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# Numerical and experimental investigation of the acoustic black hole effect for vibration damping in beams and elliptical plates

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## ABSTRACT
Flexural waves in beams and plates slow down if their thickness decreases. Such property was successfully used for establishing the theory of acoustic black holes (ABH). In fact, in the case of a sharpened edge having a power-law profile, it can be shown that the refection coefficient of a wave propagating towards the sharpened edge can be equal to zero. However, manufacturing such profiles is always related to truncations and imperfections that undermine ABH. It is known though that the use of a thin absorbing film drastically improves the damping effect of ABH. The aim of the current paper is to show numerically and experimentally the capability of ABH to provide structural damping without introducing additional mass. The dynamic behaviour of a non uniform Euler-Bernoulli beam is described using a Riccati equation for the beam impedance, which leads to the reflection matrix of the sharpened edge of the beam. The influence of length of the profile, thickness and length of the absorbing film are evaluated as realistically as possible and optimised numerically in order to reduce wave reflection from the edge. Keeping in mind the numerical results, an elliptic plate with a pit of power law profile placed at one of its focuses has been designed and tested. As a result, both numerical simulations and experimental measurements show significant reduction of vibration levels.

## 1. INTRODUCTION
Damping of structural vibrations of beams and plates has been always a challenging aim for many researchers and engineers. One of the methods for achieving this goal is by reducing wave reflections from boundaries of structures under test by treating their ends. In this regard, the fact that, flexural waves in beams and plates slow down if their thickness decreases was successfully used by Mironov[1] for establishing the theory of **A**coustic **B**lack **H**oles (ABH). The main idea of this approach is to use a power-law relationship between local thickness $h$ and the distance from the edge $x$:

$$h(x) = \varepsilon x^{m} \ (m \geq 2), \tag{1}$$

in order to reduce the local phase velocity. Furthermore, it can be shown that the travel time needed for a wave to reach the edge of a beam can be infinite when the truncated thickness of the profile tends to zero. Therefore, under such conditions the flexural waves stop propagating and the reflection coefficient becomes equal to zero. This is the basic principle of ABH as described by Mironov.

Because of technological difficulties manufactured beams and plates with power-law profiles always exhibit truncations at certain distance $x_0$ from the coordinate origin, with a non-zero thickness (see e.g. Fig. 1). This is why a reflected wave by truncated edge always occurs, which partly cancels the effect of ABH and makes its practical application unattractive. However, a recently proposed approach has combined the use of power law profile wedge (ABH) with a thin absorbing film covering fully or partially the treated area[2-4] leading to the so called ABH effect. Thus, the additional use of a conventional damping technique could overcome to some extent the unpleasant consequences of the truncated thickness profile.
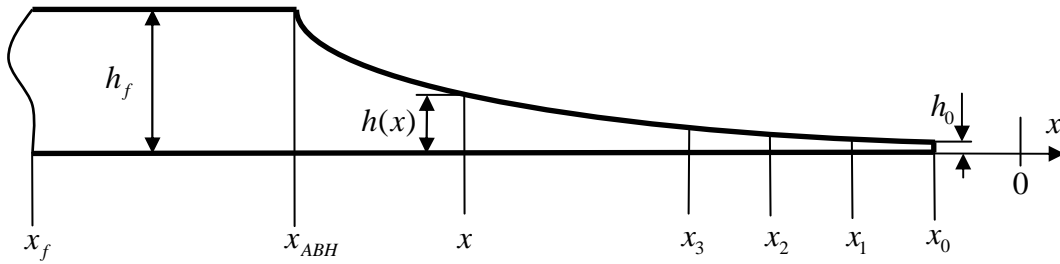


**Figure 1**: 1-D ABH configuration, corresponding geometrical values are given in Table 1.

In more details, Krylov has used a geometrical acoustic approach to describe the propagation of flexural waves towards a free edge. The effect of a thin absorbing film was taken into account using the model of Ross-Ungar-Kerwin[5] for constrained layer damping in the special case of extensional damping (no constraining layer). As a result, the reflection coefficient was analytically expressed as simplified formulae for different power law profiles of order m=2, 3, 4 and sinusoidal profile[2]. Furthermore, the first experimental results regarding the use of ABH affect were reported by Krylov and Winward[4]. The point mobility FRF's showed significant reduction of mobility in the high frequency range for the plate which power law profile was covered by an adhesive stripe compared to the same plate without absorbing material. Later on experiments on tapered rods with power law profiles[6] demonstrated the damping of flexural vibrations in 1-D structures and its application for reducing impact-induced vibrations in tennis racquets[7]. Thus, the combination between ABH and additional damping material (ABH effect) was experimentally validated as a promising measure for vibration reduction. Preliminary results to the present work were published in Ref. 8 in the case of elliptical plates where an ABH was placed in the inner 2D domain and not at the boundary of the structure taking advantage of a focalisation effect.

The aim of this paper is to establish reliable numerical and experimental approaches for designing, modelling and manufacturing ABH as an effective passive vibration damper. For this purpose, in section 2 the basic principles of the theoretical model of a 1-D ABH are presented. In section 3 the results of numerical simulations are shown. In more details, the first part illustrates ABH effect whereas the second one deals with a parametric study that allows an optimisation of the geometrical and material parameters of the absorbing film.

Section 4 encompasses the experimental investigation of elliptical plates with and without ABH, showing the applicability of the concept. Comparisons with numerical results are included as well.

## 2. MODELLING OF ONE-DIMENSIONAL ABH

The shape of an elliptical plate induces a focalization of the waves towards one of its focuses if the excitation is applied in the other focus. This geometrical focalization phenomenon could be used for studying ABH. As was mentioned above, a 1-D ABH is a non-uniform beam having a power-law profile. This 1-D ABH can be extended to a 2-D configuration consisting of an axisymmetric pit which thickness gradually decreases to (theoretically) zero towards its centre according to Eq. (1). Note that placing the ABH in the inner part of a plate instead of an edge has not been investigated in the previous ABH studies and is one of the originality of the present approach. Thus, considering an inner ABH placed in one of the focuses, all the generated waves will reach it either directly or after reflections from the free edges. The waves excited in this way could be considered as vibrational rays starting from the driving focus and ending to the damped focus, see Fig. 2. Consequently, the two dimensional vibration problems in an elliptical plate could be successfully approximated using 1-D structure.
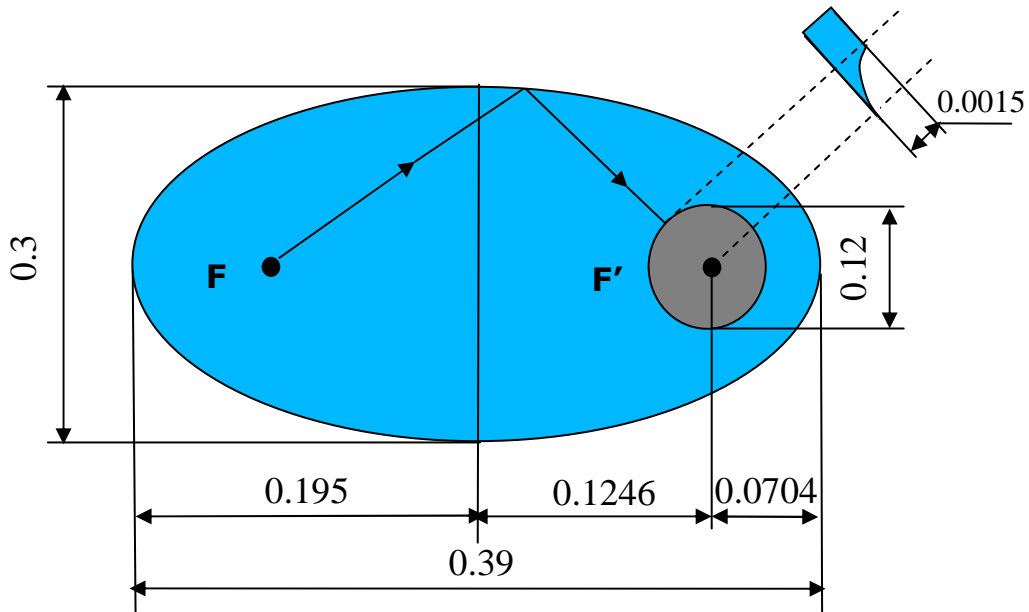


**Figure 2**: Model of elliptical plate flexural vibrations approximated by 1-D vibration rays.

The varying thickness of non-uniform 1-D structure shown in Fig. 1 is given by Eq. (1) for m=4. Its ABH model is based on classical beam theory: Euler-Bernoulli hypothesis are assumed. The vibrational state of the beam can be described by four variables: the displacement $w$, the local slope $\theta$, the shear force $F$ and the bending moment $M$. Harmonic motion is supposed (time factor $e^{i\omega t}$ is assumed) and all variables depend only on the spatial coordinate $x$. In this context, the four variables can be grouped in a state vector:

$$\mathbf{X} = \begin{bmatrix} w & \theta & F & M \end{bmatrix}^T, \tag{2}$$

Eq. (2) is a compact formulation of the Euler-Bernoulli model[9-10]. The state vector is composed of two kinematic and two force variables and the local impedance matrix $\mathbf{Z}$ can be defined as follows:

$$\begin{bmatrix} F & M \end{bmatrix}^T = j\omega\mathbf{Z}\begin{bmatrix} w & \theta \end{bmatrix}^T, \qquad \text{with} \quad \mathbf{Z} = \begin{bmatrix} Z_1 & Z_2 \\ Z_3 & Z_4 \end{bmatrix}. \tag{3}$$

It can be shown that the impedance matrix $\mathbf{Z}$ is the solution of the Riccati equation:

$$\frac{\partial \mathbf{Z}}{\partial x} = -\mathbf{Z}\mathbf{H}_1 - j\omega\mathbf{Z}\mathbf{H}_2\mathbf{Z} + \frac{\mathbf{H}_3}{j\omega} + \mathbf{H}_4\mathbf{Z}, \tag{4}$$

where

$$\mathbf{H} = \left[ \begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/E_1 I_1 \\ \hline -\rho_1 A_1 \omega^2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array} \right] = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \\ \mathbf{H}_3 & \mathbf{H}_4 \end{bmatrix}.$$

The matrices $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{H}_3$ and $\mathbf{H}_4$ are characteristic matrices of the propagating medium. If impedance $\mathbf{Z}$ can be specified at one point of the medium, solving Eq. (4) gives the way to compute $\mathbf{Z}$ at any coordinate and to derive the response of the medium to any excitation force.

Once the structural impedance $\mathbf{Z}$ is obtained, the reflection matrix $\mathbf{R}$ can be easily defined using a standard wave approach:

$$\mathbf{R} = \begin{bmatrix} j\omega\mathbf{Z}\mathbf{E}_2 - \mathbf{E}_4 \end{bmatrix}^{-1}\begin{bmatrix} \mathbf{E}_3 - j\omega\mathbf{Z}\mathbf{E}_1 \end{bmatrix}, \tag{5}$$

where the matrices $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$ and $\mathbf{E}_4$ are the components of the eigenvector matrix $\mathbf{E}$ of the matrix $\mathbf{N} = -j\mathbf{H}$ (see Ref. 10). The scalar components, $R_1$, $R_2$, $R_3$ and $R_4$, of the reflection matrix represent the reflection of evanescent and propagating flexural waves in the beam and their coupling. $R_1$ corresponds to propagating waves whereas $R_4$ corresponds to evanescent waves. The coupling between these two types of waves is characterised by $R_2$ and $R_3$.

In 1-D ABH model the damping has two origins:
(i) The visco-elastic and thermo-elastic damping inside the beam material. This kind of damping can be modelled by a complex Young's modulus $E_1$ which can be introduced using the loss factor $\eta_1$ of beam material.
(ii) And damping which can be added by a thin absorbing layer having Young's modulus $E_2$ and loss factor $\eta_2$. The model developed in this study could be extended in order to include the additional damping due to the absorbing film. The Ross-Ungar-Kerwin model[5] for constrained layer damping in the special case of extensional damping (no constraining layer) is employed here. The complex bending stiffness of the compound beam (beam covered by an absorbing film) can be expressed using the bending stiffness of the beam only:

$$EI(1+i\eta) = E_1I_1\left[(1+i\eta_1) + e_2h_2^3(1+i\eta_2) + \frac{3(1+h_2)^2 e_2h_2[1-\eta_1\eta_2 + i(\eta_1+\eta_2)]}{1+e_2h_2(1+i\eta_2)}\right], \qquad (6)$$

where $EI$ is the bending stiffness of the compound beam; $E_1I_1$ is the bending stiffness of the beam only; $\eta$ is the loss factor of the compound beam; $\eta_1$ loss factor of the beam's material; $\eta_2$ is the loss factor of the absorbing film's material; $E_1$ and $E_2$ are Young's moduli of beam's and absorbing film's materials, respectively; $e_2=E_2/E_1$; $\delta$ is the thickness of the absorbing film; $h$ is the local thickness of the beam; $h_2=\delta/h$.

In case when the thickness of absorbing film is comparable to the local thickness of the beam the mass of the absorbing film also should be taken into account, thus, the mass per unit length can be written as: $\rho A = \rho_1 A_1 + \rho_2 A_2$, where $\rho_1$ and $\rho_2$ are beam's and absorbing film's material densities, respectively; $A_1=bh$ and $A_2=b\delta$ are beam's and absorbing film's cross sections respectively; $b$ is the width of the beam.

## 3. NUMERICAL SIMULATIONS

The efficiency of ABH is estimated by the reflection matrix **R**, which can be computed from impedance matrix **Z**, as shown in Eq. (5). This latter is the solution of Eq. (4), computed with boundary conditions **Z**$(x_0)$=0, describing the fact that the end of the beam at $x=x_0$ is free. Numerical simulations are presented for a beam defined by Fig. 1 and Table 1.

**Table 1**: *Geometrical and material characteristics of the beam under consideration*

| Geometrical characteristics, (m) | | Material charateristics | |
|---|---|---|---|
| | | Beam | Absorbing film |
| $x_0$ = -0.01 | $h_f$ = -0.0015 | $E_1$ = 70 GPa | $E_2$ = 0.5 GPa |
| $x_{ABH}$ = -0.06 | $b$ = 0.0015 | $\rho_1$ = 2700 kg/m$^3$ | $\rho_2$ = 950 kg/m$^3$ |
| $x_f$ = -0.08 | $m$ = 2 | $\eta_1$ = 0.005 | $\eta_2$ = 0.05 |

### A. Illustration of ABH effect

In Fig. 3 the reflection coefficient $R_1$ is presented as a function of frequency in order to illustrate ABH effect. It is shown that reflection coefficient $R_1$ for a beam with ABH covered by 700 µm (solid curve) is about 20 % reduced at 10 kHz, whereas for beam with ABH only without any absorbing film (dashed curve) this reduction is only 7 % and for a uniform beam covered by 700 µm absorbing film is even smaller – around 4 %. Therefore, ABH effect leads to much more decrease of reflected waves compared to any of the individual treatment of the beam – ABH only or damping treatment only. Note that the oscillations of reflection
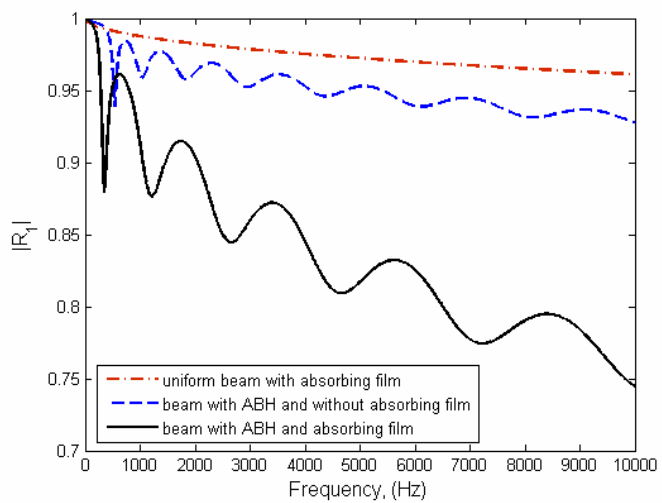


**Figure 3:** Reflection coefficient for: uniform beam (dash-dotted curve), beam with ABH (dashed curve), and beam with ABH covered by absorbing film (solid curve).

coefficient decrease and their periodicity becomes larger. Therefore, at frequencies tending to infinity the reflection coefficient does not exhibit any oscillations. The origin of these oscillations might be due to the sharpness and length of the profile.

## B. Parametric study

Using the data mentioned in Table 1, a parametric study is proposed in order to define the optimal characteristics of the absorbing film at which the reflection coefficient $R_1$ becomes minimal. ABH and absorbing film dissipate vibration energy together but using different mechanisms. The role of absorbing film is to assist for overcoming disadvantages due to the truncation, thus, the most important area is near the truncation. However, the absorbing film acts as an added mass and reduces the effect of ABH. For example, adding absorbing film leads to increase of phase velocity and wavelength in the areas where its thickness is comparable or larger to that of the beam. Therefore, the use of absorbing film in the area of ABH is not straightforward. This is why it is important to optimise the thickness and length of absorbing film by varying its geometrical and material characteristics, so that the reflection coefficient $R_1$ is minimised.

When dissipation characteristics are under consideration the most relevant parameter is the loss factor of the compound beam (beam with ABH + absorbing film). In this regard, the ratio between imaginary and real parts of the wavelength gives an estimation of this parameter. Fig. 4 shows this ratio at different thicknesses and loss factors of absorbing film. The absorbing film results in an extremum in the loss factor compared to the constant one of the beam without absorbing film see Fig. 4 (a). Moreover, increasing the thickness of the absorbing film makes the maximum to shift towards thicker part of the beam and vice versa. Note that the maximal value of the loss factor does not change when the thickness of the absorbing film varies. Thus, the position of the extremum is governed by the ratio between the thickness of the absorbing film and the thickness of the power law profile. The minimal reflection coefficient corresponds to a thickness of 0.7 mm. and this is the optimal value for this configuration. Decreasing or increasing the thickness leads to increase of reflection matrix and does not guarantee that reflection of propagating waves is minimal.
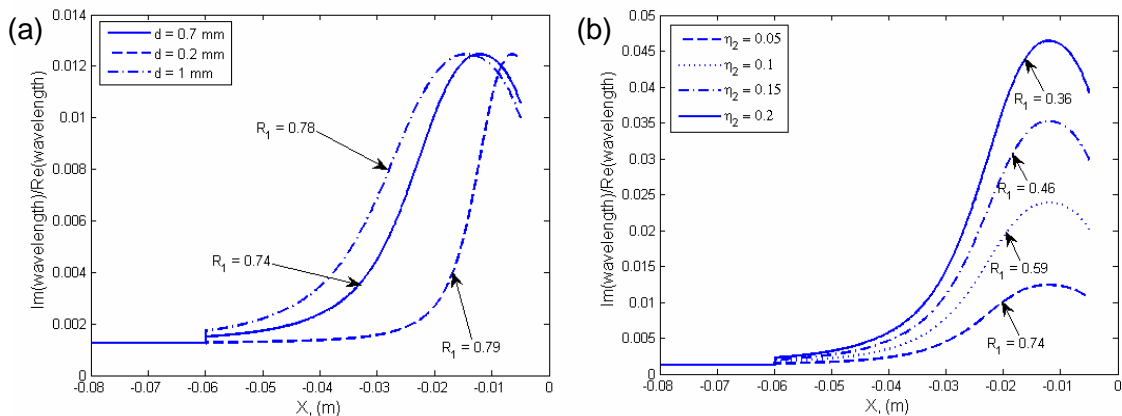


**Figure 4**: (a) – the ratio $Im(\lambda(x))/Re(\lambda(x))$ of the compound beam for different thicknesses $d$ of the absorbing film; (b) - the ratio $Im(\lambda(x))/Re(\lambda(x))$ for different values of loss factor $\eta_2$ of absorbing film.

Besides the thickness of the absorbing film, its loss factor $\eta_2$ is another parameter that could minimise reflection matrix $R$ and improve the efficiency of ABH effect. Fig. 4 (b)

shows that increasing the loss factor of the absorbing film leads to direct decrease of the reflection coefficient. For example, increasing the loss factor $\eta_2$ from 5 to 20 % leads to reduction of the reflection coefficient from 74 % to 36 %. Therefore, using a highly absorbing material as a damping layer, a passive absorber without added mass that is able to dissipate about 90 % of vibration energy of propagating waves could be designed.

## 4. EXPERIMENTAL RESULTS

Measurements have been carried out for different replicas of the aluminium elliptical plate in order to demonstrate the capabilities of the proposed passive absorbing damper. The models that were tested are as follows: elliptical plate with ABH and without ABH, elliptical plate with disk of resin placed at the location of the ABH and elliptical plate completely covered by resin. The equipment used was a Polytech Vibrometer Scanning Head – OFV 056, an impedance head Bruel&Kjaer type 8001, a Bruel&Kjaer Conditioning Amplifier, an Amplifier LM 3886, and a Shaker LDS V201. The plates were hanged vertically ('free' boundary conditions) and excited by the shaker using a periodic chirp signal. A number of point mobilities and velocity fields were measured.
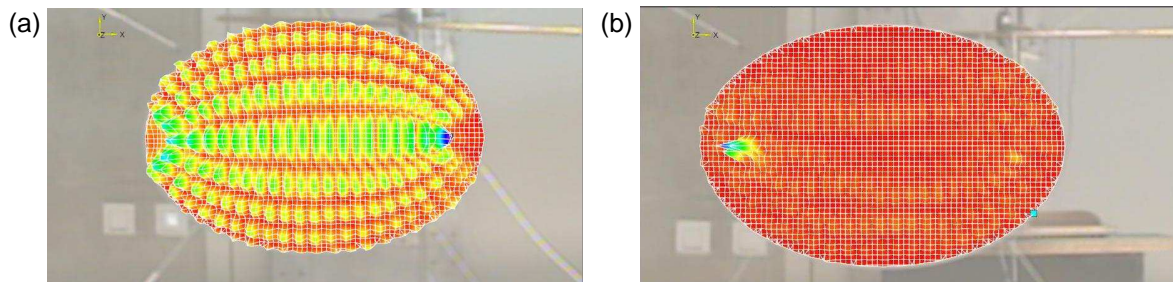


**Figure 5**: Velocity fields of an elliptical plate at 7613 Hz: (a) without ABH, and (b) with ABH.

Fig. 5 shows velocity fields of plates with ABH (b) and without ABH (a) at 7613 Hz. The excitation force was applied to the left focus whereas the ABH is in the right one. It can be seen that the spatial patterns of the plate without ABH are rather symmetric and equally distributed with some small increases in the area of the right focus, whereas those of the plate with ABH exhibit a concentration of the velocity field in the area of ABH. This concentrated activity is due to the decrease of wavelength in the area of ABH. In fact, because of the focalisation effect, the vibration energy was guided and focused in the ABH. Thus, the rest of the plate is quite silent compared to the plate without ABH. Note that due to the focalisation effect treating the right focus of the plate with absorbing film reduces plate vibrations as well. However, this decrease is much smaller compared to the reduction of the plate with ABH and this can be seen in the measured point mobility functions.

The graphs in Fig. 6 (a) represent point mobilities measured at the left focus of the plate. The measured point mobility of the plate with ABH and without an absorbing film is compared to the corresponding point mobility of the plate covered by an absorbing film. Moreover, the point mobility of the plate without ABH is presented as well. Similarly to Fig. 3, the point mobility FRF's of the plate with ABH and covered by different absorbing films exhibit the largest reduction. In the range above 2 kHz, the point mobility of the plate with ABH covered by absorbing film 2 was reduced more than 15 dB, compared to the mobility of a plate without ABH. In correspondence with Fig. 4 (b), absorbing film 1 has smaller loss factor compared to absorbing film 2 and respectively its point mobility is larger. All the
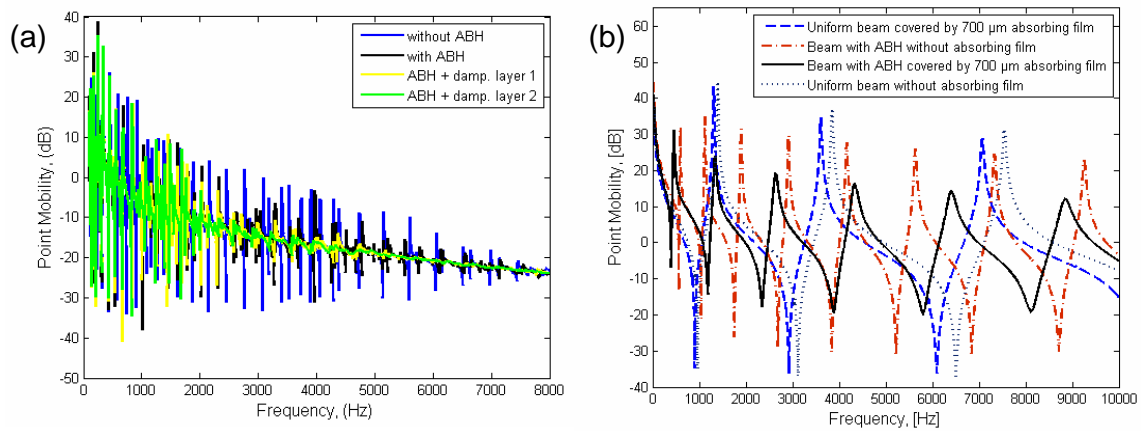
**Figure 6:** (a) Measured point mobilities of the elliptical plate and (b) calculated point mobilities of a semi-infinite beam with and without ABH, and with and without an absorbing film.

observed phenomena are clearly visible in Fig. 6 (b) as well, where the numerical results concerning a semi-infinite beam, as was discussed in Section 2, are presented. In more details, the highest peaks belong to a uniform beam with and without an absorbing film whereas a beam with ABH and without an absorbing film has yet comparable to the uniform beam resonant amplitudes though slightly reduced. Finally, a beam with ABH covered by an absorbing film exhibits the largest reduction in its point mobility similarly to Fig. 3 and Fig. 6 (a). Therefore, the experimental and numerical results confirm the effectiveness of the proposed new approach for reduction of structural vibration using ABH effect.

## 5. CONCLUSIONS

In the present paper, a new alternative approach for damping structural vibrations in beams and elliptical plates has been studied and reported. The effect of absorbing film plays a very important part in the overall design of this new type of absorber because on one hand it cancels the effect of decreasing power law profile but on the other hand reduces the reflection of waves due to the truncation. This is why a parametric study was conducted in order to specify the optimal geometrical and material properties of the absorbing film. It was shown that the thickness of an absorbing film has the optimal value of 0.7 mm for the configuration under consideration. Moreover, the larger the loss factor of an absorbing film the smaller the reflection coefficient $R_1$ without changing any other parameters. The experimental and numerical results show significant reduction (more than 15dB) of the point mobilities of the plate with ABH and covered by an absorbing film, as compared to the other configurations.

## ACKNOWLEDGEMENTS

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