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# INVESTIGATION INTO THE PHASE CHARACTERISTICS OF WAVE FIELDS

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## Abstract

The aim of the work reported in this paper is to establish a method of determining the diffusivity of an SEA subsystem using the phase of transfer functions. Specifically, this paper reports an investigation into the phase characteristics of transfer functions obtained using two flat plate structures. In the first experiment the phase characteristics of bending waves in a directional field are obtained using a perspex plate with additive damping. In the second experiment the phase characteristics of bending waves in a diffuse field are obtained from a freely suspended steel plate. The experimental data are normalised and compared to theoretical formula and limits for the diffuse field and direct field proposed.

## INTRODUCTION

Statistical Energy Analysis (SEA) is a probabilistic, energy based approach for modelling the high frequency vibrations of a structure. To construct an SEA model, the structure is conceptually divided into a number of subsystems. By measuring the energy level in each subsystem an energy balance around the structure can be established. The energy level in a given subsystem is obtained by forming an average from a number of vibration measurements at different locations on the subsystem. For the averaging procedure to be valid, a uniform energy density is assumed within the subsystem. However, in many practical structures the energy density in a subsystem may not be uniform. Hence, there is a need to establish a method of determining whether the energy density of a given subsystem is uniform. From the viewpoint of wave fields, the concept of uniform energy density is linked to that at a diffuse field. A diffuse field consists of both outgoing and reflected waves. In a totally diffuse field, waves propagate in all directions. In contrast, in a directional field the vibrational waves travel directly away from the source location.

The aim of the work reported in this paper is to establish a method to determine the diffusivity of a SEA subsystem using the phase of transfer functions. Previous research into the phase characteristics of transfer functions has included a study of the various factors affecting the phase in multi-degree of freedom systems [1] and the study of phase in beam framework

structures [2]. In this paper the phase of bending waves in two different structures, one containing a diffuse field and the other containing a direct field are investigated. The experimental data are compared to theoretical formula and limits for the direct field and the diffuse field proposed.

## THEORY

A directional field consists of outgoing, propagating waves only. There are no reflected waves as the acoustically large dimensions or high damping in the structure cause the waves to die out before they are reflected back to the source. In a directional field, the energy within the system will not necessarily be the same at any two locations.

A diffuse field consists of both outgoing and reflected waves. It assumes that at any point in the system, there are an infinite number of propagating waves in all directions, all contributing to the energy within that system. Reflected waves are due to the boundaries of the system or changes of impedance within the system. An idealised diffuse field has equal wave intensity in all directions at any point, with a resultant intensity of zero.

When a system is excited, waves will travel across the system and when the waves reach a boundary, or changed impedance in the system, they will be reflected. Therefore, the vibrational energy of every point includes two parts: incident waves and reflected waves. Near the excitation, the incident wave or directional field will dominate, and near the boundary, the reflected waves will become more important, as illustrated in Figure (1).

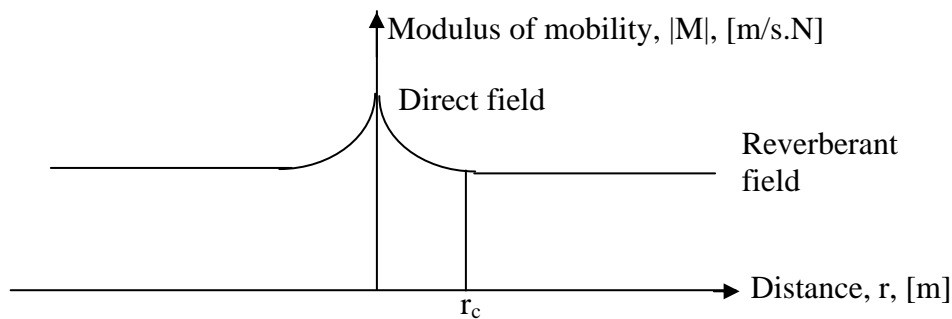


Figure (1). Relationship between mobility and distance.

The direct field is equal to the reverberant field when the critical distance,  $r_c$ , is [3]

$$r_c = \frac{\eta \omega A}{2\pi c_g} \quad (1)$$

where  $\eta$  is the loss factor,  $A$  the area of the structure and  $c_g$  the group velocity.

### Directional field

In situations where all the waves are outgoing and there are no reflected waves, then a plate can be considered to be of "infinite" extent, that is, only the directional field exists. The point mobility of an "infinite" plate is given by [3,4]

$$M_0 = \frac{1}{8\sqrt{B\mu}} \quad (2)$$

where the Bending Stiffness,  $B = \frac{Eh^3}{12(1-\nu^2)}$ , and the mass per unit area  $\mu = \rho h$ . Thus, the point mobility is a real constant and the phase between the velocity and the force is zero. The point mobility will be the same at any excitation location on the plate.

The transfer mobility is given by [4]

$$M_{QR} = M_0 \cdot \sqrt{\frac{2}{rk\pi}} \cdot e^{-i(kr - \frac{\pi}{4})} \quad (3)$$

where  $r$  is the distance between the excitation and response positions, and  $M_0$  is the corresponding point mobility of the plate. Thus, the transfer mobility consists of both amplitude and phase terms. The modulus of the transfer mobility decreases with increasing distance,  $r$ , and increasing wavenumber  $k$ . The phase at any point,  $r$ , is dependant upon the propagation phase,  $kr$ , less a constant phase shift of  $\pi/4$ . The transfer mobility will be the same for any two points separated by the same distance,  $r$ .

### Diffuse field

For a finite plate, both the directional field and the reverberant field exist. The point mobility of a finite plate varies according to the excitation location and with frequency. However, the average point mobility, whether a frequency average or spatial average, is the same as the point mobility of infinite plate [5], and is given by equation (2). The modulus of the transfer mobility is given by

$$|M_{QR}| = \sqrt{\frac{\text{Re}\{M_0\}}{\omega\eta m_0}} \quad (4)$$

where  $\text{Re}\{M_0\}$  is the real part of the point mobility and  $m_0$  is the total mass of the plate. For an idealised diffuse field the phase is given by [1]

$$\phi_{QR} = -\frac{\pi}{2} N_p \quad (5)$$

where for a flat plate the number of resonances below frequency  $\omega$