TRANSFER FUNCTION PHASE OF A DIFFUSE VIBRATIONAL FIELD

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1. INTRODUCTION

The transfer function between two points in a structure can be easily measured and is widely used to analyse the characteristics of the system. Often the phase of the transfer function is discarded because only the amplitude of the vibration is of interest. For example, when calculating the energy of a sub-system during the Statistical Energy Analysis (SEA) of a structure. Yet the phase of transfer function may also reveal information about the structure, for example, the level of diffusivity of the vibrational field.

Research into the transfer function phase characteristics in the past has received only limited attention. Lyon and Toyama having made a significant contribution through a series of investigations into the behaviour of phase accumulation for transfer functions in a multi-dimensional system [1-7]. First the relationship between the transfer function phase and the poles and zeros of the system was presented [1,2]. In the same references the phase model of one and two-dimensional systems was predicted using the theory of poles and zeros. In reference [3] the distribution of the zeros in the complex plane was investigated. It was shown that the zeros could be categorised into different two different types, minimum phase and non-minimum phase. The effect of truncation of impulse response data on phase accumulation was discussed in reference [4]. In reference [5] the transfer function phase was expressed using only the number of non-minimum phase zeros. The phase variability in a reverberant field was reported in reference [6] and the relationship of phase with group delay was discussed in reference [7]. A contribution from Fletcher and Thwaites considers the conditions under which direct field propagation phase occurs in a reverberant field [8].

In this paper, the phase of the transfer function for a thin steel plate is reported. A theoretical phase model for the plate is given and the factors likely to affect the phase accumulation are noted. An experiment to obtain data for transfer function phase from a thin steel plate is described. The experimental data are normalised and compared to the theoretical formula for the reverberant phase limit of an idealised diffuse field. Such results are of interest when trying to assess at which frequencies the SEA assumption of uniform energy density in a sub-system is valid.

2. THEORY

The mobility of a system can be generally expressed as a ratio of polynomials, which are factored according to their roots:

$$M(\omega) = const \frac{(\omega - \omega_a)(\omega - \omega_b)(\cdots)}{(\omega - \omega_1)(\omega - \omega_2)(\cdots)} \tag{1}$$

where ω_a, ω_b, ... are the zeros of M and ω_1, ω_2, ... are its poles. The roots have been ordered so that $|\omega_a| < |\omega_b| < \cdots$ and $|\omega_1| < |\omega_2| < \cdots$. From Equation (1), the phase of transfer mobility is given by

$$\phi(\omega) = \arg(\omega - \omega_a) + \arg(\omega - \omega_b) + \cdots - \arg(\omega - \omega_1) - \arg(\omega - \omega_2) - \cdots \tag{2}$$

Thus, as the frequency increases and a zero is passed the phase increases by π radians. Conversely, as a pole is passed the phase decreases by π radians. Thus, up to frequency ω, if N_p poles and N_z zeros have been passed, then the total phase of M will be approximately [1]

$$\phi(\omega) = -(N_p - N_z)\pi \pm \pi / 2 \tag{3}$$

where the last term arises from the possibility of a pole or zero near $\omega = 0$.

Therefore, the problem in estimating the phase accumulation becomes one of estimating the number of poles and zeros that will occur up to frequency ω. Tohyama and Lyon [3] have further simplified this equation by arguing that for two-dimensional system over a sufficiently large frequency range the number of zeros is half the number of poles. Thus, the limit phase of transfer functions in a diffuse field is given by

$$\phi(\omega) = -N_p \pi / 2 \tag{4}$$

Since the poles are corresponding to system resonances, the usual methods of mode count estimation used in SEA are useful. For a flat plate the number of poles or resonances below frequency ω is given by [9] as

$$N_p = \omega A / (4\pi\kappa c_l) \tag{5a}$$

$$= (A\, k^2) / (4\pi) \tag{5b}$$

where c_ℓ is the longitudinal wave speed in the material and κ is the radius of gyration of the cross-section. For a homogeneous cross-section the radius of gyration $\kappa = h / 2\sqrt{3}$, where h is the thickness of the plate. Thus, the phase limit of the diffuse field is dependent upon the plate area, A, and the bending wavenumber, k.

3. EXPERIMENTAL TECHNIQUE

To obtain a diffuse field, the steel thin plate was suspended using elastic ropes from a rigid frame as illustrated in Figure (1). Steel has relatively low internal damping, so more waves are reflected by the boundaries. The plate was excited using random excitation from an Electro dynamic exciter, located on the centre of the plate, and the input force measured with a force transducer. The response acceleration was measured with an accelerometer on the reverse side of the plate at the excitation point and at different distances from the source location. The dimension of the test plate was 600*600*1.5mm. Measurements were carried out using a Hewlett-Packard 3566A spectrum analyser. The damping of the steel plate was also measured using the reverberation time method. The damping varied with different frequencies. The damping at frequency 4000Hz was 2.9E-4.

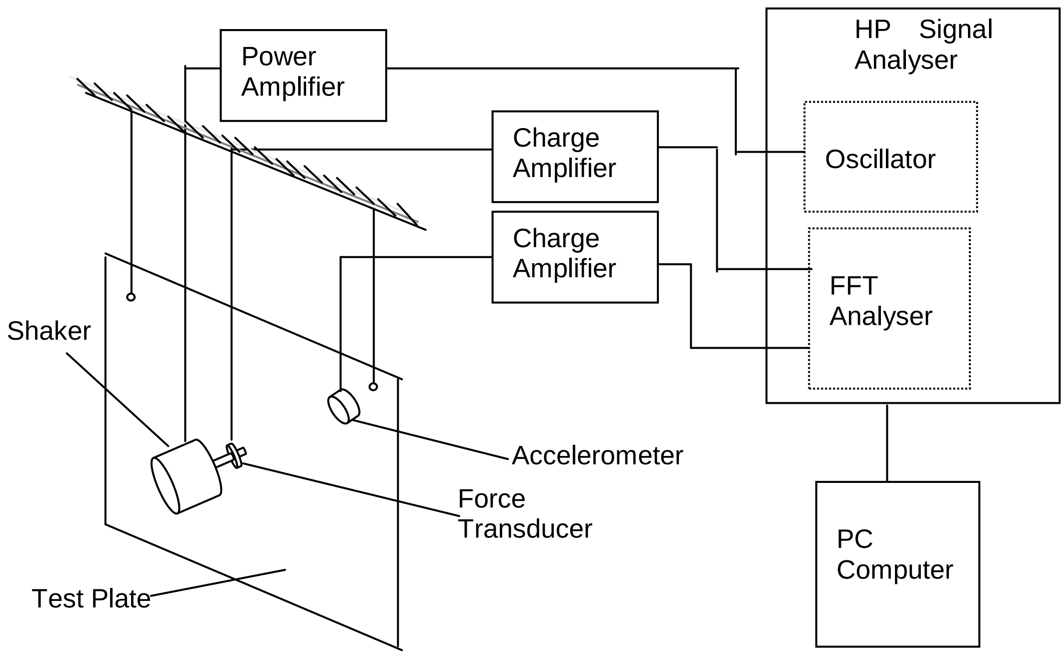

Figure (1): Measurement set-up for the diffuse field experiment.

4. RESULTS

Figure (2) shows the unwrapped transfer mobility phase, in degrees, for the steel plate plotted over a linear frequency range of 0 to 6400Hz. The distance between the source and response points was 6cm. The calculated direct field propagation phase, - kr, where k is the wave number and r is the distance between the source and the response point, is shown lying between 0 and 1000 degrees. The propagation phase on this scale appears to lie close to the zero degrees, however, when the scale is expanded, the propagation phase is, as expected, proportional to f^2. The theoretical diffuse field phase limit, calculated using equations (4) and (5), is shown as a straight line extending from 0 to 45000 degrees. The experimentally measured phase shown in Figure (2) can be seen to lie between the direct field phase and the diffuse field phase limit. Compared to the theoretical phase limit of the purely diffuse field, the slope of the experimental phase is much lower than the theoretical result. In fact it has a slope of around 40% of the value of diffuse phase limit.

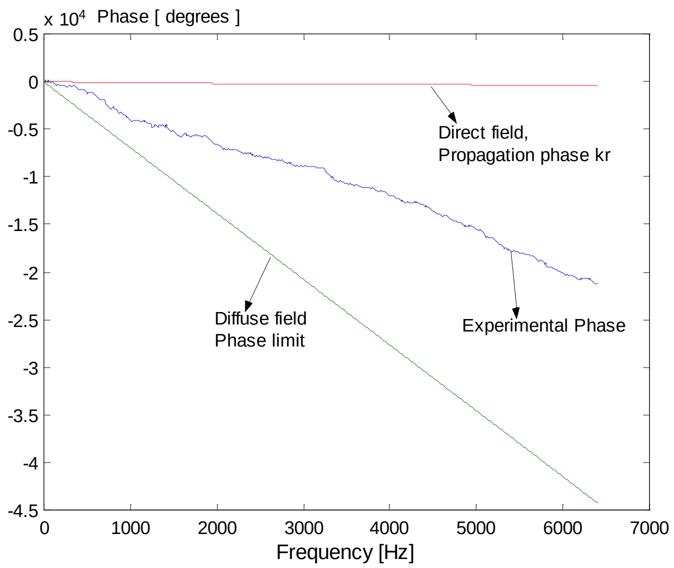

Figure (2): Transfer mobility phase of the steel plate, r=6cm.

Figure (3) shows the transfer function phase at different distances, ranging from 2cm to 14cm, from the source location. Generally, the experimental phase curves are very close to one another except for the phase of the transfer function corresponding to r=2cm.

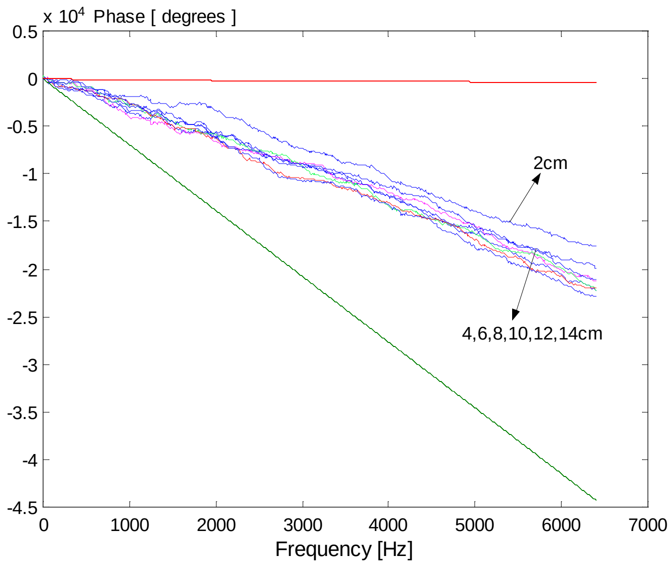

Figure (3): Transfer mobility phase of the steel plate at separation distances: r=2, 4, 6 8, 10, 12, 14cm.

Following the approach of Fletcher and Thwaites [8] the relationship between the transfer function phase and the distance from source to the response point is shown in Figure (4). It can be seen in Figure (4) that all frequencies show the same trend. That is, for distances less than 6cm the phase increases with distance. For distances greater than 6cm there is a phase plateau such that as the distance become larger, the phase stays at approximately the same value.

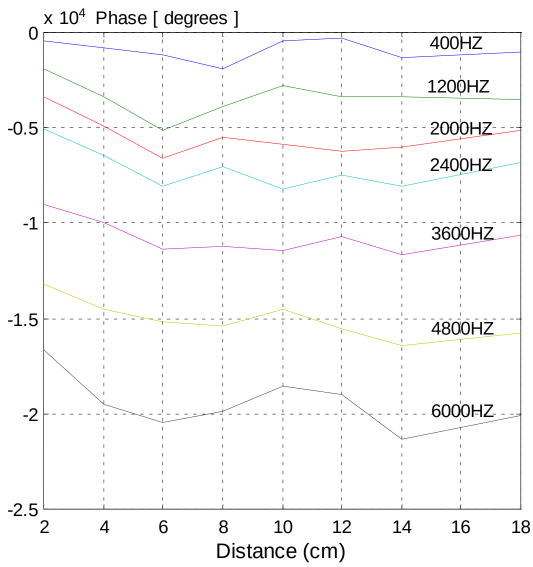

Figure (4): Relationship of transfer mobility phase with distance at selected frequencies from 400Hz to 6000Hz.

5. DISCUSSION AND CONCLUSIONS

This paper has reported an investigation into the transfer function phase characteristics of a steel plate. The phase characteristics from a number of separation distances between the source and response point have been measured and compared to theoretical formulae for the phase limit of an idealised diffuse field.

A general conclusion from the work reported above is that as the separation distance increases the transfer function phase increases towards the value of the diffuse field limit. However, at a certain distance the phase no longer increases with distance but remains approximately constant at a given frequency. Thus, the diffuse field phase limit given by equations (4) and (5) which indicates that there is no relationship between separation distance and the transfer function phase is only true beyond a certain critical separation distance. This result is in agreement with the results of Fletcher and Thwaites [8]. However, the phase curves shown in Figures (2) to (4) are only 40% of the value of the theoretical diffuse field limit. Tohyama, Lyon and Koiko proposed a possible explanation in reference [6] where the theoretical diffuse filed phase limit was shown to reduce as the damping in the structure increases. This was also observed in reference [8], but does not provide a satisfactory explanation for the lightly damped steel plate investigation reported in this paper. This will be the subject of future investigations.

6. REFERENCES

1. R. H. Lyon, "Progressive phase trends in multi-degree-of freedom systems", J. Acoust. Soc. Am., 73(4), 1223-1228 (1983)
2. R. H. Lyon, "Range and frequency dependence of transfer function phase", J. Acoust. Soc. Am., 76(5), 1433-1437 (1984)
3. M. Tohyama, R. H. Lyon, "Zeros of a transfer function in a multi-degree-of-freedom vibrating system", J. Acoust. Soc. Am., 86(5), 1854-1863 (1989)
4. M. Tohyama, R. H. Lyon, "Transfer function phase and truncated impulse response", J. Acoust. Soc. Am., 86(5), 2025-2029 (1989)
5. M. Tohyama, R. H. Lyon, "Reverberant phase in a room and zeros in the complex frequency plane", J. Acoust. Soc. Am., 89(4), 1701-1707 (1991)
6. M. Tohyama, R. H. Lyon, T. Koike, "Phase variabilities and zeros in a reverberant transfer function", J. Acoust. Soc. Am., 95(1), 286-296 (1994)
7. H. He, R. H. Lyon, "Group delay in resonant scattering", J. Acoust. Soc. Am., 99(3), 1465-1474 (1996)
8. N. H. Fletcher, S. Thwaites, "Propagant phase in reverberant environments", J. Acoust. Soc. Am., 99(1), 314-323 (1996)
9. R. H. Lyon, R. G. DeJong, *Theory and Application of Statistical Energy Analysis*. (Second Edition, Butterworth-Heinemann, 1995).