

# FLUCTUATIVE MECHANISM OF VORTEX NUCLEATION IN THE FLOW OF ${}^4\text{He}$ .\*

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We propose a mechanism of a vortex nucleation in a flow of a rotating superfluid  ${}^4\text{He}$ . The mechanism is related to the creation by critical fluctuations of a "plasma" of half-vortex rings located near the wall. The "plasma" screens the attraction of the vortex to the wall and permits vortex nucleation. In the spirit of Williams-Shenoy theory we derive the scaling laws in the critical region and estimate the scaling relation and the critical exponent  $p$  for critical velocity; we find  $V_c \sim V_0(1 - T/T_c)$ , so that  $p = 1$ . Various applications of the obtained results are discussed.

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The studies of the vortex nucleation in the flow  ${}^4\text{He}$  have a long history and many interesting results. Packard and Sanders [1] first found that the nucleation of the vortices may be related with the rotation of the  ${}^4\text{He}$ . It is, however, recognized that there is a very large barrier near the wall, which prevents the free vortices from penetrating into the volume. On the other hand Awshalom and Schwarz [2] have shown the presence of remnant vortices, pinned by the wall. It was established that the vortex nucleation in a submicron orifice is related to the activation or tunneling of the half-vortex ring into the volume of the superfluid and different mechanisms have been proposed [4–7].

However, it was, experimentally, found that the critical velocity has the scaling law of the type  $V_c = V_0(1 - T/T_0)$  [3–8] or a small deviation from this law [9,10]. For the very small orifice, the barrier height for vortex nucleation is small. Therefore, activation and tunneling processes for the single vortex generation are possible. However, when the orifice radius increases the barrier height increases. The larger barrier height occurs for penetration of vortices in rotating superfluid [1]. Therefore in these cases both the activation and the tunneling mechanisms alone for the vortex nucleation are ruled out. To improve a disagreement in the vortex nucleation rate between experiment and Iordanskii-Langer-Fisher (ILF) nucleation theory [11,12], Kawatra, Pathria [13] and Volovik [14] suggested that near the wall there is a microscopical surface vorticity sheet. More emphasis to the surface vorticity sheet has been assigned in next development by Ihas et al [4,6,7] who suggested that the surface vorticity sheet may aid to tunneling or activation through a velocity dependent barrier.

We propose a mechanism, which is very different from the one discussed in the literature [4–8]. The vortex penetrates the barrier near the wall with the aid of critical fluctuations via a creation of a half-vortex ring "plasma" ( or the surface vorticity sheet ) . There occurs a

phase transition, in which *the width of microscopical the surface vorticity sheet reaches a critical size  $R_c$* , the barrier for the vortex nucleation disappears and vortex generation is started. The vortex nucleates in a process similar to the Berezinskii-Kosterlitz-Thouless(BKT) phase transition [15–17], where instead of vortex-antivortex (V-A) pairs there near the wall many half-vortex rings (fluctuational plasma or the surface vorticity sheet are generated. The vortex coupling with the wall is screened and the half-vortex rings of a very large radius are created. The longer half-ring, then, nucleates the vortex and initiates phase slippage. With the increase of the flow velocity and the temperature, the correlation length of critical fluctuations in the surface vorticity sheet or its thickness increases. This growth is stopped at some threshold defined both by the temperature and by the flow velocity where the creation of vortices is started. Sonin [18,19] noticed that "neither tunneling nor an activation is a threshold effect, but the vortex nucleation is a threshold effect". Our mechanism describes indeed the threshold effect.

As the flow velocity increases the energy of the pinned the half-vortex ring decreases. This stimulates their activation through thermal fluctuations. In their turn the fluctuational half-vortex rings of a small radius assist in the creation of half-vortex rings of larger radius and so on. The picture is reminiscent of the scaling in BKT transition, where the coupling between the vortex and antivortex decreases as the temperature rises. The similar situation occurs in the Williams-Shenoy(WS) [20,21] model of the  $\lambda$ -phase transition where the role of V-A pairs is played by the vortex rings. The role of half-vortex rings in our mechanism are similar to the role of the vortex-antivortex pairs, whose spontaneous generation is a driving mechanism of the BKT transition.

We derive scaling relations associated with two relevant operators: the temperature and the flow velocity. In this derivation we will follow the WS approach [20,21]. We start with

an assumption that the half-vortex rings are pinned by the wall and are polarized by the flow. To create a half-vortex ring in the external flow of the velocity  $u_0$  one needs the energy

$$E = E_0 - pu_0 \cos \theta \quad (0.1)$$

where  $E_0$ ,  $p$  are the energy and the impulse of the half-vortex ring. The angle between the flow velocity  $\mathbf{u}_0$  and the normal to the vortex ring plane is  $\theta$ . The energy is equal to the half of the energy of the vortex ring [22]

$$E_0 = \pi^2 RK_0 \left( \ln \frac{R}{a_c} + C \right) \quad (0.2)$$

where  $R$  is the radius of the loop;  $C$  and  $a_c$  are core energy and radius, respectively. The constant  $K_0$  is proportional to superfluid density. The half-impulse is equal to  $p = k\pi R^2/2$ , where  $k$  is a vorticity. The probability to find a single half-vortex ring on a scale  $R$  in the area  $\pi R^2 dR d\cos \theta$  is  $dn(R, \theta) = \pi R^2 dR d\cos \theta \exp(-E(R, \theta))$ .

In the low fugacity limit [20] the interaction between vortex rings is neglected. The effective susceptibility is equal to  $\chi = \int_1^{R_c} dn(R, \theta) \alpha_\Delta$ , where  $\alpha_\Delta$  is a polarizability of these half-vortex rings, which is estimated to be a quarter of the polarizability of a vortex ring  $\alpha_\Delta = \beta p^2/12$ . The scale is measured in units of  $a_c$ . Following Williams and Shenoy one may introduce the dielectric constant  $\epsilon = 1 + 4\pi\chi$  and the screened density as  $K_r = K_0/\epsilon$ . As the result we arrive to the recursion equation:

$$\frac{1}{K_r} = \frac{1}{K} + (A/2) \int_1^{R_c} dR d\cos \theta R^6 \exp(-E(R, \theta)) \quad (0.3)$$

where  $A$  is a some constant. Following Ref. [20] we introduce the fugacity as  $y = \exp[-\pi^2 K \ln \sqrt{gK} + C]$ ; then after an angular integration the recursion relation takes the general BKT-WS form:

$$\frac{1}{K_r} = \frac{1}{K} + A \int_1^{R_c} dR R^4 y^R \exp(-\pi^2 K R \ln R) \sinh(uR^2)/u, \quad (0.4)$$

where we introduced the notations as  $u = k\pi u_0/2$ . With the aid of the vortex core rescaling technique by Jose et al [23] which consists of integrating over small distances and then rescaling as  $R \rightarrow Rb$ , we get the following relation:

$$\frac{1}{K_r} = \frac{1}{K} + Ay \ln b \frac{\sinh u}{u} + Ab^5 \int_1^{R_c} dR R^4 y^{bR} \exp(-\pi^2 K b R \ln b R) \frac{\sinh(ub^2 R^2)}{u} \quad (0.5)$$

By introducing the new variables

$$\frac{1}{K'} = \frac{1}{b} \left( \frac{1}{K} + Ay \ln b \frac{\sinh u}{u} \right) \quad (0.6)$$

$$A'y' = Ab^6 y^b \exp(-\pi^2 K b \ln b) \quad (0.7)$$

as well as  $K_r' = bK_r$  and  $u' = b^2u$  we get the same relation as eq(0.4). These rescaling eqs may be represented in differential form similar to that of Williams [20]

$$\frac{d}{dl} \left( \frac{1}{K} \right) = -\frac{1}{K} + A_0 K y \frac{\sinh u}{u} \quad (0.8)$$

$$\frac{dy}{dl} = [6 - K(\ln \sqrt{Kg} + C + 1)]y \quad (0.9)$$

with the initial value  $g_0 = 1/K_0$  or in the form similar to that of Shenoy for 3DXY model:

$$\frac{dK}{dl} = K - A_0 y K^2 \frac{\sinh u}{u} \quad (0.10)$$

$$\frac{dy}{dl} = (6 - \pi^2 K L)y \quad (0.11)$$

$$\frac{du}{dl} = 2u \quad (0.12)$$

where  $L = \ln \frac{a}{a_c} + 1$  and  $a, a_c$  are an effective size of the loop and of its core, respectively (above  $a_c = 1$ ) [21]. One sees that the difference with Williams-Shenoy equations lies in the coefficient  $A_0$  and in the strong (exponential) dependence on the flow velocity, which

appears in the second term of the first equations. In other words when  $u = 0$  the equations coincide with WS equations, but with the difference that Shenoy got the value  $A_0 = \frac{4\pi^3}{3}$ , while Williams — the value  $A_0 = \frac{\pi^5}{6}$ . Because of the wall, the half-rings are created on a half space, so the coefficient in our equations is equal to 1/8 of the Williams one,  $A_0 = \pi^5/48$ . The dependence of scaling on the superfluid flow velocity  $u$  arises because of the spontaneous generation of half-vortex rings induced by the flow. This diminishes the effective coupling  $K$  and stimulates the vortex nucleation. One sees from this equation that the flow velocity is very important for the behavior of  $K$  [24].

There are two types of a behavior of the system. The first one is the superfluid or the low temperature one, which is characterized by growing  $u_l = u_0 e^{2l}$ ,  $K_l = K_0 e^l$  and vanishing fugacity  $y = y_0 e^{-\frac{l}{\xi_0}}$ , where  $\xi_0 = \frac{1}{(\pi^2 K_0 L)}$  and valid when  $A_0 K y \frac{\sinh u}{u} \ll 1$ . The second one is a high-temperature solution, which is characterized by growing the fugacity  $y$ . One sees that the transition of the vortex nucleation depends explicitly from the flow velocity, while the transition temperature does not.

Between these two low- and high-temperature phases there is a critical point, which is associated with the nontrivial fixed point of the rescaling equations:  $u = 0$ ,

$$(6 - \pi^2 K L)y = 0 \tag{0.13}$$

$$K - A_0 y K^2 \frac{\sinh u}{u} = 0 \tag{0.14}$$

The nontrivial solution is  $K_1 = \frac{6}{\pi^2 L}$  and  $y_1 = \frac{\pi^2 L}{6 A_0}$ .

Now let us make an expansion in the vicinity of this critical point as  $K_l = K_1(1+k)$ ,  $y_l = y_1(1+y)$ , and  $u_l = u$ . Then, the scaling equations take the linear form:

$$\frac{du}{dl} = 2u \tag{0.15}$$

$$\frac{dy}{dl} = -6k \tag{0.16}$$

$$\frac{dk}{dl} = -k - y \quad (0.17)$$

The behavior of scaling near the fixed point is associated with the three eigenvalues:  $\lambda_+ = 2$ ,  $\lambda_- = -3$  and  $\lambda_3 = 2$ . The rescaling law for the free energy  $F_l$  obeys the relation

$$Z(K_0, y_0, u_0) = e^{-(F_l - F_0)L^3} Z(K_l, y_l, u_l), \quad (0.18)$$

which is associated with two relevant operators related to the temperature axis  $A_+ e^{\lambda_+ l}$  and the critical velocity  $u_+ e^{\lambda_3 l}$  and one irrelevant associated with the fugacity  $y_l$ , i.e.

$$Z(K_0, y_0, u_0) = e^{-(F_l - F_0)L^3} Z(A | \epsilon | e^{\lambda_+ l}, A_- e^{\lambda_- l}, u_+ e^{\lambda_3 l}) \quad (0.19)$$

where  $\epsilon$  is a deviation of the temperature  $T$  from  $T_c$ :  $\epsilon = (1 - T/T_c)$  and  $u_+$  is the ratio of critical velocity to the relative one  $u_+ = V_c/V_0$  [25]. Because of the two relevant operators it is nontrivial to find where the scaling must be stopped. In order to understand this we must look into the original recursion relation (0.3). With the scaling the critical velocity  $u_l$  and the coherence length  $\xi$  are growing, but the critical radius  $R_{cl}$  decreases. The scaling is stopped, when the critical radius  $R_{cl} \sim \xi$ , i.e. when  $l_- = \ln(\frac{\xi}{a_c}) \simeq \ln(\frac{R_c}{a_c})$ .

It is obvious that  $l_- = \ln(\frac{R_c}{a_c}) \rightarrow \infty$  as  $u_+ \rightarrow 0$ . On the other hand the scaling constraint on the temperature is that  $\ln \frac{\xi}{a_c} \rightarrow \infty$  as  $\epsilon \rightarrow 0$ . Setting  $l = l_- \rightarrow \infty$  into the partition function, we see that it is well defined as only if

$$R_c \cong a_c (u_+)^{-\frac{1}{\lambda_3}} \cong \xi \cong a_c |\epsilon|^{-\frac{1}{\lambda_+}} \quad (0.20)$$

whence we find that the critical velocity

$$V_c \simeq V_0 \left(1 - \frac{T}{T_c}\right)^p \quad (0.21)$$

where  $V_0 = \hbar/ma_c$  and the critical exponent  $p = \frac{\lambda_3}{\lambda_+} = 1$ , that observed in Refs [3–8].

The proposed mechanism of the vortex nucleation has a very general character. It is definitely applicable to systems like orifices with different geometry. The latter dictates the shape of the optimal vortex ring segments fluctuatively generated. For orifice of a square or a rectangular cross section the relevant will be quarter-vortex ring segments. They prefer to be nucleated at corners. There a vortex-ring segments "plasma" is spontaneously generated. Via this mechanism the barrier for such a nucleation vanishes and the vortex (or a vortex ring) is nucleated. The scaling relation for critical velocity will again take the derived universal form(0.21) with  $p = 1$ . The vortex nucleation is related to some kind of a phase transition, which occurs near the surface with the critical width  $R_c$ . In the limit when  $T \rightarrow T_c$  the width  $R_c$  increases and this "near surface phase transition" transforms into the bulk  $\lambda$ -phase transition driven by a generation of half-vortex rings but not a generation of full vortex rings as in the WS model. Here the coefficient  $A_0$  has also decreased. Such a change reduces the critical temperature but has no effect on the critical indices. Thus, our findings offer a new driven mechanism for the  $\lambda$ - phase transition – a generation of half-vortex rings . Probably, for a complete description of the  $\lambda$ - phase transition both half-vortex rings and full vortex rings must be taken into account. This may give a more reasonable value for the coefficient  $A_0$ .

There is nothing in the theory which restricts it to pure bulk  $^4\text{He}$ . Our findings give an explanation for a number of phenomena in Vycor glasses, where there are narrow channels. Then, one has to take into account the curvature of the walls of these narrow channels. Therefore, instead of the half-vortex rings for the plane geometry, the optimal shape of fluctuations will be smaller segments of the vortex rings. This shape depends on the curvature, i.e. on the radius of the narrow channel. Because of this dependence the coefficient  $A_0$  in the scaling equations and the critical temperature of the phase transition decrease while



the critical indices i.e. the universality class of the phase transition remain the same. This explains why in Vycor glasses the critical temperature decreases with the decrease of the diameter of the channels while the character of the  $\lambda$ -phase transition remains the same. Therefore the critical velocity in the Vycor is described by the same formula as in the absence of the Vycor [8]. Thus, the theory works equally well for the flow of  $^4\text{He}$  in Vycor and probably for Xerogel and Aerogel glasses [8]. However, the fractal structure of Aerogel glasses may give some peculiarities.

The other relevant systems are superfluid films of finite thickness. Ambegaokar et al [28] shown that there with the temperature occurs a crossover from  $2D$  to  $3D$ . With this crossover the character of the vortex nucleation changes. We discuss this here only qualitatively. The quantative treatment must include anisotropy effects created by the flow as it was also indicated by perturbation theory [29,30]. The behavior of the critical velocity in the films depends on whether the thickness of the film  $d$  is bigger ( $d > R_c$ ) or smaller ( $d < R_c$ ) than the critical size  $R_c$  of the surface vorticity sheet . When the the thickness of the film is smaller than the critical size  $R_c$  of the surface vorticity sheet, i.e.  $d < R_c$ , one may expect that the BKT  $2D$  V-A pair unbinding process holds [34]. In this case the critical velocity  $V_c$  is estimated from the condition that the coherence length of BKT transition at  $T < T_c$  is equal to a critical radius of the vortex pair separation, i.e.  $\xi_- = r_c$ . The coherence length estimated in Ref. [28] is  $\xi_- = a_{c2} \exp(b/\sqrt{(1 - T/T_c)})$ , while the critical radius of the vortex pair separation is  $r_c = \hbar/mV_c$  [25]. Whence we find that

$$V_c = \frac{\hbar}{ma_{c2}} \exp(-b/\sqrt{(1 - T/T_c)}). \quad (0.22)$$

where  $a_{c2}$  is a vortex core radius which depends on the thickness of the film [35]. On the other hand when the film thickness  $d > R_c$  the critical velocity obeys eq.(0.21) with  $p = 1$ .

However, the width  $R_c$  of the surface vorticity sheet depends on the temperature. The value of  $R_c$  increases when  $T \rightarrow T_c$ . Therefore, there exists the region of the temperatures (near  $T_c$ ) where the value  $R_c > d$  and the critical velocity in the film is described by the expression (0.22). With decreasing temperature it may happen that the value  $R_c$  becomes smaller than the value  $d$  ( $R_c < d$ ). Then the critical velocity is described by eq. (0.21) with  $p = 1$ . Therefore, with decreasing temperature, there occurs a crossover in the behavior of the critical velocity of the film from the regime described by eq.(0.22) to the 3D regime. Probably, to confirm this picture additional experiments are needed ( see, also, Fig.20 in Ref. [27]).

Similarly a vortex may be nucleated both in the rotating superfluid  ${}^3\text{HeB}$  and in superconductors. Indeed a surprising scaling relation for the critical velocity of the vortex nucleation in rotating  ${}^3\text{HeB}$  has been observed [36]. The huge vortex nucleation barrier rules out the conventional mechanisms like tunneling and activation. There is only one possibility here to generate the vortex: i.e. via a creation of a critical the surface vorticity sheet the attraction of the vortex to the wall is screened, the barrier vanishes and an intrinsic instability for the vortex creation observed in [36] occurs. However, the core of these  ${}^3\text{HeB}$  vortices proportional to the BCS coherence length is growing to infinity with its critical exponent when temperature rises. This makes a strong difference with the case considered in the present paper although in general the proposed physical mechanism of the vortex nucleation via fluctuative half-vortex rings are also applicable here. The critical exponents as well as the scaling equations may vary and probably be very different from the obtained ones.

In summary, we proposed the fluctuative mechanism of the vortex nucleation, which is similar to the original BKT mechanism. The nucleation of the vortex is usually prevent by

its attraction to the wall. Loosely speaking, this attraction is due to mirror forces to the mirror vortex. For the half-vortex ring discussed this will be an attraction to the other mirror part of the vortex ring behind the wall which is a mirror image of this half-vortex ring. If we consider the single half-vortex ring, its penetration into the volume, goes via transition through a very high barrier associated with this "coulomb" attraction to the wall. However these "coulomb" forces may be screened if in the neighborhood of the wall a large number of the half-vortex rings of the small sizes will be created. These half-rings will create a some kind of "plasma" located mostly near the wall, which, in turn, screens this attraction of the nucleated vortices to the wall. As a result of this screening the single vortex may easily penetrate into the volume. We have also the following main results accomplishing the proposed mechanism: 1) In a flow near a surface a critical surface vorticity sheet occurs and then a spontaneous barrierless vortex nucleation is started. It seems that a recent analysis of various experimental data supports this idea [36]. 2) Because the half vortex rings have half the energy of the full vortex rings of the same radius, they are more important both for the vortex nucleation and for  $\lambda$ -phase transition. 3) The theoretical expression for the critical velocity (main result, eq.(21)) has been obtained for the first time. In the derivation the renormalization group equations have been used. The proposed mechanism may be further confirmed or rejected by experimental studies of the critical exponents, which, according to the scaling hypothesis, obey universal relations.

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no 2D limit in this system, since it is assumed that half of the 3D space is filled with the superfluid. Therefore, eqs(10-12) and the Sujani et al equations live in different dimensions ( $3D$  and  $2D$ , respectively) and should not match each other. And therefore eqs(10-12) look like an extension of the  $3D$  WS equations, while the ( $2D$ ) Sujani et al equations look like an extension of the Kosterlitz-Thouless equations.

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