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DAMPING OF FLEXURAL VIBRATIONS IN THIN PLATES USING ONE AND TWO DIMENSIONAL ACOUSTIC BLACK HOLE EFFECT

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ABSTRACT

The reduction of flexural vibration in thin plates is examined using the acoustic black hole effect associated with nearly zero reflection of quasi-plane waves from a lightly damped wedge or tapered hole where the profile varies according to a power-law. The flexural wave propagation can be determined through the application of geometrical acoustics approximation or exact analytical solutions. For a plate of thickness of power-law profile, the wave slows down and grows in amplitude. In the ideal case of no truncation of the quadratic (or higher) profile, the phase speed asymptotically decreases to zero and the wave never reaches the end. Manufactured plates always have a truncation, leading to relatively high reflection coefficients, however, the application of small damping layers leads to substantial decreases in the reflection coefficients and thus large reductions in mobility amplitudes.

This paper contains the results of numerical models and experimental measurements of point mobility for structural plates incorporating tapered holes for validation. A rectangular plate with a 1D wedge on one end is examined, in addition to a circular plate with a quadratic profile in the centre. In both cases, the measurements show significant reductions in resonant peaks of mobility, in good agreement to numerical predictions.

1 INTRODUCTION

The traditional methods of damping flexural vibrations in structural plate elements involve the use of viscoelastic damping layers applied to the surface, which in many cases is not necessarily efficient and does not provide a substantial reduction in vibration amplitude [1-4]. More effective developments of these methods utilise constrained layer damping, modal shifting techniques and mass redistribution to focus energy into specific frequency bands of concern. However, the general topic of flexural vibration of plates still remains an important area of research and development, especially for functions where the application of damping films to the whole plate surface is unrealistic.

This paper details a series of investigations into the novel and potentially efficient method of reducing the amplitude of flexural vibration of plate structures at higher frequencies through the introduction of profiled indentations incorporating small amounts of viscoelastic damping material. The specific designs of the indentations follow a power-law relationship $h(x) = \varepsilon x^m$, where x is the distance along the plate surface [m], h is the thickness [m], ε and m are positive constants.

The use of damped indentations is designed to reduce the reflection coefficient of flexural waves from free edges. The reduction of edge reflections from 1D beams through the use of profiled beams was first proposed by Mironov [5], following work on the classical bending plate equation of motion through the application of geometrical acoustics approximation. As a flexural wave travels into a beam where the reduction in thickness is given by $h(x) = \varepsilon x^m$, the phase speed slows down and the wave grows in amplitude. It can further be shown that for a profile of $m \geq 2$, the phase speed asymptotically decreases to zero, and the wave then requires an indefinite time to reach the end of the beam at $x = 0$, never reaching the end and therefore never actually reflecting back [6]. Due to the increased amplitude of vibration towards the end of the beam, any inherent loss factor in the material causes an enhanced and highly efficient attenuation of energy through extensive and compressive motion (this energy is converted to heat).

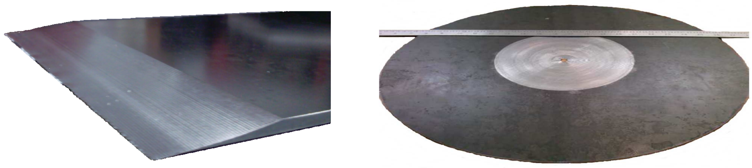
A number of problems remain with this basic theory, the first of which is that it is based on simplified classical theory, where any displacement from the equilibrium position (neutral layer) is considered small in comparison to the beam thickness. As soon as the amplitude grows in the beam, which occurs very close to the end of the profile, $x = 0$, it is necessary to take into account non-linear terms which prevent such high amplitudes that would lead to plastic deformation. It has been shown by Krylov [7] through the analysis of the geometrical acoustics approximation that for $m = 2$ the solution is valid at all distances x, and for $m > 2$ it is valid only in the area that is relatively close to the tip.

Furthermore, it is practically impossible to manufacture a wedge shape which extends to $x = 0$ following the power-law profile. At some point in the cutting process, a truncation is introduced, creating a free edge from which the reflection coefficients can be as high as 50-70 percent [7]. Krylov has suggested that one possible solution is to apply thin layers of polymeric damping film to the wedge surface, near to the truncation as a means of reducing these reflection coefficients [6-8] while still utilising the relatively high vibration amplitudes near to the wedge end.

Although the idea of using damping layers to reduce peak vibration mobility amplitudes is not new, the integration of these damping layers to a tapered geometry is new. As the thickness of the plate becomes comparable with the thickness of the damping layer, the composite loss factor increases considerably [4]. The integration of the tapered power-law profile with damping materials leads to a theoretical reduction in reflection coefficients to practically zero. From a position outside of the wedge, it appears that the wave travels into the wedge but does not significantly reflect back out, which has been termed the “acoustic black hole effect”. A similar effect could be replicated by modifying the material properties of the plate with distance [9] (for example, by modifying the material density with distance, or the Young’s modulus). Techniques have been developed to modify the density of a plate surface with distance, see for example the manufacture of metal foams, but these are more expensive than the simple milling or forging techniques required to create a power-law profile. In addition, a power-law profile can be easily incorporated into an existing structure suffering from vibration and structure-borne acoustic problems, whereas it is more difficult to modify the internal density of a beam.

In this paper, we present both the theory and experimental measurements for the integration of these tapered power-law geometries into some plate structures, specifically with the application of damping films to the tips of these wedge indentations. Section 2 contains a short theoretical discussion of the vibration of rectangular plates with wedge attachments, followed by a number of numerical predictions. Then, section 3 describes the vibration of plates with tapered cylindrical holes, then conclusions. The profiled indentations are both quadratic power-law, as the theory suggests this is the lowest order for which the phase speed will asymptotically decrease to zero in a profile extending to zero thickness.

Although this paper contains a series of results for rectangular plates with attached wedges, (see Fig. 1(a)), it is also possible to manufacture power-law profiles into other geometries. For example, in Fig. 1(b), the circular plate has been modified to include a circular inclusion of power-law profile. This geometry is important in this research work as there exist analytical solutions for cylindrical tapering profiles, which are only valid when any small hole (the circular analogy of the truncation) is placed into the centre of the plate. Otherwise the series does not converge as the phase speed in the radial direction decreases asymptotically to zero. Further power-law profiles are not considered in this paper, however, following investigation with these simple shapes, it is intended that profiled indentations can be created in commercial structural elements and engineering parts where the potential reductions can be demonstrated.



(a) (b)

Figure 1. Examples of structural plates with tapered indentations of quadratic power-law profile; (a) plate incorporating a wedge which is only tapering in one dimension, truncating at a thin free edge, (b) cylindrical plate with cylindrical profiled indentation, where the very centre has a hole creating the truncation free edge.

2 VIBRATION OF RECTANGULAR PLATES WITH WEDGE ATTACHMENTS

The numerical model of a plate with a profiled wedge section is completed in two parts, firstly the equation of motion for a constant thickness plate is developed, then the profiled test section is joined at one end, as shown in Fig. 2.

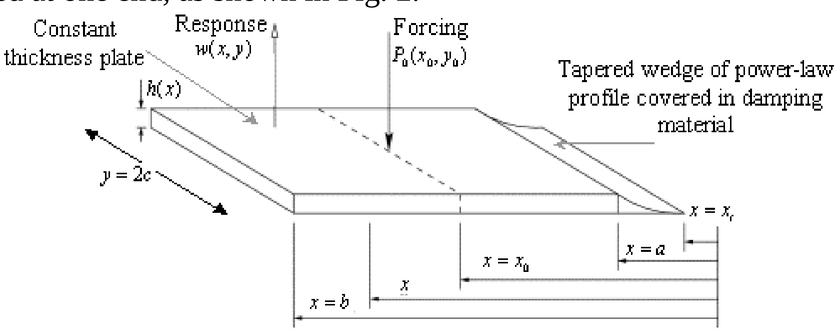


Figure 2. Numerical representation of a rectangular plate of constant thickness with a wedge of power-law attached to one end.

2.1 Numerical model of a constant thickness rectangular plate.

The classical bending plate equation of motion for flexural vibration of a rectangular plate in the (x , y) plane is given as [3],

$$\left(\nabla^2\right)\left(D\nabla^2\right)w(x,y,t)+\rho h\frac{\partial^2 w(x,y,t)}{\partial t^2}=0, \qquad (1)$$

where $D=Eh^3/12(1-\upsilon^2)$ is the bending stiffness, E, ρ and υ are the Young's modulus, density and Poisson's ratio of the material respectively and h is the plate thickness. This equation is valid for both Cartesian and polar coordinate systems and for a plate of constant thickness there are analytical solutions, employing either trigonometric or Bessel's functions [3].

In the case of rectangular plates, we look for solutions to the flexural displacement as a Fourier series of the form $w(x,y,t)=\sum A_j e^{(k_j[x-a])}e^{(ik_y[y])}e^{-i\omega t}$, where k_j and k_y are wavenumbers in the x and y directions respectively and harmonic motion at frequency ω is assumed. The amplitude of displacement is provided by the constants A. The plate is assumed to be simply supported at all times on the plate boundaries $y=\pm c$, so that zero displacement and bending moment exist here. Then the only applicable values of the wavenumber are $k_y=n\pi/2c$, where n is the mode number in the y direction, and 2c represents the width of the plate.

Substitution of these terms into the equation of motion (1) and neglecting the trivial solution of zero displacement yields a fourth order equation in the unknown wavenumbers in the x direction.

$$\left(Dk_j^4+(-2Dk_y^2)k_j^2+(Dk_y^4-\rho h\omega^2)\right)w(k_j,k_y,\omega)=0. \qquad (2)$$

The solution to this equation provides four possible solutions, which correspond to the four Fourier amplitudes A_j required to determine the displacement. In order to solve this problem and determine these Fourier amplitudes, a numerical method based on a matrix of four known boundary conditions is used. At the far ends of the plate in the x direction, the edges are free and therefore we may assume zero bending moment and shear force exist at both of these ends. These four conditions fully determine the free vibration motion of the plate, however, we are interested in the plate response when forced, therefore, at the far end of the plate $x=b$, we specify a shear force along the entire length of the edge of unit amplitude. These boundary conditions in both Cartesian and polar coordinate systems are found in the literature, see for example [3] in terms of displacements. Thus the bending moment $M=\frac{\partial^2 w}{\partial x^2}+\upsilon\frac{\partial^2 w}{\partial y^2}$ and the shear force $V=\frac{\partial^3 w}{\partial x^3}+(2-\upsilon)\frac{\partial^3 w}{\partial x\partial y^3}$.

The numerical method is used to calculate the displacement of the plate surface for a range of mode numbers in the y direction and frequencies in the range 0-8kHz. For low values of the excitation frequencies it is not possible to ensure that the excitation position does not locate on a nodal or anti-nodal point, therefore it can be misleading to review the amplitude changes in this region. We therefore predict the numerical response at the far end of the plate, $x=b$ for a range of higher frequencies.

We now have a method to determine the forced displacement of a constant thickness plate, however, since the aim is to subsequently attach a profiled wedge section onto one end, as shown in Fig. 2, then we must replace two of the boundary conditions at this end, $x=a$, representing the free edge with the numerical model of the profiled wedge, described in the following section.

2.2 Numerical model of a variable thickness rectangular plate.

There are many analytical and numerical solutions for the classical bending plate equation of motion when the thickness is constant [3], however, if this varies, the equations become much more complicated and consequently, fewer analytical solutions exist. Due to these complications, more of the work in the literature is carried out using finite series solutions to approximate the displacement of the wedge (the shape functions chosen in the series all fit the boundary conditions applied to the plate edges). In this paper, geometrical acoustics approximations are applied to determine a solution based on wave propagation.

Denoting the flexural displacement in the wedge w_w, we assume a general form to the displacement $w_w(x,y,t) = B(x)e^{ik_pS(x)}e^{ik_yy}e^{-i\omega t}$, where k_p is the plate wavenumber $k_p = \omega^2(\rho(1-\nu^2))/E$, $B(x)$ is the Fourier amplitude for the wedge displacement and $S(x)$ is the integrated phase function. Substitution of the displacement in the wedge into the equation of motion yields a far more complicated series of equations, as now the bending stiffness is a function of x, as well as the displacement. This may be simplified by application of geometrical acoustics approximations [10], such that any derivatives of order greater than two are neglected as being small in comparison to the remaining terms. The remaining equation is a fourth order quadratic equation in the derivative of the integrated phase function.

$$\left(\frac{\partial S(x)}{\partial x}\right)^4 + \left(\frac{\partial S(x)}{\partial x}\right)^2\left(\frac{2k_y^2}{k_p^2}\right) + \left(\frac{k_y^4}{k_p^4} - \frac{\rho h(x)\omega^2}{D(x)k_p^4}\right) = 0. \tag{3}$$

The four roots give four distinct solutions $\partial S(x)/\partial x = \pm(k_y/k_p)(-1 \pm \gamma x^{-m})^{1/2}$ where $\gamma = (12^{1/2}k_p/\varepsilon k_y^2)$. This solution for $\partial S(x)/\partial x$ can be expanded using a binomial expansion for any value of k_y and it can be shown that the only time the integral $S(x) = \int \partial S(x)/\partial x \, dx$ converges is when the power-law profile satisfies the condition $m < 2$ (this applies when the profile fully extends to $x = 0$). For a power-law profile with $m \geq 2$, the flexural wave travels closer to the point $x = 0$, asymptotically slowing down but never actually reaching it. As it can never reach the end of the plate, it can never reflect back and therefore the reflection coefficient for an observer point on the plate is zero.

As has previously been mentioned, real manufactured plates never extend all of the way to $x = 0$ and always include a truncation point, $x = x_t$, where the cutting tool either begins to rip the free edge of the thin metal or compresses the remaining material to create an edge which no longer follows a power-law profile. For typical commercial applications, no matter how accurate the tool path or precise the grinding, the profile will at some point tend away from a power-law, or truncate the material. As soon as this occurs, the incident flexural wave reflects from the free edge with a substantial reflection coefficient.

The developed numerical prediction method incorporates the wedge into the model for the constant thickness plate by assuming that at $x = x_t$, zero bending moment and shear force exist (it therefore becomes a free edge with a low thickness). At $x = a$, as the wedge is joined onto the constant thickness plate, we now have common displacement, slope, second and third derivatives with the constant thickness plate in terms of the boundary conditions. The overall solution matrix is increased in size from a four by four, to an eight by eight so that the amplitude coefficients from both the constant thickness plate and the wedge can be determined.

To minimise the amplitude of the reflected wave from the free edge $x = x_t$, a thin damping layer of length $x = 2$ cm and width $y = 2c$, with a high loss factor $\eta = 0.2$ and compressed thickness 0.5mm is applied to the wedge surface. As the thickness of the damping layer becomes comparable with the thickness of the wedge, the composite loss factor increases significantly. This loss factor is included through the introduction of a complex term in the Young's modulus $E = E(1+\eta i)$ [1]. The inherent loss factor of mild steel is assumed to be $\eta = 0.006$ reflecting the fact that there is little damping in the material [4], which rises when a thin layer of damping material is applied to the surface. In order to quantify this rise in the loss factor, we assume that the material properties of the elastic damping tape are E_D, η_D for the Young's modulus and loss factor, h_D for the thickness. Using the non-dimensional terms for the thickness ratio $\Gamma = h_D / h$, $\alpha = \Gamma E_D / E$ and $\beta = h_D + h_D / 2h$ and assuming that the Young's modulus of the damping tape is significantly less than the wedge material, then Ross, Kerwin and Ungar [4] determine the composite damping loss factor to be given by,

$$\eta_{comp} = \left(\eta_D \frac{E_D h_D}{Eh} \left[4\left(\frac{h_D}{h}\right)^2 + 6\left(\frac{h_D}{h}\right) + 3 \right] \right) \Bigg/ \left(1 + \frac{E_D h_D}{Eh} \left[4\left(\frac{h_D}{h}\right)^2 + 6\left(\frac{h_D}{h}\right) + 5 \right] \right). \quad (4)$$

It may be shown that the composite loss factor of a plate of constant thickness rises to approximately $\eta = 0.014$ when a thin elastic damping layer of Young's modulus 10MPa is applied to the surface. When the damping layer is applied to the wedge of power-law profile with $m = 2$, the local loss factor varies with the position on the wedge. The above equation is applied to constant thickness plates, therefore we require a method to incorporate this into a variable thickness plate. Krylov [7] has calculated the reflection coefficient from a wedge of power-law profile, taking into account the integration of the local loss factor over the length of the wedge. This reflection coefficient is used to determine the equivalent composite loss factor, which in the wedge rises to $\eta = 0.034$ as the thickness of the damping layer can become comparable to the tip. This composite damping rises further if the truncation point of the wedge is moved closer to $x = 0$, subject to the constraints of the machining technique.

2.3 Numerical predictions for a rectangular plate with a wedge attachment

The numerical predictions apply to a mild steel plate (see Fig. 2) with Young's modulus, density and Poisson's ratio E=210GPa, ρ=7800kg/m^3 and υ= 0.3 respectively, with dimensions $a = 0.05$m, $b = 0.3$m, $c = 0.03$m (and thickness 2.5mm). This plate can incorporate a wedge profile of power-law $m = 2$, with truncation position $x_t = 3$mm.

The numerical calculations provide the predicted vibration velocity response of the plate surface to an applied load (v/p)dB, at a position $(x, y) = (b,0)$ when the plate is forced along the edge at $x = b$ (see boundary conditions in [3] for shear force). The boundary conditions for a free edge on a plate are zero bending moment and zero shear force.

The velocity is obtained from the displacement for each frequency, $v = -i\omega w$. The response of the plain rectangular plate is shown in Fig. 3, where it may be seen that the first mode of vibration occurs at a frequency of approximately 1.75kHz. There then exist a series of resonances corresponding with the mode shapes in the x direction, then the second mode shape in the y direction has a resonance located at approximately 6.85kHz.

The application of a thin layer of damping tape across the whole of the plate surface yields a small reduction in peak mobility amplitude, of approximately 4-4.2dB, which applies to both longitudinal and across width mode shapes. Further reductions could be found by optimising this layer, through the use of constraining layers to enhance the amount of shearing action in the damping material, however, there may be applications where the application of damping material over the whole plate surface is not feasible or where larger reductions in peak mobility amplitude are required.

Many current engineering structures utilise this damping method, as it is relatively cheap and simple to incorporate. In order to recognise further reductions, larger thickness damping layers are required, subject to a saturation of the composite loss factor, adding mass and thickness to the structure, therefore we now examine the alternative method, of incorporating the wedge of power-law profile into the structure, utilising only a thin strip of damping material on the very wedge end.

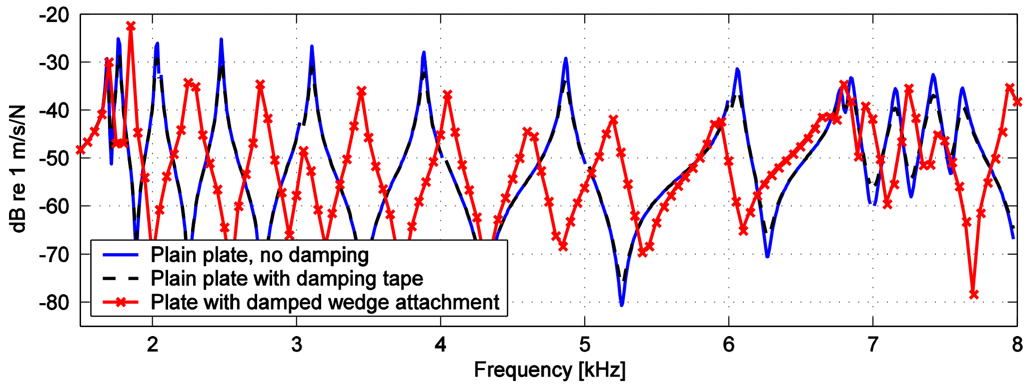


Figure 3. Comparison of the mobility predictions (v/p) dB re $1\,m/s/N$ for a plain undamped rectangular plate with the same plate when covered entirely with a thin layer of elastic damping material. The final plot is the undamped rectangular plate incorporating a wedge of quadratic power law-profile covered by a layer of damping film.

When an undamped wedge of power-law profile is attached to the end of this plate, as shown in Fig. 1(a), the wavelengths in the longitudinal direction are modified, moving the longitudinal resonant frequencies lower. The resonances in the across width direction are maintained at a frequency of approximately 1.71 and 6.83kHz. The peak amplitudes of the resonances appear to be very similar, which represents the fact that any truncation of the power-law profile leads to a significant reflection coefficient from the free edge. Therefore, we conclude that the undamped wedge cannot provide a significant reduction in amplitude.

A thin damping strip is now placed onto the wedge surface, and the response of this plate with a damped wedge attachment is compared to the plain rectangular plate in Fig. 3. The peak amplitudes of the resonances in the across directions are not modified at all, as there is no additional damping in the across width direction. However, the resonances in the longitudinal direction are significantly modified, with a reduction in peak amplitude of between 8-20dB between a frequency range of 2.75-6kHz (although there is a sharp increase in the response at one initial resonant frequency, of 5dB at 1.95kHz). We expect the effective damping from the wedge to increase at higher frequencies and mode shapes due to the shorter flexural wavelengths and this is reflected in the results.

A table containing the peak mobility amplitudes for a range of mode shapes is provided, Table 1, where the peak mobility amplitude is selected as a suitable measure of damping effectiveness as for structural vibration control, it is usually of the most interest. As the incorporation of a wedge into the basic plate manipulates the frequency location of the resonances, we cannot directly compare the amplitude at each resonance. Instead the average amplitude of peak resonance in a given frequency band are compared and reductions in peak mobility noted by using the plain undamped rectangular plate as a reference. It may be seen that the introduction of the damped wedge provides more than double the peak reduction than just covering the whole plate with damping material.

Plain plate		Plain plate with damping	Plain plate with damped wedge attachment
Frequency /kHz	*Peak mobility amplitude /dB*	*Reduction in peak mobility amplitude /dB*	*Reduction in peak mobility amplitude /dB*
2-4	-26.8	-4.2	-8.5
4-6	-31.0	-4.0	-11.0
6-8	-34.3	-4.2	-6.0

Table 1. Comparison of the peak amplitude mobility predictions at various resonant frequencies, (v/p)dB re $1\,m/s/N$. The mobilities for an undamped rectangular plate are shown, with reductions from this reference given for the damped rectangular plate and also the undamped rectangular plate incorporating a wedge of power-law profile with a thin damping layer on the wedge tip.

3 VIBRATION OF CYLINDRICAL PLATES WITH CYLINDRICAL INDENTATIONS OF POWER-LAW PROFILE

The previous section contained an analysis of the effect on the vibration amplitude of incorporating a simple rectangular wedge structure onto one end of a rectangular plate. In order to determine the numerical solution, geometrical acoustics approximation has been used. In this section, we turn our attention to power-law indentations of cylindrical profile, as shown in Fig. 1(b), as these can be easily incorporated into plate structures without the need to attach further plate structures, thus reducing the added mass.

Cylindrical indentations of power-law profile, that materialise two-dimensional “acoustic black holes”, have one particular advantage over the wedges of the rectangular geometry considered in the previous section. Namely, there exists an analytical solution to the equation of motion for flexural waves in polar coordinates applied to variable thickness plates, specifically where the power-law is quadratic. The most common application is envisaged to be an indentation applied to a rectangular plate. However, plates of other geometrical forms can be used as well, e.g. elliptical plates where focusing of flexural waves has been used to enhance the two-dimensional “acoustic black hole effect” [11]. In this section we restrict the analysis to circular plates so that this analytical solution may be used.

The equation of motion for these indentations of radius r and circumferential angle θ is now in polar cylindrical coordinates, where the flexural displacement can be written as $w(r,\theta,t) = w(r)e^{in\theta}e^{-i\omega t}$ (n is now the circumferential mode number).

$$\rho h \frac{\partial^2 w(r,\theta,t)}{\partial t^2} = (1-\upsilon)\Diamond^4 \{D, w(r,\theta,t)\} - \nabla^2\left[D\nabla^2 w(r,\theta,t)\right] \tag{5}$$

The bilinear operator is defined as,

$$\Diamond^4\{D,w\} = \frac{\partial^2 D}{\partial r^2}\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) + \frac{\partial^2 w}{\partial r^2}\left(\frac{1}{r}\frac{\partial D}{\partial r}\right). \tag{6}$$

For the case of a circular plate of constant thickness, the bending stiffness is not a function of the radius and the equation of motion reduces to the common wave equation in polar coordinates with four solutions incorporating Bessel’s functions. Four constants are included, representing amplitudes to be determined through the application of boundary conditions and the variable $\beta^4 = \rho h \omega^2 / D$.

$$w(r,\theta,\omega) = \left(c_1 J_n(\beta r) + c_2 Y_n(\beta r) + c_3 I_n(\beta r) + c_4 K_n(\beta r)\right)e^{in\theta}e^{-i\omega t}. \tag{7}$$

The centre of the circular plate is replaced with a profiled indentation, as shown in Fig.1 (b), of power law profile. The numerical method is very similar to the one introduced in the previous section, where boundary conditions on the outer edge are free (at the outer radius we apply zero bending moment and shear force [3], this time in polar coordinates) and common boundary conditions join the constant thickness plate to the profiled indentation (common displacement, slope, second and third derivatives of the displacement).

We now proceed with the analysis of the circular indentation, in particular the case of a quadratic radial power-law profile, $m = 2$, where the existence of an analytical solution by Conway [12] is used, which is valid for the case of motion independent of circumferential position $n = 0$ using a variable $K = 12\rho\omega^2(1-v^2)/E\varepsilon^2$.

$$w(r, n=0, \omega) = c_5 r^{\lambda_1} + c_6 r^{\lambda_2} + c_7 r^{\lambda_3} + c_8 r^{\lambda_4} \quad (8)$$

$$\lambda = -2 \pm \sqrt{7 - 3\upsilon \pm \sqrt{9(1-\upsilon^2) + K}} \quad (9)$$

As the radius decreases to $r = 0$, there exists a singularity similar to the acoustic black hole effect seen in beams. The $\ln(r)$ term implies an increase in amplitude through the constants c with a consequent fall in phase speed. The only way this singularity can be avoided, while still maintaining the analytical solution is through the introduction of a truncation hole in the centre of the inclusion, $r = r_t$, introducing a free edge from which wave reflections may occur. Higher order solutions for non-zero circumferential modes are subsequentially determined by numerical calculations, although the method remains the same.

In order to provide a validation to the numerical theory and explore how representative the classical plate theory is for practical plate designs, three circular plates have been machined from the same sheet of mild steel (these are a different size to the plates discussed in section 2). The first is a plain circular plate of radius 250mm by 5.04mm thickness. The second is an identical circular plate covered with a layer of damping material. The third is an undamped circular plate of outer radius 250mm and inner radius 100mm where a cylindrical indentation of quadratic power-law profile extends to a truncation radius of 5mm, created by traversing a cutting bit in a circular motion 50 times, each at a slightly different position in radius and depth. Although this is a profile made of a series of discrete thickness changes, we assume that the change in thickness is small compared to the overall radius. A layer of damping material is then applied to the surface of this indentation.

The plates are supported by thin straps on the outer diameter to represent free boundary conditions on the outer and inner surfaces. A force transducer (B&K type 8200) is connected through an electromagnetic shaker (controlled through a broadband frequency signal). An accelerometer (B&K type 4371) is attached to the surface in the same location, as we are interested in driving point mobility response of the plate $\ddot{w} = -i\omega v$, where the dot represents a differentiation with respect to time. The location of the shaker is at a radius of 175mm at the same circumferential position as the accelerometer.

The comparison of the experimental driving point mobility against frequency for the undamped plain plate is shown in Fig. 4, also shown is the equivalent plate once a layer of damping material is applied to the surface. Once a layer of damping treatment is applied to the plain plate, we expect there to be reductions in the peak mobility amplitudes, especially at the higher frequencies, but these are expected to be small as both the thickness and stiffness of the damping tape is small in comparison to the plate materials. On the same graph we also show the point mobility measurements once an indentation of power-law profile is machined into the surface (and then covered with a layer of damping film).

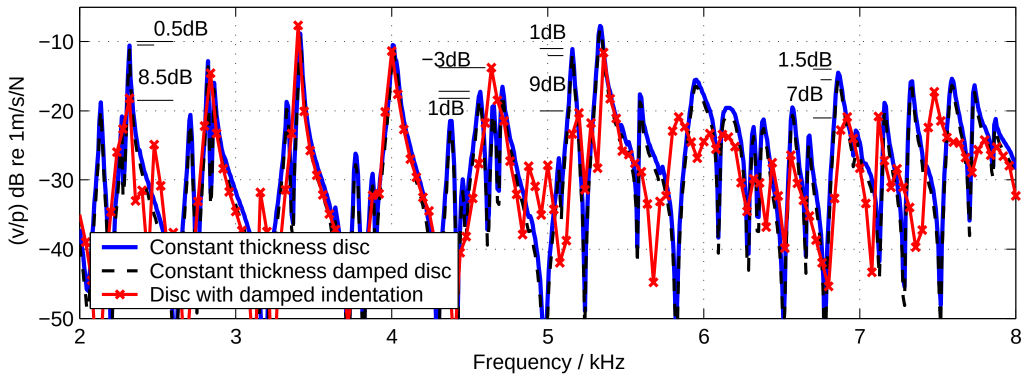


Figure 4. Comparison of the mobility predictions (v/p) dB re $1\,m/s/N$ for an undamped circular plate of radius 250mm with the same plate once a thin layer of damping material is applied to the whole surface. Also shown are the measurements once an indentation is created in the centre of the plate which is then damped.

We now compare the predictions of the point mobility of these same plates found through the use of the numerical models, shown in Fig 5.

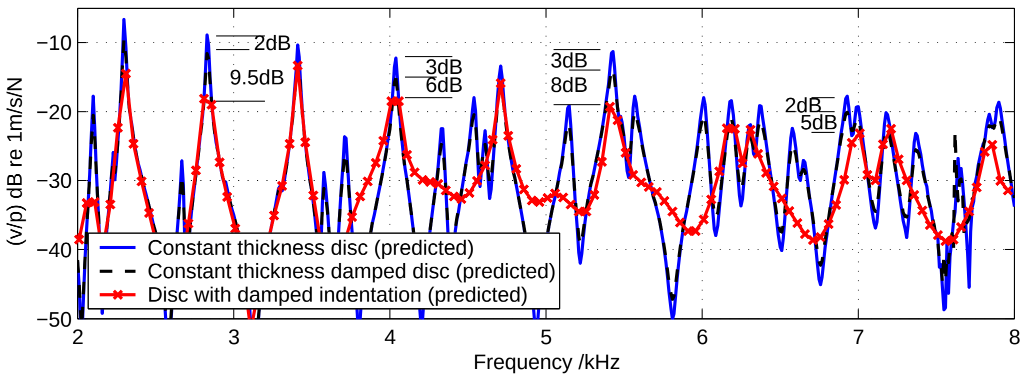


Figure 5. Comparison of the mobility predictions (v/p) dB re $1\,m/s/N$ for an undamped circular plate with the same plate which has an indentation of power-law profile machined into the surface, and a thin damping layer placed over it.

A table of peak mobility amplitudes for different ranges of resonant frequencies is shown in Table 2 for both experimental and numerical predictions. The numerical predictions show that the inclusion of a damped tapered inclusion of power-law profile can provide double the damping reduction at high frequency than that found by covering the whole cylindrical plate with a damping material. The experimental results are slightly more varied however, indicating that the inclusion of a profiled hole can lead to a slight increase in mobility amplitudes at lower frequencies around 2-3kHz.

Plain plate		Plain plate with damping	Plain plate with damped radial quadratic power-law indentation
		Experimental measurements	
Frequency /kHz	*Peak mobility amplitude /dB*	*Reduction in peak mobility amplitude /dB*	*Reduction in peak mobility amplitude /dB*
2-4	-10.63	-0.435	+0.495
4-6	-15.55	-0.85	-4.85
6-8	-16.8	-1.4	-4.3
		Numerical predictions	
2-4	-10.94	-2.7	-7.9
4-6	-15.1	-2.6	-8.3
6-8	-18.8	-2.5	-4.6

Table 2. Comparison of the peak amplitude mobility predictions at various resonant frequencies, $(v/p)\,\mathrm{dB}\ \mathrm{re}\,1\,m/s/N$. The mobilities for an undamped circular plate are shown, with reductions from this reference given for the damped rectangular plate and also the undamped rectangular plate incorporating a radial indentation of quadratic power-law profile with a thin damping layer on the surface.

The results for a cylindrical profile indicate that the inclusion of a cylindrical hole is not as efficient a damping method than the rectangular profiled wedges, most likely as the available surface area for damping is reduced. Further work will be carried out into determining the actual reflection coefficients and introducing different manufacturing methods for other power-law profiles.

CONCLUSIONS

It has been shown theoretically that the incorporation of a profile of quadratic power-law into a rectangular plate structure can reduce the peak mobility amplitude at higher frequencies by significantly more than found by covering the whole plate structure with damping material. This proves that the inclusion of power-law profile indentations can provide a valid alternative damping method where covering the whole plate structure with damping material is not possible.

It has been demonstrated both theoretically and experimentally that the introduction of a circular indentation of quadratic power-law profile that has an analytical solution associated with low reflection coefficients, also results in damping flexural vibrations. However, the damping introduced by these indentations does not appear to be as efficient as the damping due to rectangular power-law profiles, most likely due to the reduction in available surface area.

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REFERENCES

Books:

[1] M. Heckl, L. Cremer, E. Ungar, Structure Borne Sound, 2nd Edition, Springer-Verlag, 1988.

[2] D. Mead, Passive Vibration Control, Wiley, 1998.

[3] K. Graff, Wave Motion in Elastic Solids, Clarendon Press, Oxford, 1975.

Journals:

[4] D. Ross, E. Kerwin, E. Ungar, Damping of plate flexural vibrations by means of viscoelastic laminae, in Structural Damping (Ruzicka, J. E., ed) 3 (1959) 44–87.

[5] M. Mironov, Propagation of a flexural wave in a plate whose thickness decreases smoothly to zero in a finite interval, Soviet Physics - Acoustics 34 (1988) 318–319.

[6] V.V. Krylov, New type of vibration dampers utilising the effect of acoustic 'black holes', Acta Acustica united with Acustica 90 (2004) 830–837.

[7] V.V. Krylov, F. Tilman, Acoustic black holes for flexural waves as effective vibration dampers, Journal of Sound and Vibration 274 (2004) 605–619.

[8] V.V. Krylov, E.R.T.B. Winward, Experimental investigation of the acoustic black hole effect for flexural waves in tapered plates, Journal of Sound and Vibration 300 (2007) 43–49.

[9].C. Vemula, A.N. Norris, G.D. Cody, Attenuation of waves in plates and bars using a graded impedance interface at edges, Journal of Sound and Vibration 196 (1996) 107–127.

[10] V.V. Krylov, Geometrical-acoustic approach to the description of localized vibrational modes of an elastic solid wedge, Soviet Physics - Technical Physics 35 (1990) 137–140.

[11] F. Gautier, J. Cuenca, V.V. Krylov, L.Simon, Experimental investigation of the acoustic black hole effect for vibration damping in elliptical plates (Abstract for the International Conference "Acoustics 08", Paris, France, June 2008), Journal of the Acoustical Society of America, 123 (5) (2008) 3318.

[12] H.D. Conway, Some special solutions for the flexural vibrations of discs of varying thickness, Ing.-Arch. 26 (1958) 408-410.