ECCOMAS Thematic Conference on Computational Methods
in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, D.C. Charmpis, N.D. Lagaros, Y. Tsompanakis (eds.)
Rethymno, Crete, Greece, 13–16 June 2007

BRIDGE MONITORING SYSTEM BASED ON VIBRATION MEASUREMENTS

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Keywords: Bridges, Health Monitoring, Operational Modal Analysis, Model Updating, Structural Identification, Damage Detection.

Abstract. *This work outlines the main algorithms involved in a proposed bridge monitoring system based on ambient and earthquake vibration measurements. The monitoring system can be used to predict the existence, location and size of structural modifications in the bridge by monitoring the changes in the modal characteristics and updating the finite element model of the bridge based on the modal characteristics. Sophisticated system identification methods, combining information from a sensor network with the theoretical information built into a finite element model for simulating structural behaviour, are incorporated into the monitoring system in order to track structural changes and identify the location, type and extent of these changes. Emphasis in this work is given on presenting theoretical and computational issues relating to structural modal identification and structural model updating methods. Specifically, the proposed work outlines the algorithms and software that has been developed for computing the modal properties using ambient and earthquake data, as well as recent methodologies and software for finite element model updating using the modal characteristics. Various issues encountered in the optimization problems involved in model updating are demonstrated, including the existence of multiple local optima and the effects of weight values in conventional weighted modal residual methods for selecting the optimal finite element model. Selected features are demonstrated using vibration measurements from a four-span bridge of the Egnatia Odos motorway in Greece.*

1 DESCRIPTION OF BRIDGE MONITORING SYSTEM AND ALGORITHMS

Successful health monitoring of structural systems depends to a large extent on the integration of cost-effective intelligent sensing techniques, accurate physics-based computational models simulating structural behavior, effective system identification methods, sophisticated health diagnosis algorithms, as well as decision-making expert systems to guide management in planning optimal cost-effective strategies for system maintenance, inspection and repair/replacement. Structural integrity assessment of highway infrastructure can in principle be accomplished using continuous structural monitoring based on real vibration measurements. Thus, bridge monitoring equipment and identification methodologies offer an important tool for bridge maintenance, which can constitute an indispensable part of a modern intelligent bridge management system. Taking advantage of modern technological capabilities, vibration data can be obtained remotely, allowing for an on-time assessment of the bridge condition. Using these measurements, it is possible to identify the dynamic modal characteristics of the bridge and update a theoretical finite element model. The results from the identification and updating procedures are useful to examine structural integrity after severe loading events (strong winds and earthquakes), as well as bridge condition deterioration due to long-term corrosion, fatigue and water scouring.

This paper presents parts of the methodological framework required for bridge structural health monitoring combining information from structural models (e.g. finite element models) representing the behavior of bridges and vibration measurements recorded using an array of sensors. The monitoring system should incorporate algorithms related to

- Optimal experimental design,
- Experimental modal analysis from ambient and earthquake induced vibrations,
- Finite element model updating, and
- Structural damage detection.

A brief overview of these algorithms and the software developed by the Systems Dynamics Laboratory of University of Thessaly to be used for the Egnatia Odos highway system is next presented.

Optimal experimental design methods refer to algorithms for optimizing the location and number of sensors in the structure such that the measure data contain the most important information for structural identification purposes. Algorithms based on information theory and the nominal finite element model describing the behavior of the structure, have been proposed to address this problem [1]. It has been shown that optimal sensor configuration depend on several factors, including the purpose of the analysis (modal analysis, model updating or damage detection), parameterization schemes used in model updating, damage scenarios examined, and the number of contributing modes. Effective heuristic optimization tools have also been developed for efficiently solving the resulting nonlinear single and multi-objective optimization problems. More details can be found in [2].

Experimental modal analysis algorithms should be able to process ambient as well as earthquake-induced vibrations in order to identify the modal characteristics of bridges. Modal analysis algorithms based on ambient vibrations have recently been developed using various time and frequency domain methods [e.g. 3-5]. An overview of a class of operational modal analysis methods is presented in [3]. Modal analysis methods based on earthquake records have also been developed in time [6] and frequency [7] domain. Recent efforts have been concentrated on algorithms for automated modal analysis with minimum user interference. Along this direction, methods based on least-squares fit between experimentally obtained and modal model predicted response characteristics have been developed [8] and implemented in a user-friendly graphical user interface software (www.mie.uth.gr/labs/sdl/). The algorithms implemented for processing earthquake data is an extension to the non-classically damped case of the available time domain [6] and frequency [7] domain algorithms. The algorithms implemented for processing the ambient vibrations are presented in more detail in Section 3.

The structural model updating problem has recently been formulated in a general multi-objective context [9] that allows the simultaneous minimization of the multiple metrics, measuring the residuals between measured and model predicted modal data. Theoretical and computational issues arising in multi-objective identification are addressed and the correspondence between the multi-objective identification and the conventional weighted residuals identification (e.g. [10,11]] are summarized in Section 4 of this paper. More details on the effectiveness of the multi-objective identification method, the variability of the Pareto optimal models, as well as the variability of the response and reliability predicted by the Pareto optimal models can be found in [9]. A user-friendly graphical user interface has been developed (www.mie.uth.gr/labs/sdl/) for finite element model updating using a linear relationship between the global mass and stiffness matrices and the model parameters. The finite element modeling is based on the commercial COMSOL Multiphysics software.

Finally, damage detection algorithms based on reconciling finite element models with data collected before and after damage have been developed using a Bayesian methodology for model class selection and updating from a family of parameterized model classes. The structural damage identification is accomplished by associating each parameterized model class in the family to a damage pattern in the structure, indicative of the location of damage. Using a Bayesian model selection framework, the probable damage locations are ranked according to the posterior probabilities of the corresponding model classes. The severity of damage is then inferred from the posterior probability of the model parameters derived for the most probable model class. Details about the methodology and the computational issues involved can be found in [12]. Based on asymptotic approximations, the diagnosis involves solving a series of model updating problems for each model class in the family. In particular, the asymptotic estimates provide useful insight into the role of the sensor configuration on the identification of damage. In particular, it has been shown that for reliable prediction of damage, the sensor configuration should be capable of providing informative data for all model classes in the chosen family.

Due to space limitations, only experimental modal analysis algorithms based on ambient data and finite element model updating algorithms based on modal data are briefly reviewed in Sections 3 and 4, respectively. In Section 5, the methodology is applied to a four-span bridge structure with the purpose of revealing certain features and difficulties associated with model updating, such as the presence of multiple local optima, as well as irregularities and discontinuities of the Pareto front and Pareto optimal models.

2 STRUCTURAL MODAL ANALYSIS BASED ON AMBIENT VIBRATIONS

2.1 Formulation

The modal identification using ambient vibration data is based on a minimization of the measure of fit between the cross power spectral density matrix estimated from the measured acceleration time histories and the cross power spectral density matrix predicted from a parameterized non-classically modal model of the structure. Specifically, let $\hat{\boldsymbol{S}}_{\underline{\ddot{y}}}(k\Delta\omega) \in C^{N_0 \times N_0}$ be the cross power spectral density (CPSD) matrix of the measured acceleration at the N_0 measured degrees of freedom (DOF), where $\Delta\omega$ is the discretization step in the frequency domain, $k = \{1, \cdots, N_\omega\}$ is the index set corresponding to frequency values $\omega = k\Delta\omega$, and

N_ω is the number of data in the indexed set. Let also $\mathbf{S}_{\underline{\ddot{y}}}(k\Delta\omega;\underline{\psi}) \in C^{N_0 \times N_0}$ be the CPSD matrix of the acceleration response of a linear structure at the measured DOF predicted by a modal model involving a parameter set $\underline{\psi}$ and m modes. The parameters in $\underline{\psi}$ include the modal and input characteristics needed to completely define the CPSD matrix of the responses. Mathematically, the modal model identification is formulated as a problem of finding the optimal values of the set $\underline{\psi}$, along with the number of modes m, that minimizes the measure of fit

$$E(\underline{\psi}) = \sum_{k=1}^{N_\omega} tr\left[\left(\mathbf{S}_{\underline{\ddot{y}}}(k\Delta\omega;\underline{\psi}) - \hat{\mathbf{S}}_{\underline{\ddot{y}}}(k\Delta\omega)\right)^{*T} \left(\mathbf{S}_{\underline{\ddot{y}}}(k\Delta\omega;\underline{\psi}) - \hat{\mathbf{S}}_{\underline{\ddot{y}}}(k\Delta\omega)\right) \right] \quad (1)$$

The formulation for the CPSD matrix $\mathbf{S}_{\underline{\ddot{y}}}(k\Delta\omega;\underline{\psi})$ based on modal model is briefly described next. According to modal analysis, the response of the structure at the model degrees of freedom is obtained as a superposition of the modal responses. Assuming general non-classically damped modes, the CPSD matrix $\mathbf{S}_{\underline{\ddot{y}}}(k\Delta\omega;\underline{\psi})$ based on the modal model, after considerable work, can be obtained in the form [8]

$$S_{\underline{\ddot{y}}}(\omega;\underline{\psi}) = \sum_{r=1}^{m} \left[\frac{\underline{\phi}_r \underline{g}_r^T}{(j\omega) - \lambda_r} + \frac{\underline{\phi}_r^* \underline{g}_r^{*T}}{(j\omega) - \lambda_r^*} + \frac{\underline{g}_r \underline{\phi}_r^T}{-(j\omega) - \lambda_r} + \frac{\underline{g}_r^* \underline{\phi}_r^{*T}}{-(j\omega) - \lambda_r^*} \right] + A \quad (2)$$

where $\lambda_r = -\zeta_r\omega_r \pm j\omega_r\sqrt{1-\zeta_r^2}$ is the complex eigenvalue of the r-th contributing mode, ω_r is the r-th modal frequency, ζ_r is the r-th modal damping ratio, $\underline{\phi}_r \in C^{N_0}$ is the complex modeshape of the r-th mode, $A \in R^{N_0 \times N_0}$ and $\underline{g}_r \in C^{N_0}$ are matrix and vector quantities that depends on the characteristics of the modal model and the CPSD of the white noise input vector [10], while the symbol u^* denoted the complex conjugate of a complex number u.

The modal parameter set $\underline{\psi}$ to be identified contains the following modal model parameters: the modal frequencies ω_r, the modal damping ratios ζ_r, the complex modeshapes $\underline{\phi}_r$, the complex vectors $\underline{g}_r$, $r = 1,\cdots,m$, and the elements of the real matrix A. The total number of parameters is $2m(1+2N_0)+N_0^2$ for non-classically damped models.

2.2 Numerical implementation

The modal parameters in the set $\underline{\psi}$,along with the number of modes m, are obtained from a three-step approach [8]. In the first step, the complex eigenvalues λ_r are obtained as the poles of the common denominator polynomial with the coefficients of the polynomial estimated by the solution of a set of linear systems. Herein, the polynomial representation of the frequency response matrix function in the first step is based on a discrete time model formulation that numerically behaves better, avoiding ill-conditioning, as the order p of the denominator polynomial increases. Stabilization diagrams that show the variation of the modal frequencies and the modal damping ratios with respect to the order p of the common denominator polynomial, are used to identify the m real modes and ignore the rest spurious modes of the system.

In the second step, the objective function is minimized with respect to the residues $R_r = \underline{\phi}_r \underline{g}_r^T \in C^{N_0 \times N_0}$ in (2), given the values of the eigenvalues λ_r of the stabilized (real) modes identified in the first step. Noting that the objective function is quadratic in the unknown residues R_r, the problem of finding the optimal values R_r is reduced to the solution of a linear set of equations for R_r, $r = 1, \cdots, m$. Noting also that R_r are matrices of rank one, singular value decomposition is used to decompose these matrices in the form $R_r = \underline{\phi}_r \sigma \underline{g}_r^T$ after keeping only the first highest singular value σ in the decomposition.

In order to improve the estimates of the modeshapes $\underline{\phi}_r$ and the vectors $\underline{g}_r$ in the second step, the optimization of the objective function (1) can be performed simultaneously with respect to the modeshapes $\underline{\phi}_r$ and the vectors $\underline{g}_r$, given the values of the eigenvalues λ_r of the stabilized (real) modes identified in the first step. The resulting optimization problem is a non-linear least-squares optimization problem. The optimization problem can be solved efficiently, significantly reducing computational cost, by recognizing that the error function in (1) is quadratic with respect to the modeshape $\underline{\phi}_r$ and the parameters in the matrix A. This observation can be used to develop explicit expressions that relate the parameters $\underline{\phi}_r$ and A to the vectors $\underline{g}_r$, so that the number of parameters involved in the optimization is reduced from $2m(1+2N_0)+N_0^2$ to $2mN_0$. Applying the optimality conditions in (1) with respect to the components of $\underline{u}_r$ and A, a linear system of equations results for obtaining $\underline{\phi}_r$ and A with respect to the vectors $\underline{g}_r$. The resulting nonlinear optimization problem with respect to the vectors $\underline{g}_r$, $r = 1, \cdots, m$, is solved using available recursive optimization methods with initial values the ones obtained by the singular value decomposition.

In the third step, the estimates of the modal parameters $\underline{\psi}$ can be improved by solving the nonlinear optimization problem with respect to all parameters in $\underline{\psi}$ and with the initial estimates the ones provided by the first two steps. Recognizing that the objective function is quadratic in $\underline{\phi}_r$ and A, explicit relations are developed to relate $\underline{\phi}_r$ and A with respect to ω_r, ζ_r and $\underline{g}_r$, and the problem is transformed to a nonlinear optimization problem with respect to the $2m(N_0+1)$ variables ω_r, ζ_r and $\underline{g}_r$.

It is worth pointing out that a modal sweep approach can be applied to carry out the analysis separately in different frequency bands with m modes used in each band. For well separated modes, one mode per frequency band ($m=1$) can be used so that the number of modal parameters involved in the optimization within a frequency band is kept to a minimum so that the computational effort is greatly reduced.

3 FINITE ELEMENT MODEL UPDATING BASED ON MODAL RESIDUALS

The objective in a finite element model updating methodology is to estimate the values of the free parameter set $\underline{\theta} \in R^{N_\theta}$ of a class of linear finite element models so that the modal frequencies and modeshapes $\{\omega_r(\underline{\theta}),\ \underline{\phi}_r(\underline{\theta}) \in R^{N_0}, r = 1, \cdots, m\}$ predicted by the linear class of models best matches, in some sense, the experimentally obtained modal data $\{\hat{\omega}_r, \hat{\underline{\phi}}_r \in R^{N_0},\ r = 1, \cdots, m\}$ contained in the set $\underline{\psi}$, where m is the number of observed modes, and N_0 is the number of measured DOFs. For this, the measured modal properties are first grouped into n groups g_i, $i=1,\cdots,n$. Each group contains one or more modal properties. For the i th group g_i, a norm $J_i(\underline{\theta})$ is introduced to measure the residuals of the difference between the measured and the model predicted modal properties involved in the group.

The grouping of the modal properties and the selection of the residuals $J_1(\underline{\theta}),\cdots,J_n(\underline{\theta})$ are usually based on user preference. For demonstration purposes, the following grouping scheme is introduces with residuals given by

$$J_1(\underline{\theta}) = \frac{1}{m}\sum_{r=1}^{m}[\omega_r(\underline{\theta}) - \hat{\omega}_r]^2 / [\hat{\omega}_r]^2 \quad \text{and} \quad J_2(\underline{\theta}) = \frac{1}{m}\sum_{r=1}^{m}\left\|\beta_r \underline{\phi}_r(\underline{\theta}) - \hat{\underline{\phi}}_r\right\|^2 / \left\|\hat{\underline{\phi}}_r\right\|^2 \quad (3)$$

The first group contains all modal frequencies with the measure of fit $J_1(\underline{\theta})$ selected to represent the difference between the measured and the model predicted frequencies for all modes, while the second group contains the modeshape components for all modes with the measure of fit $J_2(\underline{\theta})$ selected to represents the difference between the measured and the model predicted modeshape components for all modes.

3.1 Multi-objective identification

The problem of identifying the model parameter values that minimize the modal residuals can be formulated as a multi-objective optimization problem stated as follows [9,13]. Find the values of the structural parameter set $\underline{\theta}$ that simultaneously minimizes the objectives

$$\underline{y} = \underline{J}(\underline{\theta}) = (J_1(\underline{\theta}),\cdots,J_n(\underline{\theta})) \quad (4)$$

where $\underline{\theta} = (\theta_1,\cdots,\theta_{N_\theta})$ is the parameter vector, and $\underline{y} = (y_1,\cdots,y_n)$ is the objective vector. For conflicting objectives $J_1(\underline{\theta}),\cdots,J_n(\underline{\theta})$, there is no single optimal solution, but rather a set of alternative solutions, known as Pareto optimal solutions, that are optimal in the sense that no other solutions in the parameter space are superior to them when all objectives are considered. The set of objective vectors $\underline{y} = \underline{J}(\underline{\theta})$ corresponding to the set of Pareto optimal solutions $\underline{\theta}$ is called Pareto optimal front. The characteristics of the Pareto solutions are that the modal residuals cannot be improved in any group without deteriorating the residuals in at least one other group. The multiple Pareto optimal solutions are due to modeling and measurement errors. The set of Pareto optimal solutions are obtained using available multi-objective optimization algorithms. Evolutionary algorithms, such as the strength Pareto evolutionary algorithm [14], are well-suited to solve the multi-objective optimization problem.

3.2 Weighted residuals identification

The parameter estimation problem is solved by minimizing the single objective

$$J(\underline{\theta};\underline{w}) = \sum_{i=1}^{n} w_i J_i(\underline{\theta}) \quad (5)$$

formed from the multiple objectives $J_i(\underline{\theta})$ using the weighting factors $w_i \geq 0$, $i=1,\cdots,n$, with $\sum_{i=1}^{n} w_i = 1$. The objective function $J(\underline{\theta};\underline{w})$ represents an overall measure of fit between the measured and the model predicted characteristics. The relative importance of the residual errors in the selection of the optimal model is reflected in the choice of the weights. The results of the identification depend on the weight values used. Conventional weighted least squares methods assume equal weight values, $w_1 = \cdots = w_n = 1/n$.

It can be readily shown that the optimal solution to the problem (5) is one of the Pareto optimal solutions. Thus, solving a series of single objective optimization problems and varying the values of the weights w_i from 0 to 1, excluding the case for which the values of all weights are simultaneously equal to zero, Pareto optimal solutions $\hat{\underline{\theta}}(\underline{w})$ are alternatively obtained. It should be noted, however, that there may exist Pareto optimal solutions that do not correspond to solutions of the weighted least-squares problem. For computing the Pareto solutions, solving the multi-ojective optimization problem is preferred since the optimal solutions provided by solving the series of weighted single-objective optimization problems by uniformly varying the values of the weights often results in cluster of points in parts of the Pareto front that fail to provide an adequate representation of the entire Pareto shape.

The optimization of $J(\underline{\theta};\underline{w})$ in (5) with respect to $\underline{\theta}$ for given $\underline{w}$ can readily be carried out numerically using any available algorithm for optimizing a nonlinear function of several variables. However, the optimization problems may involve multiple local/global optima. Conventional gradient-based local optimization algorithms lack reliability in dealing with the estimation of multiple local/global optima observed in structural identification problems, since convergence to the global optimum is not guaranteed. Evolution strategies (ES) [15] are more appropriate and effective to use in such cases. ES are random search algorithms that explore better the parameter space for detecting the neighborhood of the global optimum, avoiding premature convergence to a local optimum. A disadvantage of ES is their slow convergence at the neighborhood of an optimum since they do not exploit the gradient information. A hybrid optimization algorithm [13] should be used that exploits the advantages of ES and gradient-based methods. Specifically, an evolution strategy is used to explore the parameter space and detect the neighborhood of the global optimum. Then the method switches to a gradient-based algorithm starting with the best estimate obtained from the evolution strategy and using gradient information to accelerate convergence to the global optimum.

4 APPLICATIONS

The proposed framework has been applied to two R/C bridges of Egnatia Odos motorway which crosses Northern Greece in the east-west direction. The two bridges have been instrumented with special array of 24 accelerometers each. The response to ambient excitation caused by traffic and wind has been systematically monitored. Herein, results are presented for the four span Kavala bridge, shown in Figure 1a, and described in detail in [16]. The proposed modal identification algorithm, using ambient vibrations resulted in the reliable estimation of seven modes: two transverse (0.81 Hz and 2.36 Hz), one longitudinal (1.29 Hz), one torsional (1.61 Hz) and three closely spaced bending modes (3.41 Hz, 3.46 Hz and 3.51 Hz).

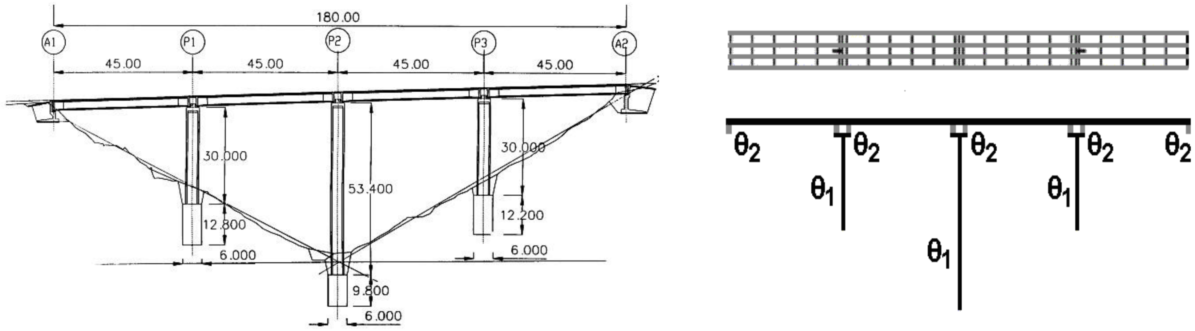

Figure 1: (a) Plan of the four-span Kavala bridge, (b) Parameterized finite element model.

To implement the model updating techniques, an appropriate parametric finite element model of the bridge is considered using three-dimensional two-node beam-type finite elements to model the deck, the piers and the bearings. This model is shown in Figure 1b and has 900 degrees of freedom. The entire simulation is performed within the COMSOL Multiphysics modeling environment. Model updating results using various parameterization schemes can be found in [16]. Herein, a two parameter model class is employed in order to demonstrate the applicability of the proposed methodologies, and point out issues and computational difficulties associated with the optimization problem. The first parameter θ_1 accounts for the stiffness of the three piers, while the second parameter θ_2 accounts for the stiffness of the elastomeric bearings at the piers and the abutments. The nominal finite element model corresponds to values of $\theta_1 = \theta_2 = 1$.

The parameterized finite element model class is updated using the seven modal frequencies and modeshapes obtained from operational modal analysis and the two modal groups with modal residuals given by (3). For demonstration purposes, results are obtained and compared for the Pareto points corresponding to the following selection of the weight parameters in equation (5): (a) $w_1 = w_2 = 1$, (b) $w_1 = 1$ and $w_2 = 0$, corresponding to the case where the fit is based on modal frequencies only, and (c) $w_1 = 0$ and $w_2 = 1$, corresponding to the case where the fit is based on modeshape components only.

4.1 Presence of multiple local optima

In order to reveal the features and the difficulties associated with model updating, the model parameters are first updated based on the seven modal properties using simulated modal data instead of the measure ones. Specifically, the “measured” modal frequencies are generated by the model corresponding to values $\theta_1 = 1$ and $\theta_2 = 1$. In this way the simulated modal data are free of measurement and model error. These simulated data are then fed to the optimization method to compute the optimal values of the model parameters θ_1 and θ_2. The contour plots of the objective function in the two-dimensional parameter space are shown in Figure 2a-c for the three cases considered. For case (a) besides the expected global optimum at $\theta_1 = \theta_2 = 1$, several local optima are also revealed. For case (b) only one optimum, the global one, is obtained at $\theta_1 = \theta_2 = 1$. For case (c) besides the global optimum at $\theta_1 = \theta_2 = 1$, several local optima exist. Comparing the first three cases, it can be concluded that the existence of the multiple local optima in the objective function is caused by the fit in the modeshape components. In order to further identify which modeshapes cause the existence of multiple local optima, Figure 2d presents the contour plots of $J(\underline{\theta})$ for the case (a) but excluding the fit in the bending modeshapes in the objective function $J_2(\underline{\theta})$, i.e. only the first four modeshapes are kept in the second group, while the bending modeshapes are removed from the group. A unique (global) and well-defined optimum is observed in Figure 2d, which verifies that the existence of the local optima is due to the three closely spaced bending modes arising for this type of bridge structure.

The model updating is repeated using the seven experimentally obtained modal frequencies and modeshapes. Contour plots of the objective functions are shown in Figures 3a and b for the cases (a) $w_1 = w_2 = 1$, and (b) $w_1 = w_2 = 1$ excluding the fit in the bending modeshapes. One can conclude that the objective function using experimental data manifests similar behavior to the objective function observed in Figures 2a and b using simulated data. The existence of multiple local optima is due to the existence of closely-spaced bending modes, while the amount of measurement error for the identified modeshape components and the model error tend to increase the number of local optima. This increase complicates further the problem of locating the global optimum. Summarizing, the previous results demonstrate the importance of using global optimizers, such as the proposed hybrid optimization algorithm, in updating finite element models.

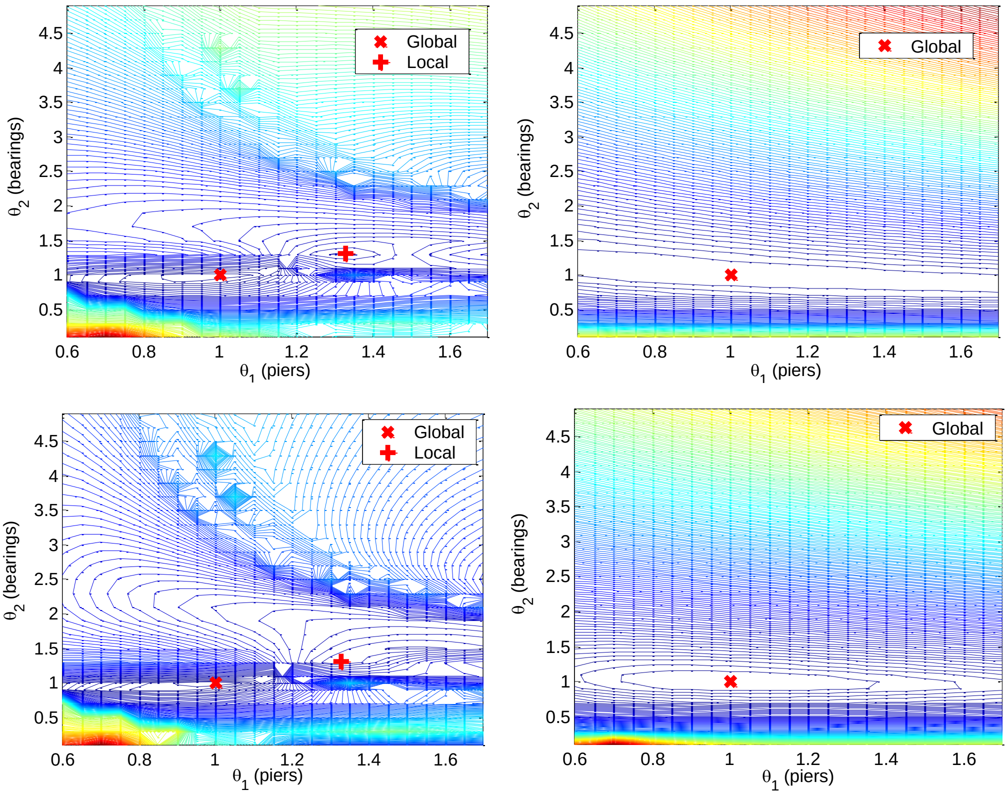

Figure 2: Contour plot of the objective function using simulated data, (a) $w_1 = w_2 = 1$, (b) $w_1 = 1$, $w_2 = 0$, (c) $w_1 = 0$, $w_2 = 1$, (d) $w_1 = w_2 = 1$ excluding the bending modeshapes from the second modal group.

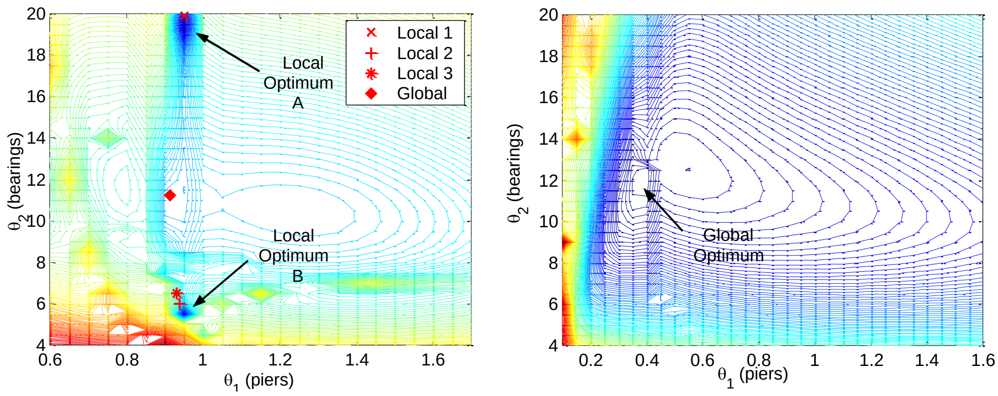

Figure 3: Contour plot of the objective function using experimental data, (a) $w_1 = w_2 = 1$, (c) $w_1 = w_2 = 1$ excluding the bending modeshapes from the second modal group.

4.2 Irregularities and discontinuities of Pareto front and Pareto optimal models

Results are next presented for the complete Pareto front and the corresponding Pareto optimal models. The objective of the analysis is to present the difficulties one may face in order to compute the Pareto front and the Pareto optimal solutions. For comparison purposes, the Pareto front and the corresponding Pareto optimal solutions are obtained using two different methods. Firstly, the Pareto front is obtained by varying the weights in the weighted residuals method. The hybrid optimization method [13], combining evolution strategies and gradient-based methods, are used to find the optimal solution of the objective function for each value of the weights w_1 and w_2. This is required in order to identify the global optimum from the existing multiple local ones.

A detailed search has been done by carefully varying the values of the weights and taking into consideration that the Pareto front is expected to be a piecewise continuous function of the weights. The corresponding Pareto front and Pareto optimal solutions in the parameter space are presented in Figure 4a and b, respectively. Pareto points are obtained inside the region denoted by I and II in these Figures. Among all Pareto points, the most important ones seem to be only two points in the objective space. Specifically, these points are $\underline{J}_I(\hat{\underline{\theta}}_I)$=[5.615e-4, 0.1426] in region I and $\underline{J}_{II}(\hat{\underline{\theta}}_{II})$=[5.95e-4, 0.1149] in region II of the objective space, corresponding to optimal Pareto solutions $\hat{\underline{\theta}}_I$=(0.8648, 11.4105) in region I and $\hat{\underline{\theta}}_{II}$=(0.9146, 11.1889) in region II in the parameter space, respectively. From the engineering point of view, the rest of the Pareto points make insignificant improvements in the fit of a modal group, while deteriorate significantly the fit in the other modal group.

Another notable region in the objective space is the one denoted by region III, confined in the domain $S \in \{5.608e-4 \leq J_1(\underline{\theta}) \leq 5.956e-4,\ 0.1426 \leq J_2(\underline{\theta}) \leq 0.1150\}$. Despite the small increments used in the weight values to generate Pareto points in the region III, the search was unsuccessful. Based on the results provided by varying the weight values in the weighted residual method, one can assume that the Pareto front is discontinuous in the region III, consisting of regions I and II only.

Next, the SPEA algorithm [13,14] is used to obtain the Pareto front and the corresponding Pareto optimal solutions. These Pareto solutions in the objective space are presented in Figure 4a for 100 and for 1000 number of generations. The solutions in the parameter space, using 100 and 1000 number of generations, respectively, are presented in Figure 4b. The number of parent and offspring elements required in the algorithm is kept constant and equal to 15 and 100, respectively. As it was expected, the Pareto front computed using 1000 generations is significantly lower than the Pareto front computed using 100 generations which means that the 100 generation are not enough to achieve convergence with satisfied accuracy.

It is worth noting that using the SPEA algorithm with 1000 generations, several Pareto points have been located in the region III. It can be shown theoretically that the first method generates solutions that correspond to Pareto points. However, the opposite is not true. That is, there are extra Pareto points that cannot be generated by varying the weight values. This seems to be the case of Pareto points generated by the SPEA algorithm in the region III in the objective space. These Pareto points do not correspond to any values of the weights and thus the first method can miss part of the Pareto solutions.

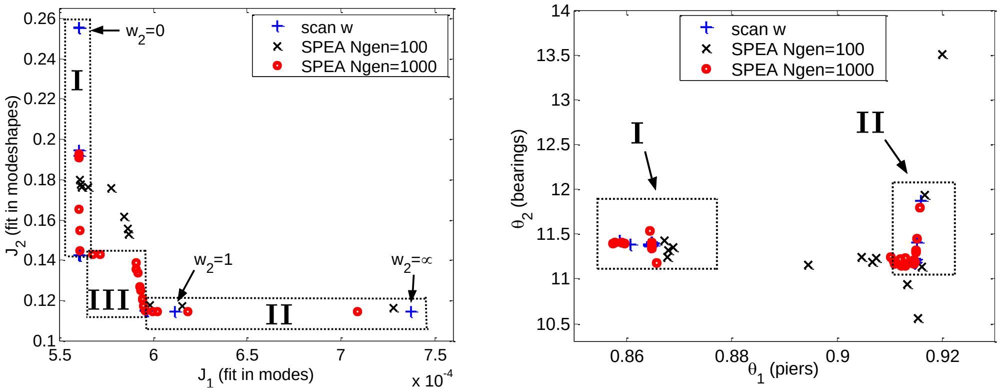

Figure 4: Pareto solutions obtained by varying the weight values and by SPEA algorithm for 100 and 1000 number of generations: (a) Objective space, (b) Parameter space.

5 CONCLUSIONS

A bridge health monitoring system using vibration measurements was outlined in this work. Algorithms for designing optimal sensor configurations, performing operational and earthquake-induced modal analysis, improving finite element models of structures and detecting structural damage, were summarized. Based on these algorithms, user friendly graphical user interface software tools has been developed to assist bridge maintenance personnel for analyzing vibrations from a sensor network and evaluating the bridge structural state. Selected results using monitoring data from a four-span instrumented bridge were presented to reveal certain features that the proposed algorithms should have in order to be effective. In particular, it is demonstrated that global optimization tools are required in order to carry out reliably the model updating and identify the global structural model from multiple local ones. Closely-spaced modes is one example for which multiple local/global solutions are manifested in the search of the optimal finite element model. The proposed multi-objective framework for model updating provides the whole spectrum of Pareto optimal finite element models and can be viewed as a generalization of the available conventional weighted modal residuals methods. It was also demonstrated that the Pareto front may have irregular behavior and may be discontinues. The irregularities and discontinuities depend on the type of the structure, the model class selected, the type and number of modal data.

The proposed framework and software can be used by highway managing authorities as a part of a intelligent bridge management system to provide a useful tool for the continuous monitoring of bridges and assessment of structural integrity.

ACKNOWLEDGEMENTS

This research was funded by the Greek Secretariat of Research and Technology and the European Research Fund, through the EPAN program framework under grant DP15. This support is gratefully acknowledged.

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