

Vortex and critical fields in charged Bose liquids and unconventional superconductors

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Abstract

A single vortex in the charged Bose gas (CBG) has a charged core and its profile different from the vortex in neutral and BCS superfluids. Lower and upper critical fields of CBG are discussed. The unusual resistive upper critical field, $H_{c2}(T)$, of many cuprates and a few other unconventional superconductors is described as the Bose-Einstein condensation field of preformed bosons-bipolarons. Its nonlinear temperature dependence follows from the scaling arguments. Exceeding the Pauli paramagnetic limit is explained. Controversy in the determination of $H_{c2}(T)$ of cuprates from kinetic and thermodynamic measurements is addressed in the framework of the bipolaron theory.

Key words: vortex, critical fields, bipolarons, cuprates

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Introduction

The seminal work by Bardeen, Cooper and Schrieffer [1] taken further by Eliashberg [2] to the intermediate coupling solved one of the major problems in Condensed Matter Physics. High-temperature superconductors present a challenge to the conventional theory. While the BCS theory provides a qualitatively correct description of some novel superconductors like magnesium diboride

and doped fullerenes (if the phonon dressing of carriers, i.e. polaron formation is properly taken into account), cuprates remain a problem. Here strong antiferromagnetic and charge fluctuations and the Fröhlich and Jahn-Teller electron-phonon interactions have been identified as an essential piece of physics. In particular, experimental [3,4,5,6,7,8,9,10] evidence for an exceptionally strong electron-phonon interaction in all high temperature superconductors is now overwhelming. Our view, which we discussed in detail elsewhere [11] is that the extension of the BCS theory towards the strong interaction

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between electrons and ion vibrations describes the phenomenon naturally. The high temperature superconductivity exists in the crossover region of the electron-phonon interaction strength from the BCS-like to bipolaronic superconductivity as was predicted before [12], and explored in greater detail after the discovery [13,14,15,16]. The low energy physics in this regime is that of a charged Bose gas of small bipolarons, which are real-space bosons dressed by phonons. They are itinerant quasiparticles existing in the Bloch states at temperatures below the characteristic phonon frequency. Here I review the bipolaron theory of the vortex state.

1 Charged vortex

CBG is an extreme type II superconductor, as shown below. We can analyse a single vortex in CBG and calculate the critical fields by solving a stationary equation for the macroscopic condensate wave function $\psi_s(\mathbf{r})$ [17],

$$\left[\frac{[\nabla + 2ie\mathbf{A}(\mathbf{r})]^2}{2m^{**}} + \mu \right] \psi_s(\mathbf{r}) = \frac{4e^2}{\epsilon_0} \int d\mathbf{r}' \frac{|\psi_s(\mathbf{r}', t)|^2 - n_b}{|\mathbf{r} - \mathbf{r}'|} \psi_s(\mathbf{r}). \quad (1)$$

Subtracting n_b in the integral of Eq.(1) explicitly takes into account the Coulomb interaction with the homogeneous charge background of the same density as the density of charged bosons n_b . Here $2e$ and m^{**} are the charge per boson and the effective mass, respectively, and

$$\hbar = c = k_B = 1).$$

The integra-differential equation (1) is quite different from the Ginsburg-Landau [18] and Gross-Pitaevskii [19] equations, describing the vortex in the BCS and neutral superfluids, respectively. While CBG shares the quantum coherence with the BCS superconductors and neutral superfluids owing to the Bose-Einstein condensate (BEC), the long-range (nonlocal) interaction leads to some peculiarities. In particular, the vortex is charged in CBG, and the coherence length is just the same as the screening radius.

Indeed, introducing dimensionless quantities $f = |\psi_s|/n_b^{1/2}$, $\rho = \mathbf{r}/\lambda(0)$, and $\mathbf{h} = 2e\xi(0)\lambda(0)\nabla \times \mathbf{A}$ for the order parameter, length and magnetic field, respectively, Eq.(1) and the Maxwell equations take the following form:

$$\frac{1}{\kappa^2 \rho} \frac{d}{d\rho} \rho \frac{df}{d\rho} - \frac{1}{f^3} \left(\frac{dh}{d\rho} \right)^2 - \phi f = 0, \quad (2)$$

$$\frac{1}{\kappa^2 \rho} \frac{d}{d\rho} \rho \frac{d\phi}{d\rho} = 1 - f^2, \quad (3)$$

$$\frac{1}{\rho} \frac{d}{d\rho} \rho \frac{dh}{f^2 d\rho} = h. \quad (4)$$

A new feature compared with the GL equations for a single vortex [20] is the electric field potential determined as

$$\phi = \frac{1}{2e\phi_c} \int d\mathbf{r}' V(\mathbf{r} - \mathbf{r}') \times [|\psi_s(\mathbf{r}')|^2 - n_b] \quad (5)$$

with a new fundamental unit $\phi_c =$

$em^{**}\xi(0)^2$. The potential is calculated using the Poisson equation (3). At $T = 0$ the coherence length is the same as the screening radius,

$$\xi(0) = (2^{1/2}m^{**}\omega_{ps})^{-1/2}, \quad (6)$$

and the London penetration depth is

$$\lambda(0) = \left(\frac{m^{**}}{16\pi n_b e^2} \right)^{1/2}. \quad (7)$$

Here $\omega_{ps} = [16\pi e^2 n_b / (\epsilon_0 m^{**})]^{1/2}$ is the plasma frequency. There are now six boundary conditions in a single-vortex problem. Four of them are the same as in the BCS superconductor [20], $h = dh/\rho = 0$, $f = 1$ for $\rho = \infty$ and the flux quantization condition, $dh/d\rho = -pf^2/\kappa\rho$ for $\rho = 0$, where p is an integer. The remaining two conditions are derived from the global charge neutrality, $\phi = 0$ for $\rho = \infty$ and

$$\phi(0) = \int_0^\infty \rho \ln(\rho)(1 - f^2)d\rho \quad (8)$$

for the electric field at the origin, $\rho = 0$. We notice that the chemical potential μ is zero at any point in the thermal equilibrium.

CBG is an extreme type II superconductor with a very large Ginsburg-Landau parameter, $\kappa = \lambda(0)/\xi(0) \gg 1$. Indeed, with the material parameters typical for oxides, such as $m^{**} = 10m_e$, $n_b = 10^{21}cm^{-3}$ and the static dielectric constant $\epsilon_0 = 10^3$ we obtain $\xi(0) \simeq 0.48nm$, $\lambda(0) \simeq 265nm$, and the Ginsburg-Landau ratio $\kappa \simeq 552$. Owing to a large dielectric con-

stant the Coulomb repulsion remains weak even for heavy bipolarons,

$$r_s = \frac{4m^{**}e^2}{\epsilon_0(4\pi n_b/3)^{1/3}} \simeq 0.46. \quad (9)$$

If $\kappa \gg 1$, Eq.(4) is reduced to the London equation with the familiar solution $h = pK_0(\rho)/\kappa$, where $K_0(\rho)$ is the Hankel function of imaginary argument of zero order. For the region $\rho \leq p$, where the order parameter and the electric field differ from unity and zero, respectively, we can use the flux quantization condition to “integrate out” the magnetic field in Eq.(2). That leaves us with two parameter-free equations written for $r = \kappa\rho$ as

$$\frac{1}{r} \frac{d}{dr} r \frac{df}{dr} - \frac{p^2 f}{r^2} - \phi f = 0, \quad (10)$$

and

$$\frac{1}{r} \frac{d}{dr} r \frac{d\phi}{dr} = 1 - f^2. \quad (11)$$

They are satisfied by regular solutions of the form $f = c_p r^p$ and $\phi = \phi(0) + (r^2/4)$, when $r \rightarrow 0$. The constants c_p and $\phi(0)$ are determined by complete numerical integration of Eqs.(10) and (11). The numerical results for $p = 1$ are $c_1 \simeq 1.5188$ and $\phi(0) \simeq -1.0515$.

In the region $p \ll r < p\kappa$ the solutions are $f = 1 + (4p^2/r^4)$ and $\phi = -p^2/r^2$. In this region f differs qualitatively from the BCS order parameter, $f_{BCS} = 1 - (p^2/r^2)$ [20]. The difference is due to a local charge redistribution caused by the

magnetic field in CBG. Quite different from the BCS superconductor, where the total density of electrons remains constant across the sample, CBG allows for flux penetration by redistributing the density of bosons within the coherence volume. This leads to an increase of the order parameter compared with the homogeneous case ($f = 1$) in the region close to the vortex core. Inside the core the order parameter is suppressed, as in the BCS superconductor. The resulting electric field, (together with the magnetic field) acts as an additional centrifugal force increasing the steepness (c_p) of the order parameter compared with the BCS superfluid, where $c_1 \simeq 1.1664$.

The breakdown of the local charge neutrality is due to the absence of any equilibrium *normal* state solution in CBG below $H_{c2}(T)$ line. Both superconducting ($\Delta_{\mathbf{k}} \neq 0$) and normal ($\Delta_{\mathbf{k}} = 0$) solutions are allowed at any temperature in the BCS superconductors. Then the system decides which of two phases (or their mixture) is energetically favorable, but the local charge neutrality is respected. In contrast, there is no equilibrium normal state solution (with $\psi_s = 0$) in CBG below $H_{c2}(T)$ -line because it does not respect the density sum rule. Hence, there are no different phases to mix, and the only way to acquire a flux in the thermal equilibrium is to redistribute the local density of bosons at the expense of their Coulomb energy. This energy determines the vortex free energy $F = E_v - E_0$, which is the difference of the energy of CBG with, E_v , and without, E_0 , magnetic flux,

$$F = \int d\mathbf{r} \frac{1}{2m^*} |\nabla + 2ie\mathbf{A}(\mathbf{r})\psi_s(\mathbf{r})|^2 + e\phi_c\phi[|\psi_s(\mathbf{r})|^2 - n_b] + \frac{(\nabla \times \mathbf{A})^2}{8\pi}.$$

Using Eqs.(2), (3) and (4) it can be written in the dimensionless form as

$$F = 2\pi \int_0^\infty [h^2 - \frac{1}{2}\phi(1 + f^2)]\rho d\rho. \quad (12)$$

In the large κ limit the main contribution comes from the region $p/\kappa < \rho < p$, where $f \simeq 1$ and $\phi \simeq -p^2/(\kappa^2\rho^2)$. The energy is thus the same as that in the BCS superconductor, $F \simeq 2\pi p^2 \ln(\kappa)/\kappa^2$. It is seen that the most stable solution is the formation of the vortex with one flux quantum, $p = 1$, and the lower critical field is the same as in the BCS superconductor, $h_{c1} \approx \ln \kappa/(2\kappa)$ [20]. However, different from the BCS superconductor, where the Ginsburg-Landau phenomenology is microscopically justified in the temperature region close to T_c , the CBG vortex structure is derived here in the low temperature region. Actually the zero temperature solution is applied in a wide temperature region well below the Bose-Einstein condensation temperature, where the depletion of the condensate remains small. The actual size of the charged core is about 4ξ .

2 Upper critical field in the strong-coupling regime

If we “switch off” the Coulomb repulsion between bosons, an ideal

CBG cannot be bose-condensed at finite temperatures in a homogeneous magnetic field because of a one-dimensional particle motion at the lowest Landau level [21]. However, an interacting charged Bose-gas is condensed in a field lower than a certain critical value $H_{c2}(T)$ [22]. Collisions between bosons and/or with impurities and phonons make the motion three-dimensional, and eliminate the one-dimensional singularity of the density of states, which prevents BEC of the ideal gas in the field. As we show below the upper critical field of CBG differs significantly from $H_{c2}(T)$ of BCS superconductors. It has an unusual positive curvature near T_c , $H_{c2}(T) \sim (T_c - T)^{3/2}$ and diverges at $T \rightarrow 0$, if there is no localisation due to a random potential. The localization can drastically change the low-temperature behavior of $H_{c2}(T)$, so that at high density of impurities a re-entry effect to the normal state might occur.

In line with the conventional definition, $H_{c2}(T)$ is a field, where a first nonzero solution of the linearized stationary equation for the macroscopic condensate wave function occurs,

$$\left[\frac{1}{2m^{**}} [\nabla - 2ie\mathbf{A}(\mathbf{r})]^2 + \mu \right] \psi_s(\mathbf{r}) = V_{scat}(\mathbf{r}) \psi_s(\mathbf{r}). \quad (13)$$

Here we introduce the “scattering” potential $V_{scat}(\mathbf{r})$ caused, for example, by particle-particle and/or particle-impurity collisions. Let us first discuss noninteracting bosons, $V_{scat}(\mathbf{r}) = 0$. Their energy spectrum

in the homogeneous magnetic field is

$$\varepsilon_n = \omega(n + 1/2) + \frac{k_z^2}{2m^{**}}, \quad (14)$$

where $\omega = 2eH_{c2}/m^{**}$ and $n = 0, 1, 2, \dots, \infty$. BEC occurs when the chemical potential “touches” the lowest band edge from below, i.e. $\mu = \omega/2$. Hence, quite different from the GL equation, the Schrödinger equation (13) does not allow for a direct determination of H_{c2} . In fact, it determines the value of the chemical potential. Then using this value the upper critical field is found from the density sum rule,

$$\int_{E_c}^{\infty} f(\epsilon) N(\epsilon, H_{c2}) d\epsilon = n_b, \quad (15)$$

where $N(\epsilon, H_{c2})$ is the density of states (DOS) of the Hamiltonian, Eq.(13), $f(\epsilon) = [\exp(\epsilon - \mu)/T - 1]^{-1}$ is the Bose-Einstein distribution function, and E_c is the lowest band edge. For ideal bosons we have $\mu = E_c = \omega/2$ and

$$N(\epsilon, H_{c2}) = \frac{\sqrt{2}(m^{**})^{3/2}\omega}{4\pi^2} \times \Re \sum_{n=0}^{\infty} \frac{1}{\sqrt{\epsilon - \omega(n + 1/2)}}.$$

Substituting this DOS into Eq.(15) yields

$$\frac{\sqrt{2}(m^{**})^{3/2}\omega}{4\pi^2} \int_0^{\infty} \frac{dx}{x^{1/2} \exp(x/T) - 1} = n_b - \tilde{n}(T), \quad (16)$$

where

$$\tilde{n}(T) = \frac{\sqrt{2}(m^{**})^{3/2}\omega}{4\pi^2} \int_0^\infty \frac{dx}{\exp(x/T) - 1} \times \sum_{n=1}^\infty \frac{1}{\sqrt{x - \omega n}} \quad (17)$$

is the number of bosons occupying the levels from $n = 1$ to $n = \infty$. This number is practically the same as in zero field, $\tilde{n}(T) = n_b(T/T_c)^{3/2}$, if $\omega \ll T_c$. On the contrary, the number of bosons on the lowest level, $n = 0$, is given by a divergent integral on the left-hand side of Eq.(18). Hence the only solution to Eq.(16) is $H_{c2}(T) = 0$.

The scattering of bosons effectively removes the one-dimensional singularity in $N_0(\epsilon, H_{c2}) \propto \omega(\epsilon - \omega/2)^{-1/2}$ leading to a finite DOS near the bottom of the lowest level,

$$N_0(\epsilon, H_{c2}) \propto \frac{H_{c2}}{\sqrt{\Gamma_0(H_{c2})}}. \quad (18)$$

Using the Fermi-Dirac golden rule the collision broadening of the lowest level $\Gamma_0(H_{c2})$ is proportional to the same DOS

$$\Gamma_0(H_{c2}) \propto N_0(\epsilon, H_{c2}), \quad (19)$$

so that Γ_0 scales with the field as $\Gamma_0(H_{c2}) \propto H_{c2}^{2/3}$. Then the number of bosons at the lowest level is estimated as

$$n_0 = \frac{\sqrt{2}(m^{**})^{3/2}\omega}{4\pi^2} \int_{\Gamma_0}^\infty \frac{dx}{x^{1/2} \exp(x/T) - 1} \propto TH_{c2}^{2/3}, \quad (20)$$

as long as $T \gg \Gamma_0$. Here we apply the one-dimensional DOS, but cut the in-

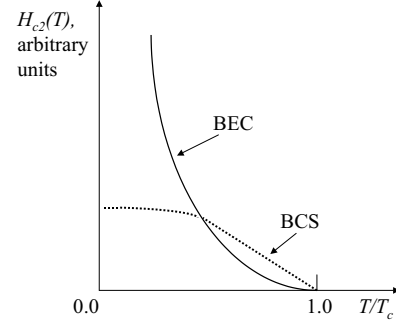


Fig. 1. Upper critical field of CBG compared with $H_{c2}(T)$ of BCS superconductors.

tegral at Γ_0 from below. Finally we arrive at

$$H_{c2}(T) = H_0(t^{-1} - t^{1/2})^{3/2}, \quad (21)$$

where $t = T/T_c$, and H_0 is a temperature independent constant. The scaling constant H_0 depends on the scattering mechanism. If we write $H_0 = \Phi_0/(2\pi\xi_0^2)$, then the characteristic length is

$$\xi_0 \approx \left(\frac{l}{n_b}\right)^{1/4}, \quad (22)$$

where l is the zero-field mean-free path of low energy bosons.

The upper critical field has a nonlinear behaviour,

$$H_{c2}(T) \propto (T_c - T)^{3/2},$$

in the vicinity of T_c , and diverges at low-temperatures as

$$H_{c2}(T) \propto T^{-3/2}.$$

These simple scaling arguments are fully confirmed by DOS calculations with impurity [22] and boson-boson [23] scattering. The “coherence” length ξ_0 of CBG, Eq.(22), depends on the mean free path l and the inter-particle distance $n_b^{-1/3}$. It has nothing to do with the size of the bipolaron, and could be as large as the coherence length of the weak-coupling BCS superconductors.

Thus $H_{c2}(T)$ of strongly-coupled superconductors has a “3/2” curvature near T_c different from the linear BCS $H_{c2}(T)$. The curvature is a universal feature of CBG, which does not depend on a particular scattering mechanism and on approximations made. Another interesting feature of strongly-coupled superconductors is a breakdown of the Pauli paramagnetic limit given by $H_p \simeq 1.84T_c$ in the weak-coupling theory. $H_{c2}(T)$ of bipolarons exceeds this limit because the singlet bipolaron binding energy Δ is much larger than their T_c . Bosons are condensed at $T = 0$ no matter what their energy spectrum is. Hence, in the charged Bose-gas model, $H_{c2}(0) = \infty$, Fig.1. For composed bosons, like bipolarons, the pair-breaking limit is given by $\mu_B H_{c2}(0) \approx \Delta$, so that $H_{c2}(0) \gg H_p$.

3 Universal upper critical field of unconventional superconductors

In cuprates [24,25,26,27,28,29,30], spin-ladders [31] and organic superconductors [32] high magnetic field studies revealed a non-BCS upward

curvature of resistive $H_{c2}(T)$. When measurements were performed on low- T_c unconventional superconductors [25,26,28,31,32], the Pauli limit was exceeded by several times. A non-linear temperature dependence in the vicinity of T_c was unambiguously observed in a few samples [27,28,29,30]. Importantly, a thermodynamically determined H_{c2} turned out much higher than the resistive H_{c2} [33] due to contrasting magnetic field dependencies of the specific heat anomaly and of resistive transition.

We believe that many unconventional superconductors are in the ‘bosonic’ limit of preformed real-space bipolarons, so their resistive H_{c2} is actually a critical field of the Bose-Einstein condensation of charged bosons [22]. Calculations carried out for the heat capacity of CBG (see below) lead to the conclusion that the resistive H_{c2} and the thermodynamically determined H_{c2} are very different in bosonic superconductors. While the magnetic field destroys the condensate of ideal bosons, it hardly shifts the specific heat anomaly as observed.

A comprehensive scaling of resistive H_{c2} measurements in unconventional superconductors is shown in Fig.2 [30] in the framework of the microscopic model of charged bosons scattered off impurities (section 2). Generalised Eq.(21) accounting for a temperature dependence of the number of delocalised bosons, $n_b(T)$, can be written as

$$H_{c2}(T) = H_0 \left[\frac{n_b(T)}{tn_b(T_c)} - t^{1/2} \right]^{3/2}. \quad (23)$$

In the vicinity of T_c one obtains the parameter-free $H_{c2}(T) \propto (1 - t)^{3/2}$ using Eq.(23), but the low-temperature behaviour depends on a particular scattering mechanism, and a detailed structure of the density of localised states. As suggested by the normal state Hall measurements in cuprates $n_b(T)$ can be parameterised as $n_b(T) = n_b(0) + \text{constant} \times T$ [34], so that $H_{c2}(T)$ is described by a single-parameter expression as

$$H_{c2}(T) = H_0 \left[\frac{b(1-t)}{t} + 1 - t^{1/2} \right]^{3/2}. \quad (24)$$

The parameter b is proportional to the number of delocalised bosons at zero temperature. We expect that this expression is applied in the whole temperature region except ultra-low temperatures, where the Fermi Golden-rule in the scaling fails. Exceeding the Pauli pair-breaking limit readily follows from the fact, that the singlet-pair binding energy is related to the normal-state pseudogap temperature T^* , rather than to T_c . T^* is higher than T_c in bosonic superconductors, and cuprates.

The universal scaling of H_{c2} near T_c is confirmed by resistive measurements of the upper critical field of many cuprates, spin-ladders, and organic superconductors, as shown in Fig.2. All measurements reveal a universal $(1 - t)^{3/2}$ behaviour in a wide temperature region (inset), when they are fitted by Eq.(24). The low-temperature behaviour of $H_{c2}(T)/H_0$ is not universal, but well described using the same equation with the single fitting parameter, b .

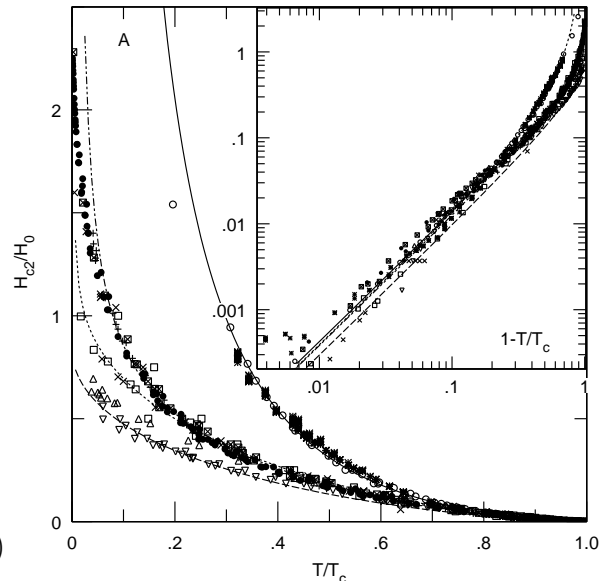


Fig. 2. Resistive upper critical field (determined at 50% of the transition) of cuprates, spin-ladders and organic superconductors scaled according to Eq.(24). The parameter b is 1 (solid line), 0.02 (dashed-dotted line), 0.0012 (dotted line), and 0 (dashed line). The inset shows a universal scaling of the same data near T_c on the logarithmic scale. Symbols correspond to $Tl - 2201$ (\bullet), $LSCO$ (Δ), $Bi - 2201$ (\times), $Bi - 2212$ ($*$), $YBCO$ (\circ), $La_{2-x}Ce_xCuO_{4-y}$ (\square), $Sr_2Ca_{12}Cu_{24}O_{41}$ ($+$), and $(TMTSF)_2PF_6$ (∇)

The parameter is close to 1 in high quality cuprates with a very narrow resistive transition [29]. It naturally becomes rather small in overdoped cuprates where randomness is more essential, so almost all bosons are localised (at least in one dimension) at zero temperature.

4 Specific heat anomaly in CBG

Bose liquids (or more precisely He^4) show the characteristic λ -point singularity of their specific heat, but superfluid Fermi liquids like BCS superconductors exhibit a sharp second order phase transition accompanied by a finite jump in the specific heat. It was established beyond doubt [35,36,37,38,39] that the anomaly in high T_c cuprates differs qualitatively from the BSC prediction. As was stressed by Salamon et al.[40] the heat capacity is logarithmic near the transition, and consequently, cannot be adequately treated by the mean-field BCS theory even including the gaussian fluctuations. In particular, estimates using the gaussian fluctuations yield an unusually small coherence volume [36], and Gi number of the order of one.

The magnetic field dependence of the anomaly [41] is also unusual, but it can be described by the bipolaron model [42,30]. Calculations of the specific heat of charged bosons in a magnetic field require an analytical DOS, $N(\epsilon, B)$ of a particle, scattered by other particles and/or by a random potential of impurities. We can use DOS in the magnetic field with an impurity scattering, which allows for an analytical result [30]. The specific heat coefficient

$$\frac{C(T, B)}{T} = \frac{d}{TdT} \int d\epsilon \frac{N(\epsilon, B)\epsilon}{\exp[(\epsilon - \mu)/T] - 1}$$

calculated with this DOS and with μ determined from $n_b = \int d\epsilon N(\epsilon, B)f(\epsilon)$

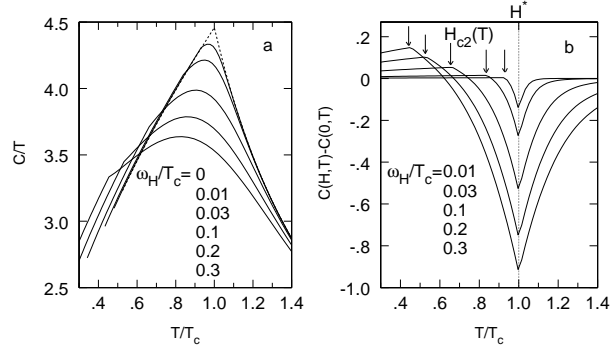


Fig. 3. Temperature dependence of the specific heat divided by temperature (arb. units) of the charged Bose-gas scattered off impurities for several fields ($\omega_H = 2eB/m^{**}$). Fig. 3b shows two anomalies, the lowest one traces resistive transition, while the highest anomaly is the normal state feature.

is shown in Fig.3.

The broad maximum at $T \approx T_c$ is practically the same as in the ideal Bose gas without scattering [42]. It barely shifts in the magnetic field. However, there is the second anomaly at lower temperatures, which is absent in the ideal gas. It shifts with the magnetic field, tracing precisely the resistive transition, as clearly seen from the difference between the specific heat in the field and zero-field curve, Fig. 3b. The specific heat, Fig. 3, is in striking resemblance with the Geneva group's experiments on $DyBa_2Cu_3O_7$ and on $YBa_2Cu_3O_7$ [41], where both anomalies were observed. Within the bipolaron model, when the magnetic field is applied, it hardly changes the temperature dependence of the chemical potential near the zero field T_c because the energy spectrum of thermally excited bosons is practically unchanged. That is because their characteristic energy (of the

order of T_c) remains huge compared with the magnetic energy of the order of $2eB/m^{**}$. In contrast, the energy spectrum of low energy bosons is strongly perturbed even by a weak magnetic field. As a result the chemical potential ‘touches’ the band edge at lower temperatures, while having almost the same ‘kink’-like temperature dependence around T_c as in zero field. While the lower anomaly corresponds to the true long-range order due to the Bose-Einstein condensation, the higher one is just a ‘memory’ about the zero-field transition. This microscopic consideration shows that a genuine phase transition into a superconducting state is related to resistive transition and to the lower specific heat anomaly, while the broad higher anomaly is the normal state feature of the bosonic system in the external magnetic field. Different from the BCS superconductor these two anomalies are well separated in the bosonic superconductor at any field but zero.

In conclusion, the bipolaron theory of the critical fields and vortex structures in strong-coupling superconductors has been reviewed. A single vortex in this regime has a charged core and its profile different from the vortex in neutral and BCS superfluids. The upper critical field is also qualitatively different from the weak and intermediate-coupling $H_{c2}(T)$. We have interpreted unusual resistive upper critical fields of many unconventional superconductors as the Bose-Einstein condensation field of preformed bosons-bipolarons. Their nonlinear temperature dependences follow from the scaling arguments.

Exceeding the Pauli paramagnetic limit has been explained, and the controversy in the determination of $H_{c2}(T)$ of cuprates from kinetic and thermodynamic measurements has been addressed in the framework of the bipolaron theory.

References

- [1] Bardeen J, Cooper L N and Schrieffer J R 1957 *Phys. Rev* **108** 1175
- [2] Eliashberg G M 1960 *Zh. Eksp. Teor. Fiz.* **38**, 966; **39**, 1437; 1960 *Sov. Phys. JETP* **11** 696; **12**, 1000.
- [3] Mihailovic D, Foster C M, Voss K and Heeger A J 1990 *Phys. Rev.* B42 7989
- [4] Calvani P, Capizzi M, Lupi S, Maselli P, Paolone A, Roy P, Cheong S-W, Sadowski W and Walker E 1994 *Solid State Commun.* **91** 113;
- [5] Timusk T, Homes C C and Reichardt W 1995 in *Anharmonic properties of High Tc cuprates* (eds. Mihailovic D, Ruani G, Kaldis E and Muller K A, Singapore: World Scientific) p.171
- [6] Zhao G, Hunt M B, Keller H, and Müller K A 1997 *Nature* **385** 236
- [7] Lanzara A, Bogdanov P V, Zhou X J, Kellar S A, Feng D L, Lu E D, Yoshida T, Eisaki H, Fujimori A, Kishio K, Shimoyama J I, Noda T, Uchida S, Hussain Z, Shen Z X 2001 *Nature* **412** 510
- [8] Egami T 1996 *J. Low Temp. Phys.* **105** 791

- [9] Temprano D R, Mesot J, Janssen S, Conder K, Furrer A, Mutka H, and Müller K A 2000 *Phys. Rev. Lett.* **84** 1990
- [10] Shen Z X, Lanzara A, Ishihara S, Nagaosa N 2002 *Phil. Mag.* **B82** 1349
- [11] Alexandrov A S 2003 *Theory of Superconductivity: from Weak to Strong Coupling* (Bristol and Philadelphia: IoP Publishing)
- [12] Alexandrov A S 1983 *Zh. Fiz. Khim.* **57** 273 ; 1983 *Russ. J. Phys. Chem.* **57** 167; 1998 *Models and Phenomenology for Conventional and High-temperature Superconductivity* (Course CXXXVI of the International School of Physics ‘Enrico Fermi’, eds. G. Iadonisi, J.R. Schrieffer and M.L. Chiofalo, Amsterdam: IOS Press), p. 309
- [13] Alexandrov A S and Mott N F 1993 *Supercond. Sci. Technol.* **6** 215; 1994 *J. Supercond.* **7** 599; 1994 *Rep. Prog. Phys.* **57** 1197
- [14] see contributions in 1995 *Polarons and Bipolarons in High- T_c Superconductors and Related Materials* (eds. Salje E K H, Alexandrov A S and Liang W Y, Cambridge: Cambridge University Press), and in 1995 *Anharmonic properties of High T_c cuprates* (eds. Mihailovic D, Ruani G, Kaldis E and Muller K A, Singapore: World Scientific)
- [15] Devreese J T 1996 in *Encyclopedia of Applied Physics*, vol. **14**, p. 383 (VCH Publishers) and references therein
- [16] Tempere J, Fomin V M and Devreese J T 1997 *Solid State Commun.* **101** 661
- [17] Alexandrov A S 1999 *Phys. Rev.* **B60** 14573
- [18] Ginzburg V L and Landau L D 1950 *Zh. Eksp. Teor. Fiz.* **20** 1064
- [19] Gross E P 1961 *Lett. Nuovo Cimento* **20** 454; Pitaevskii L P 1961 *Sov. Phys. JETP* **13** 451
- [20] Abrikosov A A 1957 *Zh. Eksp. Teor. Fiz.* **32** 1442
- [21] Schafroth M R 1955 *Phys. Rev.* **100** 463
- [22] Alexandrov A S 1984 *Doctoral thesis MIFI (Moscow)*; 1993 *Phys. Rev.* **B48**, 10571
- [23] Alexandrov A S, Beere W H, and Kabanov V V 1996 *Phys. Rev.* **B54** 15363
- [24] Bucher B, Karpinski J, Kaldis E, and Wachter P 1990 *Physica C* **167** 324
- [25] Mackenzie A P, Julian S R, Lonzarich G G, Carrington A, Hughes S D, Liu R S, Sinclair D C 1993 *Phys. Rev. Lett.* **71** 1238
- [26] Osofsky M S, Soulen R J, Wolf S A, Broto J M, Rakoto H, Ousset J C, Coffe G, Askenazy S, Pari P, Bozovic I, Eckstein J N, and Virshup G F 1993 *Phys. Rev. Lett.* **71**, 2315; *ibid* 1994 **72** 3292
- [27] Alexandrov A S, Zavaritsky V N, Liang W Y, and Nevsky P L 1996 *Phys. Rev. Lett.* **76** 983
- [28] Lawrie D D, Franck J P, Beamish J R, Molz E B, Chen W M, and Graf M J 1997 *J. Low Temp. Phys.* **107** 491
- [29] Gantmakher V F, Tsydynzhapov G E, Kozeeva L P, and Lavrov A N *Zh. Eksp.*

- Teor. Fiz. **88** 148 ; Gantmakher V F, Emel'chenko G A, Naumenko I G, and Tsydynzhapov G E 2000 *JETP Lett.* **72** 21
- [30] Zavaritsky V N, Kabanov V V and Alexandrov A S 2002 *Europhys. Lett.* **60** 127
- [31] Nakanishi T, Motoyama N, Mitamura H, Takeshita N, Takahashi H, Eisaki H, Uchida S, and Mōri N 2000 *Int. J. Mod. Phys***14** 3617
- [32] Lee I J, Chaikin P M, and Naughton M J 2000 *Phys. Rev.* **B62** R14 669
- [33] Wen H H, Li S L, and Zhao Z X 2000 *Phys. Rev.* **B62** 716
- [34] Alexandrov A S, Bratkovsky A M and Mott N F 1994 *Phys. Rev. Lett* **72** 1734
- [35] Fisher R A, Kim S, Lacy S E, Phillips N E, Morris D E, Markelz A G, Wei J Y T and Ginley D S 1988 *Phys. Rev.* **B38** 11942
- [36] Loram J W, Cooper J R, Wheatley J M, Mirza K A and Liu R S 1992 *Phil Mag.* **B65** 1405
- [37] Inderhees S E, Salamon M B, Goldenfeld N, Rice J P, Pazol B G and Ginzberg D M 1988 *Phys. Rev. Lett.* **60** 1178
- [38] Junod A, Eckert D, Triscone G, Lee V Y and Muller J 1989 *Physica C***159** 215
- [39] Schnelle W, Braun E, Broicher H, Dömel R, Ruppel S, Braunisch W, Harnischmacher J and Wohlleben D 1990 *Physica C***168** 465
- [40] Salamon M B, Inderhees S E, Rice J P and Ginsberg D M 1990 *Physica A***168** 283
- [41] Junod A, 1996 in *Studies of High Temperature Superconductors* **19** (ed. Narlikar A, New York: Nova Science, Commack) p.1. ; Revaz B, Junod A, and Erb A 1998 *Phys. Rev.* **B58** 11153 ; Roulin M, Junod A, and Walker E 1998 *Physica C***296** 137
- [42] Alexandrov A S, Beere W H, Kabanov V V and Liang W Y 1997 *Phys. Rev. Lett.* **79** 1551