LQG Control of a High Redundancy Actuator

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Abstract—**A high redundancy actuator, comprising a relatively large number of actuation elements, is being developed for safety critical applications. Some classical control results have previously been reported and this paper will focus on evaluation of the LQG control design. Three different design approaches will be presented and compared under different types of typical faults in the sub-actuation elements. Overall a LQG design using a physically motivated reduced order model appears to be the best approach.**

Index Terms—**fault tolerance, high redundancy, LQG control**

I. Introduction

High levels of availability and reliability are important objectives for the design of most modern engineering systems, especially in safety critical applications. Hence fault tolerant systems, which have the capability of tolerating component malfunctions whilst still maintaining desirable and robust performance and stability properties [1], are the solution to such problems. Such tolerance should not only be built into hardware/mechanisms but also in the controller design.

Practical examples of fault tolerant systems can be found in aerospace systems, e.g. Airbus fly-by-wire system [2] and Boeing 737 trailing edge flap drive system [3]. Here, low levels of functional redundancy in sensors and actuators (e.g. triplex and quadruplex) and even in the control computers can be used to provide the system with the capability of fault tolerance, thereby ensuring the safety and stability of the whole system.

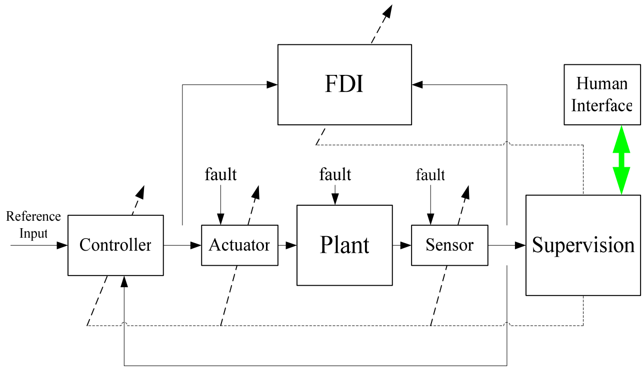

Fig. 1 Scheme of fault-tolerant control system with supervision subsystem [1]

A popular structure of fault tolerant system, as proposed in [1], includes a fault detection and isolation (FDI) unit and reconfiguration scheme which as shown in Fig 1. The solid lines represent signal flow, and the dashed lines represent adaptation. The supervision system will reconfigure the actuator and/or sensor sets, and adapt the controller, based on the fault information collected by the FDI unit together with the inputs and outputs of the system, to accommodate the fault effects. A possible disadvantage of such approaches is the possibility of faults occurring in the FDI unit and the supervision system. One might ask the question, "Who/what monitors the monitors?".

Another approach, called passive fault tolerance, uses a fixed robust controller that tolerates changes of the plant dynamics [2]. The method is called passive because fault tolerance is obtained without changing parameters or structures of the controller. However, the robust control theory is limited to a relatively small range of changes in the plant behaviour caused by faults.

This paper proposes an alternative route to fault tolerant actuation. The high redundancy actuator (HRA) suggested comprises a relatively large number of actuation elements in a matrix-like structure, and is controlled in such a way that faults in individual actuation elements are inherently accommodated. A fault detection unit may still be required for monitoring, but it is no longer strictly necessary and no reconfiguration of the controller or hardware will be needed. Instead the actuation elements work together to complete the system's objective by using the redundancy which is inherent in the structure. The HRA, working with a fixed controller, extends the limit of robust control theory in fault tolerant applications by using a more complex mechanical configuration. If this method can be proven to work, the advantage would arise through removing the possibility of faults occurring within the FDI unit and supervision system.

Several possible configurations are shown in Fig 2. At this stage no attempt has been made to find the optimal structure for the actuation elements but to concentrate upon the most appropriate control approach although some discussions relating to the configurations are discussed in a related paper [5]. Using a relatively simple (2 by 2) structure, a classical controller has been designed and reported in previous papers with a view to discovering whether controlling such structures without reconfiguration is viable [6, 7].

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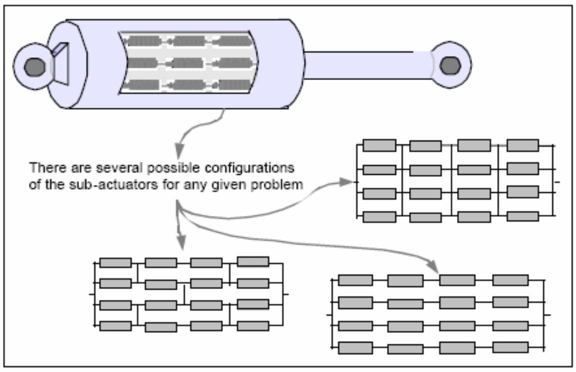

Fig. 2 Several possible configurations of the high redundancy actuator

This paper will focus on the LQG design. Hence, the paper is structured as follows: work on modelling is presented in section 2; the control design using a LQG approach is covered in section 3; then in section 4 a selection of simulation results is presented to give an indication of performance in the presence of faults alongside the fault-free case; the paper concludes in section 5, including comments on the future direction of this research.

II. Modelling Work

A. Individual Actuator Model

Electromechanical actuators are chosen as the sub-elements. The simulation model built in Simulink/MATLAB can be found in [6, 7]. It can be presented as a linear state space model in the well known form of $\dot{x} = Ax + Bu \quad y = Cx$, where

$$A = \begin{bmatrix} -\frac{R_{arm}}{L_{arm}} & -\frac{K_e}{L_{arm}} & 0 & 0 & 0 & 0 & 0 \\ \frac{K_t}{J_m} & -\frac{C_m}{J_m} & -\frac{n^2 K_m}{J_m} & 0 & \frac{nK_m}{J_m} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{K_m * n}{M_s} & -\frac{C_s}{M_s} & -\frac{K_s + K_m}{M_s} & \frac{C_s}{M_s} & \frac{K_s}{M_s} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{C_s}{M} & \frac{K_s}{M} & -\frac{C_s}{M} & -\frac{K_s}{M} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{1}{L_{arm}} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \quad C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} i_a & \dot{\theta}_m & \theta_m & \dot{x}_m & x_m & \dot{x}_{act} & x_{act} \end{bmatrix}^T, u = v_a, y = \begin{bmatrix} \dot{x}_{act} & x_{act} \end{bmatrix}^T$$

TABLE I
DEFINITION OF PARAMETERS AND STATES

R_{arm}	motor resistance	L_{arm}	inductance of the windings
K_e	voltage constant	K_t	torque constant
J_m	motor inertia	C_m	damping coefficient
K_m	motor stiffness	n	screw pitch
K_s	screw stiffness	C_s	Screw damping
M_s	screw mass	M	load mass
i_a	armature current	θ_m	angular rotation of the motor shaft
x_m	screw position	x_{act}	end-of-actuator position

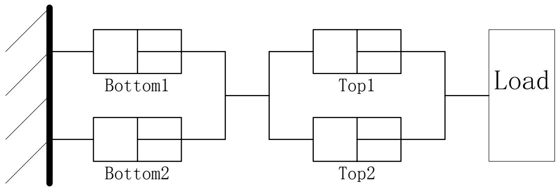

Fig. 3 Parallel in series two by two structure

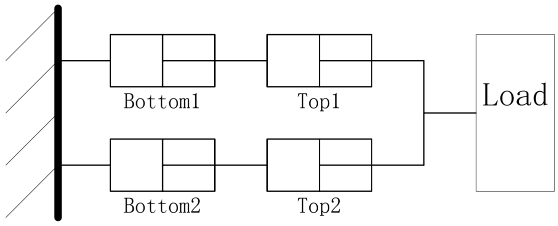

Fig. 4 Series in parallel two by two structure

The parameters and states are defined in Table 1. The actuator is composed of two parts, D.C. motor (electro part) and mechanical part, which is also the same in the state space model. The first three states represent the behaviour of the D.C. motor current, velocity and position, and the other five states allow for the mechanical behaviour transmitted to the actuator's linear motion. More details can be found in [7].

B. Two by Two Network

There are two basic multi-actuator structures. One is called parallel-in-series (PS) structure which is presented in Fig 3, and the other is called series-in-parallel (SP) structure which is presented in Fig 4. The following control design work will be applied on the series-in-parallel structure so that it can be compared with the results using the classical control approach presented in [7].

In the SP structure, there are four sub-actuation elements included, each of which includes seven states based on the individual model, so that the HRA model becomes four times larger than the individual one. However, because the final two states of the top actuators are identical (being the velocity and position of the load), there will be twenty six states in total. The control design will be based on this twenty six order system. As with the individual model, the velocity and position of load are chosen as outputs. The input is the voltage, which is applied directly to each sub-actuation element.

C. Faults Modelling

The actuator faults can be divided into two types: motor faults which occur in the D.C. motor as electrical faults, and the mechanical faults which happen in the components where the torque is transferred to the load. The faults considered are summarised below.

Overheating. Generally it will cause reduction in the voltage constant K_e and the torque constant K_t .

Open circuit. The effect is that the resistance of the rotor will be very large so that no current will flow. Hence no torque will be generated by the motor.

Backlash: Generally, the backlash effect occurs due to excessive wear in the gears of a system. Its effect can be described using the dead-zone [7].

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Lockup: The actuator becomes locked into position due to mechanical interference within the mechanism. This fault is serious as it cannot be solved (by reconfiguration) at the signal level.

III. LQG Control

Optimal control seeks to control the plant so as to get the best possible performance which is expressed in a mathematical expression (a cost function). It is based on state variable models of systems. Through mathematical methods (generally computer-assisted), a state variable feedback matrix will be designed to minimize the cost function to achieve the optimal performance. The Linear Quadratic Regulator (LQR) uses a simple quadratic cost function expressed as follows:

$$J = \int_{t_0}^{t_f} \left[x^T Q x + u^T R u \right] dt \ .$$

With modern computer-assisted control system design (CACSD) environments (such as MATLAB Control System Toolbox), the result from minimizing the cost function can be adjusted to obtain the required closed-loop performance by tuning weighting matrix Q and R. Based on the LQR controller design method, the Linear Quadratic Gaussian (LQG) approach combines optimal state estimation (via a Kalman filter) with the LQR controller design. It helps to avoid disturbance and noise, but only based on accurate plant models and information about white noise disturbances. Here the controller design will be started with an individual actuator. Then, the same process will be applied to the two by two series-in-parallel structure. In both cases the aim is to control the load position in response to a command input (tracking control).

A. Control of Individual Actuator

Previous papers [6, 7] studied the individual and multiple actuators in frequency domain. An open-loop individual actuator Nichols chart is given in Fig 5 using position as output. A high gain is needed to achieve a better performance in frequency domain, which has been proved in the classical design. The system is fully controllable and observable which gives the possibility of LQG control design.

In the LQG control design, an extra state, which is the integral of position error, is added into the previous seven states individual actuator model to ensure tracking of the position command. The corresponding weight for the extra state in Q, which is chosen as a diagonal matrix, is relatively large compared with the other seven weights, while the input weighting matrix R is chosen as single value because only one control input is considered here. Position and velocity measurements are chosen as the input to the state estimator. The simulation result with a 0.03m step input is shown in the Fig 6.

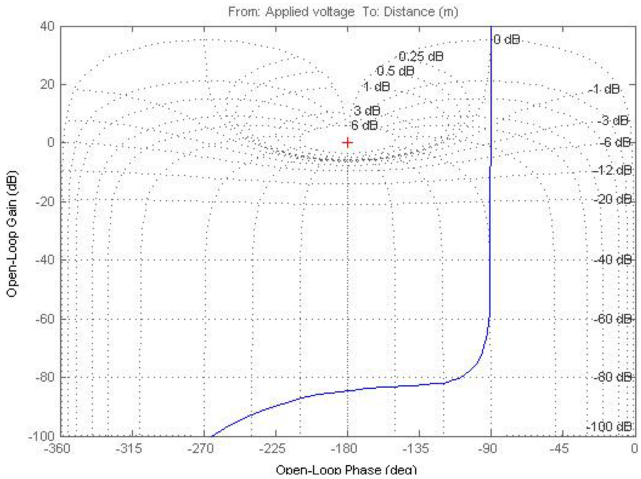

Fig. 5 Open-loop Nichols chart for individual actuator

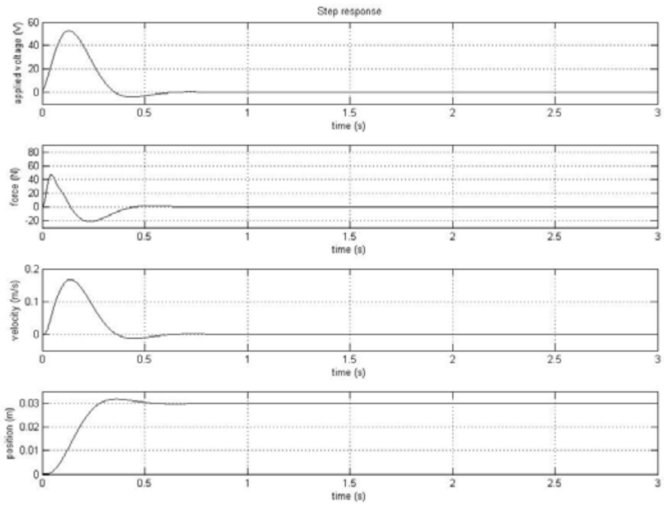

Fig. 6 Control result for individual actuator

B. Control of Multiple Actuator

The controller design is based on the series-in-parallel two by two structure which has been shown in Fig 4. In the simulation, the four sub-actuation elements are ideally the same so that the frequency performance for the SP is similar to the individual actuator. However, the repeated dynamics reduce the controllability and observability of the system which causes mathematical problems for the LQG control design. To obviate this problem, three different approaches will be presented. The first approach is to add some small variations into the parameters to avoid the repetition. The other two methods employ model reduction. The full order system includes 26 states (as introduced in section II) and the final two approaches attempt to reduce the order, thereby alleviating the problem. Of these, one uses a mathematical method called balanced realization truncation and the other uses a physical reduction (based on the physical equations).

1) Parameter Variation: Five percent variations of each parameter are introduced to each actuation element in different place. As introduced in the section II B, the HRA model includes four individual actuator models (also some connection items). The variation in actuation element helps to avoid the repeated dynamics of each sub element which

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would otherwise make calculation of the controller and observer gains difficult.

2) Balanced Realization Truncation: The central problem in model reduction is to find a low-order approximation given a high-order linear time-invariant stable model such that the infinity norm of the difference is small. The advantage is that a simpler controller can be found by reducing the number of states. The balanced realization truncation is based on the balanced realization of the model which evaluates the contributions to the response of each mode [9]. The state coordinate basis is selected as a diagonal matrix in descending order. The magnitudes of the diagonal entries reflect the contributions. Only the most effective states, which affect the input-output mostly, are kept so that similar performance still can be achieved. Again, it's relatively easy to find the balanced realization and the most effective states in the realization of the SP two by two HRA with the help of CACSD environments (MATLAB Control System Toolbox). In this case, eight states are kept in the balanced truncation model

3) Physical Reduction: As for the balanced realization, the purpose for physical reduction is to find a low-order model which has similar performance to the high-order model. The difference here is that the reduction is based on physical understanding rather than mathematical methods. In this approach, the series-in-parallel two by two actuator structure is seen as a bigger individual actuator with four times power input, but double speed and position outputs. This approach gives a state space model with only seven states, just as an individual actuator model. The simulation results show very similar performance between the full order and reduced order model in both frequency and time domain although difference can be found in high frequency which is not important in this case for the controller design.

Then, same progress as the individual actuator control design can be applied while different models are used in the state estimator (Kalman filter). Again, the integral of position error is added as an extra state into the model, with a relatively large corresponding weight in Q matrix, to ensure tracking of the position command.

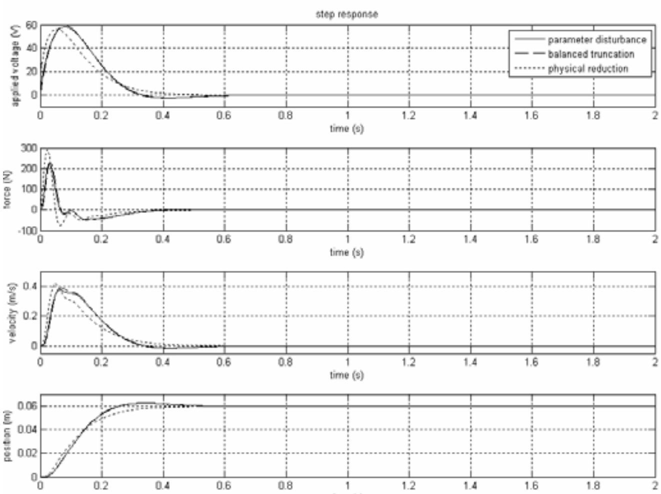

Fig. 7 Global control result for two by two structure

The simulation results with a 0.06m step input using the three different approaches described above is given in Fig 7. All four sub-actuation elements are connected to the same controller so that it can be considered a global controller [7].

IV. Simulation Results Under Faults

In this section, the three different LQG design approaches will be tested under four kinds of faults described previously. Just as in the healthy situation, a step input will be used firstly, and then, a ramp signal will be created as the input to test the tracking ability of the systems.

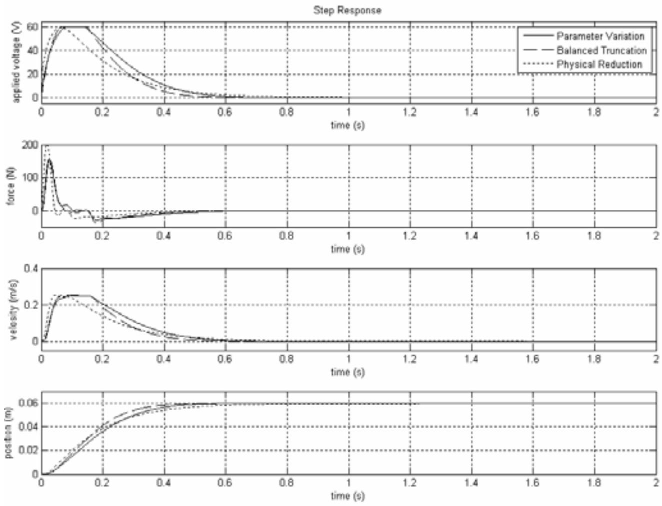

Fig. 8 Simulation result with bottom 1 actuator locked up

TABLE II
STEP RESPONSE PERFORMANCE OF SEREIS IN PARALLEL TWO BY TWO STRUCTURE IN TIME AND FREQUENCY DOMAIN

	Situations	FV(m)	RT(s)	ST(s)	OS
Healthy	Parameter variation	0.06	0.20	0.45	4.3%
	Balanced truncation	0.06	0.20	0.44	4.2%
	Physical reduction	0.06	0.23	0.36	0%
Over heating	Parameter variation	0.06	0.17	0.51	9.5%
	Balanced truncation	0.06	0.17	0.52	9.8%
	Physical reduction	0.06	0.18	0.25	0%
Open circuit	Parameter variation	0.06	0.20	0.45	4.3%
	Balanced truncation	0.06	0.20	0.44	4.0%
	Physical reduction	0.06	0.23	0.36	0%
Backlash	Parameter variation	0.06	0.20	0.45	4.3%
	Balanced truncation	0.06	0.20	0.44	4.2%
	Physical reduction	0.06	0.23	0.36	0%
Lockup	Parameter variation	0.06	0.31	0.43	0%
	Balanced truncation	0.06	0.31	0.43	0%
	Physical reduction	0.06	0.38	0.64	0%

		PM(deg)	GM(dB)	BW(Hz)
Healthy	Parameter variation	64.5	20.8	2.14
	Balanced truncation	64.3	19.3	2.18
	Physical reduction	76.2	20.7	1.72
Over heating	Parameter variation	57.8	17.8	2.7
	Balanced truncation	57.4	16.1	2.75
	Physical reduction	70.9	20.5	2.42
Open circuit	Parameter variation	63.9	16	2.14
	Balanced truncation	63.8	15.3	2.24
	Physical reduction	75.7	14.0	1.78
Backlash	Parameter variation	64.4	20.8	2.13
	Balanced truncation	64.3	19.2	2.16
	Physical reduction	76.1	20.6	1.61
Lockup	Parameter variation	72.1	26.4	1.70
	Balanced truncation	72.2	24.3	1.32
	Physical reduction	80.9	27.7	1.04

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A. Step Response

As in the healthy situation, a 0.06m step is chosen as the input command. As an example, the simulation result with one sub-actuator locked up using three design approaches is shown in Fig 8. Bottom 1 as shown in Fig 4 is chosen as the faulty sub-actuator, and the applied voltage is limited at 60V. Note that, this will be the same in the simulations for other three types of fault.

Compared with the result for the healthy situation, which was shown in Fig 6, a decrease in performance can be found (as expected). More details about the performance in both time and frequency domain is given in the Table II, where FV is Final value of position, RT is Rise time, ST is Settling time, and OT is overshoot, GM is Gain margin, PM is Phase margin, BW is Bandwidth.

From the performance table, decreases in the performance in both time and frequency domain can be found under all four types of fault. It is also noted that the performance is similar for all three design (reduction) approaches.

B. Ramp Response

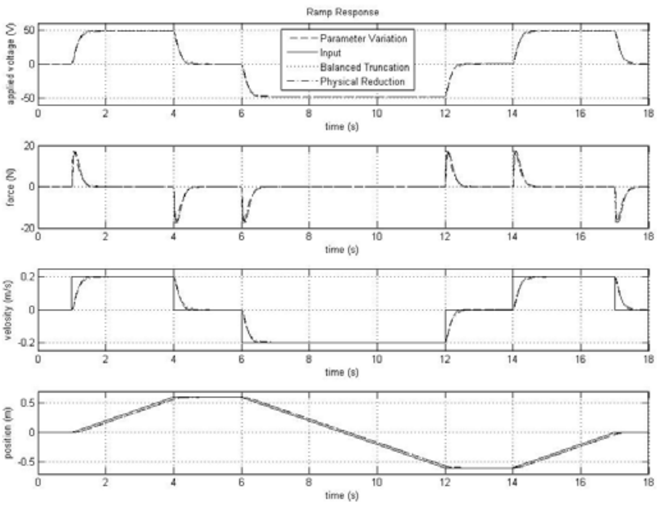

Fig. 9 Simulation result with bottom 1 actuator locked up

TABLE III
RAMP RESPONSE PERFORMANCE OF SEREIS IN PARALLEL TWO BY TWO STRUCTURE

Situations		Coefficient of determination for position	Coefficient of determination for velocity
Healthy	Parameter variation	99.79%	96.69%
	Balanced truncation	99.80%	95.72%
	Physical reduction	99.76%	95.17%
Over heating	Parameter variation	99.83%	96.85%
	Balanced truncation	99.84%	95.79%
	Physical reduction	99.81%	95.46%
Open circuit	Parameter variation	99.79%	96.62%
	Balanced truncation	99.80%	95.64%
	Physical reduction	99.76%	95.11%
Backlash	Parameter variation	99.79%	96.57%
	Balanced truncation	99.80%	95.63%
	Physical reduction	99.76%	95.07%
Lockup	Parameter variation	99.55%	94.36%
	Balanced truncation	99.56%	94.43%
	Physical reduction	99.48%	94.71%

Again, an example under locked up fault is given first. The velocity input is a step signal, while the position input is a ramp signal. Both the inputs and outputs using three different approaches are given in Fig 9

The coefficient of determination for both position and velocity tracking is calculated with all four kinds of faults. This statistic measures how well the output tracks the input command. It is given by the following equation:

$$Rt^2 = 1 - \left(\frac{e}{y_d - mean(y_d)} \right)^2$$

where e is error between the input and output, y_d is the input value.

Again, the difference between the tests results, as seen in table III, is very small, although larger changes in Rt^2 values are found in the velocity output.

Based on the results of both step and ramp responses, it is clear that there is only a small difference in the performance of the three different design approaches. They all work well under the four different kinds of faults, although performance decreases are found.

Hence, on the basis of performance alone, it is not possible to say which approach is better. However, considering the complexity of controller itself and the design process does reveal some differences: It is evident that the approach using parameter variation is the most complex because twenty six states and gains are need to be considered. Meanwhile the other two approaches both have fewer states and gains. The balance truncation approach has only five states and gains, but it is still necessary to build up a twenty six state model first. For a more complex HRA configuration, such as ten by ten, there will be hundreds of states included in the model which is very difficult to build up and deal with. Finally, the approach using physical reduction is the simplest. No matter how many sub-actuators the HRA has inside, it will be always seen as a single actuator containing only seven states so that it will be straightforward to construct and deal with.

V. Conclusion

In this paper, a novel type of high redundancy actuator with the capability of fault tolerance is introduced. An individual electro-mechanical actuator and a two by two configuration HRA have been modeled. A LQG controller has been presented for the individual actuator, followed by three different LQG designs for the HRA (series-in-parallel two by two configuration). These were tested under typical fault conditions. The results show that the high redundancy actuator does have a fault tolerance capability without the need to reconfigure. Based in the performance alone there was little difference between the three LQG design approaches. But when complexity of the design process and the controller order are taken into account, the LQG design using a physically motivated reduced order model is judged to be the best approach. Other on-going work is investigating the design of alternative actuator system structures. Future studies will extend the concepts to 3-by-3 structures and

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higher, and will evaluate other controller designs based on modern robust control methods, i.e. H_2 and H_∞ . Other structures of controller, including inner loops or local controllers [5, 6], will also be considered. It is also intended to develop a lab scale actuator to demonstrate the concept in hardware. By doing so the authors hope to gain an insight into the practicality and rough cost implications of such a system.

VI. Acknowledgement

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