Comparison of Optimal Driving Policies for Limit Handling Manoeuvres

Matthew C Best
Dept of Aeronautical and Automotive Engineering
Loughborough University, UK

Tim J Gordon
UMTRI / Dept of Mechanical Engineering
University of Michigan, USA

University of Michigan Transportation Research Institute
2901 Baxter Road, Ann Arbor, MI 48109, USA
tel. (+1) 734 936-0215, fax (+1) 734 936-1068
E-mail : tjgordon@umich.edu

This paper concerns the synthesis of optimal control inputs for automotive handling dynamics, a typical application being in the evaluation of active safety systems operating near the limits of friction. The paper considers an example of an emergency limit handling manoeuvre – combined acceleration and steering to achieve obstacle avoidance whilst also maximising speed and maintaining stability. Two independent methods are applied to the problem. The first is a general numerical optimiser for non-linear control systems (Generalised Optimal Control, or GOC). The second is an indirect Dual Model (DM) method, which has the advantage that no differential analysis of the vehicle model is required, and it can therefore be applied directly to a wide range of complex multibody dynamic models. A relatively low-order handling model is actually used within this study, since this allows comparison between the two methods and an evaluation of the general usefulness of the DM approach in the future.

Topics / Vehicle Dynamics Control, Active Safety

1. INTRODUCTION

Optimisation of handling performance, in its most general sense, has been a major goal of vehicle dynamics since its inception. For improved handling performance, safety and driving pleasure, vehicles have been greatly improved over the years, through the underlying mechanical design of the chassis, through improvements to tyre materials and construction, and more recently through the use of active controls in brakes, suspension and steering. Much of this 'optimisation' is of course informally based, and would more commonly be described as 'design evolution' or 'handling development'. However, as vehicles – and especially their chassis systems - rely increasingly on electronic controls, the design space acquires ever increasing dimensions, and a more rigorous approach to handling improvement becomes increasingly important. Formal optimisation does not come without its own limitations, perhaps the most fundamental being its critical dependence on the choice (and relative weighting) of performance metrics and constraint functions, and these in turn require a high degree of abstraction from an ill-defined 'engineering problem' to the formal mathematical and numerical definition of that problem.

Once suitable performance metrics have been defined, objective *open-loop* handling tests may be optimised in a relatively straightforward manner using simulation, even when the vehicle dynamics model is complex and nonlinear. On the other hand, many applications, including active safety and motorsport, require *closed loop* evaluation – the driver inputs must be modelled or systematically generated. Where a driver model is used, it has a potential confounding effect – simulated improvements apparently due to (say) a new chassis control system, might be largely the result of an interaction with the particular driver model used. To overcome this limitation, the driver inputs might be synthesised in some kind of optimal way. The resulting optimal nonlinear control synthesis is technically challenging even with relatively simple (nonlinear) vehicle models; for complex nonlinear models the difficulties of finding optimal control signals can be overwhelming, and approximate methods are usually considered. The aim of this paper is to explore a way of bridging between the formal optimal control of low-order vehicle models, and a 'near-optimal' driver-model approach that is compatible with the use of complex vehicle models. The study will consider the two approaches in the context of a simple collision avoidance manoeuvre.

Here we shall make use of a formal method directly based on the underlying Pontryagin Minimum Principle formulation of nonlinear optimal control [1]. This *Generalised Optimal Control* (GOC) technique [2] is applied to the collision avoidance manoeuvre –a double lane change defined by the road and obstacle boundaries - using a low order nonlinear vehicle handling model. This restriction to low order allows the GOC method to provide a performance benchmark for a second Dual Model (DM) technique [3] which is not restricted in this way. These two techniques are described in Sections 2 and 3.

The conceptual difference between the two approaches is as follows. GOC works at a single level, working with vehicle states and road/obstacle geometry limits simultaneously, to provide the optimal sequence of steering, brake and throttle actions to give the best obstacle avoidance possible; there is no internal representation of the 'driver'. DM on the other hand formally optimises the control of a simple particle model in the same geometry, and this provides a reference input to a second combined driver/vehicle model. Of course the choice of driver model may still effect the results, but if – as this study explores – the DM method can come close to matching the performance of GOC, the same approach can be used with confidence as a near-optimal 'driver' for closed loop control, even with more sophisticated vehicle models. This is precisely the motivation for this study.

The low-order vehicle model, described in Section 4, employs a fixed roll-axis, and has four vehicle degrees of freedom – yaw, sideslip, roll and (variable speed) longitudinal motion. The brakes and driveline have further degrees of freedom, including wheel-spin, and the tyres are represented as four load-dependent combined-slip Pacejka 'magic formulae' models, to impose realistic friction limits on the vehicle.

The manoeuvre, and it's formulation via GOC and DM, are given in Section 5, test results are described in Section 6, and conclusions are drawn in Section 7.

2. GENERALISED OPTIMAL CONTROL

The control optimisation is a nonlinear formulation of LQR; controls are sought to minimise a Hamiltonian which is prescribed in terms of a (nonlinear) system of costate equations over a fixed time period. Given a cost function of time, L and a residual cost associated with final states, L_T :

$$J = L_T[\mathbf{x}(T)] + \int_0^T L[\mathbf{x}(t), \mathbf{u}(t)]dt \tag{1}$$

Adding constraint equations to this with a vector of Lagrange multiplier functions, $\mathbf{p}(t)$:

$$J = L_T[\mathbf{x}(T)] + \int_0^T \{L[\mathbf{x}(t), \mathbf{u}(t)] + \mathbf{p}^T(t)[g[\mathbf{x}(t), \mathbf{u}(t)] - \dot{\mathbf{x}}(t)]\}dt \tag{2}$$

where g is given by the system equations, $\dot{\mathbf{x}} = g[\mathbf{x}(t), \mathbf{u}(t)]$. The Lagrange multipliers can be formed as a so-called costate system, and the Hamiltonian function can then be defined (see for example [1]) as

$$H = L[\mathbf{x}(t), \mathbf{u}(t)] + \mathbf{p}^T(t) g[\mathbf{x}(t), \mathbf{u}(t)] \tag{3}$$

Eqn. 2 can now be integrated by parts to give,

$$J = L_T[\mathbf{x}(T)] + \mathbf{p}^T(0)\mathbf{x}(0) - \mathbf{p}^T(T)\mathbf{x}(T) + \int_0^T \{H + \dot{\mathbf{p}}^T(t)\mathbf{x}(t)\}dt \tag{4}$$

Considering small changes δJ in the dynamic cost caused by small changes in the controls $\delta \boldsymbol{u}(t)$ and in the states $\delta \boldsymbol{x}(t)$:

$$\delta J = \left[\frac{\partial L_T}{\partial \mathbf{x}} - \mathbf{p}^T(T)\right]\delta\mathbf{x}(T) + \mathbf{p}^T(0)\delta\mathbf{x}(0) +$$

$$\int_0^T \left\{\left[\frac{\partial H}{\partial \mathbf{x}} + \dot{\mathbf{p}}^T(t)\right]\delta\mathbf{x}(t) + \frac{\partial H}{\partial \mathbf{u}}\delta\mathbf{u}(t)\right\}dt \tag{5}$$

and costates can be chosen such that δJ depends only on changes in the controls by imposing the following conditions :

$$\dot{\mathbf{p}}^T(t) = -\frac{\partial H}{\partial \mathbf{x}} = -\frac{\partial L}{\partial \mathbf{x}} - \mathbf{p}^T\frac{\partial g}{\partial \mathbf{x}}, \qquad \mathbf{p}^T(T) = \frac{\partial L_T}{\partial \mathbf{x}} \tag{6}$$

hence,
$$\delta J = \mathbf{p}^T(0)\delta\mathbf{x}(0) + \int_0^T \left\{\frac{\partial H}{\partial \mathbf{u}}\delta\mathbf{u}(t)\right\}dt \tag{7}$$

As we seek an open loop series of controls to minimise the dynamic cost J for constant conditions, $\delta \boldsymbol{x}(0) = \boldsymbol{0}$, and the minimum cost must therefore exist where

$$\frac{\partial H}{\partial \mathbf{u}} = 0, \qquad \forall t \tag{8}$$

In [4] an approximation to the continuous solution is found using a discrete sequence of controls, each held constant for a small time dt. Within the time period for each control, the cost gradient can then be identified as

$$\frac{\partial J}{\partial u_i} = \int_{t_{i-1}}^{t_i} \frac{\partial H}{\partial u_i}\mathrm{dt} \tag{9}$$

So it is feasible to establish a gradient based iteration optimisation of a sequence of discrete controls spanning the required time frame (Fig 1).

Note that, provided the control remains constant for its discretisation period, the method is valid irrespective of the duration. Also, independent controls can take different discretisations. Coupling this with the fact that in the nonlinear model, any variable can be designated a control, it is straightforward to include model parameters within the optimisation. These are defined simply as controls which remain constant over the entire simulation period, and whose gradients are thus computed as

$$\frac{\partial J}{\partial u_\eta} = \int_0^T \frac{\partial H}{\partial u_\eta}\,\mathrm{dt} \tag{10}$$

Figure 1 provides a summary of the algorithm which can be used to conduct the GOC optimisation.

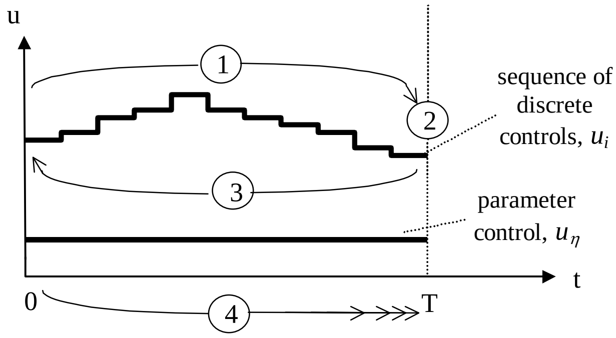

Fig. 1 Summary of GOC algorithm

1 : Using the current discrete control sequence, integrate the state-space system from $\mathbf{x}(0)$ and evaluate $J_{[0,T]}$.

2 : Evaluate the residual cost L_T and hence $\mathbf{p}(\mathrm{T})$ from Eqn. 6.

3 : Integrate the costate system and $\partial H/\partial \boldsymbol{u}$ in reverse-time from the initial condition $\mathbf{p}(\mathrm{T})$. Calculate cost gradients from Eqn 9.

4 : Update the control sequence by a line search optimisation along the steepest descent or successively conjugate gradients to minimise J (evaluated by repeating Stages 1 & 2).

3. DUAL MODEL OPTIMIZATION

This starts with the road geometry, defined by reference to a centreline C, and is assumed to be of uniform width and composed of circular arc segments. As described in reference [3] this allows one to introduce curvilinear coordinates to compute the vehicle path and define reference accelerations. Using track-based displacement coordinates (s_x, s_y) for the analysis, reference velocity components v_x and v_y determine targets for forward and lateral velocities for the driver model [5]. In the present paper, the road is a simple straight section, with additional obstacles defined – Section 5.

The 'flow optimisation' of these velocity components is broadly to maximise v_x and hence maximise the average speed along the track, choosing v_y to reduce the 'flow acceleration' and hence allow further increases in v_x. The flow acceleration is the local acceleration of a particle whose velocity components are v_x and v_y. In Cartesian coordinates the flow acceleration [5] is simply given by

$$\mathbf{a} = \left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} \right)\mathbf{v} \tag{11}$$

and a 'friction' constraint takes the form

$$\|\mathbf{a}\| \le \mu(\mathbf{s}) \tag{12}$$

For numerical representation, the track is divided into a rectangular grid based on the (s_x, s_y) coordinates, and a finite difference scheme is adopted [3]. The mean flow $\bar{v}_x$ is maximised, subject to the friction constraint (12) and a 'containment' constraint $v_y \le -v_{\min}$ on the right track boundary, and $v_y \ge v_{\min}$ on the left boundary.

$$P_0 = \sum_{(i,j)} V_{x(i,j)} \tag{13}$$

Using a penalty function approach for the optimisation, an augmented performance index is chosen in the form

$$P = P_0 - \lambda_1 P_1 - \lambda_2 P_2 \tag{14}$$

where

$$P_1 = \sum_{(i,j)} \tfrac{1}{2} e_{(i,j)} \mu_{(i,j)}^{-2} \left(\left\| \mathbf{a}_{(i,j)} \right\| - \mu_{(i,j)} \right)^2 \tag{15}$$

with $e_{(i,j)}$=0 when the friction constraint (5) is satisfied and $e_{(i,j)}$=1 otherwise. The containment term is f the form

$$P_2 = \sum_i \left(f_{i,2} V_{x(i,j_{\max})} - f_{i,1} V_{x(i,j_{\min})} \right) \tag{16}$$

with $f_{i,2}$=0 when $V_{x(i,j_{\max})} < v_{\min}$ is satisfied, and $f_{i,2}$=1 otherwise. Similar terms are used for left containment and obstacle avoidance (Section 5). A final cost function term limits the flow convergence [5]

$$P_3 = (v_{1,1})^2 + (v_{2,2})^2 + (v_{1,2})^2 + (v_{2,1})^2 \tag{17}$$

and the overall cost function is

$$P = P_0 - \lambda_1 P_1 - \lambda_2 P_2 - \lambda_3 P_3 \tag{18}$$

4. VEHICLE MODEL

The model is based on the well known three degree of freedom, yaw, roll, sideslip model, with a fixed, inclined roll axis, and using a load dependent, combined slip Pacejka tyre model. The equations are for sideslip,

$$M\dot{v} + Mh\dot{p} = \sum_{i=1,4} F_{yi} - Mur \tag{19}$$

for forward speed,

$$M\dot{u} = \sum_{i=1,4} F_{xi} + Mrv + Mhrp \quad (20)$$

for yaw,

$$I_{zz}\dot{r} - I_{xz}\dot{p} = b\sum_{i=1,2} F_{yi} - c\sum_{i=3,4} F_{yi} \quad (21)$$

for roll,

$$-I_{xz}\dot{r} + Mh\dot{v} + I_{xx}\dot{p} = -Mhur - \left(B_f + B_r\right)p + \left(Mgh - K_f - K_r\right)\phi + \left(h - h_f\right)\sum_{i=1,2} F_{yi} + \left(h - h_r\right)\sum_{i=3,4} F_{yi} \quad (22)$$

$$\dot{\phi} = p$$

And here the forces F_{xi}, F_{yi} acting on the body, are given from tyre forces

$$\begin{aligned} F_{x1,2} &= F_{tx1,2}\cos\delta - F_{ty1,2}\sin\delta, \qquad & F_{x3,4} &= F_{tx3,4} \\ F_{y1,2} &= F_{ty1,2}\cos\delta - F_{tx1,2}\sin\delta, \qquad & F_{y3,4} &= F_{ty3,4} \end{aligned} \quad (23)$$

where the tyre forces are derived using the Pacejka magic formula Ω (see for example Milliken and Milliken, 1995) :

$$F_{txi}, F_{txi} = \Omega_i\left(\frac{C_\alpha \tan\alpha_i}{\mu F_{zi}}, \frac{K_x\left(w - u\cos\alpha_i\right)}{\mu F_{zi}\left(u\cos\alpha_i\right)}, Sp\right) \quad (24)$$

with

$$\begin{aligned} \alpha_1 = \alpha_2 &= \left(\frac{-v - br}{u}\right) + \delta \\ \alpha_3 = \alpha_4 &= \left(\frac{rc - v}{u}\right) \end{aligned} \quad (25)$$

and the wheel rotation degree of freedom is modeled as

$$\dot{w}_i = \tfrac{1}{I_w}\left\{\tau_i - r_r f_{xi}\right\} \quad (26)$$

Vertical load transfer due to roll is given by

$$\begin{aligned} F_{z1,2} &= \frac{cmg}{2(b+c)} \pm \left(F_{y1,2}h_{rf} - K_f\phi - B_f p\right) \\ F_{z3,4} &= \frac{bmg}{2(b+c)} \pm \left(F_{y3,4}h_{rr} - K_r\phi - B_r p\right) \end{aligned} \quad (27)$$

And finally, vehicle position and orientation in global coordinates is then calculated using yaw angle, such that

$$\begin{aligned} \dot{\theta} &= r \\ \dot{X} &= u\cos\theta - v\sin\theta \\ \dot{Y} &= u\sin\theta + v\cos\theta \end{aligned} \quad (28)$$

Control inputs δ and τ are applied equally to both wheels at the front axle only.

Table 1 Model Nomenclature and Parameters

States, $\boldsymbol{x}$	
u	forward velocity (*m/s*)
v	sideslip velocity (*m/s*)
p	roll angular velocity (*rad/s*)
r	yaw angular velocity (*rad/s*)
ϕ	roll angle (*rad*)
θ	yaw angle *(rad)*
X,Y	Vehicle position in global space (*m*)
parameters, $\boldsymbol{\eta}$ *(values)*	
I_{xx}	roll moment of inertia (*200 kgm²*)
I_{zz}	yaw moment of inertia (*2500 kgm²*)
I_{xz}	roll/yaw cross moment of inertia (*0 kgm²*)
M	mass (*1400 kg*)
b	longitudinal Distance of C of G to front axle (*1.16 m*)
c	longitudinal Distance of C of G to rear axle (*1.54 m*)
h	C of G height above roll axis (*0.33 m*)
h_0	ground plane to roll axis distance below CofG (*0.27 m*)
h_f	roll axis to x axis vert. distance at front axle (*0.17 m*)
h_r	roll axis to x axis vert. distance at rear axle (*0.23 m*)
t_f	front track (*1.5 m*)
t_r	rear track (*1.5 m*)
K_f	front roll stiffness (*37 kNm/rad*)
K_r	rear roll stiffness (*16 kNm/rad*)
B_f	front roll damping (*790 Nms/rad*)
B_r	rear roll damping (*860 Nms/rad*)
Sp	Pacejka tyre model shape coefficients (*0.709, 1.41, 1.0, 0.0*)
C_α	zero lateral slip cornering stiffness (*64 kN*)
K_x	zero longitudinal tyre slip rate (*64 kN*)
μ	tyre friction coefficient (*1.0*)

5. OBSTACLE AVOIDANCE MANOEUVRE

Both optimisation methods were applied to the following problem. The vehicle is moving at 10 m/s, 2.5 from the left road edge, and parallel to the road as shown in Fig. 3. Track dimensions are given in metres, and the dark regions are obstacles. The vehicle must swerve to avoid the left obstacle, and swerve back to avoid the second of the right-hand obstacles whilst also accelerating. To ensure the problem is well posed, the optimisation takes the form of minimising the time to pass the 35m mark on the track. Although this is not fully representative of an active safety manoeuvre, the minimum time requirement (subject to staying on the road and avoiding the obstacles) provides a suitable case to compare the results of the two methods.

The GOC method optimises a simulation of fixed duration, so to achieve minimum time over a fixed distance it is given a sufficient total time (3.5 seconds) and targeted to maximise the distance travelled in this time. A residual, final cost function is thus set as :

$$L_T = \left(X_G - X(T)\right)^2 \quad (29)$$

with X_G set at some large, unattainable distance (in this case 50m). The time varying cost is then dictated by a track following term :

$$L_{track} = \lambda x^2 \quad (30)$$

where x is the perpendicular distance of the vehicle to the track centre, which is defined using straight-line and circular segments as shown in Fig. 2. λ is chosen, to provide the minimum track following cost whilst ensuring that the obstacles are avoided (λ=100).

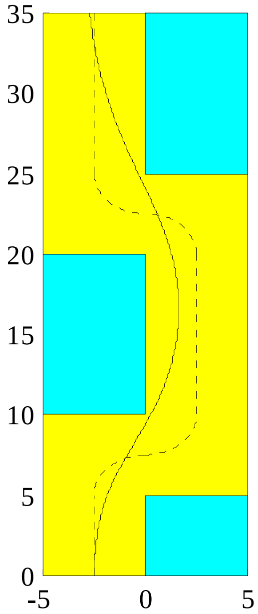

Fig. 2 Obstacle Avoidance

The overall DM method goes through a sequence of optimisations, starting with a finite difference representation of the 'flow map' of the reference field, Fig. 3. This is initially optimised without consideration of any vehicle path. An ideal vehicle would follows the reference field, while using the driver model of [5], the approximate path is achieved as shown.

This vehicle-driver result makes use of a secondary 2-parameter re-optimisation of the driver model. While the core of the model is unchanged, the 'driver' component is given the freedom to adapt 'visual preview' and reference speed according to the equation:

$$\mathbf{v}_{ref}(t) = \rho\,\mathbf{w}(\mathbf{r}_G + \mathbf{v}_G \tau) \tag{31}$$

where $\mathbf{w}(\mathbf{r})$ is the optimised reference field, τ is an anticipation factor - effectively giving a phase lead compensation in the short-term vehicle control – and ρ is the speed adaptation. In this paper and are constants and the re-optimisation uses the combined criteria of angular deviation from the reference field direction, and time to exit the track portion, leading to the values $\tau = 0.12, \sigma = 0.9$.

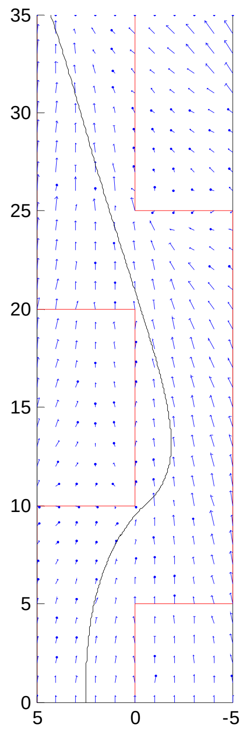

Fig. 3 DM Optimization Method

6. RESULTS

The GOC method reaches the end of the manoeuvre in 3.19 seconds, with an exit speed of 14.75m/s using the control sequence illustrated in Fig. 4. Fig. 5. shows the tyre force utilisation throughout the manoeuvre where it is immediately apparent that the front axle forces are on, or close to their peak throughout, only deviating significantly during the changeover of steering command. This is the expected result for the given, understeering vehicle design. Note also that the more lightly loaded tyre is over-slipping throughout the test, so the torque input is being kept on the point of wheel-spin. The resulting path was shown in Fig. 2.

The DM approach turns out to behave in different ways depending on the exact track position at the start of the simulation. For lateral positions up to 2 m from the centre, the results are very similar to that of GOC. However, for the 2.5 m offset shown, the path (Fig. 3) has greater initial curvature, and this coincides with high levels of initial braking (Fig. 6) not seen with GOC. Thus the field-based method appears to be more 'conservative' on the one hand, and this incurs the penalty of a later completion of the track section (4.6 seconds). The driver model also has greater steering workload in controlling the relatively large slip angles (Fig 6).

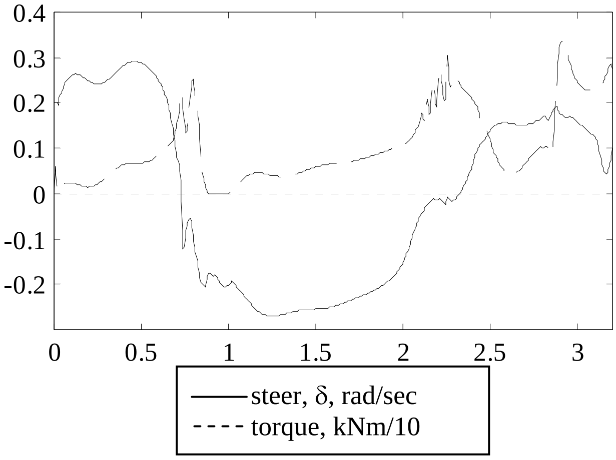

Fig. 4 GOC control inputs

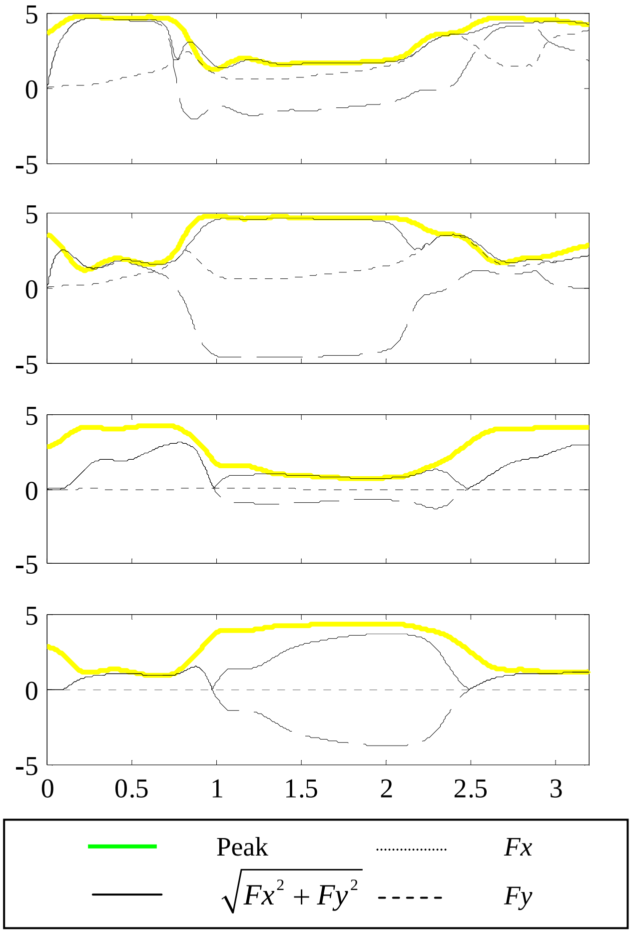

Fig. 5 GOC tyre forces (FR, FL, RR, RL), kN

7. CONCLUDING REMARKS

The two approaches to handling optimization are completely different, and yet under certain conditions their results are very similar, though the very accurate preview enjoyed by GOC allows it to make more finely-

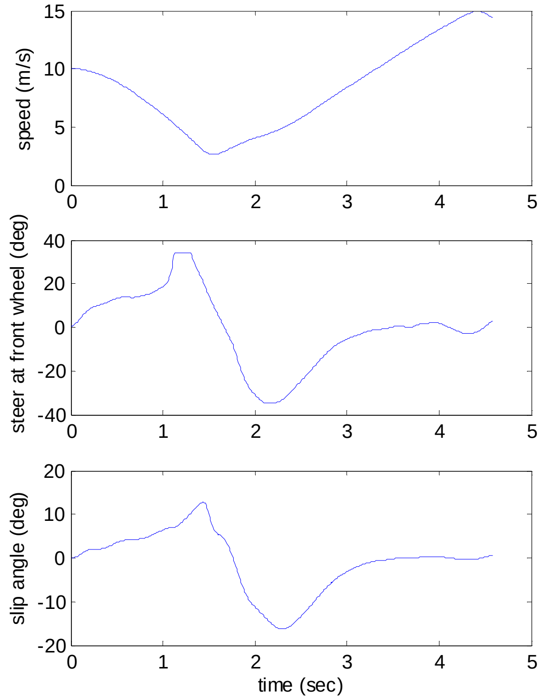

Fig. 6 DM Vehicle Response and Steering Input

judged control decisions. It is clear that the overall optimality comparison between these methods requires further objective assessment, but the general viability of both approaches is clear. Each method has strengths and weaknesses outlined in the paper, and a combined approach, at least for relatively low-order models, is likely to lead to a deeper understanding of the issue of optimal handling control.

REFERENCES

1. Bryson A.E. and Ho, Y.C., “Applied Optimal Control: Optimisation, Estimation and Control,” *Hemisphere*, New York, 1975.
2. Best, M.C. and Gordon, T.J., “Simultaneous Optimisation of Vehicle Parameter and Control Action to Examine the Validity of Handling Control Assumptions” , Proceedings of the 6th International Symposium on Advanced Vehicle Control (AVEC) , Hiroshima, Japan, September 2002, pp 87-92.
3. Gordon, T.J. and Best, M.C., "A Sequential Dual Model Approach to Lap Optimisation" , Proceedings of the 6th International Symposium on Advanced Vehicle Control (AVEC) , Hiroshima, Japan, September 2002, pp 99-104
4. Marsh C., “A Nonlinear Control Design Methodology for Computer-controlled Vehicle Suspension Systems,” *PhD Thesis*, Loughborough University, 1992.
5. Gordon, T.J. An Automated Driver Based on Convergent Vector Fields, Proc Inst Mech Engrs, vol 216, part D, pp329-347.