Simultaneous Optimisation of Vehicle Parameter and Control Action to Examine the Validity of Handling Control Assumptions

Matthew C Best and Timothy J Gordon
Loughborough University, UK

Dept. Aeronautical and Automotive Engineering, Stuart Miller Building, Loughborough University
Loughborough, Leicestershire, LE11 3TU, UK
Phone : +44 1509 227209
Fax : +44 1509 227275
E-mail : M.C.Best@lboro.ac.uk

In this paper a general method is presented for optimising system parameters and inputs. The *Generalised Optimal Control* technique involves iterative resimulation of system states, but is applicable to any (smoothly) nonlinear system, and can be operated using non-quadratic cost functions. Here it is applied to find optimal steer and torque inputs for a 2DOF vehicle handling model with a (combined slip) nonlinear tyre model. System parameters for centre of gravity and yaw inertia are simultaneously optimised, and hence the validity of some handling control assumptions – particularly the benefits of zero sideslip – is examined. The results are satisfactory, and they are mainly in keeping with expectation. The method is proven to be effective, though computationally rather expensive !

Keywords / Vehicle dynamics and control, Vehicle Handling, Optimisation

1. INTRODUCTION

Researchers who consider algorithms for controlling race-car handling, must make assumptions about what behaviour is desirable in the vehicle. For example, they frequently consider that zero sideslip velocity is desirable during steady-state cornering, and so design the vehicle to exhibit neutral steer behaviour and / or prescribe control algorithms about this reference (examples are [1] and [2]). Another assumption is that a particular 'magic' ratio between mass and yaw inertia increases control of yaw transients, and hence yields competitive advantage. However, the validity of these assumptions can be strongly influenced by combination effects between driver or automated control inputs, vehicle setup and driver dynamics.

In this paper we consider a way to combine the choice of control sequence and selected vehicle parameters in an optimisation technique which can test these assumptions, within a fairly simple model of driver capability. A *Generalised Optimal Control* (GOC) technique is applied; the process operates iteratively, solving a two point boundary value problem over a fixed time interval, using Pontryagin's Minimum Principle. The method has advantages over standard optimal control techniques in that it can be applied to any (smoothly) nonlinear plant, and the cost model is not restricted to quadratic functions. The only limitation is that model / simulation complexity must be kept at a suitably simple level, as the method uses multiple simulations to converge on optimal behaviour.

Here the technique has been adapted and extended from earlier work (eg in [3] and [4]), so that model parameters can be selected as additional control inputs, which remain constant over time; in this way the final solution can provide vehicle setup as well as control information. The revised GOC method is presented in Section 2.

The simulations use a two degree-of-freedom yaw / sideslip handling model with a stiff suspension model which allows meaningful load transfer to four independent tyre models; the tyres employ combined slip Pacejka 'magic' formulae to impose realistic friction limits. The time sequence of acceleration / braking and steering inputs are optimised along with the vehicle parameter for centre of gravity position or yaw inertia, with a cost function set to reflect the simple objective of minimum time taken over a simulated section of race track.

The model, implementation detail and cost functions are described in Section 3, and a series of tests are conducted in Section 4 which illustrate the capability of the method, and present some basic findings in combined optimisation of vehicle set-up and control.

2. GENERALISED OPTIMAL CONTROL

The control optimisation is a nonlinear formulation of LQR; controls are sought to minimise a Hamiltonian which is prescribed in terms of a (nonlinear) system of costate equations over a fixed time period. Given a cost function of time, L and a residual cost associated with final states, L_T :

$$J = L_T\left[\mathbf{x}(T)\right] + \int_0^T L\left[\mathbf{x}(t), \mathbf{u}(t)\right] dt \qquad (1)$$

Adding constraint equations to this with a vector of Lagrange multiplier functions, $\mathbf{p}(t)$:

$$J = L_T\left[\mathbf{x}(T)\right] + \int_0^T \left\{ L\left[\mathbf{x}(t), \mathbf{u}(t)\right] + \mathbf{p}^T(t)\left[g\left[\mathbf{x}(t), \mathbf{u}(t)\right] - \dot{\mathbf{x}}(t)\right] \right\} dt \qquad (2)$$

where g is given by the system equations, $\dot{\mathbf{x}} = g\left[\mathbf{x}(t), \mathbf{u}(t)\right]$. The lagrange multipliers can be formed as a so-called costate system, and the Hamiltonian function can then be defined (see for example [5]) as

$$H = L\left[\mathbf{x}(t), \mathbf{u}(t)\right] + \mathbf{p}^T(t) g\left[\mathbf{x}(t), \mathbf{u}(t)\right] \qquad (3)$$

Eqn. 2 can now be integrated by parts to give,

$$J = L_T\left[\mathbf{x}(T)\right] + \mathbf{p}^T(0)\mathbf{x}(0) - \mathbf{p}^T(T)\mathbf{x}(T) + \int_0^T \left\{ H + \dot{\mathbf{p}}^T(t)\mathbf{x}(t) \right\} dt \qquad (4)$$

Considering small changes δJ in the dynamic cost caused by small changes in the controls $\delta\mathbf{u}(t)$ and in the states $\delta\mathbf{x}(t)$:

$$\delta J = \left[\frac{\partial L_T}{\partial \mathbf{x}} - \mathbf{p}^T(T)\right]\delta\mathbf{x}(T) + \mathbf{p}^T(0)\delta\mathbf{x}(0) +$$

$$\int_0^T \left\{ \left[\frac{\partial H}{\partial \mathbf{x}} + \dot{\mathbf{p}}^T(t)\right]\delta\mathbf{x}(t) + \frac{\partial H}{\partial \mathbf{u}}\delta\mathbf{u}(t) \right\} dt \qquad (5)$$

and costates can be chosen such that δJ depends only on changes in the controls by imposing the following conditions :

$$\dot{\mathbf{p}}^T(t) = -\frac{\partial H}{\partial \mathbf{x}} = -\frac{\partial L}{\partial \mathbf{x}} - \mathbf{p}^T\frac{\partial g}{\partial \mathbf{x}}, \qquad \mathbf{p}^T(T) = \frac{\partial L_T}{\partial \mathbf{x}} \qquad (6)$$

hence, $$\delta J = \mathbf{p}^T(0)\delta\mathbf{x}(0) + \int_0^T \left\{ \frac{\partial H}{\partial \mathbf{u}}\delta\mathbf{u}(t) \right\} dt \qquad (7)$$

As we seek an open loop series of controls to minimise the dynamic cost J for constant conditions, $\delta\mathbf{x}(0) = \mathbf{0}$, and the minimum cost must therefore exist where

$$\frac{\partial H}{\partial \mathbf{u}} = 0, \qquad \forall t \qquad (8)$$

In [6] an approximation to the continuous solution is found using a discrete sequence of controls, each held constant for a small time dt. Within the time period for each control, the cost gradient can then be identified as

$$\frac{\partial J}{\partial u_i} = \int_{t_{i-1}}^{t_i} \frac{\partial H}{\partial u_i}\,\mathrm{dt} \qquad (9)$$

So it is feasible to establish a gradient based iteration optimisation of a sequence of discrete controls spanning the required time frame (Fig 1).

Note that, provided the control remains constant for its discretisation period, the method is valid irrespective of the duration. Also, independent controls can take different discretisations. Coupling this with the fact that in the nonlinear model, any variable can be designated a control, it is straightforward to include model parameters within the optimisation. These are defined simply as controls which remain constant over the entire simulation period, and whose gradients are thus computed as

$$\frac{\partial J}{\partial u_\eta} = \int_0^T \frac{\partial H}{\partial u_\eta}\,\mathrm{dt} \qquad (10)$$

Figure 1 provides a summary of the algorithm which can be used to conduct the GOC optimisation.

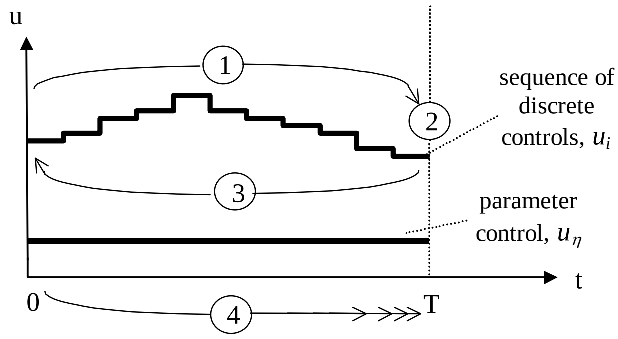


Figure 1 : *Summary of GOC algorithm*

1 : Using the current discrete control sequence, integrate the state-space system from $\mathbf{x}(0)$ and evaluate $J_{[0,T]}$.

2 : Evaluate the residual cost L_T and hence $\mathbf{p}$(T) from Eqn. 6.

3 : Integrate the costate system and $\partial H/\partial \mathbf{u}$ in reverse-time from the initial condition $\mathbf{p}$(T). Calculate cost gradients from Eqn 9.

4 : Update the control sequence by a line search optimisation along the steepest descent or successively conjugate gradients to minimise J (evaluated by repeating Stages 1 & 2).

Repeat Stages 1-4 until suitable convergence of cost and controls, and reduction of cost gradients is achieved.

3. SIMULATION

The GOC algorithm has significant benefits in flexibility, but it can be computationally expensive; the need to calculate partial derivatives of the Hamiltonian with respect to each state, coupled with iterative simulations should caution the user to favour relatively simple, low order models. (Prior papers [3,4] have used two degree of freedom ride and torsional vibration models.) However, by astute use of compiled code and automated code generation, in this paper we stretch the method's capability to a full vehicle handling model, with four independent combined slip tyre force models. This allows investigation of independent steer and torque, whilst ensuring acceptable accuracy.

3.1 Vehicle Handling Model

The well known normalised combined slip Pacejka model (eg [7]) is used to determine longitudinal and lateral tyre forces (F_x and F_y respectively in the SAE vehicle convention) :

$$F_{xi}, F_{yi} = h(u, v, r, w_i, \delta_i, Z_i) \tag{11}$$

Tyre vertical loads, Z are determined using a 'stiff suspension' model which imposes equilibrium conditions on (unmodelled) roll, pitch and bounce degrees of freedom (see also [8]). Assuming a ratio λ between the front and rear suspension roll moments, the effect of both pitch and roll load transfer is accommodated via :

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -a & -a & b-a & b-a \\ c & -c & c & -c \\ 1 & -1 & -\lambda & \lambda \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{bmatrix} = \begin{bmatrix} Mg \\ h\sum f_{xi} \\ h\sum f_{yi} \\ 0 \end{bmatrix} \tag{12}$$

and the friction coefficient is modified with respect to load, according to the simple expression :

$$\mu(Z) = \frac{\mu_0}{1 + (2Z/Mg)^3} \tag{13}$$

The system equations are then, for the rigid vehicle body:

$$\begin{aligned} \dot{u} &= \tfrac{1}{M}\sum f_{xi} + vr \\ \dot{v} &= \tfrac{1}{M}\sum f_{yi} - ur \\ \dot{r} &= \tfrac{1}{I_{zz}}\left\{ \begin{aligned} &c(f_{x1} - f_{x2} + f_{x3} - f_{x4}) + \\ &a(f_{y1} + f_{y2}) - (b-a)(f_{y1} + f_{y2}) \end{aligned} \right\} \\ \dot{\theta} &= r \\ \dot{X} &= u\cos\theta - v\sin\theta \\ \dot{Y} &= u\sin\theta + v\cos\theta \\ \dot{w}_i &= \tfrac{1}{I_w}\{\tau_i - r_r f_{xi}\} \end{aligned} \tag{14}$$

and first order lag functions are employed to simulate tyre force generation and impose a simple driver / vehicle bandwidth limitation on torque and steer inputs at the wheels :

$$\begin{aligned} &\dot{f}_{xi/yi} = \rho_t (F_{xi/yi} - f_{xi/yi}), \\ &\dot{\delta}_i = \rho_\delta(\delta_{ui} - \delta_i), \quad \dot{\tau}_i = \rho_\tau(\tau_{ui} - \tau_i) \end{aligned} \tag{15}$$

Control inputs δ_u and τ_u are applied equally to both wheels at either the front and/or rear axle.

States, x	
u	forward velocity (*m/s*)
v	sideslip velocity (*m/s*)
r	yaw angular velocity (*rad/s*)
θ	roll angle (*rad*)
X,Y	position of vehicle centre of gravity (*m*)
w_i	rotation speed of wheel i (*rad/s*)
f_i, F_i	lagged, unlagged tyre force from tyre *i* (*N*)
δ_i, δ_{ui}	lagged, unlagged steer angle of wheel *i* (*rad*)
τ_i, τ_{ui}	lagged, unlagged torque applied to wheel *i* (*Nm*)
parameters, η (*default values*)	
M	mass (*1400 kg*)
I_{zz}	yaw moment of inertia (*2300 kgm^2*)
I_w	wheel moment of inertia (*0.8 kgm^2*)
a	longitudinal Distance of CofG to front axle (*1.2m*)
b	wheelbase (*2.7 m*)
c	half track (*0.7 m*)
h	C of G height above roll axis (*0.4 m*)
r_r	wheel rolling radius (*0.3 m*)
λ	roll moment distribution factor (*0.5*)
μ_0	tyre friction coefficient (*0.9*)
ρ_t	tyre delay time constant (*100*)
ρ_δ	steer input delay time constant (*30*)
ρ_τ	torque input delay time constant (*30*)
	Pacejka tyre model shape coefficients (*0.714, 1.4, 1.0, -0.2*)
	zero lateral slip cornering stiffness (*50 kN*)
	zero longitudinal tyre slip rate (*60 kN*)

Table 1 : *Model nomenclature & parameters*

3.2 Implementing the Optimisation

The costate system (Eqn 6) for the vehicle model is prohibitively complex to establish by hand, so three techniques are employed to create accurate, yet efficient simulation code. Firstly, the equations are manipulated using an analytical math processor – the Matlab® Symbolic toolbox. Direct evaluation of the partial derivatives is then possible, but the resultant formulae are long and inefficient (eg $\partial H/\partial u$ leads to an equation comprising 134,386 characters !). These direct formulae are thus only used to validate the final code, which is generated by first breaking each partial derivative into its component parts, eg

$$\frac{\partial H}{\partial u} = \frac{\partial H'}{\partial u} + \sum_i \frac{\partial H'}{\partial f_i}\frac{\partial f_i}{\partial u} \tag{16}$$

where H′ is the Hamiltonian written in terms of the tyre forces f_i, and $\partial f_i / \partial u$ is further broken down in to component derivatives of the Pacejka formulae. The resulting derivatives are then converted into lines of computer code by an iterative extraction of common terms, to produce the shortest possible function.

To increase the efficiency of time integration of the states and costates, a discrete-time integration algorithm is employed; this is the Cash/Karp $5^{th}/6^{th}$ order algorithm (described in [9]). The timestep is kept constant within each control (u_i) time interval, and to ensure accuracy the code is written to monitor state errors and adjust the timestep duration accordingly. Finally, the integration and derivative codes are compiled to achieve the fastest possible simulation execution time.

One further modification is made to improve optimisation of the handling model; although the (u_i) controls remain functions of time, they are held constant for a specific *distance* which the vehicle travels – this improves the speed of convergence when steer and torque inputs are to be optimised simultaneously.

3.3 Cost Functions

In all these simulations the target is to gain maximum distance in a given direction, whilst maintaining yaw (velocity and angle) control of the vehicle. The residual, final cost function is set as :

$$L_T = 0.001\left(X_G - X(T)\right)^2 + 100r^2(T) + 100\theta^2(T) \quad (17)$$

with X_G set at some large, unattainable distance (in this case 200m). The coefficients are chosen to ensure that the maximum distance term dominates by a factor of approximately ten.

The time varying cost is dominated by track following terms :

$$\begin{aligned} L_{track} &= \chi^2 , && \text{if } |\chi| < \ell \\ L_{track} &= \chi^2 + 0.01\left(|\chi| - \ell\right)^4 , && \text{if } |\chi| \geq \ell \end{aligned} \quad (18)$$

where χ is the perpendicular distance of the vehicle to the track centre, which is defined using straight-line segments. ℓ is the half track width, set ℓ = 2.5. An additional term is also set, at a relatively low level, to guard against excessive wheel spin, such that

$$L = L_{track} + 10\sum_i \frac{\left(w_i r_r - u\right)^2}{u^2} \quad (19)$$

4. EXPERIMENTS

4.1 Steer only

First consider a simple 90° turn; the vehicle sets off at 20m/s in the positive X direction ($\theta(0) = 0$), and the distance objective is to maximise Y (with no 'track'). Figures 2 compares two conditions. In the first, front steer angle is optimised, with rear steer and all torque inputs set to zero. The parameter control seeks the best fore / aft centre of gravity position, $u_\eta = a$. The experiment is then repeated with both front and rear steer (and a) optimised.

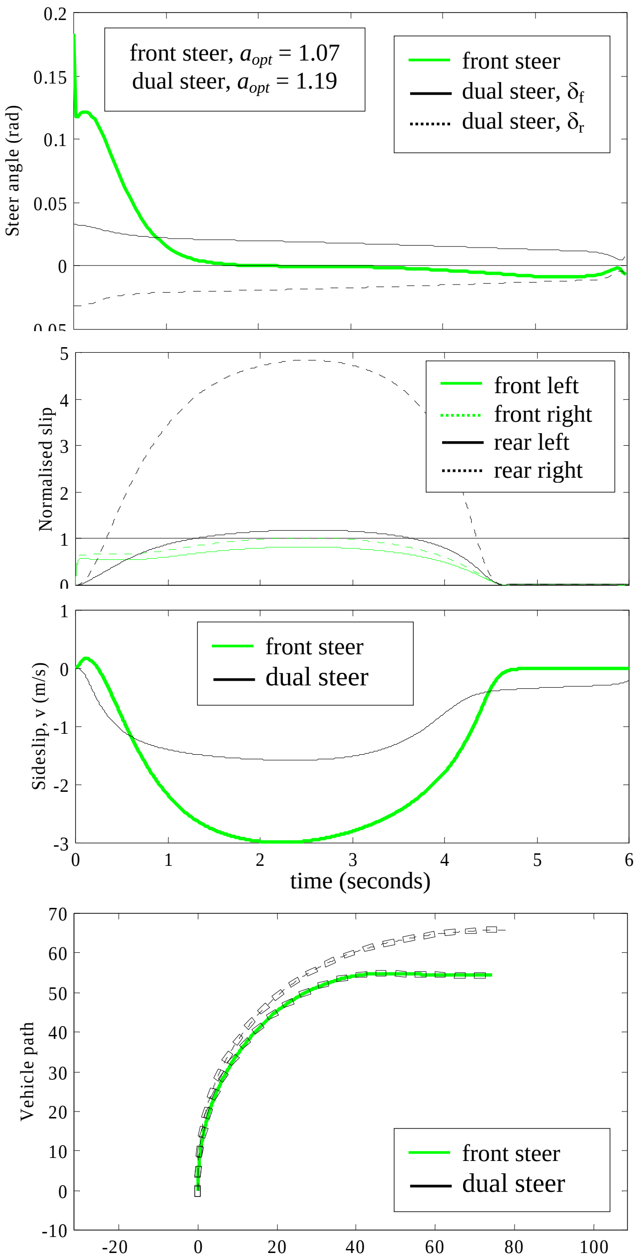


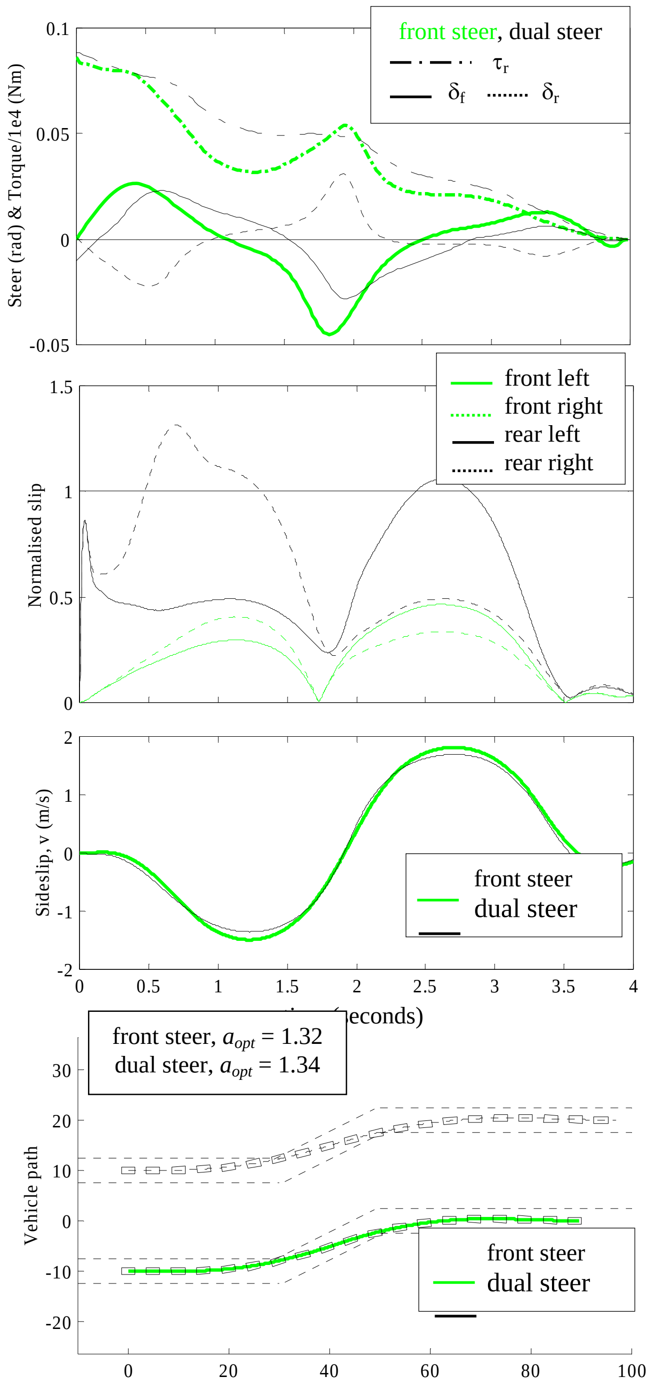
Figure 2 : Optimised performance – steer only

In the front steer case, a high steer angle (7°) is first applied to drive the vehicle into a limit friction yaw rate (the normalised slip plot shows the front steer case). This is recovered using a small opposite steer action around 5 sec. The centre of gravity optimises well ahead of the centre of the vehicle (a_{centre}=1.35), though this provides good balance given that the roll moment distribution is set to induce oversteer (λ=0.5).

The dual steer vehicle adopts less dramatic steering, achieving a greater distance to the right (see vehicle path) with less vehicle sideslip. The centre of gravity is slightly further back here, which is intuitively sensible as the rear steer prevents terminal oversteering, allowing a more even weight distribution.

4.2 Combined steer and torque – lanechange

Here we introduce a torque input, at the rear; this should alter the optimal fore/aft balance – we might expect a larger value of a to increase acceleration capacity. Figure 3 shows how the model tackles a track-imposed lane-change manoeuvre over four seconds, with the initial speed again set to 20m/s. The results are again presented with and without the inclusion of rear steer.



Here the dual steer vehicle achieves greater distance due to a slightly increased torque at the rear wheels, but note that the transient vehicle sideslip trace is very similar to the front steer car. Interestingly, with the exception of the inner rear, the tyres remain well within their friction limits through both turns. This is true for both cases, though again only the front steer vehicle is shown here. As expected the centre of gravity position has converged further back – almost at the centre of the vehicle in both cases.

4.3 Yaw inertia experiment

If a realistic driver model were included in the system to be optimised, it might be possible to examine the performance value of tuning the ratio of yaw inertia to mass (the dynamic index). A ratio of 0.92 is thought by many to offer a good compromise between rapid vehicle yaw response and driver controllability.

Here we have rather too basic a driver model to make a judgement, but it is interesting to note the results of optimisation on I_{zz}; Figure 4 shows how the parameter and cost converges against iteration number for a test which is again executed on the lane-change track, though in this instance with only front wheel drive and steer.

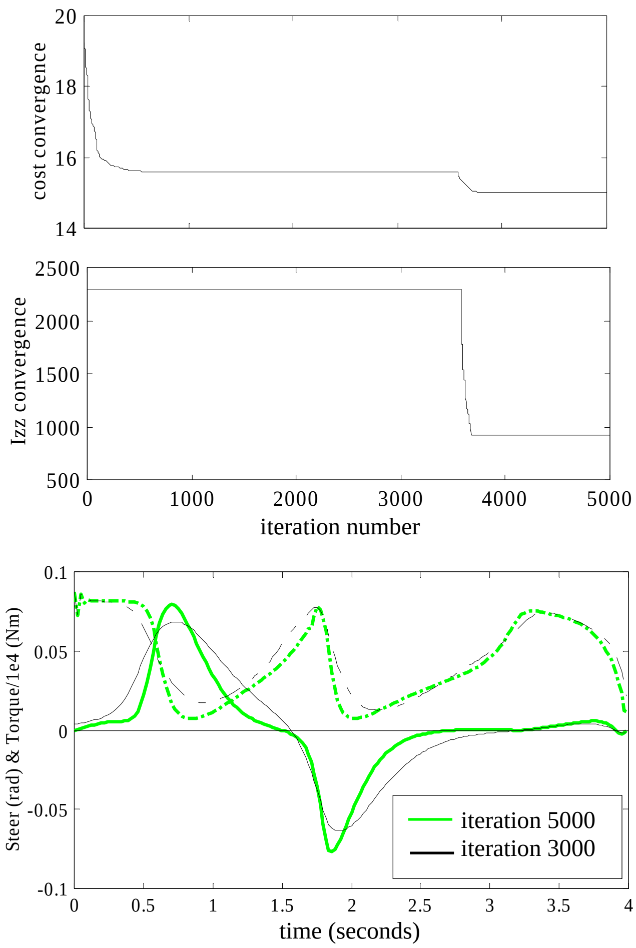


Figure 4 : *Optimised performance – varying* I_{zz}

Note how I_{zz} changes within only around 200 iterations, remaining almost unchanged during the other iterations; this occurs (although to a lesser extent) with all the optimisations – the time varying controls converge almost independently of u_η until they become 'aligned' in such a way that cost gain can be achieved by change in all controls.

The bottom plot of Figure 4 shows how lower I_{zz} has allowed more rapid change in control, with the first steer event delayed, allowing for a longer period of acceleration initially, and hence lower cost. This might be a pathological case for the method, as the controlled parameter has a relatively small effect on cost. It does serve to caution the user to diligence in the monitoring of convergence however; this should principally be based on successive iterations yielding consistently low control gradients.

5. CONCLUDING REMARKS

These results show the scope of this adapted GOC method for examining optimal behaviour, though the relatively small set of scenarios do not provide strong enough insight into specific vehicle set-up or sideslip requirements. The drawback with the method (in this guise) is in its very slow convergence.

For these simulations each iteration took approximately 30 seconds on a modern (1.5GHz) PC, so some of the results needed more than one day to converge. The convergence issue is also exacerbated when the tyres are close to, or beyond their limit of friction; this makes it inefficient to carry out 'follow-on' optimisations – for example using the front steer results as a basis on which to optimise the dual steer case. As a result it was necessary to re-optimise all the cases presented above, from very simple initial conditions.

The method is effective if not efficient however, so further research is planned to reduce computational load in handling optimisations. Simplification of the tyre model, perhaps by spline fitting the magic formula, is one possibility. Another is to further reduce the control's dependence on time, by restating the two point boundary value problem in terms of distance travelled. If each control action is posed as a function of δx rather than δt, the torque and steer controls will be more successfully decoupled.

REFERENCES

1. Abe, M., Ohkubo N., and Kano, Y., "Comparison of 4WS and Direct Yaw Moment Control (DYC) for Improvement of Vehicle Handling Performance," *proceedings from the 2nd International Symposium on Advanced Vehicle Control (AVEC),* Tsukuba, Japan, October 1994, pp 159-164.
2. Ryu, J., Lee, J. and Kim, A., "Evaluation of a Direct Yaw Moment Control Algorithm by Brake Hardware-in-the-loop," *proceedings from the 4th International Symposium on Advanced Vehicle Control (AVEC)*, Nagoya, Japan, September 1998, pp 231-236.
3. Best M.C., "Nonlinear Optimal Control of Vehicle Driveline Vibrations," *proceedings from the UKACC International Conference on CONTROL '98*, Swansea, UK, September 1998, pp 658-663.
4. Gordon T.J. and Sharp R.S. "On Improving the Performance of Automotive Semi-Active Suspension Systems Through Road Preview" *Journal of Sound and Vibration*,' Vol 217(1), pp 163-182, 1998.
5. Bryson A.E. and Ho, Y.C., "Applied Optimal Control: Optimisation, Estimation and Control," *Hemisphere*, New York, 1975.
6. Marsh C., "A Nonlinear Control Design Methodology for Computer-controlled Vehicle Suspension Systems," *PhD Thesis*, Loughborough University, 1992.
7. Milliken W.F. and Milliken D.L., "Race Car Vehicle Dynamics" *SAE International*, 1995
8. Gordon T.J and Best M.C., "Stability Augmentation of Handling Dynamics for Uncertain Road Friction," *proceedings from the 4th International Symposium on Advanced Vehicle Control (AVEC)*, Nagoya, Japan, September 1998, pp 105-110.
9. Press, W.H. Teukolsky, S.A. Vetterling W.T. and Flannery, B.P., "Numerical Recipes : The Art of Scientific Computing," *Cambridge University Press*, Cambridge, 1992.