Reflection of a Rayleigh wave from the edge of a wedge in oblique incidence

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The reflection of a Rayleigh wave from the edge of a sharp elastic wedge in oblique incidence is analyzed. It is shown that the nature of the angular dependence of the modulus of the reflection coefficient is strongly affected by the phase shift of the symmetric field mode associated with reflection from a caustic. The results are tested experimentally.

The interaction of a Rayleigh wave with the edge of an elastic wedge plays an important part in various branches of solid-state acoustics, includng ultrasonic flaw detection and acoustoelectronics.[1-3] An approximate theory has been proposed[3] for the reflection and transmission of Rayleigh waves in a sharp wedge. This theory is based on the representation of the incident Rayleigh wave on the edge of the wedge by the sum of the two lowest (symmetric and antisymmetric) Lamb (plate-wave) modes for a plate of variable thickness. The propagation of these modes to and from the edge is treated separately with the application of the known solutions of the reflection problems for each of them at the free edge of an infinitesimally thin plate. Then allowance is made for the fact that the symmetric and antisymmetric Lamb modes propagate with different velocities in the vicinity of the edge of the wedge. As a result, a difference in the phase shifts, which depends on the wedge angle θ, builds up between these modes in propagation from the driving point to the reception point. This phase difference is responsible for the experimentally observed[1] multiple oscillations of the moduli of the reflection coefficient $|R|$ and the transmission coefficient $|T|$ as a function of θ for a Rayleigh wave in normal incidence on the edge. Simple geometrical considerations are also employed in Ref. 3 to analyze the case of oblique incidence of a Rayleigh wave on the edge at small angles α. The oblique-incidence problem is reduced to the problem of normal wave incidence on a wedge with an equivalent wedge angle θ' smaller than θ. The possibility of oscillations of $|R|$ and $|T|$ as a function of the angle of incidence θ for a fixed wedge angle α is predicted in this work.

In the present article we investigate the reflection of a Rayleigh wave from the edge of a wedge theoretically for the case of oblique incidence at arbitrary angles α. We also give the results of experimental work on the investigated effect.

As in the case of normal incidence, we proceed from the representation of the incident Rayleigh wave by the sum of a symmetric (longitudinal) and an antisymmetric (flexural) Lamb mode propagating in a plate of variable thickness h. The Rayleigh-wave reflection and transmission coefficients can also be expressed in terms of the difference in the total phase shift $\Delta\phi(\alpha) = \phi_a(\alpha) - \phi_s(\alpha)$ of the antisymmetric and symmetric modes during their propagation to the edge of the wedge and back again, and in terms of the difference in the phase shifts $\Delta\Phi(\alpha) = \Phi_a(\alpha) - \Phi_s(\alpha)$ associated with the reflection of each mode separately from the edge.[3] We calculate the phase shifts $\phi_a(\alpha)$ and $\phi_s(\alpha)$ in the geometrical-optics (geometrical-acoustics) approximation, which generalizes the WKB approximation used in Ref. 3 to the case of oblique incidence. These calculations, of course, are more complicated than in the case of normal incidence.

First of all, it must be borne in mind that the symmetric and antisymmetric modes are both subjected to refraction as they approach the edge, where the nature of their refraction differs as a result of the difference in the laws governing the variation of their velocities near the edge. The velocity of the antisymmetric (flexural) mode decreases from the Rayleigh-wave velocity c_R to zero (as $h \to 0$), so that this mode approaches the edge practically in the normal direction (Fig. 1). The velocity of the symmetric (longitudinal) mode, conversely, increases in the direction of decreasing h from the velocity c_R to the so-called "plate" velocity $c_p = 2c_t(1 - c_t^2/c_\ell^2)^{1/2}$, where $c_{\ell,t}$ denotes the velocities of longitudnal and shear bulk waves. As a result, the symmetric mode is incident on the edge at a more oblique angle. Second, it must be taken into consideration that the ray path of the symmetric mode has a turning point at a certain angle of incidence α_0, i.e., the symmetric mode on longer reaches the edge (Fig. 1). Consequently, rays with angles of incidence $\alpha \geq \alpha_0$ from a simple caustic, at which the symmetric mode acquires an additional (caustic) phase shift $-\pi/2$ in reflection from it.[4]

Using the well-known geometrical-optics (geometrical-acoustics) relation for the phase of a wave propagating in a medium that is inhomogeneous in one direction (see, e.g., Refs. 4 and 5), we can write an equation for the difference in the phase shifts of the symmetric and antisymmetric modes excited and received at a point infinitely far from the edge:

$$\Delta\varphi(\alpha) = -2k_R \left\{ \int_0^\infty \left[\frac{k_a^2(x)}{k_R^2} - \sin^2\alpha \right]^{1/2} dx \right. - \tag{1}$$

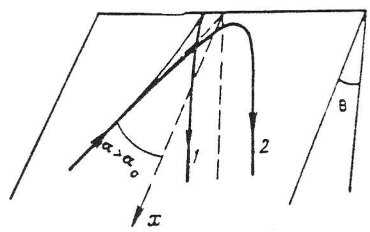

FIG. 1. Oblique incidence of a Rayleigh wave on the edge of a wedge. 1) Antisymmetric mode; 2) symmetric mode.

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$$-\int_{x_{\Pi}(\alpha)}^{\infty}\left[\frac{k_s^2(x)}{k_R^2}-\sin^2\alpha\right]^{1/2}dx\Big\}.$$

Here $k_{s,a}(x)$ denotes the wave numbers of the symmetric and antisymmetric modes as a function of the coordinate x, which is directed away from the edge of the wedge; k_R is the Rayleigh wave number in a half-space; and $x_t(\alpha)$ is the coordinate of the turning point. Making use of the fact that $h(x, \theta) = 2x\tan(\theta/2)$, we can rewrite Eq. (1) in the convenient form

$$\Delta\varphi(\alpha)=-2\delta(\alpha)/\mathrm{tg}(\theta/2),$$

$$\delta(\alpha)=\frac{k_R}{2}\Big\{\int_0^{\infty}\left[\frac{k_a^2(h)}{k_R^2}-\sin^2\alpha\right]^{1/2}dh$$

$$-\int_{h_{\Pi}(\alpha)}^{\infty}\left[\frac{k_s^2(h)}{k_R^2}-\sin^2\alpha\right]^{1/2}dh\Big\}. \quad (2)$$

The functions $k_{s,a}(h)$ do not have analytical expressions, and so it is often necessary in calculations to use suitable approximations for the corresponding dispersion curves calculated by numerical methods (see, e.g., Refs. 6 and 7). We use the approximations proposed in Ref. 3. The value of the local wedge thickness $h_t(\alpha)$ characterizing the turning point of the symmetric mode is determined from the equation

$$k_s^2(h_{\Pi})/k_R^2-\sin^2\alpha=0. \quad (3)$$

The value of the critical angle α_0, at which this point emerges, corresponds to the replacement of $k_s(h_t)$ in Eq. (3) by $k_s(0) = k_p = \omega/c_p$. It follows from this result, in particular, that $\alpha_0 \approx 32°$ for aluminum.

We now discuss the behavior of the phase shifts $\Phi_{s,a}(\alpha)$ of the symmetric and antisymmetric modes in the reflection of each from the edge of the wedge for the case of oblique incidence. Inasmuch as the antisymmetric mode is incident on the edge practically at a right angle due to refraction, its phase shift in reflection does not depend on the angle of incidence α and coincides with the phse shift in the case of normal incidence on a wedge with vertex angle θ, i.e., $\Phi_a = (\pi+\theta)/2$ (Ref. 3). The phase shift for the symmetric mode is readily determined from the solution of the problem of oblique incidence of the symmetric mode on the free edge of a thin plate This problem is exactly analogous to the problem of oblique incidence of a plane longitudinal wave on the boundary of an elastic half-space. It follows from the solution that the value of $\Phi_s(\alpha)$ is identically zero at $\alpha < \alpha_0$ (we recall that only under this condition does the symmetric mode reach the edge), as in the case of normal incidence.[7] The modulus of the symmetric-mode reflection coefficient, however, differs from unity for $0 < \alpha < \alpha_0$, because part of the energy of the symmetric mode is converted into the energy of shear SH waves, which are generated in reflection and are polarized in the plane of the wedge. The difference does not exceed ~10% for the majority of materials in the investigated range of angles, and this is consistent with the error limits of the approximate theory used here. We shall therefore ignore the indicated difference from now on. When $\alpha \geq \alpha_0$, i.e., when a turning point is present, SH waves are not generated. In this case, as mentioned, the reflected symmetric node acquires a caustic phase shift equal to $-\pi/2$.

The expression for the modulus of the coefficient of reflection of a Rayleigh wave from the edge of a wedge in oblique incidence can be written as follows in light of the foregoing considerations:

$$|R|=\left|\sin\left[\frac{\delta(\alpha)}{\mathrm{tg}(\theta/2)}-\frac{\pi-\theta}{4}+\frac{\Delta\psi_k}{2}\right]\right|, \quad (4)$$

where $\Delta\psi_k$ is the caustic phase shift, which is equal to zero for $\alpha < \alpha_0$ and is equal to $-\pi/2$ for $\alpha \geq \alpha_0$. It follows from Eq. (4) that the function $R(\theta, \alpha)$ has a complex oscillating behavior with respect to both θ and α. When $\alpha = 0$, we obtain the equation for the reflection coefficient in normal incidence[3] from Eq. (4):

$$|R|=\left|\sin\left[\frac{\delta}{\mathrm{tg}(\theta/2)}-\frac{\pi-\theta}{4}\right]\right|, \quad (5)$$

where

$$\delta=\delta(0)=(1/2)\int_0^{\infty}[k_a(h)-k_s(h)]dh.$$

In the case of small angles α (smaller than α_0), the expression for $\delta(\alpha)$ from Eqs. (2) can be simplified by replacing the sines with their arguments and expanding the square roots in a power series, which are then restricted to the first two terms. Here

$$\delta(\alpha)\approx\frac{1}{2}\Big\{\int_0^{\infty}[k_a(h)-k_s(h)]dh+\frac{\alpha^2}{2}$$

$$\times\int_0^{\infty}[k_a(h)-k_s(h)]\frac{k_R^2}{k_a(h)k_s(h)}dh\Big\}. \quad (6)$$

According to the theorem of the mean, which is well known in analysis, the second integral in Eq. (6) can be represented in the form $f(h_0)\int_0^{\infty}[k_a(h)-k_s(h)]dh$, where $f(h_0) = k_R^2/k_a^0(h_0)k_s(h_0)$ is the value of the function $f(h)$ at some point h_0 belonging to the domain of integration. A numerical calculation using approximative functions $k_{s,a}(h)$ shows that $f(h_0) \approx 0.75$. Making use of the fact that $(1/2)\int_0^{\infty}[k_a(h)-k_s(h)]dh=\delta(0)$, we can now write Eq. (6) in the form

$$\delta(\alpha)\approx\delta(0)(1+0.75\alpha^2/2). \quad (7)$$

We note that the function (7) differs somewhat from the result obtained[3] for small α by reduction of the problem of oblique incidence of a Rayleigh wave on a wedge at small angles α to the problem of normal incidence on an equivalent wedge with angle $\theta' = 2\tan^{-1}[\tan(\theta/2)\cos\alpha]$. But the coefficient in front of $\delta(0)\alpha^2/2$ in Ref. 3 is equal to unity. This implies that the influence of refraction is considerably smaller in the first approximation with respect to α^2.

The approximation (7) becomes invalid for angles $\alpha \gtrsim \alpha_0$, and it is necessary to carry out a complete numerical calculation of the function $\delta(\alpha)$ and the quantity $|R|$. We have performed such a calculation on a computer for an aluminum wedge for two vertex angles $\theta = 30$ and $60°$. The calculated function $\delta(\alpha)$ was multiplied by a correction factor of 0.915, which was determined from the condition of exact agreement of the theoretical value of $|R|$ in normal incidence on a wedge with angle $\theta = 30°$ and

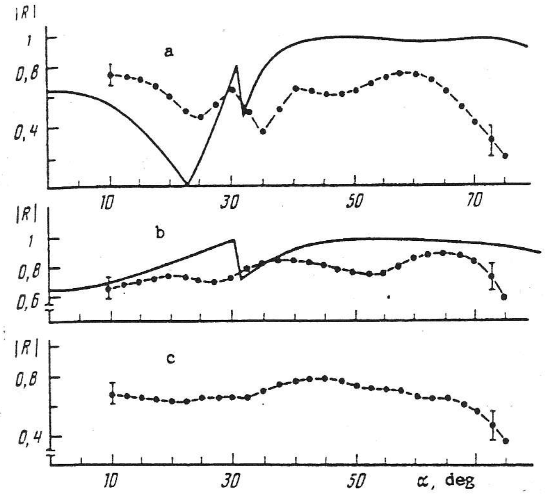

FIG. 2. Theoretical and experimental graphs of the modulus of the reflection coefficient $|R|$ vs angle of incidence α. a) θ = 30°; b) 60°; c) 90°.

the experimental value for this case.[1] This corresponded to replacement of the value $\delta(0) = 2.75$ calculated in Ref. 3 according to the approximate functions by the quantity $\delta(0) = 2.52$. Such a correlation was necessitated by the establishment of the initial correspondence in a comparison of the calculations with the above-described experiments on Rayleigh-wave reflection from the edge of a wedge in oblique incidence. The comparison would be meaningless if the theoretical values were already in disagreement with the experimental at $\alpha = 0$.

An aluminum prism with a height of 16 cm and a base in the shape of a right triangle with 30, 60, and 90° angles was used for the experimental determination of the modulus of the reflection coefficient $|R|$ as a function of the angle of incidence α. This object enabled us to measure the functions $|R(\alpha)|$ for three values of the angle θ on a single sample. The measurement were carried out in the pulsed regime at a frequency of 1 MHz; the pulse duration was ~10 μs. Rayleigh waves were generated and received by means of two Plexiglas wedge transducers, which were moved along the surface of the sample and were bonded to it acoustically through an epoxy resin layer. The diameters of the longitudinally vibrating piezoceramic wafers on the wedge transducers were equal to 1 cm. All the measurements were repeated several times and were processed statistically.

The experimental curves of $|R(\alpha)|$ for θ = 30 and 60° are shown in Fig. 2 together with the corresponding theoretical curves calculated according to Eqs. (2)-(4). We see that the curves undergo oscillations as α is varied, and the amplitude of the oscillations is smaller for θ = 60° than for θ = 30°.

Figure 2 also shows the experimental curve of $|R(\alpha)|$ for θ = 90°, which is practically nonoscillating. Calculations were not carried out for this case, because it exceeds the limits of validity of the given theory.

A comparison of the experimental data with the results of the calculations for θ = 30° indicates that they are in satisfactory qualitative agreement, as witnessed by the good correspondence between the positions of the maxima and minima of the compared curves. The theoretical curve for the case θ = 60° only very remotely resembles the experimental curve. This is what we should expect insofar as the given value of θ corresponds to the limit of validity of the given theory, which holds in the interval $0 \leq \theta \leq 60°$, i.e., for sufficiently sharp wedges.

We call attention to the fact that the occurence of the caustic phase shift for the symmetric mode (at $\alpha \geq 32°$) has a strong influence on the behavior of the theoretical curves of $|R(\alpha)|$; in particular, it induces an abrupt variation of $|R(\alpha)|$ at $\alpha \sim 32°$. It is readily discerned from Fig. 2 that a similar anomaly is clearly observed in the experiment for the case θ = 30°. In contrast with the theory, however, the experiment yields a smoother decay of the reflection coefficient at $30 < \alpha < 35°$. This result can be explained in part by the fact that the theory involves plane surface waves, whereas all the investigated processes in the experimental work took place for cylindrical waves, from which the receiving transducer extracted a narrow bundle of rays with an angular width of ~3°. This tended to average the experimental curves over the indicated intervals. It is important to note that the caustic phase shift does not occur in experiments in the usual practical situations, e.g., in problems of ocean acoustics, and so it does not have any practical significance. The above-considered case of Rayleigh-wave reflection from the edge of an elastic wedge, where this shift affects the modulus of the reflection coefficient, is unique in this respect by virtue of the presence of a reference wave (the antisymmetric mode) that is not reflected from the caustic.

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