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**Department of Economics**

*Business Cycle Volatility and Economic Growth*  
*Research Paper No. 01/1*

*MULTIVARIATE MARKOV SWITCHING*  
*COMMON FACTOR MODELS FOR THE U.K.*

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## **ABSTRACT**

We estimate a model that incorporates two key features of business cycles, comovement among economic variables and switching between regimes of boom and slump, to quarterly U.K. data for the last four decades. A common factor, interpreted as a composite indicator of coincident variables, and estimates of turning points from one regime to the other, are extracted from the data by using the Kalman filter and maximum likelihood estimation. Both comovement and regime switching are found to be important features of the U.K. business cycle. The composite indicator produces a sensible representation of the cycle and the estimated turning points agree fairly well with independently determined chronologies. These estimates are sharper than those produced by a univariate Markov switching model of GDP alone. A fairly typical stylised fact of business cycles is confirmed by this model - recessions are steeper and shorter than recoveries.

**Keywords:** BUSINESS CYCLES, REGIME SWITCHING, MARKOV MODELS, COMOVEMENT.

**J.E.L. Classification:** C5, E3

## 1. INTRODUCTION

The two empirical regularities of business cycles highlighted by Burns and Mitchell (1946) - comovement among economic variables through the cycle and asymmetry in the evolution of the cycle - have undergone a resurgence of interest in recent years, prompted by the development of new time series techniques. Two of the most influential models of the business cycle are Stock and Watson's (1989, 1991, 1993, 1999) linear common factor model and Hamilton's (1989) regime switching model. Stock and Watson develop a linear dynamic factor model where business cycles are measured by comovements in various components of economic activity. Using several macroeconomic time series, they extract a single unobserved variable and interpret it as the "state of the economy". They then compare this variable with the U.S. Department of Commerce (DOC) composite index, and find that the similarity between the two is striking, especially over the business-cycle horizon. The disadvantage of their model, however, is that its linearity cannot capture business cycle asymmetry, and forces expansions and contractions to have the same amplitude and duration.

To capture such asymmetry, Hamilton (1989) develops a regime switching model in which output growth switches between two states according to a first order Markov process. Expansions can therefore be gradual and drawn out while recessions may be shorter and steeper - the 'stylised facts' of modern business cycles. Applying this model to the U.S., he shows that shifts between positive and negative output growth accord well with the NBER's chronology of business cycle peaks and troughs. Being based on a single time series, however, Hamilton's model cannot capture the notion of economic fluctuations corresponding to comovements of many aggregate and sectoral variables. It may well be impossible for only one coincident variable to capture all underlying business cycle information, which is the conclusion of both Filardo (1994) and Diebold and Rudebusch (1996).

Indeed, Diebold and Rudebusch provide both empirical and theoretical support for combining these two key features of the business cycle, although they do not fully estimate a model. Building on this research, however, several studies do estimate these two features simultaneously within the regime switching common factor model: for example, Chauvet (1998), Kim and Yoo (1995), and Kim and Nelson (1998). The common factor is defined to be an unobserved variable that summarises the common cyclical movements of a set of coincident macroeconomic variables, as in Stock and Watson (1991). However, it is also subject to discrete shifts so that it can capture the asymmetric nature of business cycle phases, as in Hamilton (1989). Within a multivariate framework, all three papers report that inferences about the state of the

economy obtained from the model exhibit significantly higher correlations with the NBER reference dates than if just a single variable, such as output growth, was used.

The basic idea behind these studies is that information about business cycles can be extracted from a group of series rather than a single series, so that estimated business cycles reflect information from various economic sectors. Furthermore, the extracted factor can be compared with, for example, the DOC coincidence index, and more importantly, it can be used for real time assessment of the economy.

Previous research using these models has typically used data from the U.S., and few studies of other economies have been undertaken. In the U.K., research on the asymmetry of business cycles have been based on the univariate Hamilton regime switching model, for example, Krolzig and Sensier (2000) and Simpson, Osborn and Sensier (2001). However, none of these have attempted to combine asymmetry with a common factor derived from a set of indicator series. We think such an extension is important for two related reasons. First, if a set of indicators can correctly provide signals of changes in aggregate economic activity, then this would be helpful to any business or government in their decision making, as they are typically affected by economic expansions and contractions. Second, in studying aggregate fluctuations like business cycles, it is useful to be able to analyse a group of important economic time series. Individual series measure only one aspect of economic activity, so they cannot capture the idea of cyclical fluctuations corresponding to comovements of many aggregate and sectoral variables. Knowledge of these features for the U.K. economy is therefore important for policy makers and forecasters.

## 2. MODEL SPECIFICATION

As stated in the introduction, our model combines the common factor model with regime switching. Suppose that  $Y_{it}$  is (the logarithm of) a macroeconomic variable that moves contemporaneously with overall economic conditions. It can be modelled as consisting of two stochastic autoregressive processes - a single unobserved component, which corresponds to the common factor, and an idiosyncratic component. Defining  $\Delta y_{it} = \Delta(Y_{it} - \bar{Y}_i)$ , the model can be written as follows,

$$\Delta y_{it} = \lambda_i \Delta c_t + z_{it}, \quad i = 1, \dots, n, \quad (1)$$

$$\phi(L)\Delta c_t = \mu_{s_t} + v_t, \quad v_t \sim i.i.d. N(0,1), \quad (2)$$

$$\psi_i(L)z_{it} = \varepsilon_{it}, \quad \varepsilon_{it} \sim i.i.d. N(0, \sigma_i^2) \quad (3)$$

$\Delta c_t$  is the growth rate of the common factor, which is dependent on whether the economy is in expansion or recession, and it enters each of the  $n$  equations with a different weight  $\lambda_i$ , which measures the sensitivity of the  $i$ th variable to the business cycle. The variables  $z_{it}$  are idiosyncratic terms having an AR representation. Their innovations  $\varepsilon_{it}$  can be thought of as measurement errors and  $v_t$  is the innovation to the common factor. The functions  $\psi_i(L)$  and  $\phi(L)$  are polynomials in the lag operator, where  $L$  is the lag operator and  $\Delta = 1 - L$ .

To incorporate the asymmetry of business cycles, the common factor is assumed to be generated by a Markov switching process of the type proposed by Hamilton (1989), so that

$$\mu_{S_t} = \mu_0(1 - S_t) + \mu_1 S_t \quad (4)$$

where  $S_t$  is an unobservable state variable that switches between state 0 (recession) and state 1 (expansion) with transition probabilities governed by the Markov process

$$\begin{aligned} P[S_t = i | S_{t-1} = i] &= p_{ii} \\ P[S_t = j | S_{t-1} = i] &= 1 - p_{ii} \\ 0 < p_{ii} < 1, \quad & i, j = 0, 1 \end{aligned}$$

In the absence of equation (4), we have the Stock and Watson dynamic factor model. For the identification of the model, it is assumed that the variance of  $v_t$  is unity. The innovations  $v_t$  and  $\varepsilon_{it}$  are assumed to be independent for all  $t$  and  $i$ .

With the availability of the estimation method developed by Kim (1994), the model can be estimated by maximising the likelihood function. Inferences about the unobserved nonlinear factor and the latent Markov state can then be obtained at the same time. The method consists of a combination of Hamilton's algorithm and the nonlinear discrete version of the Kalman filter: we refer to Kim (1994) for technical details.

To facilitate estimation, the model can be expressed in state-space representation. With AR(2) processes for both the common factor and idiosyncratic term, and with  $n = 4$  (as in the application below), the model can be expressed as the measurement and transition equations

$$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \Delta y_{3t} \\ \Delta y_{4t} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta c_t \\ \Delta c_{t-1} \\ e_{1t} \\ e_{1,t-1} \\ e_{2t} \\ e_{2,t-1} \\ e_{3t} \\ e_{3,t-1} \\ e_{4t} \\ e_{4,t-1} \end{bmatrix} \quad (5)$$

and

$$\begin{bmatrix} \Delta c_t \\ \Delta c_{t-1} \\ z_{1t} \\ z_{1,t-1} \\ z_{2t} \\ z_{2,t-1} \\ \dots \\ z_{3t} \\ z_{3,t-1} \\ z_{4t} \\ z_{4,t-1} \end{bmatrix} = \begin{bmatrix} \mu_{s_t} \\ 0 \\ 0 \\ \dots \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \psi_{11} & \psi_{12} & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & \psi_{41} & \psi_{42} \\ 0 & 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta c_{t-1} \\ \Delta c_{t-2} \\ z_{1,t-1} \\ z_{1,t-2} \\ z_{2,t-1} \\ z_{2,t-2} \\ \dots \\ z_{3,t-1} \\ z_{3,t-2} \\ z_{4,t-1} \\ z_{4,t-2} \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \\ \varepsilon_{1t} \\ 0 \\ \varepsilon_{2t} \\ 0 \\ \dots \\ \varepsilon_{3t} \\ 0 \\ \varepsilon_{4t} \\ 0 \end{bmatrix} \quad (6)$$

### 3. DATA AND RESULTS

We chose four time series that are representative coincident economic indicators: output, income, sales and employment. These series are GDP at factor cost, real household disposable income, retail sales, and employee jobs.<sup>1</sup> All series are seasonally adjusted quarterly observations and logarithms are used. The sample period is from 1959Q2 to 2000Q2.<sup>2</sup> Graphs of the four series are shown in Figure 1.

We first test whether the four series are individually integrated and, if they are, whether they are cointegrated.<sup>3</sup> We find that we cannot reject the hypothesis that each of the series is integrated, and neither can we reject the hypothesis of no cointegration

<sup>1</sup> Except for the retail sales series, taken from Datastream, all other data are from the Office of National Statistics. The series codes are YBHH, NRJR, UKRETTOTG and BCAJ, respectively. We also tried workforce rather than employees, producing results similar to those reported here.

<sup>2</sup> Monthly income is only available after 1986Q1.

<sup>3</sup> Results are available upon request.

among these variables. Therefore, we use the first differences of the variables (multiplied by one hundred) as is implied by the model set out in equations (1) to (6). As in the model, all series are demeaned by subtracting the sample mean from each difference.

As in equation (6), we began with second order autoregressive specifications for both the common component and the four idiosyncratic components, producing the results presented in Table 1. The estimated model suggests that the common factor and GDP are, in fact, white noise. Restricting the appropriate four parameters to zero produced the estimates shown in Table 2. As the likelihood ratio statistic from the two models has a value of only 0.32, the restricted model is taken as our preferred one.

Table 1 Estimates of the dynamic factor model with Markov switching

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<u>Common factor</u>					
$\phi_1$	$\phi_2$	$\mu_0$	$\mu_1$	$P_{00}$	$P_{11}$
-0.0477	0.0482	-1.0965	0.3581	0.8076	0.9407
(0.1477)	(0.1238)	(0.4164)	(0.2090)	(0.1406)	(0.0527)
<u>Idiosyncratic component</u>					
	$\psi_{i1}$	$\psi_{i2}$	$\sigma_i^2$	$\lambda_i$	
$\Delta y_{1t}$	-0.0208 (0.0786)	-0.0001 (0.0083)	0.8310 (0.0593)	0.5091 (0.0857)	
$\Delta y_{2t}$	-0.3513 (0.0874)	-0.0308 (0.0153)	1.3063 (0.0838)	0.6701 (0.1213)	
$\Delta y_{3t}$	-0.5864 (0.1252)	-0.0860 (0.0367)	0.7993 (0.1002)	0.7664 (0.1226)	
$\Delta y_{4t}$	0.4623 (0.0815)	0.1405 (0.0821)	0.3940 (0.0243)	0.1202 (0.0375)	
Log-likelihood	-254.33				

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Note: The order of the variables in  $y_{it}$  is GDP, income, sales and employment.  
Standard deviations are in parentheses



Table 2 Estimates of dynamic factor model with Markov switching with restrictions

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<u>Common factor</u>					
$\phi_1$	$\phi_2$	$\mu_0$	$\mu_1$	$p_{00}$	$p_{11}$
-	-	-1.0668 (0.3632)	0.3448 (0.1804)	0.8213 (0.1149)	0.9457 (0.0466)
<u>Idiosyncratic component</u>					
	$\psi_{i1}$	$\psi_{i2}$	$\sigma_i^2$	$\lambda_i$	
$\Delta y_{1t}$	-	-	0.8333 (0.0587)	0.5116 (0.0833)	
$\Delta y_{2t}$	-0.3543 (0.0869)	-0.0314 (0.0154)	1.3028 (0.0834)	0.6751 (0.1174)	
$\Delta y_{3t}$	-0.5957 (0.1242)	-0.0887 (0.0370)	0.7974 (0.0990)	0.7667 (0.1148)	
$\Delta y_{4t}$	0.4615 (0.0812)	0.1408 (0.0821)	0.3943 (0.0242)	0.1214 (0.0365)	
Log-likelihood value	-254.49				

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Note: The order of the variables in  $y_{it}$  is GDP, income, sales and employment.  
Standard deviations are in parentheses

The estimated model seems successful in extracting information about fluctuations in economic activity. The results support the presence of asymmetric business cycles that switch between two different states, with state 0 having a significantly negative mean and state 1 a significantly positive mean. The transition probabilities associated with these two regimes of recession and expansion are 0.821 and 0.946 respectively. These estimates imply that the average duration of the expansionary regime is  $(1 - p_{11})^{-1} = 18.4$  quarters, which may be contrasted with  $(1 - p_{00})^{-1} = 5.6$  quarters for the average duration of the recessionary regime. The estimates of the mean growth rates of the business cycle common factor are  $-1.07$  and  $0.34$ . Therefore, recessions on average are both steeper and shorter, both by a factor of approximately three, than expansions, which is consistent with the findings of Kim and Nelson (1998) for the U.S. Figure 2 plots the extracted Markov switching

common factor in both levels and first differences.<sup>4</sup> The levels of this series, which may be interpreted as an index of the business cycle, accurately reproduce the stylised facts of the U.K. experience, while the differences show clearly the volatility of the 1970s and the relative stability of the 1990s.

Moving to the idiosyncratic component, our estimates show that sales has the highest weighting on the common factor, suggesting that this series is the most sensitive coincident variable. This is consistent with the common observation that sales respond immediately to changes in economic conditions. The next most sensitive series is income, followed by GDP and employment. Using monthly data from the U.S., Kim and Nelson (1998) found that industrial production had the highest weighting, followed by income, sales and, finally, employment. We suspect that our different ordering is because industrial production responds more swiftly than GDP to economic conditions, especially when the economy is close to turning points. Employment is the least sensitive to business cycle movements and also has the smallest innovation variance among the four variables. The negative coefficients of  $\psi_{i1}$  and  $\psi_{i2}$  for income and sales indicate that the idiosyncratic components of these series exhibit negative serial correlation, while the employment series behaves differently with positive idiosyncratic autocorrelation.

Figure 3 plots the probability that the economy is in a recession: panel (a) shows the filtered probability conditional on information available through  $t$ ,  $\Pr[S_t = 1 | \Psi_t]$ , ( $t = 1, 2, \dots, T$ ), while panel (b) shows the smoothed probability based on the complete set of information up to  $T$ ,  $\Pr[S_t = 1 | \Psi_T]$ , ( $t = 1, 2, \dots, T$ ). The two graphs are very similar and clearly pick out and date correctly the three major recessions that the U.K. economy has experienced during the last four decades. Apart from these, the plots identify several brief recessions during the 1960s, typically related to an overvalued exchange rate and consequent balance of payments deficits.

Unfortunately, there is no official U.K. business cycle chronology that we may relate our results to. We have thus compared our inferred probabilities of recessions with the chronology provided by Artis, Kontolemis and Osborn (1997), where they use both their own procedure and one provided by Bry and Boschan (1971)<sup>5</sup>. Their dating is based on monthly industrial production and finishes in 1993, and so can only be used for rough comparisons. We find that our recession probabilities are more closely related to Bry and Boschan's dating, which are shown as shaded areas on the plots of Figure 3. Almost all of their recessions are picked up by our model, although the durations of each recession are somewhat different.

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<sup>4</sup> The details of how to obtain the levels of the common factor are described in Stock and Watson (1991).

<sup>5</sup> Both chronologies are presented in Table D1 of Artis, Kontolemis and Osborn (1997)

We also estimate a univariate Markov switching model using GDP data only, where the growth rate of GDP is assumed to follow an AR(4) process, as in Hamilton (1989). The results are shown in Table 3. It is interesting to find that the transition probabilities associated with the two regimes of expansion and recession are 0.957 and 0.701 respectively, and the associated average durations are 23.04 and 3.35 quarters for expansions and recessions respectively. These estimates thus support previous findings that univariate Markov switching models tend to find lower probabilities of recession and hence shorter average recession durations. Note that the mean growth of GDP in recessions is  $-3.2\%$  per annum, while it is  $2.6\%$  per annum during expansions.

Figure 4 plots the filtered and smoothed recession probabilities from the univariate Markov switching model, again with the Bry and Boschan business cycle dating. As before, the three major recessions are identified, but the model fails to account for several of the recessions during the 1960s.

Table 3 Estimates from the univariate Markov switching model

Parameter	Estimates
$\mu_0$	-0.7838 (0.0688)
$\mu_1$	0.6626 (0.2575)
$p_{00}$	0.7011 (0.1333)
$p_{11}$	0.9566 (0.0236)
$\sigma$	0.8654 (0.0562)
$\phi_1$	-0.1697 (0.0893)
$\phi_2$	-0.0534 (0.0954)
$\phi_3$	0.0165 (0.1073)
$\phi_4$	-0.0451 (0.0887)
Log-likelihood value	-223.96353

## 4. CONCLUSIONS

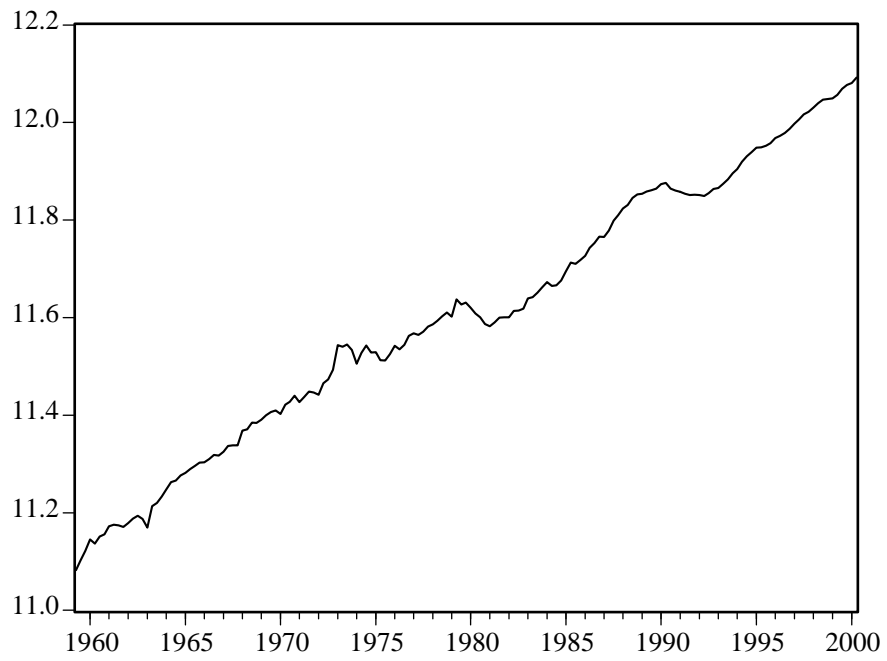
We have estimated a model that incorporates two key features of business cycles, comovement among economic variables and switching between regimes of boom and slump, to quarterly U.K. data for the last four decades. A common factor, interpreted as a composite indicator of coincident variables, and estimates of turning points from one regime to the other, were extracted from the data by using the Kalman filter and maximum likelihood estimation approach of Kim (1994). Both comovement and regime switching are found to be important features of the U.K. business cycle. The composite indicator produces a sensible representation of the cycle and the estimated turning points agree fairly well with independently determined chronologies. These estimates are sharper than those produced by a univariate Markov switching model of GDP alone. A fairly typical stylised fact of business cycles is confirmed by this model - recessions are steeper and shorter than recoveries.

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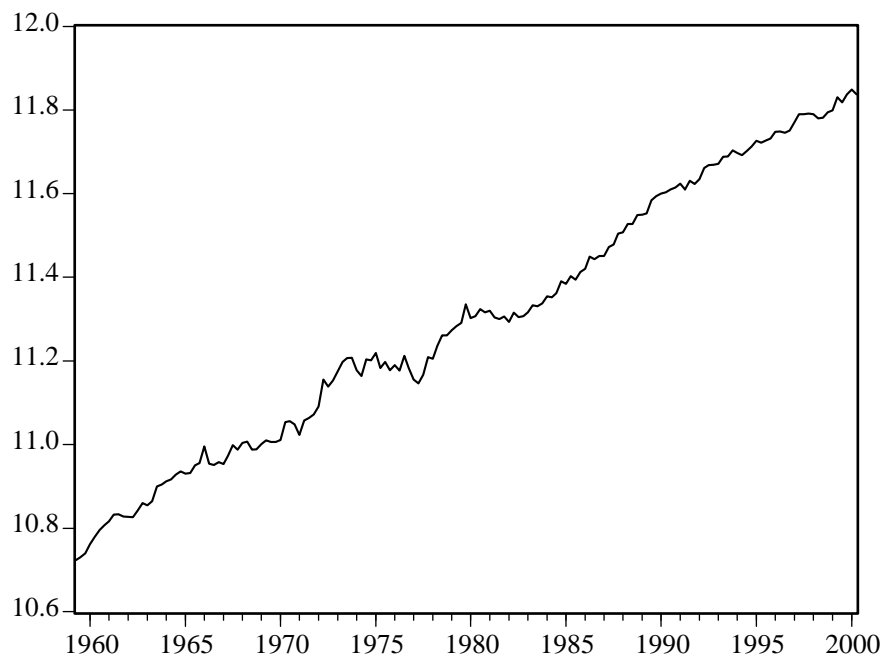
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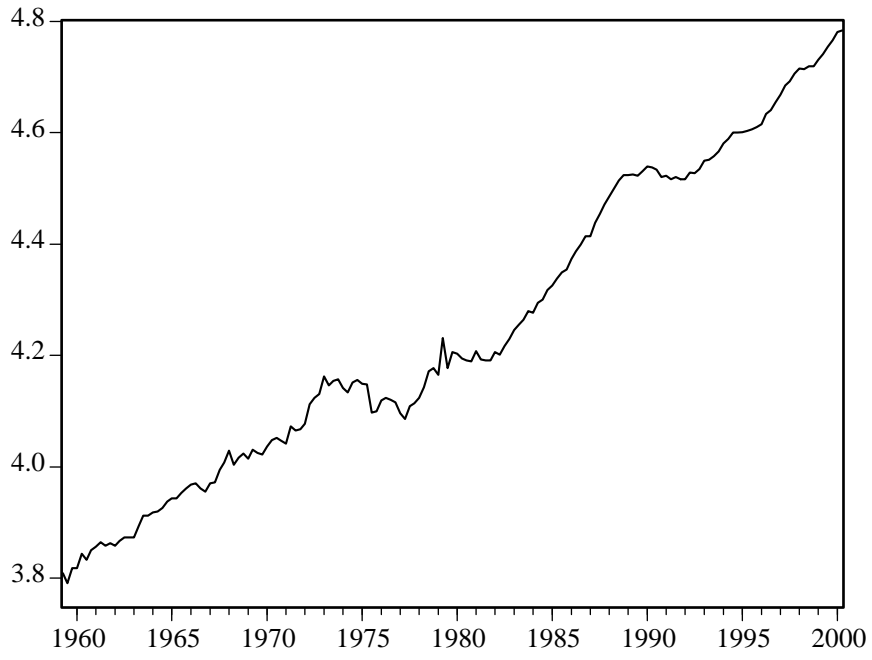
Figure 1. Time series of the four coincident variables



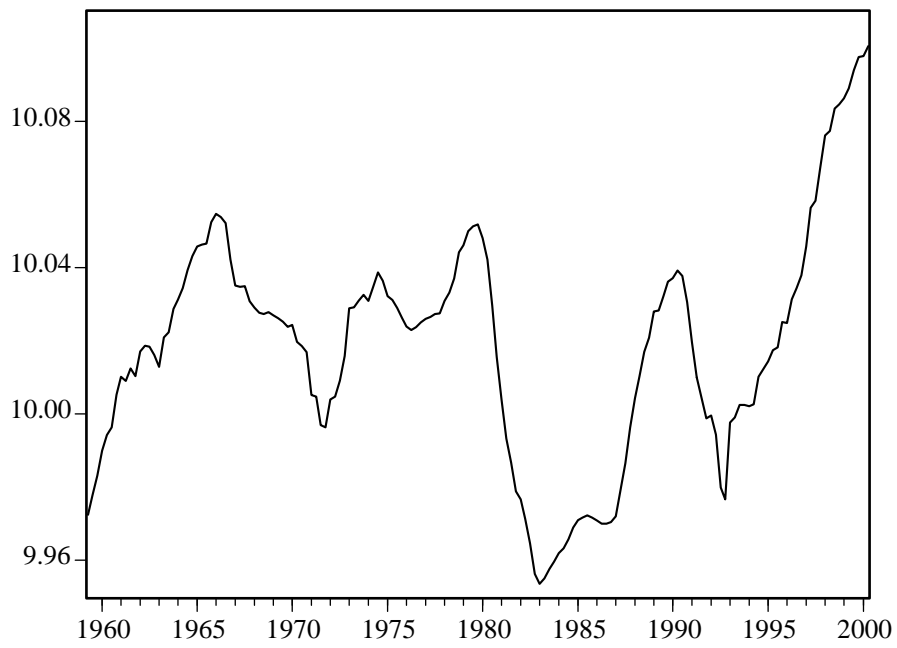
(a) Logarithm of GDP



(b) Logarithm of income

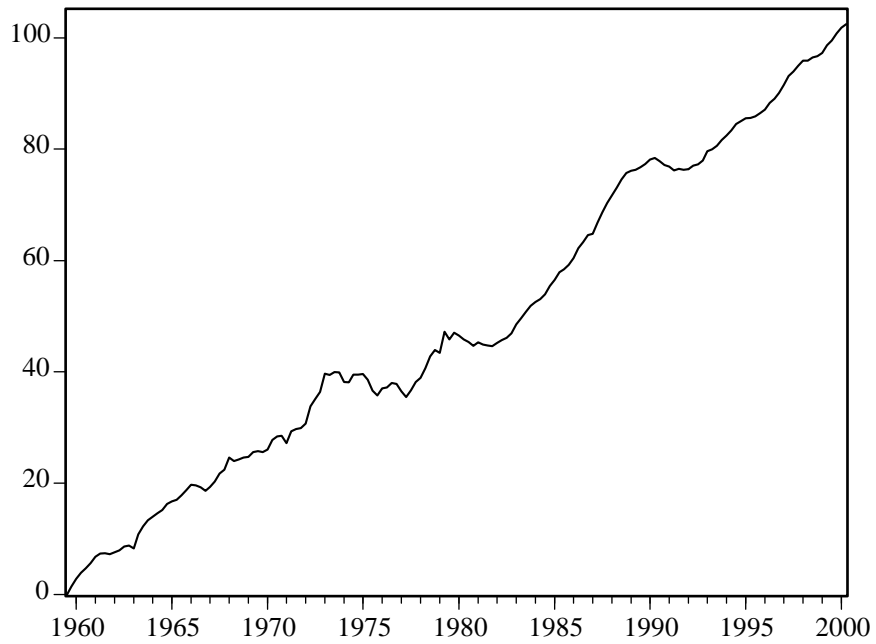


(c) Logarithm of sales

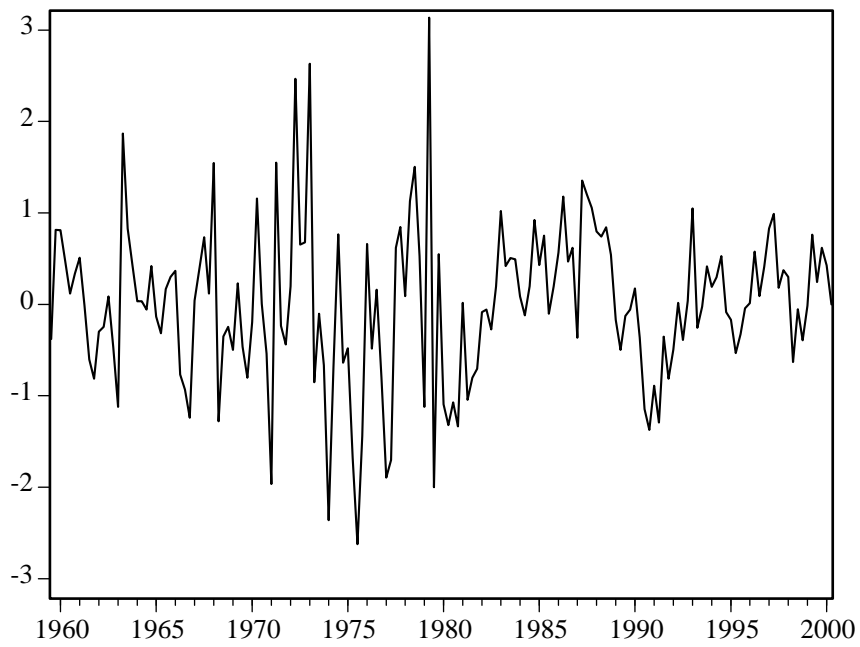


(d) Logarithm of employee jobs

Figure 2. Extracted Markov switching common factor



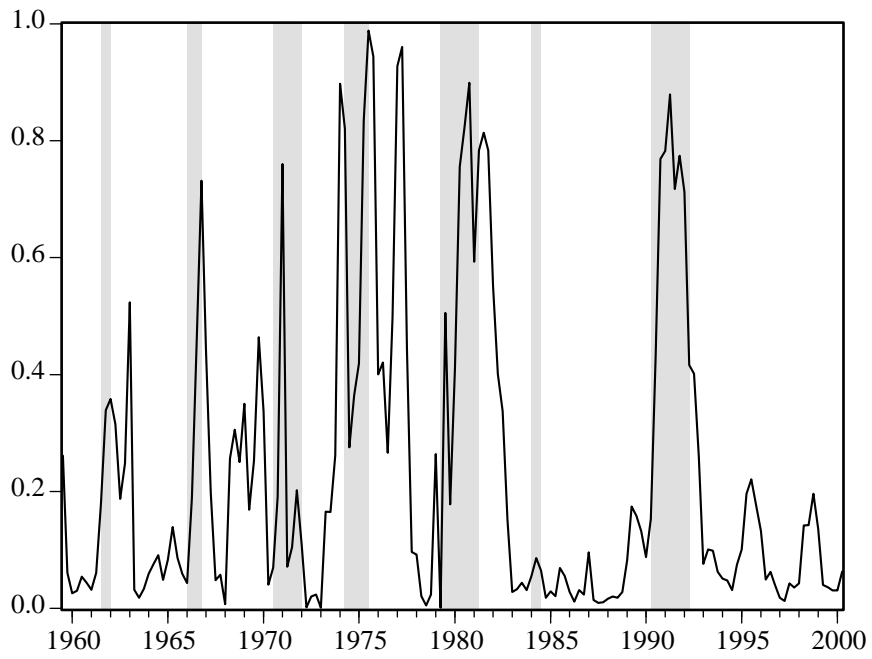
(a) Common factor,  $c_t$



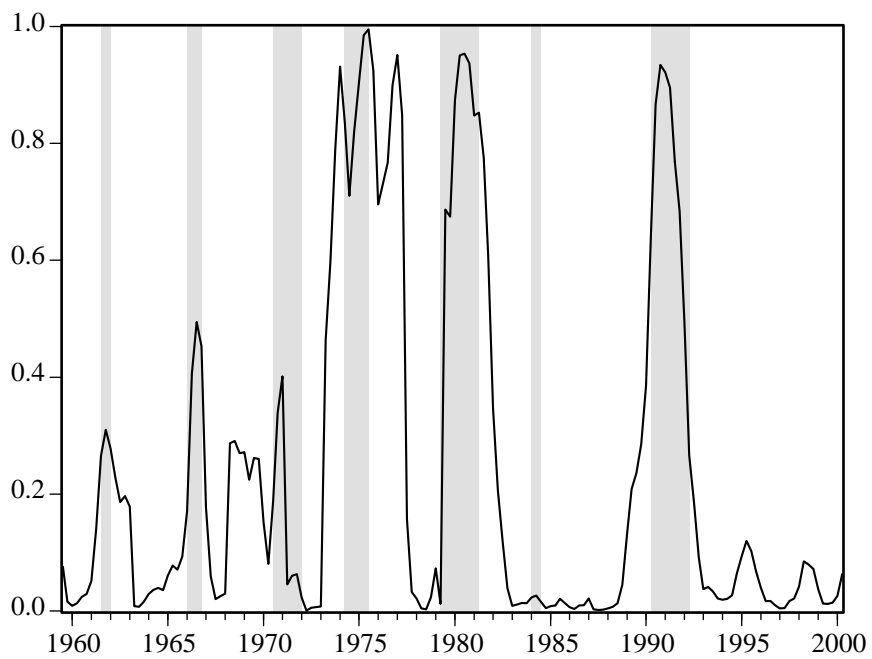
(b) Growth rate of common factor,  $\Delta c_t$



Figure 3. Filtered and smoothed recession probabilities from Markov switching and common factor model.

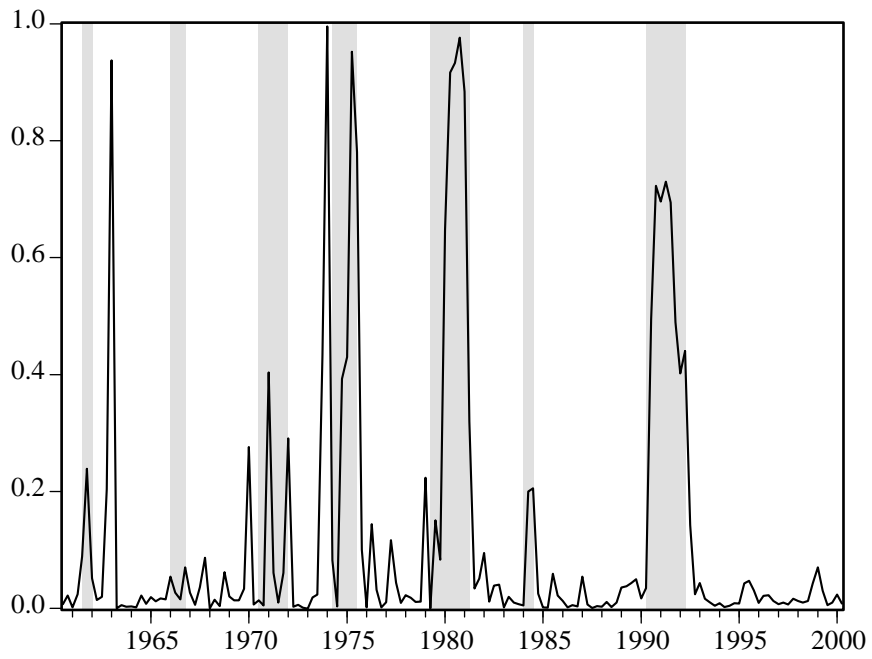


(a) Filtered probabilities

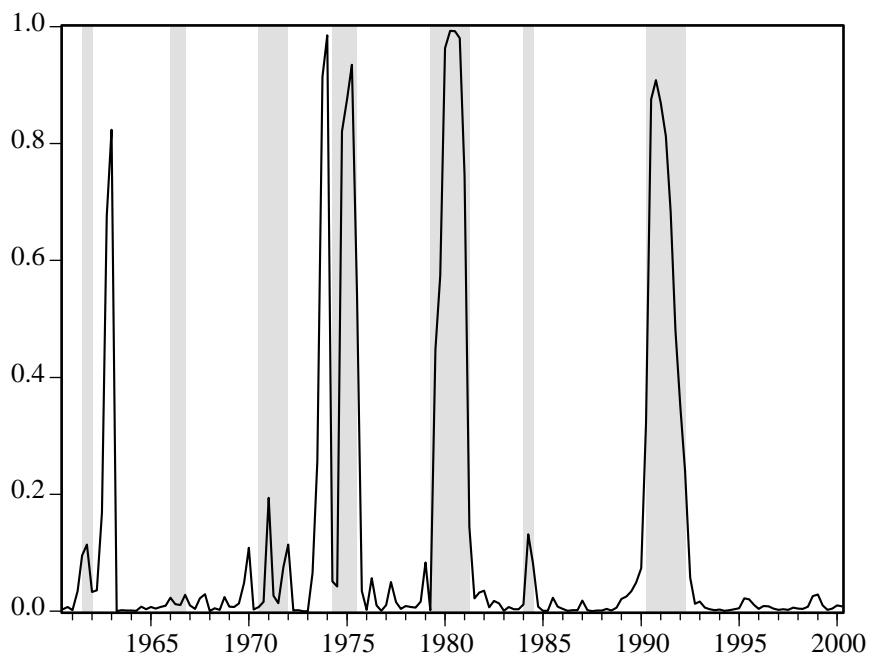


(b) Smoothed probabilities

Figure 4. Filtered and smoothed recession probabilities from univariate model of GDP



(a) Filtered probabilities



(b) Smoothed probabilities