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H_∞ Voltage Control of a Direct High-Frequency Converter

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Abstract: Providing a secure power network is a demanding task but as network complexity is expected to grow with the connection of large amounts of distributed generation so the problem of integration, not just connection, of each additional generator becomes more protracted. A fundamental change to contemporary network architectures may eventually become necessary and this will provide new opportunities for power electronic converters to deliver advanced management and new power flow control features. Direct resonant converters (Dang 2005), could be used in novel devices such as the Active Transformers (Garlick 2008). The key to the successful exploitation of these devices will be their versatility, controllability and cost efficiency.

The Active Transformer model, Fig. 1, is an a.c. link system using high frequency, direct conversion and consists of a resonant, supply-side converter, a high frequency transformer and a resonant, load-side converter. The work presented in this paper compares the performance of the PI output voltage controller for step changes in load resistance (disturbance) with an alternative H_∞ controller design using the same linearised converter voltage model and then validating on the non-linear model of the converter.

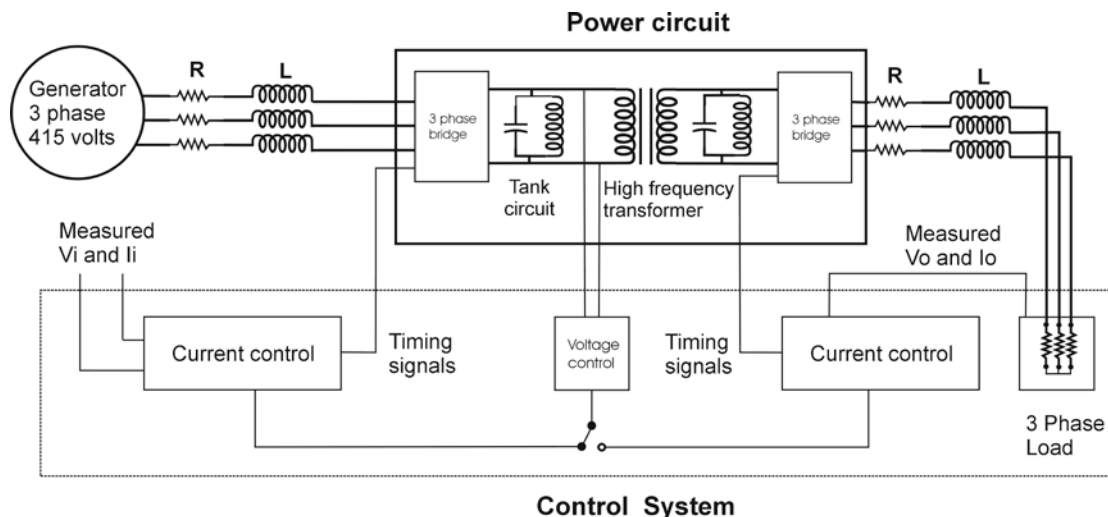


Fig. 1 Active transformer schematic diagram

Keywords: Converter control, high-frequency converter, H_∞ robust control, resonant converter.

1. INTRODUCTION

There are almost as many converter topologies as there are applications. However, the application of power electronic converter technologies in high-voltage power networks is a relatively recent development and includes such applications as High Voltage Direct Current (HVDC) or Flexible A.C. Transmission System (FACTS) devices. High-voltage direct converters that use silicon technologies are not very practicable or cost effective at Transmission or Distribution voltages but perhaps with the development of silicon carbide power devices a.c. link techniques, such as that described by Sood (1986), and direct conversion techniques may be more

realizable. This makes direct converter techniques, topologies, control and potential applications a relevant and interesting topic for research.

A model of a 5 kW prototype direct, Fig. 2, resonant converter design was used to demonstrate voltage control, (Dang 2005). The analysis and design of the output voltage controller was based on a linearised converter model operating at a full load condition; this design method is valid for small perturbations around the operating point, but in power systems applications the converter load will be changing continually and therefore, the converter must remain stable over a wide range of load conditions.

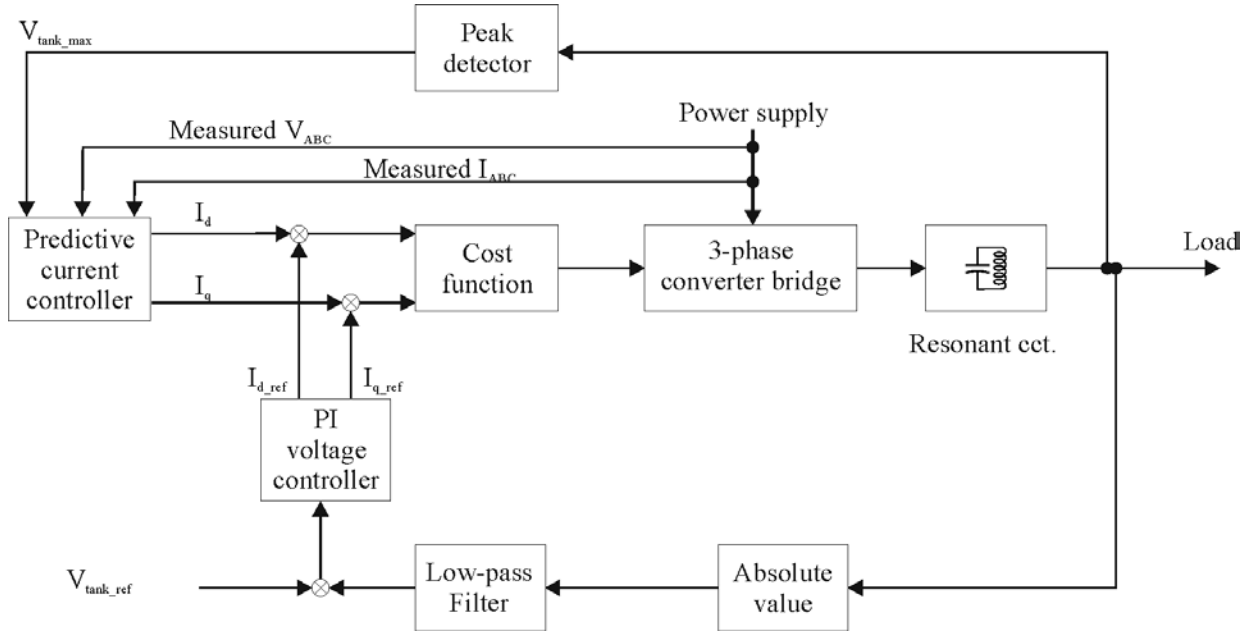


Fig. 2 Converter block diagram

2. LINEARISED VOLTAGE CONTROL MODEL

The voltage control loop was designed using a linearised converter transfer function, (2).

$$G(s) = \frac{\Delta V_{\text{tank_avg}}}{\Delta I_d} \quad (1)$$

$$G(s) = \frac{-\frac{3}{2} L_s I_d^* s + (\frac{3}{2} V_d - 3 R_s I_d^*)}{C V_{\text{tank_avg}}^* s + \frac{2 V_{\text{tank_avg}}^*}{R_L}} \quad (2)$$

where: s is the Laplace transform, $G(s)$ is the Converter transfer function (note: its non-minimum phase characteristics), L_s is the input inductance, R_s is the input resistance, V_d is the peak supply phase voltage in the d - q plane, C_{eq} is the resonant tank capacitance, R_L is the load resistance, $V_{\text{tank_avg}}^*$ is the nominal and $\Delta V_{\text{tank_avg}}$ is a small change of value of the average half-cycle tank voltage, I_d^* is the nominal, and ΔI_d is a small change of value of the input phase current in the d - q plane. Notably, the measured voltages and currents in (2) do not contain any dynamic terms. By inspection, (2) has an open loop pole that moves towards the origin of the s -plane for light loads, i.e. for increasing values of R_L , thus creating the possibility for marginal instability. A PI controller was designed using the Root Locus method, which resulted in the following controller transfer function:

$$K_{PI} = \frac{0.002s + 100}{s} \quad (3)$$

The effect of varying R_L , and hence I_d , in (2) is shown in the closed-loop step responses, but keeping the controller gains constant, Fig. 3. The large overshoot for the lightest load is of particular concern for power electronics designers as it may approach voltage breakdown levels. The likelihood of

instability at light loads is somewhat mitigated by the fact that the loop gain also reduces for smaller loads and the two effects tend to cancel each other to some extent, but the potential for instability remains. Thus, a controller, robust by design, with a lower overshoot at light loads is sought.

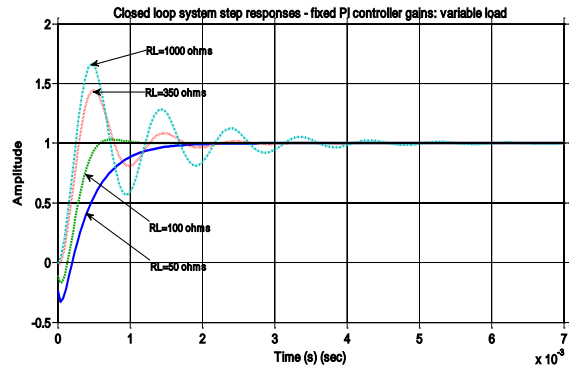


Fig. 3 Linearised closed-loop system step responses: R_L 50 – 1000 ohms

3. ROBUST STABILITY

3.1 Background

A robust control system is one that exhibits the required performance in the presence of significant uncertainty. A control system is robust when:

- i) there is low sensitivity to environmental effects
- ii) it is stable over the required range of parameters
- iii) its performance continues to meet specification in the presence of defined parameter variation.

In operation, power electronic converters are expected to cope with many uncertainties, not just variation in component parameters, but also in the environmental conditions, such as

wide variations in temperature, load and supply voltages, which are often unpredictable. The conventional approach is to over design and over rate the converter for its particular application to ensure that a robust performance is maintained over a wide range of conditions. Substantial test regimes are then used to verify performance, which tend to be costly and lengthy.

3.2 \mathcal{H}_∞ loop-shaping voltage control

This paper concentrates on \mathcal{H}_∞ loop-shaping, which is a modern design method that extends classical control design into a robust framework, (Skogestad 2005). The \mathcal{H}_∞ norm of a stable scalar transfer function $f(s)$ is the peak value of $|f(j\omega)|$:

$$\|f(s)\|_\infty = \max_{\omega} |f(j\omega)| \quad (4)$$

For a single-input-single-output system this is the system's frequency response. The design intent for the resonant converter was for a multivariable controller to control output voltage and input current and power factor, but in the first instance, control of the output voltage was developed to verify the effectiveness of the \mathcal{H}_∞ loop-shaping design procedure for this class of converters. Classical gain and phase margins are generally unreliable indications of stability in multivariable systems because simultaneous changes may be occurring in each loop. The step-by-step \mathcal{H}_∞ loop-shaping design procedure, Skogestad (2005) pages 371/2, was used to design a voltage controller, which uses a combination of loop-shaping and robust stabilization. Robust stabilization on its own is not very practicable because the designer is not able to specify performance requirements. The design process had two stages:

- i) the open-loop system was augmented by pre- and post-compensators to give the required open-loop frequency response or performance. In order to make suitable comparisons with 6, the pre-compensator or weighting function was chosen as the PI controller transfer function.
- ii) the resulting shaped system was robustly stabilized with respect to the general class of co-prime factor uncertainty using H_∞ optimisation.

4. MODELS AND INITIAL SIMULATIONS

4.1 Converter models

The control system design used the MATLAB[®] toolboxes and a linear model of the converter based on Equation (2), but with the addition of a low pass filter for the measurement of the peak converter output voltage. A non-linear model based on Fig. 1 was used to evaluate and compare controller designs.

4.2 Non-linear simulations

The non-linear model of the converter was also developed to better represent the switching behaviour of the converter and to include the predictive or sliding mode current control

shown in Fig. 2. The PI controller design from Dang (2005) was initially used to verify the non-linear model by comparing the results of a step change in load from 100 Ω to 50 Ω and a change of demand from 1000 V to 1300 V. A copy of the results from Dang (2005) are shown in Fig. 4 and compare well with the Simulink[®] model results, Fig. 5.

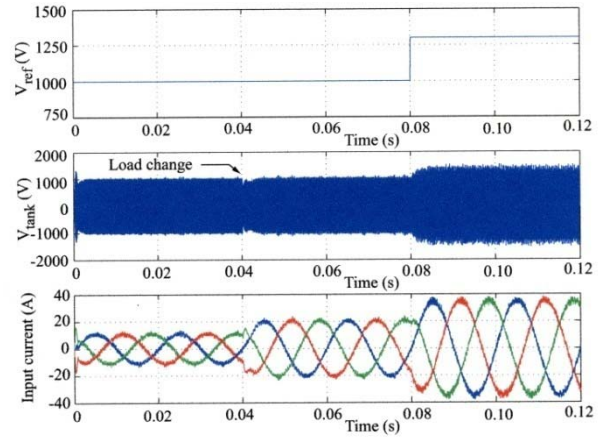


Fig. 4 Dang (2005) – PI controller: 50 – 100 Ω Load and 1000 – 1300 V demand changes

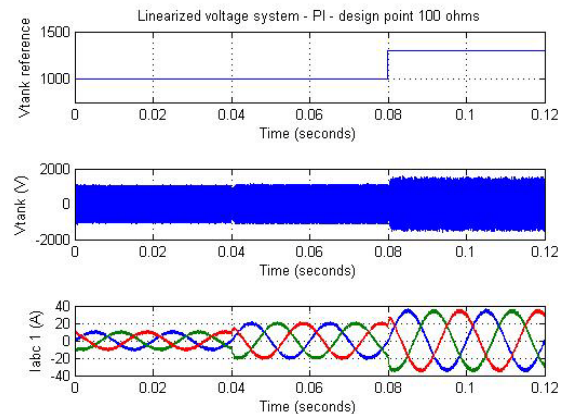


Fig. 5 Simulink[®] model results – PI controller: 50 – 100 Ω Load and 1000 – 1300 V demand changes

5. DESIGN OF AN H_∞ CONTROLLER FOR THE LINEARISED CONVERTER MODEL

5.1 Uncertainty modelling

An \mathcal{H}_∞ controller, designed using Glover and McFarlane's loop shaping approach, was chosen to provide improved robustness over that provided by a classical PI controller. One measure of the robustness of a control system is its insensitivity to differences between the model used for analysis and control design and the real system, or to paraphrase, its uncertainty model. Its robustness properties, in dealing with the uncertainties of the converter load that appear in the linearised transfer function of the converter, are a key consideration in choosing an appropriate control methodology.

Real systems contain frequency-dependant or dynamic uncertainty where the model lacks unknown system dynamics

or lacks a true understanding of the detailed system characteristics. A power converter when connected to a varying load is a good example of such a type of model uncertainty. The uncertain load generates an uncertain pole and hence an uncertain supply current term in the transfer functions.

Multiplicative (relative) uncertainty is often preferred and is expressed as:

$$l_I(\omega) = \frac{\max_{G_p \in \Pi} |G_p(j\omega) - G(j\omega)|}{|G(j\omega)|} \quad (5)$$

With a rational weight:

$$|\omega_I(j\omega)| \geq l_I(\omega), \forall \omega \quad (6)$$

where l_I is the value of the relative errors of all possible systems as a function of ω , G_p is the transfer function of the perturbed system, G is the transfer function of the original system. Equation 6 then represents a weight, or desired transfer function, that embraces all the family of possible systems due to the uncertainty.

Converter control systems are often designed at their full load point, because this is where they are most efficient and at which they are most likely to be operated. The control system is then tuned to give good performance at full load and an acceptable performance at a range of lower loads. The converter to be used in the Active Transformer must be able to provide robust performance across a wide range of loads to the choice of design point is not so clear, so designs were evaluated for 100 and 10% load.

The choice of a design point at 10% load was an arbitrary one, being of sufficiently low resistance to demonstrate a large step change in load. In a practical design, an analysis with the real load would be necessary to provide a more optimum design point or a limit to load excursions.

For multiplicative uncertainty in the converter, the relative errors response between the nominal and the perturb system transfer functions is shown in Fig. 6 for the full load, 100 Ω , and a light load, 1000 Ω conditions. The converter control system was designed using positive feedback and, for the nominal system designed at full load, the relative errors response lies above 0 dB for low frequencies. This indicates that an integrator in the \mathcal{H}_∞ weighting function, WI , is not appropriate, because when closing the control loop, the system would have positive gain at these frequencies. Alternatively, with the nominal system designed at 10% load, the relative errors response lies below 0 dB for low frequencies, indicating an integrator in the \mathcal{H}_∞ weighting function, WI , is feasible. For the converter controller, integral action is required in order to provide zero error between the converter output voltage and reference.

Skogestad (2005) suggests a better alternative approach, inverse multiplicative uncertainty, for cases of pole uncertainty, as is the case of the converter. A system with inverse multiplicative uncertainty may be represented by:

$$G(p) = G(s)(1 + \omega_{il}(s)\Delta_{il}(s))^{-1}; |\Delta_{il}(j\omega)| \leq \forall \omega \quad (7)$$

Where $G_p(s)$ is the perturb model and $\Delta_{il}(s)$ is any stable transfer function where its magnitude at any frequency is ≤ 1 . ω_{il} is the magnitude of the system uncertainty expressed at each frequency. The subscript I denotes ‘‘input’’, but for a SISO system the perturbation may be considered at the system input or output. The condition for robust stability with inverse multiplicative uncertainty gives an upper bound:

$$|S| < \frac{1}{|\omega_{il}|}, \quad \forall \omega \quad (8)$$

where S is the system sensitivity function. A family of inverse multiplicative responses exist for the range of converter loads. The response for 100 and 10% load (100 and 1000 ohms) cases are shown in Fig. 7. At frequencies where the uncertainty is large and $|\omega_{il}|$ is greater than 1, the system sensitivity must be made small.

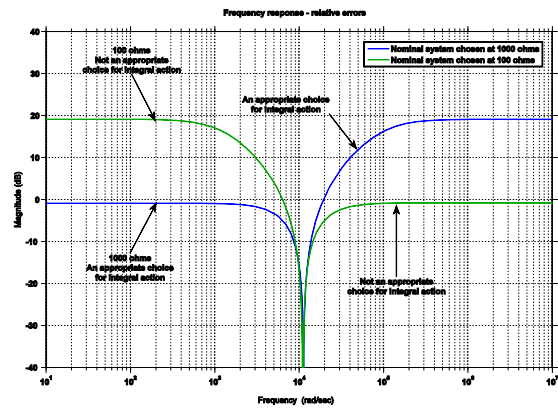


Fig. 6 System uncertainty responses for 1000 and 100 ohm loads

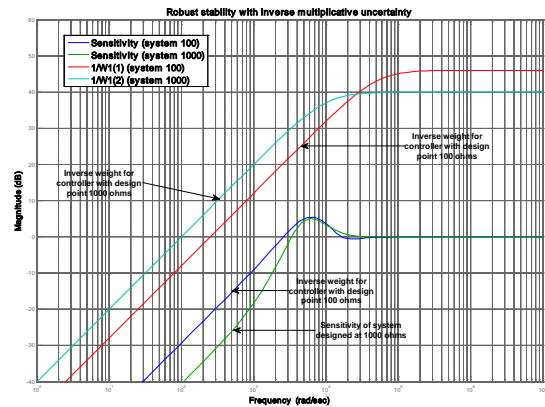


Fig. 7 Inverse multiplicative uncertainty responses for 1000 and 100 ohms designs

For a system such as the power converter, this is not always possible because of the right-hand-plane-zero (RHP-zero), which constrains $S = I$ and therefore requires that, $|\omega_{il}| \leq 1$. The result is that there cannot be large pole uncertainty where a system has a RHP-zero. In the case of the converter, this requirement is met as the sensitivity of the closed-loop system is always less than $1/|\omega_{il}|$.

These results indicate that an \mathcal{H}_∞ controller, with integral action, designed for a 10% load will be robust and not experience the problems of the controller designed at 100% load.

5.2 \mathcal{H}_∞ controller design

The H_∞ design procedure shapes the converter response using pre- and post-filters. The shaped system transfer function $G(s)$ is shown in Fig. 8a, and the system implementation, with appropriate weighting functions absorbed into the controller, is shown in Fig. 8b. The designer adjusts $W1$ and $W2$ for the required system performance.

This is a rather arbitrary task. $W1$ should contain all the dynamic shaping and the logical shaping to use was the PI gains from the controller as these produced a stable system at

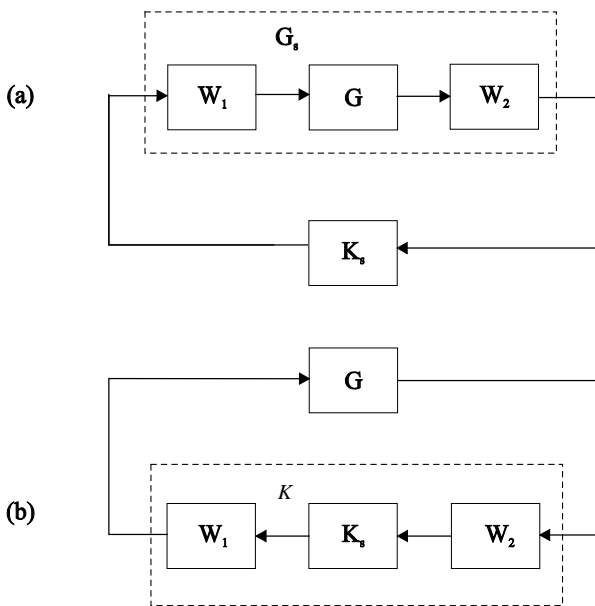


Fig. 8 Shaped system and controller

full load. Their use also was useful in making comparisons with original system performance. For a single input/single output system, $W_2=1$.

The resulting H_∞ controller transfer function at 10% load design point was:

$$K = \frac{-8401s^3 - 1.36 \times 10^7 s^2 - 6.15 \times 10^{10} s - 9.32 \times 10^{13}}{s^4 + 4.38 \times 10^4 s^3 + 5.84 \times 10^8 s^2 + 2.80 \times 10^{12}} \quad (9)$$

This is recognised as a more complex controller than the PI controller, (3) and the final controller design, K_s , is shown in Fig. 8b.

The loop shapes were plotted and are shown in Fig. 9. With the 10% load controller design, the cross-over frequency of the loop is 4.5 krads/s and was close to that of the shaped system as required by the design procedure. The angle of the response at cross-over is low indicating good bandwidth, gain and phase margins. The corresponding step response, Fig. 10, shows a stable system in both cases at 100Ω load, but with greater damping, and slower rise times, at all loads for

the \mathcal{H}_∞ controller. So the cost of damping the light load oscillations has led to a slower converter response when compared to the PI controller.

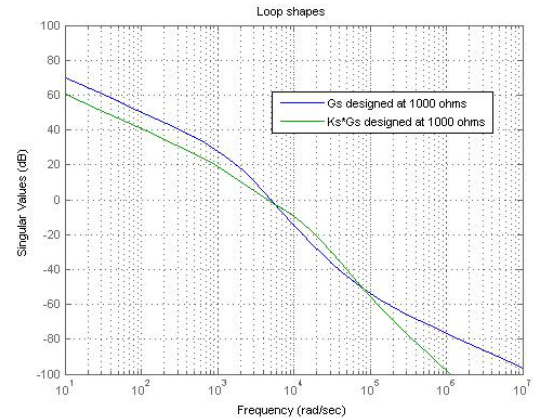


Fig. 9 Converter model (design point 1 kΩ) – loop shapes

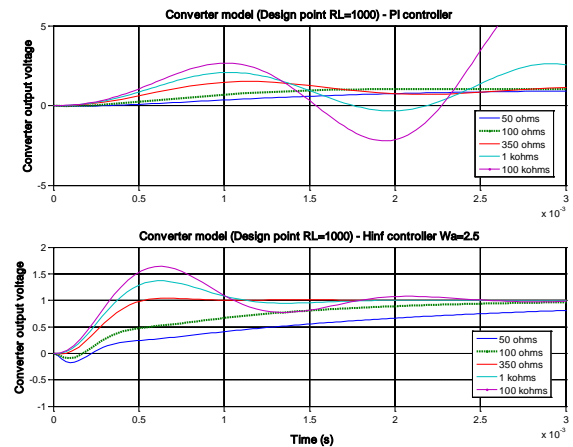


Fig. 10 Converter model (design point 1 kΩ) –step responses PI and \mathcal{H}_∞

5.3 Non-linear modelling

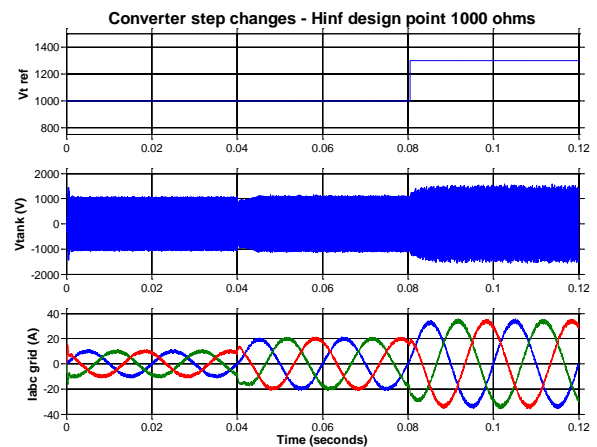


Fig. 11 Simulink® model - \mathcal{H}_∞ controller: 50 – 100 Ω Load and 1000 – 1300 V demand changes

The \mathcal{H}_∞ controller design was then used in the non-linear converter model and the previous step response tests repeated, Fig. 11. The simulation was started from zero initial conditions, which resulted in the initial variation in responses. As the control became effective, the converter output stabilized. With the load change at 40 ms the output voltage is well controlled prior to and after the event. After the step change to 1300 V the converter output voltage peaks are a little more irregular, but the output voltage is still controlled.

As a test of robustness, the PI and \mathcal{H}_∞ controllers models were subjected to a step change of load from full load, 100 Ω , to a light load, 1000 Ω , which is equivalent to 100 -10% load change. Figs 12 and 13 show the converter output voltage and supply currents for each controller. Both controllers produced satisfactory steady state control at full load but after the change of load, but the \mathcal{H}_∞ control also continued to control the output voltage at 10% load whereas the PI control converter is clear unstable.

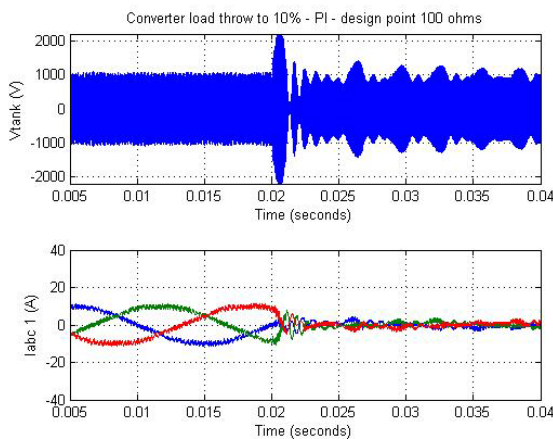


Fig. 12 Non-linear model - PI controller: step responses from R_L 100 Ω to 1000 Ω

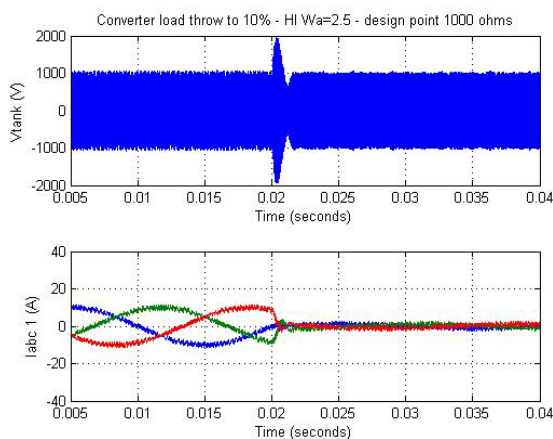


Fig. 13 Non-linear system model - H_∞ controllers Design point $R_L=1000\Omega$, step 100 -10% load

The step change also produced an initial over-shoot, as did the linear analysis, in excess of 200%, which would necessitate de-rating and over voltage protection for the

power devices. Further investigation is required to identify and limit the overshoot to less than 20%.

6. SUMMARY

The PI controller proved effective at full load conditions, but ineffective for large step changes in load. The H_∞ controller was effective at both full and 10% loads but at a cost of a more damped response. However, an initial overshoot of 200% remained. This feature is related to the non-minimum phase value of the system and is the subject of the next phase or the research.

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