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# 13. BUILDING AND SUSTAINING INQUIRY COMMUNITIES IN MATHEMATICS TEACHING DEVELOPMENT

Teachers and Didacticians in Collaboration

Teachers and didacticians both bring areas of expertise, forms of knowing and relevant experience to collaboration in mathematics teaching development. The notion of inquiry community, provides a theoretical and practical foundation for development. Within an inquiry community all participants are researchers (taking a broad definition). With reference to a research and development project in Norway (Learning Communities in Mathematics – LCM) this chapter explains the theoretical notions, discusses how one community was conceived and emerged in practice and addresses the issues contingent on emergence and sustaining of inquiry practices. In doing so it provides examples of collaborative activity and the reciprocal forms of expertise, knowing and experience that have contributed to community building. It illuminates issues and tensions that have been central to the developmental process and shows how an activity theory analysis can help to navigate the complexity in characterizing development.

# INTRODUCTION

This chapter focuses on *co-learning inquiry*, a mode of developmental research in which knowledge and practice develop through the inquiry activity of the people engaged (Jaworski, 2004a, 2006). This involves the creation of *inquiry communities between didacticians and teachers* to explore ways of improving learning environments for students in mathematics classrooms. Research both charts the developmental process and is a tool for development. The chapter draws on a research and development project in Norway,<sup>1</sup> *Learning Communities in Mathematics* (LCM), for which co-learning inquiry and communities of practice have formed a theoretical basis. The nature of inquiry, development and research in the project is used as a basis for extracting more general principles and issues.

The LCM project focused on how learners of mathematics at any level of schooling can develop conceptual understanding of mathematics that is reflected in nationally and internationally measured success. The project was rooted in

<sup>&</sup>lt;sup>1</sup> The LCM project was funded by the Research Council of Norway (RCN) in their advertised programme Kunnskap, Utdanning og Laering (Knowledge, Education and Learning – KUL): Project number 157949/S20.

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established systems and communities in which education is formalised and mathematics learning and teaching take place.<sup>2</sup>

The chapter weaves theory and practice to address meanings and roots of colearning inquiry and inquiry community and issues in creating and sustaining inquiry communities for development of learning and teaching mathematics.

# THEORETICAL BACKGROUND

# Knowledge in Sociocultural Settings

Knowledge is seen to be both brought by people engaged in the educational process and embedded in the practices and ways of being of these people – students in classrooms, teachers of mathematics in schools, and mathematics didacticians in a university.

According to Lave and Wenger (1991), knowledge is in participation in the practice or activity, and not in the individual consciousness of the participants. "The unit of analysis is thus not the individual, nor the environment, but a relation between the two" (Nardi, 1996, p. 71). So, the practice, or activity, in which participants engage is crucial to a situated (social practice theory) perspective. Wenger (1998) talks of *belonging* to a community of practice involving engagement, imagination and alignment. The terms *participation, belonging*, *engagement* and *alignment* all point towards the situatedness of activity and the growth of knowledge in practice.

Within the communities of our project we recognize both *individuals* and *groups*: that is we ascribe *identity* to both. Holland, Lachicotte, Skinner, and Cain (1998, p. 5) write, "Identity is a concept that figuratively combines the intimate or personal world with the collective space of cultural forms and social relations". Identity refers to ways of being (Holland et al., 1998). We talk about ways of being in the LCM project community and in the other various communities of which project members are a part, leading to a concept of *inquiry as a way of being* (Jaworski, 2004a). Inquiry is first of all a tool used by participants in a community of practice in consideration and development of the practice, that of mathematics learning and teaching in classrooms. Inquiry mediates between the activity of the classroom and the developmental goals of participants. Participants engage in action that involves inquiry and learn from the outcomes of their action relative to established ways of being. Relationships between individuals and the communities in which they are participants are complex with respect to the forms of knowledge they encompass and growth of knowledge within the communities.

Wertsch (1991, p. 12) emphasises that "the relationship between action and mediational means is so fundamental that it is more appropriate, when referring to the agent involved, to speak of 'individual(s)-acting-with-mediational-means' than

<sup>&</sup>lt;sup>2</sup> A copy of the project proposal can be obtained from the author by direct communication.

to speak simply of 'individual(s)'". Wertsch refers to Vygotsky's (1978, p. 57; emphasis in original) well known law of cultural development which states:

Every function of a child's cultural development appears twice: first, on the social level, and later, on the individual level; first *between* people (*interpsychological*), and then *inside* the child (*intrapsychological*). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relations between human individuals.

Such a perspective sees learning as *participation* in social practice or activity. As we participate we "take part" in the practices or activities involved, grow into those practices or activities, and learn through our doing and acting. We *engage* mentally and physically, and communicate with those around us. We use the language, words or gestures, of the practice or activity to engage and communicate. Different social groups use language in different ways and within any group we speak or learn to speak the group language.

Leont'ev (1979, pp. 47-48) writes,

in a society, humans do not simply find external conditions to which they must adapt their activity. Rather these social conditions bear with them the motives and goals of their activity, its means and modes. In a word, society produces the activity of the individuals it forms.

Thus, activity is necessarily motivated; actions have explicit goals, and individuals engage in activity with goal-directed action leading to integral formation of "the intermental plane". Mediation is central to this formation, with the mediational means (tools, signs or other) a key focus in activity theory (Leont'ev, 1979; Wertsch, 1991).

Thus, starting from identity as meaning belonging in practice, with knowledge firmly rooted in practice (Wenger, 1998), we move to identity as the mediational formation of the intramental plane through goal-directed action (Wertsch, 1991). This extension of belonging through goal-directed action offers a theoretical grounding for the extension of alignment to critical alignment through processes of inquiry. I shall return to this below.

# Co-Learning Inquiry

Co-learning inquiry means people learning together through inquiry; inquiry being a mediational tool as indicated above. The term "co-learning" comes from Wagner (1997, p. 16) who writes

In a co-learning agreement, researchers and practitioners are both participants in processes of education and systems of schooling. Both are engaged in action and reflection. By working together, each might learn something about the world of the other. Of equal importance, however, each may learn

something more about his or her own world and its connections to institutions and schooling.

An aim of the LCM project was that didacticians from the university and teachers from schools would work together to explore and develop mathematics learning and teaching in classrooms. In such collaboration, both groups are practitioners and, since both engage in exploration and inquiry, both are researchers. We thus adapted slightly the words from Wagner (1997, p. 16) to read: "teachers and didacticians are both practitioners and researchers in processes of education and systems of schooling". The simple aim, that didacticians and teachers would work together as both practitioners and researchers, was both a guiding force for LCM and a source of tension in relation to power and hierarchy. Didacticians conceptualized the project, gained the funding, invited participation from schools, and set up the basic project design. Given such clear "ownership" of the project, could it be possible to redress the obvious hierarchy and create some kind of sharing of power and responsibility? This question will be addressed throughout the chapter with relation to the developmental project (LCM) and the theoretical perspectives outlined above.

# Inquiry Community

*Inquiry community* was part of didacticians' vision for the LCM project; a theoretical concept rooted in wide previous experience and a number of key sources. According to Chambers Dictionary, inquiry means to ask a question; to make an investigation; to acquire information; to search for knowledge. Wells (1999, p. 122) speaks of "dialogic inquiry" as "a willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them".

He emphasizes the importance of *dialogue* to the inquiry process in which questioning, exploring, investigating, and researching are key activities or roles of teachers and didacticians (and ultimately students). These activities can be discerned through the analysis of dialogue in interactions within the community.

Didacticians had distinguished between use of inquiry *as a tool* in teaching and learning, and developing inquiry as *a way of being*, so that the identity of individual or group within an inquiry community would be rooted in inquiry (Jaworski, 2004a). Developing inquiry as a way of being involves *becoming*, *or taking the role of*, an inquirer; becoming a person who questions, explores, investigates and researches within everyday, normal practice. The vision has much in common with what Cochran-Smith and Lytle (1999) speak of as "inquiry as stance" – the stance of teachers who engage in an inquiry way of being, an inquiry identity, in engagement in practice. A focus of the LCM project was to explore what inquiry could mean in mathematics classrooms and in the activity of teachers and didacticians trying to explore development of mathematics teaching and learning.

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These words suggest that we do not necessarily have inquiry ways of being in "normal" practice. Brown and McIntyre (1993), researching teaching in classrooms from observation of classroom activity and interviews with teachers, suggested that teaching and learning in classrooms develops "normal desirable states". Teachers and students find ways of working together that fit as well as possible with expectations of educational and social systems and groups and allow a workable environment. The workable environment comes from an implicit agreement between teachers and students about what is expected, and what is acceptable in classroom activity – a sort of didactic contract (Brousseau, 1984). Such ways of working and being in classrooms might be characterized as *communities of practice*<sup>3</sup> (Lave & Wenger, 1991), in which participants align themselves with the normal desirable state. However, the normal desirable state does not necessarily foster the kinds of mathematical achievement didacticians, and society more broadly, would like to see.<sup>4</sup>

In terms of Wenger's (1998) theory, that *belonging to a community of practice* involves *engagement, imagination* and *alignment,* we might see the normal desirable state as *engaging* students and teachers in forms of practice and ways of being in practice with which they *align* their actions and conform to expectations. *Imagination* ensures comfortable existence within the broader social expectations and acceptable or desirable patterns of activity.

One of the reasons for introducing inquiry as a tool – for example, in designing inquiry tasks to stimulate inquiry in the classroom – is to challenge the normal (desirable) state and question what it is achieving. For example, if students are learning mathematics through text book exercises, in which the goal is to practise skills and become fluent with operations, we might ask questions about the degree of conceptual understanding that is afforded by this practice. If the normal desirable state is to be sure that students can *do* what is required, and not to worry too much about understanding, then it could be that we are denying students an important opportunity – to understand the mathematics they are learning, and to relate particular ideas more widely, both in mathematics and in real world applications. So, we might ask, what can we do in classrooms to enable students to understand better the mathematics they meet in text book exercises? This is a developmental question. As soon as we strive to address such a question, we enter an inquiry or a research process.

In an inquiry community, we are not satisfied with the normal (desirable) state, but we approach our practice with a questioning attitude, not to change everything overnight, but to start to explore what else is possible; to wonder, to ask questions,

<sup>&</sup>lt;sup>3</sup> The *practice* is that of engaging in classroom activity according to the norms and expectations of the particular setting in which activity takes place. Such practice is often referred to as mathematics teaching and/or learning.

<sup>&</sup>lt;sup>4</sup> The TIMSS and PISA studies provide ample evidence of this, for Norway and for many other countries. See, for example, Kjærnsli, Lie, Olsen, and Turmo, 2004; Grønmo, Bergem, Kjærnsli, Lie, and Turmo (2004); Mullis, Martin, Gonzalez, and Chrostowski (2004); Mullis, Martin, Beaton, Gonzalez, Kelly, and Smith (1998).

and to seek to understand by collaborating with others in the attempt to provide answers to them (Wells, 1999). In this activity, if our questioning is systematic and we set out purposefully to inquire into our practices, we become researchers.

The community of the LCM project, set up to generate a community of inquiry, had to learn, to grow into, to come to know what it could mean to work in inquiry ways, to develop questioning attitudes, to design inquiry tasks and to foster students' own inquiry. Thus the community of inquiry was an *emergent* rather than an *established* form of practice. Inquiry practices in schools bring new elements to established practices. Thus, in order to move from a community of practice to a community of inquiry, participants will engage in existing practices, aligning to some extent with those practices, but in a questioning or inquiry mode. This has been termed *"critical alignment"* (Jaworski, 2006). It involves a recognition that within existing practices, alignment (in Wenger's terms) is essential, but if we bring a critical attitude to alignment – that is we question, we explore, we seek alternatives while engaging – then we have possibilities to develop and change the normal states.

# Activity Theory as an Analytical Tool

The theoretical ideas outlined above have allowed us to conceptualise the roots of inquiry communities; we have found, however, that they do not go far enough in allowing us to analyse the various forms of data we have generated in order to cut through complexities in the various communities in which the LCM project has been embedded. For this reason we have turned to activity theory which has allowed us to inter-relate concepts of community, inquiry and critical alignment in seeking to explain issues and tensions in the project and emergent growth of knowledge.

We start here from transitions between intermental and intramental planes (Vygotsky, 1978; Wertsch, 1991) and the roles of didacticians and teachers in promoting development in mathematics classrooms. As I shall explain below, practices within the LCM project, although goal-directed, were not pre-designated. It is one thing to propose creation of a community of inquiry and quite another to realize it. A major part of our developmental activity and associated research involved exploring the creation and nature of an inquiry community. The inquiry community was emergent in the project as were the knowledge and learning associated with it. As the people of the project engaged with the activity of the project within the project community, also working simultaneously in other communities of practice (schools or university), people learned and knowledge grew. From knowledge and activity within existing communities of practice, and activity within the project, new understandings, and new ways of being and acting, emerged. In Wertsch's (1991) terms, people acting with mediational means within their respective communities, with goals relating to developing mathematics learning and teaching in classrooms, form, as part of their communicative interaction, their intermental plane. We see the intermental plane to be the learning and knowing that occurs within the community as a whole, with the formation of *intra*mental planes as individuals participate in mediated action. Leont'ev's (1979) concepts of motivated activity and goal-directed action have been employed in analysis of data to chart learning in LCM (Goodchild & Jaworski, 2005; Jaworski & Goodchild, 2006), along with Engeström's (1999) *mediational triangle* and concept of *expansive learning* (see below).

# CREATING AN INQUIRY COMMUNITY

# Starting Points: From Motives to Goal-Directed Action

The term "community" designates a group of people identifiable by who they are in terms of how they relate to each other, their common activities and ways of thinking, beliefs and values. Activities are likely to be explicit, whereas ways of thinking, beliefs and values are more implicit. Wenger (1998, p. 5) describes community as "a way of talking about the social configurations in which our enterprises are defined as worth pursuing and our participation is recognisable as competence".

According to Rogoff, Matusov, and White (1996, p. 388), in a *learning community*, "learning involves transformation of participation in collaborative endeavour". The idea of *inquiry* community makes the nature of transformation more explicit: didacticians and teachers (and ultimately students) will engage together *in inquiry activity*. What such activity should or could consist of, and how it should or could relate to activity in existing communities of practice, the classrooms, schools and university settings was a focus of research in LCM.

LCM was motivated by developmental aims from which project activity was designed. In submitting a proposal to seek funding, didacticians proposed certain forms of action which would give shape to the project. These included workshops for teachers and didacticians in university settings, design of tasks for workshops and classrooms, teacher teams in schools for design of classroom activity, and collection of data from all activity. Thus, realization, or operationalization of the project required activity in which this design was implemented into project practice. We proposed engagement in an *inquiry cycle* (plan, act & observe, reflect and analyse, feedback) in the design process as the basis for our practical realization of a developmental research paradigm – more of this below.

The *nature* of the inquiry cycle was something that emerged in project activity. The proposed practices set out in the initial design were what engaged us initially along with the philosophy of co-learning inquiry. We (didacticians) wished to collaborate with teachers as *partners* in developing and researching mathematics teaching in classrooms (Jaworski, 1999). We wanted to try to avoid positions of offering teachers models of practice and supporting their implementation, or of bringing teachers into developmental practice after the design stage and including them only then in the action (Jaworski, 2004b). Nevertheless, the project had been conceived by didacticians: the philosophical basis of the project (in co-learning inquiry) was not negotiable but was clearly open to interpretation; the more practical aspects of project design could be negotiated but award of funding

brought with it a responsibility for didacticians to achieve what had been set out, so at some levels it was not possible to start to (re)negotiate the ground with teachers. So, the initial position was that motivation for the project was in place together with some designated action and goals. A major developmental question at this stage was how to bring teachers into the project.

### Action and Inquiry

In creating an inquiry community, the participants have to come together in goaldirected action. Establishing goals within a community is itself a developmental task, and goes with initial action. In their invitation to schools and teachers to participate, didacticians set out the principles of the project and outlined its operation based on workshops at the university and innovation in schools and classrooms. Schools were recruited for two years, with the possibility of a third year.<sup>5</sup>

So, with regard to action and goals some things were taken as basic (e.g., workshops and co-learning inquiry) and (many) others were open to negotiation and experimentation in project activity. The motivating principle on which we all agreed (didacticians and teachers) was our desire to develop better learning environments for students in mathematics at the levels of schooling with which we were associated. Unsurprisingly, the ways of thinking about this principle were deeply related to the communities of practice from which we came, and these varied across the schools and between schools and university. The knowledge we brought to the project initially was also deeply embedded in our established communities with sociohistorical precedents and cultural practices forming identities in the project.<sup>6</sup>

Two examples from LCM illustrate the initial position. Didacticians' planning for workshops was rooted in their philosophy for the project and their knowledge of educational literature and research relating to teacher education and developmental practice in mathematics classrooms. Some had pioneered small group problem solving in mathematics teaching at the university (Borgersen, 1994) and all believed strongly in investigative approaches to teaching mathematics at any educational level. This embedded knowledge was highly motivational in the activity that didacticians planned for workshops. Teachers came from an educational system in which initial teacher education was provided in a university with practice in schools, and continuing teacher education was provided through workshops and seminars led by teachers from the university. There was expectation by at least some teachers that didacticians would lead the way in proposing developmental activity. A quotation from a teacher Agnes, in a focus

<sup>&</sup>lt;sup>5</sup> Eight schools from early primary to upper secondary joined the project and 40 teachers participated during three years. Some funding was given to schools to support teachers in attending workshops in school time. Further details can be found in Jaworski (2005).

<sup>&</sup>lt;sup>6</sup> Norwegian culture and society along with educational values and systems were in common for most project participants (teachers and didacticians).

group interview at the end of two years of classroom activity indicates that she struggled in the beginning because didacticians did not seem to show teachers what to do.

Agnes: [...] in the beginning I struggled, had a bit of a problem with this because then I thought very much about you should come and tell us how we should run the mathematics teaching. This was how I thought, you are the great teachers [...] (FG\_060313. Translated from the Norwegian by Espen Daland)

Thus teachers found it difficult in the initial stages to initiate innovative activity in their schools because it had not been their custom to think of doing so and they expected a clear lead from didacticians. There were other barriers as well, which I shall come to.

In LCM, particularly in the first year, it was the workshops which led the way in bringing participants together to build community and create an inquiry approach to thinking about mathematics, teaching and learning. Didacticians planned tasks to stimulate thinking and action: these were increasingly influenced over three years by teachers' comments, suggestions and requests for particular forms of activity. In any workshop, teachers and didacticians worked side by side on tasks, usually in small groups, in a mode intended to create genuine collaboration in doing mathematics and talking about associated classroom issues. Groups were organized sometimes to cross school levels, at other times to align with levels more or less finely. Plenary sessions allowed input on relevant topics, presentations from school activity (often using video recordings from classrooms), and feedback from group activity and discussion.

In all workshops, mathematical tasks were chosen or designed carefully (mainly by didacticians – discussed further below) for their mathematical or didactical appropriateness for the stage of the project. In the first workshops, problems were chosen which had rich potential for stimulating mathematical thinking and which were accessible to people with widely different mathematical experience. Later, problems or tasks were designed related to curriculum topics. All work on tasks led to discussions, in both small group and plenary, around the didactics and pedagogy of creating tasks for classrooms and the associated issues.<sup>7</sup>

The workshops were spaced throughout the school year so that, between workshops, school activity and innovation could take place. Two forms of activity in school emerged from this opportunity. In some cases, teachers took tasks from the workshops and, with suitable modification, used them in their classrooms with students. Frequently, reporting at a workshop included presentations from such student activity. In other cases, the teacher team in a school designed a task or set of tasks to bring an inquiry approach to a curriculum topic. Varying degrees of collaboration between teachers and didacticians were involved in designing and

<sup>&</sup>lt;sup>7</sup> A special issue of the Journal of Mathematics Teacher Education (JMTE 4-6, 2007) is devoted to research into the design and use of mathematical related tasks in teacher education.

planning such tasks. Didacticians often recorded classroom activity on video when such innovation took place, and video became an important medium in the project for sharing experience of task design and use in schools. We see here clear examples of mediation between inter- and intra-mental planes.

Thus early action took place in workshops in the university and in school activity stimulated by the workshops. Inquiry was evident in the planning process, in ways in which teachers took workshop ideas back to schools and tried out ideas in classrooms and in the developing relationships between the participants as activity progressed. I shall talk later about the outcomes of such activity in terms of participants' learning and issues and tensions which arose.

# An Inquiry Cycle in the Design Process

From the beginning of the projects, design was a central factor in creating workshop or classroom activity and innovation. Didacticians followed loosely a design research approach to creating activity in workshops (Kelly, 2003; Wood & Berry, 2003). The approach was inquiry-based and iterative (plan, act and observe, reflect and analyse, feedback to planning) and was in Kelly's terms "generative and transformative" (2003, p. 3). Typically, following an initial planning meeting, a small team of didacticians took on the design of tasks according to agreed criteria for the coming workshop: for example, tasks relating to algebra at a range of levels including opportunities for generalization and justification of conjectures. The small team circulated the outcomes of their design process and these were discussed in a subsequent meeting. After the workshop, one meeting of didacticians was dedicated to reflecting on the workshop activity including outcomes from the use of tasks; these reflections feeding into subsequent decision making and planning. This inquiry process was centrally important in sharing knowledge and expertise among didacticians, stimulating creativity, generating a group outcome in terms of tasks for a workshop, and building new knowledge within the didactician community.

Didacticians envisaged a similar process for school teams planning for the classroom. Unsurprisingly, the outcomes were very variable, and related to particular school circumstances. While, in at least one school, the design cycle, in planning and implementing tasks and reflecting on their use by students, was exemplary (Fuglestad, Goodchild, & Jaworski, 2006; Hundeland, Erfjord, Grevholm, & Breiteig, 2007) in other schools planning was more ad hoc, often individual and relating to one class only (Daland, 2007). The most common practice observed was teachers' use of workshop tasks, modified for classrooms. Teachers reported in subsequent workshops from their classroom activity and the engagement of their students, and video extracts showed evidence of classroom innovation. The words of Agnes, continuing from the quotation above, testify to teacher growth through this process:

[...] but now I see that my view has gradually changed because I see that you are participants in this as much as we are even though it is you that organise.

Nevertheless I experience that you are participating and are just as interested as we are to solve the tasks on our level and find possibilities, find tasks, that may be appropriate for the students, and that I think is very nice. So I have changed my view during this time. (FG\_060313. Translated from the Norwegian by Espen Daland)

Project activity in schools proved a major learning experience for didacticians as I discuss below.

# A Developmental Research Paradigm

The central use of *design*, as in *design of tasks* and activity for workshops and classrooms suggested a design research approach to the project. Didacticians would design for workshops, albeit taking into account strongly the views and suggestions of teachers. Teachers would design for the classroom, drawing on experiences in workshops and inviting didacticians' contributions as appropriate. However, the theory of design research (see Wood & Berry, 2003) proved too "clinical". The design cycle, even in the activity of didacticians, was rarely conceived "up front", and emerged largely from human interactivity around the aims of the project. In schools, it was often hard to recognize clearly the elements of a cycle, intertwined as they were with the multitude of factors that make up teachers' lives.

Here we recognize the developmental nature of the projects – activity emerged from engagement. *Action, observation, reflection* and *analysis* in the inquiry/design cycle led to growing awareness of the nature of co-learning in the projects. This inquiry cycle was overtly a learning process for all participants who acted as insider researchers, inquiring into their own practices and feeding back what they learned into future action (Bassey, 1995; Jaworski, 2004b; Goodchild, 2007). The systematic nature of such inquiry varied considerably across the project.

From the beginning, didacticians collected data as far as possible from all activity – all meetings at which didacticians were present were recorded on audio, all workshop activity on video or audio, photographs were taken and documents carefully stored. Some school meetings were audio recorded and some classroom activity video recorded. A large data bank was organized to which all didacticians had access. These data were not related to particular research questions; rather research questions evolved through activity and data was used according to need. As didacticians followed up initial research questions in analysis of data and writing of papers, more refined questions emerged which then fed into future activity and further research. In this way, the emergent nature of research in the project became centrally visible, and it was possible to trace links between research activity and developmental progress.<sup>8</sup>

<sup>&</sup>lt;sup>8</sup> See Gravemeijer (1994) and Goodchild (Volume 4 of this Handbook) for related and extended accounts of developmental research.

#### INQUIRY COMMUNITY AND OUTCOMES IN THE LCM PROJECT

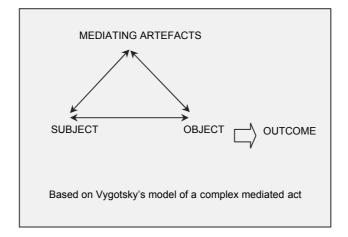
The essence of an inquiry community is that, through goal-directed action in communities of practice, participants explore, inquire into, their own practice with the motive of learning how to improve the practice (see also Benke, Hošpesová, & Tichá, this volume). All participants engaged in the project community, but they were also a part of other communities which made demands on their work and lives, and the inquiry process resulted in a more critical scrutiny of the range of practices and possibilities they afforded. Thus, didacticians and teachers, in their respective established communities both aligned with the practices of those communities and looked critically at their engagement. Teachers participated in the day to day life of their schools and, integrally, explored the use of inquiry-based tasks in their classrooms and observed their students' mathematical activity and learning. Didacticians collected and analysed data and wrote research papers, as expected of university academics and, integrally, explored the design of tasks for workshops and their work with teachers in school environments to support teachers in their project activity. Activity in the project community emerged from action. Action in the form of task design led to action in a workshop, which led in its turn to action in schools, each of these feeding back to inform succeeding stages of activity. The inquiry community of the project could not be separated from the established communities of which project members were a part. Interaction between established communities, their joint enterprise, mutual engagement and shared repertoire (Eriksen 2007; Wenger, 1998), and the emerging project community led to recognition of a complexity of inter-relations, issues and tensions as the project progressed. As indicated earlier, didacticians used activity theory to try to make sense of the complexity and address issues and tensions.

# Mediated Action and Engeström's Triangle

Relating to Vygotsky's (1978) law of cultural development and ideas from Leont'ev (1979) and Wertsch (1991), expressed above, a simple triangle (see Figure 1) expresses the mediational process as individuals or groups (the subject of activity) engage in action to achieve goals (the object of activity).

In all cases, according to the theory, activity is mediated: for example, activity in workshops is mediated by the tasks in which teachers and didacticians engage; activity in schools is mediated by the ideas teachers' bring from workshops related to tasks for the classroom and approaches to working with students.

However, according to Engeström (1998) this "simple" mediational triangle ignores the "hidden curriculum", the factors in education in schools that influence fundamentally what is possible for teachers and their students, and ultimately for didacticians in a developmental project such as LCM. Engeström (1998, 1999) extended the simple triangle to the more complex version (see Figure 2).



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Figure 1. The simple mediational triangle.

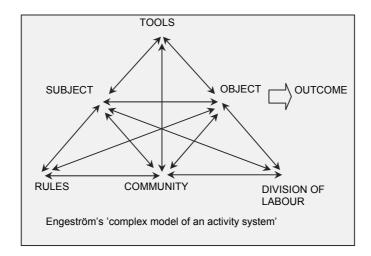


Figure 2. Engeström's mediational triangle including the hidden curriculum.

The progression from subject to object can be achieved in mediation through any of the paths indicated. Rules include the curriculum and its assessment, the ways in which school and educational systems operate, the societal and political

expectations of schools and teachers. Community includes the established communities discussed above, as well as the project community in LCM. Division of labour includes the roles of participants, teachers in their school system, didacticians within a university setting; new roles developing through the project.

Issues and tensions arise when elements of the hidden curriculum challenge the achievement of goals. I use the theory of mediated action within communities of inquiry and the hidden curriculum expressed in Engeström's triangle to present some of the outcomes of the LCM project (in the rest of this section) and lead to more general observations concerning development in communities of inquiry (in the final section of this chapter).

# Didacticians' Roles

A tension with which didacticians have grappled since the beginning of the project concerns a didactician's role in working with teachers, either in a workshop or in a school environment. To what extent were we to offer our own thinking, viewpoint or expertise? In one early meeting, considering our role in a workshop small group, the term "coordinator" was used and rejected. Someone equated it with being "the boss". The words "facilitator" was preferred (Cestari, Daland, Eriksen, & Jaworski, 2006). It was clear that the didactician in a group had some responsibility to ensure the smooth working of the group according to the declared task. This might mean ensuring that all participants were included in dialogue and activity. It might mean helping to keep the group focused. It might mean taking initiative to suggest roles for participants. It was agreed that it should not mean explaining the mathematics, or giving the solution of a problem. However, to what extent should a didactician participate in the mathematics? To what extent should he or she present a personal point of view in discussion? We had no clear answers to such questions. It remained for us to work according to broadly agreed principles and respond to particular circumstances. Activity was mediated through workshops tasks, experience from our activity in other communities, responses from the community of teachers present and so on. Mediation through subsequent sharing of experience with the didactician team enabled our awareness to grow and strengthened our ability to act knowledgeably according to agreed principles. For example, after one workshop, a didactician praised the actions of one colleague in enabling discussion in a small group. A further meeting was planned to watch a video recording of this group and to synthesise from the praised actions. From such interactivity over time, we learned both to live with uncertainty and to recognize the nature of growth in being a didactician in such a project. Despite saying these things so simply, this was not always a comfortable process.

# The Locus of Power and Control

This issue of the didactician's role in activity with teachers adumbrates a fundamental issue that underpinned much of the LCM project – that of where the power and responsibility in the project was located, and its implications.

Undeniably, the project originated with the didacticians; they were responsible to the research council, owned both conceptualization and operationalisation to a high degree, and controlled funding. Schools had volunteered to be in the project and signed a contract with the university regarding their participation (Jaworski, 2005). Teachers participated with willingness and enthusiasm, and there was also much evidence of enjoyment. Teachers were also critical of what they experienced, and expressed points of view that were not always in accord with didacticians' concepts of events.

For example, although workshop activity in small groups which crossed school levels was presented by didacticians as valuable for understanding students' experience beyond one's own level, teachers preferred overwhelmingly to work with colleagues at the same school level, and said so! After the very early months, small groups were usually same-level (and sometimes same-school) groups. Some teachers were critical of mathematical problems that were not clearly related to a topic in their own curriculum. They indicated that demands of curriculum and available time meant there was no possibility for them to use such problems, even though the problems were interesting and often fun to engage with. One teacher expressed this point of view after having chosen himself to engage with a 'fun' problem in a workshop. The implication was that in his lessons there was no time for 'fun'. Didacticians responded to such comments by designing mathematical tasks which could be seen as clearly curriculum-related, but nevertheless might be fun to engage with. Teachers responded that such tasks could be seen as valuable, but were much more time consuming than the text book tasks they used. However, the teachers expressing this point of view in one school invited didacticians to engage with them in designing more open tasks that could engage students conceptually. This resulted in a set of lessons, according to the teachers, quite different from those they held normally. They reported that students had seemed to have a better understanding of the mathematical concepts than earlier groups. Nevertheless, they were clear that they could not afford generally the amount of time demanded by these tasks (Fuglestad et al., 2006; Hundeland et al., 2007).

We see here clear examples of critical alignment by both didacticians and teachers – a complex set of actions and reactions in and to project activity closely related to school activity. On the one hand, activity was led by design of tasks and group organization designed by didacticians. On the other, teachers' responses and perspectives led to reconceptualization and redesign; for example, groups became mainly same-level groups; tasks were increasingly curriculum-related. Teachers spoke from their own experiences and perspectives rooted in their normal activity in school communities and from the demands of the rules of schooling, for example the pressure of needing to "cover" the national curriculum. Rules and communities mediating the thinking and actions of teachers impinged on the project and mediated the design of tasks and workshop groupings. In order to achieve project goals didacticians needed to recognize and respond to teachers' concerns. Teachers surprised didacticians nevertheless by engaging in activity in ways that showed workshop goals being achieved in classrooms. Thus, control

shifted between didacticians and teachers in interesting ways showing a complex division of labour in the project.

## Mutual Adaptation and Learning

The first year of activity with schools constituted Phase 1 of the project.<sup>9</sup> Before the start of the second year (Phase 2), didacticians responded to teachers' comments on workshops by holding a consultative meeting. Teachers were invited to express frankly their views on workshops and to make suggestions for workshops in the coming phase of activity. Many indicated that finding time in school for the kinds of planning meetings they needed to design activity for classrooms was extremely difficult. School structures militated against such meetings and time was limited. They would like the opportunity to plan together with colleagues from other schools at the same level, to produce classroom tasks and to report on the classroom activity on a future occasion. These suggestions were so strongly supported across school levels, that Phase 2 of the project became structured accordingly. The Norwegian phrase "planlegge et opplegg" (devise the lesson plan) became a watchword for Phase 2. Here didacticians could be seen clearly to take on board teachers' perspectives and to build these into ongoing activity in workshops. Increasingly in Phase 2, input from teachers relating to activity in classrooms became a central feature of plenary sessions. Curriculum topics were used explicitly as a focus for mathematical activity. Same-level groups predominated. Feedback from a focus group interview with each school team at the end of Phase 2 indicated that teachers had appreciated didacticians' accommodation to their perspectives in a range of factors and showed corresponding activity in classrooms. Invitations from teachers to didacticians to videorecord innovative activity in classrooms resulted in a bank of videodata charting development in classrooms. We might see, in retrospect the meeting between Phases 1 and 2 as a watershed in project activity. Engeström's (1999) theory of expansive learning might be seen to capture this watershed.

# Expansive Learning

The outcome of tensions, such as those expressed above, in the LCM project was that activity went on. We did not see a breakdown. Trust, good will and positive intentions led to realization (both recognition and making-real) of ways of working that enabled some achievement of some goals (on both sides) to some extent. In this process, there was some event or initiative which acted as a force to resolve tensions – expressed by Engeström (1999) as *"expansive learning"*. For example, during the first phase we had seen a build up of tension as teachers engaged with activity, provided clear evidence of valuing the project and their participation, yet

<sup>&</sup>lt;sup>9</sup> The LCM project was funded for four years. During this time, there were three Phases of activity, each of one school year, in which didacticians and teachers worked together as described here.

at the same time increasingly expressed a wish for modified forms of action (such as the nature of small groups or the kinds of mathematical tasks). The meeting between the phases allowed overt expression of desire for alternative action and clear suggestions for the form such action might take.

Expansive learning is rooted in the activity theory concepts expressed above – notably goal directed mediated action, based in Vygotsky (1978) and Leont'ev (1979). Engeström (1999, p. 382), following Leont'ev (1979), expresses it as a dialectic of "ascending from the abstract to the concrete" and adds (pp. 382–383):

A method of grasping the essence of an object by tracing and reproducing theoretically the logic of its development, of its historical formation through the emergence and resolution of inner contradictions. [...] The initial simple idea is transformed into a complex object, a new form of practice. [...] The expansive cycle begins with individual subjects questioning the accepted practice, and it gradually expands into a collective movement or institution.

Through complex interactions traceable to all three elements of the hidden curriculum, participants in the project are able to recognize and isolate the *inner contradictions* expressed by Engeström (1999). In the case above, the concerns about groups and about tasks in workshops, through the between-phases meeting, led to the emergence of the new idea of *planlegge et opplegg* through which planning in homogenous groups in a workshop with didacticians' support could lead to teachers having suitable activity for their classrooms leading to development for students' learning of mathematics. What started as internal rumblings within activity resulted in an external development explicit for all to engage with. Such analysis enables us to trace our activity, noting its historical development and becoming clearer about the issues, tensions or contradictions inherent in the developmental process.

As a further example, I refer to an event with took place in Phase 3 of the project. This phase was introduced (by didacticians) as focusing on declaring and achieving school goals for development of mathematics learning and teaching within a school. Activity in Phase 3 proceeded along familiar lines with engagement in workshops and associated work in schools and with associated issues and tensions acknowledged but not resolved. The focus on school goals was elusive and progression towards school goals not achieved. Then one didactician suggested a task that was to have important consequences for the goals of Phase 3. The task was connected to a series of three workshops focusing on algebra. It involved teachers in undertaking some focused observation of some of their own students related to work on algebra. Teachers were asked to bring to the next workshop some input from their observations. The workshop was organized to develop a "red thread" through observations at different levels of students' algebraic understanding across the range of school levels. In order to visualize teachers' judgement on the quality of students' algebraic thinking, teachers were asked to pin their written observations to a line which was strung across the workshop's main room. The coffee break allowed all to view the line and think about its contents. The quality of teachers' perceptions expressed in the final

plenary discussion and comments received from several teachers after the workshop indicated that this had been an important experience for teachers: most significant had been their insights into the thinking and understanding of their own students and recognition of the task as a research event with serious learning outcomes for themselves. The task had provided the opportunity for expansion and for a breakthrough in activity.

Teachers' participation and comments in and from this activity suggested to didacticians that certain goals had been achieved. Teachers had engaged overtly in a research task, conducting activity in their schools, findings the time to do so, recognizing their learning, and valuing their insights into students' perceptions/thinking/understandings of algebra. This signified for didacticians strong developmental outcomes from their own activity and participation – with evidence of both teachers' learning and didacticians' associated learning. For example, teachers suddenly came to see, through their study of students' thinking and activity in algebra, how they could explore in their school environment ways to develop teaching and learning; didacticians saw the nature of a task that could lead to teachers' effective recognition of the nature of school goals for students' development and learning in mathematics.

Seeing the enterprise in terms of an activity system made it possible to pick out elements in the complexity and trace developmental patterns for participants in the project (see Goodchild & Jaworski, 2005; Jaworski & Goodchild, 2006). In this process, tensions became evident as catalysts providing opportunity for learning. We see the nature of community as central to this provision of opportunity. During the three years, the members of the project community came to know each other as colleagues, appreciating good intentions, trusting good will, recognizing differences, respecting alternative points of view and becoming aware of developing thinking and associated possibility for action. This is not to claim hugely visible changes to the everyday practices in which established communities were rooted, but rather to recognize a relationship between developmental aims and the realities of normal working life. For example, the structure in a school could not change to suit the aims of the project; nor should the project fail because these aims could not be met. So, in what ways might we accommodate to achieve the aims? Through such recognition also, the aims become more understandable and perhaps more open to flexibility in their achievement - that is we were able to relate the aims to real settings and work out alternative approaches compatible with the aims.

# A MORE GENERAL PERSPECTIVE

This chapter has interwoven complex aspects of theory and a specific developmental research project to illuminate notions of the development of

mathematics learning and teaching through developmental research in inquiry communities involving teachers and didacticians.<sup>10</sup>

In this final section of the chapter my purpose is to pull out to a more general viewpoint on communities of inquiry and the associated theoretical perspectives. Key areas of theory have been

- Communities of practice with notions of belonging through engagement, imagination and alignment (Wenger, 1998) shifting to critical alignment through inquiry (Jaworski, 2006);
- Mediated activity between people involving individuals acting with mediational means (Vygotsky, 1978; Wertsch, 1991);
- The motivated nature of activity involving goal-directed action (Leont'ev, 1979);
- Engeström's expanded mediational triangle and the concept of expansive learning (Engeström, 1998, 1999).

The concept of *community* is clearly central in all of these and needs no further comment. The place of *inquiry* perhaps needs further elucidation. Inquiry brings the critical element to community of practice through which participants can inquire into existing practices with possibility to modify and improve. Inquiry can be seen as a mediational tool in social settings enabling development of knowing between people and hence of participative individuals. Inquiry as in the design/inquiry cycle promotes goal-directed action leading to developmental outcomes. Inquiry ways of being allow the possibility of contradictions emerging as powerful motivators for expansion within an activity system.

The *inquiry community* starts with intentions to use inquiry as a tool for learning and development. Through engagement with an inquiry cycle in the design of tasks and opportunity for participation, a community grows into inquiry ways of being which encourage mediation of complexity within the hidden curriculum of systems and structures that constrain development. As compared to established communities of practice, in which norms of practice nurture undesirable states, the inquiry community is emergent. It does not avoid issues, tensions and contradictions, but deals with them as part of emergent recognition and understanding leading to possibilities for expansive learning. Inquiry ways of being accept the unfinished nature of learning and development. There is not an end point.

<sup>&</sup>lt;sup>10</sup> For those interested in knowing more about the LCM project, the website http://fag.hia.no/lcm/ contains a list of relevant publications and the book (Jaworski, Fuglestad, Bjuland, Breiteig, Goodchild, & Grevholm, 2007) charts the project as a whole.

# EPILOGUE

LCM ended in December 2007. However, in 2006, an extension to LCM was already started in the form of a new project, TBM, *Teaching Better Mathematics*, funded again by the RCN. This new project involves a consortium including five centres in different parts of Norway linking didacticians with schools and rooted in a philosophy of inquiry communities. At Agder University, TBM is linked to LBM (*Learning Better Mathematics*), a parallel project owned by schools. LBM and TBM work in concert with a managing committee including school leaders and didacticians. Schools pay for the work of one didactician based at the university with a responsibility for liaising between the two projects and supporting teachers' participation. Both schools and didacticians contribute to conceptualization, planning and engagement in project activity in workshops and classrooms.

The consortium has come about through didacticians in institutions in the five regions recognizing shared goals rooted in developing inquiry communities between didacticians and teachers in their own region. Each regional group has their own specific project with its own clear focus and goals, but all share the same theoretical basis. The research council has seen value in such collaboration in supporting the project. Its invitation to the Agder community to offer a dedicated day conference in Oslo in October 2007 was a further indication of its support. We see this as very positive encouragement from an important part of the establishment to continue this developmental approach.

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