Fault Detection in Model Predictive Controller

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Abstract

Real-time monitoring and maintaining model predictive controller (MPC) is becoming an important issue with its wide implementation in the industries. In this paper, a measure is proposed to detect faults in MPCs by comparing the performance of the actual controller with the performance of the ideal controller. The ideal controller is derived from the dynamic matrix control (DMC) in an ideal work situation and treated as a measure benchmark. A detection index based on the comparison is proposed to detect the state change of the target controller. This measure is illustrated through the implementation for a water tank process.

1 Introduction

With its wide implementation in the industries, the MPC performance monitoring and assessment have been a hot research field recently. Huang and Shah [1] illustrated the LQG control as the benchmark to assess the model predictive controller. Patwardan et al. [2] proposed the use of the historical objective functions as a practical benchmarking technique and Ko and Edgar [3] presented a benchmark based on the finite horizon Minimum Variance Controller (MVC) to monitor the target controller performance.

Most of researches in this field in fact assume that the controllers are in the normal work state. However, MPC software is becoming more and more complicated with MPC strategies widely incorporated into industrial computer based control systems to solve actual difficult multiple input, multiple output (MIMO) control problems. The steadily growing size and complexity of control software are making it extremely difficult to exhaustively test the software to ensure that it will adequately perform its specified function. It has been shown [4] that no matter how much effort has been put into the early stages of the software development, building large fault-free software systems has proven nearly impossible in practice. Even if these software initially perform well, some factors can contribute to them abrupt or gradual performance deterioration after their releases. So it is important to introduce a measure to real time detect the MPC faults.

So far, very little research has touched in this field. The most related work is the model based fault detection for plants/machines [5]. The method utilized an explicit mathematical model of the system under test. A reference model is obtained by first identifying the system in a fault-free situation, and then repeatedly identified. Deviations from the reference model parameters serve as the basis for fault detection.

Different from plants or machines, MPCs are linear time-varying controllers, and their parameters are real time updated in response to controlled process dynamics to minimize a time-varying objective function; therefore, it is hard to establish an accurate mathematical model as the reference model to precisely describe controller behaviors in practice.

In this work, a novel measure is presented to detect faults of MPCs. The metric is based on the comparison between the ideal and actual achieved control efforts under the identical inputs. The ideal controller is derived from the dynamic matrix control (DMC). The ideal controller is thought to work in an ideal situation and to be free of constraints of control process and to have enough power to achieve the control objective. A detection index based on the comparison is used to detect the state change of the actual controller. A water tank process is chosen as a case study to illustrate the proposed measure. The main advantage of this measure is that it takes into account the structure of the MPC application along with its design specification and the detection index is quite straightforward and easy to be implemented.

The rest of this paper is outlined as follows. A brief introduction of MPC preliminaries is given in Section 2, followed by a discussion of the ideal controller design in Section 3. A measure to detect the state change of MPCs is presented in Section 4. Section 5 contains a case study, in which the proposed measure is employed in a water tank process. Section 6 gives the conclusions.

2 Ideal Controller Design

2.1 MPC Preliminaries

The various MPC algorithms propose different functions for obtaining the control law. The underling philosophy of the MPC is that future output on the considered horizon should follow a determined reference signal and at the same time, the control effort necessary for doing so should be penalized. The general objective function in MPCs is the sum of (i) a weighted norm of the control errors over a prediction horizon, p; and (ii) a weighted norm of the control moves over a control horizon, m:

$$J = \sum_{j=1}^{p}[\hat{y}(t+j\,|\,t) - w(t+j)]^2 + \sum_{j=1}^{m}\lambda[\Delta u(t+j-1)]^2 \quad (1)$$

where $\hat{y}(t+j\,|\,t)$ is the estimation of the output $y(t+j)$ at the instant $t+j$, $w(t+j)$ is the reference signal at the instant $t+j$, λ is relative weight used to achieve a smooth control; $\Delta u(t+j-1)$ is control effort at the instant $t+j-1$.

The corner-stone component of any predictive control scheme is a predictor, which should be precise enough to fully capture the process dynamics and predict the process response over the prediction horizon, based on the current and past measurements. The different strategies of MPCs can use various models to describe the process behavior.

The general truncated form of step response model of stable systems is given by:

$$y(t) = y_0 + \sum_{i=1}^{N} g_i \Delta(t-i) \quad (2)$$

where g_i are the sample output values for the step input. The value of y_0 can be set to 0 without loss of generality, so the predictor will be

$$\hat{y}(t+k\,|\,i) = \sum_{i=1}^{N} g_i \Delta u(t+k-i\,|\,t) \quad (3)$$

One great advantage of the method is that no prior information about the process is needed, so that the identification process is simplified and at the same time it allows complex dynamics to be described easily; however, this method needs the large number of parameters as N is usually a high value.

The predicted values along the prediction horizon can be expressed in the vector form as follows:

$$\hat{\mathbf{y}} = G\Delta\mathbf{u} + \mathbf{f} \quad (4)$$

where the system's dynamic matrix, G is

$$G = \begin{bmatrix} g_1 & 0 & 0 & \cdots & 0 \\ g_2 & g_1 & 0 & \cdots & 0 \\ \cdots & & & & \\ g_m & g_{m-1} & \cdots & g_2 & g_1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ g_p & g_{p-1} & \cdots & g_{p-m+2} & g_{p-m+1} \end{bmatrix}_{p\times m} \quad (5)$$

$$\Delta\mathbf{u} = [\Delta u(t) \quad \Delta u(t+1) \quad \ldots \quad \Delta u(t+m-1)]^T \quad (6)$$

$\mathbf{f}$ is the free response vector of the system and does not depend on the future control actions.

2.2 Ideal Controller Design

In order to assess the performance and work state of the target controller, an ideal controller is designed here in an ideal situation: without noise, disturbance and model mismatch and having enough power to achieve control task.

With MPC quadratic objective function, the design requirements are quantified by:

$$J = \mathbf{e}\mathbf{e}^T + \lambda\Delta\mathbf{u}\Delta\mathbf{u}^T \quad (7)$$

where $\mathbf{e}$ is the vector of the future errors along the prediction horizon;

$$\mathbf{e} = [\hat{y}(t+1\,|\,t) - w(t+1) \quad \ldots \quad \hat{y}(t+p\,|\,t) - w(t+p)]^T \quad (8)$$

The DMC strategy is obtained by computing the derivative of J and making it equal to 0, and $\Delta\mathbf{u}$ is given by:

$$\Delta\mathbf{u} = (G^T G + \lambda I)^{-1} G^T(\mathbf{w} - \mathbf{f}) \quad (9)$$

where $\mathbf{w} = [w(t+1) \quad w(t+2) \quad \ldots \quad w(t+p)]^T$.

The MPC calculates the optimal control efforts by minimizing this objective function over the feasible control moves and a ratio between the optimal control efforts and inputs of the ideal controller are given by:

$$\frac{\Delta\mathbf{u}}{\mathbf{w}-\hat{\mathbf{y}}} = -\frac{\Delta\mathbf{u}}{G\Delta\mathbf{u}+\mathbf{f}-\mathbf{w}} = -\frac{(G^T G+\lambda I)^{-1}G^T}{G(G^T G+\lambda I)^{-1}G^T - I}$$
$$= -\frac{I}{G - [(G^T G+\lambda I)^{-1}G^T]^{-1}} = \frac{1}{\lambda}G^T$$
$$= \frac{1}{\lambda}\begin{bmatrix} g_1 & g_2 & g_3 & \cdots & g_{p-1} & g_p \\ 0 & g_1 & \cdots & g_{m-1} & \cdots & g_{p-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & g_2 & \cdots & g_{p-m+2} \\ 0 & 0 & 0 & g_1 & \cdots & g_{p-m+1} \end{bmatrix}_{m\times p} \quad (10)$$

During the implementation of the MPC strategy, we often only utilize the first element of $\Delta\mathbf{u}$. With the first row in Equation 10, $\Delta u(t)$ can be calculated by $\Delta u(t) = \tilde{\mathbf{g}}(t) \times \tilde{\mathbf{e}}(t)$, where $\tilde{\mathbf{g}}(\mathbf{t})$ is a vector of control gain of the MPC at instant t, $\tilde{\mathbf{e}}(\mathbf{t})$ is a vector of future error along the prediction horizon from instant t.

$$\begin{cases} \tilde{\mathbf{g}}(t) = [g_1 \quad g_2 \quad \cdots \quad g_p] \\ \tilde{\mathbf{e}}(\mathbf{t}) = [w(t+1) - \hat{y}(t+1\,|\,t) \quad \ldots \quad w(t+p) - \hat{y}(t+p\,|\,t)]^T \end{cases} \quad (11)$$

Assuming the ideal controller works in an ideal situation and the process model is identical to the actual process, the predicted output is equal to the actual output, $\hat{y}(t) = y(t)$, then $\Delta u(t+1)$ in the ideal situation can be obtained by implementing $\Delta u(t)$ into Equation 4 and repeating the calculation procedure in Equation 9. Therefore, the control efforts of the ideal controller can be calculated by:

$$
\begin{cases}
\Delta\tilde{u}_{ideal}(t) = \tilde{\mathbf{g}}(t) \times \tilde{\mathbf{e}}(t) \\
\Delta\tilde{u}_{ideal}(t+1) = \tilde{\mathbf{g}}(t+1) \times \tilde{\mathbf{e}}(t+1) \\
\quad \ldots\ldots \\
\Delta\tilde{u}_{ideal}(t+h) = \tilde{\mathbf{g}}(t+h) \times \tilde{\mathbf{e}}(t+h)
\end{cases} \tag{12}
$$

where the ideal control effort at the instant t is denoted as $\Delta\tilde{u}_{ideal}(t)$, h stands for the moving horizon of the observation.

3 Fault Detection

A simple measure of fault detection then can be obtained by computing the variance of the difference between the actual and ideal control efforts.

The actual control efforts can be expressed by:

$$
\begin{cases}
\Delta\tilde{u}_{act}(t) = [\tilde{\mathbf{g}}(t) + \Delta\tilde{\mathbf{g}}(t)] \times \tilde{\mathbf{e}}(t) \\
\Delta\tilde{u}_{act}(t+1) = [\tilde{\mathbf{g}}(t+1) + \Delta\tilde{\mathbf{g}}(t+1)] \times \tilde{\mathbf{e}}(t+1) \\
\quad \ldots\ldots \\
\Delta\tilde{u}_{act}(t+h) = [\tilde{\mathbf{g}}(t+h) + \Delta\tilde{\mathbf{g}}(t+h)] \times \tilde{\mathbf{e}}(t+h)
\end{cases} \tag{13}
$$

where $\Delta u(t)_{act}$ is the control effort of the actual controller at the instant t. $\Delta\tilde{\mathbf{g}}(t)$ is a vector of the discrepancies between the actual control gain and the ideal gain at the instant t.

The detection index, η, is given by:

$$
\begin{aligned}
\eta &= \mathrm{var}[\sum_{i=0}^{h} \Delta u_{act}(t+i) - \sum_{i=0}^{h} \Delta u_{ideal}(t+i)] \\
&= \mathrm{var}[\sum_{i=0}^{h} \Delta\tilde{\mathbf{g}}(t+i) \times \tilde{\mathbf{e}}(t+i)]
\end{aligned} \tag{14}
$$

where var is the variance of the sample data.

This detection index is equal to 0 when the actual work situation is identical to the ideal situation. As mentioned above, the ideal situation is free of model structure, nonlinearities and modeling uncertainty. The actual situation is hard to meet these requirements and the actual control efforts should differ from the ideal control efforts. However, the ideal situation can be obtained from an theoretical design environment; the ideal controller does not put into actual implementations, it is derived from theoretical deduction and served as the basis for the detection.

Taking the moving window to on-line observe the actual controller behaviors, a fault occurring in the actual controller can be caught through observing the value of the index. A significant increase of the index value indicates the controller in a faulty work state.

4 A Case Study

In order to illustrate and evaluate how the proposed detecting metric monitors state change of MPCs, a water tank process has been used as a case study.

4.1 Water Tank Process

Function of the water tank is to match two flows, inlet and outlet flows. A MPC at the outlet is properly tuned to maintain an expected level to the process. The transfer function of the water tank process is $\frac{0.25}{1.399s-1}$, where s is the Laplace transfer symbol.

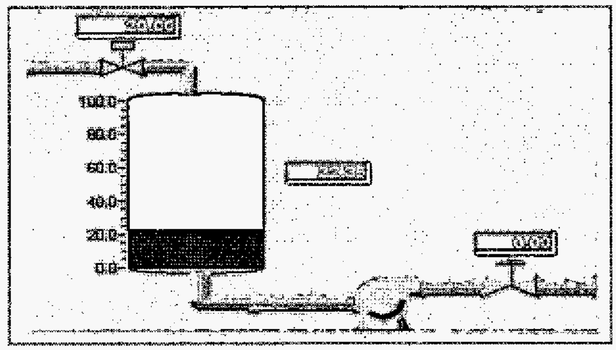

Figure 1: Water tank process

Coefficients g_i of system's dynamic matrix can obtain from the response of the model shown in Figure 2.

$$
G = \begin{bmatrix}
0.1786 & 0 & 0 & 0 & 0 \\
0.3062 & 0.1786 & 0 & 0 & 0 \\
0.3973 & 0.3062 & 0.1786 & 0 & 0 \\
0.4624 & 0.3973 & 0.3062 & 0.1786 & 0 \\
0.5089 & 0.4624 & 0.3973 & 0.3062 & 0.1786
\end{bmatrix} \tag{15}
$$

For the MPC, the prediction horizon and control horizon are set 5, $m = p = 5$ and the relative weight is set 1, $\lambda = 1$. A sine wave with frequency 0.2 is given in this process as noise.

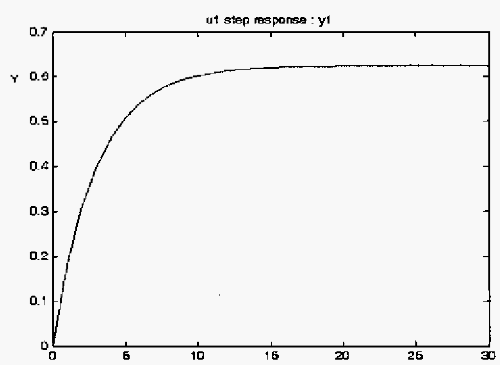

Figure 2: Step response of the water tank

Table 1. Work situations for the MPC and measure results

Situation	Description	Index
Situation #1	Without noise. Setpoint: a step unit	$\eta_1 = 3.5276\times 10^{-6}$
Situation #2	Noise: a sine wave, amplitude=0.5 Setpoint: a square wave, amplitude= [0 1]	$\eta_2 = 0.0110$
Situation #3	Noise: a sine wave, amplitude=1.5 Setpoint: a square wave, amplitude= [0 1]	$\eta_3 = 0.0145$
Situation #4 (Faulty controller)	Noise: a sine wave, amplitude=1 Setpoint: a square wave, amplitude= [0 1]	$\eta_4 = 0.0068$

4.2 Simulation Design and Results Analysis

To test the proposed measure, we design four work situations for the MPC, which are shown in Table 1. The simulation is implemented using the MPC toolbox of MATLAB. The designed faults in the controller result in the control gain of the MPC deviating from the normal gain. For this MPC, the normal control gain is $Kmpc$:

$$Kmpc = [0 \quad 0.4277 \quad 0.0722 \quad -0.0061 \quad -0.0045]$$

The faulty control gain is set as $K'mpc$:

$$K'mpc = [0 \quad 0.04 \quad 0.0072 \quad -0.0006 \quad -0.000045]$$

The results of the MPC working in the above four situations are shown in Figures 3 to 6.

In the situation 1, which can be considered as an ideal situation for the controller, the achieved outputs almost exactly track the setpoint as shown in Figure 3 and the corresponding index from Table 1 is close to 0. It reveals that the actual performance almost meets the desired performance. This result demonstrates the validity of theoretical deduction of the ideal controller as well. With the increase of noises from the situations 2 to 3, the achieved outputs begin to deviate away from the setpoint as shown in Figures 4 and 5 and the corresponding indices increase; however, the increment is in a quantitative level. When the designed fault has taken place in the controller as shown in Figure 6, the index increases significantly and it clearly indicates that the controller is in a faulty work state by comparing the index value with the others.

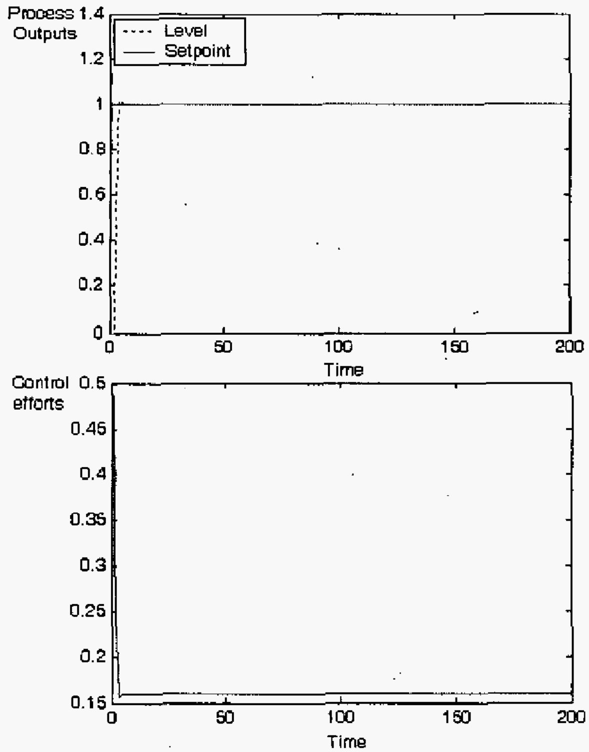

Figure 3: MPC working in situation #1

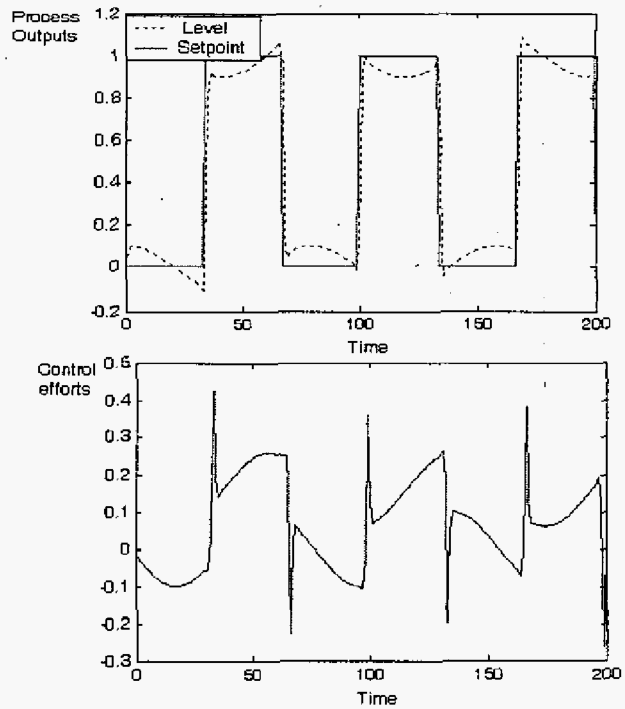

Figure 4: MPC working in situation #2

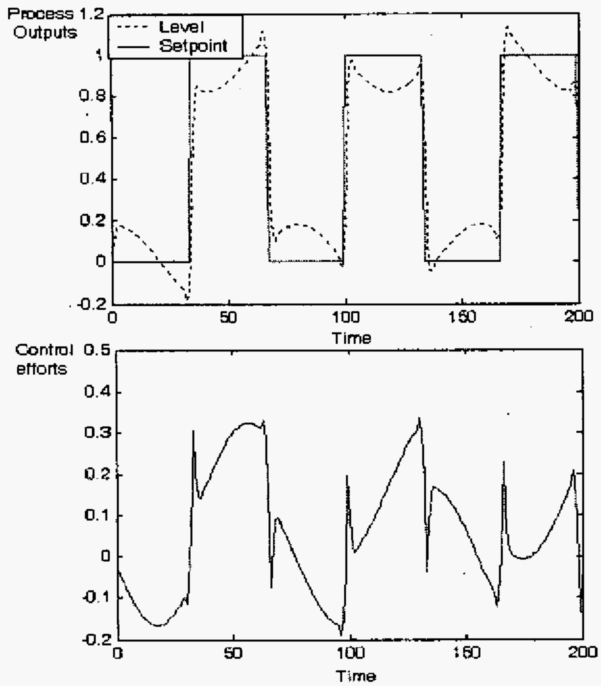

Figure 5: MPC working in situation #3

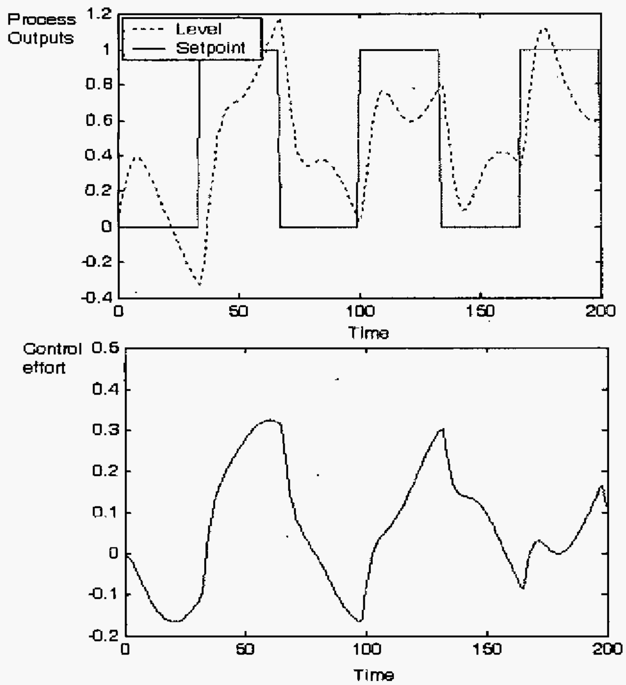

Figure 6: MPC working in situation #4

5 Conclusions

Monitoring and maintaining of controller is an important issue due to the curial role of controller in the control system. In this paper, a novel measure to detect the faults of MPCs based on an ideal controller is presented. The ideal controller is treated as a benchmark and served as the basis for the comparison between the ideal and actual control efforts. The ideal controller is derived from DMC in an ideal work situation. A detection index is proposed here to detect the work state change of the target MPC. The proposed metric is implemented in a simulated water tank process. Four different situations are adopted to test the measure. Simulation results demonstrate its validity and effectiveness.

Acknowledgement

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