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The influence of the interface coefficient of friction upon the propensity to judder in automotive clutches

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Abstract: This paper presents an investigation of the driveline torsional vibration behaviour, referred to as judder, which takes place during the clutch engagement process, particularly on small trucks with diesel engines. A non-linear multibody dynamic model of the clutch mechanism is employed to study the effect of various clutch system and driveline components on the clutch actuation performance. The paper demonstrates that judder is affected by driveline inertial changes, variation in the coefficient of friction, μ, of the friction disc linings with slip speed, v, and the loss of clamp load. The results of the simulations show that various friction materials with different μ–v characteristics produce torsional self-excited vibrations of the driveline. The results also show that loss of clamp load relating to the speed of clutch actuation also contributes to judder. Furthermore, it is shown that the simulation results conform closely to the experimental findings.

Keywords: judder, clutch, dynamic model, torsional vibrations, driveline

NOTATION

c	effective drivetrain damping
F_f	friction force
F_n	clamp force
i_{dif}	differential ratio
i_{gbx}	first gear ratio
J	mass moment of inertia
J_{red}	reduced mass moment of inertia
k	effective drivetrain stiffness
m	mass of vehicle
M_f	friction torque
R	mean radius of friction lining
R_e	outer radius of friction lining
R_i	inner radius of friction lining
v	slip speed
θ	relative angle between the friction disc and the hub
μ	coefficient of friction
ω	relative angular slip velocity

The MS was received on 20 November 1997 and was accepted after revision for publication on 9 October 1998.
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Abbreviations

brg	bearing
cbll	clutch cable
crks	crankshaft
cvr	clutch cover
diff	differential
fdsc	friction disc
flw	flywheel
gbx	gearbox
hsg	bell housing
lvr	release lever
prpl	pressure plate
qua	clutch pedal quadrant
sft	transmission input shaft
slv	sleeve
tors_damp	torsional damper
vhc	vehicle
whe	wheel

Joints/constraints

coup	coupler
cvcv	curve to curve constraint
cy	cylindrical
fx	fixed
inp	in-plane
rv	revolute
tr	translational

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1 INTRODUCTION

The combustion process in engine cylinders induces a torsional fluctuation on the crankshaft rotational speed. Engine vibrations are transmitted to the passenger compartment through the engine mounts and through the driveline components. The clutch system, mounted between the flywheel and the gearbox, influences the driveline vibrations and noise perceived by the driver. These cannot be totally eliminated. However, it is expected that clutch design should make the necessary provisions in order to reduce noise and vibrations to an acceptable level.

Clutch judder is a back and forth vibration of a vehicle in the frequency range 5–20 Hz, caused by the torsional oscillations of the driveline that occur during the clutch engagement process, usually in the start-up process. Judder is essentially considered to be influenced by frictional characteristics of the clutch. It is also related to the inertia of the driveline. The severity of the clutch judder phenomenon is influenced by the way the vehicle is driven.

The modelling of the engagement process has been studied by different authors. Jania [**1**] presents equations of the transmitted torque during clutch engagement and an analysis of the performance of friction clutches. Lucas and Mizon [**2**, **3**] built a model of clutch engagement that incorporated the coefficient of friction as a function of rubbing speed, temperature and load and represented driver behaviour in the manner in which the clutch is operated and the engine throttle is applied. However, their study does not deal with the clutch judder problem.

The vibrations induced by dry friction have been studied by Jarvis and Mills [**4**]. By means of numerical analysis they showed, theoretically, that the variation in the coefficient of friction with the relative velocity is insufficient to cause vibrations and that the instability is due to the manner in which the motions of the components take place. The self-excited oscillations that occur when two elastic half-spaces are sliding against each other with a constant coefficient of friction has been studied by Adams [**5**]. He concluded that self-excited oscillations exist for a wide range of material combinations, friction coefficients and sliding speeds. The self-oscillations of a mechanical system containing an engine and a friction clutch can be simulated using the theoretical model proposed by Plakhtienko and Yasinskii [**6**], whose results were confirmed by computer simulations.

The relationship between the coefficient of friction and the relative velocity has been studied extensively by Armstrong-Hélouvry [**7**, **8**]. Heap [**9**] considers the coefficient of static friction only as a function of pressure, while the coefficient of kinetic friction is considered as a function of pressure and velocity. The static and the dynamic coefficients of friction and their variation have also been studied by Herscovici [**10**]. Raghavan and Jayachandran [**11**] considered that the coefficient of friction varies with the sliding velocity, as well as with the number of clutch engagements, the generated contact pressure and temperature.

Kani *et al.* [**12**] have proposed that judder is significantly related to the μ–v characteristics (where μ is the coefficient of friction and v is the slip speed) of an interface friction material. Using an experimental tester, they found that $d\mu/dv$ has a negative gradient when judder occurs and that the value of $d\mu/dv$ depends on the type and the amount of film formed on the friction surface. Maucher [**13**] also studied the basic principles governing the vibrations that occur in the clutch system owing to the frictional characteristics of the clutch facing, i.e. the damping value, the clamp load, the mass moment of inertia and the torsional spring rate of the drivetrain. He concluded that frictional vibrations occur in the presence of low drivetrain damping values and a negative gradient of the coefficient of friction. Drexl [**14**] found that, when judder occurred, the lowest natural frequency of his rigid body model was excited. The simulation results showed that a negative value of the variation in the coefficient of friction with slip speed induced self-excited oscillations, while a positive gradient of the coefficient of friction versus slip speed (or differential speed) exhibited a damped vibration response. Newcomb and Spurr [**15**] agree that, although most published work shows that judder has generally been attributed to a particular type of variation in the coefficient of friction with slip speed, this is not a necessary condition for judder to occur. Using a dynamic model of the clutch, Jarvis and Oldershaw [**16**] concluded that judder was a resonance of the system that was excited at the frequency of slipping of the driven plate. Rabeih and Crolla [**17**, **18**] developed a mathematical model including torsional vibrations of the driveline, vehicle body fore-aft vibrations and vertical vehicle vibrations and concluded that high values of system damping tend to discourage self-excited vibrations and that a decreasing gradient of friction causes system instability. Centea [**19**] describes a multiple degrees of freedom non-linear dynamic model of a diesel engine light truck clutch system that incorporates the non-linear friction characteristics of the clutch lining and engine torque characteristics. The numerical investigations reported in [**19**] were instigated by the Ford Motor Company whose extensive on-vehicle observations have shown that judder is a complex phenomenon affected by the gradient of the μ–v characteristics, as also observed in references [**12**] to [**14**], [**17**] and [**18**]. However, these observations show that although these characteristics play a significant role in judder, they are not a necessary condition for judder to occur, as also observed in references [**4**] and [**15**]. In practice, judder has been observed even with a positive gradient of μ–v characteristics, depending on the manner in which the clutch is engaged by the driver and the associated loss of clamp load. Therefore, it is clear that, to study the clutch judder problem, a multibody mechanism model is required in order to incorporate both the clutch pedal effort and the generated clamp load, as well as the transmission route for the clamp load to the pressure plate during the take-up process and in the presence of stick–slip oscillations at the friction material interface. Such a detailed model, not hitherto reported in the literature, is necessary in order to be able to compare on-vehicle observations with simulation results.

This paper reports on some of the findings in reference [**19**] and introduces driver behaviour in the speed of clutch actuation and its effect on the propensity to judder. A simple analytic model is also presented which is used to explain the validity of the simulation results for both the μ–v characteristics and the loss of clamp load.

2 TORSIONAL VIBRATIONS OF CLUTCH

The energy necessary for the motion of a vehicle is transmitted by the engine to the wheels through the flywheel, clutch and the driveline. The clutch takes the energy from the flywheel and transmits it to the driveline. During the engagement process, on the friction surfaces of the clutch the friction torque acts as an engaging force for the driveline. A part of the energy transmitted through the driveline is transformed into other forms of energy by positive damping effects. If for some reason the damping becomes negative, a part of the energy transmitted by the clutch could induce self-excited torsional vibrations of the driveline, contributing to judder.

In the Coulomb friction region, the friction torque, M_f, can be defined as

$$M_f = 2F_f R = 2\,\mu\,F_n\,R \tag{1}$$

where μ is the coefficient of friction, F_n is the clamp load (normal force acting on the friction surfaces) and R is the mean radius of the friction surface, defined by Wilson [**20**] and by Herscovici [**10**] as

$$R = \frac{2}{3}\frac{R_e^3 - R_i^3}{R_e^2 - R_i^2} \tag{2}$$

Equation (1) shows that the friction torque M_f depends on the coefficient of friction μ, the clamp load F_n and the mean radius of the friction surface R. The damping coefficient of the driveline can become negative only if the friction torque has a variation caused by changes in μ, F_n or R. For a constant mean friction radius, the torsional vibrations of the driveline can be caused by a loss of clamp load or by variations in the interface coefficient of friction. For a constant clamp load, a cause of variation for the friction torque M_f is the change in the value of the coefficient of friction during the engagement process.

Clutch engagement occurs gradually, bringing the driveline (through the friction disc) and the crankshaft (through the flywheel and the pressure plate) to the same rotational speed. During engagement, the relative angular velocity of the discs diminishes. It is therefore important to study the variation in the coefficient of friction μ with the relative angular velocity of the clutch discs, ω, by finding the variation in $\mathrm{d}\mu/\mathrm{d}\omega$ during the development of the friction torque M_f.

The gradient of the friction torque against the relative angular velocity $\mathrm{d}M_f/\mathrm{d}\omega$ can be obtained using equation (1), assuming that the clamp load F_n is independent of the slip speed:

$$\frac{\mathrm{d}M_f}{\mathrm{d}\omega} = \frac{\mathrm{d}(2\mu F_n R)}{\mathrm{d}\omega} = 2F_n R\,\frac{\mathrm{d}\mu}{\mathrm{d}\omega} \tag{3}$$

where M_f is the friction torque, ω is the relative rotational speed, μ is the coefficient of friction, F_n is the clamp load and R is the mean friction radius. For a constant mean friction radius R, the gradient of the coefficient of friction against relative rotational velocity $\mathrm{d}\mu/\mathrm{d}\omega$ can be expressed through the variation in the gradient of the coefficient of friction with slip speed $\mathrm{d}\mu/\mathrm{d}v$:

$$\frac{\mathrm{d}\mu}{\mathrm{d}\omega} = \frac{\mathrm{d}\mu}{\mathrm{d}v}\frac{\mathrm{d}v}{\mathrm{d}\omega} = \frac{\mathrm{d}\mu}{\mathrm{d}v}R \tag{4}$$

Using equations (3) and (4), the variation in the friction torque against the relative angular velocity $\mathrm{d}M_f/\mathrm{d}\omega$ can be obtained:

$$\frac{\mathrm{d}M_f}{\mathrm{d}\omega} = 2F_n R\,\frac{\mathrm{d}\mu}{\mathrm{d}\omega} = 2F_n R^2\,\frac{\mathrm{d}\mu}{\mathrm{d}v} \tag{5}$$

where M_f is the friction torque, ω is the relative rotational speed, v is the relative linear velocity at the mean friction radius R, μ is the coefficient of friction and F_n is the clamp load. Equation (5) shows the variation in the friction torque during the engagement process (after the moment when the clamp load reaches a constant value). This can be studied by means of the gradient of the coefficient of friction with slip speed. According to Kani *et al.* [**12**] the general equation of motion of the vehicle during clutch slipping is

$$m\,\ddot{x} + \left[c + \frac{\mathrm{d}F_f(v)}{\mathrm{d}v}\right]\dot{x} + kx = 0 \tag{6}$$

where m is the vehicle mass, c is the damping coefficient of the vehicle, k is the total stiffness, v is the relative speed and $F_f(v)$ is the friction force that depends on the slip velocity. The term $\mathrm{d}F_f(v)/\mathrm{d}v$ represents the damping created by the variation in the coefficient of friction μ with relative velocity v between the clutch facings. The friction force F_f depends on the value of the coefficient of friction μ and also on the clamp load F_n:

$$F_{\mathrm{f}} = \mu F_{\mathrm{n}} \tag{7}$$

Assuming that the clamp load F_{n} is constant, the variation in the friction force F_{f} with the slip speed v becomes

$$\frac{\mathrm{d}F_{\mathrm{f}}}{\mathrm{d}v} = \frac{\mathrm{d}(\mu F_{\mathrm{n}})}{\mathrm{d}v} = F_{\mathrm{n}} \frac{\mathrm{d}\mu}{\mathrm{d}v} \tag{8}$$

The free vibrations of a damped system can be studied using Newton's law, which yields the equation of motion:

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{9}$$

where m is the mass, $\ddot{x}$ is the acceleration, c is the viscous damping coefficient, $\dot{x}$ is the velocity, k is the system stiffness and x is the mass displacement due to spring deflection. The solution of equation (9) can be found assuming that it is in the form

$$x(t) = C\,\mathrm{e}^{st} \tag{10}$$

where C is a constant, s is an exponential coefficient and t is time. Substitution of equation (10) in equation (9) gives the characteristic equation

$$ms^2 + cs + k = 0 \tag{11}$$

The solution to equation (11) is provided by

$$s = -\frac{c}{2m} \pm \sqrt{\left[\left(\frac{c}{2m}\right)^2 - \frac{k}{m}\right]} \tag{12}$$

Substitution of equation (12) in equation (10) gives two solutions. The general solution of equation (9) is obtained by superposition of these two solutions:

$$x(t) = C_1 \exp\left\{-\frac{c}{2m} + \sqrt{\left[\left(\frac{c}{2m}\right)^2 - \frac{k}{m}\right]}\right\} + C_2 \exp\left\{-\frac{c}{2m} - \sqrt{\left[\left(\frac{c}{2m}\right)^2 - \frac{k}{m}\right]}\right\} \tag{13}$$

where C_1 and C_2 are constants that can be determined from the initial conditions of system vibrations.

Using equation (8), the solution to equation (6) is in the form given by equation (13):

$$x(t) = C_1 \exp\left\{-\frac{c + F_{\mathrm{n}}\,\mathrm{d}\mu/\mathrm{d}v}{2m} + \sqrt{\left[\left(\frac{c + F_{\mathrm{n}}\,\mathrm{d}\mu/\mathrm{d}v}{2m}\right)^2 - \frac{k}{m}\right]}\right\} + C_2 \exp\left\{-\frac{c + F_{\mathrm{n}}\,\mathrm{d}\mu/\mathrm{d}v}{2m} - \sqrt{\left[\left(\frac{c + F_{\mathrm{n}}\,\mathrm{d}\mu/\mathrm{d}v}{2m}\right)^2 - \frac{k}{m}\right]}\right\} \tag{14}$$

The solution should be considered in the case of positive and negative damping. If the damping is positive, then

$$c + F_{\mathrm{n}} \frac{\mathrm{d}\mu}{\mathrm{d}v} > 0 \tag{15}$$

The solution form given by equation (14) contains negative exponents. Thus, the displacement history forms an oscillatory decay and converges to a stable cycle for all the gradients of the coefficient of friction with slip speed. If the damping is negative, then

$$c + F_{\mathrm{n}} \frac{\mathrm{d}\mu}{\mathrm{d}v} < 0 \tag{16}$$

Solution bifurcation results depend on the sign of the 'quantity' under the radical in equation (14):

1. If this 'quantity' is positive or equals zero, then

$$\left(c + F_{\mathrm{n}} \frac{\mathrm{d}\mu}{\mathrm{d}v}\right)^2 \geqslant \frac{k}{m} \tag{17}$$

   The exponents in equation (17) are positive and the solution indicates a diverging motion, leading to system instability.
2. If the 'quantity' is negative, then

$$\left(c + F_{\mathrm{n}} \frac{\mathrm{d}\mu}{\mathrm{d}v}\right)^2 < \frac{k}{m} \tag{18}$$

   The exponents in equation (18) are complex conjugates and it can be proved that the solution of the equation of motion includes a diverging oscillatory solution and hence an unstable system can emerge. The solution of the equation of motion applied to the driveline indicates that, if the gradient of the coefficient of friction with slip speed is positive, the damping of the driveline and friction disc system as defined in equation (16) is positive and the system is stable. No self-excited oscillations will occur. Thus, no judder will emerge.

If the gradient of the coefficient of friction with slip speed is negative, the damping of the driveline defined by equation (17) can be positive or negative. If

$$\frac{\mathrm{d}\mu}{\mathrm{d}v} \geqslant \frac{-c}{F_{\mathrm{n}}} \tag{19}$$

then the damping is positive and the system is stable. If

$$\frac{\mathrm{d}\mu}{\mathrm{d}v} < \frac{-c}{F_{\mathrm{n}}} \tag{20}$$

then the damping coefficient of the driveline becomes negative and the vibration system becomes unstable. The system will be self-excited, probably inducing judder. The results obtained experimentally by Kani *et al.* [**12**], Maucher [**13**] and Drexl [**14**] demonstrate that the conclusions obtained from relationships (14) and (15) are correct, showing that, for negative values of the gradient of the coefficient of friction with slip speed, when a certain value is reached the vehicle is more prone to judder.

The value of the damping coefficient for the vehicle [c in equation (20)] is quite difficult if not impossible to obtain. Therefore, it is practically impossible, using equation (20), to find the precise value of the critical damping coefficient or the value of the critical variation in the coefficient of friction with slip speed. However, using simulation techniques it can be shown that, starting from a certain value of the gradient of the coefficient of friction with relative velocity, the torsional vibrations of the driveline have a large enough amplitude to be felt in the passenger compartment as fore and aft vibrations of the entire vehicle.

3 DESCRIPTION OF THE CLUTCH TYPE

The clutch studied is a light truck clutch mounted in the gearbox housing between the flywheel 2 and the input shaft 10, as shown in Fig. 1. The main parts of this clutch are situated between the flywheel and the diaphragm spring. The pressure plate 4 is mounted by the clutch manufacturer to the clutch cover 7 with straps 5. These straps keep the pressure plate and the clutch cover rotating with the same speed and also, through their longitudinal compliance, permit an axial displacement of the pressure plate against the cover. The friction disc 3 is free to float between the flywheel and the pressure plate through a hub splined to the input shaft of the gearbox. The friction disc is pressed between the pressure plate and the flywheel by the clamp force F_n, provided by the diaphragm spring 6. The clutch engagement is obtained through the application of the clamp force provided by the diaphragm spring when the clutch is mounted on to the flywheel.

In the disengagement process, the force applied by the driver to the pedal is transmitted through the pedal quadrant to one end of the cable. The other end of the cable is mounted through a spherical type joint to the release lever 9. The motion of the cable is transferred to the release lever, which rotates and pushes the release bearing 8 against the diaphragm spring fingers. The diaphragm spring pivots on a fulcrum ring which is riveted on to the cover and the clamp force is subsequently reduced. The cushion spring and the straps pull back the pressure plate from the friction disc. The reducing friction torque permits a progressive braking of the torque transmitted by the engine through the flywheel to the driveline. The engagement process is similar to the disengagement process and occurs when the driver decreases the applied pedal force to zero.

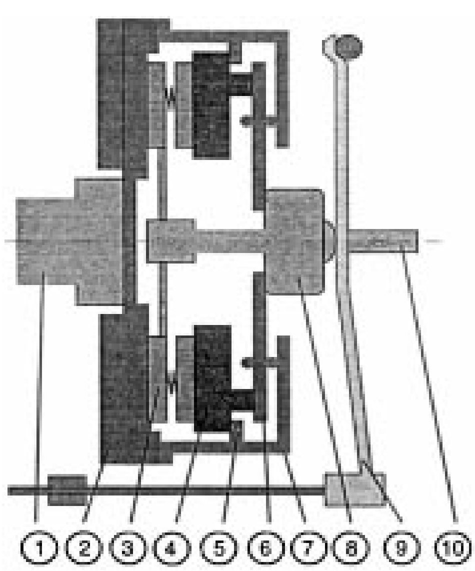

Fig. 1 Main parts of the clutch used for studying the judder phenomenon

4 CLUTCH JUDDER MODEL

The model of the clutch engagement, built in order to study the take-up judder problem, is a multibody nonlinear dynamic model. The parts incorporated in the model, according to the clutch components described in Section 3, are detailed in Table 1. The parts subjected to torsional motion are characterized by their inertial properties. The inertia of the differential has to be reduced to the input shaft of the gearbox according to a first gear ratio of 3.89 using the equation

$$J_{\text{red}} = \frac{J}{i_{\text{gbx}}^2} \tag{21}$$

where J is the inertia, J_{red} is the reduced inertia at the input shaft of the gearbox and i_{gbx} denotes the first gear ratio.

The inertias of the road wheel and of the vehicle are also reduced to the input shaft according to a first gear ratio i_{gbx} of 3.89 and a differential ratio i_{dif} of 4.11 using the equation

$$J_{\text{red}} = \frac{J}{i_{\text{gbx}}^2 \, i_{\text{dif}}^2} \tag{22}$$

In multibody formulation, constraint functions have to be formulated in order to assemble the mechanism. For this purpose the constraint functions, in the form of joints and joint primitives, have to be chosen in a manner that restricts undesired motions. For the clutch and driveline system studied here and subjected to torsional vibrations in the engagement process, the constraints that have been chosen for the model are described in Table 2. Two motion constraints are specified in the dynamic model: the release motion of the pedal (the driver behaviour) and the rotation of the flywheel. The release motion is transmitted in the engagement process to the quadrant, cable, lever, release bearing, pressure plate and to the friction disc material (which in the model is attached to the pressure plate). It starts at the position where the pedal is totally depressed. The displacement of the pedal takes 5 s,

Table 1 Inertial parts in the clutch multibody model

Number	Part name	Abbreviation	Mass (kg)	Inertia (kg m^2)	Ratio	Referred inertia (kg m^2)
1	Crankshaft	crks	(10)*	1	1	1
2	Flywheel	flw	(14.5)	0.25	1	0.25
3	Cover	cvr	(1.6)	0.03	1	0.03
4	Pressure plate	prpl	(4.15)	0.04	1	0.04
5	Friction disc	fdsc	(1.45)	0.065	1	0.065
6	Hub	hub	(1)	0.00001	1	0.00001
7	Shaft	sft	(1.5)	0.0025	1	0.0025
8	Gearbox	gbx	(20)	0.002	1	0.002
9	Differential	dif	(20)	0.045	3.89	0.003
10	Wheels	whe	(10)	2	3.89×4.56	0.0064
11	Vehicle	vhc	(2900)	210	3.89×4.56	0.67
12	Housing	hsg	(10)	–	–	–
13	Sleeve	slv	(0.5)	–	–	–
14	Bearing	brg	0.2	–	–	–
15	Lever	lvr	1.5	–	–	–
16	Cable lvr	cbll	0.2	–	–	–
17	Cable guide	gid	~0.1	–	–	–
18	Cable qua	cblq	~0.2	–	–	–
19	Quadrant	qua	~0.2	–	–	–
20	Pedal	pdl	~1.5	–	–	–
21	Ground	gnd	–	–	–	–

* The numbers in parenthesis provide representative values.

allowing a translational displacement of the pressure plate by 4.5 mm. The engagement starts only in the last 0.7 mm of the pressure plate travel, when the cushion spring is compressed and the induced clamp load (see Fig. 2a) produces the necessary friction torque. The speed of actuation has a profound effect on the history of the clamp load application and, as can be seen later on, can increase the propensity to judder, even with a desired positive slope for the μ–v characteristics. The model incorporates sources of compliance as well as forces and torques, as described in Table 3.

The characteristics of the springs mounted in the friction disc are usually provided by the clutch manufacturer. The characteristics show two levels of stiffness. In order to represent these, the model includes a frictional torque which is dependent on the relative angle θ between the friction disc and the hub and is defined as follows:

$$\begin{aligned}
M &= -k_{1-}\,\theta_{-} - k_{2-}\,\theta && \text{if } \theta < -\theta_{-} \\
M &= -k_{1-}\,\theta && \text{if } -\theta_{-} < \theta < 0 \\
M &= k_{1+}\,\theta && \text{if } 0 < \theta < \theta_{+} \\
M &= k_{1+}\,\theta_{+} + k_{1+}\theta && \text{if } \theta > \theta_{+}
\end{aligned} \tag{23}$$

where M is the torque, k_{1-} is the stiffness of the torsional springs (situated between the friction disc and the hub) on the negative side of the characteristic curve when the angle varies between zero θ_{-}, k_{2-} is the torsional stiffness on the negative side of the characteristic when the angle is smaller than zero, k_{1+} and k_{2+} are the corresponding values on the positive side of the characteristic curve and θ_{-} and θ_{+} are the angles where the characteristics change. The values for all four stiffnesses and both angles are defined as input values in the model. The characteristics obtained by running the model with the torque function described above is shown in Fig. 2b. The parts of the vehicle that have a torsional displacement are presented in Fig. 3.

The values for all of the stiffness components in the model are given in Table 3. The described model has seven degrees of freedom: the angular displacements of the flywheel, friction disc, hub, gearbox, differential,

Table 2 Constraints in the multibody model

Number	Part I	Part J	Constraint type	Constraint name
1	Housing	Ground	Fix	fx_hsg_lvr
2	Flywheel	Housing	Revolute	rv_flw_hsg
3	Friction disc	Flywheel	Inplane	inpl_fdsc_flw
4	Hub	Friction disc	Revolute	rv_hub_fdsc
5	Hub	Shaft	Translational	tr_hub_sft
6	Shaft	Housing	Revolute	rv_sft_hsg
7	Gearbox	Shaft	Revolute	rv_gbx_sft
8	Differential	Gearbox	Revolute	rv_dif_gbx
9	Wheels	Differential	Revolute	rv_whe_dif
10	Vehicle	Wheels	Revolute	rv_vhc_whe
11	Cover	Flywheel	Fix	fx_cvr_flw
12	Pressure plate	Cover	Translational	tr_prpl_cvr
13	Pressure plate	Bearing	Coupler	cou_brg_prpl
14	Bearing	Sleeve	Translational	tr_brg_slv
15	Sleeve	Housing	Fix	fx_slv_hsg
16	Bearing	Lever	Curve–curve	cvcv_brg_lvr
17	Lever	Housing	Cylindrical	cy_lvr_hsg
18	Cable lvr	Lever	Cylindrical	cy_cbll_lvr
19	Cable lvr	Cable guide	Translational	tr_cbll_gid
20	Cable guide	Housing	Spherical	sph_gid_hsg
21	Cable lvr	Cable pdl	Coupler	cou_cbll_cblp
22	Cable pdl	Ground	Translational	tr_cblp_gnd
23	Cable pdl	Quadrant	Rack–pin	rp_cblp_qua
24	Quadrant	Pedal	Fix	fx_qua_pdl
25	Pedal	Ground	Revolute	rv_pdl_gnd
26	Crankshaft	Flywheel	Fix	fx_crks_flw
27	Motion	Pedal	Ground	mo_pdl_gnd

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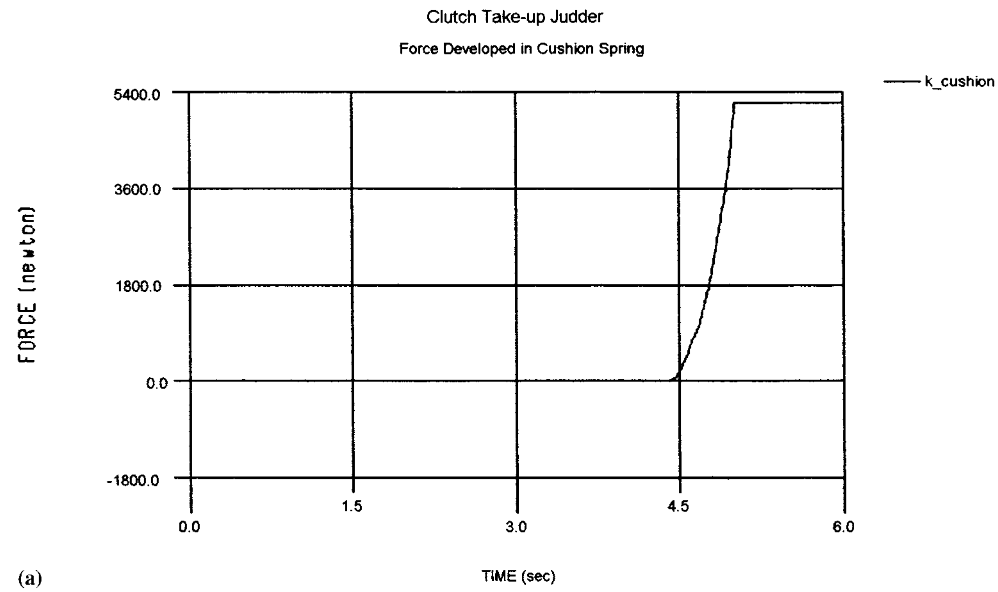
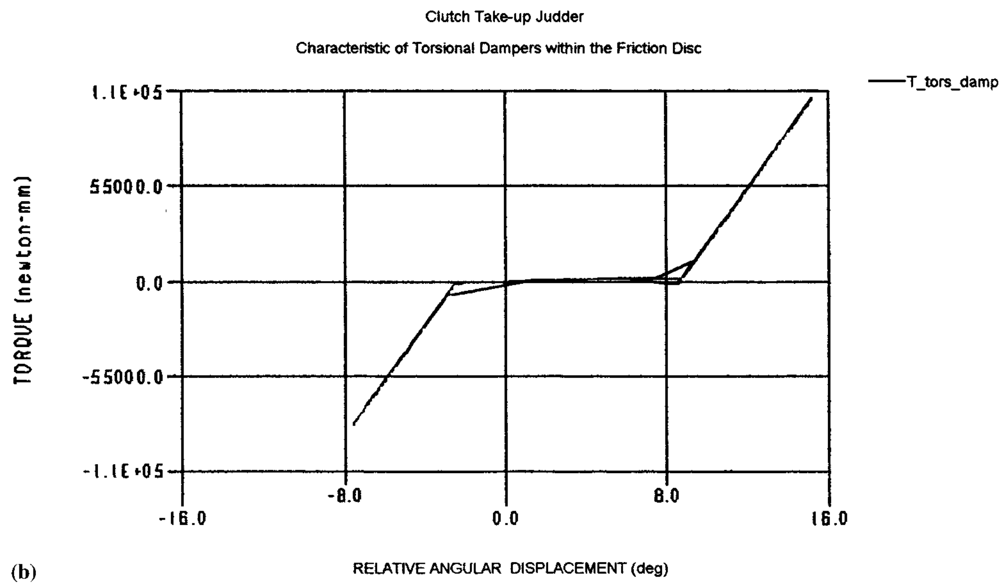


(a)



(b)

Fig. 2 (a) The clamp load–time history. (b) Characteristics of the torsional spring dampers in the model

Table 3 Forces and stiffnesses from the multibody dynamic model

Number	Part I	Part J	Stiffness type	Stiffness name	Stiffness (N m/deg–N/mm)	Ratio	Referred stiffness (N m/deg–N/mm)
1	Hub	Friction_disc	Torsional	k_tors_damp	Equation (23)	1	Equation (23)
2	Gearbox	Shaft	Torsional	k_gearbox	150	1	0.150
3	Differential	Gearbox	Torsional	k_driveshaft	43.5	3.89	140
4	Wheels	Differential	Torsional	k_halfshafts	270	3.89×4.56	0.868
5	Vehicle	Wheels	Torsional	k_tyres	700	3.89×4.56	2.222
6	Pressure_plate	Friction_disc	Transl.	k_cushion	Non-linear	–	–
7	Pressure_plate	Cover	Transl.	k_straps	50	–	–
8	Pressure_plate	Cover	Transl.	k_diaphragm	Non-linear	–	–
9	Bearing	Cover	Transl.	k_fingers	Non-linear	–	–
10	Cover	Housing	Transl.	k_cover	32 000	–	–

wheels and the fore–aft motion of the vehicle (see Fig. 4). The crankshaft, clutch cover and pressure plate have the same displacements as the flywheel. The translational displacement of the actuation route formed by the pedal, quadrant, cable, lever, bearing and pressure plate is not an independent motion because it is governed

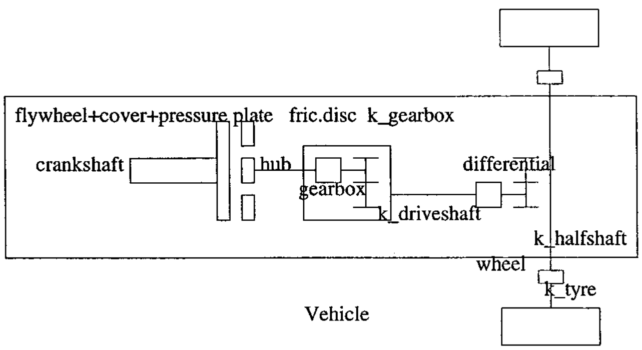


Fig. 3 Parts of the vehicle modelled for studying the torsional vibrations

by the actuation motion defined in Table 2 at position 27. This actuation motion is governed by the driver behaviour during the clutch engagement process.

5 SIMULATIONS RESULTS

The importance of the variation in the interface coefficient of friction with slip speed upon the propensity to judder in automotive clutches has been experimentally established and reported in references [**12**] to [**14**]. The analytical model presented in Section 2 indicates that when the variation in the friction coefficient with slip speed becomes negative the vibration system becomes self-excited, probably inducing judder [see equation (15)]. The non-linear multibody model is therefore employed to verify the behaviour of various measured friction material characteristics in the occurrence of judder, in conjunction with driver behaviour during the engagement process. Practical positive and negative gradients of the coefficient of friction with slip speed, obtained through experimental measurements, are considered.

A simulation run of the clutch judder model has been made for a period of 6 s, with 1024 integration time steps. This sample size (i.e. 1024) together with the simulation run time enables a frequency spectrum of response up to and including a highest frequency contribution of 83.5 Hz to be obtained. This frequency range is sufficient for the investigation of judder. However, the sample size may be altered to include a larger number of steps, thereby capturing an even larger bandwidth of frequencies.

Figure 5a shows the angular velocity of the flywheel and the friction disc during the engagement process. The results indicate a decreasing value for the angular velocity of the flywheel and a corresponding rise in the angular velocity of the friction disc until the two members move in concert with the same angular velocity. The portion of the response prior to the stick region of the friction torque characteristics indicates fluctuations or judder of the driven inertia's angular velocity (i.e. from the friction disc to the vehicle inertia in Fig. 5a). The amplitude of oscillations in this take-up region are governed by the damping characteristics at the friction material interface and the clutch pedal effort, determining the corresponding clamp load history. The required value should include the damping characteristics of all the inertial members of the drivetrain. The amplitude of oscillations is therefore larger than would otherwise be expected (since not all damping characteristics of the drivetrain system are included in the model). However, drivetrain inertia components are usually quite low, thus not significantly affecting the frequency response characteristics of the model but affecting the amplitude of oscillations owing to a gradual logarithmic decrement effect.

Maucher [**13**] has measured a negative gradient of the coefficient of friction $d\mu/dv$ of -0.0075 s/m. The dependence of the coefficient of friction on slip speed is shown to be

$$\mu = -0.0075v + 0.43 \tag{24}$$

where μ is the coefficient of friction and v is the slip speed (m/s).

In order to ascertain the influence of the friction interface on the amplitude of torsional vibrations during the engagement process, eleven analyses have been carried out. The gradients of the coefficient of friction with slip speed that are used in the simulations have been chosen around the value found by Maucher. Therefore, a constant coefficient of friction of 0.43 is considered, as well as positive and negative values of $d\mu/dv$ of 0.004, 0.008, 0.012 and 0.016 s/m.

Figure 5a shows the results of the analysis carried out using a constant value of 0.43 for the coefficient of friction. There are some torsional vibrations of the driveline at a frequency of around 7 Hz (this also being the same frequency obtained experimentally for the modelled 'judder vehicle'). This value can easily be deduced from the time response history of oscillations. Figure 5b shows the results obtained using an increasing slope for of the coefficient of friction with slip speed of 0.004 s/m. The take-up oscillations in the engagement

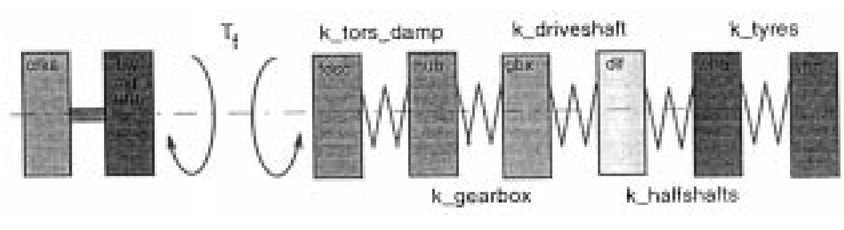


Fig. 4 Clutch model for torsional vibrations

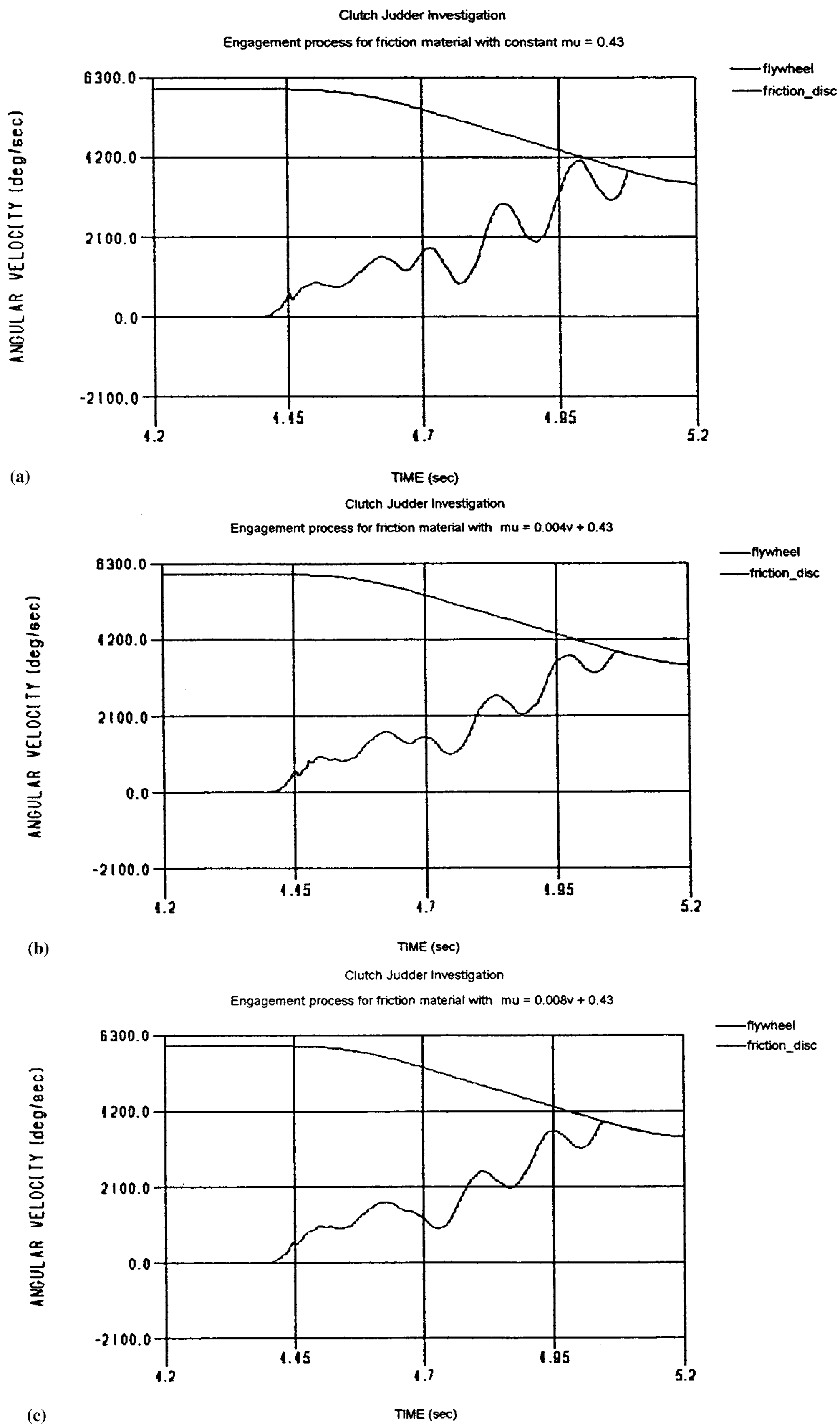
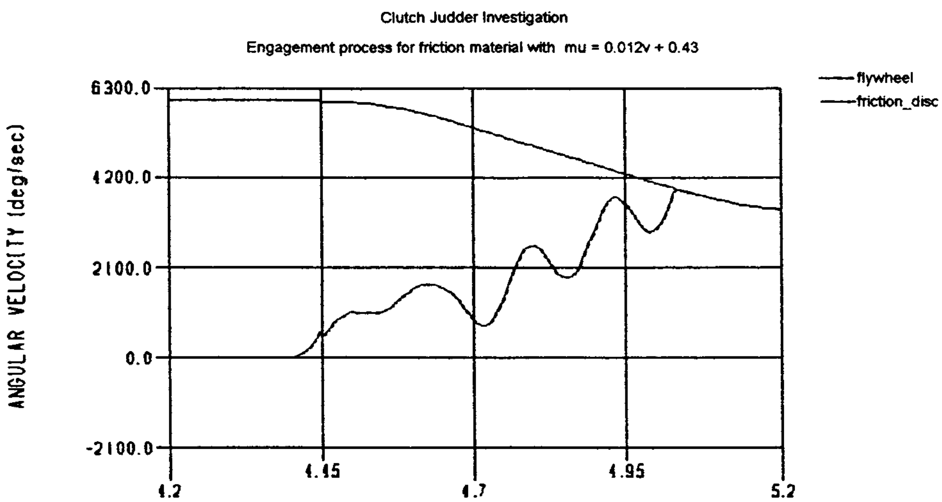
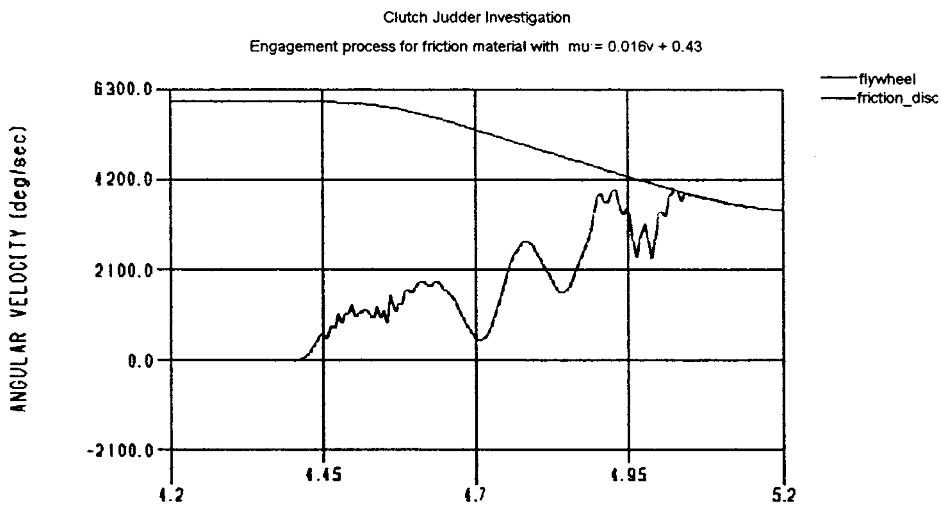


Fig. 5 (continued over)



(d)



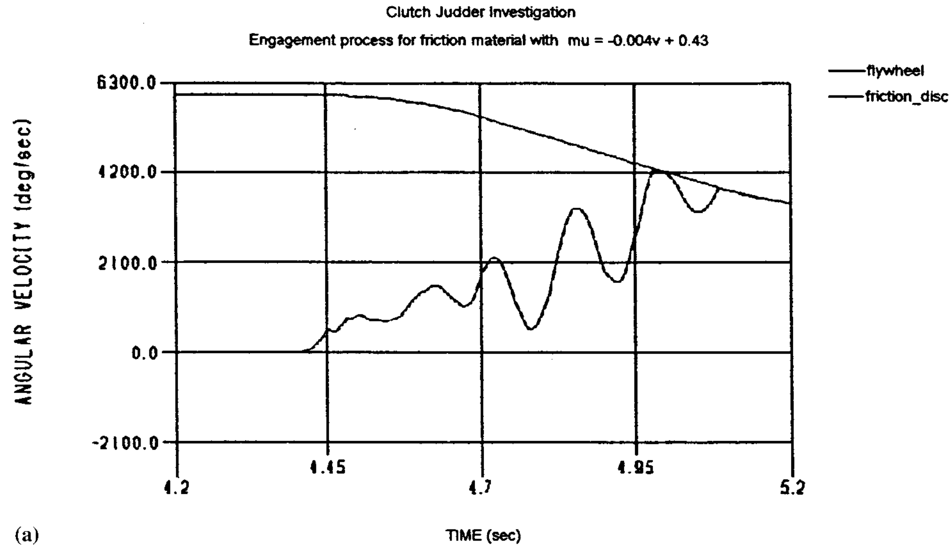
(e)

Fig. 5 Engagement process for a material (a) with a constant coefficient of friction of 0.43 and with a coefficient of friction with a positive gradient of (b) 0.004, (c) 0.008, (d) 0.012 and (e) 0.016 s/m

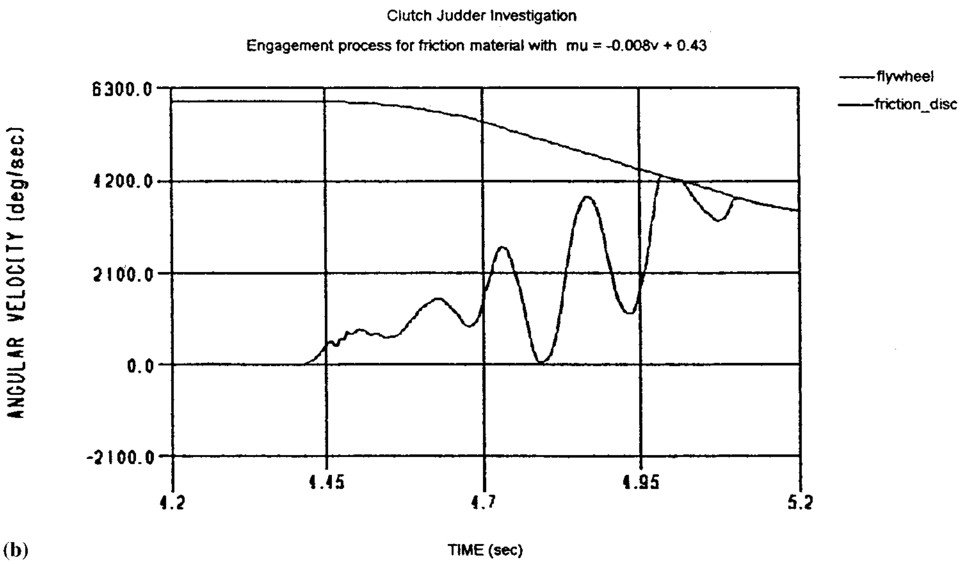
process are still there, but the amplitude is smaller than the case for constant μ (see Fig. 5a) occurring at the same frequency. Obviously a positive gradient of μ produces a damping effect. Therefore, the propensity to judder is diminished. However, it should be noted that a positive gradient for the coefficient of friction with slip speed is not the only condition required to alleviate the clutch judder problem. It should be noted that the overall drivetrain damping and the driver clutch actuation behaviour also play important roles.

The simulations obtained for higher positive gradients of the coefficient of friction [0.008 s/m (see Fig. 5c) and 0.012 s/m (see Fig. 5d)] do not exhibit any significant improvement when compared with a positive gradient of 0.004 s/m (see Fig. 5b). However, the results for the highest positive gradient exhibit a deviation from this trend. At first glance this deviation may be regarded as anomalous. However, it should be noted that the propensity to judder is also directly affected by the driver clutch actuation effort, affecting the clamp load application history. This can be seen by referring to equation (19), where with a certain combination of actuation speed and drivetrain damping (the latter affected by the friction characteristics) the slope $d\mu/dv$ can become less than the ratio $-c/F_n$ as F_n is reduced with hasty driver behaviour. This is best illustrated by comparison of results for various driver actuation speeds, which are described later on. This trend demonstrates that positive gradients of the coefficient of friction reduce the torsional oscillations of the driveline during the engagement process. These findings are in agreement with equation (15) obtained analytically in Section 2. However, it should be noted that a positive gradient for the μ–v characteristics is not the only condition guarding against the propensity to judder, as described below.

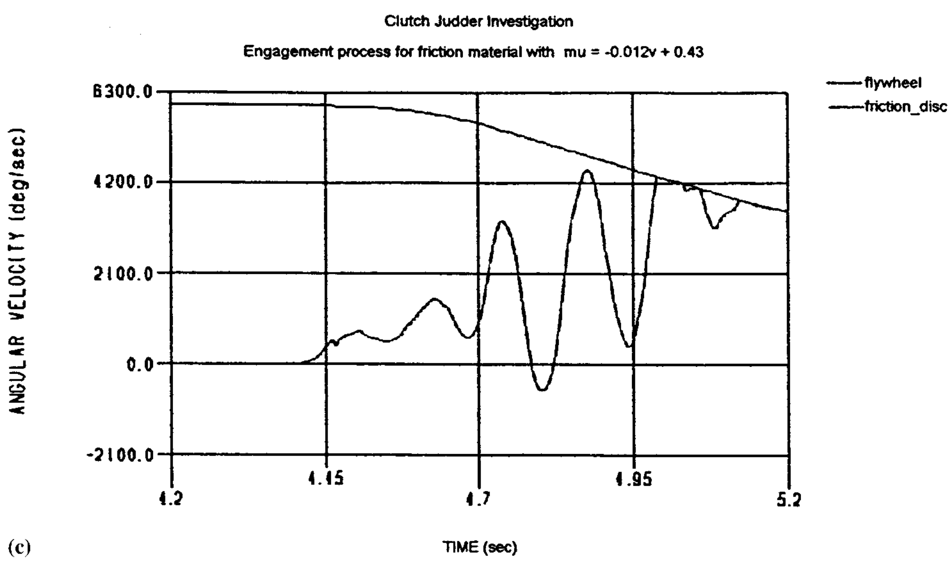
Figure 6a shows the numerical output using the negative gradient $d\mu/dv = -0.004$ s/m. The amplitudes of oscillations are considerably larger than those for the



(a)

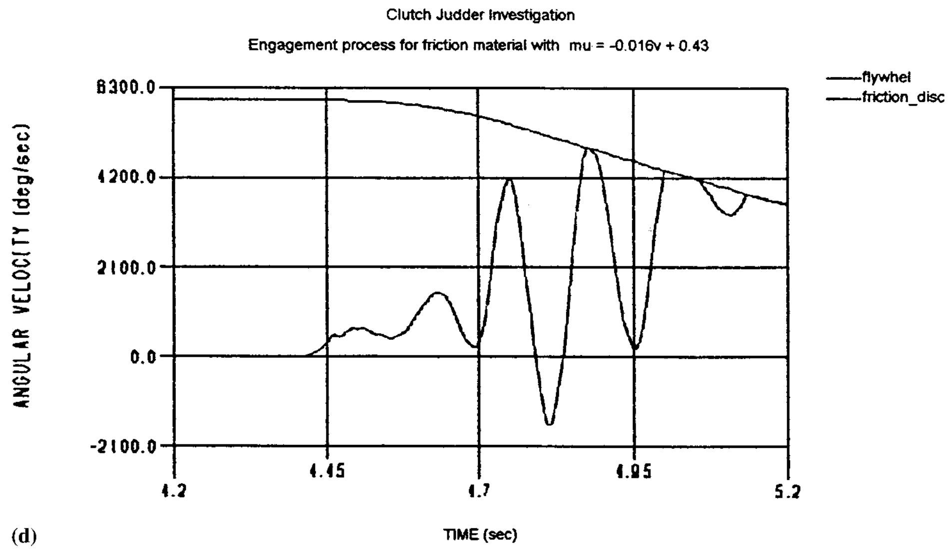


(b)



(c)

Fig. 6 (continued over)



(d)

Fig. 6 Engagement process for a material with a coefficient of friction with a negative gradient of (a) -0.004, (b) -0.008, (c) -0.012 and (d) -0.016 s/m

case of a constant coefficient of friction (see Fig. 5a), indicating a strong tendency to judder. The oscillations that occur in the engagement process can be seen in any part of the driveline modelled. The vehicle undergoes torsional oscillation at tyre contact patches at the same frequency of 7 Hz. These vibrations will also be felt by the driver and physically occur under judder conditions.

The simulations obtained for negative gradients of the coefficient of friction [-0.008 s/m (see Fig. 6b), -0.012 s/m (see Fig. 6c) and -0.016 s/m (see Fig. 6d)] show that the amplitude of the torsional vibrations of the driveline occur around 7 Hz during the engagement process and increase with larger negative values of the gradient of the coefficient of friction with slip speed. These findings are in agreement with equation (20) obtained analytically in Section 2.

Clutch judder is also considered to be dependent upon the manner in which the clutch is actuated. In order to see the response of the multibody model in respect of different clutch actuations, two analyses have been carried out. In one, the actuation speed of the clutch pedal is halved. The simulation results for a positive gradient of the coefficient of friction (see Fig. 7a) show that the amplitude of the take-up torsional oscillations has a much lower value than for the case of the higher actuation speed (see Fig. 5e). Similar results are also obtained in the case of a negative gradient of the coefficient of friction (see Fig. 7b), when compared with the results obtained for the same gradient but with the normal clutch actuation speed. However, even if in both cases the amplitude of torsional vibration is found to be lower, the simulations show that in the case of positive gradients the amplitude of oscillations is quite low and is unlikely to be transmitted through the drivetrain and therefore will not induce judder. Now, referring back to the argument that a positive gradient of the coefficient of friction is one, but not the only condition for the diminution of judder, one can observe that the results for the highest value of $d\mu/dv$, which can be regarded as anomalous by itself, are in fact indicative of the interactive nature of the driver behaviour and the friction interface conditions. These findings are in keeping with the suggestions made in references [**4**] and [**15**] and with on-vehicle observations. Furthermore, it is commonly experienced by most drivers when the clutch is engaged in a hasty manner. A partial loss of clamp load can ensue under these conditions. The numerical results presented here conform well with a significant amount of on-vehicle tests, showing the influence of both friction lining material and driver behaviour upon judder on the basis of subjective ratings given by test drivers.

6 CONCLUSIONS

The current model indicates clutch take-up judder when the engaging inertias are slipping with respect to one another. Judder can be initiated by the friction material characteristics owing to an overall reduction in the driveline damping. This argument is corroborated by the fact that, in all simulations obtained numerically here and experimentally measured in tests at Ford, the response frequency is found to be 7 Hz.

The response frequency of 7 Hz is readily transmitted to the vehicle, as shown by the results of all the simulations, and is uncomfortably close to many other significant vehicle driveline frequencies such as that of tip-in and back-out at approximately 5–6 Hz and driveline shuffle at around 3–5 Hz. In a sister study carried out on driveline vibration for the same vehicle, a coupling action of axial and torsional modes was

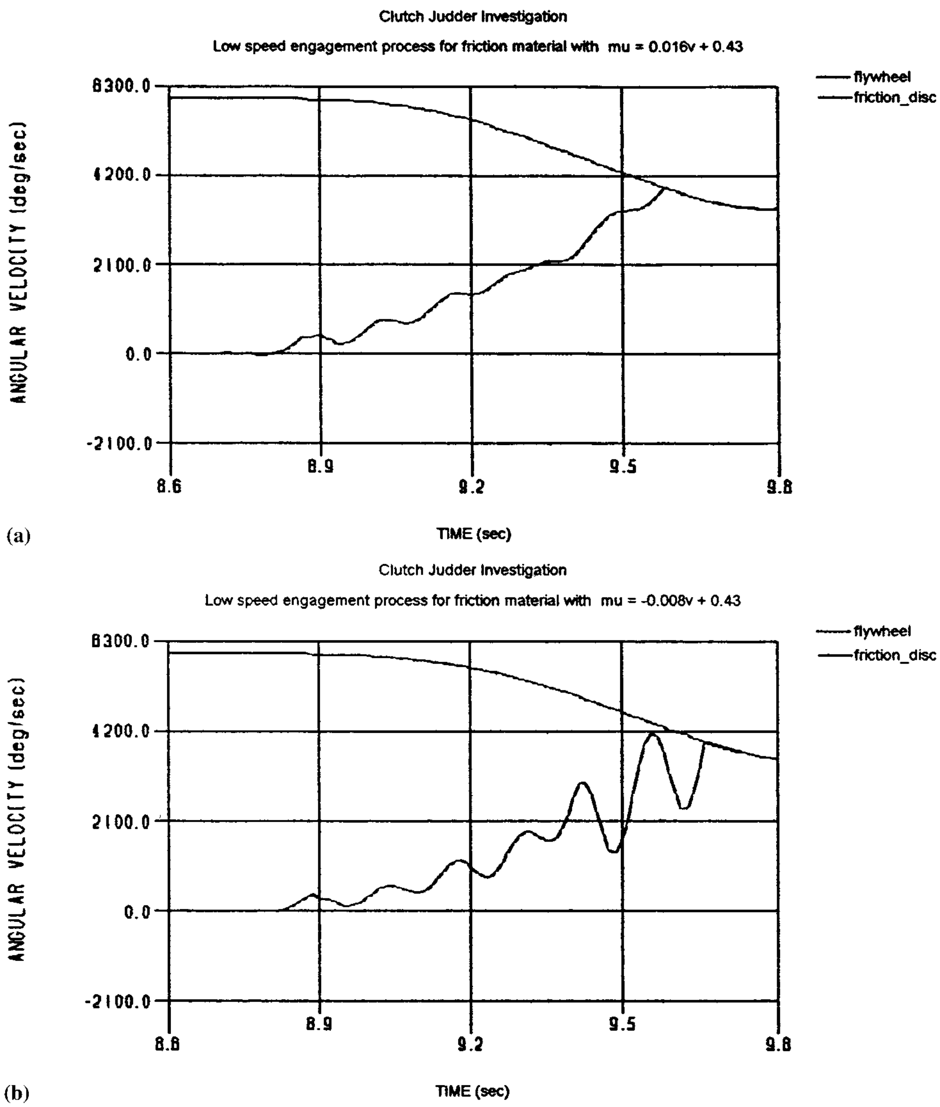


Fig. 7 Low-speed engagement process for a material with a coefficient of friction with (a) a positive gradient and (b) a negative gradient

observed in the same low frequency range of 3–18 Hz, as reported by Rahnejat *et al.* [**21**].

The choice of the friction material can be quite significant in order to damp out the effect of clutch judder in as short a window of oscillations as possible. It shows that friction materials with a positive gradient of coefficient of friction with slip speed provide a better damping effect and little or no self-excited vibrations occur and that friction material characteristics with negative slopes increase the propensity to judder. The test procedure reported here can therefore be employed as a 'sign-off' quality test for the choice of friction materials.

The limitations of the multibody approach are threefold:

1. The development of a mechanism model has traditionally been a long process but can now be managed through a parameterization process, although this approach does not lend itself to the inclusion of non-linear functions such as splines describing the clamp load variation.
2. The computation time is necessarily long as small time steps are required to describe the sharp variations in some parameters of the model such as the sharp rise in the clamp load time history.
3. The problem can best be observed by the simulation of quite fast clutch actuation speeds which lead to integration problems with very small time steps required at the onset of stick–slip motion.

ACKNOWLEDGEMENT

The authors would like to express their gratitude for the financial support extended to this research project by the Ford Motor Company.

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