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# Bending vibration measurement on rotors by laser vibrometry

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A new technique is proposed for noncontact measurement of bending vibration directly from a rotating component. This notoriously difficult and previously unattained measurement is a further development of laser Doppler vibrometry. Simultaneously the technique provides an accurate measure of shaft torsional vibration in situations in which measurements of torsional vibration have shown significant sensitivity to bending vibration. Experimental results validate the theory developed, and a conservative estimate of the minimum measurable bending vibration is made at 20 millidegrees. © 1996 Optical Society of America

The application of laser Doppler velocimetry<sup>1</sup> (LDV) to vibration velocity measurement is now established as a technique complementing the use of more traditional transducers. For vibration measurement on rotating machines, transducer selection depends on whether measurement directly onto rotating components is required or whether measurement of vibration transmitted onto nonrotating components will suffice. For lateral vibrations the relative simplicity of the latter type of measurement and the versatility of piezoelectric accelerometers have meant that this combination is the most frequently used. In many cases, however, the ideal measurement would be made directly from the rotating component, and LDV has been successfully applied to this problem.<sup>2</sup> Importantly, torsional vibration measurement has recently become a practical possibility with the invention of a versatile laser instrument operating on the same Doppler principle.<sup>3</sup>

Recent research has described the effects of angular lateral, or bending, vibrations on measurements made with a laser torsional vibrometer (LTV). The aim was to reduce the instrument's sensitivity to bending vibration to a negligible level by choice of optical configuration.<sup>4</sup> In this way complete immunity to all lateral shaft vibrations can be ensured, since the LTV's optical geometry provides inherent immunity to other lateral vibrations.<sup>3</sup>

Building on this research, in this Letter we describe a novel measurement technique that uses two LTV's capable of providing simultaneous measurements of both pure torsional vibration and pure bending vibrations. Bending vibration has been notoriously difficult to measure, and most reported measurements have used accelerometers attached to a stationary housing carried by a retrofitted bearing mounted on the shaft of interest.<sup>5</sup> In what follows we report what are to our knowledge the first noncontact measurements of bending vibration, representing a further step forward in the use of laser technology for machinery diagnostics.

Operation of the LTV relies on the Doppler shift in the light backscattered from a rotating target. As shown in Fig. 1, two parallel beams are incident upon the target shaft before recombination to produce a beat on the photodetector with a frequency equal to the difference frequency between the individual beams.

Demodulation of this photodetector output then produces a voltage analog of the target rotation speed. Torsional vibration manifests itself as a fluctuation in the rotation speed of the illuminated cross-sectional element, with a corresponding fluctuation in instrument output voltage. This beat frequency can be written as follows<sup>4</sup>:

$$f_{\text{beat}} = (4\mu\pi d/\lambda)N \cos \beta \sin \alpha, \quad (1)$$

where  $\mu$  is refractive index ( $\mu = 1$  in air),  $N$  is the rotation speed in hertz,  $\lambda$  is the laser wavelength, and  $d$  is the perpendicular beam separation. Torsional vibration will be seen as fluctuations in  $N$ . The important angles  $\alpha$  and  $\beta$  are shown in Fig. 2.  $\alpha$  is the angle between the direction of incidence of the laser beams and the direction of the undeflected shaft rotation axis.  $\beta$  is the angle between the plane of the cross section of the target shaft (perpendicular to the undeflected rotation axis) and the plane of the incident laser beams.

Angular lateral vibration, in which the target shaft rotation axis undergoes a change in direction, causes fluctuations in the angles  $\alpha$  and  $\beta$ , producing a corresponding fluctuation in the instrument output indistinguishable from genuine torsional vibration. It should be noted that such angular lateral vibration includes not only the shaft bending vibration of interest here but also solid body motions such as an engine block rocking in its mounts. If such solid body motions are of concern, then accelerometer measurements should be made on the block itself for comparison with the data taken directly from the rotating shaft.

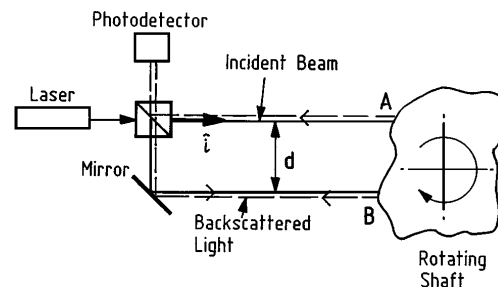


Fig. 1. Optical configuration of the LTV.

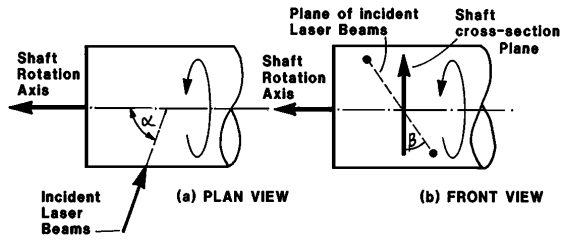


Fig. 2. Definition of instrument-target incidence angles.

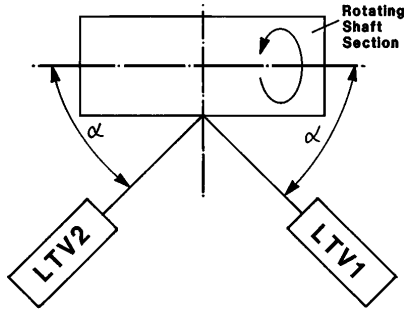


Fig. 3. Experimental arrangement of two LTV's.

It was previously shown that the sensitivity to bending vibration is dependent on the incidence angles  $\alpha$  and  $\beta$  themselves.<sup>4</sup> For torsional vibration measurement with a single LTV it is preferable to arrange for  $\alpha = 90^\circ$  and  $\beta = 0^\circ$ , in which case the instrument sensitivity to torsional vibrations is maximized and the error induced by bending vibrations is minimized. Experience has shown that, although it is common to operate with  $\beta = 0^\circ$ , restricted access often demands that the LTV be used with  $\alpha$  at an angle other than  $90^\circ$ . In this case there will be, in general, some measurable sensitivity to bending vibrations, and it will not be possible to make an unambiguous measure of torsional vibration through use of a single LTV. This sensitivity to bending vibration of the target shaft can, however, be used to advantage to provide a measure of that bending vibration.

A solution has therefore been developed capable of providing not only an unambiguous measure of torsional vibration but also a simultaneous measurement of bending vibration—a measurement not previously realized by noncontact means. In this new configuration two LTV's are arranged symmetrically on either side of the normal to the shaft rotation axis, as shown in Fig. 3. With  $\beta = 0^\circ$  in both cases the sensitivity to bending vibrations producing a variation  $\Delta\beta(t)$ , known as pitch, is negligible. Bending vibrations producing a variation  $\Delta\alpha(t)$ , known as yaw, will modulate the beat frequencies measured:

$$f_1 = (4\mu\pi d/\lambda)N \sin[\alpha + \Delta\alpha(t)], \quad (2a)$$

$$f_2 = (4\mu\pi d/\lambda)N \sin[\alpha - \Delta\alpha(t)]. \quad (2b)$$

Since the angular vibration will be small, appropriate approximations can be applied:

$$f_1 = (4\mu\pi d/\lambda)N[\sin \alpha + \Delta\alpha(t)\cos \alpha], \quad (3a)$$

$$f_2 = (4\mu\pi d/\lambda)N[\sin \alpha - \Delta\alpha(t)\cos \alpha]. \quad (3b)$$

The sum of the outputs provides the real-time measurement of the rotation speed,  $N$ :

$$f_1 + f_2 = (4\mu\pi d/\lambda)2N \sin \alpha, \quad (4)$$

where the mean value is proportional to the mean rotation speed and the fluctuating component provides the unambiguous measure of torsional vibration velocity. The difference in the outputs, normalized by the signal above, provides the new measurement of the bending vibration:

$$(f_1 - f_2)/(f_1 + f_2) = \Delta\alpha(t)/\tan \alpha. \quad (5)$$

Equations (4) and (5) indicate the influence of the incidence angle  $\alpha$  on the sensitivity of the measurement. At  $\alpha = 90^\circ$  there is maximum sensitivity to torsional vibrations and effective immunity to bending vibrations, whereas when  $\alpha$  approaches  $0^\circ$  this situation is reversed. At intermediate values there is acceptable sensitivity to both vibrations, and an experimental validation of this newly proposed technique is given below.

Two LTV's were arranged as shown in Fig. 3, with  $\alpha$  nominally set to  $45^\circ$ . This angle was chosen as a satisfactory compromise of three factors: sensitivity to torsional vibration, sensitivity to bending vibration, and to ensure sufficient intensity of light collected. The laser beams illuminated a section of shaft driven by a dc motor. The motor was mounted directly onto a gear wheel to which a small angular vibration displacement was imparted by means of a rack, meshed with the gear wheel and driven back and forth by an electrodynamic shaker. The motion of the rack, measured by an accelerometer, provides an independent measure of the angular displacement of the shaft rotation axis. This simulates the bending of a target shaft rotation axis. An ac ripple was added to the dc motor drive, causing its speed to fluctuate and thereby simulating a torsional vibration. An independent measure of this torsional vibration was provided by a third LTV set

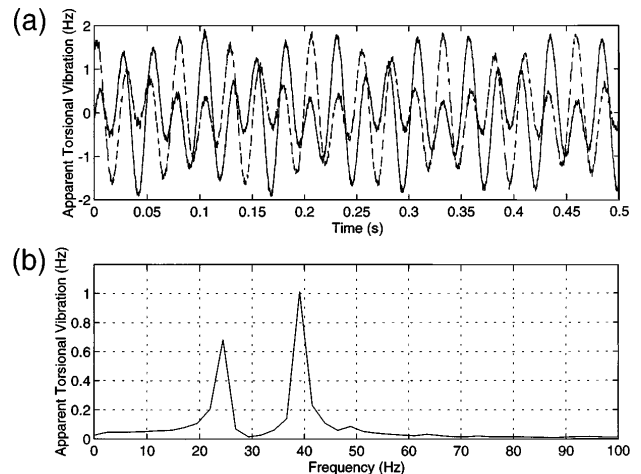


Fig. 4. LTV outputs showing combined torsional and bending vibrations: (a) LTV1 (solid curve) and LTV2 (dashed curve); (b) spectrum of LTV1 output.

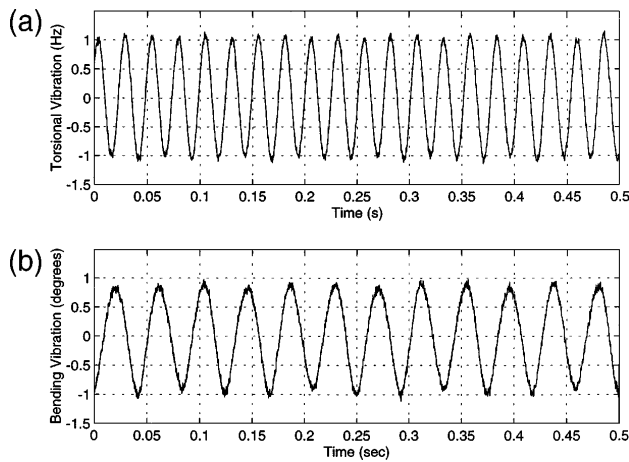


Fig. 5. Processed time-domain signals: (a) resolved torsional vibration, (b) resolved bending vibration.

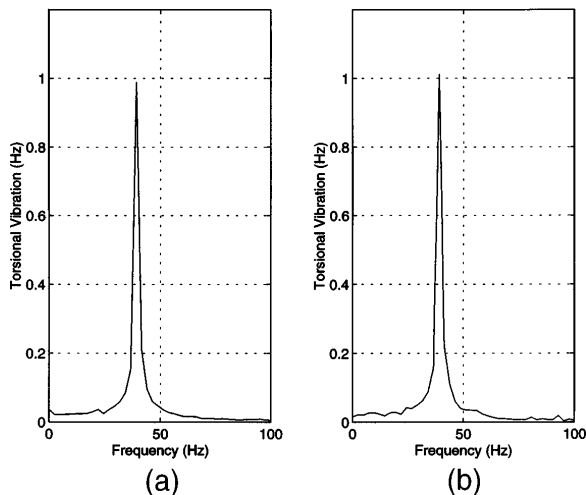


Fig. 6. Torsional vibration spectra: (a) resolved data, (b) independently measured data.

with  $\alpha = 90^\circ$  such that the measurement is effectively insensitive to the bending vibration.

Figure 4(a) shows simultaneous time-domain signals from the two LTV's in Fig. 3. The spectrum of one of the signals is presented in Fig. 4(b), which shows the vibration frequencies of, nominally, 25 Hz for bending vibration and 40 Hz for torsional vibration. The combination of these motions is apparent in Fig. 4(a), but it is not possible to distinguish one motion from the other.

Figure 5(a) shows the resolved torsional vibration obtained by processing the two LTV signals in accordance with Eq. (4). Figure 5(b) shows the bending vibration measurement resolved in accordance with Eq. (5). Comparison with the independent measurements is possible by consideration of the spectra

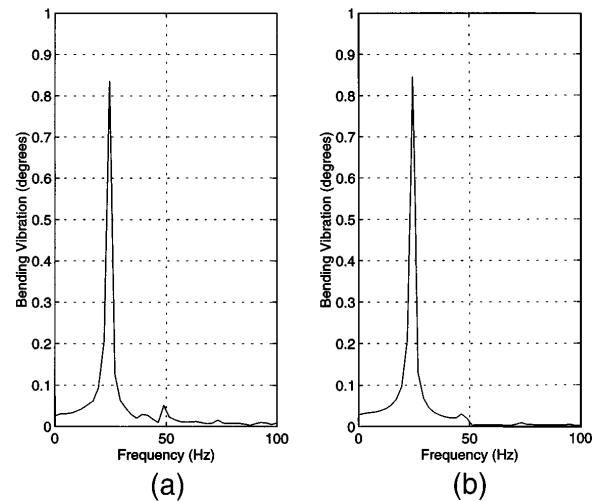


Fig. 7. Bending vibration spectra: (a) resolved data, (b) independently measured data.

shown in Figs. 6 and 7 for the torsional and bending vibrations, respectively. Clear agreement is seen between the resolved levels and the genuine levels. A conservative estimate of the lowest distinguishable bending from Fig. 5(b) is 20 millidegrees, but if necessary this can be improved on by decreasing  $\alpha$ .

In conclusion, a new optical measurement technique has been described that permits, for the first time to our knowledge, noncontact measurement of bending vibrations from rotating components. In addition, the technique provides a simultaneous measure of pure torsional vibration in situations in which a measurement by a single laser torsional vibrometer would be ambiguous through the presence of bending vibrations. The technique demonstrated here uses two laser torsional vibrometers arranged symmetrically about the normal to the rotation axis of the shaft under investigation, and the experimental research undertaken validates the method. It is expected that bending vibrations lower than 20 millidegrees could be measured.

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