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Cycles in U.K. Investment *

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Abstract

This paper investigates the cyclical behaviour of quarterly U.K. investment using an unobserved components framework formulated in continuous time. Comparisons are made between the results from two models that differ in the specification of the cyclical component.

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1 Introduction

Unobserved components models, also known as structural time series models, can be used to decompose a time series into its most salient features. Here a continuous time unobserved components framework is employed to investigate the nature of cycles in U.K. investment. The series used in this application is the logarithm of quarterly U.K. Total Gross Capital Formation covering the period 1955q1 to 2001q2. Two models are estimated and both include trend, seasonal and cyclical components. The models differ only in the specification of the non-seasonal cycle. In one model the cyclical component is written as a differential equation, in the form specified in Harvey and Stock (1993), henceforth the H-S component. In the second model the cyclical component is formulated as a differential-difference equation in the form specified by Chambers and McGarry (2002), henceforth the DDE component. Each represents cyclical behaviour in a very different way and here the primary interest are the estimated investment cycle durations. The trend and seasonal components are the same in both models and are written as differential equations.

Henceforth, the continuous time unobserved components models that incorporate the H-S and DDE cyclical components will be referred to as the H-S model and the DDE model respectively.

2 The Unobserved Components Models

The continuous time investment process $y(t)$ is written here as a function of the three components $y(t) = \mu(t) + \gamma(t) + \phi(t)$ where $\mu(t)$ represents the trend, $\gamma(t)$ the seasonal and $\phi(t)$ the cycle. The two models differ only in the specification of $\phi(t)$, and we concentrate on this component here. The H-S specification is given by the following system of differential equations

$$d \begin{bmatrix} \phi(t) \\ \phi^*(t) \end{bmatrix} = \begin{bmatrix} \ln \rho & \lambda_c \\ -\lambda_c & \ln \rho \end{bmatrix} \begin{bmatrix} \phi(t) \\ \phi^*(t) \end{bmatrix} dt + \begin{bmatrix} \kappa(dt) \\ \kappa^*(dt) \end{bmatrix} \quad (1)$$

for which $\kappa(dt)$ and $\kappa^*(dt)$ are mutually and serially uncorrelated random measures with common variance $\sigma_\kappa^2 dt$, for stationarity $0 < \rho < 1$ and λ_c is the frequency at which the cycle oscillates. The unknown parameters are ρ, λ_c and σ_κ^2 . The discrete time representation of the H-S component is

$$\begin{bmatrix} \phi_t \\ \phi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos(\lambda_c) & \sin(\lambda_c) \\ -\sin(\lambda_c) & \cos(\lambda_c) \end{bmatrix} \begin{bmatrix} \phi_{t-1} \\ \phi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix} \quad (2)$$

where $[\kappa_t, \kappa_t^*]'$ is a moving average disturbance vector. The moving average representation arises because investment is a flow process. The non-stochastic form of this system is $\phi_t = \alpha \rho^t \cos(t\lambda_c) + \beta \rho^t \sin(t\lambda_c)$, where $\phi_0 = \alpha$ and $\phi_0^* = \beta$, and it exhibits a cyclical pattern with a period $\frac{2\pi}{\lambda_c}$. System (2) simply introduces stochastic behaviour into this cyclical pattern. Hence the H-S component produces a single cycle with a period $\frac{2\pi}{\lambda_c}$. In contrast, the DDE component can generate an infinite number of cycles, although only those with longer durations (such as business cycles) are likely to be of interest to economists. The simple form of DDE considered here is given by

$$d\phi(t) = [a_0\phi(t) + a_1\phi(t - \nu)]dt + \kappa(dt) \quad (3)$$

where again $\kappa(dt)$ is a random measure with variance $\sigma_\kappa^2 dt$. The unknown parameters are a_0, a_1, ν and σ_κ^2 . This DDE specification was originally derived by Kalecki (1935) as a model of the macrodynamic theory of business cycles. Variations of the DDE specification have arisen more recently in, for example, Ioannides and Taub (1992) on time-to-build investment models and Boucekkine et al (1997) on vintage capital growth models.

The cycle durations from the DDE component are calculated from a function of the estimated values for a_0, a_1 and the lag parameter ν . The reader is referred to Chambers and McGarry (2002) for a detailed discussion of the process of determining the cycle durations. In practice, the lag parameter ν in (3), which Kalecki (1935) interprets as a time-to-build factor or gestation period, will be of unknown value. However, the estimation procedure developed

in Chambers and McGarry (2002) will estimate ν as well as the coefficient parameters a_0 and a_1 .

The seasonal component, common to both H-S and DDE models, takes the form $\gamma(t) = \gamma_1(t) + \gamma_2(t)$ where both $\gamma_1(t)$ and $\gamma_2(t)$ are specified in a similar way to the H-S cyclical component (1) but with $\rho = 1$ and in terms of the respective frequencies $\lambda_1 = \frac{\pi}{2}$ and $\lambda_2 = \pi$. These are the seasonal frequencies for quarterly data and $\gamma(t)$ contains seasonal unit roots at these frequencies. The common variance of the random measures in $\gamma_1(t)$ will be denoted $\sigma_{\omega_1}^2 dt$ and that for $\gamma_2(t)$ as $\sigma_{\omega_2}^2 dt$. The trend component in both models takes the form $d\mu(t) = \eta(dt)$ where $\eta(dt)$ is a random measure with variance $\sigma_{\eta}^2 dt$. The analogous representation in discrete time is as a random walk $\mu_t = \mu_{t-1} + \eta_t$. The unknown parameters in the seasonal component are $\sigma_{\omega_1}^2$ and $\sigma_{\omega_2}^2$ and in the trend it is σ_{η}^2 .

McGarry (2000) and Chambers and McGarry (2002) consider a frequency domain maximum likelihood estimator for the estimation of the parameters in DDEs. This form of estimator is employed here to estimate all of the unknown parameters in both H-S and DDE models. The reader is referred to Chambers and McGarry (2002) for details about this procedure.

3 Results

For the H-S model, the parameter vector is $\theta = \{\rho, \lambda_c, \sigma_{\kappa}^2, \sigma_{\eta}^2, \sigma_{\omega_1}^2, \sigma_{\omega_2}^2, \sigma_{\epsilon}^2\}$ and for the DDE model it is $\theta = \{a_0, a_1, \nu, \sigma_{\kappa}^2, \sigma_{\eta}^2, \sigma_{\omega_1}^2, \sigma_{\omega_2}^2, \sigma_{\epsilon}^2\}$. The estimates of the structural parameters of both models are given in the table, where t statistics are given in bold. In both models, only the parameters that determine the cycle durations are statistically significant. The variance parameters, although insignificant, are estimated with similar values in both models. Of primary interest however are the cycle duration estimates. Using the H-S model, a single cycle of length $\frac{2\pi}{\lambda_c} = 13.5$ quarters (3.4 years) is detected. As stated earlier, the DDE component can identify more than one cycle and in this application several low frequency cycles are estimated. The durations of these cycles are not estimated directly and hence do not appear in the table.

Table 1: Parameter Estimates

Parameter	H-S model	DDE model
λ_c	0.464479 5.649774	-
ρ	0.999984 1.430622	-
a_0	-	-1.593572 -3.603930
a_1	-	-2.015947 -3.985691
ν	-	19.969402 35.304580
σ_κ^2	0.000002 0.099673	0.000001 0.086071
σ_η^2	0.000416 0.613775	0.000453 0.650771
$\sigma_{\omega_1}^2$	0.000054 0.480161	0.000050 0.441680
$\sigma_{\omega_2}^2$	0.000066 0.465939	0.000065 0.434446
σ_ϵ^2	0 0	0 0

They are computed from a complex function of the estimated values for a_0, a_1 and ν . The largest of these cycles has a duration of 41 quarters (10.25 years). This fulfils the Kalecki definition of a business cycle, which must have length greater than twice the lag value, ν . The lag value in this case is highly significant and represents a period of 19.97 quarters (5 years). Other shorter cycles that are found using the DDE model have lengths 5.1 years and 3.4 years, the latter being the same cycle as that estimated from the H-S model. The H-S representation has not detected the longer cycles (10 and 5 years) in this investment series. It is likely that the longer durations are more difficult to estimate because there are fewer complete cycles in any fixed sample of data.

The results from the DDE model compare favourably with those found in Reiter and Woitek (1999), in which the cyclical behaviour of fixed investment (amongst other variables)

is analysed for 15 OECD countries using spectral techniques. For the U.K. they find that the spectral density is concentrated in a cycle range of 5-7 years, although it also has some mass in the ranges 7-10 and 10-15 years. The range 3-5 years has little spectral mass, which is not consistent with the single cycle estimated by the H-S model. Wen (1998) estimates a cycle of 7 years in the U.S. aggregate fixed investment to output ratio. It is suggested that the long period of this cycle is due to the length of time taken in the production of fixed capital and that this time-to-build factor generates persistent demand for investment goods. This reflects our findings from the DDE cyclical component, where the longer cycle of just over 10 years is generated by the long time-to-build factor of 5 years (the lag parameter).

In both models, the cycle is non-stationary. In the H-S case, this is shown by the fact that the damping parameter is not statistically different from unity. This implies the presence of a unit root and that the cycle is actually undamped. The DDE parameter combination is also non-stationary, which is inferred also from damping factors of 1.25 for the major 10-year cycle and 1.2 for the cycle of 3.4 years.

4 Conclusion

In conclusion, the H-S model has identified one cycle of duration 3.4 years and the DDE model has identified three important long length cycles of duration 10.25, 5.1 and 3.4 years. The 10-year cycle accords with evidence from previous studies and has been detected by the DDE specification despite just four of these cycles having been completed during the timespan of the dataset.

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