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Vehicle System Dynamics
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Model Predictive Control based on Mixed $\mathcal{H}_2/\mathcal{H}_\infty$ Control Approach for Active Vibration Control of Railway Vehicles

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This paper investigates the application of Model Predictive Control (MPC) technology based on mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control approach for active suspension control of a railway vehicle, the aim being to improve the ride quality of the railway vehicle. Comparisons are made with more conventional control approaches, and the applicability of the linear matrix inequality approach is illustrated via the railway vehicle example.

Keywords: Railway vehicles; active suspension; model predictive control; $\mathcal{H}_2$ norm; $\mathcal{H}_\infty$ norm; linear matrix inequality (LMI)

1 Introduction

The technology of active suspension control has been well developed for both automobiles and railway vehicles because passive suspensions have reached their performance limits due to the trade-offs in the design process. Active suspension control of railway vehicles aims to improve the ride quality, and in this context significant performance benefits have been demonstrated through both classical and modern control design methods [1, 3]. Future railway design is envisaged to be more cost-effective and energy-efficient, with faster speed and lighter bodies introduced. Thus, the controller will be desired to be more robust, i.e. to operate effectively through a full range of operational conditions.

In this paper Model Predictive Control (MPC) technology based on mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control approach [2] is utilized for active vibration control of a railway vehicle. This new approach is suitable as both performance and robustness issues are handled within a unified framework. The method presented in this paper develops a Linear Matrix Inequality (LMI) which is linear in the state feedback (LMIs are matrix inequalities which are linear in a set of matrix variables). From a control engineering point of view, one of the main attractions of LMIs is that they can be used to solve problems which involve several matrix variables, and moreover, different structures can be imposed on these matrix variables. Another attractive feature of LMI methods is that they are flexible, thus it is usually relatively straightforward to pose a variety of problems as LMI problems. An advantage of LMIs is that they are able to unite many previous existing results in a common framework. The potential of improving the ride quality of the railway vehicle and overall performances of the system will be investigated.

2 Active suspension model of a railway vehicle

The mathematical model of the system is based on the side-view of a railway vehicle as shown in Figure 1, considering both the bounce and pitch motions of the vehicle body and only the bounce motion of the bogie masses. The suspensions, which include the primary suspensions and secondary suspensions, are represented by dampers and springs in parallel. In fact, the primary suspension is mainly for providing

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guidance of the vehicle and the secondary suspension is aimed to improve the ride quality of the vehicle. Active control is provided by actuators placed across the front and rear secondary suspensions. The control objective is to achieve good ride quality while maintaining adequate suspension clearance, i.e. minimizing the acceleration of the vehicle body experienced by passengers without causing large suspension deflections. The dynamics of the model is given by [4]

$$\mathbf{x} = \boldsymbol{A}\mathbf{x} + \boldsymbol{B_1}\mathbf{w} + \boldsymbol{B_2}\mathbf{u}$$

Only the rigid motion of the railway vehicle body are considered, where the states are chosen as translational velocities of the three masses, the rotational velocity of the vehicle body, and deflections across the various springs, represented as

$$x = [\dot{Z}_{3c} \quad \dot{\theta} \quad \dot{Z}_{1l} \quad \dot{Z}_{1r} \quad Z_{3l} - Z_{1l} \quad Z_{3r} - Z_{1r} \quad Z_{2l} - Z_{1l} \\ Z_{2r} - Z_{1r} \quad Z_{1l} - Z_{0l} \quad Z_{1r} - Z_{0r}] \tag{1}$$

The suspension control inputs is given as

$$\mathbf{u} = [U_l \quad U_r] \tag{2}$$

And the track disturbance input is

$$\mathbf{w} = [\dot{Z}_{0l} \quad \dot{Z}_{0R}] \tag{3}$$

3 Control design

MPC is a form of control in which the current control action is obtained by solving on-line, at each sampling instant, an open-loop optimal control problem. Using the current states of the plant as the initial states; the optimization yields an optimal control sequence and the first control in this sequence is applied to the plant. The main advantages of MPC include the ability to handle constraints and the capability for controlling multivariable plants.

This section presents details of the application of MPC based on mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control approach for active vibration control of a railway vehicle.

3.1 *System Description*

We consider the following discrete-time state-space representation of the linear time invariant railway vehicle model from Section 2

$$\begin{aligned} x(k+1) &= Ax(k) + B_1 w(k) + B_2 u(k) \\ y(k) &= Cx(k) + D_{1w} w(k) + D_{1u} u(k) \\ z(k) &= \begin{bmatrix} \sqrt{R_{LQ}}u(k) \\ \sqrt{Q_{LQ}}y(k) \end{bmatrix} \\ x(0) &= x_0 \end{aligned} \tag{4}$$

where the state $x(k) \in \mathcal{R}^n$, the track disturbance $w(k) \in \mathcal{R}^l$, and the control inputs, $u(k) \in \mathcal{R}^m$ are same as given in Section 2. $y(k) \in \mathcal{R}^r$ is the output, which is chosen as

$$y = [\ddot{Z}_{3C} \quad \ddot{Z}_{3l} \quad \ddot{Z}_{3r} \quad Z_{Ld} \quad Z_{Rd}] \tag{5}$$

where

$$\begin{aligned} Z_{Ld} &= Z_{3l} - Z_{1l} \\ Z_{Rd} &= Z_{3r} - Z_{1r}. \end{aligned} \tag{6}$$

$z(k) \in \mathcal{R}^s$ the controlled output, i.e. a combination of inputs and outputs of the system, and A, B_1, B_2, $\sqrt{R_{LQ}}$, $\sqrt{R_{LQ}}$, C, D_{1w} and D_{1u} are matrices of appropriate dimensions based on system dynamics and physical constraints.

The aim is to find a control strategy $\{u(k)\}$ in $\mathcal{L}_2$ to minimize the $\mathcal{H}_2$ cost function J, where

$$J = \sum_{k=0}^{\infty} z^T(k) z(k), \tag{7}$$

subject to a specified level of disturbance rejection γ, i.e.

$$\|T_{zw}\|_\infty < \gamma \tag{8}$$

for any $\gamma > 0$, and where T_{zw} represents the transfer matrix from w to z.

Input and state constraints can be added of the form below:

For given $H_1,\ldots,H_{m_u} \in \mathcal{R}^{n_h \times n_u}$ and $\bar{u}_1,\ldots,\bar{u}_{m_u} > 0$ the general quadratic input constraints

$$u^T(k) H_j^T H_j u(k) \leq \bar{u}_j^2, \quad \forall k; \text{ for } \ j = 1,..,m_u, \tag{9}$$

are satisfied. Also, for given $E_1,...,E_{m_x} \in \mathcal{R}^{n_e \times n}$ and $\bar{x}_1,\ldots,\bar{x}_{m_x} > 0$ the general quadratic state/output constraints

$$x^T(k+1) E_j^T E_j x(k+1) \leq \bar{x}_j^2, \ \forall k; \text{ for } j = 1,\ldots,m_x, \tag{10}$$

are satisfied.

3.2 *Problem Formulation*

For state feedback $u(k) = Fx(k)$, the state equation (4) becomes

$$x(k+1) = \overbrace{(A + B_2 F)}^{A_{cl}} x(k) + B_1 w(k).$$

In this paper, the cost in (7) is considered. Moreover, to accommodate the disturbance rejection level $\gamma > 0$, we have

$$\begin{aligned} J &= \sum_{k=0}^{\infty} u^T(k) R_{LQ} u(k) + y^T(k) Q_{LQ} y(k) \\ &= \sum_{k=0}^{\infty} [x^T(k)(F^T R_{LQ} F + C^T Q_{LQ} C + F^T D_{1u}^T Q_{LQ} D_{1u} F + 2F^T D_{1u}^T Q_{LQ} C) x(k) \\ &\quad + w^T(k) D_{1w}^T Q_{LQ} D_{1w} w(k) + 2w^T(k)(D_{1w}^T Q_{LQ} C + D_{1w}^T Q_{LQ} D_{1u} F) x(k) - \gamma^2 w^T(k) w(k)] \\ &\quad + \gamma^2 \sum_{k=0}^{\infty} w^T(k) w(k). \end{aligned} \tag{11}$$

The disturbance is assumed to be bounded with

$$\sum_{k=0}^{\infty} w^T(k)w(k) \leq \bar{w}^2,$$

or $\|w\|_2 \leq \bar{w}$.

3.3 *Derivation of upper bound*

Consider a quadratic function $V(x) = x^T Px$, $P > 0$ of the state $x(k)$ of (4). Now,

$$V(x(k+1)) - V(x(k)) = x^T(k+1)Px(k+1) - x^T(k)Px(k). \tag{12}$$

It follows from (4) and $u(k) = Fx(k)$ that

$$\begin{aligned} V(x(k+1)) - V(x(k)) = &\begin{bmatrix} x(k) \\ w^T(k) \end{bmatrix}^T K \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} - 2w^T(k)[D_{1w}^T Q_{LQ} D_{1u} F + C_2^T D_{1w}^T Q_{LQ} C]x(k) \\ &- x^T(k)[C^T Q_{LQ} C + F^T R_{LQ} F + F^T D_{1u}^T Q_{LQ} D_{1u} F + 2F^T D_{1u}^T Q_{LQ} C]x(k) \\ &+ \gamma^2 w^T(k)w(k) - w^T(k) D_{1w}^T Q_{LQ} D_{1w} w(k), \end{aligned} \tag{13}$$

where

$$K = \begin{bmatrix} A_{cl}^T P A_{cl} - P + C^T Q_{LQ} C + F^T R_{LQ} F & A_{cl}^T P B_1 + F^T D_{1u}^T Q_{LQ} D_{1w} + C^T Q_{LQ} D_{1w} \\ +2F^T D_{1u}^T Q_{LQ} C + F^T D_{1u}^T Q_{LQ} D_{1u} F & \\ B_1^T P A_{cl} + D_{1w}^T Q_{LQ} D_{1u} F + D_{1w}^T Q_{LQ} C & B_1^T P B_1 + D_{1w}^T Q_{LQ} D_{1w} - \gamma^2 I \end{bmatrix}. \tag{14}$$

Consider the sum (12) from $k = 0$ to $k = \infty$, to get

$$\sum_{k=0}^{\infty} [x^T(k+1)Px(k+1) - x^T(k)Px(k)] = -x_0^T P x_0, \tag{15}$$

assuming that

$$\lim_{k \to \infty} x(k) = 0.$$

Adding (11) and (15) gives

$$J = x_0^T P x_0 + \gamma^2 \sum_{k=0}^{\infty} w^T(k)w(k) + \sum_{k=0}^{\infty} \begin{bmatrix} x(k) \\ w(k) \end{bmatrix}^T K \begin{bmatrix} x(k) \\ w(k) \end{bmatrix}$$

where K is defined in (14). An application of the bounded real lemma [5] shows that A_{cl} is stable and (8) is satisfied if and only if $K < 0$ and $P > 0$. These condition can be reduced to LMIs as shown in Theorem 3.1 below. Thus,

$$J \leq x_0^T P x_0 + \gamma^2 \sum_{k=0}^{\infty} w^T(k)w(k) \leq x_0^T P x_0 + \gamma^2 \bar{w}^2. \tag{16}$$

This is an upper bound on the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ performance objective. Thus the goal of the robust mixed $\mathcal{H}_2/\mathcal{H}_\infty$ MPC is to compute, at each time step, a state-feedback control law $u(k) = Fx(k)$, to minimize this upper bound and the first of the computed inputs is implemented. We are concerned with finding state feedback control law $u = Fx$ with F of appropriate dimension, such that

(i) For stability, the matrix $A_{cl} = A + B_2F$ is stable;
(ii) For disturbance rejection, the closed loop transfer function, T_{zw} from w to z satisfies (8) for $\gamma > 0$, or equivalently, $K < 0$;
(iii) For performance, the upper bound (16) is minimized.

Such an F is called an admissible state feedback gain.

The next result gives sufficient conditions in the form of easily computable LMIs for the existence of an admissible state feedback gain.

Theorem 3.1 Consider the system defined in (4). Then, F is an admissible state feedback gain if $F = YQ^{-1}$ where $Q>0$ and Y are obtained from the solution, if it exists, of the following LMI minimization problem:

$$\min_{\alpha^2,Q,Y} \alpha^2$$

$$\text{subject to} \quad \begin{bmatrix} 1 & \star & \star \\ \gamma^2\bar{w}^2 & \alpha^2\gamma^2\bar{w}^2 & \star \\ x_0 & 0 & Q \end{bmatrix} \geq 0 \tag{17}$$

$$\begin{bmatrix} -Q & \star & \star & \star & \star \\ 0 & -\alpha^2\gamma^2 I & \star & \star & \star \\ AQ + B_2Y & \alpha^2 B_1 & -Q & \star & \star \\ D_{1u}Y + C_2Q & \alpha^2 D_{1w} & 0 & -\alpha^2 I & \star \\ C_1Q & 0 & 0 & 0 & -\alpha^2 I \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} \bar{u}_j^2 I & \star \\ Y^T H_j^T & Q \end{bmatrix} \geq 0, \; j = 1, ..., m_u \tag{19}$$

$$\begin{bmatrix} \bar{x}_j^2 I - (1+\bar{w}^2)E_jB_1B_1^TE_j^T & \star \\ QA^TE_j^T + Y^TB_2^TE_j^T & \frac{Q}{(1+\bar{w}^2)} \end{bmatrix} \geq 0, \; j = 1, ..., m_x \tag{20}$$

where $\star$ represents terms readily inferred from symmetry.

The proof is similar to the proof of Theorem 1 of [2] and, hence, is omitted.

Remark 3.2 The variables F, Y and Q in LMI minimization problem of Theorem 3.1 are computed at time k, the subscript k is omitted for convenience.

Remark 3.3 In this formulation, γ^2 is a parameter that can be tuned using simulations, while α^2, Q and Y are variables determined by the solution to the LMI. Moreover, α^2 is a measure of the contribution of the H_2 cost, while γ^2 measures the contribution of the H_∞ cost, and determines the disturbance rejection level of the control system.

Remark 3.4 We can give the following interpretation to the result obtained. Since the upper bound on J is given by $x_0^TPx_0 + \gamma^2\bar{w}^2$, and since $x_0^TPx_0 + \gamma^2\bar{w}^2 \leq \alpha^2$, it follows that $x_0^TPx_0$ may be regarded as the contribution of the initial state to the total cost while $\gamma^2\bar{w}^2$ may be regarded as the corresponding contribution of the disturbance. By guaranteeing $x_0^T\alpha^{-2}Px_0 + \alpha^{-2}\gamma^2\bar{w}^2 \leq 1$, this is a normalized mixed performance objective, where $\alpha^{-2}\gamma^2\bar{w}^2$ is the contribution of the $\mathcal{H}_\infty$ cost and $x_0^T\alpha^{-2}Px_0$ is the contribution of the $\mathcal{H}_2$ cost. Therefore condition (17) might be considered as the normalized mixed objective function. This is a more natural combination of the control objectives since it emphasizes the trade-off between the normalized $\mathcal{H}_2$ cost and the normalized $\mathcal{H}_\infty$ bound.

4 Simulation Results

In this simulation, we considered the discrete-time model of an active railway vehicle as given by (4), with a sampling time of 0.001s. The initial values of the states are

$$\boldsymbol{x_0} = [0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0]. \tag{21}$$

The disturbances of the track inputs were approximated by Gaussian white noise with a flat spectrum as given in [1], with a delay of 0.35s between the left and right track inputs.

There are different ways to quantify ride comfort; this includes a consideration of root-mean-square (RMS) values of vertical accelerations measured at vehicle floor or occupant's seat location [6]. Here the RMS values of the bounce accelerations in gravity%, deflections in millimeters and control in Newton, will be tabulated for the different γ^2 considered. Table 1 gives the RMS values for the passive system.

4.1 *Parameter tuning*

4.1.1 *Tuning of R_{LQ} and γ^2.* The term R_{LQ} is given by

$$R_{LQ} = \begin{bmatrix} r_1 & 0 \\ 0 & r_1 \end{bmatrix} \tag{22}$$

r_1 is tuned in order to achieve proper scaled control inputs and the best performance for the system. The choice of γ^2 is based on Remark 3.3. It measures the contribution of the H_∞ cost in (7), and determines the disturbance rejection level of the control system. It is found that γ^2 needs to be adjusted accordingly so that LMI could find a feasible solution while it is desired to be small to maintain high disturbance rejection level. For example, when stricter constraints on control inputs are imposed with a relatively bigger value for r_1, γ^2 may need to be increased too if LMI becomes infeasible.

4.1.2 *Tuning of Q_{LQ}.* The term Q_{LQ} is given by

$$Q_{LQ} = \begin{bmatrix} q_1 & 0 & 0 & 0 & 0 \\ 0 & q_1 & 0 & 0 & 0 \\ 0 & 0 & q_1 & 0 & 0 \\ 0 & 0 & 0 & q_2 & 0 \\ 0 & 0 & 0 & 0 & q_2 \end{bmatrix} \tag{23}$$

where q_1 is the weight for accelerations and q_2 is the weight for suspension deflections. They are initially determined based on the physical constraints of the system, e.g. $q_1 = 1/(0.15^2)$ and $q_2 = 1/(0.010^2)$, and further tuned to achieve the best performance.

From experience, it is known that the requirements for acceleration and suspension deflections are in conflict with each other. So when it shows there is space for the deflections to be increased, weight q_2 can be decreased in order to achieve better ride quality. After changing the value for q_2, it may be necessary to go back to tune R_{LQ} again as q_2 will have an effect on the control inputs too.

4.2 *Results*

RMS results for different values for q_2 and r_1 with $\gamma^2 = 1000$ and q_1 fixed to $1/(0.15^2)$ are shown in Table 2. They are also compared with results from a classical control method with skyhook damping. The classical control method is widely adopted for vehicle suspension control. The force provided by an ideal skyhook damping is dependent upon the absolute velocity of the vehicle body. In practice, the velocity required by skyhook damper can be obtained by integrating the accelerations that are to be measured. A similar control structure as in [1] is used.

As shown in Table 2, the controller with $r_1 = 10^{-8}$ and $q_2 = 1/(0.02)^2$ provides the best performance for the system. The responses of bounce accelerations, deflections to the track inputs and control inputs are shown in Figure 2.

Comparing with the classical controller, the proposed new method of MPC based on a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control approach is quite effective in active vibration control of a typical passenger railway vehicle, and is more capable in minimizing the accelerations while keeping the suspension deflection small. Better ride quality is achieved.

5 Conclusion

This paper presented the application of a new MPC based on a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control approach for active suspension control of a railway vehicle. Parameter tuning to optimize the performance of the system is discussed, and the its performance of the controller designed is compared with a classical skyhook controller. This modern control method shows the capability of achieving a more complicated multi-objective control and balance well between different or even conflicting performance requirements. It provides the freedom to minimize both the accelerations and suspension deflections by tuning the parameters.

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Table 1. RMS results for a passive system

Left acc $\ddot{Z}_{3l}$(%g)	Center acc $\ddot{Z}_{3c}$(%g)	Right acc $\ddot{Z}_{3r}$(%g)	Left def Z_{Ld}(mm)	Right def Z_{Rd}(mm)
2.66	1.66	3.41	7.78	11.00

Table 2. RMS results for MPC control approach

r_1	q_2	$\ddot{Z}_{3l}$(%g)	$\ddot{Z}_{3c}$(%g)	$\ddot{Z}_{3r}$(%g)	Z_{Ld}(mm)	Z_{Rd}(mm)	U_L(kN)	U_R(kN)
10^{-9}	$1/(0.01)^2$	1.83	1.27	1.79	8.71	8.71	3.12	3.32
10^{-8}	$1/(0.01)^2$	1.91	1.50	1.87	6.31	6.40	2.77	2.87
10^{-8}	$1/(0.02)^2$	0.61	0.50	0.72	8.66	6.52	3.21	2.88

Table 3. RMS results for classical method

$\ddot{Z}_{3l}$(%g)	$\ddot{Z}_{3c}$(%g)	$\ddot{Z}_{3r}$(%g)	Z_{Ld}(mm)	Z_{Rd}(mm)	U_L(kN)	U_R(kN)
1.85	1.67	1.83	11.93	9.21	3.03	3.05

Figure captions

Figure 1: Sideview model of a typical high speed passenger railway vehicles;

Figure 2: Simulation results for $r_1 = 10^{-8}$, $q_2 = 1/(0.02)^2$ (a) Center bounce accelerations (b)Left bounce accelerations (c)Right bounce accelerations (d) Left suspension deflections (e)Right suspension deflections (f) Left actuator control force;

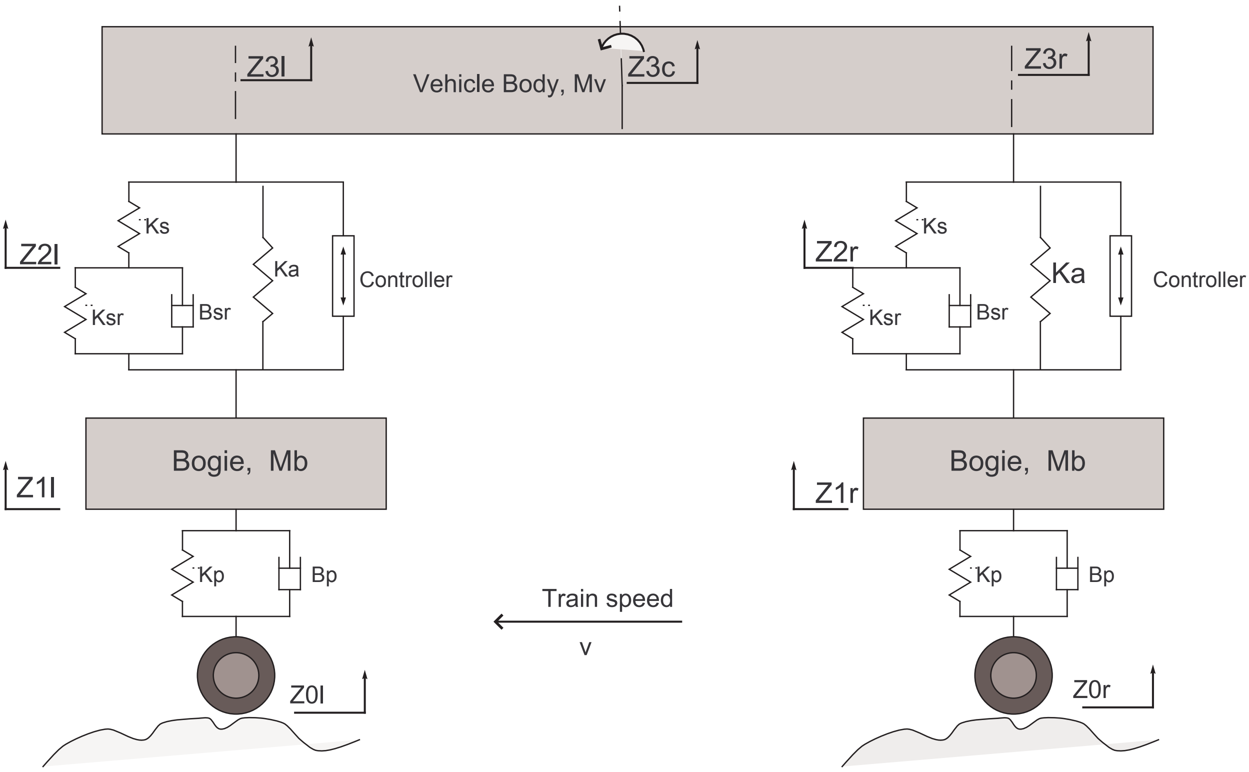
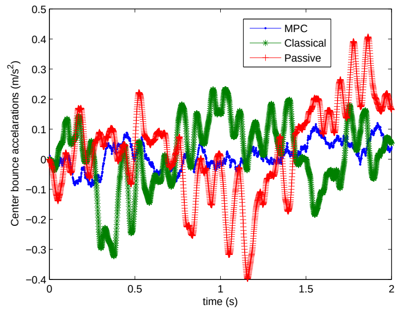
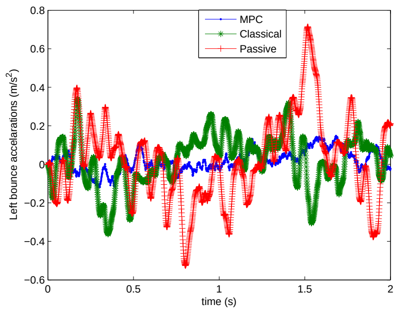


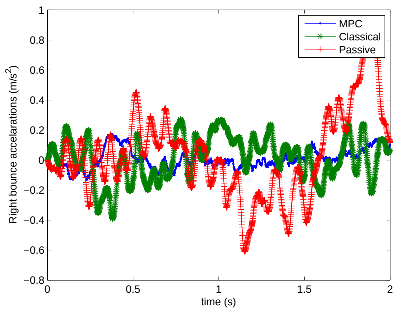
Figure 1. Sideview model of a typical high-speed passenger railway vehicles



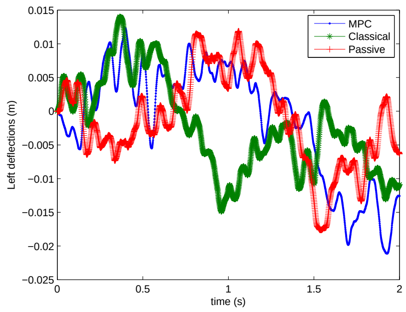
(a) Center Bounce Accelerations



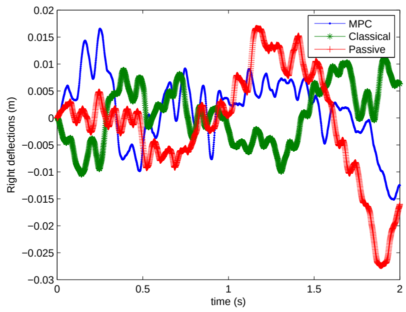
(b) Left Bounce Accelerations



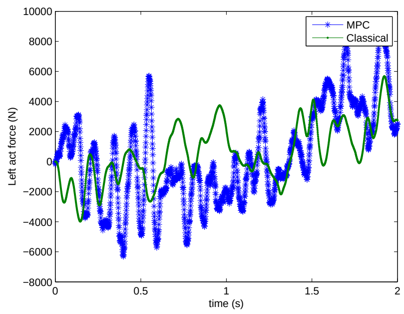
(c) Right Bounce Accelerations



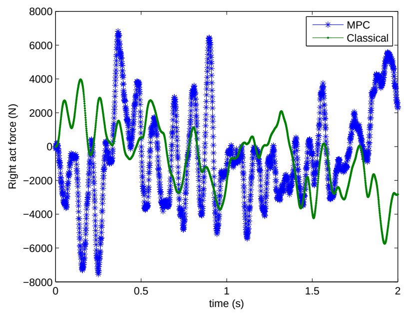
(d) Left Suspension Deflections



(e) Right Suspension Deflections



(f) Left Actuator Control Force



(g) Right Actuator Control Force

Figure 2. Vehicle body rigid model responses for $r_1 = 0.6 \times 10^{-7}$, $q_2 = 1/(1.1)^2$.