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# Effects of the embankment topography and track curvature on ground vibration boom from high-speed trains

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**ABSTRACT:** The present paper investigates the effects of the embankment topography and track curvature on ground vibrations generated by high speed trains travelling faster than Rayleigh waves in the supporting ground. It is shown that the presence of the embankment can result in waveguide propagation of generated ground vibrations at certain range of train speeds. The presence of a track curvature (to provide the possibility of changing direction of train movement) can result in focusing of generated Rayleigh waves along the caustic line which is associated with the increase in amplitudes of generated ground vibrations.

## 1 INTRODUCTION

According to numerous practical observations, the intensity of railway-generated ground vibrations generally becomes larger if train speeds increase. In particular, it has been predicted that especially large increase in ground vibration level should occur if a train speed  $v$  exceeds the velocity of Rayleigh surface waves in the ground  $c_R$  (Krylov, 1995, 1998a,b, 2001). If this happens, a *ground vibration boom* takes place, similar to a sonic boom from supersonic aircraft.

In the fall of 1997 the above mentioned theoretical prediction of ground vibration boom has been experimentally confirmed by a research team working on behalf of the Swedish National Railway Administration (Banverket) on their newly opened West Coast Line - from Gothenburg to Malmö (see Madshus et al. 1998). The speeds achievable by the X2000 high-speed trains operating on this line (up to 200 km/h) can be larger than Rayleigh wave velocities in this part of south-western Sweden characterised by very soft ground. In particular, at the location near Ledsgård (25 km South of Gothenburg) the Rayleigh wave velocity in the ground was as low as 42 m/s, so that an increase in train speed from 140 to 180 km/h lead to about 10 times (20 dB) increase in generated ground vibrations. This indicated that 'supersonic' or (more precisely) 'trans-Rayleigh' trains have become today's reality. The experimental investigations in Sweden stimulated an increased activity of researchers both in theory and experiment

(see, e.g. Degrande et al. 1998, 2001; Takemiya 1998; Petyt et al. 1999; Sheng et al 1999).

The aim of the present paper is to describe the basic principles of the theoretical model earlier developed by the present author for generation of ground vibrations by high-speed trains and to discuss the ways of making the model more accurate. The two specific aspects being considered are taking into account possible waveguide effects of the embankments and wave focusing due to the track curvature. It will be demonstrated that both these aspects are important at certain practical situations and can result in noticeable changes in generated ground vibrations.

## 2 GENERAL APPROACH

The main mechanisms of railway-generated ground vibrations are the wheel-axle pressure onto the track, the effects of joints in unwelded rails, and the dynamically induced forces of carriage- and wheel-axle vibrations excited due to unevenness of wheels and rails. In this paper we consider only the most common generation mechanism which is present even for ideally flat rails and wheels - the quasi-static pressure of wheel axles onto the track, which is also responsible for railway-generated ground vibration boom.

According to the earlier developed theoretical model (see, e.g. Krylov, 1995, 1998a,b, 2001), to describe generation of ground vibrations by moving

trains, one needs to know the frequency spectra of dynamic forces applied from each sleeper to the ground. Since the dynamic forces associated with any particular sleepers are related to each other via the distance between them and the train speed, it is sufficient to consider only the force associated with a single sleeper, e.g. the one located at  $x = 0$ .

To calculate ground vibrations generated by a moving train one needs superposition of waves generated by each sleeper activated by wheel axles of all carriages, with the time and space differences between sources (activated sleepers) being taken into account. To derive this in a rigorous way one can use the Green's function approach. The corresponding analytical formulae relating the frequency spectra of vertical component of surface ground vibration velocity at any point of observation with the sleeper force spectra  $P(\omega)$  and geometrical parameters of track and train can be found in the literature (Krylov 1995, 1998a,b, 2001; Krylov et al. 2000). In the recent development of this model carried out by other researchers the reciprocity principle has been applied to achieve more efficient results in calculating ground vibrations (Degrande et al 2001).

For 'trans-Rayleigh trains', i.e. for trains travelling at speeds  $v$  higher than Rayleigh wave velocity in the ground  $c_R$ , the analysis shows that maximum radiation of ground vibrations takes place if the train speed  $v$  is larger than Rayleigh wave velocity  $c_R$  (Krylov 1995, 1998a). Under this condition, a *ground vibration boom* takes place, i.e., ground vibrations are generated as quasi-plane Rayleigh surface waves symmetrically propagating at angles  $\Theta = \cos^{-1}(c_R/v)$  in respect to the track, and with amplitudes much larger than in the case of conventional trains.

Note that for trans-Rayleigh trains these Rayleigh surface waves are generated equally well on tracks with and without railway sleepers, whereas for conventional trains the presence of sleepers is paramount (Krylov 1995, 1998a). Without them no propagating waves are generated in the framework of the quasi-static pressure generation mechanism.

### 3. EFFECT OF GROUND STRATIFICATION AND MEASUREMENTS AT LEDSGÄRD

The first and most important modification of the above-mentioned basic theoretical model for real conditions was taking into account the ground stratification. This has been done using the simplified engineering approach based on constructing the approximate Green's function for a

layered ground (Krylov 1998b). The approach is semi-analytical and takes into account the effects of layered structure on the amplitude and phase velocity of the lowest order surface mode only.

We remind the reader that in layered media the lowest order surface modes are dispersive, i.e. their phase velocities  $c_R$  depend on frequency  $\omega$ :  $c_R = c_R(\omega)$ . According to the said engineering approach, the modified Green's function  $G_{zz}^L(\rho, \omega)$  for a layered ground was assumed to have a structure similar to that of the function for a homogeneous half space  $G_{zz}(\rho, \omega)$ . However, the wavenumber of a Rayleigh wave for a homogeneous half space,  $k_R = \omega/c_R$ , has been replaced in  $G_{zz}^L(\rho, \omega)$  by the wavenumber of the lowest order Rayleigh mode in a layered ground propagating with frequency-dependent velocity  $c_R(\omega)$ :  $k_R^L = \omega/c_R(\omega)$ , and wavenumbers of longitudinal and shear bulk waves have been replaced by the so-called 'effective' wavenumbers of longitudinal and shear bulk elastic waves at given frequency  $\omega$ :  $k_l^L = \omega/c_l^L(\omega)$  and  $k_t^L = \omega/c_t^L(\omega)$ , where  $c_l^L(\omega)$  and  $c_t^L(\omega)$  are longitudinal and shear wave velocities averaged over the 'effective' depth of Rayleigh wave penetration into the ground at given frequency  $\omega$ .

The approximate analytical Green's function constructed in this way has been validated via finite element calculations using the commercial package LUSAS (Mohammad et al 2000).

Calculations using this Green's function in the general model show that the most significant effect of layered ground structure on generated vibrations is due to the phase variations caused by Rayleigh wave dispersion. In particular, for the case of simple monotonous layered system with a soft upper layer, the increase in Rayleigh wave velocities at very low frequencies, associated with a deeper wave penetration into the ground, might violate the trans-Rayleigh condition  $v > c_R$  responsible for generating ground vibration boom, thus causing a reduction in the low-frequency components of generated ground vibration spectra.

According to the published geological data, the ground stratification at Ledsgård can be approximated by a non-monotonous four-layered system, with the 'slowest' layer of organic clay being positioned beneath the top layer. In such a system the Rayleigh wave velocity may have minimum at a certain frequency and a decrease in levels of ground vibrations generated by a trans-Rayleigh train may take place for both low and high frequencies. Using the developed theoretical model, calculations of the vertical ground vibration velocity averaged over the

frequency range 0-50 Hz have been carried out (this roughly corresponds to a peak level of vibration velocity used in the experimental observations). Also, the reported lowest value of Rayleigh wave velocity in the ground ( $c_R = 42 \text{ m/s}$  at frequency 5

Hz) has been used. A typical result of the calculations for an X2000 train travelling on a layered ground is shown on Fig. 1. For comparison, on the same figure the result for a homogeneous elastic ground with  $c_R = 42 \text{ m/s}$  is shown as well.

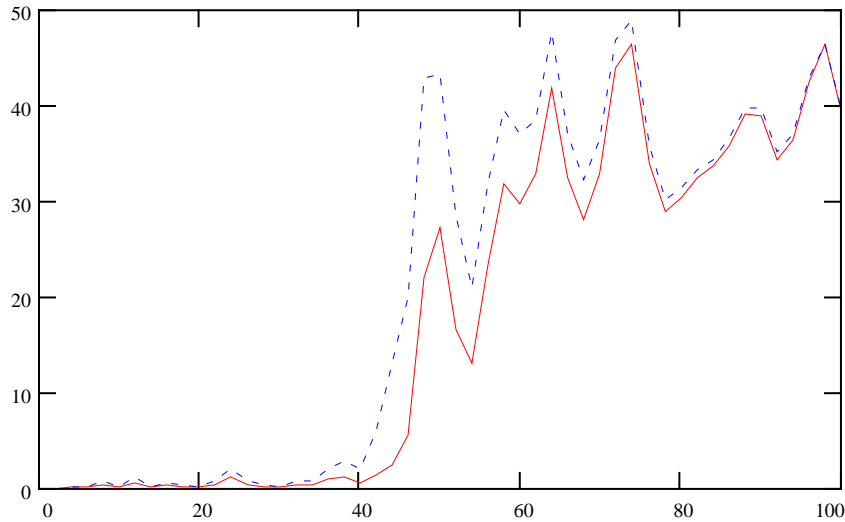


Fig.1. Effect of train speed (in m/s) on the averaged ground vibration velocity (in 0.01 mm/s) generated by X2000 trains in a layered ground (solid curve) and in a homogeneous ground (dashed curve)

As one can see from Fig. 1, the ground stratification reduces the growth rate of generated vibration amplitudes with the increase of train speed. The predicted amplitudes of the peak vertical velocity change from about  $1 \times 10^{-5} \text{ m/s}$  at  $v = 140 \text{ km/h}$  ( $38.8 \text{ m/s}$ ) to  $15 \times 10^{-5} \text{ m/s}$  at  $v = 180 \text{ km/h}$  ( $50 \text{ m/s}$ ). Thus, the estimated 15 times increase in ground vibration level following from the above theory for the given train speeds and Rayleigh wave velocity is in reasonable agreement with the 10 times increase observed experimentally for typical runs of the X2000 train at Ledsgård.

Calculations on Fig. 1 also took into account the effect of track critical velocity  $c_{min}$  for train speed approaching or exceeding its value ( $c_{min} = 60 \text{ m/s}$  in this example). This effect means that the level of generated ground vibrations becomes larger (by approximately 1.5 times, as compared to the case of absence of track dynamics effects).

#### 4 WAVEGUIDE EFFECTS OF THE EMBANKMENTS

An interesting aspect that has to be taken into account on certain locations along high-speed railway lines is possible waveguide effects of the embankments on generated ground vibrations. The

possibility of the waveguide effects playing a noticeable role in reshaping the ground vibration fields generated by trans-Rayleigh trains can be explained by the fact that Rayleigh waves are generated in this case predominantly at small angles  $\Theta = \cos^{-1}(c_R/v)$  versus the track direction. This is why, if a track is placed on the top of the embankment, a large part of radiated energy can be trapped and dissipated within the embankment itself, with little effect on the outside area. Physical principle of these so-called ‘topographic waveguides’ has been described by the present author in the earlier publications (see e.g. Biryukov et al. 1995, p. 220-224, 233-242). Some indication of the presence of such waveguide effects follows from the experiments at Ledsgård.

The effect of railway embankments on generated ground vibrations can be analysed via constructing the specific Green's function for an elastic half space with an embankment acting as a topographic waveguide for surface waves. It was assumed that such a Green's function should take into account the internal reflections of generated surface Rayleigh waves from the geometric boundaries between the embankment's top flat and side slop surfaces, and between side slop surfaces and the low ground. The approximate analytical theory of Rayleigh wave reflection from the boundary between two surfaces

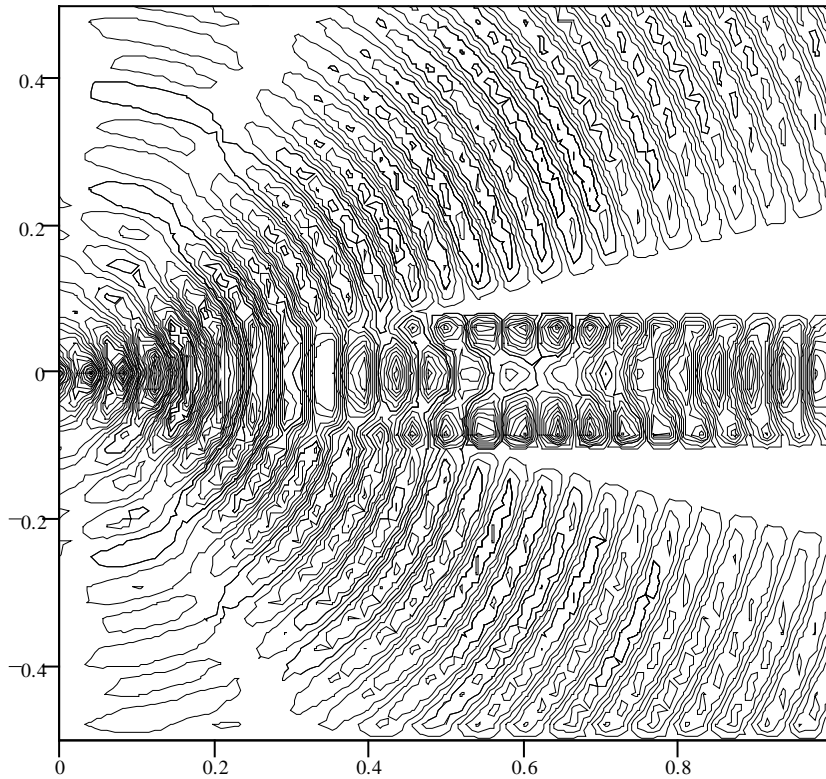
intersecting at an obtuse angle has been earlier developed by the present author (see, Biryukov et al 1995, p. 277). For the problem under consideration the corresponding expression for the reflection coefficient was modified to describe the reflection at all angles of Rayleigh wave incidence  $\alpha$  on geometric boundaries.

The resulting reflection coefficient  $R_i(\alpha)$  has been used to construct the Green's function for a top area of the embankment. This has been done in a way similar to a ray approach to the representation of the field inside an ideal waveguide, i.e. by considering the waveguide contribution to the Green's function in question as an infinite sum of Rayleigh waves radiated by imaginary sources with the amplitudes defined by multiples of the reflection coefficient  $R_i(\alpha)$ . For points of observation outside the embankment, the expressions for the Green's function have been constructed in a similar way using also the transmission coefficient for Rayleigh

waves passing through the boundary between flat top and side areas of the embankment.

Using this Green's function in the general model allows calculation of ground vibrations generated by high-speed trains travelling along the embankments. Since the resulting expressions are rather bulky and contain triple summation - over sleepers, carriage axles, and imaginary sources, only the simplest examples of a single axle load travelling along the embankment have been considered so far to illustrate the principle.

Figure 2 shows the spatial distribution of ground vibration field (in arbitrary units) generated by a single axle load travelling at trans-Rayleigh speed  $v = 126 \text{ m/s}$  over 10 sleepers placed on the embankment of  $8 \text{ m}$  width and with the slop angle  $\Theta_s = 150^\circ$  versus the flat top. The area considered is  $48 \times 48 \text{ m}^2$ . The Rayleigh wave velocity used in calculations was  $c_R = 125 \text{ m/s}$ .



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Fig.2. Spatial distribution of ground vibration field (in arbitrary units) generated by a single axle load travelling along the embankment at trans-Rayleigh speed

One can see that generated ground vibrations propagate predominantly along the embankment where their amplitudes are much larger than in the outside area. This demonstrates that the embankments can act as waveguides for generated ground vibrations, thus reducing their impact outside the embankments for a range of trans-Rayleigh train

speeds for which the total internal reflection occurs. In the same time, the waveguide effects may result in a large increase of ground vibrations inside the embankment. These waveguide-induced additional vibrations, which propagate at speed of the train, can amplify the existing quasi-static vertical deflections in the system track-ground accompanying moving

axle loads. As a result, very large vibrations of the embankment can be observed at train speeds around the velocity of Rayleigh waves, i.e. before achieving the value of track critical velocity.

## 5 EFFECT OF TRACK CURVATURE

If a track has curved parts to provide the possibility of changing direction of train movement, the wave fronts of ground surface waves generated under the conditions of ground vibration boom may become concave at one side of the track, instead of being convex (under usual circumstances) or plane (under the condition of ground vibration boom). This may result in focusing of generated ground vibrations along the simple caustic line at one side of the track and in the corresponding increase of their amplitudes. According to the geometrical acoustics

approximation, this increase is up to infinity. However, because of the diffraction limit, the real increase is much more modest and does not exceed 2-4 times.

It is easy to show that if a train moves along a circle of radius  $R$  at speed  $v$  (with  $v > c_R$ ) then the caustic line will be a concentric circle of a smaller radius  $r$  defined by the expression:  $r = R(c_R/v)$ .

The wave analysis of this problem is rather straightforward and is based on the rewriting the expression for a distributed axle loads that takes into account the track curvature.

The results of calculations of the spatial distribution of ground vibrations generated at the frequency component  $f = 30 \text{ Hz}$  by a single axle load travelling along the curved track with the radius of curvature  $R_0 = 100 \text{ m}$  is shown on Fig. 3. The load speed was  $v = 50 \text{ m/s}$ , and the velocity of Rayleigh wave in the ground was set as  $c_R = 45 \text{ m/s}$ .

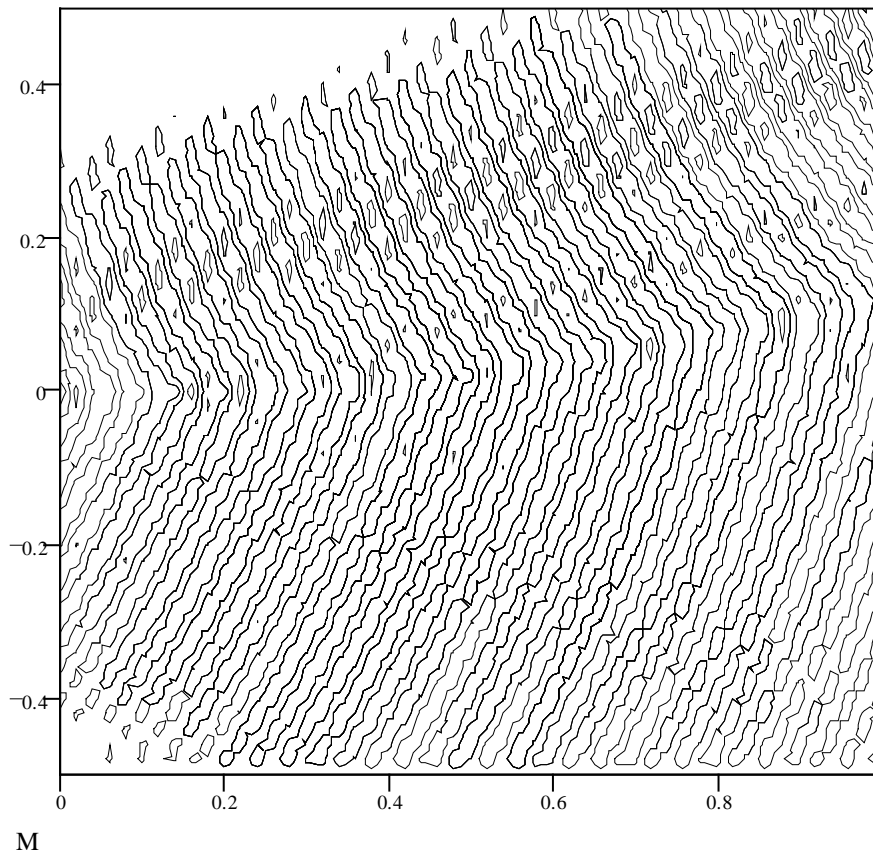


Fig.3. Spatial distribution of ground vibration field (in arbitrary units) generated by a single axle load travelling along a curved track at trans-Rayleigh speed

It is clearly seen from the figure that on the left-hand side of the curved track (the load is moving from left to right) a focusing of ground vibrations takes place. This results in the increase in ground vibration amplitudes by 1.5-2 times.

Note that the effect of Rayleigh wave focusing similar to the one caused by track curvature may occur also in the case of a train accelerating along a straight line if its current speed,  $v(t) = v_0 + at$ , is higher than Rayleigh wave velocity in the ground  $c_R$

(here  $v_0$  is the train initial speed and  $a$  is the acceleration). However, the caustic line in this case is not confined to the area near the train path and moves away from it as the train speed increases.

## 6 CONCLUSIONS

The basic theoretical model of ground vibration boom from high speed trains earlier developed by the present author and modified to take into account the ground stratification agrees well with the recent measurements at Ledsgård. Effect of layered ground structure on ground vibration boom reveals mainly through Rayleigh wave velocity dispersion  $c_R(\omega)$ . In particular, for rather complex cases of three- or four-layer ground structures typical for the location near Ledsgård, with the softest layer being in the depth of the ground, the Rayleigh wave velocity may have minimum at a certain frequency. In such cases a decrease in levels of ground vibrations generated by trans-Rayleigh trains may take place for both low and high frequencies, the high-frequency reduction being the most essential.

Waveguide effects of the embankments can cause concentration of radiated ground vibration energy around the track. This can reduce ground vibration amplitudes outside the embankment. However, the associated vibrations of the track itself may increase tremendously in this case, even at train speeds below the track critical velocity  $c_{min}$ . Therefore, very large rail deflections can be observed on the embankments at train speeds approaching the velocity of Rayleigh waves, rather than track critical velocity.

Under the condition of ground vibration boom, the effect of track curvature may result in focusing of generated ground vibrations at one side of the track and in the corresponding increase of their amplitudes. The focusing effect similar to the one caused by track curvature may occur also in the case of a train accelerating along a straight line.

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