Chaos and rectification of electromagnetic wave in a lateral semiconductor superlattice

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We find the conditions for a rectification of electromagnetic wave in a lateral semiconductor superlattice with a high mobility of electrons. The rectification is assisted by a transition to a dissipative chaos at a very high mobility. We show that mechanism responsible for the rectification is a creation of warm electrons in the superlattice miniband caused by an interplay of the effects of nonlinearity and finite band width.

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The research of nonlinear transport properties of the strongly coupled semiconductor superlattices (SLs) under the action of constant and alternating fields is the subject of traditional interest associated with the great potential of numerous applications in microwave and terahertz technologies [1]. In recent years the primary goal has been investigations of novel *strongly nonlinear phenomena* at the miniband transport in SLs, such as dissipative chaos [2] and excitation of Bloch oscillations under the action of a pure ac electric field [3, 4]. The last effect can be considered as a new mechanism of the rectification of THz radiation; the generated dc voltage is either integer [3] or fractionally quantized [4].

The recent experimental progress in the observation of the influence of intensive THz electromagnetic radiation on voltage-current characteristics of SLs [5] has stimulated advances in the theory [2, 3, 4]. Observations of strongly nonlinear properties of the miniband transport in SLs subjected to an intensive electromagnetic field require a relatively high level of the carrier density together with the high carrier mobility [2, 3, 4]. However, due to the experimental conditions associated with bulk SLs it is difficult to meet simultaneously these two requirements, because of a relatively low electron mobility and a formation of high field domains inside heavy doped SLs.

These limitations can be overcome in the lateral semiconductor superlattices (LSSLs) [6], and especially in those LSSLs which are fabricated using the cleaved edge overgrowth (CEO) technique [7]. In these LSSL devices a two-dimensional electron gas resides in an atomically precise one-dimensional superlattice potential. CEO LSSLs are most suitable for an observation of Bloch oscillations: they have high electron mobility and can deliver high currents without formation of high field and low field domains [7]. In these respects the LSSLs are promising objects for investigations of various strongly nonlinear phenomena.

In this Letter, we describe a novel mechanism of rectification of electromagnetic radiation in a superlattice that is different from the mechanism of AC-induced Bloch oscillations [3] and that always results in a generation of an unquantized bias. We consider an interaction of a plane electromagnetic wave with 2D electrons in a single miniband of LSSL, and incorporate the feedback effects of electron motion on the incident field. These feedback effects are responsible for the nonlinearity of the problem. We show that for some field strengths and frequencies, the interplay of effects of the nonlinearity and the finite miniband width causes a weak asymmetry of a distribution function of electrons in the momentum space with respect to the center of Brillouin zone. This deformation of nonequilibrium distribution function creates a direct current that is the reason of an appearance of bias in the wave transmitted through the LSSL. In mathematical description, this effect corresponds to the symmetry-breaking bifurcation in nonlinear dynamical systems [8]. We also show that the interaction of the electromagnetic radiation with miniband electrons of the very high mobility can result in a dissipative chaos instead of a rectification.

Consider a plane electromagnetic wave $E_{in} = E_0 \cos(\Omega t - kz)$, which is polarized along the superlattice axis, being incident normally to the surface of LSSL located at $z = 0$. The LSSL sits on a semi-insulating background with an average dielectric constant ϵ_0. We assume that both the superlattice thickness in the z-direction and the superlattice length along its axis are much less than the characteristic scale of the electromagnetic wave in the medium, $2\pi/k$. To study the electron transport through a single miniband, a spatially homogeneous LSSL with the period a and the miniband width Δ, we use the tight-binding energy-quasimomentum dispersion relation $\varepsilon(k_{el}) = (\Delta/2)\left[1 - \cos(k_{el} a)\right]$ (k_{el} is the electron wave vector along the superlattice axis). The dynamics of miniband electrons is described by the following nonlinear balance equations [9]

$$\dot{v} = -Uw - \gamma v, \tag{1a}$$
$$\dot{w} = Uv - \gamma(w - w_{eq}), \tag{1b}$$
$$U = U_{in}(t) - \kappa v, \tag{1c}$$

where $U_{in}(t) = (2ea/\hbar(1 + \epsilon_0^{1/2}))E_{in}(t, z = 0)$ and $U(t) = (ea/\hbar)E_{out}(t, z = 0)$ ($E_{in}(t, z)$ and $E_{out}(t, z)$ are the fields of the incident wave and the wave transmitted by LSSL, respectively). The relaxation processes for miniband electrons are characterized by a scattering constant γ. The electron variables $v = m_0\overline{V}a/\hbar$, $w = (\overline{\varepsilon} - \Delta/2)(\Delta/2)^{-1}$ and w_{eq} are the scaled electron velocity, the scaled electron energy, and the equilibrium value of the scaled electron energy, respectively, and $m_0 = (2\hbar^2)/(\Delta a^2)$ is the effective mass at the bottom of the miniband. The lower (upper) edge of the miniband corresponds to $w = -1$ ($w = +1$), and the value of w_{eq} ($w_{eq} < 0$) is a function of lattice temperature. The scaled variables $v(t)$ and $w(t)$ are proportional to the variables $\overline{V}(t)$ and $\overline{\varepsilon}(t)$, which are the electron velocity and the energy averaged over the time-dependent distribution function satisfying the Boltzmann equation.

The first two Eqs. of set (1) are well-known superlattice balance equations [10]; their solution for a given field $U(t)$ is exactly the same as the solution of the time-dependent Boltzmann equation for a tight-binding lattice. Eq. (1a) describes the acceleration of electrons under the action of the electric field $U(t)$ and their slowing down caused by an effective friction due to the scattering, while the second Eq. (1b) describes the balance of electron's energy gain under the action of the electric field and the energy loss due to scattering. These equations are valid if the standard conditions of semiclassical approach, $eE_0a < \Delta$, $\hbar\omega < \Delta_g$, $eE_0a < \Delta_g$ (Δ_g is the width of minigap) [1], are satisfied.

The Eq. (1c) is derived by modeling the LSSL as an equivalent current screen of infinitesimal thickness with the use of the Maxwell equations and appropriate boundary conditions [9, 11]. Such approach is valid if superlattice thickness is much less than the wavelength $2\pi/k$ [9]; this condition is easily satisfied for typical LSSL irradiated by submillimeter or millimeter electromagnetic waves. This Eq. (1c) describes the back influence of the electron current in LSSL on the dynamics of miniband electrons making the whole set of Eqs. (1) nonlinear. The degree of nonlinearity is controlled by the value of parameter $\kappa = (4\pi e^2 N_s)/m_0 c(1 + \epsilon_0^{1/2})$, where c is the speed of light and N_s is the areal electron density.

The dynamics of electrons in LSSL as well as the time-dependence of the transmitted and reflected waves is determined by an interplay of the relaxation ($\propto \gamma$) and the nonlinearity ($\propto \kappa \sim N_s$). Therefore, it is convenient to introduce a single parameter, $\Gamma = (\gamma/\kappa)^{1/2}$, controlling the relative contribution of these two basic processes. For this we introduce the scaled time $\tau = \omega_0 t$, $\omega_0 = (\kappa\gamma)^{1/2}$, scaled fields $u = U/\omega_0$, $u_{in} = U_{in}/\omega_0$, and rewrite Eqs. (1) as

$$\begin{aligned} \dot{v} &= -\left(u_{in}(\tau) - \Gamma^{-1}v\right) w - \Gamma v, \\ \dot{w} &= \left(u_{in}(\tau) - \Gamma^{-1}v\right) v - \Gamma(w - w_{eq}), \end{aligned} \tag{2}$$

where the overdot means the differentiation with respect to the value τ, $u_{in}(\tau) = u_0 \cos\omega\tau$ with $\omega = \Omega/\omega_0$ and $u_0 = (2eaE_0)/\hbar\omega_0(1 + \epsilon_0^{1/2})$. The wave transmitted by

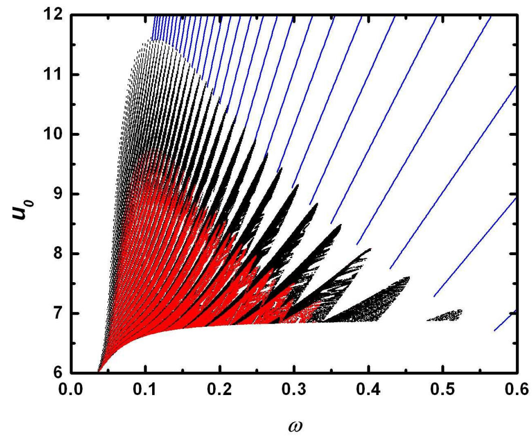

FIG. 1: (color). The regions of the rectification (black) and the chaos (red) in the frequency – field strength plane for $\Gamma = 0.1$. Straight blue lines mark roots of Bessel function.

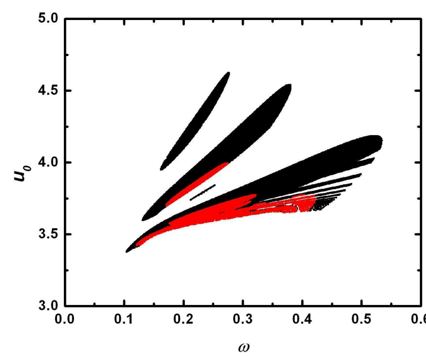

FIG. 2: (color). Same as in Fig.1, but for $\Gamma = 0.2$.

LSSL is $u(\tau) = -\Gamma^{-1}v + u_0 \cos\omega\tau$.

We have performed a systematic numerical search for stationary solutions ($t \gg \Gamma^{-1}$) of Eqs. (2) which have a nonzero dc component of the outgoing wave, $\langle u \rangle \neq 0$ (the averaging is made over a period of the ingoing wave $T = 2\pi/\omega$). We also detect the chaotic attractors characterized by a positive value of the maximal Lyapunov exponent [2]. In contrast, the Lyapunov exponent is always zero for the limit cycles. The locations of chaotic attractors and symmetry-broken periodic attractors with $|\langle u \rangle| \geq 10^{-4}$ in the ωu_0-plane for different values of Γ are shown in Figs. 1-3. The structure of the ωu_0-plot shown in Fig. 1 for $\Gamma = 0.1$ is quite typical for the behavior of our system at weak damping. It consists of the stripes for solutions supporting rectification and some area of chaotic solutions located at the lower values of u_0. For a weak damping and a low frequency, "symmetry-broken stripes" are located near the lines of zero order Bessel function roots defined as $J_0\left(u_0/\omega\right) = 0$ (Fig. 1, only about 30 roots are shown). With an increase of damping ($\Gamma = 0.2$, Fig.2), the region for an existence of chaotic attractors in the ωu_0-plane shrinks, while the regions of nonchaotic attractors with $\langle u \rangle \neq 0$ occupy a few wide stripes (cf. Figs.1 and 2). Finally, with a further increase of Γ, chaos completely disappears and attractors with $\langle u \rangle \neq 0$ are located in a single compact region in the ωu_0-plane (Fig.3). For $\Gamma \geq \Gamma_{cr} \approx 0.4$ we didn't observe the effect of rectification anymore. The generated dc voltage is unquantized (Fig.4). This is in contrast to the case of rectification making use of the mechanism of ac-induced Bloch oscillations, where the ratio $\langle u \rangle/\omega$ can be either an integer or a fractional number for a weak damping [3, 4]. The efficiency of transformation of ac field to dc field, $\langle u \rangle/u_0$, is rather low; in conditions of Fig.3 it is less than four percents overall.

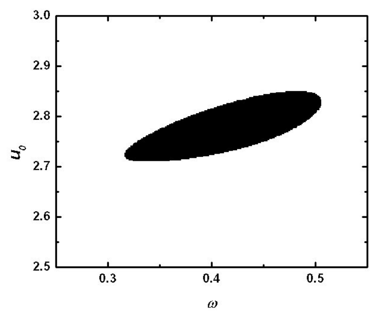

FIG. 3: The region of the rectification in the frequency – field strength plane for $\Gamma = 0.3$.

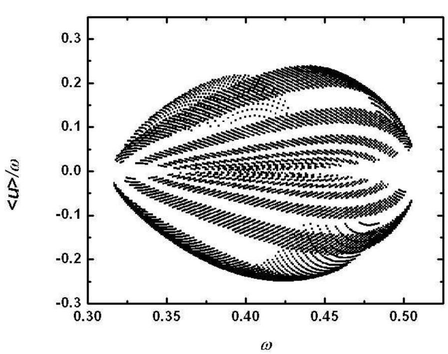

FIG. 4: The dependence of the spontaneously generated dc bias $\langle u \rangle/\omega$ on frequency ω for $\Gamma = 0.3$.

Physical mechanism responsible for the rectification of electromagnetic wave in LSSL will be our focus in the remainder of this paper. There are three main distinct physical factors which can contribute to the generation of unquantized dc voltage in our problem: finite band width, nonlinearity, and dissipation. In order to distinguish an influence of these factors, we consider the limit $\Gamma \to 0$ in the Eqs. (2) and get

$$\dot{\theta} + \Gamma^{-1} \sin\theta = -u_0 \cos\omega\tau, \tag{3}$$

where $v = -\sin\theta$, $w = -\cos\theta$ and $u = -\dot{\theta}$. Eq. (3) describes a time evolution of electrons within the first Brillouin zone ($|\theta| < \pi$) for a negligible scattering but with an account of nonlinearity. Unbiased AC field can not create *rotational states* with $\langle \dot{\theta} \rangle \neq 0$ in the overdamped pendulum, therefore the AC-induced Bloch oscillations [3] can not exist here. However, the ac field can excite symmetry-broken swinging states with $\theta_0 \equiv \langle \theta \rangle \neq 0$ [8]. In a small vicinity of the symmetry-breaking bifurcation, a stationary solution of Eq. (3) can be presented in the form [12] $\theta = \theta_0 + \alpha \sin(\omega\tau + \mu)$, where α and μ are constants dependent on the parameters of LSSL. Substituting this expression in Eq. (3) and equating zero harmonics, we have

$$J_0(\alpha) \sin\theta_0 = 0. \tag{4}$$

Eq. (4) shows that either $\theta_0 = 0$ (a symmetric solution) or $\theta_0 \neq 0$, but α should satisfy $J_0(\alpha) = 0$ (a boundary of symmetry-breaking, *cf.* ref. [12]). Analogously, equating first harmonics, we can find the dependence of amplitude α on u_0 and ω for a symmetric solution, as well as the dependence θ_0 on u_0 and ω at the boundary of symmetry-breaking [13]. This analysis shows that the behavior of symmetric and symmetry-broken solutions is quite different: For a symmetric solution α increases with increase of u_0, while for an asymmetric regime θ_0 increases but α stays constant.

Therefore, we should distinguish two regimes of the wave transmission through a LSSL. In the first regime, the interaction of ac field with an electron gas can led only to a difference between the amplitudes of incident, u_0, and transmitted, $u_{max} = \omega\alpha$, waves due to a nonlinear dependence of α on u_0 (*cf.* [9]). The value of θ_0 is zero indicating that the distribution function of electrons in k_{el}-space is symmetric in respect to the center of Brillouin zone. With an increase of u_0 this regime continues until α reaches the value corresponding to one of the Bessel roots. In this second regime of a strong interaction between electrons and radiation, a variation of amplitude of the incident wave practically doesn't result in a change of amplitude of the transmitted wave: $u_{max} \propto \alpha$=const independently on u_0, at least near the boundary of such regime. Instead, the energy of absorbed radiation goes to an effective heating of electron gas and a creation of "warm electrons". The distribution function of these "warm electrons" is slightly asymmetric in respect to the center of Brillouin zone ($\theta_0 \neq 0$). *In the presence of scattering the asymmetry of distribution function results in a directed current and in an appearance of dc component of the outgoing wave.*

Consider the physics of a rectification of the wave with a low frequency, $\omega \ll 1$, and a large amplitude, $u_0 \gtrsim \Gamma^{-1}$, in more detail. In this case our analysis shows [13] that $\alpha \to u_0/\omega$ and therefore the condition for symmetry-breaking bifurcation in Eq. (3) becomes $J_0(u_0/\omega) = 0$ (we also have checked it solving Eq. (3) numerically). On other hand, this condition is in a reasonable agreement with the results of numerical simulations of rectification at small Γ employing balance equations (2) (see Fig.1 for $\Gamma = 0.1$). Further, we find that the large amplitude of ac field is almost unchanged after passing trough LSSL, $u_{max} = \omega\alpha \approx \omega(u_0/\omega) = u_0$. The absolute value of generated dc bias is very small: $|\langle u\rangle| \simeq 10^{-4}$ for $\Gamma = 0.1$ (however, such values are still quite distinguishable numerically from artificial dc effects arising due to round off errors that result in $|\langle u\rangle| \lesssim 10^{-8}$). The expression $J_0(u_0/\omega) = 0$ determines *the condition for dynamic localization* of miniband electrons driven by *a given ac field* [14]. Both quantum mechanical [15] and semiclassical [16] considerations of the dynamic localization show that a forward motion of an electron is transformed to its local oscillations. These localized electrons strongly interact with radiation. The localization of every electron under the action of a strong ac field is a coherent control effect [14]. However, different oscillating electrons have different and random phases in the presence of scattering; interaction of many electrons with the radiation results in an effective electron's heating [16]. It was shown recently [17] that the overheating may lead to the symmetry-breaking instability resulting in dc. The development of this instability and its next stabilization is a rather complicated process which is associated with a nonlinear feedback action of electrons on the incident field (the terms $+\Gamma^{-1}vw$ and $-\Gamma^{-1}v^2$ in Eqs. (2)). Importantly, the nonlinear effects can not be ignored anymore in the conditions of localization even for a low electron density N_s. Thus, when a dissipation is weak and the self-consistent field inside LSSL is only slightly different from the incident ac field, just an interplay of the effects of dynamic localization and nonlinearity provides a conversion of radiation into a very small dc bias.

For available LSSLs fabricated with the use of CEO technique, $a = 15$ nm, $\Delta = 12$ meV, $\Delta_g = 85$ meV, $\epsilon_0 = 13$ (GaAs), $N_s = 6\times10^{11}$ cm^{-2} and $\gamma^{-1} = 10$ ps at temperature 0.3 K [7], we estimate $\Gamma = 0.6 > \Gamma_{cr} \approx 0.4$. Because $\Gamma \propto (N_s\tau\Delta a^2)^{-1/2}$, a further increase of the electron's density, the mobility, as well as an increase of the miniband width should result in a possibility to observe the predicted effects of rectification and chaos. In physical units, the desirable field strength $u_0^{(sb)} \approx 2.75$ and the frequency $\omega \approx 0.35$ (cf. Fig. 3) correspond to $E_0^{(sb)} \approx 825$ V/cm and $\Omega/2\pi \approx 9$ GHz, that can be easily reached within the range for existing Gunn diodes. At the same time, the conditions of applicability of the semiclassical superlattice equations are well satisfied for such values of parameters (in particular, $eaE_0/\Delta \approx u_0(\hbar\omega_0/\Delta) \simeq 10^{-2} \ll 1$).

In conclusion, we should notice that likely similar effects of a resonant photovoltaic response of 2D electron gas in semiconductor heterojunctions to microwave and far-infrared radiations have been observed in the experiments [18, 19, 20]. These photovoltaic effects were attributed in Refs. [18, 19, 20] to a "resonant heating" of intraband electrons confined by the different methods, such as an excitation of the discrete 2D plasmons [18], an application of the quantized magnetic field [19] or a creation of the field-effect confined quantum dots [20]. In contrast, we consider the localization of electrons in the superlattice miniband induced by the ac electric field itself (dynamic localization). Possible realizations of the symmetry-breaking bifurcations in the particular experimental situations [18, 19, 20] must be further investigated.

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