizon. The disadvantage of their model, however, is that its linearity cannot capture business cycle asymmetry, and forces expansions and contractions to have the same amplitude and duration.

To capture such asymmetry, Hamilton (1989) develops a regime switching model in which output growth switches between two states according to a first order Markov process. Expansions can therefore be gradual and drawn out while recessions may be shorter and steeper - the 'stylised facts' of modern business cycles. Applying this model to the U.S., he shows that shifts between positive and negative output growth accord well with the NBER's chronology of business cycle peaks and troughs. Being based on a single time series, however, Hamilton's model cannot capture the notion of economic fluctuations corresponding to comovements of many aggregate and sectoral variables. It may well be impossible for only one coincident variable to capture all underlying business cycle information, which is the conclusion of both Filardo (1994) and Diebold and Rudebusch (1996).

Indeed, Diebold and Rudebusch provide both empirical and theoretical support for combining these two key features of the business cycle, although they do not fully estimate a model. Building on this research, however, several studies do estimate these two features simultaneously within the regime switching common factor model: for example, Chauvet (1998), Kim and Yoo (1995), and Kim and Nelson (1998), using U.S. data, and Mills and Wang (2001a) using U.K. data. The common factor is defined to be an unobserved variable that summarises the common cyclical movements of a set of coincident macroeconomic variables, as in Stock and Watson (1991). However, it is also subject to discrete shifts so that it can capture the asymmetric nature of business cycle phases, as in Hamilton (1989). Within a multivariate framework, all papers report that inferences about the state of the

economy obtained from the model exhibit significantly higher correlations with the NBER reference dates than if just a single variable, such as output growth, was used.

The basic idea behind these studies is that information about business cycles can be extracted from a group of series rather than a single series, so that estimated business cycles reflect information from various economic sectors. Furthermore, the extracted factor can be compared with, for example, the DOC coincidence index, and more importantly, it can be used for real time assessment of the economy.

One problem associated with this framework is that the models cannot capture the peak-reversion feature, or 'plucking behaviour', of business cycle movements, first suggested by Milton Friedman more than 30 years ago. Friedman (1964, 1993) pointed out that the amplitude of cyclical contractions in U.S. output tended to be strongly correlated with succeeding expansions, but that these expansions were uncorrelated with the amplitude of subsequent contractions, thus producing an asymmetry between succeeding phases of the business cycle. Friedman (1993) provides some basic statistical evidence to support the plucking model.

Kim and Nelson (1999) propose a framework that enables both asymmetry and an output ceiling to be captured within a single model containing shifts in regime. Their nonlinear model incorporates asymmetric movements of output away from trend and asymmetric persistence of shocks during recessionary and normal times. This framework is able to estimate the importance of downward shocks to both trend and cycle, and to test the plucking hypothesis against symmetric trend-plus-cycle alternatives such as Clark (1987). Kim and Nelson show that the stochastic behaviour of U.S. output is well characterised by Friedman's plucking model, i.e., output is occasionally plucked down by recession and the cyclical or transitory component exhibits asymmetric behaviour. Mills and Wang (2001b) further extend the analysis to the G7 countries and find a variety of results.

While Kim and Nelson (1999) focus only on the business cycle of a single variable, that of output, in this paper we generalise the Diebold and Rudebusch (1996) model to include a common transitory component. This common transitory component is subject to the type of regime switching advocated by Kim and Nelson (1999). We think that this extension is important for three related reasons. First, if a set of indicators can correctly provide signals of changes in aggregate economic activity, then this would be helpful to any business or government in their decision making, as they are typically affected by economic expansions and contractions. Second, in studying aggregate fluctuations like business cycles, it is useful to be able to analyse a group of important economic time series. Individual series measure only one aspect of economic activity, so they cannot capture the idea of cyclical fluctuations corresponding to comovements of many aggregate and sectoral variables.

Third, if there is a transitory component that plucks the economy down then, as suggested by Sichel (1994), there may exist a high-growth recovery phase following a recession. Knowledge of these features for the U.K. economy is therefore important for both policy makers and forecasters.

The rest of the paper is organised as follows. In section 2 we illustrate the multivariate dynamic factor model which incorporates independent regime switching of permanent and transitory components. The data sets used in our analysis are introduced in Section 3, where we also report the empirical results of our modelling exercises. Section 4 draws implications and concludes.

2. MODEL SPECIFICATION

Suppose that Y_{it} is (the logarithm of) a macroeconomic variable that moves contemporaneously with overall economic conditions. It can be modelled as consisting of three stochastic autoregressive processes - a single unobserved permanent component, C_t, a single unobserved transitory component, x_t, and an idiosyncratic component, z_t. Defining $\Delta y_{it} = \Delta\left(Y_{it} - \overline{Y}_i\right)$, where $\overline{Y}_i$ is the sample mean of Y_{it}, the model can be written as follows,[1]

$$\Delta y_{it} = \gamma_i \Delta c_t + \beta_i \Delta x_t + z_{it}, \qquad i = 1, \ldots, n, \qquad (1)$$

Δc_t is the demeaned growth rate of the common permanent component, which is dependent on whether the economy is in expansion or recession, and it enters each of the n equations with a different weight γ_i, which measures the sensitivity of the ith variable to the common permanent component. Similarly, the factor loadings β_i indicate the extent to which each series is affected by the common transitory component, Δx_t. The variables z_{it} are idiosyncratic terms.

To incorporate the asymmetry of business cycles, the common permanent component is assumed to be generated by a Markov switching process of the type proposed by Hamilton (1989), so that

$$\phi(L)\left(\Delta c_t - \mu_{S_{1t}}\right) = v_t, \qquad v_t \sim i.i.d. \, N(0,1), \qquad (2)$$

$$\mu_{S_{1t}} = \mu_0\left(1 - S_{1t}\right) + \mu_1 S_{1t}$$

where S_{1t} is an unobservable state variable that switches between state 1 (recession) and state 0 (expansion) with transition probabilities governed by the first-order Markov switching process

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[1] Writing the model in deviations from means allows identification of the model: see Kim and Nelson (1998).

$$P\left[S_{1t}=0 \mid S_{1,t-1}=0\right]=p_1$$
$$P\left[S_{1t}=1 \mid S_{1,t-1}=1\right]=q_1$$

To capture peak-reversion behaviour, the common transitory component is subject to the type of regime switching advocated by Kim and Nelson (1999):

$$\phi^*(L)x_t = \mu_{S_{2t}} + u_t, \qquad u_t \sim i.i.d.\, N(0,1), \tag{3}$$

$$\mu_{S_{2t}} = \tau S_{2t}, \quad \tau \leq 0,$$

Here $\mu_{S_{2t}}$ is an asymmetric discrete shock, which is dependent upon an unobserved variable, S_{2t}, an indicator variable that switches between state 1 (recession) and state 0 (expansion) and also evolves according to a first-order Markov-switching process:

$$P[S_{2t}=0 \mid S_{2,t-1}=0]=p_2$$
$$P[S_{2t}=1 \mid S_{2,t-1}=1]=q_2$$

where S_{2t} is independent of S_{1t}. During 'normal times', $S_{2t}=0$ and the economy is near to potential or trend output. During 'recessions', however, $S_{2t}=1$ and the economy is hit by a transitory shock and plucked down by the size of τ ($\tau \leq 0$). The idiosyncratic component is assumed to have the following autoregressive representation,

$$\psi_i(L)z_{it} = \varepsilon_{it}, \qquad \varepsilon_{it} \sim i.i.d.\, N(0,\sigma_i^2) \tag{4}$$

The innovations ε_{it} can be thought of as measurement errors, while v_t and u_t are the innovations to the common permanent and transitory components, respectively. The functions $\psi_i(L)$, $\phi(L)$ and $\phi^*(L)$ are polynomials in the lag operator L and $\Delta = 1 - L$. For the identification of the model, it is assumed that the variances of v_t and u_t are unity. The innovations v_t, u_t and ε_{it} are assumed to be independent for all t and i.

The model can be thought of as a generalised dynamic factor model, and has been estimated by Kim and Murray (2001) using U.S. data. With appropriate restrictions, it can reproduce many of the models that have appeared in the literature. For example, if $n=1$, it is a univariate model and with $\tau=0$, we have Hamilton's (1989) model. With $\psi_1=0$, we have Kim and Nelson's (1999) model. If $n \geq 1$, it becomes a multivariate model. In the absence of equation (3), we have the Diebold and Rudebusch (1996) model that has been estimated by Chauvet (1998), Kim and Yoo (1995), and Kim and Nelson (1998) for the U.S., and by Mills and Wang (2001b)

for the U.K. On the other hand, in the absence of equations (2) and (3), we have the Stock and Watson (1989, 1991, 1993) linear dynamic factor model.

To facilitate estimation, the model can be given a state-space representation. With AR(1) processes for the common permanent and transitory components and the idiosyncratic term, and with $n = 4$ (as in the application below), the model can be expressed as the measurement and transition equations

Measurement equation

$$
\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \end{bmatrix} = \begin{bmatrix} \gamma_1 & \beta_1 & \beta_1 & 1 & 0 & 0 & 0 \\ \gamma_2 & \beta_2 & \beta_2 & 0 & 1 & 0 & 0 \\ \gamma_3 & \beta_3 & \beta_3 & 0 & 0 & 1 & 0 \\ \gamma_4 & \beta_4 & \beta_4 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_t \\ x_t \\ x_{t-1} \\ z_{1t} \\ z_{2t} \\ z_{3t} \\ z_{4t} \end{bmatrix} = H \xi_t
$$

Transition equation

$$
\begin{bmatrix} c_t \\ x_t \\ x_{t-1} \\ z_{1t} \\ z_{2t} \\ z_{3t} \\ z_{4t} \end{bmatrix} = \begin{bmatrix} s_{1t} \\ s_{2t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \phi_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_1^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \psi_4 \end{bmatrix} \xi_{t-1} + \begin{bmatrix} v_t \\ u_u \\ 0 \\ \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix} = S_t + F \xi_{t-1} + V_t
$$

where
$$E(V_t V_t^{'}) = Q$$

and

$$
Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_4 \end{bmatrix}
$$

With the availability of the estimation method developed by Kim (1994), the state space model can be estimated by maximising the likelihood function. Inferences about the unobserved nonlinear permanent and transitory components and the latent Markov state variables can then be obtained at the same time. The method consists of a combination of Hamilton's algorithm and the nonlinear discrete version of the Kalman filter: we refer to Kim (1994) for technical details.[2]

3. DATA AND RESULTS

We chose four time series that are representative coincident economic indicators: output, income, sales and employment. These series are GDP at factor cost, real household disposable income, retail sales, and employee jobs.[3] All series are seasonally adjusted quarterly observations and logarithms are used. The sample period is from 1959Q2 to 2000Q2.[4] Graphs of the four series are shown in Figure 1.

We first test whether the four series are individually integrated and, if they are, whether they are cointegrated.[5] We find that we cannot reject the hypothesis that each of the series is integrated, and neither can we reject the hypothesis of no cointegration among these variables. Therefore, we use the first differences of the variables (multiplied by one hundred) as is implied by the model set out in equations (1) to (4). As indicated in section 2, all series are demeaned by subtracting the sample mean from each difference.

For the model specification, we initially fitted AR(2) processes for the common permanent and transitory components and the four idiosyncratic components in equations (2), (3) and (4). Based on various diagnostic tests, however, we chose a parsimonious AR(1) representation for all components, producing the estimates presented in Table 1. Before discussing our results, a further diagnostic test was carried out to assess the robustness of these estimates. We estimated the model with the restriction $S_{1t} ? S_{2t}$, which assumes that both permanent and transitory components switch at the same time. Under this restriction, we obtain the estimates shown in Table 2. Several of the parameter estimates now become insignificant and a comparison of the two models produces a likelihood ratio of 11.6. Although standard critical values are not appropriate here, the combination of such a high likelihood ratio for the imposition of one restriction, coupled with the poorer set of restricted

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[2] Estimation was performed using routines written in GAUSS. No constraint was placed on the sign of ?. Details are available on request.

[3] Except for the retail sales series, taken from Datastream, all other data are from the Office of National Statistics. The series codes are YBHH, NRJR, UKRETTOTG and BCAJ, respectively. We also tried workforce rather than employees, producing results similar to those reported here.

[4] Monthly income is only available after 1986Q1.

[5] Results are available upon request.

estimates, leads us to prefer the unrestricted model, which implies that the common permanent and transitory components switch at different times. Consequently, our discussion is based on the estimates reported in Table 1.

The estimated model seems successful in extracting information about fluctuations in economic activity. Both common permanent and transitory components exhibit first order autocorrelation as the estimates of $?_1$ and $?_1^*$ are significant at the 1% level. Consider first the common permanent component. The results support the presence of asymmetric business cycles that switch between two different states, with state 1 having a significantly negative mean and state 0 a significantly positive mean. The transition probabilities associated with these two regimes of recession and expansion are 0.832 and 0.967 respectively. These estimates imply that the average duration of the expansionary regime is $(1?p_1)^{?1}?30.3$ quarters, which may be contrasted with $(1?q_1)^{?1}?6.0$ quarters for the average duration of the recessionary regime.

Since the mean of $?c_t$ is 0.63, equivalent to a 'trend' growth of $2\frac{1}{2}\%$ per annum, the estimates of $?_0$ and $?_1$ imply mean growth rates of the business cycle common permanent component in the two regimes of $0.63?1.56??0.93$ and $0.63?0.33?0.96$, i.e., approximately $?3\frac{3}{4}\%$ per annum. Therefore, since $q_1?p_1$ and $|?_1|?|?_0|$, recessions on average are both steeper and shorter, than expansions. Figure 2 plots the extracted Markov switching common permanent component.[6] This series, which may be interpreted as an index of the business cycle, accurately reproduces the stylised facts of the U.K. experience, that is, the volatility of the 1970s and the relative stability of the 1990s. Regarding the factor loadings indicated by the $?_i$, they are all significant, suggesting that these macroeconomic variables are explained by the common permanent component of business cycles. As each weighting is positive, all variables move pro-cyclically. This is not surprising and is in agreement with conventional views of the business cycle (see, for example, Dow, 1998). Our estimates show that employment has the highest weighting in the common permanent component, followed by output, sales and income.

On the other hand, the estimated common transitory component is consistent with peak-reversion behaviour of the business cycle, as the plucking term, $?$, is significantly negative, thus supporting Friedman's hypothesis that the economy is temporarily plucked down by negative shocks. The estimated transition probabilities are 0.972 and 0.385 for expansions and recessions, respectively. Therefore, the average durations of the expansionary and recessionary regimes are 35.7 and 1.6 quarters respectively, thus producing an even sharper contrast than the transition

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[6] The details of how to obtain the levels of the common permanent component are described in Stock and Watson (1991).

probabilities from the common permanent component. As far as the factor loadings, $?_i$, are concerned, they are all significant except $?_4$, which corresponds to employment, implying that the common transitory component also plays a significant role in explaining business cycle fluctuations. Sales have the highest weighting in the common transitory component, followed by income, output and employment, suggesting that declines in sales are the major factor in producing temporary negative shocks to the economy. Figure 3 plots the common transitory component. It clearly shows that a succession of transitory shocks played a major role during the recessions of the 1970s and 1980s, but that they were less of a factor in the recesssion of the early 1990s.

Moving to the idiosyncratic component, the negative coefficients of $?_i$ indicate that the idiosyncratic components of these series exhibit negative serial correlation. While income has the highest innovation variance among the four variables, employment has the smallest.

Figures 4 and 5 plot the probabilities that the economy is in a recession: panel (a) shows the filtered probability conditional on information available through t, $\Pr\Big(S_{it} ? 1 \big| ?_t ?\Big), (t ? 1,2, \quad ,T), (i ? 1,2)$, while panel (b) shows the smoothed probability based on the complete set of information up to T, $\Pr\Big(S_{it} ? 1 \big| ?_T ?\Big), (t ? 1,2, \quad ,T), (i ? 1,2)$. While the two pairs of filtered and smoothed graphs are very similar, the timing and duration of the permanent and transitory components are different. As there is no official U.K. business cycle chronology that we may relate our results to, we have thus compared our inferred probabilities of recession with the chronology provided by Artis, Kontolemis and Osborn (1997)[7]. Their dating is based on monthly industrial production and finishes in 1993, and so can only be used for rough comparisons. For the permanent component, we find that our recession probabilities are closely related to their dating. This component clearly picks out and dates correctly the several major recessions that the U.K. economy has experienced during the last four decades, which are shown as shaded areas on the plots of Figure 4. For the transitory component, however, the recession probabilities only pick up the recessions of the 1970s, and totally miss the recession in the early 1990s. Interestingly, Birchenhall, Osborn and Sensier (2001), using a logistic model, have dated the major classical troughs of GDP to be 1975Q2, 1981Q1 and 1992Q2. It can be seen from Figures 4 and 5 that the first of these is identified via the transitory component, whereas the other two are picked up by the permanent component. This suggests that the features of each recession are different: for example, following a recession, a high-recovery phase is not always found, which is also confirmed by Kim and Murray (2001) for the U.S.

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[7] The chronology is presented in Table D1 of Artis, Kontolemis and Osborn (1997)

4. CONCLUSIONS

In this study we have extended Diebold and Rudebusch's (1996) multivariate Markov switching business cycle factor model to include a common transitory component. This common transitory component is subject to regime switching, so it can capture the peak-reversion, or plucking, behaviour of the business cycle. Applying the model to the quarterly U.K. for the last four decades, we found the model captures the important features of the U.K. business cycle fairly well. The common permanent component, interpreted as a composite indicator of coincident variables, switches between regimes of boom and slump. On the other hand, the estimated common transitory component supports the peak-reversion behaviour of the business cycle movement and is particularly influenced by retail sales.

In addition, we also found significant timing differences between permanent and transitory components of recessions, notably the lack of the usual high-growth recovery phase following the early 1990s recession. Our results thus suggest that each recession is indeed different.

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Table 1 Estimates of the dynamic common permanent and transitory model with Markov switching (S_{1t} ? S_{2t})

Common permanent component

$?_1$	$?_0$	$?_1$	p_1	q_1
0.4458	0.3302	-1.5636	0.9669	0.8319
(0.1254)	(0.1376)	(0.4360)	(0.0208)	(0.0871)

Common transitory component

$?_1^*$	?	p_2	q_2
0.9477	-4.2406	0.9718	0.3845
(0.0201)	(0.8486)	(0.0144)	(0.2070)

Idiosyncratic component and factor loadings

|	$?_i$	$?_i$	$?_i$	$?_i$
$?y_{1t}$	-0.0595	0.8174	0.3132	0.2460
	(0.0865)	(0.0504)	(0.0711)	(0.0554)
$?y_{2t}$	-0.3441	0.8235	0.3515	0.1250
	(0.0795)	(0.0527)	(0.0613)	(0.0421)
$?y_{3t}$	-0.5831	0.5307	0.5261	0.1648
	(0.0993)	(0.0789)	(0.0803)	(0.0421)
$?y_{4t}$	-0.0877	0.4069	0.0523	0.5617
	(0.2458)	(0.1375)	(0.0601)	(0.1086)

Log-likelihood	-368.788

Note: The order of the variables in y_{it} is GDP, income, sales and employment.
Standard deviations are in parentheses

Table 2 Estimates of the dynamic common permanent and transitory model with Markov switching (S_{1t} ? S_{2t})

Common permanent component

$?_1$	$?_0$	$?_1$	p_1	q_1
0.6081	0.2837	-1.2093	0.9464	0.8172
(0.1664)	(0.1523)	(0.4793)	(0.0331)	(0.1061)

Common transitory component

$?_1^*$	$?$	p_2	q_2
0.9330	-1.4350	-	-
(0.0271)	(0.4034)		

Idiosyncratic component and factor loadings

	$?_i$	$?_i$	$?_i$	$?_i$
$? y_{1t}$	-0.0685	0.8094	0.4047	0.1515
	(0.0882)	(0.0508)	(0.0600)	(0.0616)
$? y_{2t}$	-0.3299	0.8285	0.3992	0.0297
	(0.0797)	(0.0521)	(0.0590)	(0.0450)
$? y_{3t}$	-0.5861	0.5340	0.6236	0.0278
	(0.1057)	(0.0896)	(0.0838)	(0.0562)
$? y_{4t}$	-0.0605	0.4672	0.1009	0.5106
	(0.2205)	(0.1543)	(0.0778)	(0.1489)

Log-likelihood   -374.605

Note: The order of the variables in y_{it} is GDP, income, sales and employment.
Standard deviations are in parentheses

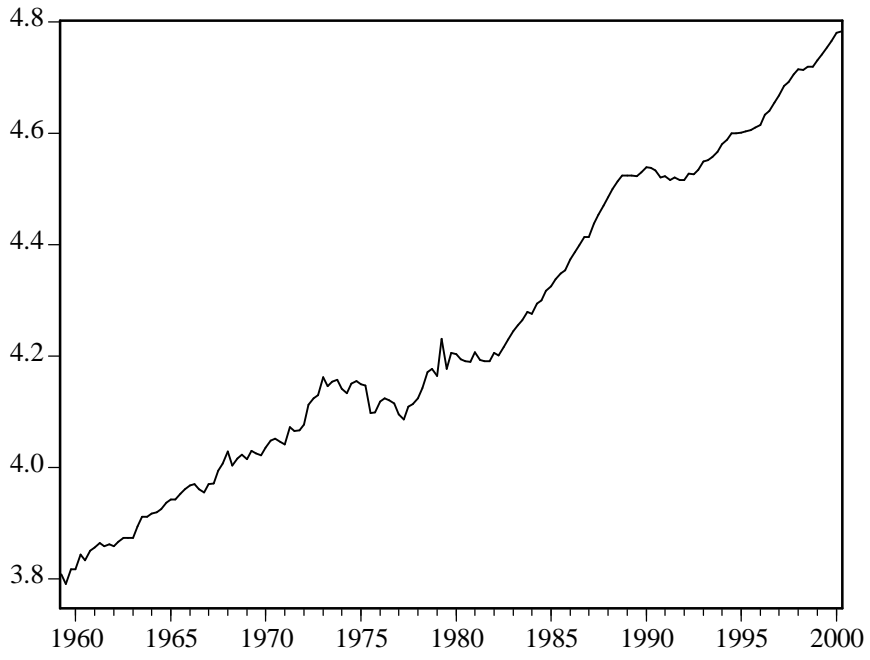Figure 1. Time series of the four coincident variables
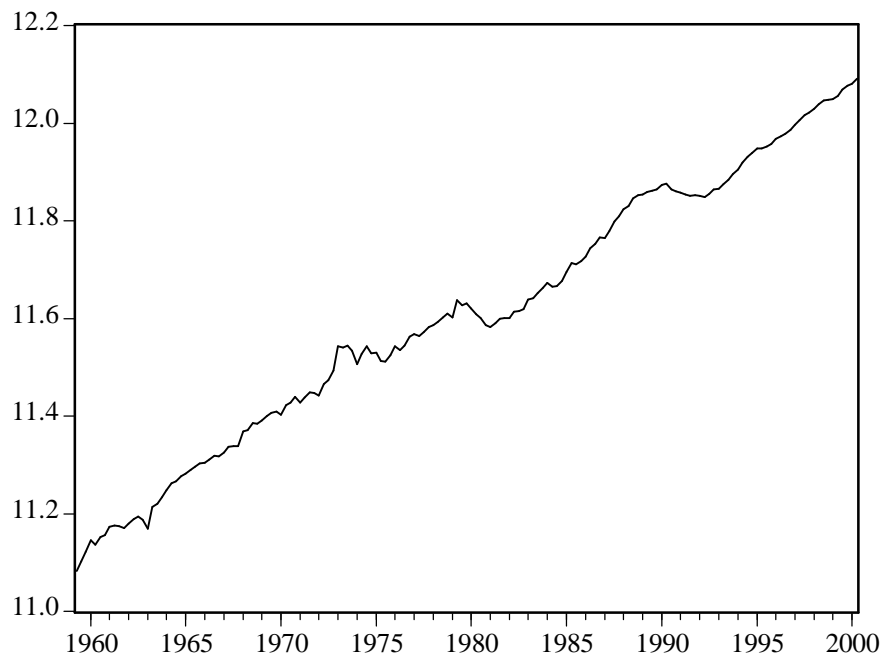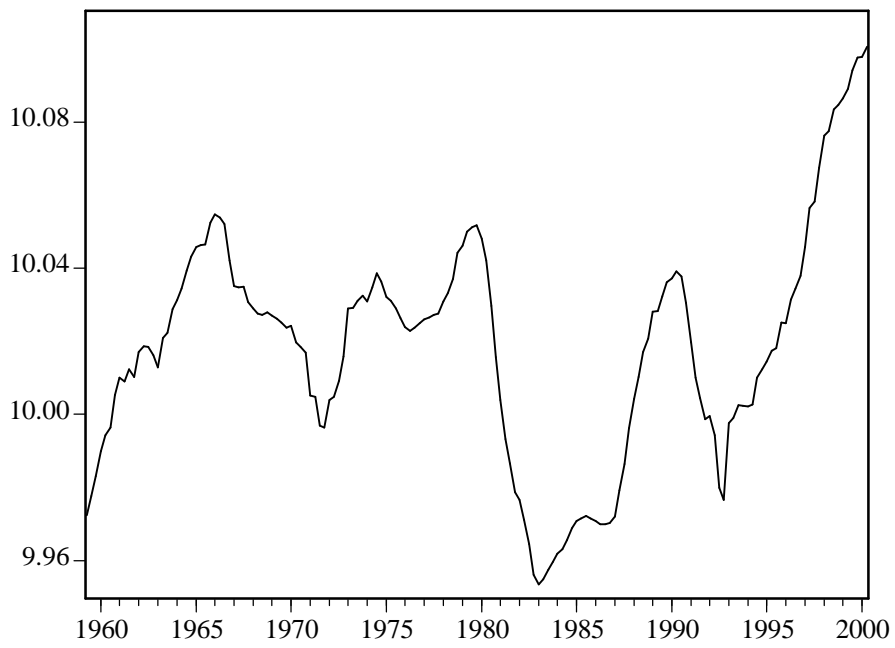

(a) Logarithm of GDP
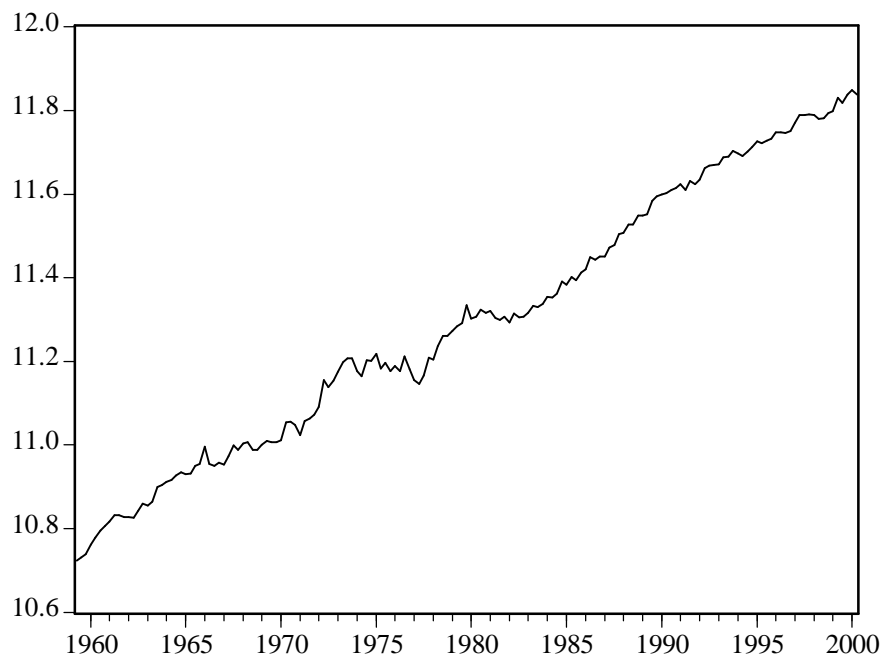
(b) Logarithm of income

(c) Logarithm of sales

(d) Logarithm of employee jobs

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Figure 2. Extracted Markov switching common permanent component
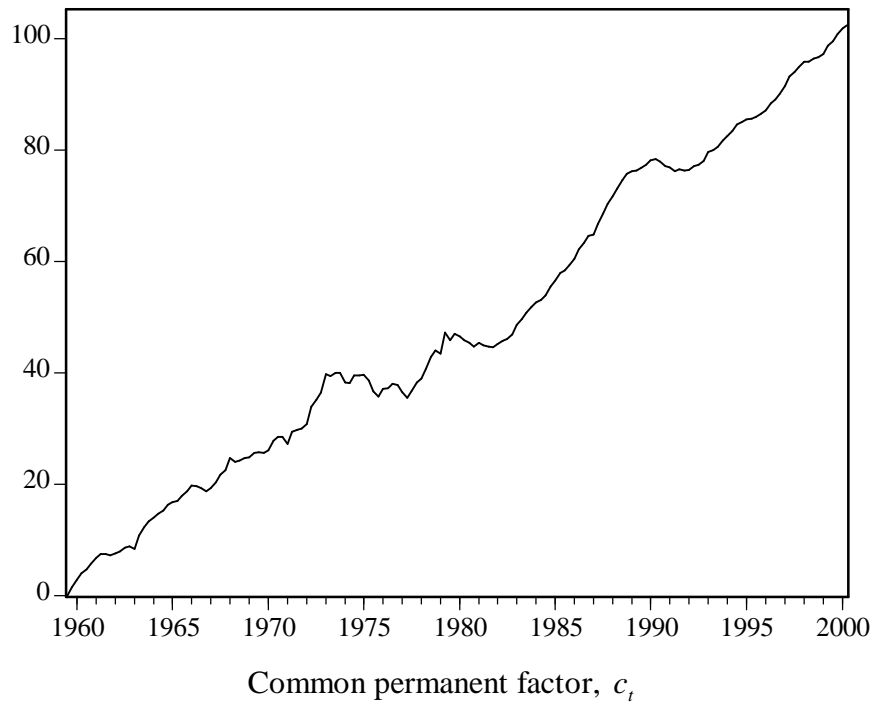

Common permanent factor, c_t

Figure 3. Extracted Markov switching common transitory component
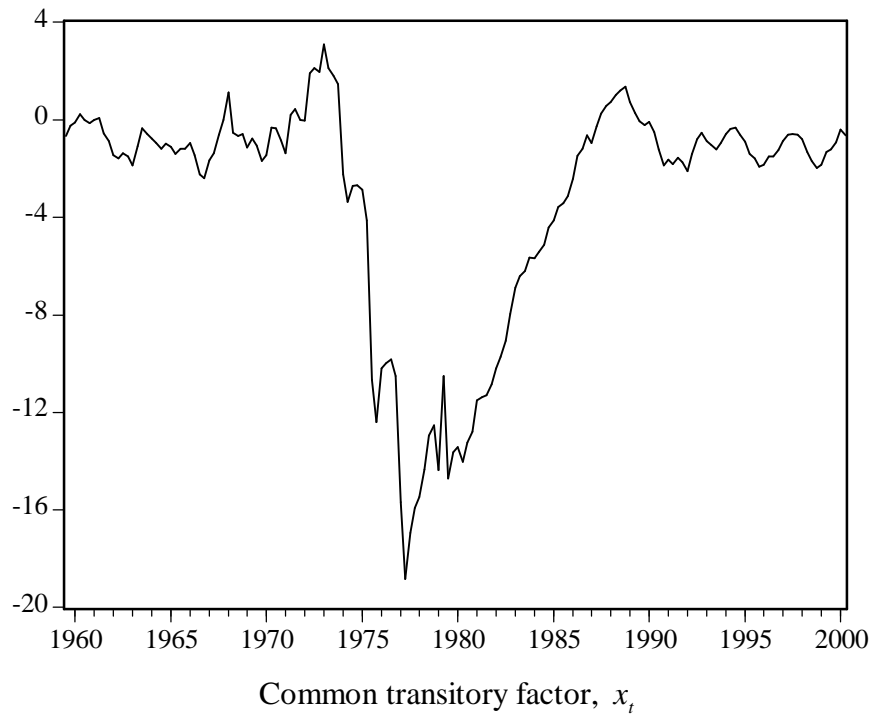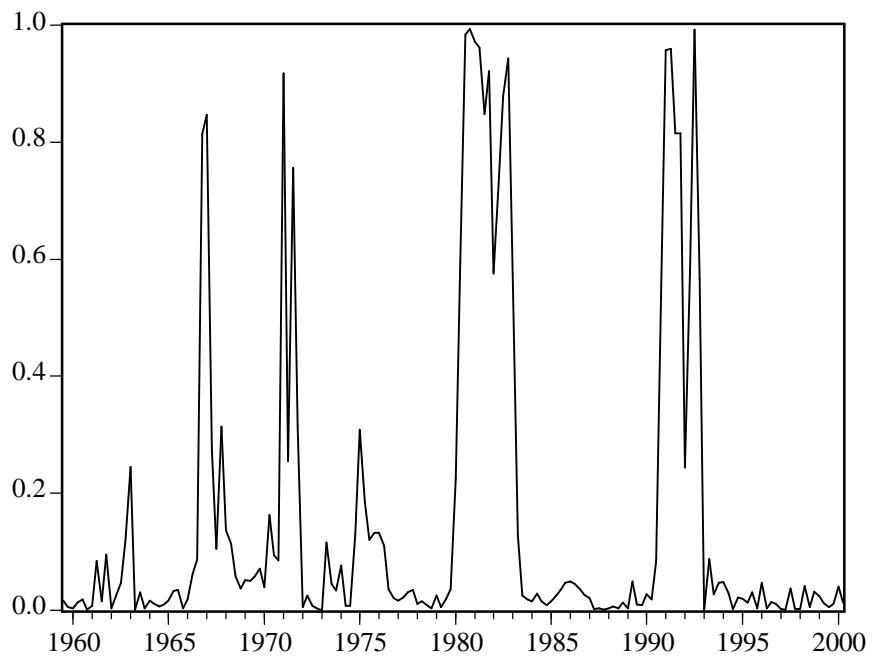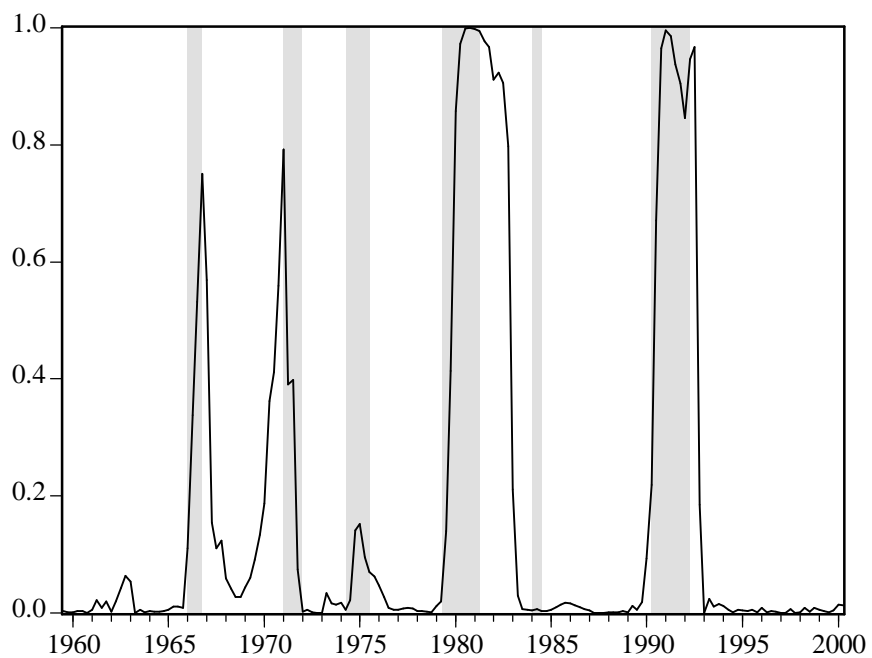

Common transitory factor, x_t

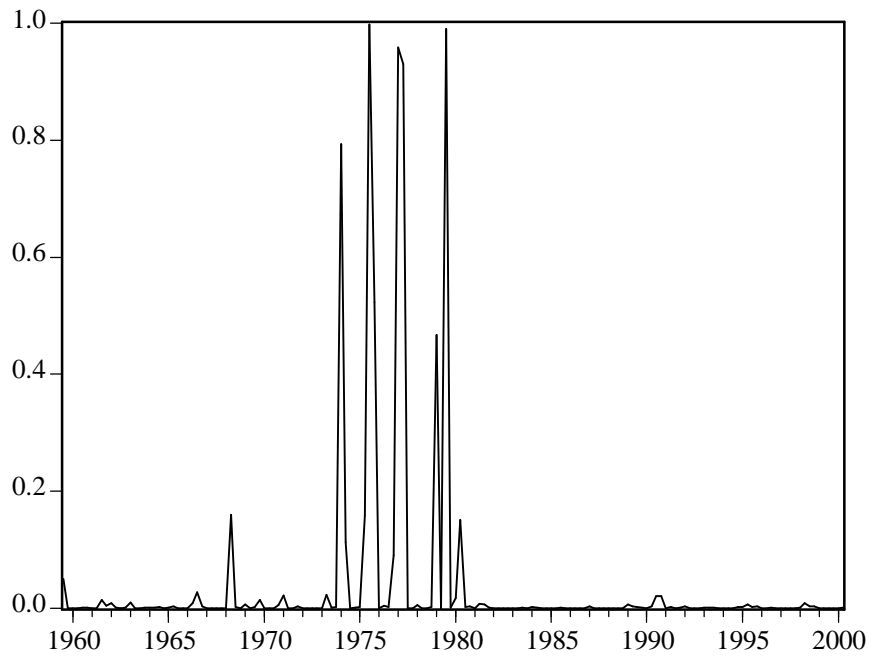Figure 4 Filtered and smoothed recession probability of common permanent
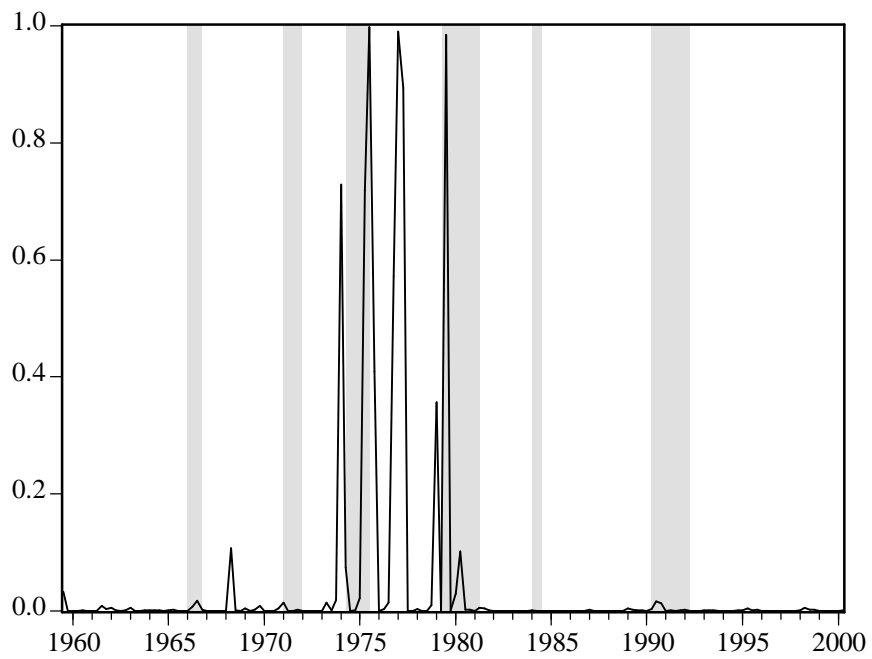component

(a) Filtered probability

(b) Smoothed probability

Figure 5 Filtered and smoothed recession probability of common transitory component

(a) Filtered probability

(b) Smoothed probability