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Cover page

Title: Optimal Sensor Location Methodology for Structural Identification and Damage Detection

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ABSTRACT

Theoretical and computational issues arising in the selection of the optimal sensor configuration in structural dynamics are addressed. The information entropy is introduced to measure the performance of a sensor configuration. Asymptotic estimates are used to rigorously justify that selections of optimal sensor configurations can be based solely on nominal structural models, ignoring the time history details of the measured data that are not available in the initial experimental design stage. Heuristic algorithms are proposed for constructing effective sensor configurations that are superior, in terms of computational efficiency and accuracy, to the sensor configurations provided by available algorithms suitable for solving general optimisation problems. The theoretical developments and the effectiveness of the proposed algorithms are illustrated by designing the optimal configuration for an array of acceleration sensors placed on a bridge structure.

INTRODUCTION

Structural model identification using measured dynamic data has received much attention over the years because of its importance in structural model updating, health monitoring, damage detection and control. The quality of information that can be extracted from the measured data for structural identification purposes depends on the type, number and location of sensors. The objective in this work is to optimise the number and location of sensors in the structure such that the resulting measured data are most informative for estimating the parameters of a family of mathematical model classes used for structural identification and damage detection.

Information theory based approaches [e.g. 1-5] have been developed to provide rational solutions to several issues encountered in the problem of selecting the optimal sensor configuration. In references [1-2] the optimal sensor configuration is taken as the one that maximizes a norm (determinant or trace) of the Fisher information matrix (FIM). Reference [4] treats the case of large model uncertainties expected in model

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updating. The optimal sensor configuration is chosen as the one that minimizes the expected Bayesian loss function involving the trace of the inverse of the FIM for each model. Papadimitriou et al. [4] introduced the information entropy norm as the measure that best corresponds to the objective of structural testing, which is to minimize the uncertainty in the model parameter estimates. Specifically, the optimal sensor configuration is selected as the one that minimizes the information entropy measure since it gives a direct measure of this uncertainty. It has been shown [5] that, asymptotically for very large number of data, the information entropy depends on the determinant of the Fisher information matrix. An important advantage of the information entropy measure is that it allows us to make comparisons between sensor configurations involving a different number of sensors in each configuration. Furthermore, it has been used to design the optimal characteristics of the excitation (e.g. amplitude and frequency content) useful in the identification of linear and strongly nonlinear models [6]. The methodology has also been extended in [7] to design optimal sensor locations for updating multiple model classes useful for damage detection purposes. Finally, heuristic algorithms [5,7] have been proposed that are computationally much more effective and accurate for selecting the optimal sensor location.

In this work, the problem of optimally placing the sensors in the structure is revisited and the information entropy approach is used to design the optimal sensor configurations for two type of problems: (i) identification of structural model (e.g. finite element) parameters or modal model parameters (modal frequencies and modal damping ratios) based on acceleration time histories, and (ii) identification of structural model parameters based on modal data. Analytical expressions are developed that show the relative effect of model and measurement error on the design of the optimal sensor configuration. Results on a four-span bridge structure are used to illustrate the theoretical developments.

STRUCTURAL IDENTIFICATION METHODOLOGY

Consider a parameterized class $\mathcal{M}$ of structural models (e.g. a class of finite element models or a class of modal models) chosen to describe the input-output behavior of a structure. Let $\boldsymbol{\theta} \in R^{N_\theta}$ be the vector of free parameters (physical or modal) in the model class. A Bayesian statistical system identification methodology [8,9] is used to estimate the values of the parameter set $\boldsymbol{\theta}$ and their associated uncertainties using the information provided from dynamic test data. For this, the uncertainties in the values of the structural model parameters $\boldsymbol{\theta}$ are quantified by probability density functions (PDF) that are updated using the dynamic test data. The updated PDF is then used for designing the optimal sensor configuration.

Identification Based on Response Time History Data

Let $D = \{\hat{x}_j(k\Delta t) \in R^{N_0},\ j = 1,\cdots,N_0,\ k = 1,\cdots,N_D\}$ be the measured sampled response time history data from a structure, consisting of acceleration, velocity or displacement response at N_0 measured DOFs, where N_D is the number of the sampled data using a sampling rate Δt. The measured DOFs are usually referred to translational DOFs. Let also $\{x_j(k;\boldsymbol{\theta}) \in R^{N_d},\ j=1,\cdots,N_d,\ k=1,\cdots,N_D\}$, where N_d is the number of model degrees of freedom (DOF), be the predictions of the sampled response time histories obtained from a particular model corresponding to a specific value of the parameter set $\boldsymbol{\theta}$. The prediction error $e_j(k)$ between the sampled measured response time histories and the corresponding response time histories predicted from a model, for the jth measured DOF and the kth sampled data, is given by the prediction error equation

$$e_j(k) = \hat{x}_j(k) - x_j(k;\boldsymbol{\theta}) \tag{1}$$

where $j=1,\ldots,N_0$ and $k=1,\ldots,N_D$. The predictions errors at different time instants are modeled by independent (identically distributed) zero-mean Gaussian variables. Specifically, the prediction error $e_j(k)$ for the jth measured DOF is assumed to be a zero mean Gaussian variable, $e_j(k) \sim N(0,\sigma_j^2)$ with variance σ_j^2. The model prediction error is due to modeling error and measurement noise.

Applying the Bayesian system identification methodology [8,9], assuming independence of the prediction errors $e_j(k)$, the updating PDF $p(\boldsymbol{\theta},\boldsymbol{\sigma} \mid D)$ of the parameter sets $\boldsymbol{\theta}$ and $\boldsymbol{\sigma} = (\sigma_1,\cdots,\sigma_{N_o})$, given the measured data D and the class of models $\mathcal{M}$, takes the form:

$$p(\boldsymbol{\theta},\boldsymbol{\sigma} \mid D) = \frac{\tilde{c}}{\sqrt{2\pi}^{\,N_D N_0}\,\rho(\boldsymbol{\sigma})} \exp\left\{-\frac{N_D N}{2} J(\boldsymbol{\theta};\boldsymbol{\sigma})\right\} \pi_{\boldsymbol{\theta}}(\boldsymbol{\theta})\pi_{\sigma}(\boldsymbol{\sigma}) \tag{2}$$

where

$$J(\boldsymbol{\theta};\boldsymbol{\sigma},D) = \frac{1}{N_0}\sum_{j=1}^{N_0}\frac{1}{\sigma_j^2} J_j(\boldsymbol{\theta}), \qquad J_j(\boldsymbol{\theta}) = \frac{1}{N_D}\sum_{k=1}^{N_D}\left[\hat{x}_j(k) - x_j(k;\boldsymbol{\theta})\right]^2 \tag{3}$$

is the overall weighted measure of fit between measured and model predicted response time histories for all measured DOFs, $\rho(\boldsymbol{\sigma}) = \prod_{j=1}^{N_0}\sigma_j^{N_D}$ is a scalar function of the prediction error parameter set $\boldsymbol{\sigma}$, $\pi_{\boldsymbol{\theta}}(\boldsymbol{\theta})$ and $\pi_{\sigma}(\boldsymbol{\sigma})$ are the prior distribution for the parameter sets $\boldsymbol{\theta}$ and $\boldsymbol{\sigma}$, respectively, $N = N_0$ and $\tilde{c}$ is a normalizing constant chosen such that the PDF in (2) integrates to one.

Identification Based on Modal Data

The methodology is next extended to the case where the dynamic data consist of modal data. Let $D = \{\hat{\omega}_r^{(k)}, \hat{\boldsymbol{\phi}}_r^{(k)} \in R^{N_0}, r=1,\cdots,m,\ k=1,\cdots,N_D\}$ be the measured modal data from a structure, consisting of modal frequencies $\hat{\omega}_r^{(k)}$ and modeshape components $\hat{\boldsymbol{\phi}}_r^{(k)}$ at N_0 measured DOFs, where m is the number of observed modes and N_D is the number of modal data sets available. Let also $\{\omega_r(\boldsymbol{\theta}), \boldsymbol{\phi}_r(\boldsymbol{\theta}) \in R^{N_d}, r=1,\cdots,m\}$ be the predictions of the modal frequencies and modeshapes obtained for a particular value of the model parameter set $\boldsymbol{\theta}$ by solving the eigenvalue problem corresponding to the model mass and stiffness matrices.

The prediction error $\boldsymbol{e}_r^{(k)} = [e_{\omega_r}^{(k)} \;\; \boldsymbol{e}_{\phi_r}^{(k)}]$ between the measured modal data and the corresponding modal quantities predicted by the model is given separately for the modal frequencies and the modeshapes by the prediction error equations:

$$e_{\omega_r}^{(k)} = \hat{\omega}_r^{(k)} - \omega_r(\boldsymbol{\theta}) \quad \text{and} \quad \boldsymbol{e}_{\phi_r}^{(k)} = \hat{\boldsymbol{\phi}}_r^{(k)} - \beta_r^{(k)} L_0 \boldsymbol{\phi}_r(\boldsymbol{\theta}) \tag{4}$$

$r = 1, \cdots, m$, where $e_{\omega_r}^{(k)}$ and $\boldsymbol{e}_{\phi_r}^{(k)} \in R^{N_d}$ are respectively the prediction errors for the modal frequency and modeshape components of the r-th mode, $k = 1, \cdots, N_D$, $\beta_r^{(k)} = \hat{\boldsymbol{\phi}}_r^{(k)T} \boldsymbol{\phi}_r / \boldsymbol{\phi}_r^T \boldsymbol{\phi}_r$ is a normalization constant that accounts for the different scaling between the measured and the predicted modeshape, and L_0 is a $N_0 \times N_d$ matrix of ones and zeros that maps the model DOFs to the measured degrees of freedom. The model prediction error is due to modeling error and measurement noise.

The prediction error $e_{\omega_r}^{(k)}$ for the r-th modal frequency is assumed to be a zero mean Gaussian variable, $e_{\omega_r}^{(k)} \sim N(0, \sigma_{\omega_r}^2 \hat{\omega}_r^{(k)2})$, with standard deviation $\sigma_{\omega_r} \hat{\omega}_r^{(k)}$. The prediction error for the r-th truncated modeshape vector $\boldsymbol{e}_{\phi_r}^{(k)} \in R^{N_0}$ is also assumed to be zero mean Gaussian vector, $\boldsymbol{e}_{\phi_r}^{(k)} \sim N(\mathbf{0}, C_{\phi_r}^{(k)})$, with diagonal covariance matrix $C_{\phi_r}^{(k)} = \sigma_{\phi_r}^2 \left\| \hat{\boldsymbol{\phi}}_r^{(k)} \right\|_{N_0}^2 I \in R^{N_0 \times N_0}$, where $\left\| \hat{\boldsymbol{\phi}}_r^{(k)} \right\|_{N_0}^2 = \left\| \hat{\boldsymbol{\phi}}_r^{(k)} \right\|^2 / N_0$, $\|\cdot\|$ is the usual Euclidian norm and I is the identity matrix. The parameters σ_{ω_r} and σ_{ϕ_r}, represent the prediction error estimates of the measured modal frequencies and modeshapes involved in D.

Applying the Bayesian identification, assuming independence of the prediction errors $e_{\omega_r}^{(k)}$ and $\boldsymbol{e}_{\phi_r}^{(k)}$, the updating PDF $p(\boldsymbol{\theta}, \sigma \mid D)$ of the parameter sets $\boldsymbol{\theta}$ and $\boldsymbol{\sigma} = \{\sigma_{\omega_r}, \sigma_{\phi_r}, r = 1, \ldots, m\}$, given the data D and the class of models $\mathcal{M}$, takes the form (2), where

$$J(\boldsymbol{\theta}; \boldsymbol{\sigma}) = \frac{1}{N_D N} \sum_{r=1}^{m} \left[\frac{1}{\sigma_{\omega_r}^2} \sum_{k=1}^{N_D} \frac{[\omega_r(\boldsymbol{\theta}) - \hat{\omega}_r^{(k)}]^2}{[\hat{\omega}_r^{(k)}]^2} + \frac{N_0}{\sigma_{\phi_r}^2} \sum_{k=1}^{N_D} \frac{\left\| \beta_r^{(k)} L_0 \boldsymbol{\phi}_r(\boldsymbol{\theta}) - \hat{\boldsymbol{\phi}}_r^{(k)} \right\|^2}{\left\| \hat{\boldsymbol{\phi}}_r^{(k)} \right\|^2} \right] \tag{5}$$

represents the weighted measure of fit between the measured modal data and the modal data predicted by a particular model within the selected model class, $N = m\, N_0 + 1$ is the number of measured data per modal set, and $\rho(\boldsymbol{\sigma}) = \prod_{r=1}^{m} (\sigma_{\omega_r})^{N_D} (\sigma_{\phi_r})^{N_0 N_D}$ is a function of the prediction error parameters $\boldsymbol{\sigma}$.

OPTIMAL SENSOR LOCATION BASED ON INFORMATION ENTROPY

The marginal updated PDF $p(\boldsymbol{\theta} \mid D)$ specifies the plausibility of each possible value of the structural model parameters. It provides a spread of the uncertainty in the structural model parameter values based on the information contained in the measured data. A unique scalar measure of the uncertainty in the estimate of the structural parameters $\boldsymbol{\theta}$ is provided by the information entropy, defined by [4]:

$$H\ \boldsymbol{\delta},D\ = E_{\boldsymbol{\theta}}\left[-\ln p\ \theta|D\ \right] = -\int\ \ln p\ \theta|D\ \ p\ \theta|D\ d\theta \qquad (6)$$

The information entropy depends on the available data $D \equiv D(\boldsymbol{\delta})$ and the sensor configuration vector $\boldsymbol{\delta}$.

An asymptotic approximation of the information entropy, valid for large number of data ($N_D N \to \infty$), is available [5] which is useful in the experimental stage of designing an optimal sensor configuration. The asymptotic approximation is obtained by substituting $p(\boldsymbol{\theta} \mid D) = \int p(\boldsymbol{\theta},\boldsymbol{\sigma} \mid D)\ d\boldsymbol{\sigma}$ and (2) into (6) and observing that the resulting integral can be re-written as Laplace-type integrals which can be approximated by applying Laplace method of asymptotic approximation [10]. Specifically, it can be shown that for a large number of measured data, i.e. as $N_D N \to \infty$, the following asymptotic results hold for the information entropy [5]

$$H(\boldsymbol{\delta},D) \sim H(\boldsymbol{\delta};\hat{\boldsymbol{\theta}},\hat{\sigma}) = \frac{1}{2} N_{\boldsymbol{\theta}} \ln(2\pi) - \frac{1}{2}\ln[\det \mathbf{h}(\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\sigma}};\boldsymbol{\delta})] \qquad (7)$$

where $\hat{\boldsymbol{\theta}} \equiv \hat{\boldsymbol{\theta}}(\boldsymbol{\delta},D) = \arg\min_{\boldsymbol{\theta}} J(\boldsymbol{\theta};D)$ is the optimal value of the parameter set $\boldsymbol{\theta}$ that minimizes the measure of fit $J(\boldsymbol{\theta};D)$ given in (3), $\hat{\boldsymbol{\sigma}}^2$ is the optimal prediction error given by $\hat{\boldsymbol{\sigma}}^2 = [J_1(\hat{\boldsymbol{\theta}};D),\cdots,J_{N_0}(\hat{\boldsymbol{\theta}};D)]$, and $\mathbf{h}(\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\sigma}};\boldsymbol{\delta})$ is an $N_\theta \times N_\theta$ positive definite matrix defined and asymptotically approximated by

$$\mathbf{h}(\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\sigma}};\boldsymbol{\delta}) = -\boldsymbol{\nabla}_{\boldsymbol{\theta}}\boldsymbol{\nabla}_{\boldsymbol{\theta}}^T \ln[J(\boldsymbol{\theta};D)]^{-N_D N}\Big|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} \sim \mathbf{Q}(\boldsymbol{\delta},\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\sigma}}) \qquad \text{as} \quad N_D N \to \infty \quad (8)$$

in which $\boldsymbol{\nabla}_{\boldsymbol{\theta}} = [\partial / \partial\theta_1,\cdots,\partial / \partial\theta_{N_\theta}]^T$ is the usual gradient vector with respect to the parameter set $\boldsymbol{\theta}$. For response time history data, the matrix $\mathbf{Q}(\boldsymbol{\delta},\boldsymbol{\theta})$ appearing in (8) is a positive semi-definite matrix of the form

$$\mathbf{Q}(\boldsymbol{\delta},\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\sigma}}) = \frac{N_D}{2}\sum_{j=1}^{N_d} \delta_j \frac{1}{\hat{\sigma}_j^2}\mathbf{P}^{(j)}(\hat{\boldsymbol{\theta}}) \qquad (9)$$

known as the Fisher information matrix [1] and containing the information about the values of the parameters $\boldsymbol{\theta}$ based on the data from all measured positions specified in $\boldsymbol{\theta}$. The matrix $\mathbf{P}^{(j)}(\boldsymbol{\theta})$ is a positive semi-definite matrix given by

$$\mathbf{P}^{(j)}(\boldsymbol{\theta}) = \sum_{k=1}^{N_D} \boldsymbol{\nabla}_{\boldsymbol{\theta}} x_j(k;\boldsymbol{\theta})\ \boldsymbol{\nabla}_{\boldsymbol{\theta}}^T x_j(k;\boldsymbol{\theta}) \qquad (10)$$

containing the information about the values of the parameters $\boldsymbol{\theta}$ based on the data from one sensor placed at the j-th DOF. For given excitation characteristics, the matrix $\mathbf{P}^{(j)}(\boldsymbol{\theta})$ depends only on the response of the optimal model at the particular DOF j, while it is independent of the sensor configuration vector $\boldsymbol{\delta}$.

The only dependence of the resulting asymptotic value of the information entropy (7) on the data comes implicitly through the optimal values $\hat{\boldsymbol{\theta}} \equiv \hat{\boldsymbol{\theta}}(\boldsymbol{\delta},D)$ and the prediction errors $\hat{\sigma}_j^2 = J_j(\hat{\boldsymbol{\theta}};D)$. Consequently, the information entropy (7) is completely defined by the optimal value $\hat{\boldsymbol{\theta}}$ of the model parameters and the optimal prediction error $\hat{\sigma}_j^2 = J_j(\hat{\boldsymbol{\theta}};D)$, $j = 1,\cdots,N_0$, expected for a set of test data, while the time history details of the measured data do not enter explicitly the formulation. The prediction errors can be written in the form $\hat{\sigma}_j^2 = J_j(\hat{\boldsymbol{\theta}};D) = s_1^2 + s_2^2 g_j(\hat{\boldsymbol{\theta}})$, where s_1^2 gives the contribution from measurement error assumed to be constant for all measurements, and s_2^2 gives the contribution from model error assumed to be proportional to the average strength $g_j(\hat{\boldsymbol{\theta}}) = (1/N_D)\sum_{k=1}^{N_D} x_j^2(k;\hat{\boldsymbol{\theta}})$ of the response at the j-th DOF. The optimal sensor location depends on the optimal model $\hat{\boldsymbol{\theta}}$ and the values of s_1^2 and s_2^2 assumed for the measurement and model errors, respectively.

For modal data, following a similar analysis, the matrix $\mathbf{Q}(\boldsymbol{\delta},\boldsymbol{\theta})$ is a positive semi-definite matrix given by

$$\mathbf{Q}(\boldsymbol{\delta},\boldsymbol{\theta}) = \frac{N_D}{2}\sum_{r=1}^{m}\left[\frac{\nabla_{\boldsymbol{\theta}}\omega_r(\boldsymbol{\theta})\ \nabla_{\boldsymbol{\theta}}^T\omega_r(\boldsymbol{\theta})}{s_1^2 + s_2^2\omega_r^2(\boldsymbol{\theta})} + \sum_{j=1}^{N_d}\delta_j\frac{\nabla_{\boldsymbol{\theta}}L_0\phi_{jr}(\boldsymbol{\theta})\ \nabla_{\boldsymbol{\theta}}^T L_0\phi_{jr}(\boldsymbol{\theta})}{s_1^2 + s_2^2\left\|L_0\phi_r(\boldsymbol{\theta})\right\|^2/N_0}\right] \quad (11)$$

containing the information about the values of the model parameters $\boldsymbol{\theta}$ based on the modal data from all sensors placed in the structure.

Based on the asymptotic analysis, two heuristic sequential sensor placement (SSP) algorithms, the forward (FSSP) and the backward (BSSP), were proposed [5,6] for constructing predictions of the optimal and worst sensor configurations. According to FSSP, the positions of N_0 sensors are computed sequentially by placing one sensor at a time in the structure at a position that result in the highest reduction in information entropy. The BSSP algorithm is used in an inverse order, starting with N_d sensors placed at all DOFs of the structure and removing successively one sensor at a time from the position that results in the smallest increase in the information entropy. The computations involved in the SSP algorithms are an infinitesimal fraction of the ones involved in the exhaustive search method and can be done in realistic time, independently of the number of sensors and the number of model DOFs. It was found that for essentially the same accuracy, genetic algorithms, well-suited for solving the resulting discrete optimization problem, require significantly more computational effort than the heuristic SSP algorithms. In almost all cases considered, the estimate from the GA algorithm did not improve the estimate provided by the SSP algorithms. Thus, although the SSP algorithms are not guaranteed to give the optimal solution, they were found to be effective and computationally attractive alternatives to the GAs. In particular, SSP algorithms provide with minimal computational effort the variation of the lower and upper bounds of the information entropy as a function of the number of sensors. Such bounds are useful in evaluating the effectiveness of a sensor configuration as well as in guiding the cost-effective selection of the number of sensors, trading-off information provided from extra sensors with cost of instrumentation.

ILLUSTRATIVE EXAMPLE

In order to demonstrate the theoretical developments and illustrate the effectiveness of the proposed algorithms the methodology is applied to the design of the optimal configuration for an array of acceleration sensors placed on the 180-meter-long 13-meter-wide four-span bridge structure, located at Kavala (Greece). The deck of the bridge, consisting of four prestressed beams supporting the 20-cm thick deck, “floats” on laminated elastomeric bearings located at the top of the three piers and the abutments. A 900-DOF finite element model of the bridge consisting of 3-d beam elements is shown in Fig. 1. The structure is parameterized using three parameters, with the first parameter modeling the stiffness of the deck, the second parameter modeling the stiffness of all bearings and the third parameter modeling the stiffness of the three columns of the bridge. The nominal structure stiffnesses are chosen such that the 1st (0.54 Hz), 3rd (0.67 Hz), 4th (1.07 Hz), 5th (1.77 Hz), 6th (2.08 Hz) and 8th (2.72 Hz) modes are transverse, the 2nd (0.58 Hz) mode is longitudinal, the 7th (2.50 Hz) is local bending mode of the central pier, and the 9th to 12th modes are closely spaced (2.80, 2.824, 2.825 and 2.84 Hz) bending modes of the deck.

The optimal sensor locations for 1-12 sensors based on modal data, for the case of model error only ($s_1 = 0$ and $s_2 = s$) are shown in Figs. 1(a) and 1(b) for 4 and 12 observable modes, respectively, while for the case of measurement error only ($s_1 = s$ and $s_2 = 0$) the optimal sensor locations are shown in Figs. 2(a) and 2(b) for 4 and 12 observable modes, respectively. The minimum and maximum information entropy values as a function of the sensors computed by the exhaustive search method (exact method) for up to two sensors and the FSSP and BSSP algorithms are shown in Figs. 3(a) and 3(b) for 4 and 12 observable modes, respectively.

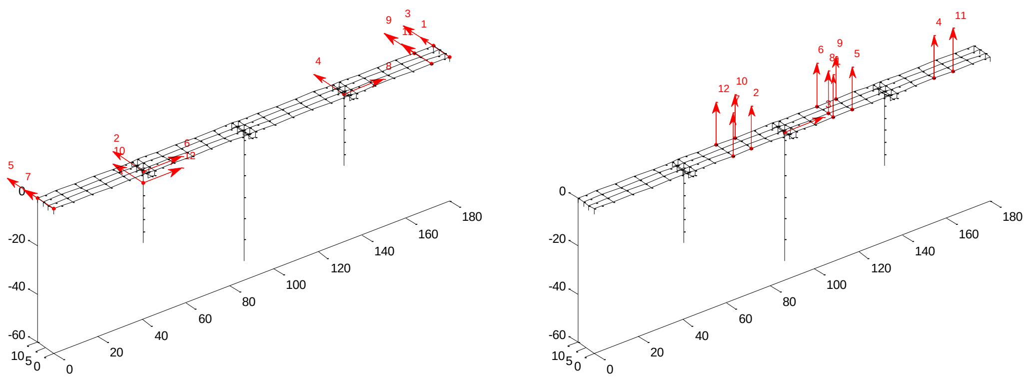

Fig. 1. Optimal locations for 1 to 12 sensors assuming model error for 4 and 12 observable modes.

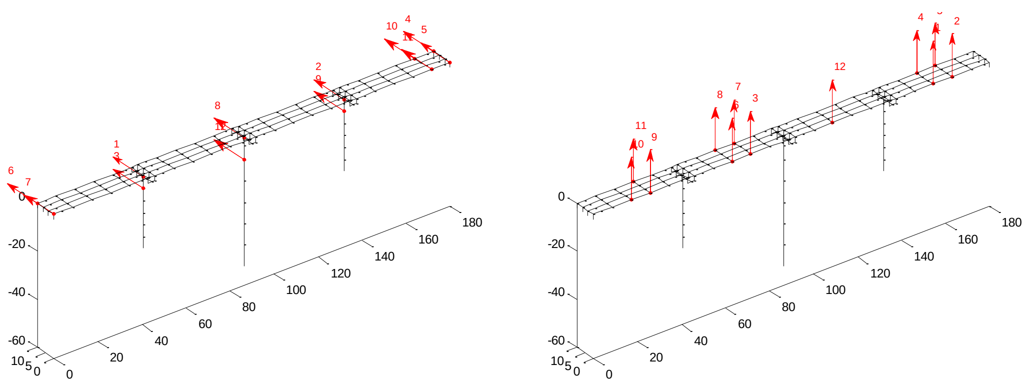

Fig. 2. Optimal locations for 12 sensors assuming measurement error for 4 and 12 observable modes.

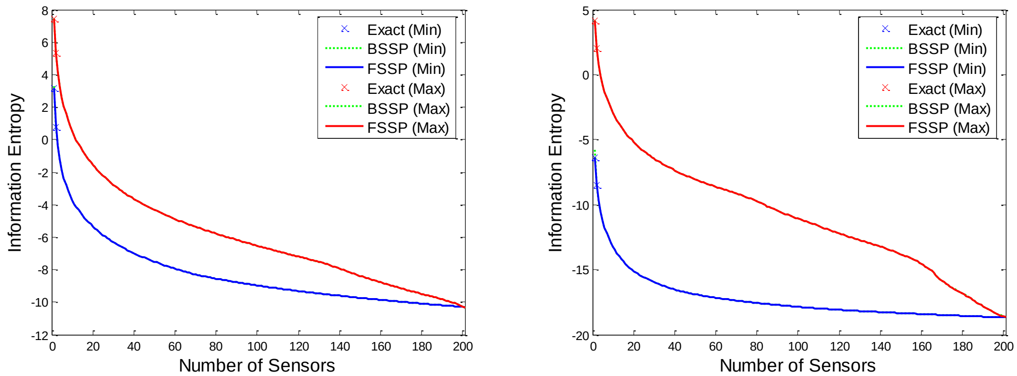

Fig. 3. Minimum and maximum information entropy values for 4 and 12 observable modes.

CONCLUSIONS

A rigorous formulation of the optimal sensor placement problem for structural identification was presented based on the information entropy measure of parameter uncertainty for two type of problems: (i) identification of structural model (e.g. finite element) parameters or modal model parameters (modal frequencies and modal damping ratios) based on acceleration time histories, and (ii) identification of structural model parameters based on modal data. An asymptotic estimate, valid for large number of data, was derived and used to justify that the sensor placement design can be based solely on a nominal model, ignoring the details in the measured data. Analytical expressions and numerical results showed the effect of model and measurement error on the design of the optimal sensor configuration. The analysis also showed that the lower and upper bounds of the information entropy values, corresponding respectively to the optimal and worst sensor configuration, is a decreasing function of the number of sensors.

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REFERENCES

[1] Udwadia F.E. 1994. “Methodology for Optimal Sensor Locations for Parameter Identification in Dynamic Systems,” *Journal of Engineering Mechanics (ASCE)*, 120(2):368-390.

[2] Kirkegaard P.H., R. Brincker. 1994. “On the Optimal Locations of Sensors for Parametric Identification of Linear Structural Systems,” *Mechanical Systems and Signal Processing*, 8:639-647.

[3] Heredia-Zavoni E., L. Esteva. 1998. “Optimal Instrumentation of Uncertain Structural Systems subject to Earthquake Motions,” *Earthquake Engineering and Structural Dynamics*, 27(4):343-362.

[4] Papadimitriou, C., J. L. Beck, and S. K. Au. 2000. "Entropy-Based Optimal Sensor Location for Structural Model Updating," *Journal of Vibration and Control*, 6(5):781-800.

[5] Papadimitriou, C. 2004. "Optimal Sensor Placement Methodology for Parametric Identification of Structural Systems." *Journal of Sound and Vibration*, 278(4):923-947.

[6] Metallidis P., G. Verros, S. Natsiavas, C. Papadimitriou 2003. "Identification, Fault Detection and Optimal Sensor Location in Vehicle Suspensions," *Journal of Vibration and Control*, 9(3-4):337-359.

[7] Papadimitriou, C. 2004. "Pareto Optimal Sensor Locations for Structural Identification." *Computer Methods in Applied Mechanics and Engineering*, 196(12-16):1655-1673.

[8] Beck, J. L. and L. S. Katafygiotis. 1998. "Updating Models and their Uncertainties - Bayesian Statistical Framework," *Journal of Engineering Mechanics (ASCE)*, 124(4):455-461.

[9] Katafygiotis, L. S., C. Papadimitriou, and H. F. Lam. 1998. "A Probabilistic Approach to Structural Model Updating," *International Journal of Soil Dynamics and Earthquake Engineering*, 17(7-8):495-507.

[10] Bleistein, N. and R. Handelsman, 1986. *Asymptotic Expansions for Integrals*. Dover, New York.