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Increasing Reliability by Means of Efficient Configurations for High Redundancy Actuators

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Abstract: A high redundancy actuator (HRA) is composed of a high number of actuation elements, increasing both the travel and the force above the capability of an individual element. This provides inherent fault tolerance: if one of the elements fails, the capabilities of the actuator may be reduced, but it does not become dysfunctional. This paper analyses the likelihood of reductions in capabilities. The actuator is considered as a multi-state system, and the approach for k -out-of- n :G systems can be extended to cover the case of the HRA. The result is a probability distribution that quantifies the capability of the HRA. By comparing the distribution for different configurations, it is possible to identify the optimal configuration of an HRA for a given situation.

Keywords: high redundancy actuator (HRA), fault-tolerance, fault mode and effect analysis (FMEA), multi-state system, k -out-of- n :G system, failure probability, dependable systems.

1. INTRODUCTION

1.1 Fault Tolerance

Fault tolerance is about dealing with faults in technical systems (Blanke et al., 2006). Its goal is to prevent a component fault from becoming a system failure (Blanke et al., 2001).

So far, most theoretical considerations have focused on sensor and controller faults. These redundant structures are very efficient. Obviously, the probability of a fault in several identical components is much lower than the probability of a fault in a single one. In order to avoid common causes of failures in redundant components, redundant diversity approaches are applicable. This could mean e.g. to measure the same physical quantity by different principles, or to measure different physical quantities with a known correlation. Whilst significant achievements have been made for sensors and controllers, many of these results are not directly applicable to faults in actuators.

The reason for the difference is the effect of redundancy for actuators. Whereas redundancy for sensors and controllers is always realized by parallel configurations, the adequate configuration of actuators depends on the failure mode. For instance, a blocked valve in the closed position can be tolerated by means of a redundant valve in parallel, but a blocked valve in the open position by means of a redundant valve in series. Therefore, networks of redundant actuators with respect to their specific faults and failure modes have to be investigated.

Most existing approaches for the treatment of actuator failures are derived from the information view used to handle sensor faults. For example, the observer based approach has been ex-

tended to cover actuator faults in the form of the virtual actuator (Steffen, 2005). Likewise, the idea of analytical redundancies in sensors (Frank, 1990) has its equivalent for actuators in the form of dynamic gain scheduling and control allocation (Oppenheimer and Doman, 2006).

Consequently, the classical fault tolerant approach for actuation is replication, the same strategy usually used for sensors. Typically, 2, 3 or 4 actuators are used in parallel, very much like redundant sensors. Each actuator is strong enough to meet the performance requirements by itself. This leads to a significant amount of over-engineering and consequently a less efficient system (e.g. because of a higher weight). Also these parallel arrangements fail if one element locks up, and additional counter-measures are necessary to reduce the impact of such lock-up faults.

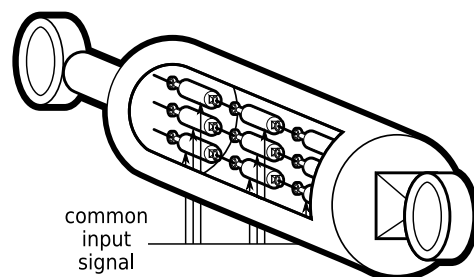


Fig. 1. High Redundancy Actuator

1.2 High Redundancy Actuator

The most general way to improve reliability in an efficient way is to use a greater number of smaller actuation elements. For example, a system with ten elements may still work with only eight of them operational. The reliability improves because two faults can be accommodated. At the same time, the overall capacity is only over-dimensioned by 25 %, making the system more efficient. This is the central idea of the high redundancy actuator (HRA).

This idea is inspired by the human musculature. A muscle is composed of many individual fibres, each of which provides only a minute contribution to the force and the travel of the muscle. This allows the muscle as a whole to be highly resilient to damage of individual fibres.

In an HRA, actuation elements are used both in parallel and in series (see Fig. 1). This increases the available travel and force over the capability of an individual element, and it makes the actuator resilient to faults where an element becomes loose or locked up. These faults will reduce the overall capability, but they do not render the assembly functionless.

So far, the research has focused on the modelling and control of simple configurations with four elements (Du et al., 2006, 2007). Previous studies on the reliability of complicated electromechanical assemblies are rare: the reliability of electro-mechanical steering is discussed by Blanke and Thomsen (2006), and electrical machines and power electronics are analysed by Ribeiro et al. (2004).

This paper presents a method to analyse the reliability of an HRA of any size, as long as it can be interpreted as a hierarchy of parallel and series configurations. The difficulty with analysing an HRA is that many faults can occur simultaneously, and the system may be still be functional. Conventional methods of reliability analysis (fault tree, event tree, stochastic automaton etc) suffer from an extreme increase of complexity, which renders the analysis infeasible even for reasonable small systems such as 10×10 .

The approach presented here is based on the concepts developed using graph theory in Steffen et al. (2007, 2008). It avoids the issue complexity by ignoring the temporal dimension of the problem, and by abstracting from individual faults. Using the principle of divide and conquer, the system is decomposed level by level, relying on simple aggregation equations. This leads to an analysis of low computational complexity. As an example, this approach is applied to different 4×4 configuration for comparison.

1.3 List of Symbols

This paper follows the notation used in the first part of Pham (2003), supplemented by the application specific interpretation of the capability c . This leads to the following symbols.

- $P(\cdot)$ probability of an event,
- q failure probability (unreliability) of an element, typically close to 0,
- p reliability of an element, typically close to 1,
- c generic capability, in multiples of a single element,
- \mathbf{c} vector of capabilities for several elements,
- c_t travel (or velocity) capability,
- c_f force capability,

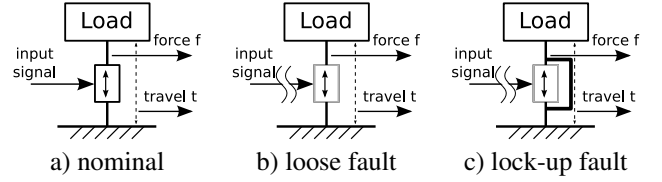


Fig. 2. A Single Actuation Element

$n!$ faculty of n , $n! = \prod_{i=1..n} i$,

$\binom{i}{n}$ binomial i over n , $\binom{i}{n} = \frac{i!}{n!(n-i)!}$,

$r_x(c)$ probability of capability c of system x :

$$r_x(c) = P(c_x = c),$$

$R_x(c)$ reliability of system x wrt. the requirement c ,

$$R(c) = P(c_x \geq c).$$

$R_{fx}(c_f)$ reliability of x wrt. the force requirement c_f .

$R_{tx}(c_t)$ reliability of x wrt. the travel requirement c_t .

$R_{fTx}(c_f, c_t)$ reliability of x wrt. the force requirement c_f and travel requirement c_t .

1.4 Structure of the Paper

Section 2 deals with the basic terms and concepts used for the reliability assessment, and it defines the behaviour of individual actuation elements. In Section 3, the effect of series or parallel arrangement of elements on reliability is investigated. In Section 4, the special cases of series-in-parallel and parallel-in-series configuration is analysed for a simple 2×2 system. In Section 5, this concept is extended to configuration with multiple layers, and an exhaustive study of 4×4 systems is presented. The paper finishes with some conclusions in Section 6.

2. SPECIFICATION OF ACTUATION ELEMENTS

The individual actuation elements of the HRA are specified using a number of different measures. From an abstract perspective, they can be divided into two types: physical measures and reliability measures. The first kind contains physical parameters related to the mechanical movement, such as force, speed, acceleration, or distance. The second kind of parameters describes the probability of a fault.

2.1 Specification of the Nominal Performance

An actuation element can perform a one-dimensional mechanical movement (expansion or contraction) in response to a control input as shown in Fig. 2a. To simplify the analysis, only the static case is considered in the following. So the central performance measurements of an element are the force f it can produce and the amount of travel t it can provide.

While it is entirely possible to use the measurements in physical units (Newton for the force and meter for the travel), this paper will use normalised values instead. The force capability c_f and travel capability c_t of a nominal element are defined to be one (without unit). The use of integer values simplifies the probability analysis significantly, because discrete distributions can be used.

2.2 Specification of Faults

The two capability measures lead to two main fault modes of an element: loss of force (loose fault, see Fig. 2b) and loss of

Table 1. Influence of Faults on Capabilities

Fault	Force Capability	Travel Capability
None	nominal (1)	nominal (1)
Loose	affected (0)	nominal (1)
Lock-Up	nominal (1)	affected (0)
Both	affected (0)	affected (0)

travel (lock-up fault, see Fig. 2c). Both faults are assumed to be complete: a fault reduces the relevant capability to zero (see Table 1).

Because both faults are considered to be independent, they can also appear together. It may seem impossible to have an element that is both loose and locked-up at the same time. However, this analysis is concerned with the guaranteed performance of an element, and it is perfectly possible that it cannot reliably provide neither force nor travel.

It is also assumed that a locked-up element is fixed in its neutral position (this would be the medium length if the nominal travel is symmetric to both sides). This requirement is for convenience only and can be relaxed later.

2.3 Specification of Reliability

In practical applications, different ways can be used to describe the reliability of an element, such as mean time to failure (MTTF), availability, failure probability over a given time, or failure probability during a specified mission. The relevant specification depends very much on the application. However, all measures are based on probabilities or probability densities over time. These functions over time can then be interpreted using any of the above measures. Therefore, this paper will use fault probabilities as a generic way to measure reliability:

$$P(\text{loose}) = P(c_f = 0) = q_f$$

$$P(\text{lock-up}) = P(c_t = 0) = q_t$$

2.4 Capability Distributions

Together with the corresponding OK-probability $P(c_f = 1) = p_f = 1 - q_f$ and $P(c_t = 1) = p_t = 1 - q_t$, these values span the two capability distributions

$$r_f(i) = P(c_f = i)$$

$$r_t(j) = P(c_t = j)$$

Because there are two capabilities, the state space is two-dimensional. However, to avoid the complexity of two-dimensional distributions, this paper deals with one capability at a time in the following. This separation is possible because both fault modes are assumed to be statistically independent.

In some cases, the cumulative capability distributions

$$R_f(i) = P(c_f \geq i) = \sum_{k=i}^{c_{f,\max}} P(c_f = k) = \sum_{k=i}^{c_{f,\max}} r_f(k)$$

$$R_t(j) = P(c_t \geq j) = \sum_{k=j}^{c_{t,\max}} P(c_t = k) = \sum_{k=j}^{c_{t,\max}} r_t(k)$$

are used for determining the reliability of more complex configurations.

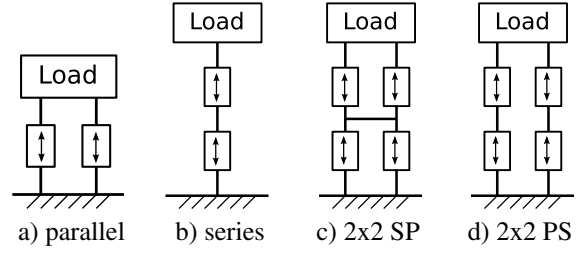


Fig. 3. Basic Configurations

Table 2. Configurations and Capabilities

Configuration	Force Capability	Travel Capability
Parallel	increased (sum)	unchanged (min)
Series	unchanged (min)	increased (sum)
Grid	increased (\times columns)	increased (\times rows)

3. AGGREGATION ON A SINGLE LEVEL

The main reason for using several elements is that they serve to increase the capabilities (see Fig. 3, Table 1). Two elements in parallel can produce twice the force, and two elements in series can achieve twice the travel. In the following, it is assumed that n equal elements are combined, and that the capability distribution for one individual element is known.

3.1 Limiting Capabilities

Some capabilities do not increase when subsystems are combined. Instead, the capability of the resulting system is determined by the weakest part. This happens e.g. with the force capability c_f for actuation elements used in series (see Fig. 3b)

$$c_{fS}(\mathbf{c}_f) = \min\{c_{f1}, c_{f2}\} \quad (1)$$

where \mathbf{c}_f denotes the vector $(c_{f1} \ c_{f2})^T$. The same equation also applies to the travel capability of elements in parallel

$$c_{tP}(\mathbf{c}_t) = \min\{c_{t1}, c_{t2}\} \quad (2)$$

(see Fig. 3a). These equations follow directly from the specification and physical laws, so they will be assumed as given for the reliability analysis.

In both cases, the capability of such a combined system is the minimum capability over all the subsystems or elements:

$$c_{\lim}(\mathbf{c}) = \min\{c_1, \dots, c_n\} \quad (3)$$

This represents a classic series arrangement of multi-state subsystems (MSS), and the reliability has been well studied in the literature. Here, a new operator is introduced to calculate the new cumulative reliability distribution for the overall system.

Theorem 1: If n elements with the cumulative reliability distributions $R_i(c)$ are connected so that the overall capability is limited by the weakest element according to Eqn. (3), the cumulative reliability distribution $R_{\lim}(c_{\lim})$ of the new system can be calculated as

$$R_{\lim}(c) = R_1 \oplus R_2 \oplus \dots \oplus R_n(c) \quad (4)$$

with the operator

$$(R_1 \oplus R_2)(c) = R_1(c)R_2(c) \quad (5)$$

Proof: According to the definition, the reliability $R_{\lim}(c)$ is the probability that the overall capability is at least c :

$$c_{\lim} \geq c$$

Because of Eqn. (3), this inequality holds if and only if all elements have at least this reliability:

$$\forall_i : c_i \geq c \quad .$$

Since the capability of the elements c_i are considered to be independent, the probability of this condition can be calculated as the product of the probabilities of the individual terms:

$$P(\forall_i : c_i \geq c) = \prod_i P(c_i \geq c) = \prod_i R_i(c) \quad .$$

This is exactly the result defined by the operator \oplus .

Since the original Eqn. (3) is applicable in two cases, the same is true for the resulting operator \oplus . It can be used to describe the force of elements in series

$$R_{fS} = R_{f1} \oplus R_{f2} \quad (6)$$

or the travel for elements in parallel

$$R_{tP} = R_{t1} \oplus R_{t2} \quad . \quad (7)$$

3.2 Additive Capabilities

If several actuation elements are used together, the capability of the combined system may increase above the capability of any element. In fact, this increase is the motivation for using several element in the first place.

In contrast to the maximum operator in Eqn. (1), the sum applies to the force capability of two elements in parallel (see Fig. 3a),

$$c_{fP}(\mathbf{c}_f) = c_{f1} + c_{f2} \quad (8)$$

and to the travel capability of two elements in series (see Fig. 3b)

$$c_{tS}(\mathbf{c}_t) = c_{t1} + c_{t2} \quad . \quad (9)$$

In both cases, the relevant capabilities of the elements add up to the capability of the overall system:

$$c_{\text{add}}(\mathbf{c}) = c_1 + c_2 + \dots + c_n \quad . \quad (10)$$

This is unlike typical multi-state systems (Jenab and Dhillon, 2006), because the state space of the system c_{add} can be larger than the state space of any element c_i . Again, a new operator \otimes is introduced to calculate the cumulative reliability distribution of the combined system of two elements.

Theorem 2: If n elements with cumulative reliability distributions $R_i(c_i)$ are arranged so that the capabilities add up according to Eqn. (10), the cumulative reliability distribution $R_{\text{add}}(c_{\text{add}})$ of the resulting system is defined by

$$R_{\text{add}}(c) = R_1 \otimes R_2 \otimes \dots \otimes R_n(c) \quad (11)$$

with the operator

$$(R_1 \otimes R_2)(c) = \sum_{i=0}^c (R_1(i) - R_1(i+1)) R_2(c-i) \quad . \quad (12)$$

Proof: It is easier to work with the same statement in terms of reliability distributions r . Because only integer capabilities are used, it follows from the definition of R and r that $r(i) = R(i) - R(i+1)$. Therefore, the following equation is equivalent to (12):

$$r_{\text{add}}(c) = \sum_{i=0}^c r_1(i) r_2(c-i) \quad . \quad (13)$$

Central to this proof is the set of all capability combinations c_1 and c_2 that lead to the same overall capability $c_{\text{add}} = c$. According to Eqn. (10), this set is

$$\mathcal{C}(c) = \{(c_1, c_2) \in \mathbb{N}_0^2 : c_1 + c_2 = c\} \quad .$$

The probability of the two elements to have the capabilities (c_1, c_2) is

$$P(c_1, c_2) = P(c_1)P(c_2) = r_1(c_1)r_2(c_2)$$

because both are considered to be independent. Now the probability of a given overall capability of c can be calculated as:

$$P(c_{\text{add}} = c) = \sum_{(c_1, c_2) \in \mathcal{C}(c)} P(c_1)P(c_2)$$

which is equivalent to Eqn. (13).

This operator \otimes is applicable in two situations: the force of elements in parallel

$$R_{fP} = R_{f1} \otimes R_{f2} \quad (14)$$

and the travel for elements in series

$$R_{tS} = R_{t1} \otimes R_{t2} \quad . \quad (15)$$

4. HIERARCHICAL AGGREGATION

An HRA contains elements in series and in parallel. Thus it is important to analyse the reliability resulting from multiple levels of aggregations. Assuming that the configuration is given, this section explains how to find the reliability distribution of the overall system by combining the operators defined above.

Any structure can be analysed using an iterative bottom-up approach. From the capability distribution of the individual elements, it is possible to calculate the distributions for the basic subsystems, which are either parallel or series arrangements of elements. Basic subsystems can be aggregated to more complex subsystems, and this can be repeated until the reliability of the overall system is found. For a successful application of this iterative approach, it is required that the actuator configuration is described as a series-parallel network.

4.1 Notation and Formalism

For the examples used here, it is assumed that two equal subsystems are used in series or in parallel. A series configuration is denoted with the letter S, and the parallel configuration with the letter P (cf. Section 3). A sequence of letters denotes a hierarchical configuration, from the bottom level of aggregating individual elements up to the complete system.

So two series elements, duplicated in parallel, are called SP. The dual configuration (two parallel elements, and two of these blocks arranged in series) is denoted as PS. Using two SP systems in series leads to an SPS configuration and so on. It is also possible to have identical levels following each other, for example a PP configuration consists of 4 elements in parallel.

Several examples are shown in Fig. 4. All systems defined by this notation are highly regular and symmetrical, which simplifies the analysis considerably. Following the notation from Section 3, the cumulative force capability of a configuration x is denoted with $R_{fx}(c_f)$, and the cumulative travel capability with $R_{tx}(c_t)$. This allows an easy comparison between different configurations. In the following, all elements are assumed to be identical as specified using the properties defined in Section 2.

4.2 Iterative Reliability Calculation

In each iterative step, two equal subsystems with a known reliability distribution are combined to a new system. The

configuration of a subsystem is assumed to be x , and the cumulative force and travel reliability distributions are $R_{fx}(c_f)$ and $R_{tx}(c_t)$.

For a parallel configuration (xP) of two identical subsystems x , the force increases ($c_{f1} + c_{f2}$), and the travel is limited by the weaker subsystem ($\min\{c_{t1}, c_{t2}\}$). As discussed in Section 3, the following two operators can be used to calculate the cumulative reliability distributions.

Theorem 3: The cumulative reliability distributions for a system of two identical parallel subsystems are

$$R_{fxP} = R_{fx} \otimes R_{fx} \quad (16)$$

$$R_{txP} = R_{tx} \oplus R_{tx} \quad (17)$$

Similarly, in a series configuration (xS), the force is limited by the weakest element ($\min\{c_{f1}, c_{f2}\}$), and the travel increases ($c_{t1} + c_{t2}$). So the cumulative reliability distributions are determined by the other operator, respectively.

Theorem 4: The cumulative reliability distributions for a system of two identical subsystems in series are

$$R_{fxS} = R_{fx} \oplus R_{fx} \quad (18)$$

$$R_{txS} = R_{tx} \otimes R_{tx} \quad (19)$$

The proofs for these two theorems are analogue to the proofs of Theorems 1 and 2 in Section 3. Instead of the two individual elements assumed there, two identical subsystems specified by R_{fx} and R_{tx} are used. These subsystems satisfies all the assumptions made about the elements, including the independence.

5. EXAMPLES

Some representation examples will be discussed in this section. A comprehensive study of further symmetrical configuration will be presented in a forthcoming paper.

Each level combines two subsystems, therefore each configuration consists of four levels, two of which are series connections, and while the other two parallel connections. All six possible configurations are shown in Fig. 4. In the nominal state, all configurations are identical: both force and travel capability are four times the value of a single element.

However, the response to faults differs significantly. The high number of layers makes the reliability slightly more complicated to analyse than in the examples above, but the procedure is still the same: the two Eqns. (16) and (18) are used to determine the cumulative reliability distribution for series and parallel connections.

The values of $1 - R_f(2)$ (allowing two effective element faults) are calculated (they are all polynomials in q_f of order 16) and plotted over q_f on a logarithmic scale in Fig. 5. A number of observations are interesting from the point of high redundancy actuation.

- (1) All reliabilities have the same polynomial structure: they start at 1, the first non-constant term is a factor of q_f^2 . This is a consequence of the basic requirements, which can be fulfilled in every configuration with none or one faulty element.
- (2) The reliabilities maintain a partial order over the configuration

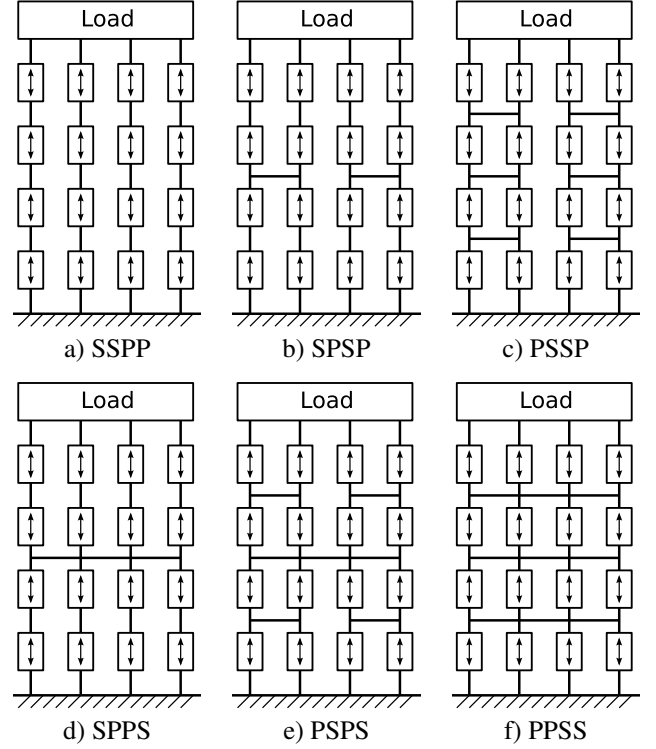


Fig. 4. All Symmetrical 4×4 Configurations

$$R_{fSSPP}(2) > R_{fSPSP}(2) > R_{fPSSP}(2),$$

$$R_{fSPPS}(2) > R_{fPPS}(2) > R_{fPPSS}(2)$$

for all $0 < q_f < 1$. (There is no clear relation between $R_{fPSSP}(2)$ and $R_{fSPPS}(2)$.)

- (3) The reliability of travel ($R_t(2)$) follows the opposite order, because of the correspondence $R_{fSSPP} = R_{tPPSS}$ (for corresponding $q_t = q_f$) etc.

Based on these results, it is possible to calculate the failure probability due to insufficient force and travel, and then select the best configuration for given reliability values q_t and q_f .

The same results are shown in Figure 6 over time. The assumption is that each element fails with a constant rate of 1 faults every τ second (shown by the reference line), leading to $q_f(t) = 1 - e^{-t/\tau}$. Inserting this function into the polynomial $R_{fx}(2)$ leads to a combination of exponential functions describing the system reliability over time. By calculating this result in two steps, the complexity of the problem is greatly reduced over approaches that work in the time domain directly (such as stochastic automaton). The reliability of a single element ($1 - q_f$) is also shown in the Figure as 1×1 for comparison.

6. CONCLUSIONS

This document has shown how to calculate the reliability of an HRA. Due to the high number of actuation elements, a new generic approach had to be developed. Using probability distributions, the problem can be solved with a low computational effort and using well understood operations.

Different configurations consist of several levels series and parallel connections are considered and modelled using multi-state systems. The results show that even with the same number of elements in the same two dimensional arrangement, the selection of the best suitable configuration (as determined by the

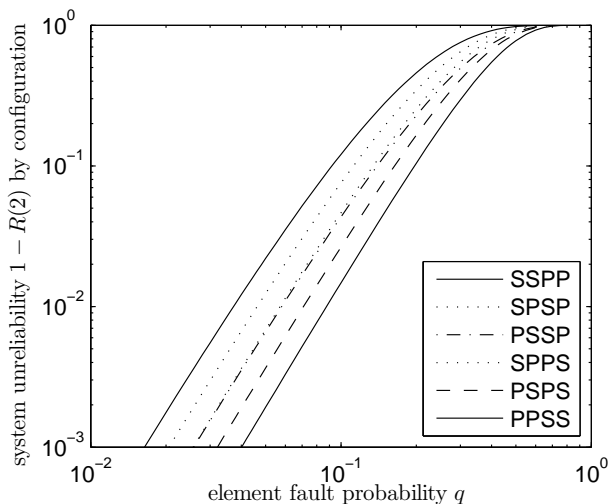


Fig. 5. Reliability $R_{fx}(2)$ on a Logarithmic Scale

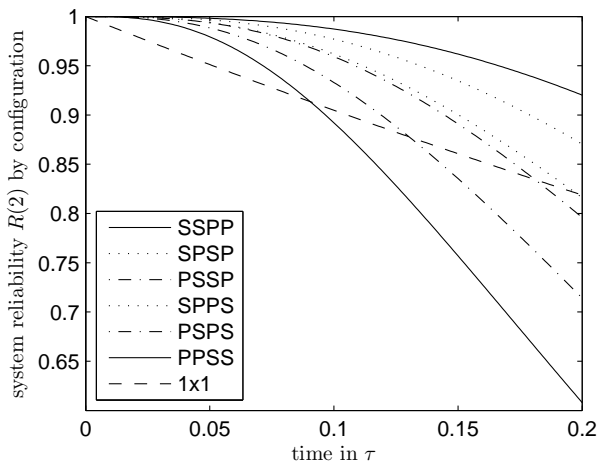


Fig. 6. Reliability $R_{fx}(2)$ over time

lateral connections) has a significant influence on the reliability of the HRA. The influence is especially important when high numbers of elements are used, as planned for the HRA. A more comprehensive analysis will be provided in a forthcoming journal paper.

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