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Experimental investigation of damping flexural vibrations using two-dimensional acoustic ‘black holes’

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Abstract

In the present paper, we report the results of the experimental investigation of damping flexural vibrations in rectangular plates containing tapered indentations (pits) of power-law profile, with the addition of a small amount of absorbing material. In the case of quadratic or higher-order profiles, such indentations materialise two-dimensional ‘black holes’ for flexural waves. In the present investigation, pits have been made in different locations of rectangular plates. It has been found that basic power-law indentations that are just protruding over the opposite surface cause rather small reduction in resonant peak amplitudes, which may be due to their relatively small absorption cross-section. To increase damping in the present investigation, the absorption cross-section has been enlarged by increasing the size of the central hole in the pit, while keeping the edges sharp. As expected, such pits, being in fact curved power-law wedges, result in substantially increased damping comparable with that achieved by one-dimensional wedges of power-law profile.

1 Introduction

Passive damping of structural vibrations is usually achieved by adding layers of highly absorbing materials to the structure in order to increase energy dissipation of propagating (mostly flexural) waves [1-3]. Another well-known approach to suppression of resonant vibrations of different structures is to reduce reflections of structural waves from their free edges [4].

To implement the latter approach in a more efficient way, a new method of damping flexural vibrations based on the so-called acoustic black hole effect has been recently developed and investigated [5-7]. This method has been initially applied to one-dimensional plates of power-law profile (wedges) that had to be covered by narrow strips of absorbing layers near sharp edges. Ideally, if the power-law exponent is equal or larger than two, the flexural wave never reaches the sharp edge and therefore never reflects back [5-8], which constitutes the acoustic black hole effect. It has been established theoretically [5,6] and confirmed experimentally [7] that this method of damping structural vibrations is very efficient even in the presence of edge truncations and other imperfections.

The focus of the present work is on the experimental investigation of damping flexural vibrations in plates containing two-dimensional tapered indentations (pits) of power-law profile, with the addition of a small amount of absorbing material. In the case of quadratic or higher-order profiles, such pits materialise two-dimensional ‘black holes’ for flexural waves. To understand basic principles of such two-dimensional black holes, a geometrical acoustics approach to analysing flexural wave interaction with power-law indentations has been developed [9]. The results of this approach show that, if a flexural wave is captured

by a black hole, its reflection coefficient can be calculated in the same way as in the case of one-dimensional wedges. The first experimental investigation of two-dimensional acoustic black holes has been described in the paper [10] dealing with flexural vibration damping in elliptical plates. It has been demonstrated in this paper that such indentations can be very efficient dampers if they are placed in one of the plate's foci. In this case, the energy of converging flexural waves is focused at the black hole, and the problem can be described by a one-dimensional theoretical model based on a tapered beam [11]. In the paper [12], circular indentations of power-law profile have been placed in the centre of a circular plate. It has been shown both theoretically and experimentally that such indentations also act as vibration dampers, albeit not as efficient as in the above mentioning case [10, 11] utilising focusing of flexural waves in elliptical plates.

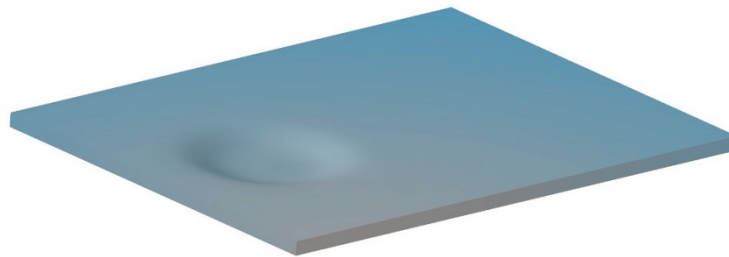


Figure 1: Illustration of a circular power-law indentation in a rectangular plate designed to suppress flexural vibrations

In the present paper, we describe experimental measurements of damping flexural vibrations in rectangular plates containing circular indentations of power-law profile placed in arbitrary locations (see Figure 1). For rectangular plates, this configuration offers a range of benefits in comparison with the case of one-dimensional acoustic black holes (wedges of power-law profile).

First of all, the potentially dangerous sharp edges of power-law wedges are eliminated, which brings a safety benefit. Secondly, two-dimensional pits can be applied to suppress just some selected resonant peaks, when placed in certain positions. In comparison with the above mentioned two-dimensional black holes in elliptical and circular plates, the arbitrarily located pits in rectangular plates open flexible solutions that could be more easily applied to various practical engineering structures.

In following sections of this paper, the manufacturing method used to produce the experimental plates with two-dimensional power-law indentations will be described, followed by the description of the experimental set-up. Then, the obtained measurements' results will be discussed, followed by the conclusions.

It will be demonstrated in this paper that basic power-law pits that are just protruding over the opposite plate surface cause rather small reduction in resonant peak amplitudes, which may be due to their relatively small absorption cross-section capturing a relatively small number of flexural wave rays. Note that for elliptical plates this disadvantage has been overcome by focusing of flexural waves in the pit [10, 11].

To increase damping in the present investigation, the absorption cross-section has been enlarged by increasing the size of the central hole in the plate while keeping the edges sharp. As expected, such large pits, being in fact curved power-law wedges, result in substantially increased damping comparable with that achieved by one-dimensional wedges of power-law profile.

2 Manufacturing of experimental samples

Experimental samples in the present work were manufactured from 5 mm thick hot drawn mild steel sheets; which are more resistant to mechanical stresses incurred in the manufacturing process than cold drawn steel sheets, resulting in fewer internal defects. Dimensions of the produced rectangular plates were 400 x 300mm. Material properties of plates and damping layers (electrical tapes) are listed in Table 1.

	Thickness	Young's modulus	Density	Poisson's ratio	Loss factor
Plate	5.04 mm	190 GPa	7000 kg/m ³	0.3	0.6 %
Damping layer	0.08 mm	-	300 kg/m ³	-	6 %

Table 1: Geometrical and material properties of plates and damping layers.

A CNC (Computer Numerically Controlled) milling machine operating at a cutter speed of 1200 rpm was used to produce circular indentations of power law profile with the exponent $m = 2$ into the plates, through an initial 'roughing out' of the indentation area, where material is removed from the central location of the hole with adequate tolerances to reduce stresses on the cutter and plate during the machining process.

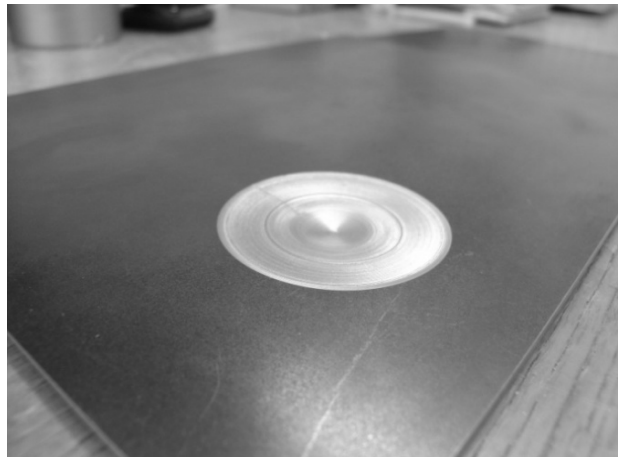


Figure 2: Plate containing a circular indentation of power law profile with $m = 2$

Three types of experimental samples were produced for this investigation, a plate with a singular circular indentation (Figure 2), a plate with a singular circular indentation with a drilled central hole (Figure 3(a)), and a plate containing three profiled circular indentations with central holes (Figure 3(b)). All these indentations have a power law (quadratic) profile with $m = 2$.

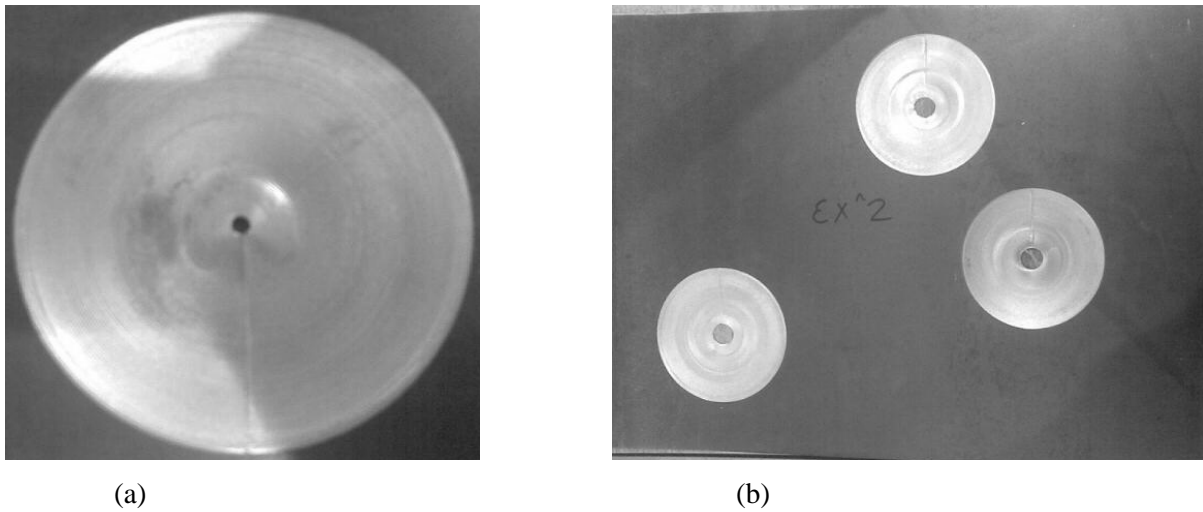


Figure 3: A singular circular indentation with a drilled central hole (a),
Three profiled circular indentations with central holes (b)

There are three main problems encountered when utilizing this method of manufacturing. The first being that at the centre of the indentation, where the machining stress and resulting heat are high, the material thickness is less than 0.4mm resulting in blistering, see Figure 4, leading to inaccurate results during test. Secondly, it is the formation of a machine line, as the cutters movement through the indentation is computer controlled, it merely moves from one programmed height to another, gouging the material and creating a raised line in the indentation, which, as with blistering, could lead to increased elastic wave scattering. Finally, additional damage can occur when a hole is drilled into the centre of the circular indentation. Due to the thickness of the material at this point, it is more susceptible to tearing.



Figure 4: Machine damage to a circular indentation

3 Experimental setup

The experimental set-up has been designed to allow nearly free vibration of the sample plates (i.e. to eliminate clamping of edges), take the weight off the plate and introduce minimal damping to the system, see Figure 5.



Figure 5: Experimental setup

The excitation force was applied to the centre of the plate via an electromagnetic shaker attached to the plate using wax and fed via a broadband signal amplifier. The response was recorded by an accelerometer (B&K Type 4371) that was attached to the upper surface, directly above the force transducer (B&K Type 8200), also via wax (see Figure 6). The acquisition of the point accelerance was utilised using a Bruel & Kjaer 2035 analyser and amplifier over a frequency range of 0-9 kHz, a schematic is shown in Figure 7 .

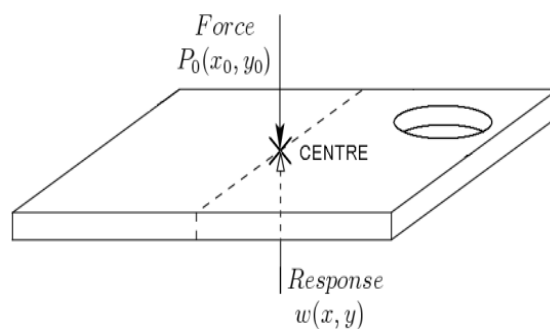


Figure 6: Locations of the shaker (Force) and of the accelerometer (Response) on an experimental sample

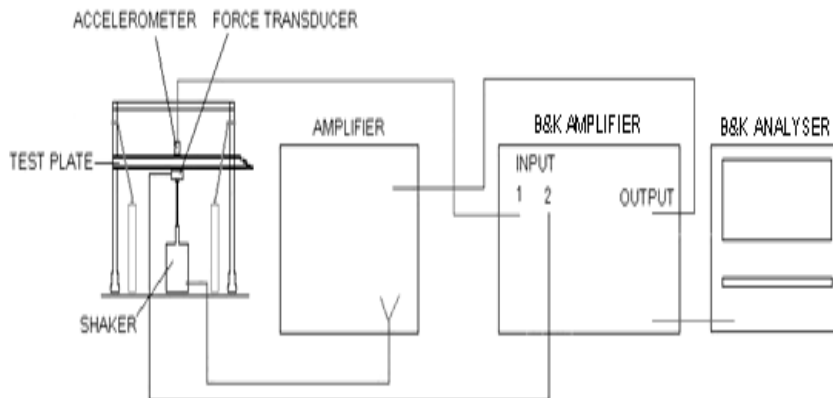


Figure 7: Schematic of the experimental setup utilising the Bruel & Kjaer Analyser

Two different styles of reference plate were considered in order to determine if there is any significant difference between them. A plain plate and a plate with a punched through hole were considered as both had merit as a reference. Both reference plates were of the same dimensions as the profiled plates, the punched hole plate contained a through cylindrical hole of the same diameter and position as the machined power-law profiles.

The plain plate more accurately represents a practical situation, with the profile being cut into an existing structure, it also has reduced internal defects resulting from machining stress, when compared to the punched hole plate. However, it does not account for the reduction in mass and equivalent stiffness of the plate. Therefore, a punched hole reference plate was used for comparison in this paper.

4. Experimental results and discussion

4.1 A profiled circular indentation with and without a small central hole

Two sets of experimental results are described in this section: the effect of adding a circular indentation of power law profile into a plate, when compared to a reference plate, and the effect of drilling a 2mm hole in the centre of the indentation.

Our initial measurements with a single circular indentation without a central hole (that are not shown here for brevity) have demonstrated that in this case there are no noticeable damping effects when compared to a reference plate. There was little to no improvement in damping by the addition of a damping layer. To the contrary, in some frequency ranges the presence of the indentation actually increased the level of vibrations in the plate. From these initial observations it has been concluded that a central hole has to be drilled into the centre of the indentation.

Note that due to manufacturing limitations at the centre of the circular pit, there is an area of almost equal thickness that extends from the central point out to a diameter of approximately 3 mm. A hole of 2mm diameter can therefore be drilled without affecting the minimum tip thickness.

Figure 8 shows the measured accelerance for an indentation of quadratic profile ($m = 2$) with a 2mm central hole and an added damping layer compared to a reference plate. As it can be seen, below 3.2 kHz

there is little to no damping, a slight increase is seen in some of the peaks between 3.4 – 4.5 kHz, this increase is most likely due to a nodal point. A maximum damping of 4.5 dB occurs at 6.6 kHz.

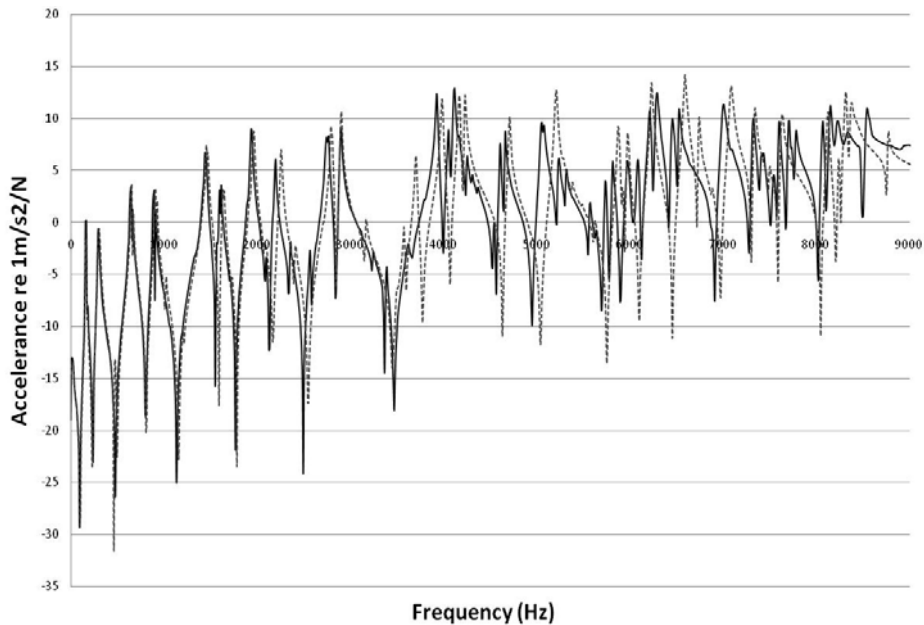


Figure 8: Measured accelerance for a $m=2$ profiled circular indentation with a 2mm central hole and an added damping layer (solid line) compared to a reference plate (dashed line)

Measurements' results show that the damping effect of drilling a hole in the center of the circular inclusion is immediately obvious. These measurements, however, also show an unusual behavior as a result of adding a damping layer to the tip of the circular inclusion. Earlier, it has been shown for power-law wedges [7] that in order to achieve significant damping there is a requirement to add an additional damping layer to the tip of the wedge. However, this is not the clear case for a circular indentation with a 2mm central hole; the addition of a damping layer in fact reduced the damping performance of the indentation. Nevertheless, if to make comparison with the reference plate, some damping occurs.

4.2 A profiled circular indentation with a large central hole

In attempts to improve the damping efficiency of the profiled indentations, the central hole size was increased progressively by 2mm until a central hole size of 14mm and an indentation diameter of 100mm was produced. As the central hole size increased so did the damping performance of the circular indentation.

The results are shown in Figure 9. As expected, Figure 9 shows that the damping layer increases the damping performance of the inclusion up until 8 kHz, after which it has a reduced effect. This pattern of varying damping is consistent with the theory of power-law profiled wedge damping [5-7]. The greatest increase in damping performance was achieved in the frequency range between 3.5- 6 kHz, when an additional damping layer was attached. The maximum increase in damping of 9dB occurs at 5.2 kHz.

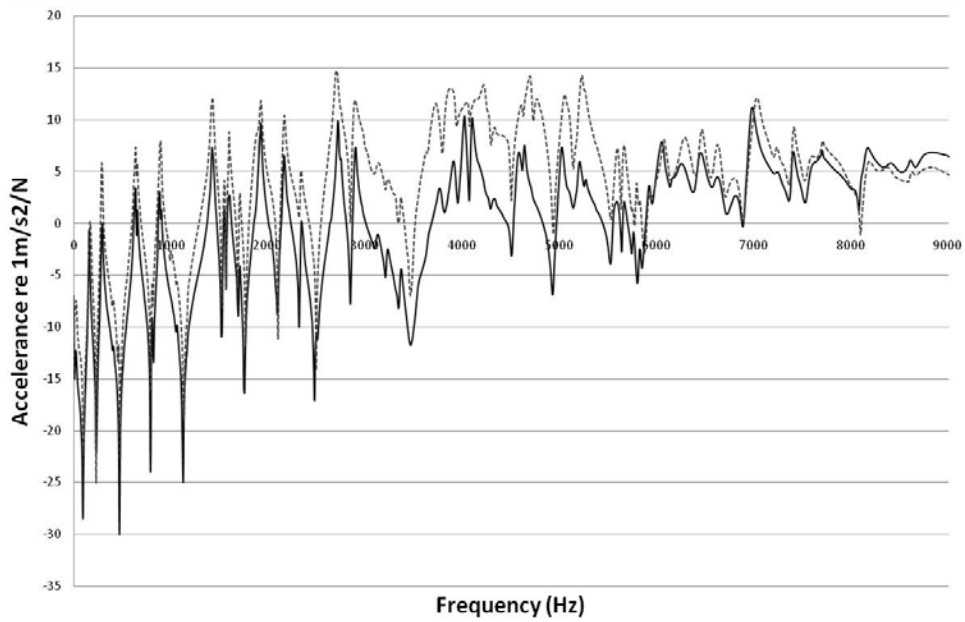


Figure 9: Measured acceleration for a profiled circular indentation with a 14mm central hole with (solid line) and without (dashed line) an additional damping layer

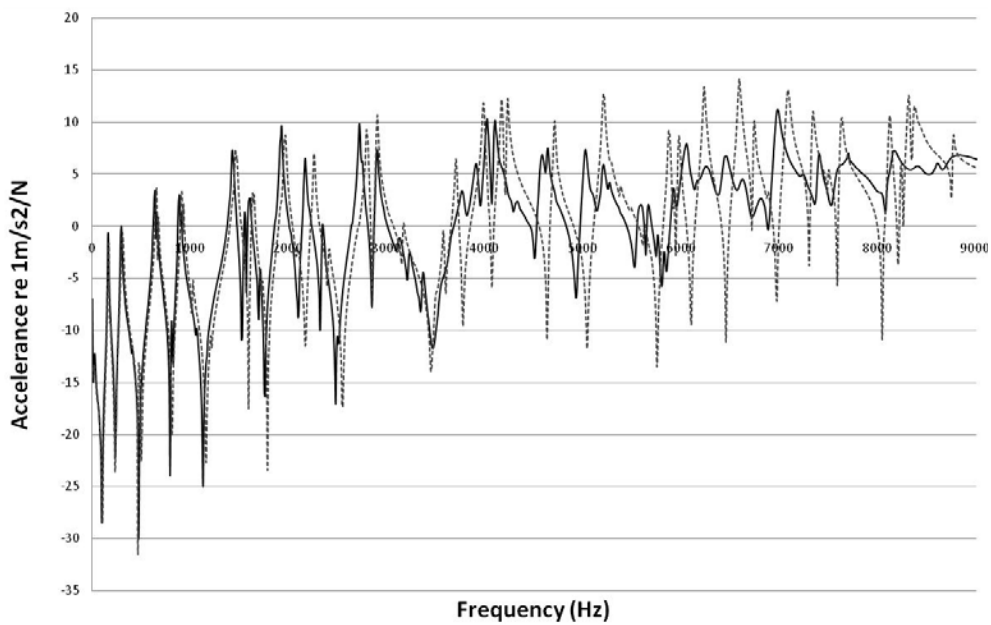


Figure 10: Measured acceleration for a profiled circular indentation with a 14mm central hole with an additional damping layer (solid line) compared to a reference plate (dashed line)

A comparison of the results for a profiled circular indentation with a 14mm central hole and an additional damping layer to the results for a reference plate are shown in Figure 10. Again, Figure 10 shows that below 3 kHz the circular pit provides little to no damping. In the region of 3.8 – 9 kHz, damping varies between 3 – 8 dB, and maximum damping occurs at 6.6 kHz.

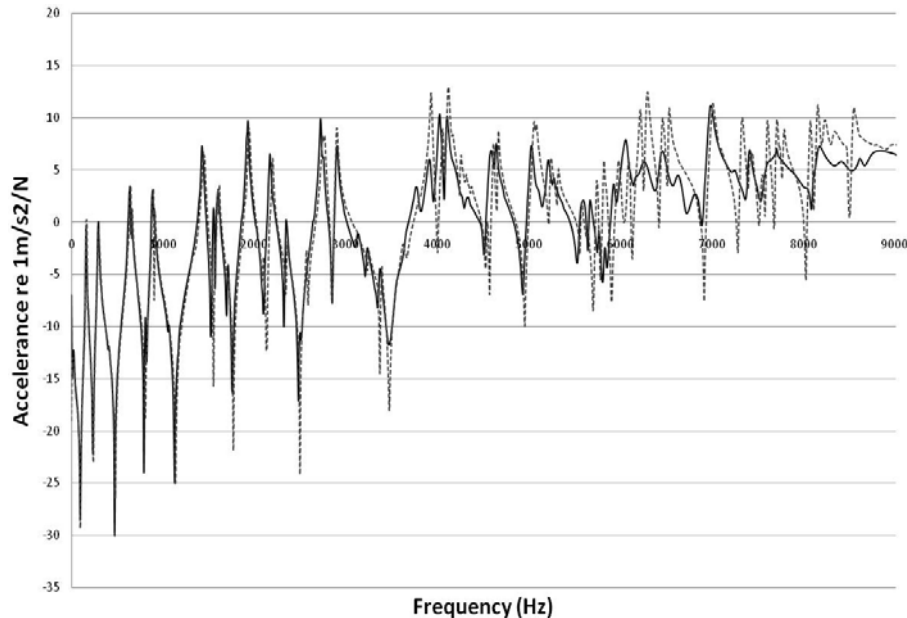


Figure 11: Measured acceleration for a damped circular indentation with a 14mm central hole (solid line) compared to a damped circular indentation with 2mm central hole (dashed line)

Figure 11 shows a direct comparison between the two $m = 2$ profiled circular indentations with 2mm and 14mm central holes. Below 3.8 kHz, the response of both samples is almost identical, varying by no more than approximately 1-2dB. The response is again identical at 7 kHz. In all other regions the 14 mm central hole has an increased damping performance compared to the 2mm central hole. In the region 6-7 kHz, the maximum difference can be seen, with the response of the 14mm central hole showing a greater reduction in amplitude than the 2mm central hole sample by a maximum of 7dB. Increasing the central hole diameter increases damping performance of the circular indentation as it increasingly resembles a curved wedge as the diameter of the central hole is enlarged.

4.3 Multiple circular indentations

This section describes the combined effect of three damped profiled ($m = 2$) circular indentations positioned randomly about the central excitation point, as compared to a reference plate and a plate containing a damped singular inclusion with a 14mm central hole. This multiple-hole sample was expected to perform better than the plates with one circular indentation. But still it was not expected to exceed the damping level of the plates with a power law wedge. The diameters of the circular inclusions were 100 mm and the central holes were 14mm in diameter.

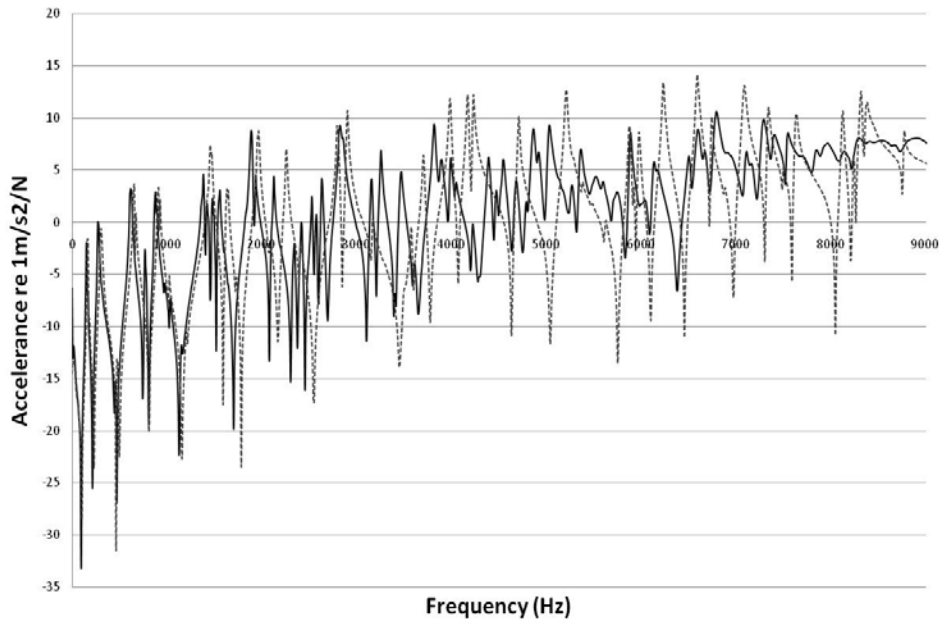


Figure 12: Measured acceleration for a plate containing three profiled circular indentations with 14mm central holes and additional damping layers, as compared to a reference plate (dashed line)

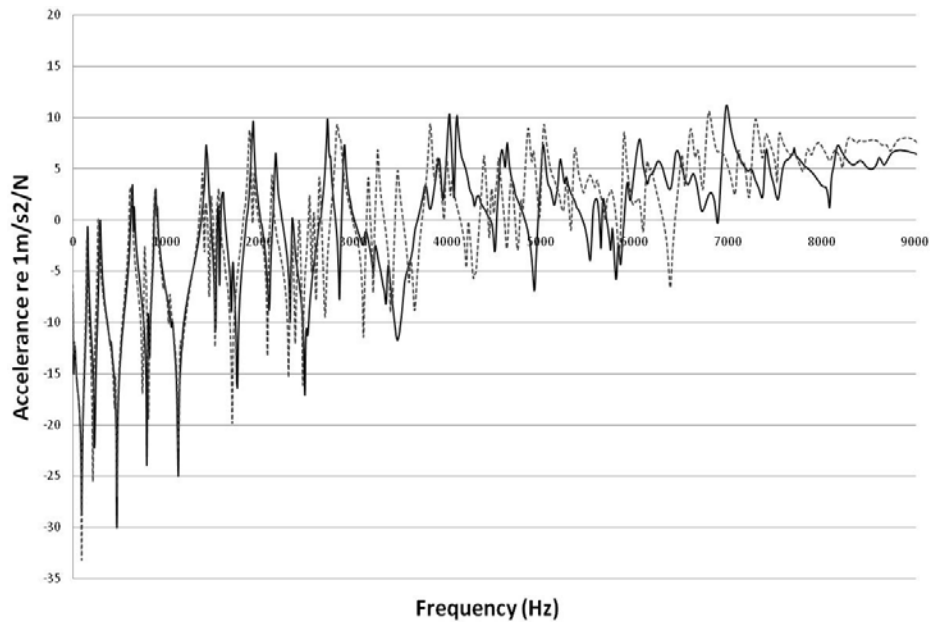


Figure 13: Measured acceleration for a plate containing three profiled circular indentations with 14mm central holes and additional damping layers (dashed line), as compared to a singular circular indentation with a 14 mm central hole and additional damping layer (solid line)

The results for the plate with three profiled circular indentations with 14mm central holes compared to a reference plate are shown in Figure 12. As expected, at low frequencies (below 1.5 kHz) there is little to no damping, between 1.5 -3 kHz, the amplitude of the response is reduced by approximately 1-2 dB. As before, there is an increased response between 3-3.8 dB, again more than likely due to a node. This response is more pronounced than in the case of a singular pit responses. Between 3.8 and 9 kHz reductions are in the range of 3-8 dB, with the maximum reduction occurring at 6.2 kHz.

The comparison of the effect of three indentations with that of a single one is shown in Figure 13. Surprisingly, it can be seen that almost no increased damping can be gained by including three indentations in the current configuration as opposed to one indentation. In fact, over the frequency range tested, the single inclusion sample showed even a greater damping ability. There is however an unfavourable increase of approximately 8 dB in the resonance peaks in the frequency range 3-8 kHz. The multiple indentations do however show a slightly increased damping performance in the range 4-5.2 kHz. As expected, the damping performance of the multiple or singular indentation plates is not greater than the performance of 1D wedges of the same profile. Also, the position of the holes is linked to performance, and different combinations may result in greater levels of damping.

5. Conclusions

It has been demonstrated in this paper that basic power-law indentations that are just protruding over the opposite plate surface cause very small or no reduction in resonant peak amplitudes, which may be due to their relatively small absorption cross-section capturing a relatively small number of flexural wave rays. Note that for elliptical plates this disadvantage has been overcome by focusing of flexural waves in the pit [10, 11]. Introduction of a 2mm central hole improved the situation and increased damping.

To increase damping even more, the absorption cross-section has been enlarged by increasing the size of the central hole in the indentation up to 14mm, while keeping the edges sharp. As expected, such pits, being in fact curved power-law wedges, resulted in substantially increased damping performance that was comparable with that achieved by one-dimensional wedges of power-law profile.

Contrary to the expectations, the introduction of multiple hole plates in the current layout did not clearly increase the damping performance of the two-dimensional indentations of power-law profile. Further research is required to clarify the multiple-hole performance.

Acknowledgements

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