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# Mode-Selective Amplification in a Waveguide Free Electron Laser with a Two-Sectioned Wiggler

Michael G. Kong, *Member, IEEE*

**Abstract**— One important issue in waveguide free electron lasers (FEL's) involves an interaction of the electron beam with one waveguide mode at two different resonant frequencies. Since the low-frequency mode often has a higher gain, the usually preferred high-frequency mode is suppressed as a result of mode competition. In this paper, possible control of this mode competition is considered using a nonstandard wiggler magnet consisting of two cascaded wiggler sections with different periods and field strengths. It is demonstrated that with an appropriate differentiation between the two wiggler sections the high-frequency mode may be amplified preferentially. This mode-selective amplification may be used to suppress the low-frequency mode. A small signal gain formulation is developed for a waveguide FEL with such a two-sectioned wiggler arrangement and numerical examples are used to demonstrate its applicability to mode control in waveguide FEL's. Effects of wiggler field errors and electron energy spread are also considered. It is shown that the requirement for wiggler field errors and electron energy spread in the two-sectioned wiggler arrangement is similar to that in the usual straight wiggler configuration.

**Index Terms**— Free electron lasers, gain control, millimeter-wave generation, mode competition, undulator, waveguides.

## I. INTRODUCTION

COMPACT free electron lasers (FEL's) in millimeter and the infrared regions have recently commanded much attention largely because of their small size and low cost realized by using low-energy and/or low-current electron beams [1]–[8]. Their output power is modest, usually at the kilowatt level rather than megawatt level, but nevertheless adequate for many applications in medicine and industry. Waveguides are almost always employed to form the interaction region in compact FEL's and, as such, the electron beam can interact strongly with many waveguide modes. Without an appropriate control, unwanted modes may grow at the expense of a preferred mode.

One type of mode competition involves an interaction of the electron beam with one waveguide mode at two different resonant frequencies, as shown in Fig. 1, where the electron beam line intersects the dispersion curve of the waveguide mode twice at point A and point B. This suggests that the waveguide mode may grow at the two resonant frequencies [1], [2], and hence there is a mode competition between the low-frequency mode and the high-frequency mode. Although radiation at high frequency is usually preferred, small signal

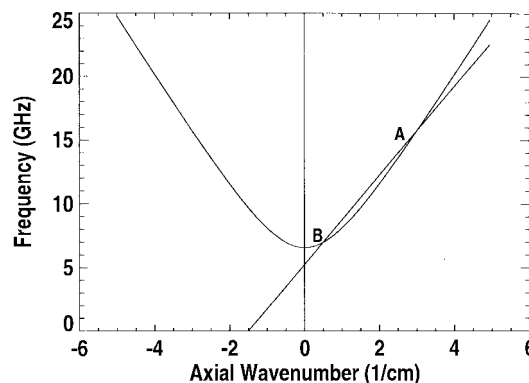


Fig. 1. Dispersion diagram in a waveguide FEL indicating two resonant frequencies.

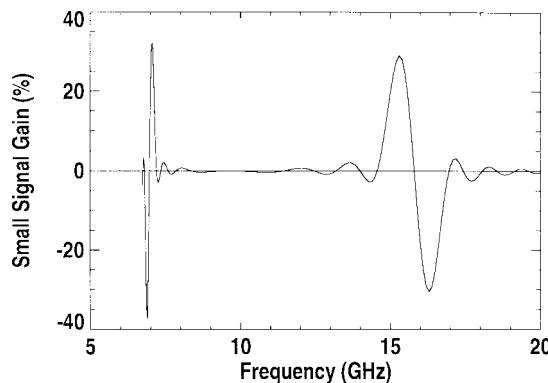


Fig. 2. Small signal gain of a waveguide FEL showing a higher gain for the low-frequency mode.

analysis suggests that the low-frequency mode often has a higher gain than the high-frequency mode [2], [9], [10], as shown in Fig. 2. As a result, the low-frequency mode grows at the expense of the desired high-frequency mode. Depending on their relative strength, the competition between the two resonant modes may extend into the large signal regime where they have been observed to undergo a highly nonlinear coupling [11]–[13]. This nonlinear interaction has been shown experimentally to result in a substantial oscillation at low frequency [11]–[13], although a recent theoretical study suggests the possibility of utilizing the nonlinear mode interaction to achieve an appreciable oscillation at high frequency [14]. This contradiction highlights the fact that the parametric dependence of the mode competition is yet to be fully understood in the large signal regime, and therefore it is

Manuscript received May 20, 1997; revised November 13, 1997.

The author is with the Department of Electrical Engineering and Electronics, University of Liverpool, Liverpool L69 3GJ, U.K.

Publisher Item Identifier S 0093-3813(98)01475-1.

preferable for the low-frequency mode to be suppressed in the small signal regime.

It is worth considering a possible variation of Fig. 1 for which point B is moved to the left of the origin by, for instance, using a shorter wiggler period. In this case, the mode competition is no longer related to the high-frequency forward wave mode of point A competing with the low-frequency forward wave mode but instead with a low-frequency backward wave mode [15], [16]. Since the backward wave mode is related to the absolute instability, its suppression may be achieved by employing a short-pulse electron beam [15]. However, for waveguide FEL's driven by long-pulse or CW electron beams, this backward wave mode can lead to a significant absolute instability and, as such, the mode competition is likely to be favorable to the low-frequency mode even more [15], [16]. In other words, the mode competition between two forward wave modes shown in Fig. 1 is in fact a less severe problem for long-pulse waveguide FEL's.

Techniques for the control of waveguide mode competition are not likely to be useful for the competition between the low-frequency mode and the high-frequency mode since they belong to the same waveguide mode. One possible solution is to choose a quality factor  $Q$  for the low-frequency mode that is sufficiently small for its suppression, but this technique is not applicable to the case where the wavenumber of the high frequency is an integer multiple of that of the low frequency. On the other hand, it should be possible to derive more sophisticated cavity-based techniques capable of achieving a better control of mode competition [17], [18]. With their control mechanisms based on the cavity's frequency selectivity, these techniques can suffer from their limitation on the tunability of the eventual FEL system [10] as well as their sensitivity to beam/cavity misalignment especially at high frequencies [18]. An alternative approach is to control the mode competition without enhancing the frequency selectivity of the device's radio frequency (RF) structure, and one example is to employ a nonstandard wiggler consisting of two cascaded wiggler sections with different periods and field strengths [19]. It was shown that this magnet-based technique is capable of suppressing the low-frequency mode considerably in the small signal regime while maintaining approximately the same linear gain for the high-frequency mode [19]. However, the proposed technique was only discussed phenomenologically without gain formulation. In this paper, the FEL interaction in such a two-sectioned wiggler is analyzed with a small signal gain formulation. It is shown that with an appropriate differentiation between the two wiggler sections, the high-frequency mode may be amplified preferentially, and this mode-selective amplification may be used to control mode competition in compact FEL's. The proposed technique is further examined for its gain dependence upon wiggler field error and electron energy spread in comparison with the case of a straight wiggler.

## II. MODE CONTROL WITH A TWO-SECTIONED WIGGLER

Suppose in a waveguide free electron laser the electron beam only interacts strongly with one waveguide mode with all other waveguide modes suppressed. With a sufficient electron

energy, the electron beam intersects the waveguide dispersion curve at two resonant frequencies in a straight wiggler magnet, as shown in Fig. 1. The slope of the electron-beam line may be controlled by the wiggler field strength, and its intersection on the wavenumber axis of Fig. 1 is given by  $-2\pi/\lambda_w$ . Suppose this waveguide FEL now employs a two-sectioned wiggler, and in both of its wiggler sections the electron beam interacts with the waveguide mode at two resonant frequencies. With different wiggler periods and field strengths for the two wiggler sections, the single electron-beam line for the case of a straight wiggler in Fig. 1 splits into two beam lines of different slopes and different intersections on the wavenumber axis, as shown in Fig. 3. It is therefore possible to adjust the periods and field strengths of the two wiggler sections so that the two beam lines intersect the dispersion curve at an identical high resonant frequency, whereas their low resonant frequency intersections are apart. The implication of this arrangement is that either of the two low-frequency modes receives an appreciable gain only in one wiggler section and undergoes no significant interaction in the other, while the desired high-frequency mode is amplified in both wiggler sections. Therefore, by limiting the amplification of the low-frequency modes to within only one wiggler section, their growth should be suppressed.

This two-sectioned wiggler technique is conceptually very similar to the compound wiggler technique proposed for lasing optical free electron lasers at a selected harmonic [20]. However, the compound wiggler technique realizes harmonic selection by differentiating the combination of wiggler period and harmonic number in its two constituent parts, while the two-sectioned wiggler technique achieves mode control by differentiating the combination of wiggler period and field strength of its two wiggler sections. It is also of interest to note that both configurations may be considered as a special types of tapered wigglers [21], [22] or multicomponent wigglers [23], [24], usually conceived to achieve a substantial improvement in FEL device performance.

## III. SMALL-SIGNAL ANALYSIS

Beam-wave interaction in waveguide FEL's is, in general, three dimensional, and usually the space charge effects are important. For compact FEL's driven by low current electron beams, however, the space charge effects are not important, and the interaction mechanism is dominated by waveguide dispersion with a negligible field variation in the transverse dimensions of the waveguide [1], [2], [4]–[6]. Hence, compact waveguide FEL's may be analyzed in the single-particle regime with both the wiggler field and the radiation field approximated by their on-axis values.

Consider a two-sectioned wiggler magnet with both sections of the same length  $L$ . We assume that a rectangular waveguide is used to form the interaction region and the electron beam only interacts with its  $TE_{10}$  mode. The electric field of  $TE_{10}$  mode has one transverse field component on axis given by [2]

$$\vec{E}_r = \hat{x}E_0 \cos(\omega t - k_z z + \varphi) \quad (1)$$

where  $\varphi$  is the electron entrance phase with respect to the electromagnetic wave. The wiggler field may be expressed by

its on-axis expression

$$\vec{B}_{w1} = \hat{y}B_{w1} \sin k_{w1}z \quad (2)$$

$$\vec{B}_{w2} = \hat{y}B_{w2} \sin k_{w2}(z - L) \quad (3)$$

for the first and second wiggler sections, respectively. To the first order of the radiation field, a direct integration of the energy conservation law gives the energy change of a reference electron at the exit of the first wiggler section as

$$\Delta\gamma_1|_{1\text{st sec}} = -\frac{a_{w1}a_s}{2\gamma_0} \left( \frac{\omega L}{\beta_{z1}c} \right) \left[ \frac{\sin(\Delta k_1 L/2)}{(\Delta k_1 L/2)} \right] \cdot \sin(\phi + \Delta k_1 L/2) \quad (4)$$

where  $a_{w1} = eB_{w1}/mck_{w1}$  and  $a_s = eE_0/mc\omega$  are the dimensionless field strength of the first wiggler section and the electromagnetic wave, respectively;  $\beta_{z1}$  is the average axial component of the normalized electron velocity; and  $\Delta k_1 L = [\omega/\beta_{z1}c - (k_z + k_{w1})]L$  is the FEL detuning parameter of the first wiggler section.

Now we consider the energy change of the reference electron in the second wiggler section. To this end, we need to know the electron's phase at the entrance of the second wiggler section. Note that since the electron travels slower than the electromagnetic wave, its phase lags behind the latter in the first wiggler section. Under the synchronism condition  $\Delta k_1 L = 0$ , the electron slips a distance of  $N_1\lambda_r$  behind the wave at the exit of the first wiggler section with  $\lambda_r$  and  $N_1$  being the radiation wavelength and the number of wiggler periods in the first section, respectively. This suggests that the electron phase at the entrance of the second wiggler section is, in general,  $\phi + 2N_1\pi + \Delta k_1 L$  with respect to the electromagnetic wave. Thus to the first order of the radiation field, the electron energy change in the second wiggler section may be expressed as

$$\Delta\gamma_1|_{2\text{nd sec}} = -\frac{a_{w2}a_s}{2\gamma_0} \left( \frac{\omega L}{\beta_{z2}c} \right) \left[ \frac{\sin(\Delta k_2 L/2)}{(\Delta k_2 L/2)} \right] \cdot \sin(\phi + \Delta k_1 L + \Delta k_2 L/2) \quad (5)$$

where  $\beta_{z2}$  is the average axial component of the normalized electron velocity in the second wiggler section,  $a_{w2} = eB_{w2}/mck_{w2}$  is its dimensionless field strength, and  $\Delta k_2 L = [\omega/\beta_{z2}c - (k_z + k_{w2})]L$  is its FEL detuning parameter.

The total energy change of the reference electron over the entire wiggler magnet is the sum of its energy change in each section. Therefore, from (4) and (5), we have

$$\Delta\gamma_1|_{\text{total}} = -\frac{a_{w1}a_s}{2\gamma_0} \left( \frac{\omega L}{\beta_{z1}c} \right) \left\{ \begin{aligned} &\text{sinc}(x_1) \sin(\phi + x_1) \\ &+ \frac{\beta_{z1}a_{w2}}{\beta_{z2}a_{w1}} \text{sinc}(x_2) \\ &\cdot \sin(\phi + 2x_1 + x_2) \end{aligned} \right\} \quad (6)$$

where  $\text{sinc}(x) = \sin(x)/x$ , and for compact expression of derivation  $x_1 = \Delta k_1 L/2$ , and  $x_2 = \Delta k_2 L/2$  are introduced. From (6), it is clear that to the first order of the radiation field, the net energy change of the electron beam is zero. Therefore, the electron energy change needs to be formulated

to the second order of the radiation field, and one method is to use the classical FEL treatment of the second-order perturbation analysis [25]. However, it has been shown that the simple relationship between the first and the second order perturbations of the electron energy suggested by Madey's theorem [26] is applicable to waveguide optical klystrons [27]. Since waveguide FEL's with a two-section wiggler may be considered as a special version of waveguide optical klystrons, Madey's theorem should apply to the former as well. Therefore, with

$$\langle (\Delta\gamma_1)^2 \rangle = \frac{1}{8} \frac{a_s^2}{\gamma_0^2} \left( \frac{\omega L}{c} \right)^2 \left\{ \begin{aligned} &\frac{a_{w1}^2}{\beta_{z1}^2} \text{sinc}^2(x_1) + \frac{a_{w2}^2}{\beta_{z2}^2} \text{sinc}^2(x_2) \\ &+ \frac{2a_{w1}a_{w2}}{\beta_{z1}\beta_{z2}} \text{sinc}(x_1) \\ &\cdot \text{sinc}(x_2) \cos(x_1 + x_2) \end{aligned} \right\} \quad (7)$$

formulated from (6) and Madey's theorem [26]

$$\langle \Delta\gamma_2 \rangle = \frac{1}{2} \frac{d}{d\gamma} \langle (\Delta\gamma_1)^2 \rangle \quad (8)$$

we have

$$\langle \Delta\gamma_2 \rangle = \frac{1}{16} \frac{a_s^2}{\gamma_0^2} \left( \frac{\omega L}{c} \right)^2 g(x_1, x_2) \quad (9)$$

with the gain function given by

$$g(x_1, x_2) = \frac{d}{d\gamma} \left\{ \begin{aligned} &\frac{a_{w1}^2}{\beta_{z1}^2} \text{sinc}^2 x_1 + \frac{a_{w2}^2}{\beta_{z2}^2} \text{sinc}^2 x_1 \\ &+ 2 \frac{a_{w2}a_{w1}}{\beta_{z1}\beta_{z2}} \text{sinc} x_1 \text{sinc} x_1 \\ &\cdot \cos(x_1 + x_2) \end{aligned} \right\}. \quad (10)$$

The power gain for waveguide FEL's is given by

$$G_p = \frac{-\langle \Delta\gamma_2 \rangle mc^2 I / |e|}{P_{em}} \quad (11)$$

where  $I$  is the current of the electron beam and  $P_{em}$  is the propagating power of the radiation field. For TE<sub>10</sub> mode,  $P_{em} = (k_z/2\omega\mu_0)E_0^2 A_{em}$ , with  $A_{em}$  being an effective cross-sectional area of the radiation field. Consequently, the small signal gain is formulated as

$$G_p = -\frac{1}{8\gamma_0^2} \left[ \frac{\omega\mu_0}{k_z} \right] \left[ \frac{Ie}{mc^2} \right] \left[ \frac{L^2}{A_{em}} \right] g(x_1, x_2). \quad (12)$$

If the above equation is specified for the case of a straight wiggler under the conditions of  $a_{w1} = a_{w2}$ ,  $\beta_{z1} = \beta_{z2}$ , and  $x_1 = x_2$ , the gain function becomes

$$g(x_1, x_2) = -4 \frac{a_{w1}^2(1 + a_{w1}^2)}{\gamma_0^3 \beta_{z1}^5} \left[ \frac{\omega L}{c} \right] \frac{d}{dx} \text{sinc}^2 x \Big|_{x=2x_1}. \quad (13)$$

Consequently, the small signal gain is reduced to

$$G_p = \frac{1}{16} \sqrt{\frac{\mu_0}{\epsilon_0}} \left( \frac{|e|I}{mc^2} \right) \left( \frac{\omega}{k_z c} \right) \frac{a_{w1}^2(1 + a_{w1}^2) (2L)^2}{\gamma_0^5 \beta_{z1}^5 A_{em}} \cdot \left[ \frac{\omega(2L)}{c} \right] \frac{d}{dx} \text{sinc}^2 x \Big|_{x=2x_1}. \quad (14)$$

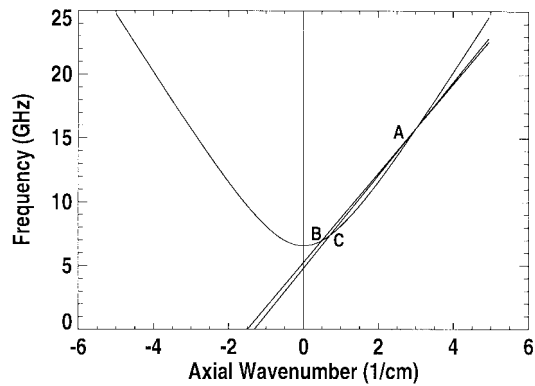


Fig. 3. Dispersion diagram in a waveguide FEL with a two-sectioned wiggler magnet.

Note that  $2L$  is the total wiggler length in this case. The above gain expression is therefore the same as that derived for conventional waveguide FEL's in [1] and [2].

#### IV. NUMERICAL EXAMPLES

Based on the gain formulation developed in the preceding section, a waveguide free electron laser is used to investigate numerically the suggested mode-selective amplification in a two-sectioned wiggler magnet. To this end, we consider first a waveguide FEL with a straight wiggler magnet. Suppose we employ an electron beam of 300 kV and 50 mA, an  $X$  band rectangular waveguide with internal waveguide dimensions of  $a = 2.286$  cm and  $b = 1.016$  cm, and a wiggler magnet of  $\lambda_w = 4.2$  cm and  $a_w = 0.4$ . Fig. 2 shows its small signal gain as a function of frequency calculated from (14) for a wiggler length of 96 cm. The high-frequency mode has a peak gain of 29% at 15.3 GHz, less than 32% for the low-frequency mode at 7.1 GHz. To obtain a mode-selective amplification for the high-frequency mode, the straight wiggler magnet is replaced by a two-sectioned wiggler of the same total length with each wiggler section 48 cm long. The first wiggler section has a period of  $\lambda_{w1} = 4.8$  cm and a field strength of  $a_{w1} = 0.2$ , whereas the second wiggler section is essentially half of the straight wiggler with  $\lambda_{w2} = 4.2$  cm and  $a_{w2} = 0.4$ . For the parameters of this two-sectioned wiggler, the small signal gain is calculated for the first section, the second section, and the entire wiggler magnet using (10) and (12) and plotted as a function of frequency in Fig. 4. The dispersion diagram of this waveguide FEL is that shown in Fig. 3.

It is clear from Fig. 4 that the low-frequency mode obtains a peak gain of 1.13% at 7.72 GHz in the first wiggler section, whereas in the second its peak gain is about 3.84% at 7.13 GHz. The frequency band for positive gain (signal amplification) is very different for the low-frequency mode in the two wiggler sections. In the first section, this amplification frequency band is from 7.4 to 8.7 GHz, whereas in the second section the interaction gain is positive from 7.0 to 7.6 GHz. Therefore, the low-frequency mode is amplified over very different frequency regions in the two wiggler sections and, as such, its two individual amplification frequency bands overlap for only 0.2 GHz from 7.4 to 7.6 GHz. This narrow frequency range is similar to the overall amplification frequency band

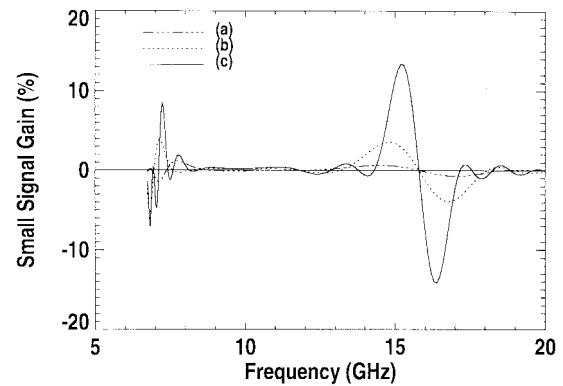


Fig. 4. FEL gain in (a) the first section; (b) the second section; and (c) the whole of a two-sectioned wiggler magnet.

of about 0.3 GHz from 7.1 to 7.4 GHz for the total gain of the low-frequency mode. It is of interest to note that over the whole wiggler magnet, the low-frequency mode reaches its maximum gain of 8.41% at 7.23 GHz, where the low-frequency mode obtains a negative gain in the first wiggler section. This may be understood from the fact that the low-frequency mode interacts strongly with the electron beam over rather different frequency regions in the two wiggler sections and, as such, the cross term (the third term) in the gain function of (10) can be a large positive number at frequencies outside the amplification frequency band of one wiggler section. Hence it is possible for the total interaction gain to peak at a frequency where the low-frequency mode has a negative gain in the first wiggler section.

In contrast, the high-frequency mode has a very similar frequency dependence of gain in the two wiggler sections. Its peak gain is about 0.6% at 14.5 GHz in the first wiggler section, and 3.5% at 14.8 GHz in the second. The amplification frequency band is from 12.1 to 15.8 GHz for the first section and from 13.2 to 15.8 GHz for the second. Thus these two amplification frequency bands overlap almost perfectly from 13.2 to 15.8 GHz. It is therefore conceivable that the two radiation fields generated in the two wiggler sections have very similar characteristics, and their interference with each other leads to a positive superimposition of the two radiation fields. As a result, it is possible to achieve a total interaction gain that is greater than a simple summation of the two individual gains achieved separately in the two wiggler sections. Indeed, Fig. 4 indicates that over the whole wiggler, the high-frequency mode is amplified from 14.4 to 15.8 GHz with a peak gain of 13.8% at 15.2 GHz, almost four times as large as the higher value of the peak gains achievable in one wiggler section. This is also much higher than the peak gain of 8.41% for the low-frequency mode. It is therefore evident that with an appropriate differentiation of the two wiggler sections, the high-frequency mode may be amplified preferentially.

The high-frequency mode achieves a greater gain than its low-frequency counterpart because it experiences a significant beam-wave interaction in both wiggler sections. Thus for the high-frequency mode, the two-sectioned wiggler may be considered as a straight wiggler twice as long as one wiggler section. Based on such a consideration and the fact that the

small signal gain is proportional to the cube of the wiggler length, the total gain for the high-frequency mode over the whole wiggler magnet was estimated to be eight times as large as that achieved in one wiggler section [19]. This hypothesis can be shown analytically if  $x_1 = x_2$  and  $a_{w1}/\beta_{z1} = a_{w2}/\beta_{z2}$  are assumed in (10). However, its prediction does not agree with the numerical results shown in Fig. 4, where the high-frequency mode has a total peak gain about four times as large as that achievable within one wiggler section. Note that the concerned numerical example does not satisfy the condition of  $x_1 = x_2$  and  $a_{w1}/\beta_{z1} = a_{w2}/\beta_{z2}$ , and thus mathematically its interaction gain should be different from the prediction of the hypothesis. It should be noted, however, that the condition of  $x_1 = x_2$  and  $a_{w1}/\beta_{z1} = a_{w2}/\beta_{z2}$  implies two identical wiggler sections in which the electron beam would radiate equally and the radiation fields generated from the two wiggler sections would interfere with each other most effectively to give a maximum interaction gain. In other words,  $x_1 = x_2$  and  $a_{w1}/\beta_{z1} = a_{w2}/\beta_{z2}$  represent the condition for the maximum achievable gain over a given wiggler length. However, such a condition does not support any mode-selective amplification since both the low-frequency mode and the high-frequency mode achieve their maximum possible gain simultaneously. Therefore, the mode selective amplification in a two-sectioned wiggler is realized at the expense of maximum possible gain with a greater gain reduction for the low-frequency mode.

## V. WIGGLER FIELD ERRORS AND ENERGY SPREAD

In the preceding section, mode-selective amplification is discussed for a monochromatic electron beam and a perfect wiggler magnet with no field errors. However, any realistic wiggler magnet has a finite field error, and any realistic electron beam has a finite energy spread. These variations in system parameters can reduce the FEL gain significantly. Therefore, it is important to understand whether or not the two-sectioned wiggler arrangement is more sensitive to these variations than the usual straight wiggler configuration. To this end, the small signal gain of the waveguide FEL of Fig. 4 is calculated for different wiggler field strengths and plotted as a function of frequency in Fig. 5. When the wiggler field of the second wiggler section varies  $\pm 2\%$  around its nominal value of  $a_{w2} = 0.4$ , the high-frequency mode experiences a visible change in both the peak gain and the peak gain frequency, whereas there is very little change for the low-frequency mode. At  $a_{w2} = 98\% \times 0.4$ , the peak gain of the high-frequency mode decreases to 13.5% from its nominal value of 13.8% with the peak gain frequency shifted to 15.35 GHz from the nominal value of 15.21 GHz. When the wiggler field increases to  $a_{w2} = 102\% \times 0.4$ , the peak gain becomes 14.02% at 15.08 GHz. Thus as the wiggler field changes  $\pm 2\%$  around the nominal value of  $a_{w2} = 0.4$ , the peak gain changes approximately the same amount of  $\pm 2\%$  around the nominal peak gain of 13.8%, whereas the peak gain frequency moves within  $\mp 0.14$  GHz. Similar field variation in the first wiggler section is found to result in a much smaller change in both the peak gain and the peak gain frequency. To compare with the case of a straight wiggler, the waveguide FEL of Fig. 2

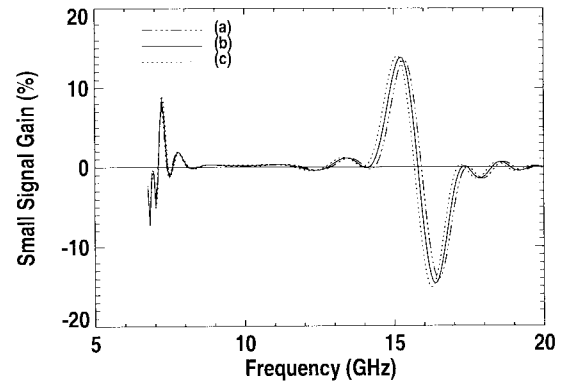


Fig. 5. FEL gain in a two-sectioned wiggler with (a)  $a_{w2} = 98\% \times 0.4$ , (b)  $a_{w2} = 0.4$ , and (c)  $a_{w2} = 102\% \times 0.4$ .

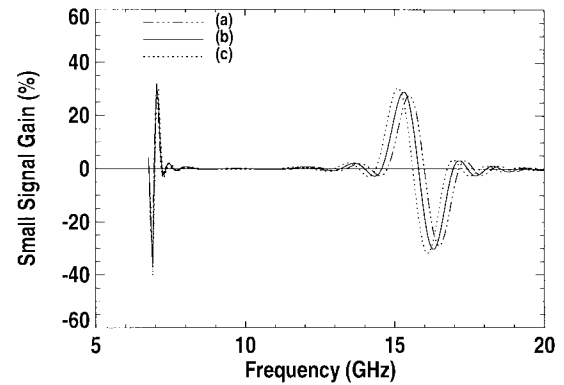


Fig. 6. FEL gain in a straight wiggler with (a)  $a_{w2} = 98\% \times 0.4$ , (b)  $a_{w2} = 0.4$ , and (c)  $a_{w2} = 102\% \times 0.4$ .

is studied for the same amount of field variation, and its effect on the small signal gain is illustrated in Fig. 6. For the wiggler field variation of  $\pm 2\%$ , the peak gain is found to change approximately  $\pm 5\%$  around the nominal value of 29% and the peak gain frequency approximately  $\mp 0.2$  GHz around the nominal value of 15.3 GHz. These calculated changes in the peak gain and peak gain frequency suggest that the effect of variations in both the peak gain and peak gain frequency should be less severe in a two-sectioned wiggler than in a straight wiggler.

Similarly the electron energy spread effect may be studied. Figs. 7 and 8 show the small signal gains calculated at the nominal electron energy of 300 keV, 1% less than 300 keV and 1% greater than 300 keV for the two-sectioned wiggler and its corresponding straight wiggler, respectively. For a 1% variation in electron energy in the two-sectioned wiggler, the peak gain of the high-frequency mode changes about 2% around its nominal value of 13.8% and the peak gain frequency varies about 0.26 GHz around the nominal value of 15.2 GHz. With the usual straight wiggler configuration, on the other hand, 1% variation in electron energy leads to a 2.1% change in the peak gain (around its nominal value of 29%) and a 0.23 GHz change in the peak gain frequency (around the nominal value of 15.3 GHz). Therefore, the requirement for electron energy spread and wiggler field error in a two-sectioned wiggler is comparable to that in a corresponding straight

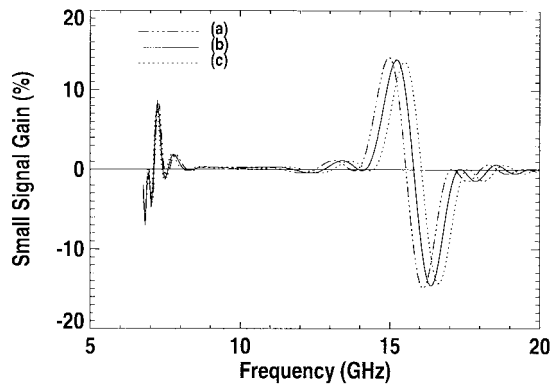


Fig. 7. FEL gain in a two-sectioned wiggler with (a)  $E_b = 99\% \times 300$  keV, (b)  $E_b = 300$  keV, and (c)  $E_b = 101\% \times 300$  keV.

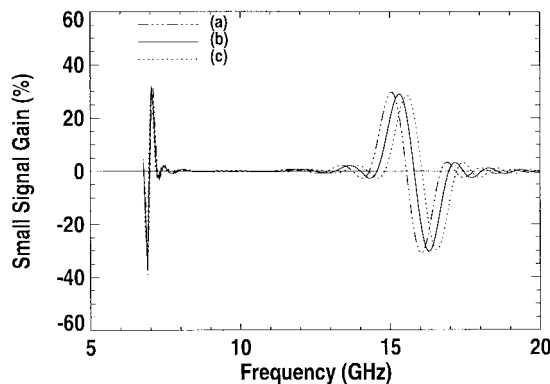


Fig. 8. FEL gain in a straight wiggler with (a)  $E_b = 99\% \times 300$  keV, (b)  $E_b = 300$  keV, and (c)  $E_b = 101\% \times 300$  keV.

wiggler. This implies that the usual design consideration for the latter in terms of its dependence on electron energy spread and wiggler field error may be adapted directly for the former.

It is of interest to note that the 1% electron energy change results in very little change in the peak gain and peak gain frequency for the low-frequency mode in both the two-sectioned wiggler and the straight wiggler. This may be understood from the fact that the slope difference between the beam line and the dispersion curve around the high-frequency intersection (point A in Fig. 3) is much smaller than that near the low-frequency intersection (point B or C in Fig. 3). This implies that a given amount of electron energy change results in a greater frequency shift for the high-frequency intersection and, as such, the high-frequency mode is more sensitive to the electron energy spread. Therefore, the high-frequency mode is likely to suffer a greater gain reduction for a given electron energy spread. One implication is that it is even more difficult in practice for the high-frequency mode to compete with the low-frequency mode in a straight wiggler magnet.

## VI. CONCLUSIONS

In this paper, a small signal gain formulation was developed to analyze a waveguide FEL with a two-sectioned wiggler magnet. It was demonstrated that with an appropriate differentiation between the two wiggler sections, the high-frequency mode can be amplified preferentially so as to provide an effective

means to suppress the unwanted low-frequency mode. In addition, it was shown that the mode-selective amplification was achieved by introducing different gain reductions to the two modes. In some cases, this can reduce the gain of the high-frequency mode considerably. Such an undesired gain reduction may be overcome with gain enhancement by introducing a drift section between the two wiggler sections to form a waveguide optical klystron configuration [27], [28]. As an alternative to cavity-based mode control techniques, this magnet-based technique is capable of suppressing the low-frequency mode in the small-signal regime, and its practical implementation is straightforward.

Numerical examples were used to demonstrate that the two-sectioned wiggler arrangement has a similar requirement for electron energy spread and wiggler field error to the usual straight wiggler configuration in waveguide FEL's. Therefore, the usual design consideration for the latter in terms of its dependence on electron energy spread and wiggler field error may be adapted directly for the former. It was also found that for a given electron energy spread and a given wiggler field error, the high-frequency mode would suffer a much greater gain degradation than the low-frequency mode and, as such, the actual mode competition would be much less favorable to the high-frequency mode than predicted in Fig. 2. This further emphasizes the need to control linear gains of the low-frequency mode and the high-frequency mode. It is worth mentioning that with a similar gain control for a preferred waveguide mode and any unwanted ones, the application of this two-sectioned wiggler technique may be extended to mode competition among different waveguide modes.

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**Michael G. Kong** (M'95) received the B.Sc. and M.Sc. degrees, both in physical electronics, from Zhejiang University, China, in 1984 and 1987, respectively, and Ph.D. degree in electrical engineering from Liverpool University, U.K., in 1992.

From 1992 to 1994, he worked as a Postdoctoral Research Fellow on a compact free electron laser project and a computational plasma project at Liverpool and Nottingham Universities, respectively. In 1995, he returned to Liverpool University as a Lecturer. His present research interests include free

electron lasers, switchgear arc plasmas, material processing plasmas, and optical fiber sensing.