RELIABILITY APPLIED TO MAINTENANCE

by

David John Sherwin

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of

Doctor of Philosophy of the Loughborough University of Technology

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ACKNOWLEDGEMENTS

Acknowledgements and thanks are due to

1. Professor F.P. Lees for supervision and guidance when needed (and only when needed)
2. R.J. Aird who first raised the possibility that hyper-exponentially distributed failures might be due to poor maintenance and who collected the 1975 data for the chemical plant study at Section 10. He and other Loughborough colleagues also gave valuable advice.
3. The Science Research Council for financial support
4. Rita Mathews who did the typing
5. Mr. R. Bullen of Ashton Paper Mill, South Wales and Mr.H.T.Hoskins of Kent and Canterbury Hospital for data collection facilities. Other firms and individuals who helped to collect data remain anonymous by their own wishes.
6. The author's wife and family for their patience during the period of composition.

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KEYWORDS

Terotechnology, Reliability, Maintenance, Applied Statistics, Optimisation, Data Systems, Pareto Analysis, Operational Research, Mathematical Models.

ABSTRACT

The thesis covers studies conducted during 1976-79 under a Science Research Council contract to examine the uses of reliability information in decision-making in maintenance in the process industries.

After a discussion of the ideal data system, four practical studies of process plants are described involving both Pareto and distribution analysis. In two of these studies the maintenance policy was changed and the effect on failure modes and frequency observed. Hyper-exponentially distributed failure intervals were found to be common and were explained after observation of maintenance work practices and development of theory as being due to poor workmanship and parts. The fallacy that constant failure rate necessarily implies the optimality of maintenance only at failure is discussed.

Two models for the optimisation of inspection intervals are developed; both assume items give detectable warning of impending failure. The first is based upon constant risk of failure between successive inspections and Weibull base failure distribution. Results show that an inspection/on-condition maintenance regime can be cost effective even when the failure rate is falling and may be better than periodic renewals for an increasing failure situation. The second model is first-order Markov. Transition rate matrices are developed and solved to compare continuous monitoring with inspections/on-condition maintenance on a cost basis. The models incorporate planning delay in starting maintenance after impending failure is detected.

The relationships between plant output and maintenance policy as affected by the presence of redundancy and/or storage between stages are examined, mainly through the literature but with some original theoretical proposals.

It is concluded that reliability techniques have many applications in the improvement of plant maintenance policy. Techniques abound, but few firms are willing to take the step of faith to set up, even temporarily, the data-collection facilities required to apply them.

There are over 350 references, many of which are reviewed in the text, divided into chapter-related sections.

Appendices include a review of Reliability Engineering Theory, based on the author's draft for BS 5760(2) a discussion of the 'bath-tub curves' applicability to maintained systems and the theory connecting hyper-exponentially distributed failures with poor maintenance practices.

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CHAPTER 1 - INTRODUCTION

1. CONSPECTUS

In this introductory chapter an overview of the contents of the remaining chapters is given. This opening conspectus is not strictly a summary of a summary but an attempt to put the subjects discussed later into perspective.

Consider very briefly some historical connections between Reliability, Maintenance and Operational Research.

Reliability has its origins in Lüsser's analysis of the series reliability of the V1 missile of the Second World War. This was the first unmanned flying weapon; there was no pilot to correct the course if anything failed in flight. The subject developed quickly in Aerospace and Electronics and from there is spreading slowly through the Mechanical, Electrical and Production Engineering fields where it is seen mainly as a part of product Quality Assurance. It was introduced into the Process Industries through early applications in connection with nuclear safety and operability. Process Industry applications are still mainly in the field of Safety and Loss Prevention, but designers are beginning to consider the economic trade-offs between efficiency of operation when not failed, freedom from failures (Reliability) and ease of maintenance (Maintainability).

Also during the Second World War, the early exponents of Operational Research (OR) were engaged upon studies designed to maximise the availability of fighting ships and aircraft. Waddington (1.1) first describes how aircraft flying hours could be constrained by Manpower, Spare Parts, Maintenance Schedules and the Flying Programme. He continues with a report of investigations into the periodicity and content of various maintenance routines which led, where the recommedations were adopted, to more hours being flown against the enemy in a given period. Fighting services do a lot of preventive maintenance (pm) in peacetime because then their objective is to be in as high a state of readiness for war as possible. In war the objective immediately changes and requires adjustment of the maintenance schedule to maximise the attainment of operational goals. This often means that maximum long-term availability is required and that equipment operates unless required for maintenance or repair, but it can also mean building up a reserve of serviceable equipment for a large operation, by refraining from operations for a while in favour of maintenance.

Commerce, from an OR viewpoint, is more closely analogous to war than to peace. If demand equals or exceeds production capacity the parallel is obvious and maximum availability over the long-term is the objective. It is tempting to maximise availability by skimping maintenance, but this can only be a short-term expedient analogous to the serviceman's large operation because if it is continued then delivery will suffer and repeat orders will be lost. Where there is competition for orders, producers should reduce prices and improve quality and delivery to try to increase their share of a shrinking market. Jenney (1.2) has proposed the conceptual equation

Value = Quality x Delivery/Price

Maintenance considerations can affect all three factors. Without changing existing plant, one way to cut costs and improve delivery is to raise the level of maintenance to reduce unscheduled downtime. Downtime and hence delivery become more predictable as the ratio of planned and preventive maintenance to breakdown repairs is increased. It is possible that British Industry's poor reputation for delivery is at least partly due to the prevalence of failure-only maintenance -

(fm)as a deliberate, if usually mistaken,management policy. Laying off workers during a lull in business is equivalent to suing for peace, and is not necessarily justified. Justification or otherwise, depends upon the scope for improving Jenney's measure of Value for money to the customer without significant capital expenditure. Using some of the spare time to improve the plant's material state, operating availability and efficiency by maintenance and minor modifications to plant and procedures may be better policy because it permits higher quality productsto be offered at shorter delivery for lower prices. Furthermore, a plant which is allowed to stand idle or deteriorate during a slack period will be unable to take prompt advantage of market recovery.

The wider the horizons are drawn in time and space the more convincing the arguments for organised maintenance become. At a national level neglect of machinery probably lowers product quality and diverts export capacity to premature renewal. On a world-wide basis it is an unnecessary waste of irreplacable resources because even if the scrap is recycled energy must be expended and some materials dispersed beyond economic recovery. See also Sherwin (1.3).

What material and procedural changes are likely to influence maintenance efficiency in the context given above? What maintenance schedule will maximise long-term availability? These questions can be answered by processing accurate numerical data and qualitative information through a model. Given an accurate model of the reactions of the plant to imposed variations of treatment such as adjustment of the maintenance effort, only one data-set need be processed. In practice however, some uncertainty ,will always remain and a full programme, even under steady external conditions of market, taxation, and competition should include a check. This is achieved by collecting more data after perturbing the system to see whether it has reacted as predicted. Under fluctuating conditions, re-assessment of optima may need to be continuous, periodic, or in wake of major changes. If either the data or the models are inaccurate, the changes will not have the calculated effect. If the data is inaccurate and not known to be so, the position is even more serious, because the check may not reveal that the policy changes have been ineffective.

A full programme of data collection, modelling analysis, synthesis of optimal conditions, change of policy to calculated optima, and back to data collection to check that it is working out as planned is time-consuming and expensive. Its cost must therefore be set against the potential benefits. The major expense is data collection. But some data collection is necessary to meet legal accountancy requirements and to retain primary control of the plant. Data for Reliability and Maintenance optimization purposes should be costed as marginal to the essential data costs, and other benefits of a comprehensive data system (possibly computerised) should be considered, before rejecting the whole idea as too risky and expensive. Pilot schemes may aid confidence.

Maintenance is generally agreed to be a Cinderella among engineering functions (1.4). In the recent relatively hard times it has begun to attract more attention, particularly through the fairly new concepts of Terotechnology and Life Cycle Costing. The measurement of maintenance performance by calculating ratios of man-hours costs etc., has become quite popular (1.5,2.41) but in too many cases the objective has been to cut maintenance costs without due regard to the effect upon the plant durability and availability. The approach through Reliability Engineering has the advantage that these factors are not omitted. There has been much recent activity in this field also as readers of "Management Science", "Operations Research", Operational Research Quarterly", "IEEE Transactions on Reliability", and "Microelectronics and Reliability", to name but a few, will have noted. Many potentially useful techniques of optimization await the collection and analysis of data, without which they cannot be effectively applied.

The rapid development of techniques, some very sophisticated, is in stark contrast to the simplistic view taken by many industrial maintenance managers. They aver that pm only disturbs equipment unnecessarily and leads to maintenance-induced failures. Much better, they say, to wait until it fails. Actually, training of maintenance personnel and inspection of work by competent supervisors can substantially reduce maintenance-induced and secondary failures, which are more likely to occur under the pressures arising from failure-only maintenance (fm) than under pm. There is theoretical and practical evidence that scheduled or periodic pm (ppm) is effective for components which wear and that scheduled inspections with on-condition repairs (ocpm) is an economic policy for components which fail in a more random fashion but which give some warning of impending failure. Provided that a fm action costs in toto on average more than a pm action optimization either by ppm or ocpm will be possible given relevant data. This means that 'laissez fail' policies (fm) are seldom justified. Some plants in both manufacturing and process industries have standby machinery for vital functions to improve availability and safety and to provide for maintenance without shutting down. Other plants may consist of duplicate lines giving partial redundancy i.e. reduced output during repairs and maintenance. Another way of providing a standby is to keep buffer stores of intermediate products, to decouple the stages of a process. Economics of production without regard to terotechnological considerations suggest large single-line plant, whereas Reliability Engineering counsels caution in increasing the scale of plant from one generation to the next.

The overall conclusion is that Reliability theory can be applied with advantage to Maintenance problems using Operational Research techniques and models, but that a sound data base is necessary.

2. SYMBOLS AND ABBREVIATIONS

A coherent system of symbols and abbreviations is used throughout. These are listed at Appendix D. In addition all such symbols and abbreviations are defined in context on the first occasion of use.

3. DATA REQUIREMENTS AND COLLECTION

The section under this heading begins by discussing the ideal data system, ideal that is from the viewpoint of the analyst whose aim is to derive statistics upon which to base terotechnological decisions. Statistics need also to be related to qualitative information about the results of inspections and the manner of failures because schedules must state what is to be done as well as when. Recognising that complete data will not always be available, an analysis is given of what derived statistics can be obtained from various classes of data and what in turn can be derived from various combinations of such statistics which will help to improve maintenance performance.

This discussion is followed by the first major report and analysis of the thesis which concerns a three-year project at a chemical plant. In this experiment, as in the minor study of autoclaves at a hospital which follows it, data was collected and analysed both before and after the maintenance system and plant configuration had been altered, so allowing objective comparison. The first study shows that it is possible to improve plant availability by adjustment of maintenance policy although it also demonstrates how difficult it can be to make truly scientific field experiments in maintenance.

Both studies involved frequency analysis as well as calculation of mean time between failures (mtbf) and showed that tbf frequency distribution, mtbf and relative prevalence of various failures modes could be affected by maintenance policy. Many instances of hyper-exponentially distributed, (hyper-exponential in the sense that standard deviation exceeded mean, rather than of any particular parameterisation having this feature), failures under failure-only maintenance (fm) were found. Other workers have remarked this phenomenon and some of them have offered much the same explanation, that it arises from maintenance deficiencies (1.6), but these are believed to be the first experiments to show that a hyper-exponential distribution may indicate short-comings in maintenance practice. Even then the evidence is somewhat inconclusive, due to management at the plants concerned declining to implement all recommendations prior to the second period of data collection, in particular that repair work should be inspected before 'closing up'. A paper (2.48) has been prepared for publication on this work. Two more analyses also include the hyper-exponential distribution.

The data analysis methods used in these studies are not new, (although the cumulative hazard method of Weibull analysis is not as widely known as its usefulness would seem to warrant). A document(1.7) prepared for the British Standards Institution for incorporation in a forthcoming Standard Guide on Reliability is reproduced in part as Appendix A. This gives the author's views on the analysis of Reliability data under ideal circumstances while the procedures actually used recognize the limitations of both the data quality and the facilities which were available to the maintenance staff at the works concerned.

Appendix B is a theoretical discussion of how exponential, hyper-exponential and wearout tbf frequency distributions might be expected to arise in data analyses. It also contains a demonstration that only a very small proportion of early failures is required to give a distribution plot which is initially hyper-exponential. There is little really original material in Appendix B, but a new chain of reasoning has been forged from existing theoretical links.

4. MODELS FOR MAINTENANCE OPTIMIZATION

In the chapter under this heading is a review of some of the extensive literature of maintenance models. The papers reviewed are those considered important, those not reviewed elsewhere and those relevant to the two new models which are later developed. A list of other papers is also given in the References Section.

The first model assumes that inspections at intervals of constant risk are approximately optimal and that the probability of failures despite inspection is a function only of the risk. It is revealed by working many examples through this model that it is not necessary for the hazard rate function (failure rate) to be increasing for an optimization to be possible. The model is designed for auto-correction of the schedule as the reliability and cost parameters change with equipment age. The work models the practical observation that equipment reliability in service is improved if repair and maintenance work is independently inspected before closing up. If an initial inspection coincident with fm or ocpm is omitted, model cost—rate usually rises. The criteria of optimality in this model are that the ratio of the mean cycle cost to the mean cycle time should be a minimum. (a cycle runs from one repair or ocpm to the next) and that this cost rate should be less than that for the best ppm policy which is otherwise preferred. Clearly, as the pdf of the distribution of tbf's under fm becomes more peaky, ppm is more likely to be optimal but an important result is that optimal schedules are possible for exponential and hyper-exponential distributions. As is so often the case in Reliability, there are simplifications if an exponential distribution can be assumed.

The second model compares ccm and ocpm for constant failure and repair rates (exponential distributions). In order to obtain first order Markov matrix models it is assumed that all the other state transition rates are constant. This, for example, makes the time between inspections an exponentially distributed random variable rather than a constant. The algebra of this model turns out to be relatively simple and an efficient computer code for making a cost comparison between the models was easily written. Markov models have the disadvantage that when transition rates reach the limiting values of 0 or 1 the matrix must be altered and the algebra with it. It was therefore necessary to consider some special cases separately. This model was made to solve a practical problem and has since found other potential applications in maintenance policy-making and scheduling.

Papers have been published on both models.(3.230 , 3.231).

5. REDUNDANCY AVAILABILITY AND INTERSTAGE STORAGE

5.1 General.

Any review of reliability applied to maintenance would be incomplete without some mention of the effects of redundancy, and storage of intermediate products. In this thesis the coverage of these aspects is intentionally thinner than that given to data analysis and maintained system modelling. This is not because these topics are considered in any way less important.

Established redundancy theory is reviewed in Appendix A. The following topics are briefly discussed below and at greater length in Chapter IV.

a) Reliability of large single-stream plants

b) Throughput and availability

c) Interstage Storage effects.

5.2. Large Single-Stream Plants

Large single stream plants get ordered because of the well-known roughly two-thirds power relation between initial cost and rated output, coupled with increasing demand and price pressure from competitors' newer (and usually larger) plant, (1.8). The reasoning here does not include Reliability considerations, but the capital savings are large and the 'big-is-best' argument is difficult to resist. Increase in size or speed must, however, involve a risk that the economies of scale will be eaten up by unreliability, particularly in new technology items. In the short-term, any'teething problems'with new equipment extend the pay-back period, and long-term unreliability erodes the carefully-calculated profit margins.

A multi-streamed plant on the other hand can be built one stream at a time,expanding to meet demand using initial profits as secondary capital. Cross-connections can be provided to make use of partial redundancy to counter the effects of breakdowns and to enable preventive maintenance to be performed without a total shutdown. Delivery is more predictable and the plant more flexible in the face of fluctuating demand.

5.3 Throughput and Availability

Plant managers usually define the 'availability' of a plant as the ratio of achieved output to rated output rather than as mtbf/(mtbf+mttr) This 'throughput availability' is more difficult to calculate in a partially redundant system than the usual reliability engineer's availability because for every possible plant state of items available, or not, the probability and output must be calculated. Systems which have redundancy at full output or only partial redundancy (effective at reduced outputs) have a higher and less variable throughput availability than a simple series system.

5.4 Interstage Storage

The provision of interstage storage for intermediate products allows production to continue behind a failure until stores there are full, and ahead of it until downstream stores are empty. The effect is to decouple, to an extent depending upon the relative capacity of the stores, the series dependency of the stages, so giving an increase in long-term average rate of output. The marginal capital and maintenance costs of extra storage probably decrease with store capacity because of the two-thirds power law mentioned above but the availability gain is subject to rapidly diminishing returns.

6. CONCLUSIONS IN BRIEF

6.1 Layout

The conclusions drawn from the work presented in the thesis are explained in detail in Chapter V. The principal conclusions are repeated without explanation below. They fall under four headings, namely

a) those depending mainly upon Chapter II with theoretical support from Appendices A and B.

b) Those depending mainly upon Chapter III which is to say that they depend only upon theoretical considerations and should be considered tentative until practical application excercises have been conducted.

c) Those depending mainly upon Chapter IV and subject to the same stricture as above(b).

d) Those which draw upon evidence from more than one chapter.

6.2 Conclusions based on Chapter II

a) The introduction of preventive maintenance to a system which has been operating under failure maintenance is likely to result in overall savings.

b) The expected form of the observed distribution of times between failures of a complex item is exponential. Departures from this form are prima facie evidence of room for improvement in the maintenance regime.

c) The overall observed failure rate of a complex item is sensitive to the maintenance regime under which it operates and its useful life is affected by the number of components or modes of failure which are covered by preventive maintenance.

d) The collection of data on failure modes, times between failures, times to repair, and maintenance work is of potential benefit to operator, maintainer and manufacturer. Reliability data collection should be costed as marginal to the cost of collection of basic management information and should be done by means of a common system which avoids duplication of effort and in which all calculations are made centrally, preferably by computer.

e) Numerical data analyses can be misleading by themselves. It is necessary to observe what is done when equipment is maintained if mainteanance practices are to be improved. Pareto or failure modes analysis is necessary for the assessment of scheduled maintenance.

6.3 Conclusions based on Chapter III

a) The usual O.R. conclusion that scheduled maintenance is not worthwhile if the hazard rate is constant or falling is not always applicable to complex items or where impending failure can be detected by inspections or continuous monitoring.

b) Preventive maintenance cannot be worthwhile unless the mean total cost of a failure including downtime etc. is greater than the cost of preventive action.

c) The cost-optimal maintenance policy for a given item may be failure maintenance(fm), periodic preventive maintenance (ppm) or scheduled inspections and on-condition maintenance (ocpm) depending upon the base distribution of times between failures (tbf's) and the various costs involved. It is not usually possible to determine which type of policy will be best without calculation.

6.4. Conclusions based on Chapter IV

a) It is possible that reliability considerations are not given sufficient weight in deciding between multi-stream plants using existing technology and large single-stream plants which inevitably call for new designs which may have to be installed without reliability testing.

b) The provision of storage for intermediate products is an alternative to redundancy for raising plant throughput availability which should be investigated in suitable cases. Many factors are involved besides availability such as safety and working capital aspects of large inventories, in coming to a decision between redundancy and interstage storage.

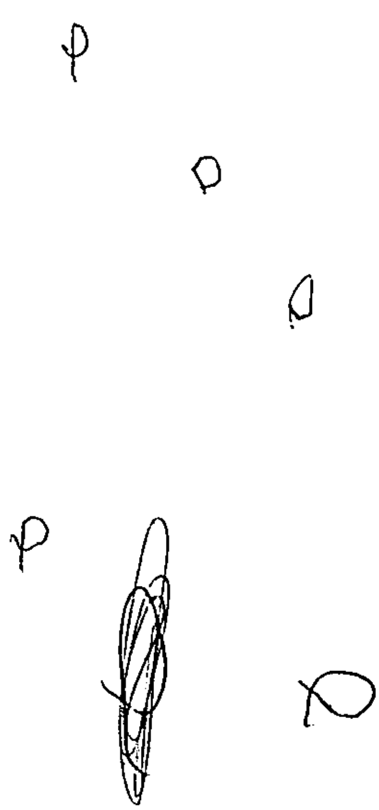
c) The literature contains papers which explain how to calculate the optimum distribution of interstage storage. Against most criteria increase in storage is subject to diminishing returns.

d) Generally, more storage should be placed immediately downstream of a stage with poor intrinsic availability (mtbf/(mtbf+mttr) than after one with high availability.

6.5 Conclusions from More than one Chapter

a) Contrary to the prevailing trend in British process industry practice, preventive maintenance according to schedules based upon the continuous collection and analysis and feed-back of failure and maintenance data is on balance considered more likely than not to lead to financially beneficial outcomes for both the operator and the manufacturer of the plant.

b) A lot of theory has been developed for optimising maintenance



regimes with respect to either cost or plant availability which can be usefully applied only where there is a data collection system capable of generating the parameter values required to produce schedules from the theoretical models.

c) The maintenance regime imposed upon capital plant together with the operational intensity largely determine equipment life. The availability of differential tax concessions and grants may be obscuring the underlying cases for more intensive preventive maintenance of existing plant and for better inherent long-term reliability and maintainability in replacement machinery. The total effect of maintenance activities upon the nation's prosperity (or upon even wider economic matters) must rely ultimately upon the real resource costs undistorted by such grants and taxes.

7 REFERENCES

Lastly there is a classified list of references to papers and books relevant to the subjects discussed in the Chapters. It was always a part of the project to compile a list of references useful in the study of Reliability and Maintenance Optimisation. More references are given than are cited and reviewed in the text and the author does not pretend to have read further than the abstract in some cases. The classification follows the chapter headings with an additional section concerned with matters discussed in Appendices A and B. Specific references are not given in Appendix A because this is based upon a text prepared for the British Standards Institution and it is not their custom to cite sources in standards. The references are in alphabetical order within the classes into which they have been divided, apart from a very few late additions.

CHAPTER II - DATA REQUIREMENTS AND COLLECTION

8. INTRODUCTION

8.1 Order of Presentation

It is usual to review the literature before describing ones own experiments and to propose an ideal system on the basis of both. In this case the order of writing has been reversed. The 'ideal' system described in Section 9 was born out of experience rather than reading. It is described first lest it be thought that the methods actually used in the data collection and analysis experiments described in Sections 10 and 11 were thought to be in any way ideal. They were not ideal but they were the best that could be managed with the co-öperation of the plant managements concerned. The work of others is placed last because it is used mainly to confirm that features observed in Sections 10 and 11 were not extraordinary and to point out and discuss various nuances of interpretation. It is submitted therefore that reversal of the usual order is convenient and logical in this particular case.

8.2. Appendices

Theory relating to this chapter is described in Appendices A and B. Appendix A is based on part 2 of the forthcoming British Standard Guide on Reliability (BS5760) and describes methods of data analysis. It was drafted for BSI Committee QMS 2/3 by the author. Appendix B deals with an amended theory of the 'bath-tub curve' with the interpretation of the hyper-exponential distribution as an indicator of maintenance deficiencies,and with the estimation of the proportion of early failures in bimodal and hyper-exponential distributions.

8.3. British Standard 5760 Part 1.

The forthcoming British Standard Guide on Reliability (BS5760-Part I "Reliability Programme Management") is a general guide on reliability management written mainly but not solely for the manufacturer of goods for sale rather than the engineer charged with the maintenance of existing plant. The writer is a member of the responsible British Standards Institution (BSI) Committee (QMS 2/3) and was responsible for the inclusion of some passages which deal specifically with maintained reliability and for altering some others so that they covered both maintained and consumed items, recognising the connections between maintenance, plant availability, and product quality. At the time of writing the committee had agreed the content of the Standard Guide but the BSI's editors had not produced a final text. So the phraseology may differ in the published version, from paragraph 8.4 below, which in any case expands a little on the draft.

8.4. Benefits of Data Collection and Analysis

Knowledge of the behaviour of an item or plant is required so that:

a) Effective action can be taken to improve the reliability of present and future items and plants. Reports are necessary to see whether the specified reliability and related factors have been achieved. These reports may lead to modifications to improve Reliability, in which case further reports will be required to monitor progress. Reliability data feed-back from service is vital to effective Quality Assurance of current production of durable items.

b) Improvements can be incorporated in future designs. Detailed qualitative data on the various ways in which items have failed in service and their effects (failure modes and effects) is required together with their absolute and relative frequencies of occurrence. This can lead to modifications to improve the Reliability of the item which are of commercial advantage to its manufacturer and his customers.

c) Safety can be objectively measured, monitored and improved. In particular, such safety surveys can be compared with initial assessments as an aid to improving the accuracy of future pre-service safety assessments.

d) Maintenance schedules, which may involve both periodic and on-condition actions can be improved with respect to plant availability or cycle costs.

e) Holdings of spare parts and special repair tools can be adjusted to an economic level.

9.THE IDEAL DATA SYSTEM

9.1 Ideal for What?

The ideal data system for a plant is that which gives the best return on the investment and running costs. Paradoxically the ideal system cannot be found without first imposing a data system which, for a period, collects and analyses more data than the ideal system would later require. This more detailed data is required to set up and validate a model of the plant's reactions to changes in maintenance policy. If the underlying reliability and maintainability characteristics of items did not change, albeit slowly, over the life of the plant a once-for-all exercise would be sufficient to devise the ideal maintenance policy.

In practice,however, plant is modified and those components which are not renewed by any routine maintenance eventually start to wear out. To retain optimality in the maintenance schedule it is therefore necessary to continue to monitor plant item reliability and maintainability characteristics on a more or less continuous basis. It is strictly not necessary for this feed-back to contain all the parameters of the model upon which the maintenance policy is based, it is sufficient that there are enough to signal significant changes. Special data collection exercises can then be mounted to find out why and subsequently to re-optimise the policy. Of course, the time constant of this data feed-back control loop will be shorter if all the model parameters are catered for in the day-to-day system. During the delay the policy is sub-optimal,which involves a loss, to be set against the cost of extra data collection and analysis.

Because most plant items must be highly reliable if the reliability of the plant as a whole is to be acceptable, it will in general take a long time to collect a viable sample of extra data. On the whole, therefore,it is considered better to collect all the data that might be needed all the time. Parsons (2.37) disagrees, he believes that the British Army overdoes data collection. For control purposes simple analyses giving easily-understood figures of merit such as mtbf and mttr will usually be sufficient, it is only when these simple statistics are seen to be changing that deeper analysis is required to reveal the reasons and regain control.

On these premisses the ideal data system is one which most efficiently provides all the quantitative and qualitative data that might be required for finding and updating the optimal upkeep schedule. This is an ideal which is seldom if ever realised because managers are unwilling to take the step of faith required to set up the system.

9.2 Organisation

The means of collecting reliability and maintenance (R & M) data should ideally be centred on an independent department or section which also collects other data and information such as costs and spares usage and ordering needed for the management of the plant. The existence of two or more collecting and collating agencies for data and information is likely to lead to inefficiency, duplication and anomalies both real and apparent. Where R & M data are not already collected, expansion of the existing management information system is usually preferable to setting up an entirely new system if only because the marginal cost of adding to an existing system is likely to be less.

The results of data analysis should be fed back not only to the maintenance organisation but also to the designers and manufacturers of the plant. It is particularly important that the R & M data sets failure in the proper context relative to the schedule of preventive maintenance and the extent of its achievement. In particular it is vital to the progressive improvement of maintenance schedules that records are kept of when inspection routines led to on-condition repairs and that scheduled component renewals are recorded especially when the schedule is not strictly observed. Experience has shown that it is difficult to collate separate records of failures and preventive maintenance and dangerous to assume that a maintenance schedule is being observed without positive checks. A Naval Rating was recently court-martialled for falsifying preventive maintenance records.

9.3. Communication and Labour Relations

It is not unknown for Trade Unions to forbid their members to fill in the forms required for a R & M data system. This is an extreme symptom of lack of management sensitivity to the need for people to understand what they are doing and why it is necessary. Direct contacts between data system personnel and the craft supervisors (who usually end up doing the bulk of the paperwork) are necessary. Education and Training programmes for the technicians and for those whose efforts are being monitored are also necessary. The aim should be initially to convince those concerned that the system offers no threat to their way of life and work and later to show them that worthwhile benefits have resulted from the effort made earlier.

Duplication of recording should be carefully avoided because experience (the writer was a member of a team developing a computer-based upkeep data collecting system for the Royal Navy(SUIS)), has shown that this leads to the most resentment. In an integrated system where R & M data is collected with spares usage, hours worked, lost time, costs and other management information it is easier to avoid duplication. Nor should the system require the unnecessary collection of information that is seldom used. It is quite possible in most cases to restrict the regularly required information to a form of about A5 size as was done for the Royal Navy Jobcard, but many other systems (e.g. the Army R.A.F, and British Airways systems) involve much larger forms. A form of reasonable size given the circumstances but above all a single form covering all requirements is considered best.

9.4 Types of Data

Note: This section is also based loosely on the draft of BS5760 Part I.

a) Library File

A comprehensive description of each item of plant is required for unique identification. In analysis it is frequently necessary to form populations of more-or-less-like items in order to obtain a statistically viable sample. It should however be possible to be precise about differences of build, construction, and environment between individual members of such groupings in case the analysis reveals that there are sub-groups distinguishable by their different behaviour in service. For example, one might analyse data arising from all centrifugal pumps at a plant and find that the distribution of times between failures had two modes, one for fresh water pumps and the other pumps handling more corrosive fluids.

Given this file, items can be identified by a simple Yard Number or a brief description (unambiguous) on subsequent paperwork.

b) Configuration Control

As a result of service experience, machinery may be modified. As a result of maintenance or repair by replacement (R x R) an item of plant may be returned to a different service. Plant layout may be altered. A system for recording these additions, modifications and movements is required as well as the history file. In the present context it is important to monitor changes in R & M characteristics following modifications to item and plant configuration.

c) Maintenance Schedule

A detailed record should be kept of the schedule of maintenance involving ppm, ocpm and ccm which the management wish to be performed on each item. Changes in the schedule can affect the availability of the items concerned and it is important to be able to associate such changes unambiguously. An advantage of scheduled items is that they can be exactly described in the schedule and then referred to easily by code numbers. It is however, essential to a full record that the actual achievement of the schedule is recorded in the History or Event File (see below) and that when a conditional routine leads to a renewal or adjustment this is also so recorded. It is possible to make out a case for changing the planned periodicity of ppm events if the condition of the components pre-emptively renewed or the clearances or conditions before adjustment are noted. Otherwise such a case must rest upon evidence of failures between renewals which will take a long time to acquire if the schedule is more frequent than optimal and may be confused by early failures induced by poor maintenance (see Appendix B).

d) <u>History (Event) File</u>

R & M characteristics can be calculated from a record of the running maintenance and failure history of each item of plant. The statistics which may be needed can be calculated from records of the calendar times at which items moved from one to another of the following states:

i) Running satisfactorily

ii) Running with performance impaired(may be further subdivided)

iii) Failed and under repair

iv) Stopped and under <u>pm</u>

v) Failed awaiting spares

vi) Failed awaiting labour

vii) Failed awaiting administrative clearance, or job stopped overnight etc.

viii)Shut down, not required, or standby (Free time)

For purposes of easy reference and the calculation of ratios useful in comparative studies define

Uptime = (i) +(ii)+(viii) t_u

Downtime = (iii)+(iv)+(v)+(vi)+(vii) t_d

Running or Operational time = (i) +(ii) t_{op}

Waiting Time = (v)+(vi)+(vii) t_w

Active Repair Time = (iii) t_{ar}

Active pm time = (iv) t_{pm}

Active Maintenance Time = (iii)+(iv) t_{am}

Lost time = (ii) where there is a loss of production t_L

NOTE: Where there is only partial loss of production t_L is the notional time that the whole plant would have been stopped for to give same loss.

In some systems the information on what exactly went wrong or was found wrong on inspection is coded (as in the major data exercise described below). This may be the best that can be managed in a large hand-recorded system but a computer can be programmed to pick out key words from a short plain language description of the defects and their effects on plant operation. This method allows more freedom when making out reports, and avoids some of the ambiguity and errors of a coded system because the whole text can be checked in cases of doubt.

9.5. Integrated Management Information Systems

Referring to Figure 9.2 a system which facilitates all the calculations which may be required for R & M purposes also incidentally makes possible other calculations in the general area of Management Information. If a little more data is recorded a single form need be the only document returned by maintenance staff covering accounting, wages, job control and planning, R & M, and spare parts usage, re-ordering and recording. A suitable format is shown in Figure 9.1 .

Such a system allows more flexibility to the accountant, who can attribute costs by items, or groups of items, by individuals or groups of workers. e.g. by trades, or skills, to preventive and corrective measures separately and so on. It should not therefore be difficult to persuade the accounting function at a particular plant that such a system would be desirable. Actually accountants' attitudes vary from enthusiasm (2.43) to downright obstruction.

As discussed above Trade Union objections may be more difficult to overcome than those of the accountants. The workers' representatives will suspect that the personal attribution of jobs in a data system would lead to 'victimisation' and that wages might be held back until paperwork which 'infringed their privacy' was completed. Actually, plant is rarely over-maintained and the likely result of full control of the maintenance system is more jobs, higher wages or more overtime or all three. The personal attribution of jobs allows those workers who need it to be retrained, not dismissed, and those who cannot cope to be found more congenial work. It also permits the company to reward workers in relation to the value of work done.

No management was found willing to risk trying such a system, and the reason given was always anticipation of labour relations problems, sometimes with accountants' objections as well.

9.6 Input Requirements

9.6.1 The Types of Job to be reported may be classified as follows:

a) Urgent emergency repair of failures, usually those where the failure causes the whole plant or a production line or important auxiliary function to stop. They are dealt with as soon as labour and materials can be mustered.

b) Planned repair of less urgent failures, where the equipment has a standby or the function is not vital. Usually pm will be brought forward to save or reduce a later routine stoppage.

c) Planned repair of defects not amounting to failure. The item runs at reduced but tolerable performance until convenient to repair. These also are usually repaired in conjunction with pm, possibly at the scheduled time for

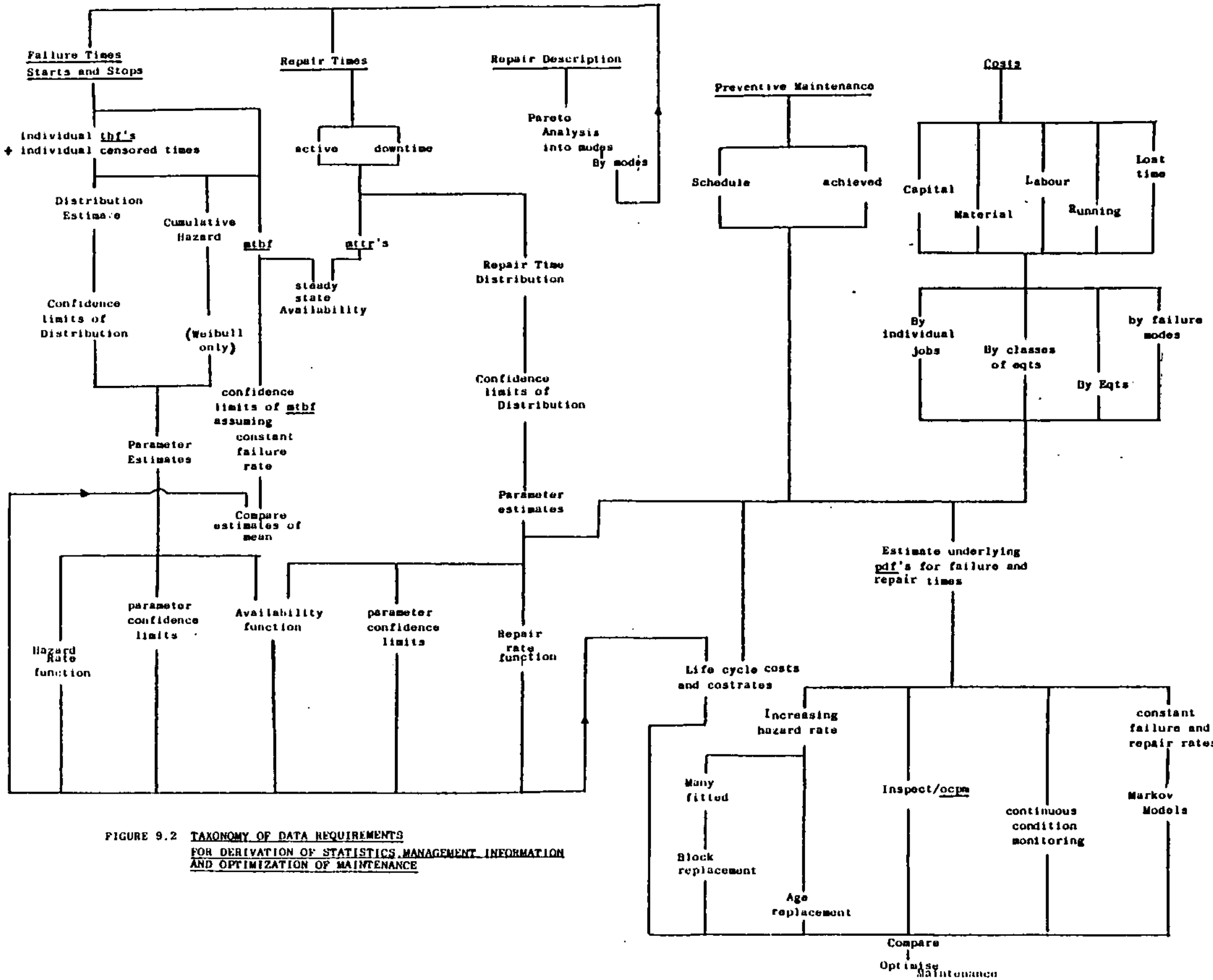


FIGURE 9.2 TAXONOMY OF DATA REQUIREMENTS FOR DERIVATION OF STATISTICS MANAGEMENT INFORMATION AND OPTIMIZATION OF MAINTENANCE

the pm, possibly at some intermediate compromise time. There is often the chance that such a defect will degenerate to a failure of type (a) before the convenient planned time is reached.

d) Periodic Preventive Maintenance (ppm). In this type of maintenance parts are renewed, adjustments made and other actions taken without regard to the condition of the item on a strictly periodic and pre-emptive basis.

e) Inspection and On-Condition Preventive Maintenance(ocpm). In this type of maintenance the item is inspected, possibly involving some dismantling, but its condition is not altered unless this is judged to be necessary as a result of the test or inspection. For example the condition of fan bearings might be judged by comparing the time taken to run down after switching off power with the corresponding time taken just after the last renewal of the bearings. If the time has shortened by more than a prescribed percentage the bearings would be greased and renewed if worn. (Example from Canadian Armed Forces practice).

f) pm brought forward to coincide with failures and defects. (see above)

g) Opportunity Maintenance i.e. doing deferred work (see (b) and (c) above) when the line or plant is stopped for an urgent repair to another item.

9.6.2. The Costs involved in a Job may be classified as follows:

a) Labour costs - subdivided by trade and degree of skill, ordinary and overtime, contract, payment by results etc.

b) Materials - subdivided into consumables and specific spare parts.

c) Maintenance overheads- calculated as a factor or multiple of (a) or (b) or (a + b)

d) Lost Time Costs - The true cost of lost production due to a failure is the marginal profit on the production foregone. Some pm items also cause lost time and not all failures do so. Where the loss is partial or shared with other jobs due allowances should be made by apportionment. Where all members of a set of redundant items are down together the Lost Time is calculated from the last failure to the first re-start.

9.7. Job Card Format

The suggested Job card Format at Figure 9.1 is designed to cover all the requirements discussed above whilst asking least of the supervisor or foreman who must fill it in. It can be used for failure repairs, preventive maintenance and for jobs which are a mixture of the two. If front and back of the form are used it need be no more than 180 mm x 148 mm (A5). If copies are considered necessary, and strictly they are not, it is probably better to have the form as one (A4) sheet.

The supervisors who fill in the form are not asked to make any calculations. Apart from matters which only they can know they do none of the coding in the right hand column; this is intended to be done by specially trained people with technical knowledge.

The form is designed for computer use, it being intended that the computer keep the records and provide the derived statistics as required. The computer would also be coded with the system reliability diagram and so able to work out from the cards whether or not system lost time occurs and for how long. The idea is to make the computer do the work rather than burdening busy supervisors with calculations and unnecessary questions. A disadvantage of this system is that there are no independent checks of data by comparing results from two collection systems. It should therefore be impressed upon all that a lot depends upon accurate returns, including the accuracy of their wage calculations!

When standbys are changed over or items stopped or started without work being done it will be necessary to raise a Job card or another special form.

The Jobcard is not intended to cover modifications and system alterations which would need a special Configuration Control form for the change plus a Jobcard for the actual work.

In the example shown in the pro-forma at Figure 9.1 , the following styles are used to distinguish whose function it is to enter the required data.

Maintenance Supervisor	I M Charge.
Coding Office Technician	79 MM3
Stores Clerk	J V Nostock

Some examples of calculations which can be made on the basis of this form alone are

Time awaiting labour (skilled mechanical) 19 hrs 30 minutes

Active Repair Time 6.30 + 1.25 = 8 hrs 5 minutes

Down Time 22 hours

Admin.Time (Stopped Overnight) 22 hrs - 8 hrs 5 mins = 13 hrs 55 mins

If the pump is reported started before JC No. 87655 is completed the computer will obtain the time running with performance impaired from the two cards. The total running time since the last failure will be calculated and added to the file of tbf's from which the mtbf's and distributions for this pump or for any grouping of pumps can be obtained on request. Similarly for the active repair times and down times.

FIGURE 9.1 JOB CARD FORMAT

Front

Serial Number/Sheet No/No.of sheets	87653/1/1
Machine Identification Pump, Water	
Machine Stopped Date 31.10.79 Time 18.50	79311-1850
Failure/PM/Deferred Failure Deferred.	FD
Start Job Date 1.11 79 Time 14.20	79312.1420
Reason for Delay Mech. fitters all busy	LMF
Finish Job Date 2.11.79 Time 12.20	79313.1220
State of Machine at Finish. Run/Standby/Failed	S
Serial Number of Further Job Card	87655
* Description of Work New rotating assembly fitted, balance checked New mechanical seal fitted. Still needs new rings	PM Routines 4M3 8M1 2A2
Possible Causes Oil contamination leading to bearing failure	BOC

Back

Name	Staff No.	Ordinary Hrs	Ordinary Mins	Overtime Hrs	Overtime Mins	Overtime Rate	
Bloggs T	MF63	6	30				MF 6.30
Jones B	MM 72	6	30				MM 6.30
Sparks E	EF 83	1	25				EF 1.25

Signed.. Supervisor I M Charge.

* Spares Used Description.	No.	Identification	Real Cost
Shaft	1	768943	14.50
Impeller	1	768945	22.35
Mechanical Seal	1	768900	[illegible]
Bearings	2	768902	14.63
Spares Ordered for further Jobcard			
Static Rings	2	768903	
Signed.. [illegible] Stores Clerk			

* Continue on second sheet if required.

9.8 Supporting Pro-Formas

9.8.1 Suggested Formats for the forms discussed below are not presented because it is felt that they should for preference be specially designed to suit the industry or even the individual factory. Where control is local, that is decisions on maintenance and renewal policy are taken on site, the Jobcard may be sufficient for all purposes and could perhaps be even further simplified. Where the works is part of a conglomerate or international company with many interests and financial decisions are taken elsewhere, there is a clear need for standardised reporting documentation to ensure that all relevant facts are objectively reported to the decision-maker when he is urged to make a change. The central controlling department is able to assess data coming from other similar plants and may wish to initiate reports from these plants in order to judge whether a problem is local or widespread. Forms are suggested for the following purposes.

9.8.2. Configuration Control. It is not possible to make objective judgements about comparable items unless the extent of their comparability is known for certain. All modifications to plant should therefore be known. Much time is wasted every day by Maintenance Engineers whose drawings and configuration documentation is out-of-date or missing. Where safety is concerned such documentation is a vital management safeguard and needs to be formal.

9.8.3. Reports of Defective Manufacture or Design. Detailed reports of repetitious failure modes with full technical assessment, the results to be fed back to designers and purchasing engineers and plants instructed as to action both immediate and long-term. Central control should not act on an isolated report but seek supporting evidence by requiring other plants to report on the same equipment using the same form.

9.8.4. Proposals to Amend the Maintenance Schedule. Whilst some such proposals will arise from data analysis revealing that an inspection is being performed at non-optimal times, the case of ppm requires reports when the schedule is too frequent. It is also desirable that individual maintenance managers feel able to influence the schedule directly. The most important use of this form however is to influence what is done and how to go about it rather than how often.

9.8.5. Alteration or Modification Proposals. It is important not to alter a plant layout or modify a machine without formally consulting the designers who alone know why matters were arranged as they are. The form should contain objective information about the present deficiencies and the advantages expected from the change. There should be a section for technical details of the proposed change which may be filled in by the proposer or left to be completed by the design section.

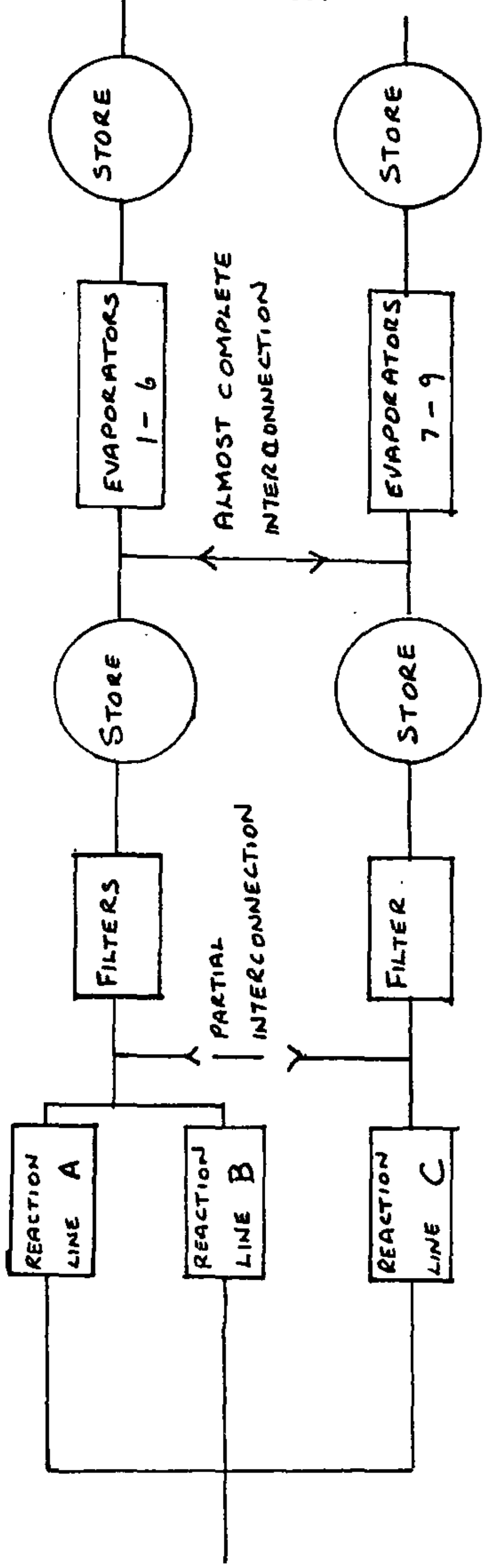


Figure 10.1 Chemical plant flow diagram

10. MAJOR DATA COLLECTION AND ANALYSIS EXPERIMENT

10.1 Introduction

The principal study was conducted on a plant making an acid. The phases of the study were those described above, namely the specification and establishment of a data collection system, the determination of the failure regime of particular equipments, a review and, where appropriate the modification of the maintenance policies, and the validation of the modifications made. The description and analysis which follows is based on a joint paper (2.48). The co-author, Professor F.P. Lees, played a full part with the present author in the writing of this paper which is reproduced with his consent. Vagueness is due to obfuscation at the behest of the firm involved.

10.2 The Process and the Plant

The process involves the treatment of ore with feed acid to produce product acid and waste solids. The plant studied was not the whole complex of plants but a part of it and was chosen 1) because it most often limited total production from the complex and 2) because it gave rise to most maintenance problems. The plant dates from about 1950.

A flow diagram of the plant is shown in Figure 10.1. The plant consists of three reaction lines A, B, and C. A and B are followed by a common bank of seven filters, while C has its own filters. These lines feed a bank of nine evaporators, which concentrate the product acid. All the evaporators can be fed from more than one reaction line and some from all three lines. The rated capacities of the various units expressed as percentages of the total rated output of the plant are

Reaction lines A, B = 25% each	Reaction Line C = 50%
Filters(mainly for reaction lines A and B) = 8.5% each	Filters(only for reaction line C) = 50%

Evaporators 1,2,3,4,5,6, = 10% each Evaporators 7,8,9, = 20% each

Many different throughputs are possible under various failure conditions.

Two features of the plant are particularly significant for maintenance problems. One is that there is some redundancy of equipment. The other is that output tends to fall off due to blockage of pipes and openings with waste solid. These factors mean that there is opportunity for maintenance without limiting output; that is, some plant can be maintained in rotation and some when it is stopped for cleaning.

The market for the product is such that the whole plant output can be readily sold. The direct costs of maintenance are low, being only about 5% of total costs. As a first approximation, therefore, profit is maximised by maximising plant availability.

10.3 Original Maintenance Policy

The existing maintenance policy for the plant was essentially one of breakdown maintenance with preventive maintenance (pm) largely confined to an annual shutdown period.

For most of the year the plant was operated until a failure occurred which made a shutdown unavoidable. During the period of the repair necessitated by the failure, it was possible to undertake other preventive work, but only if it was certain not to hold up the restart.

During the annual shutdown period of two weeks some preventive maintenance work was possible. The priorities during this period were

1) To carry out essential repairs of deferred defects.
2) To incorporate modifications designed to increase rated output (debottlenecking).

ENGINEERING LOST TIME

DAILY SUMMARY

Plant: EVAPORATOR

Month: AUGUST, 197

DATE	PLANNED ENG SHUTDOWN	ENGINEERING BREAKDOWN					TOTAL ENG LOST TIME	REASON FOR LOST TIME
		MECH.	ELECT.	INST'S	SERVICES	TOTAL		
1								
2		7.00/1.00				8.00	8.00	Condensate leaks u/m - . discharge reducer leak u/m
3		1.00				1.00	1.00	LP steam leaks
4		12.30				12.30	12.30	Steam and condensate leaks
5		10.00/1.00				11.00	11.00	Steam and condensate leakd u/m / pump u/m
6								
7								
8		3.45/9.30				13.15	13.15	. belts off - . line blockage
9		1.00				1.00	1.00	. . . u/m
10								
11		1.30				1.30	1.30	belts adrift
12		1.15/11.30				12.45	12.45	belts adrift and pump u/m
13								
14								
15								
16								
17		7.15/2.00/4.30				13.45	13.45	. . acid in cond./412 valve u/m / . . . joint leak
18								
19		6.00				6.00	6.00	Condensate leaks and auto feed valve u/m
20		3.00				3.00	3.00	pump / line u/m
21								
22								
23	1.45						1.45	Planned
24		6.00/6.00				12.00	12.00	D/water line u/m / low vacuum
25								
26								
27								
28								
29		22.00				22.00	22.00	Vacuum loss
30		10.00				10.00	10.00	Seal tank flexes u/m
31								o/flow stand pipe blockage
	1.75	127.75				127.75	129.50	

TOTAL HOURS AVAILABLE/MONTH 744

MONTH PLANT AVAILABILITY = $\frac{\text{TOTAL HOURS AVAILABLE* - TOTAL ENG. LOST TIME (7)}}{\text{TOTAL HOURS AVAILABLE*}} \times 100 = 82.66\%$

Figure 10.2a Engineering Lost Time Sheet

3) To carry out plant manufacturer's recommended maintenance.

Modifications to improve the availability of the existing equipments and/or to ease maintenance had a low priority.

The overall plant availability, defined as the ratio of achieved output to rated output, cannot be quoted, but was relatively low and was thought to be capable of considerable improvement.

A factor in the situation was that the plant was old, whilst finance for modifications to ease maintenance was generally unavailable.

10.4 The Investigation

It was agreed between the works and the authors that data should be collected and analysed with a view to determining the effectiveness of the existing maintenance policies and to making any appropriate recommendations for modifications of these policies.

10.5 Data Collection

The principal item of information which was related to failure and which was collected on a regular basis by the existing engineering information system in the works was the Engineering Lost Time Sheet (Figure 10.2). This sheet records all plant downtime where this involves loss of production and the equipment which is the cause of the downtime. It is not possible, however, to derive reliable equipment failure data from these sheets, because, as already described, the plant contains partial redundancy so that some equipment failures occur which do not cause downtime and these are not recorded. The data do provide however, a record of plant availability.

It was decided therefore, that it was essential to obtain more positive data on equipment failure. For the purposes of the investigation collection of data on equipment failure was initiated using a Breakdown Record Sheet (Figure 10.3). Each of the blocks in Figure 10.1 was

PLANT: RECORD OF BREAKDOWNS - MECHANICAL FAULT LIST   MONTH:

SHEET 1 OF 1

KEY: B = BLOCKAGES | L = LEAKS | D = DRIVE | E = ELECTRICAL | H = HOLES, BREAKS | I = INSTRUMENTS | V = VALVE FAULT | G = GENERAL

ITEM NO.	DESCRIPTION	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
602	MIXING TANK																															
603	FEED PUMP																															
604	DIAPHRAGM SLUDGE PUMP																															
605	HEAT EXCHANGER						B																									
605A	HEAT EXCHANGER																															
606A	CENTRIFUGE												D																			
606B	CENTRIFUGE																															
607	TRANSFER PUMP																															
608	SLURRYING PUMP																															
609	SLUDGE PUMP																															
612	TRANSFER TANK									L																						
613	SLURRYING TANK AND AGITATOR																															
614A	SLURRY PUMP																															
614B	SLURRY PUMP																															
616	WASH WATER TANK																															
617	FRESH WATER BOOSTER PUMP																															
618	INSTRUMENT AIR COMPRESSOR																															
621	ALL ACID AND WASHWATER LINES																															
622	ALL DOCK WATER LINES																															
623	ALL COMPRESSED AIR LINES																															

Figure 10.3b Breakdown Record Sheet

subdivided into 20 to 80 items and records were kept of the days on which these items were unavailable for any period due to failure. Failures were classified under one of the following eight broad headings: Blockages, Leaks, (joints and seals), Drives (gears and couplings), Electrical, Holes/Breaks (fracture and erosion and corrosion) Instruments, Valves and General (everything else).

These breakdown records were kept for a year (1975)(Period 1). A preliminary analysis of these data were made. The results were unexpected. The breakdown records were therefore kept for a further seven months (1976) (Period 2). A further analysis of the data was then carried out. Proposals were then formulated for the modification of the maintenance policies and were put to the works management. Discussions were held on these proposals. Modifications were then made to the maintenance policies by the works management but not all the proposals were adopted. During the period in which these modifications were being implemented the keeping of breakdown records was interrupted by a shortage of clerical staff, although the collection of the lost time data continued. The breakdown records were then restarted and were kept for a further 9 months (1978) (Period 3). Again a full analysis of the data was carried out and was used to determine the effect of the modified maintenance policies.

The data collection system described was a compromise between the ideal and the practical. Ideally, it would have been desirable to use a more rigorous classification of failures. However, the works were not able to provide records more detailed than those described. This is a fairly typical situation however, and it is of some interest to consider what use can be made of records with this level of detail.

10.6 Analysis of Initial Data (1976)

The principal results of the analyses carried out in 1976 (Period 2) and in 1978 (Period 3) are given in Tables10.1 and 10.2.

It is convenient to consider first the results for 1976. Table 10.1 gives a failure modes (Pareto) analysis for the individual plant and the mtbf's. The table shows the failure modes which were responsible for most of the failures in each equipment. Table 10.2 gives the mtbf's calculated from time and failures and also from Weibull analyses, and the Weibull shape and scale parameters β and η for the individual plant items.

The analysis actually carried out in 1976 was not as detailed as that shown in the tables but was limited to the overall equipment failure analyses, the failure modes analysis of some of the equipment and the determination of some of the mtbf and shape parameter data. The overall equipment failure analysis shows that 61% of the failures on the plant were due to pumps. The mtbf's of the pumps were in the range 45-80 days, which appeared low, and the shape parameters were in the range 0.6 - 0.7, which indicated early failure. It was decided, therefore to concentrate attention particularly on the pumps.

The failure modes analysis of the acid pumps showed that the main failure modes were Holes/Breaks, Leaks and Blockages with 156, 107 and 49 failures (out of 432) respectively. Other significant failure modes were Drives and Electrical with 38 and 28 failures respectively. Some of these latter failures were also attributed to blockages, which can cause overload.

For the water pumps the failure modes analysis showed that the main failure modes were Holes/Breaks and Leaks with 23 and 21 failures (out of 79) respectively. Other significant failure modes were Drives and

Electrical with 12 failures each.

For the vacuum pumps the failure modes analysis again showed that the main failure modes were Leaks and Holes/Breaks with 21 and 17 failures (out of 56).

10.7 Observation of Maintenance Methods

The statistical analysis was supplemented by an investigation of the maintenance situation on the plant, including observation of maintenance tasks and discussion with supervisors without criticism or comment.

Many of the maintenance tasks were carried out on the plant in very dirty working conditions with a high probability of contaminating the work. Supervision in many cases considered inadequate. Corners were cut to resume operation as soon as possible. Standards of workmanship were low, by Marine standards although not especially so for the Chemical Industry.

10.8 Modification of Maintenance Policy

Proposals aimed at reducing the number of failures on pumps in the main failure modes were discussed with the works management. Over a period of time modifications were made to the maintenance policies.

In general terms, the important features of these modifications were greater emphasis on preventive maintenance with regular inspection of certain equipment, training for and supervision of maintenance tasks, and recruitment of some additional maintenance personnel in order to make it possible to implement preventive maintenance.

More specifically, the following steps were taken:

1) Preventive maintenance routines have been re-introduced after a 7-year lapse. The ratio of manhours of preventive maintenance and work on deferred defects to manhours of breakdown maintenance has changed

from 20:80 to 63:37. Almost all service failures are now dealt with by a reduced number of shift fitters.

2) All plant is now periodically maintained 3-4 times as often as previously. Outstanding preventive maintenance and deferred defects are cleared before plant is restarted. Critical path analysis is used to minimise outages. In timing the outages the partial redundancy available is exploited. Planned outage time has approximately doubled but the overall availability has increased markedly. The major annual shutdown has been abolished except on C line by sharing the work between 3 or 4 scheduled outages. Most plant is maintained at either 9 or 12 week intervals.

3) More effort is made to find and record minor defects. An engineer or supervisor walks round the plant daily. The shift workers are also encouraged to report defects.

4) There is a greater effort on scheduled servicing such as checking lubricants for level and contamination and checking on pump suction and discharge pressures.

5) Repair and refit by replacement has been introduced for a number of common items such as pumps and valves. The objectives are to limit outages and to allow the repair or refit to be carried out under clean conditions and without hurry. To facilitate this policy some extra pumps were bought.

6) Specific training in certain maintenance tasks which have commonly been done poorly is given to both skilled and semi-skilled personnel and corresponding induction training is given to new personnel. An example is the instruction given to all fitters in the renewal of pump mechanical seals. The result in this case has been that most such repairs are now successful whereas previously many failed again within a week.

9) Maintenance manpower has been increased by 12% to allow these preventive maintenance policies to be implemented and to clear the backlog of work.

The policy of preventive maintenance has been implemented in different ways depending on whether or not there is redundancy in the plant concerned. Where redundancy exists, preventive maintenance is carried out in rotation on the non-operating equipment. Where redundancy does not exist, preventive maintenance is carried out mainly during breakdowns. In this latter case the previous policy of returning the plant to production as soon as the breakdown is repaired has been somewhat relaxed to allow important preventive maintenance tasks to be carried out in parallel with the repair of the breakdown failure.

The regular inspections by an engineer or supervisor constitute ocpm. On the acid pumps specific changes made include replacement of corroded pipe, measures to reduce seal leaks and regular hot washes to remove partial blockages. The latter measure is intended to reduce Electrical failures due to pump overload as well as Blockage failures and also to raise the instantaneous output of the plant.

Specific changes on the other pumps include measures to reduce the seal leaks on the water pumps and measures to reduce leaks on the vacuum pumps.

Although the proposals put forward by the investigators were concerned mainly with the maintenance of the pumps, the changes to the maintenance policies by the works management went wider than this as the above account indicates. There has been a marked shift in the direction of planned preventive and deferred maintenance.

The implementation of these changes occurred over a period of about 18 months. At the end of this period in 1978 further data were collected and analysed as already described.

10.9 Analysis of Final Data

The analysis carried out in 1978 was more detailed than that done in 1976 and included further analysis of the 1976 (Period 2) data as well as analysis of the 1978 (Period 3) data. The principal results for both periods are given in Tables 10.1 and 10.2.

The overall equipment failure analysis given in Table 1 and the mtbf's given in Table 3 show that between the two periods there was a marked improvement (reduced failure rates, increased mtbf's) for 7 equipments.

Acid Pumps	Heat Exchangers
Water Pumps	Pipes and Ducts
Vacuum Pumps	Other items
Agitators	

a small improvement for two equipments

Fans	Other Vessels and Tanks

and a marked deterioration for three equipments

Screw Conveyors	Evaporator flash vessels
Filters	

There was also a marked improvement for the plant overall with a reduction in the failure rate from 132.9 failures /month to 84.3 failures month.

It was expected that if the changes in the maintenance policy were successful there would be a significant reduction in the number of early failures, resulting in an increase in mtbf and in the value of the shape parameter β. In the event, these expectations were only partially fulfilled.

Overall, the exercise was undoubtedly successful, since marked reductions were achieved in the number of failures on most of the equipments and in the plant as a whole. This overall success was marred however, by the failure to achieve convincing increases in the shape parameter β and by decreases in the mtbf's of three equipments. This former outcome is not entirely negative, however, in that it has led to a better understanding of the behaviour of the shape parameter.

It is convenient to discuss first the shape parameter β. The confidence which can be placed in the estimate of β depends on several factors. The principal factor is the number of failures recorded. The number of failures in some of the data sets are fairly small.

Another factor which affects the estimate of β is the relation between the mtbf and the observation period. The effect has been investigated by Aird (unpublished work 1977). If the mtbf is a high proportion of the observation period, or even exceeds it, inaccuracy is introduced into the estimation of β.

The equipments which are least affected by these difficulties are the acid pumps. A Weibull plot for the acid pumps for 1976 and 1978 is shown in Figure 10.3.For these pumps the number of failures recorded is large and the ratio of the mtbf to the observation period is moderate. The pumps do indeed show both the expected increase in mtbf and in the value of β.

A Weibull plot for the vacuum pumps for 1976 and 1978 is shown in Figure 10.4.These pumps show a somewhat similar picture except that in this case the Weibull plot for 1978 gave a bimodal distribution so that it was not possible to obtain a single value of β for comparison with the 1976 value. The authors' interpretation of this is that there are still a few early failures but that the remaining failures show a marked wearout regime. From the failure modes analysis there is possibly a Blockage mode which could be alleviated by a periodic maintenance routine.

The water pumps exhibit a significant increase in mtbf, but a decrease in the value of β. In this case, however, the increase in the mtbf for 1978 is such as to make it greater than the observation period. It is considered, therefore, that less confidence can be placed in the β value for this period.

The water pumps also illustrate the problem of bimodal distributions. The β value given in Table 10.2 for 1976 was derived from a cumulative hazard Weibull plot which gave no clear indication of such a distribution. If, however, the data are plotted on the conventional Weibull plot as shown in Figure 10.5, the bimodal nature of the distribution is clearer.

The failure distribution given in Figure 10.5 is a good example of a hyper-exponential distribution.(see also Appendix B).

The evaporator flash vessels, on the other hand, show a marked decrease in the mtbf and a marked decrease in the value of β. Although this is an undesirable change, the decrease of the β value with the decrease in mtbf does accord with expectations. The number of failures recorded for these vessels is relatively large and the ratio of the mtbf to the observation window is moderate.

The heat exchangers show a marked increase in mtbf, but only a slight increase in the β value. In this case, however, the increase in the mtbf for 1978 is such as to make a greater that the observation window. Moreover, the value of β in 1976 is 0.9 which indicates only a very weak early failure effect.

The filters show a marked decrease in mtbf and a slight decrease in the β value. This change in the β value is not considered particularly significant.

The shape parameter β was also determined for some of the individual failure modes (as opposed to overall equipment failures). Since the failure of an equipment is a function of the failure of its individual failure modes, or its components, it is information on these latter which is most useful for the formulation of maintenance policies. The determination of the β value of the failure modes, however, is possible only if there is a sufficient number of failures in each mode and thus only if there is a relatively large number of failures in the equipment overall.

In the present case it was possible to determine the β value of the failure modes only for the acid pumps and then only for certain modes. The values obtained are shown in Table 10.3. The confidence which can be placed in these values is less than that which can be placed in the overall β value for the pumps. Nevertheless, the overall picture appears fairly clear.

In each failure mode the β value is less than unity. Since for individual failure modes the β value is less likely to be affected by a multi-modal combined distribution the β value probably indicates genuine early failures. There is, however, an increase in the β values for each failure mode, indicating reductions in the proportion of early failures

Before discussing the behaviour of the β value further, it is convenient to consider the three equipments which showed marked deterioration. The failures of the filters and screw conveyors were mainly Blockage and Drive and the failure rates were strongly influenced by the nature of the raw materials processed. There was a significant change in the raw materials used between the two periods studied and this change appears to be the best explanation available for the decreases in the mtbf's.

The reason for the increase in the failure rate of the evaporator flash vessels is also unclear. Here the best explanation appears to be that in consequence of the general shift towards preventive maintenance the repair policy on these vessels has altered from one of breakdown repairs to one of more frequent preventive repairs, which are erroneously recorded by the system as failures even when they are deferred to the next scheduled maintenance period.

It is also of interest to consider the effect of the reduction in the number of failures on the overall availability of the plant. The unavailability considered is the downtime due to breakdown failures and to maintenance work, including preventive maintenance work. Downtime due to other causes such as raw materials shortage or failure on interlinked plants is not included. Even on this basis the availability of the plant was relatively low.

The effect of failure on plant availability is complicated by the storage units on the plant. For present purposes the availability considered is a synthetic value calculated from the reliability diagram ignoring the decoupling effects of storage. In practice, the storage has the effect of increasing the overall plant availability by an approximately constant percentage, but its existence does not alter the basic arguments.

Actual figures for plant availability cannot be quoted, but the management confirmed that a worthwhile improvement was obtained. A marked decrease in downtime has been achieved on all the units and the overall downtime on the plant has been reduced by a third.

The reduction in downtime on the filters and on the evaporators is particularly interesting in view of the fact that these two types of equipment exhibited a marked increase in the number of 'failures'. This result bears out the comments made earlier that the increase in failures on these equipments is probably the result of the greater emphasis on preventive maintenance. Certainly the combination of a greater number of 'failures' and of reduced downtime must mean a marked decrease in downtime attributable to breakdown repairs.

TABLE 10.1. MAIN STUDY FAILURE MODES ANALYSIS

1976-7months 214 days 1978 -9 months 273 days % of Total at bottom or right.		PUMPS								BELT FILTERS		AGITATORS		EVAP. FLASH VESSELS		OTHER TANKS AND VESSELS		EVAP HEAT EXCHANGERS		FANS		PIPES & DUCTS		SCREW CONVEYORS		OTHER		TOTAL(EXCEPT PIPES AND DUCTS)		GRAND TOTAL		% CHANGE 1976-78
		ACID		WATER		VACUUM		TOTAL																								
Number Fitted		83		28		15		126		7		34		9		86		29		14		-		9		20		334		-		
Year		76	78	76	78	76	78	76	78	76	78	76	78	76	78	76	78	76	78	76	78	76	78	76	78	76	78	76	78	76	78	
1.Blockages	No.Fails	49	32	5	0	11	13	65	45	11	46	0	0	2	27	21	10	1	6	0	0	5	7	1	11	7	8	108	153	113	160	-
	No/Month	7	3.6	0.7	0	1.6	1.4	9.3	5.0	1.6	5.1	0	0	0.3	3	3	1.1	0.1	0.7	0	0	0.7	0.8	0.1	1.2	1.0	0.9	15.4	17.0	16.1	17.8	+10.6
	%	11.3	12.2	6.3	0	19.6	37.1	11.5	14.2	16.7	33.3	0	0	10.0	27	47.7	18.5	1.3	14.3	0	0	7.0	28	11.1	57.8	13.0	17.0	12.5	20.8	12.1	21.1	+9.0
	MTBF	362	708	1198	-	292	315	415	764	136	41.5	-	-	963	91	876	2348	5206	1319	-	-	-	-	1926	223	61.1	682	662	596	-	-	-10.0
2.Leaks	No.Fails	107	28	21	7	21	4	149	39	4	7	0	0	4	7	6	9	62	12	0	0	30	8	0	0	6	20	231	94	261	102	-
	No/Month	15.3	3.1	3.0	0.8	3.0	0.4	21.3	4.3	0.6	0.8	0	0	0.6	0.8	0.9	1.0	8.9	1.3	0	0	4.3	0.9	0	0	0.9	2.2	33.0	10.4	37.3	11.3	-69.7
	%	24.6	10.6	26.6	35	37.5	11.4	26.3	12.3	6.1	5.1	0	0	20.0	7	13.6	16.7	79.5	28.6	0	0	42.2	32	0	0	11.1	42.5	26.9	12.8	28.1	13.4	-14.7
	MTBF	166	809	285	1092	153	1024	181	882	374	273	-	-	481	351	1067	2609	100	660	-	-	-	-	-	-	713	273	309	970	-	-	+213.9
3. Drive	No.Fails	38	48	12	5	2	6	52	59	13	26	6	2	0	0	0	0	0	0	9	6	0	0	3	6	2	3	85	102	85	102	-
	No/month	5.4	5.3	1.7	0.6	0.3	0.7	7.4	6.5	1.9	2.9	0.9	0.2	0	0	0	0	0	0	1.3	0.7	0	0	0.4	0.7	0.3	0.3	12.1	11.3	12.1	11.3	-6.6
	%	8.8	18.2	15.2	25	3.6	17.1	9.2	18.6	19.7	18.8	75	100	0	0	0	0	0	0	69	40	0	0	33.3	31.6	3.7	6.4	9.1	13.9	9.1	13.4	+4.3
	MTBF	467	472	499	1529	1605	682	519	583	115	73.5	1213	4641	-	-	-	-	-	-	333	637	-	-	642	409	2140	1820	841	893	-	-	+6.2
4.Electrical	No.Fails	28	20	12	0	4	6	44	26	2	1	2	0	0	0	0	0	0	0	4	6	0	0	4	2	5	4	61	39	61	39	-
	No/Month	4.0	2.2	1.7	0	0.6	0.7	6.3	2.9	0.3	0.1	0.3	0	0	0	0	0	0	0	0.6	0.7	0	0	0.6	0.2	0.7	0.4	8.7	4.3	8.7	4.3	-50.6
	%	6.5	7.6	15.2	0	7.1	17.1	7.8	8.2	3.0	0.7	25	0	0	0	0	0	0	0	31	40	0	0	44.4	10.5	9.3	8.5	7.1	5.3	6.6	5.1	-1.5
	MTBF	634	1133	499	-	802	682	613	1323	749	1911	3638	-	-	-	-	-	-	-	749	637	-	-	481	1228	856	1365	1172	2338	-	-	+99.5
5. Holes and Breaks	No.Fails	156	54	23	6	17	6	196	66	16	13	0	0	11	40	8	14	14	21	0	0	29	5	0	0	29	9	270	163	299	168	-
	No/Month	22.3	6	3.3	0.7	2.4	0.7	28	7.3	2.3	1.4	0	0	1.6	4.4	1.1	1.6	2.0	2.3	0	0	4.1	0.6	0	0	3.6	1.0	38.6	18.1	42.7	18.7	-56.2
	%	36.1	20.5	29.1	30	30.4	17.1	34.6	20.8	24.2	9.4	0	0	55	40	18.2	25.9	17.9	50.0	0	0	40.8	20	0	0	46.3	19.1	31.4	22.2	32.1	22.1	-10.0
	MTBF	114	420	261	1274	189	682	138	521	93.6	147	-	-	175	61.4	2300	1677	443	377	-	-	-	-	-	-	171	607	265	559	-	-	+111
6.Instruments	No.Fails	2	5	0	0	1	0	3	5	0	8	0	0	2	8	3	5	0	0	0	0	0	2	1	0	2	1	11	27	11	29	-
	No/Month	0.3	0.5	0	0	0.1	0	0.4	0.6	0	0.9	0	0	0.3	0.9	0.4	0.6	0	0	0	0	0	0.2	0.1	0	0.3	0.1	1.6	3.0	1.6	3.2	+100
	%	0.5	1.9	0	0	1.8	0	0.5	1.6	0	5.8	0	0	10.0	8	6.8	9.3	0	0	0	0	0	8	11.1	0	3.7	2.1	1.3	3.7	1.2	3.8	+2.6
	MTBF	8881	4532	-	-	3210	-	8988	6880	-	239	-	-	963	307	6135	4696	-	-	-	-	-	-	1926	-	2140	5460	6498	3377	-	-	-48
7.Valves	No.Fails	38	51	2	2	0	0	40	53	20	25	0	0	1	16	2	7	1	2	0	0	3	2	0	0	2	1	66	104	69	106	-
	No/Month	5.4	5.7	0.3	0.2	0	0	5.7	5.9	2.9	2.8	0	0	0.1	1.8	0.3	0.8	0.1	0.2	0	0	0.4	0.2	0	0	0.3	0.1	9.4	11.6	9.9	11.8	+19.2
	%	8.8	19.4	2.5	10	0	0	7.1	16.7	30.3	18.1	0	0	5.0	16	4.5	13.0	1.3	4.8	0	0	4.2	8	0	0	3.7	2.1	7.7	14.2	7.4	14.0	+89.2
	MTBF	467	444	2996	3822	-	-	674	649	74.9	76.4	-	-	1926	154	9202	3354	6206	3958	-	-	-	-	-	-	2140	5460	1083	877	-	-	-19.0
8. Other	No.Fails	14	25	4	0	0	0	18	25	0	12	0	0	0	1	4	9	0	1	0	3	4	1	0	0	5	1	27	52	31	53	-
	No/Month	2	2.8	0.6	0	0	0	2.6	2.8	0	1.3	0	0	0	0.1	0.6	1.0	0	0.1	0	0.3	0.6	0.1	0	0	0.7	0.1	3.9	5.8	4.4	5.9	+34.1
	%	3.2	9.5	5.1	0	0	0	3.2	7.9	0	8.7	0	0	0	1.0	9.1	16.7	0	2.4	0	20	5.6	4	0	0	9.3	2.1	3.1	7.1	3.3	7.0	+112
	MTBF	1269	906	1498	-	-	-	1498	1376	-	159	-	-	-	2457	4601	2609	-	7917	-	1274	-	-	-	-	856	5460	2647	1753	-	-	-33.8
TOTAL	No.Fails	432	263	79	20	56	35	567	318	66	138	8	2	20	99	44	54	78	42	13	15	71	25	9	19	54	47	859	734	930	759	-
	NO/Month	61.7	29.2	11.3	2.2	8	3.9	81.0	35.3	9.4	15.3	1.1	0.2	2.9	11.0	6.3	6.0	11.1	4.7	1.9	1.7	10.1	2.8	1.1	2.1	7.7	5.2	123	81.6	132.9	84.3	-36.6
	%	76.2	82.7	13.9	6.3	9.9	11.0	60.1	41.9	7.1	18.2	0.9	0.3	2.1	13.0	4.7	7.1	8.4	5.5	1.4	1.9	7.6	3.3	1.0	2.5	5.8	6.2	92.4	96.7	100	100	-
	MTBF	47.5	108	75.8	382	57.3	117	47.5	108.2	22.7	13.8	910	464	196.3	24.8	418	435	79.6	188.5	230	4255	-	-	214	129	79.2	116.1	83.2	124.2	-	-	+49.3
MOST COMMON MODES	1-8	5,2	5	5,2	2,3,5	2	1	5	5,7,3	7	1	3,4	3	5	5,1	1	5	2	5,2	3	3,4	2,5	2,1	4,3	1	5	2	2,5	5,2	5,2	5,2	
% Change 76-78	Fail/Month	-52.7		-80.5		-51.2		-56.4		+62.8		-81.8		+279		-4.8		-57.8		-10.5		-72.3		+90.9		-32.5		-33.7		-36.6		
	%	+ 6.5		-7.6		+1.1		-18.2		+11.1		-0.6		+10.9		+2.4		-34.5		+0.5		-4.3		+1.5		+0.4		+4.3		-		
	MTBF	+127		+404		+104		+128		-39.2		+410		-74.2		+4.1		+137		+10.7		-		-39.7		+46.6		+49.3		-		

TABLE 10.2 SUMMARY OF MAIN STUDY FREQUENCY ANALYSIS

DATA SETS 1975-First-Pre-action 1976-Second Pre-action 1978-Post-action		Statistic Symbol	ALL PUMPS	ACID PUMPS						WATER PUMPS			VACUMM PUMPS AND AIR EJECTORS	BELT FILTERS	TANKS AND VESSELS				HEAT ECHANGERS
				ALL MODE'S	LEAKS	HOLES	DRIVES	VALVES	BLOCKAGES	ALL MODE'S	LEAKS	HOLES			AGITATORS	FLASH VESSELS	OTHER	ALL	
Number Fitted	Year	N	126	83	83	83	83	83	83	28	28	28	15	7	34	9	86	95	29
Number of Failures	1975	n	863	186	-	-	-	-	-	22	-	-	9	-	-	-	-	-	-
	1976		567	432	107	156	38	38	49	79	21	23	56	66	8	20	44	72	78
	1978		318	263	28	54	48	51	32	20	7	6	35	138	2	99	54	156	42
Data collection Period (days)	1975	T	365	90	-	-	-	-	-	90	-	-	90	-	-	-	-	-	-
	1976		214	214	214	214	214	214	214	214	214	214	214	214	214	214	214	214	214
	1978		273	273	273	273	273	273	273	273	273	273	273	273	273	273	273	273	273
Mean time between failures NT/n days	1975	$\hat{\theta}$	53.3	40	-	-	-	-	-	114	-	-	150	-	-	-	-	-	-
	1976		47.5	41	166	114	467	467	362	76	285	261	57	23	910	96	418	282	80
	1978		108.2	86	809	420	472	444	708	382	1092	1274	117	14	4641	25	435	166	188
Weibull MTBF $\eta\Gamma(1+1/\beta)+\gamma$ days	1975	$\tilde{\theta}$	50	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	1976		39.2	36	225	120	-	1995	4200	52	567	416	72	19	-	84	-	-	67
	1978		103.7	65	1080	549	-	1064	4740	412	-	-	115	16	-	22	-	-	166
Weibull shape parameter	1975	β	0.50	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	1976		0.64	0.63	0.69	.76	-	0.66	0.48	0.68	0.65	0.68	0.61	0.88	-	1.07	-	-	0.90
	1978		0.67	0.76	.74	.81	-	0.67	.58	0.56	-	-	0.42) 3.15)	0.85	-	0.65	-	-	0.94
Weibull Scale Parameter days	1975	η	25	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	1976		28	25	175	102	-	1500	2000	40	420	320	49	18	-	86	-	-	63
	1978		78	55	900	490	-	800	3000	250	-	-	1750) 112)	15	-	16	-	-	158
Weibull Location Parameter days	1975	γ	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	1976		0	0	0	0	-	0	0	0	0	5	0	0	-	0	-	-	0
	1978		0	0	0	0	-	8	0	0	-	-	0	0	-	0	-	-	0

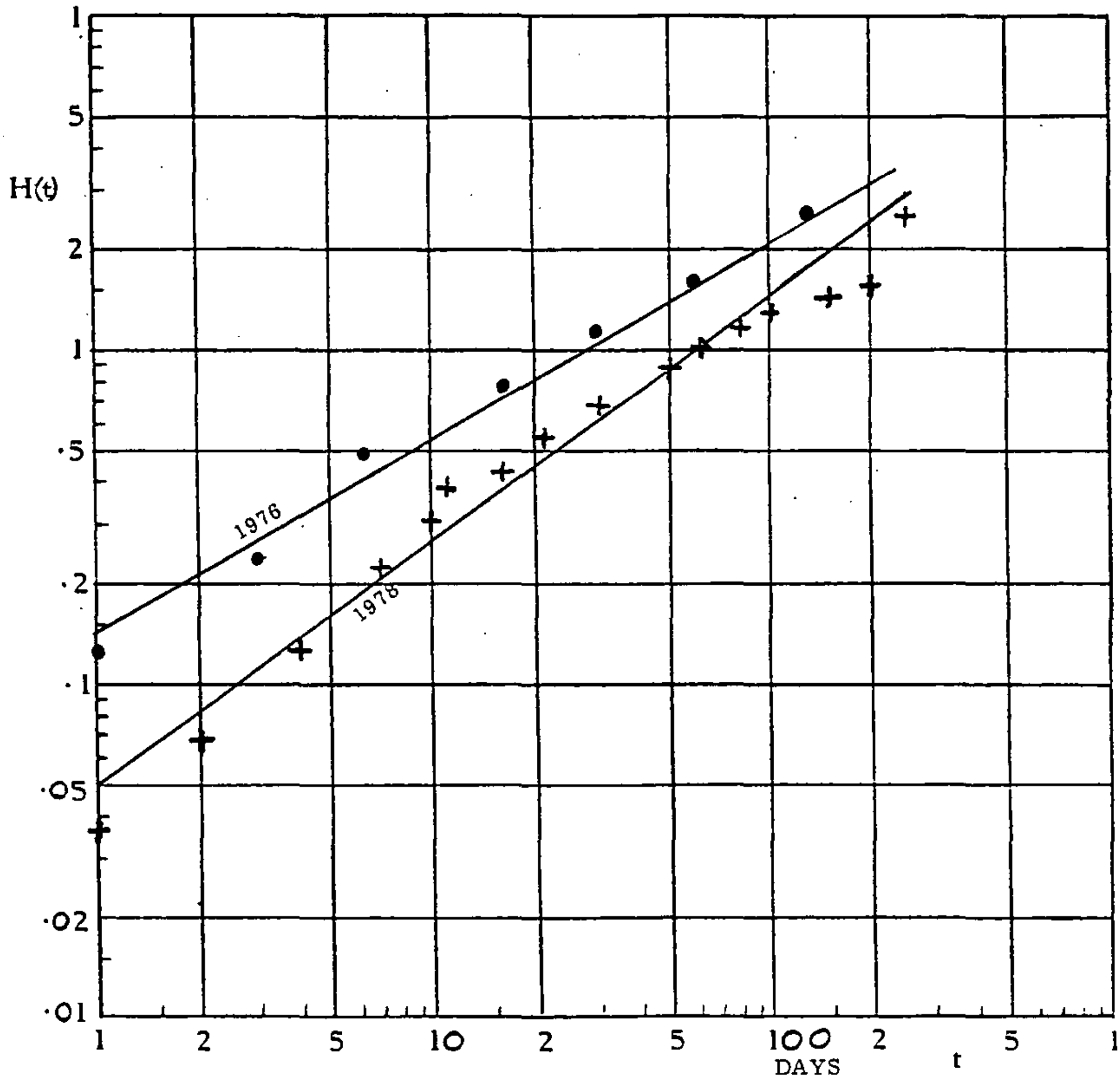


Figure 10. 3 Main Study Acid Pumps Cumulative Hazard Plots for 1976 and 1978 showing improvement in both Beta and mtbf.

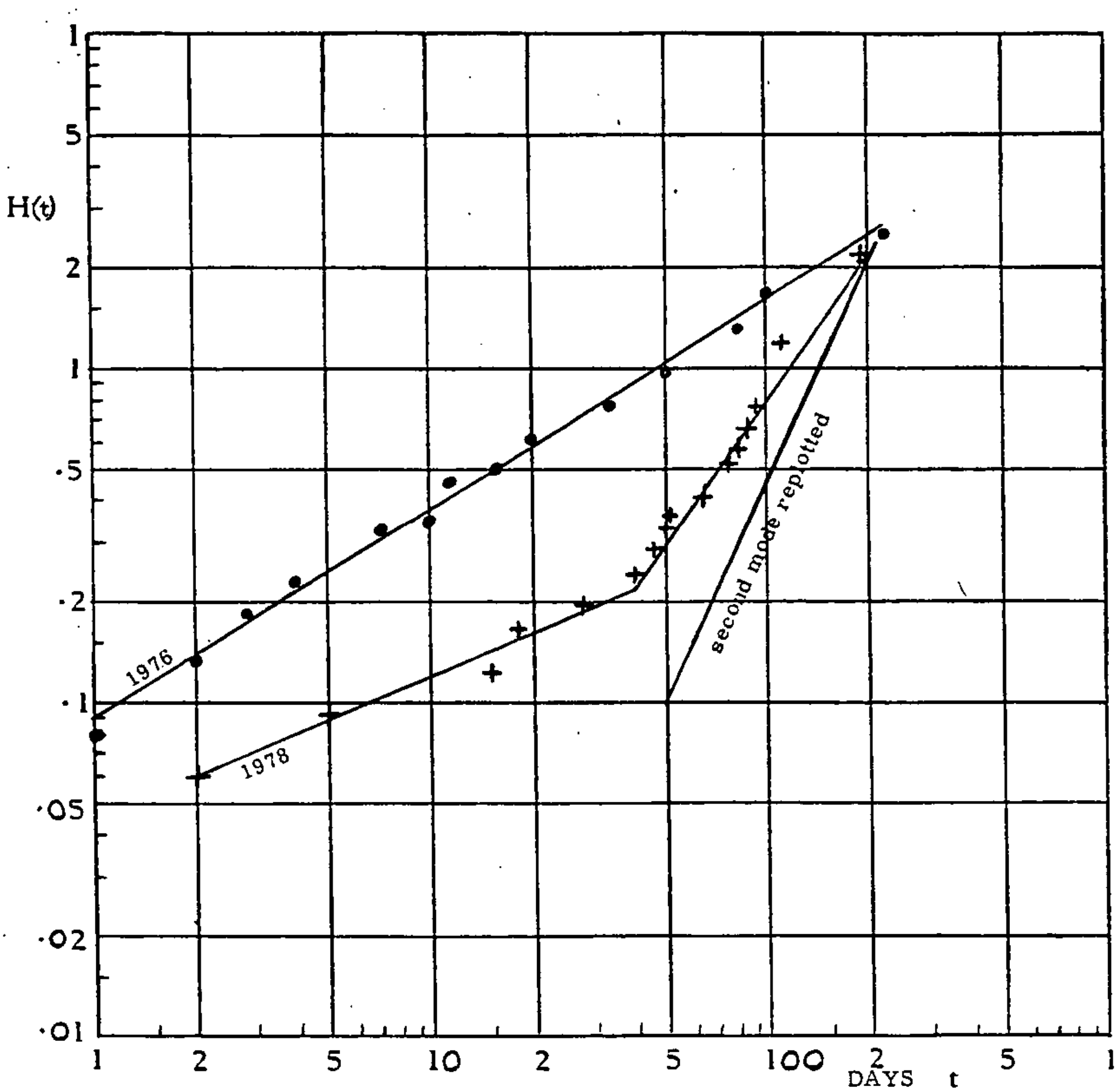


Figure 10.4 Main Study Vacuum Pumps Cumulative Hazard Plots for 1976 and 1978 showing improvement in mtbf and severely bimodal plot for 1978.

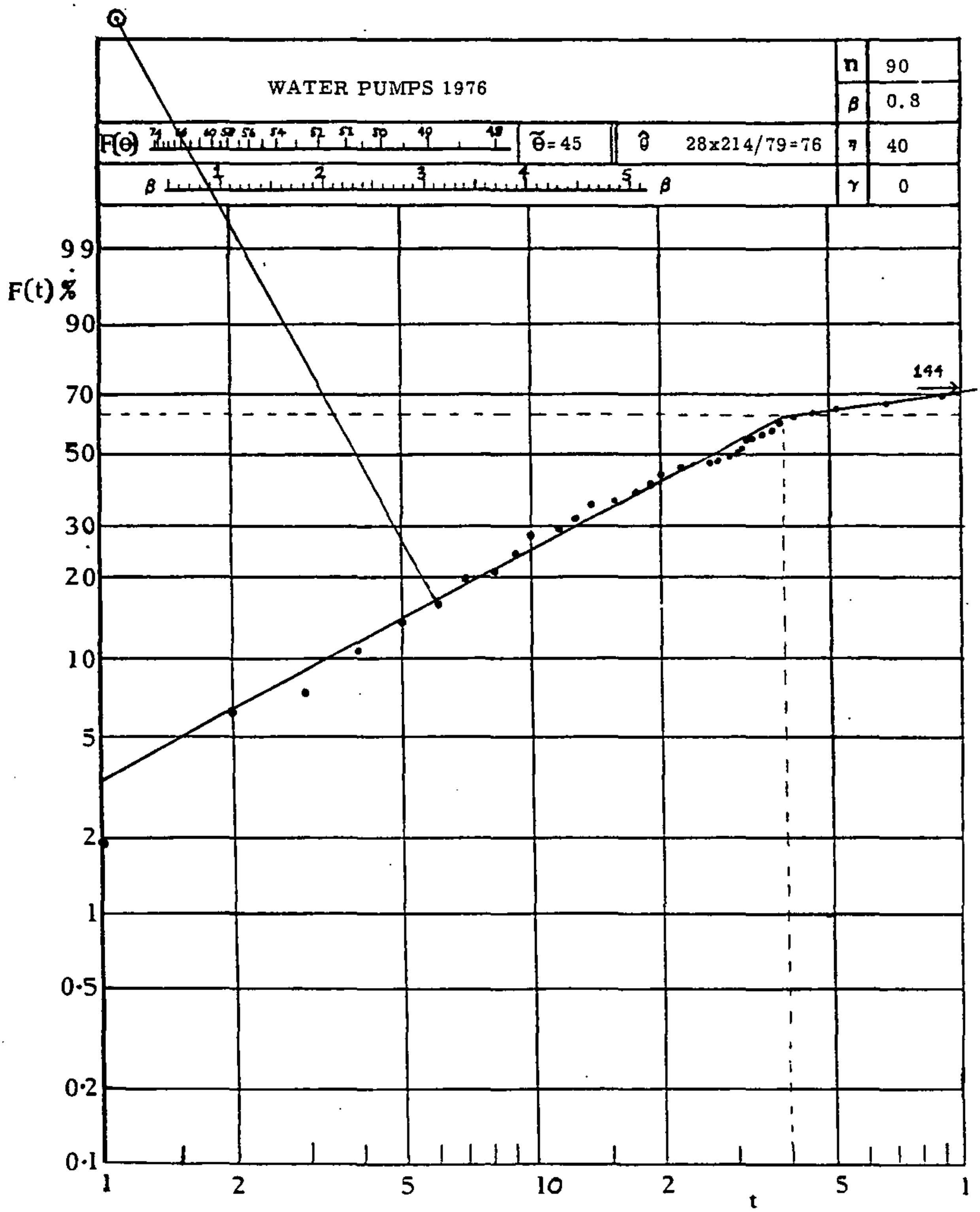


Figure 10.5 Example of cdf (Weibull) Plot. Bimoda lity which did not show up on Cumulative Hazard Plot, explains discrepancy in mean values.

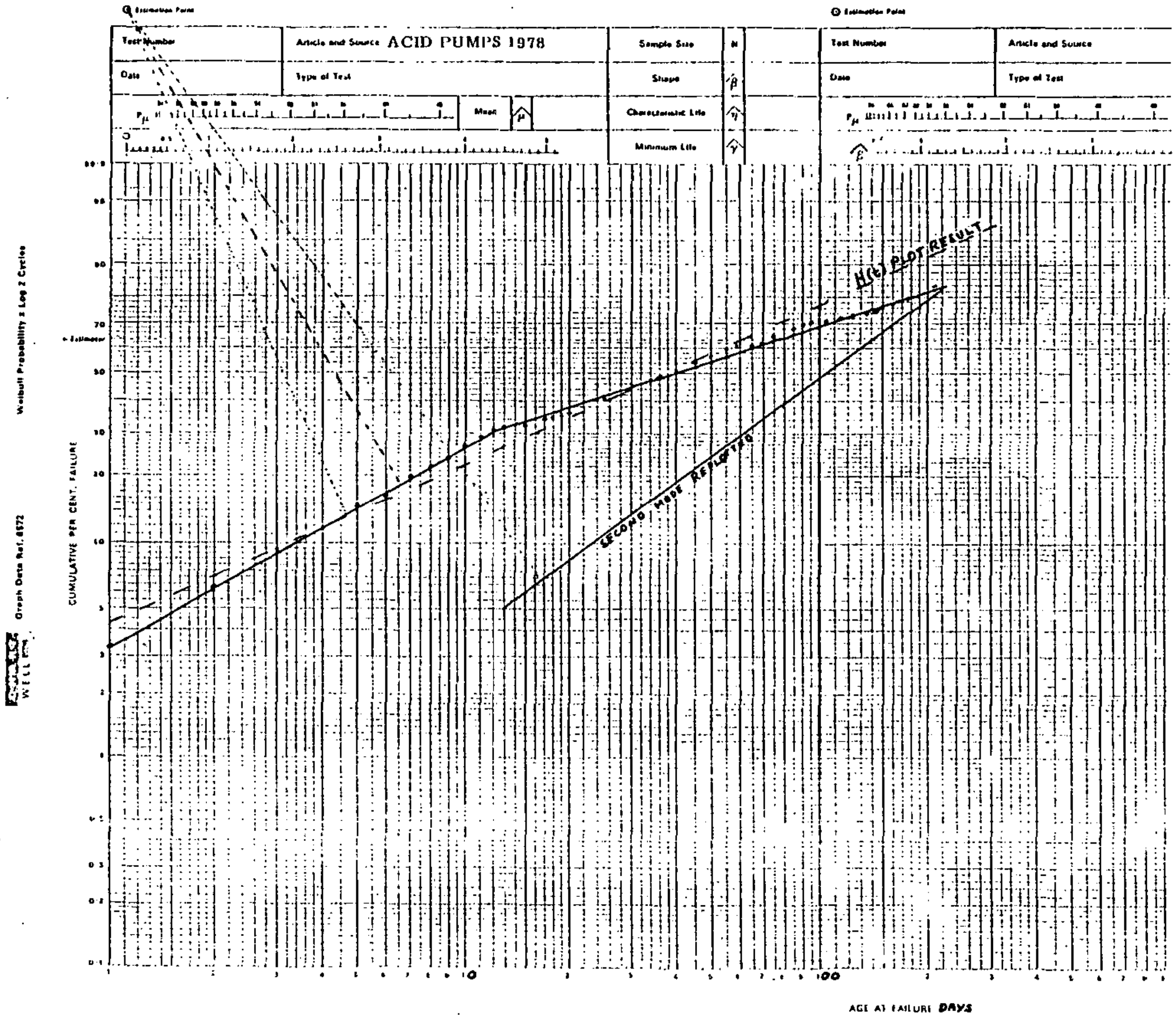


Figure 10.6 As in Figure 10.5 bimodality of 'true' hyperexponential is more obvious on a F(t) than a H(t) plot.

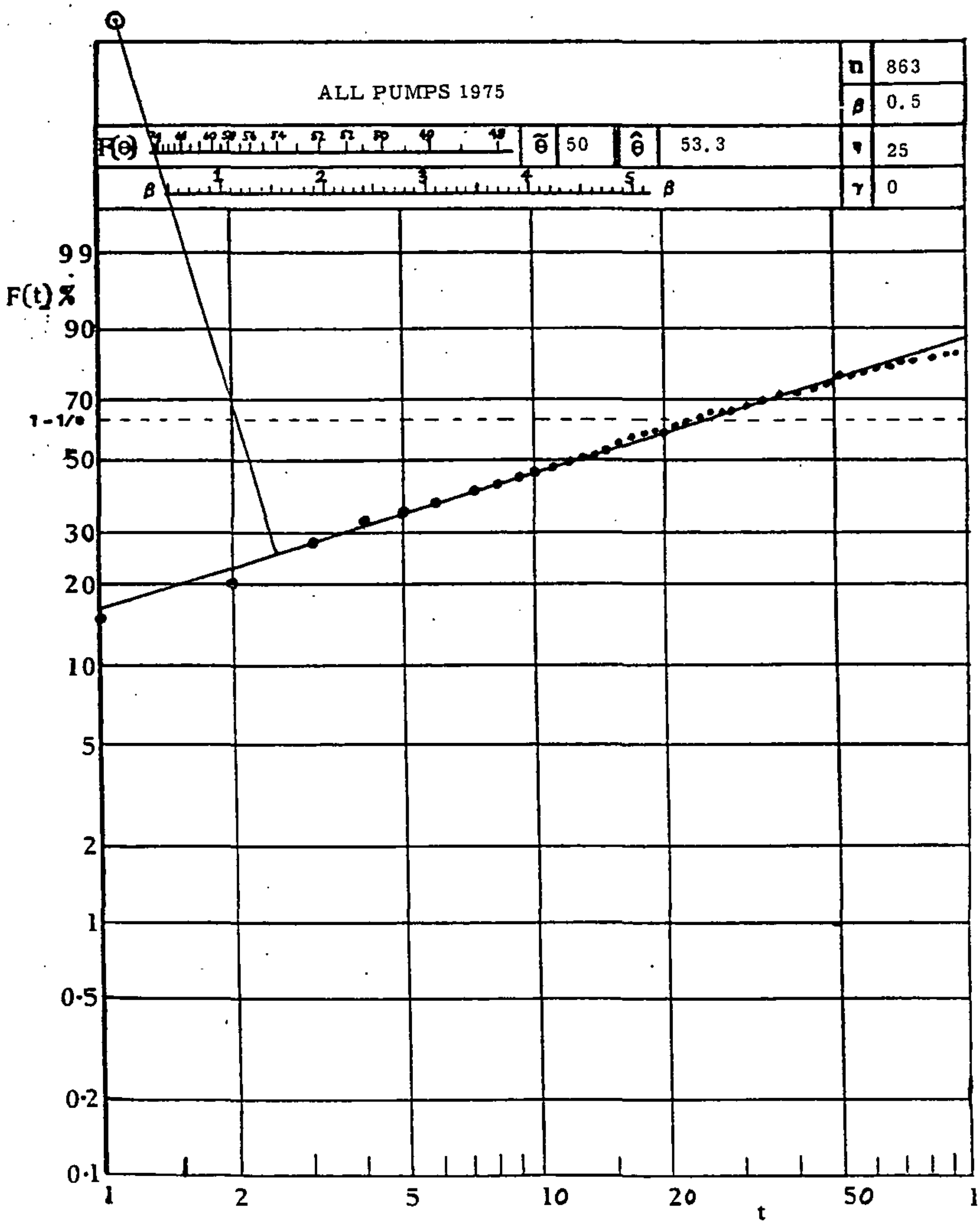


Figure 10.7 Main Study: Early Pump Data (after Aird)

11. MINOR DATA COLLECTION AND ANALYSIS PROJECTS

11.1 Hospital Autoclaves

11.1.1 Introduction. Failure of an autoclave to sterilise can have tragic consequences. Great danger exists if the failure is undetected. In hospitals autoclaves are used to sterilise fluids used in operating theatres and intravenous drips. Another type is used to sterilise dressings, towels and other items of a porous nature. The principal pharmacist of one hospital group was sufficiently worried about autoclave reliability to collect data and later contact Professor F.P. Lees of Loughborough University who delegated the investigation to the author. Apart from the risk associated with unsterile fluids, failure of an autoclave cycle also costs money inasmuch as more fluids must be obtained by the hospital from commercial sources. Often, the failed batch must be thrown away because prolonged exposure to temperature destroys the beneficial properties of the fluids. The investigation provided examples of a number of uses to which reliability information may be put, and is a cameo case history supporting the theory advanced in Appendix B2 and B3. Salient features were as follows:

a) Pareto analysis, pointing up the most common causes of failure, led to modifications to autoclaves.

b) Analysis of failure data known to be complete and of high accuracy showed that poor reliability might partly be blamed on inadequate maintenance.

c) Early failures to recently modified equipment showed low mtbf and sharply falling hazard rate, which recovered to a constant, much lower failure rate when installation and design faults had been corrected.

d) The inseparability of reliability and quality control, particularly process capability considerations in a case where failure is defined in terms of quality or uncertainty as to quality.

e) Hazard analysis to assess the probability of sending out an unsterile batch not knowing it to be unsterile.

f) Reliability was expressed in terms of operating cycles rather than time. There was no vagueness about the amount of use - every cycle had to be recorded for safety records.

11.1.2 Initial Data. The pharmacists had already collected failure data from 17 hospitals for periods of 3, 6 or 12 months when they approached the University. These data were classified into modes of failure based upon the parts found to be defective. It was possible to calculate average failure rates but the numbers of cycles between individual failures were not recorded and so it was not possible to estimate the failure distributions. A few data collected from industrial autoclaves were compatible with the hospital data. However, it was worrying to hear that other industrial concerns reported that they had no failures. This might indicate highly reliable equipment but is considered more likely to point to a careless attitude to sterility. The data from the 17 hospitals is recorded at Table 11.1 (two sheets) and a Pareto analysis appears at Table 11.2. The salient features of the analysis are:

a) Overall failure rates at different hospitals varied between 0.1 and 0.01 per cycle and averaged 0.048. This is about 1 failure every 21 cycles.

b) No one make of autoclave was outstandingly better or worse than the others.

c) Failures to electrical components, steam and water valves and instruments predominated, accounting in all for 75%

TABLE 11.1 TABLE IN RANK ORDER OF CYLES PER FAULT

(Showing equipment and prevalent faults)

Hospital Code	Equipment	Cycles per Fault	Prevalent Faults (No. per 3 months)
1	3 Manlove Tullis Mk.3	14	Electrical(13)Valves(9) Gauges (2)
2	1 Drayton Castle	13	Valves(9),Gauges(2)
3	1 Drayton Castle and 2 British Steriliser	22	Electrical(3),Valves(3) Gauges(4)
4	2 British Steriliser	51	Probe(3) Valves(1)
5	1 Allan & Hanbury	23	Electrical(3),Valves(6)
6	3 Drayton Castle	16	Electrical(3),Valves(6) Gauges(6)
7	1 Chas.Thackray and 1 Allan & Hanbury	22	Door(5), Probe(1)
8	2 Drayton Castle	22	Electrical(2)Valves(1)
9	2 Pharmacist	24	Electrical(3)
10	1 Chas.Thackray and 2 British Steriliser	10	Electrical(4)Valves(8) Door(6) Probe(3)
11	1 Drayton Castle and 2 British Steriliser	28	Electrical(10)Valves(6)
12	1 British Steriliser	100	Probe(1)
13	2 Drayton Castle	22	Valves(2) Probe(1) Compressor(1)
14	1 British Steriliser	28	Electrical(3)Valves(4)
15	1 British Steriliser	48	Water Pump (1)
16	1 Drayton Castle	91	Electrical(1)
17	3 British Steriliser	22	Compressor(2)Door Seals(1) Probe(1)

Faults	Hospital 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	Total Faults	%
Electrical	15	0	3.5	0	3	3	0	2	3	4.5	10	0	0	3	0	1	0	48	28
Steam Valves	4.5	7	1	1	0	7	0	1	0	5.75	2	0	1	4	0	0	0	34	20
Water Valves	3	2	2	0	6	0	0	0	0	1.75	2	0	1	0	0	0	0	19	11
Air Vales	1.5	0	0.25	0	0	2	0	0	0	0	2	0	0	0	0	0	0	6	4
Gauges	2	2	3.75	0	0	6	0	0	0	0	0	0	0	1	0	0	0	14	8
Door Seals	1	0	0.25	0	0	0	1	0	0	5.75	1	0	0	0	0	0	1	10	6
Door Action	1	0	0	0	0	0	4	0	0	1.5	0	0	0	0	0	0	0	6	4
Compressor	1	0	0.25	0	0	0	0	0	0	0	0	0	1	0	0	0	2	4	
Vacuum Pump	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	2	
Probe	0.5	1	1	3	0	0	1	0	0	3.25	1	1	1	0	0	0	1	14	8
Drain Trap	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	2)	
Cooling Cycle	0.5	0	3.25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4)	
Spray Jets	0	0	0	0	1	0	0	0	0	1.75	0	0	1	0	0	0	0	4)	8
Recorder	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1)	
Water Pump	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1)	
Others	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	2)	

Overall Failure rate 0.048 faults/cycle

TABLE 11.2 NUMBERS OF FAULTS REPORTED FOR A 3-MONTH PERIOD BY 17 HOSPITAL STERILE PRODUCTION UNITS USING 34 SPRAY COOLED AUTOCLAVES of failures. The dominance of the 'vital few' was not so marked as is usual in equipment with a high reliability requirement, and, with the high overall failure rate, indicated an unsatisfactory situation with regard to inherent or design reliability.

11.1.3 Pre-modification Data. Table 11.3 shows data collected over a 17 month period for a single 5-year-old autoclave at Canterbury Hospital. The figures are for cycles between failures of the same type so that censored analysis is avoided. Taking all failures there were 38 faults in 581 cycles, an average rate of 0.065 per cycle. This is not significantly different from the initial data. Of the 38 failures 20 were described as 'electrical' and 7 concerned the steam valve. A Weibull plot of all the failures indicated a shape parameter $\beta = 0.79$ and a characteristic life $\eta = 13.7$. However, an exponential (Weibull $\beta = 1$) through the mean is contained by the 90% confidence limits, so $\beta < 1$ is not conclusive, see Figure 11.1 . 50% of failures occurred within 8 cycles of previous failure and 16% before 3 cycles. This hyper-exponential pattern is typical of inadequate maintenance. Early failures may occur if

a) imminent faults are ignored during repairs

b) incompetent work initiates future failures

c) poor quality parts and consumables are used.

For further details of the theory of the early failures and maintenance see Appendix B2 and B3.

The electrical faults plot best to a lognormal distribution but also fit quite well to an exponential. The lognormal is characterised by initially rising subsequently falling hazard rate function (failure rate) and there is no doubt that the failures tends to bunch around

TABLE 11.3 DATA FROM A SINGLE 5 YEAR OLD RAPID-COOL AUTOCLAVE, 17 MONTHS PERIOD.

t cycles between failures of type named	Number of failures at cycle t	Median Rank F(t)
All faults		
1	4	0.096
2	2	0.148
3	3	0.227
4	1	0.253
5	3	0.331
6	3	0.409
7	3	0.487
8	1	0.513
9	2	0.565
10	1	0.591
11	2	0.643
12	1	0.669
14	1	0.695
16	1	0.721
18	2	0.773
23	1	0.799
29	1	0.826
37	2	0.878
41	1	0.904
53	1	0.930
56	1	0.956
93	1	0.982
581 Total	38	
Electrical Faults		
1	1	0.032
3	1	0.078
4	1	0.125
5	2	0.218
6	1	0.265
7	2	0.359
10	1	0.408
12	1	0.453
16	1	0.500
18	1	0.546
22	1	0.593
23	1	0.640
29	1	0.687
34	1	0.734
37	1	0.781
41	1	0.827
53	1	0.879
95	1	0.921
416	20 + 1 survivor at 165	
Steam Valve Faults		
6	1	0.095
15	1	0.230
29	1	0.365
35	1	0.500
95	1	0.635
187	1	0.770
214	1	0.905
581	7 (No survivor)	

6 cycles. The final series of 53, 95, and 165 + indicates that the hazard function definitely falls away with increasing cycles. On the other hand the sample is small and 'electrical' probably covers many different modes of failure so that an exponential distribution would be expected (see Appendix B).

The steam valve faults plot to $\beta = 0.8$ $\eta = 80$ but an exponential through the mean of 83 cycles is almost equally plausible. In so small a sample the confidence limits are very wide and it is not possible to confirm the tendency to early failure.

<u>11.1.4 Post-Modification Data (Table 11.4 and Figure 11.5)</u>. As a result of the record of failures analysed above, modifications were made to three autoclaves at Canterbury. Included in this work were new steam and water valves of different designs, a new design of printed circuit card for the controls, new instruments, new recorders of a different type and new test facilities. This work cost over £1000 per autoclave so a considerable improvement was expected.

The manufacturers had not conducted any experiments at their own works with any of these modifications. The hospital was therefore being used as a test facility and moreover asked to bear the costs of the experiments!

The result of fitting so many innovations at once was predictable by reliability engineering principles but appears to have been a surprise to both the hospital and the manufacturers. A Weibull plot of the data gives $\beta = 0.45$ $\eta = 6.7$.

The first autoclave to be modified ran for 139 cycles without failure and it is impossible not to suspect that the modification work on the two others was not so meticulously carried out. There is also evidence in the data of the same fault recurring after 1,2 or 3

No.	Autoclave	Cycle No.	Interval	Fault	Same as	Category
1)	Autoclave 1.	0-139	139	Fault on exhaust		Electrical
2)		140	1	" " "	(1)	Electrical
	Autoclave 2					
1)		0-25	25	Filter holed		Miscellaneous
2)		33	8	" "	(1)	Miscellaneous
3)		35	2	Failed to start		Electrical
4)		36	1	Failed on exhaust		Door Seal
5)		51	15	" " "	(4)	" "
6)		52	1	Failed to sterilise	(5)	" "
7)		52	1	Cycle counter failed		Miscellaneous
	Autoclave 3					
1)		0-3	3	Failed to reach temp		Electrical
2)		17	14	Filter holed		Miscellaneous
3)		19	2	Faulted on exhaust		Electrical
4)		21	2	" " "	(3)	Electrical
5)		21	1	Cycle counter failed		Miscellaneous
6)		26	5	Faulty recording		Recorder
7)		54	28	" "	(6)	Recorder

Cycle Interval	Faults	F(t) (Median Rank)
1	5	0.287
2	3	0.470
3	1	0.531 From Weibull
5	1	0.591 distribution
8	1	0.652 $\beta = 0.45$
14	1	0.713
15	1	0.774 Characteristic life
25	1	0.83
28	1	0.896 $\eta = 6.7$ cycles
139	1	0.957
	Total 16	

TABLE 11.4 POST-MODIFICATION DATA (1)

TABLE 11.5

POST MODIFICATION DATA (2). (collected after further action described in para. 11.1.5)

Cycles	Interval.	Description of Failure & Cause
0-90	90	Filter Blocked
139	49	Fault on exhaust
248	109	Very slow to Cool
421	173	Temperature probe cable broken
479	58	Cooling water reservoir boiling
		Temperature probe cable broken
		Main inlet stop cock leaking
		Probably all caused by central failure.
481	2	Not identified - Probe renewed
		and downstream pressure regulating
		valve (steam)
509	28	Steam leak around temperature Probe.

cycles only which points to inexpert maintenance.

11.1.5 Further Action. In view of the analysis of Table 11.4 shown in Figure 11.5 and discussed above, the manufacturers representatives were recalled to investigate the repetitive faults on the new exhaust valves, filters, recorders, and cycle counters. As a result two fitting faults were rectified on Autoclaves 2 and 3 and a modification design fault corrected on the filters. Also some training was given to the hospital electrical maintenance crew. These men were line electricians, and solid-state electronics was outside their previous experience and training. On the other hand nothing was done about the cycle counting mechanisms. The recorders remained unsatisfactory also and they are the subject of the next paragraph.

Subsequent to these further actions data was again collected. Seven failures occurred in 509 cycles. The Weibull shape factor for these failures was estimated at 1.2, but with so few, all that can really be said is that the distribution is more or less exponential.

11.1.6 Recorders - Quality Control of Product. Hoskins and Diffey, (2.25) both of the hospital concerned, describe a new suggestion for a time-temperature integral '∇' to measure the degree of sterilisation. (∇ is the time integral of temperature above 80°C for the cycle time). Any criterion for measuring sterilisation must depend heavily upon temperature measurements which must be both precise (repeatable) and accurate (correct). The sterilisation temperature for bottles of fluids must, avoiding pharmocopoeal detail, be controlled between 121°C and 124°C. If it is too low the time requirement exceeds the capacity of the autoclave time control (discharge of an electrical capacitance), if it is too high the bottle contents may suffer thermal degradation leading to lowered potency in use.

The difference between a satisfactory and an unsatisfactory cycle is such that for normal quality control charting procedures to be effective it is necessary to discriminate to about 0.25°C which is about half a standard deviation if the 3°C band is taken to cover the usual 6σ with the set-point in the centre i.e. 122.5°C.

Manufacturers' literature confirmed that the control circuit of the autoclaves relies upon the equivalence of pressure and temperature at saturation and simply maintains the chamber steam pressure constant during the 'hold' period of the cycle. The start of the hold period is signalled by a thermocouple which is placed in the coolest bottle of the load. This thermocouple is also connected to the temperature recorder which produces a time-temperature graph for the cycle. At the temperatures involved 0.25°C equates to about 18.8 mm Hg pressure and may be taken as linear over the 3°C band of interest. It is clearly much easier to control by pressure than by temperature. However equivalence can be upset by even a small air partial pressure. For quality control purposes therefore it is prudent to measure temperature directly, and this is what was done both before and after modification.

The arrangements for temperature measurement and recording were not considered satisfactory. Moreover, their inadequacy exacerbates the failure rate because a load may be thrown away because the recorder did not function correctly and so the hospital could not be sure that the cycle had been correct. The recorder pen-line was almost 2°C wide. The recorders tend to produce wavering lines, perhaps due to friction or mechanical hysteresis in their lever mechanisms. With these defects it was very difficult to tell whether the cycle has been successful or not.

Clearly, an expanded scale was required covering only the range of interest i.e. 80°C to say 130°C or whatever temperature corresponds

to the chamber safety valve set pressure. Control initiation by null points or equalities is generally to be preferred to initiation by set points. It was suggested that the thermocouple be provided with two constant voltage virtual junctions - one at 80°C and the other at 122.5°C. The 80°C null-point could be used to start the recorder and an integration circuit to measure ∇ . At the same time the second virtual junction would become operative. The second achievement of nullity would cause conditions to be held constant until a set value of ∇ had been reached.

Quality control charts for ∇ were also investigated but the precision required (+ ov - 0.25°C) exceeded the process capability with the charting arrangements then fitted. At the authors suggestion the physics section of the hospital designed built and tested a 'nablometer' to measure ∇ with a control system to initiate timing then hold the temperature at 121°C until ∇ reached a pre-set value and finally to operate the water spray for rapid cooling. The design was centred on a microprocessor. The meter could detect and identify some of the more common faults. It was also possible to obtain a record of temperature versus time in digital form, which was more accurate than the chart. As a result of this work the pharmacists at the hospital concerned are to propose changes in the Health Service standard procedure.

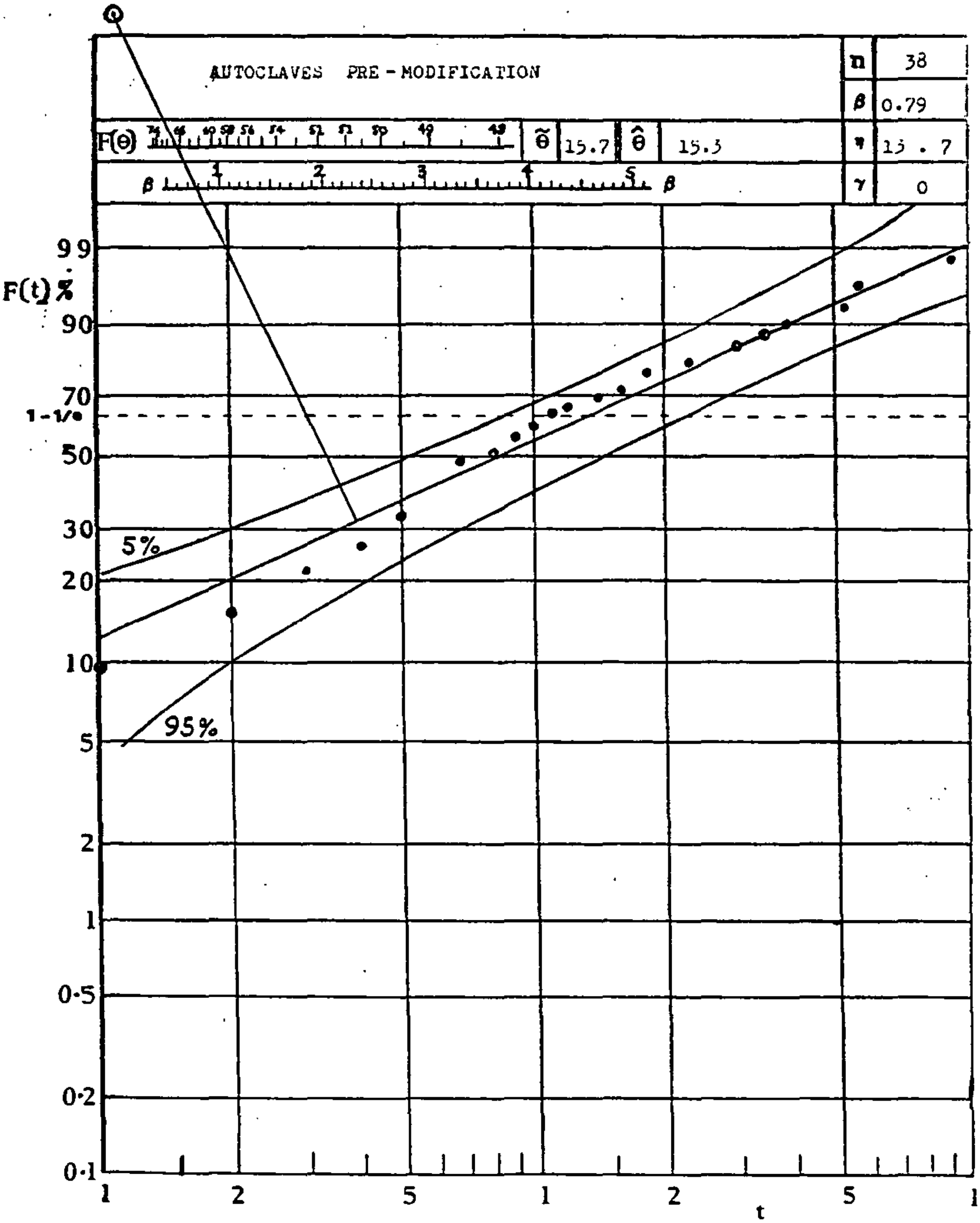

AUTOCLAVES PRE - MODIFICATION
n 38
β 0.79
η 13 . 7
γ 0
F(θ)
θ̃ 15.7
θ̂ 15.3
β 1 2 3 4 5 β
F(t) %
99
90
70
1-1/e
50
30
20
10
5
2
1
0·5
0·2
0·1
5%
95%
1 2 5 1 2 5 1
t

Figure 11 .1.1

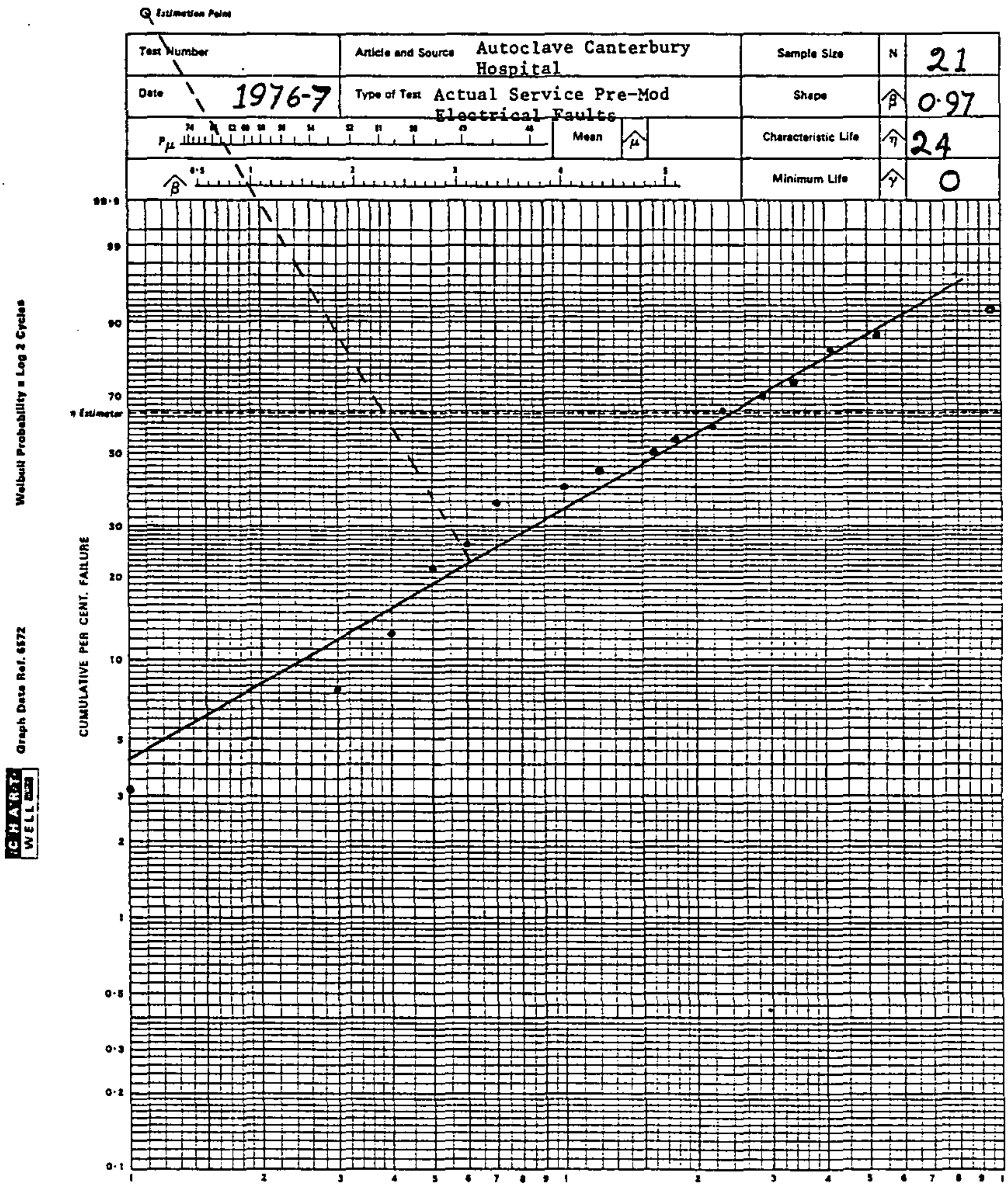

Estimation Point
Test Number
Date
1976-7
Article and Source
Autoclave Canterbury Hospital
Type of Test
Actual Service Pre-Mod Electrical Faults
Sample Size
N
21
Shape
β̂
0·97
Characteristic Life
η̂
24
Minimum Life
γ̂
0
Mean
μ̂
η Estimator
CUMULATIVE PER CENT. FAILURE
AGE AT FAILURE
Weibull Probability x Log 2 Cycles
Graph Data Ref. 6572
CHARTWELL

Figure 11.1.2

Canterbury Hospital Pre-Mod Electrical Faults 1976-7

Lognormal: m = 14·5 σ = 1·3

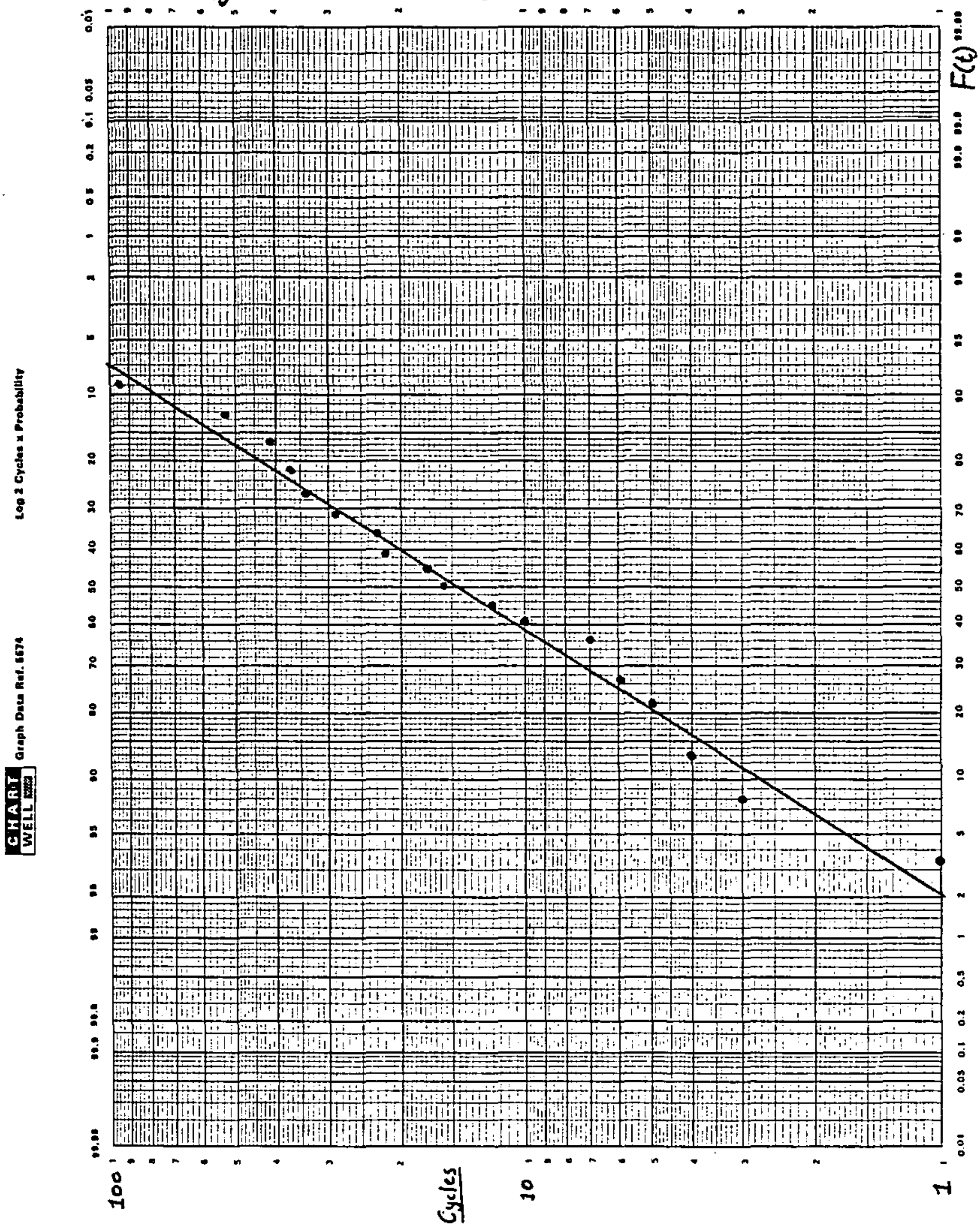


Figure 11.1.3

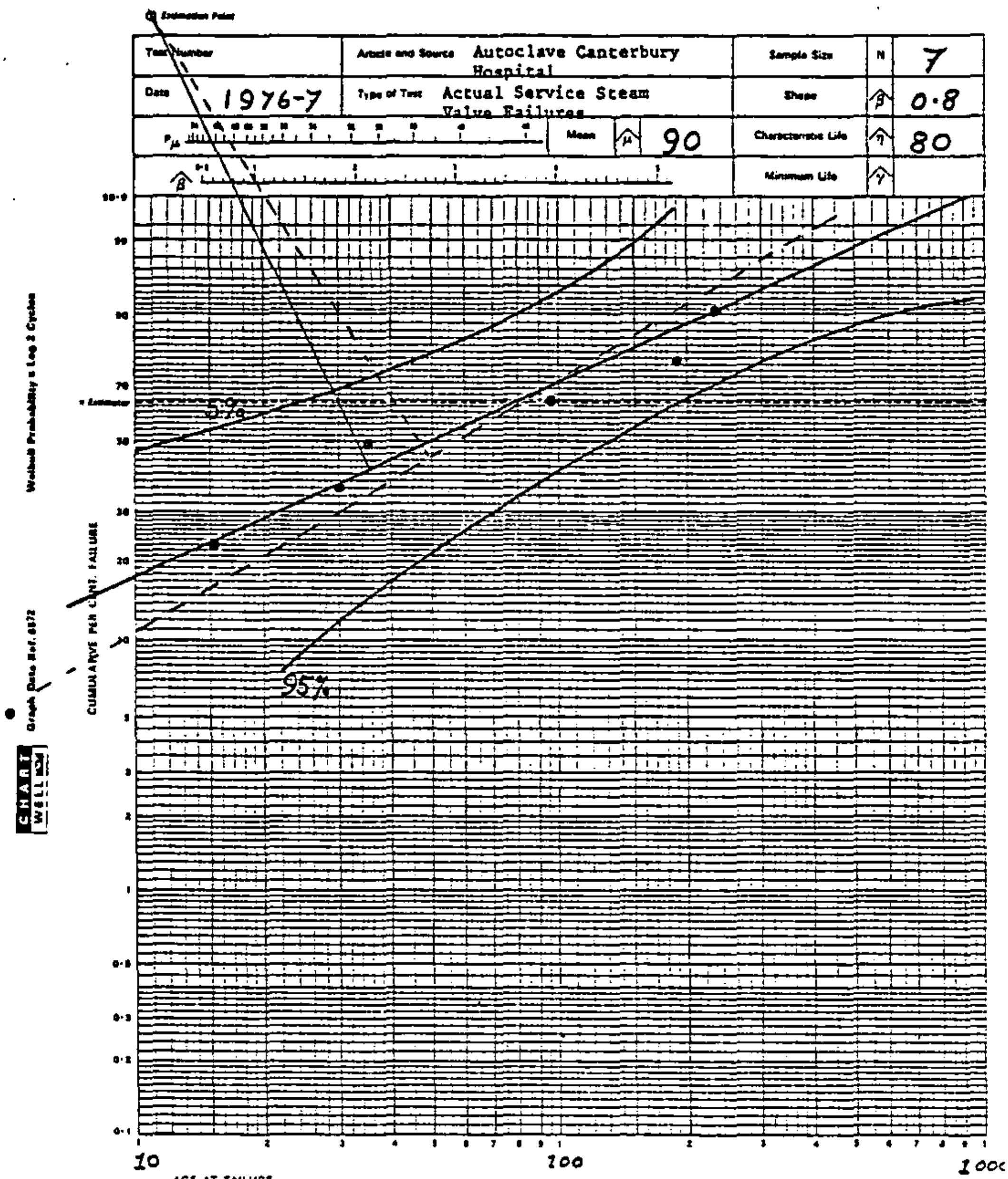

Estimation Point
Test Number
Article and Source
Autoclave Canterbury Hospital
Sample Size
N
7
Date
1976-7
Type of Test
Actual Service Steam Valve Failures
Shape
0·8
Mean
90
Characteristic Life
80
Minimum Life
Weibull Probability x Log 2 Cycles
Graph Data Ref. 6572
CHARTWELL
CUMULATIVE PER CENT. FAILURE
5%
95%
10
100
1000
AGE AT FAILURE

Figure 11.1.4

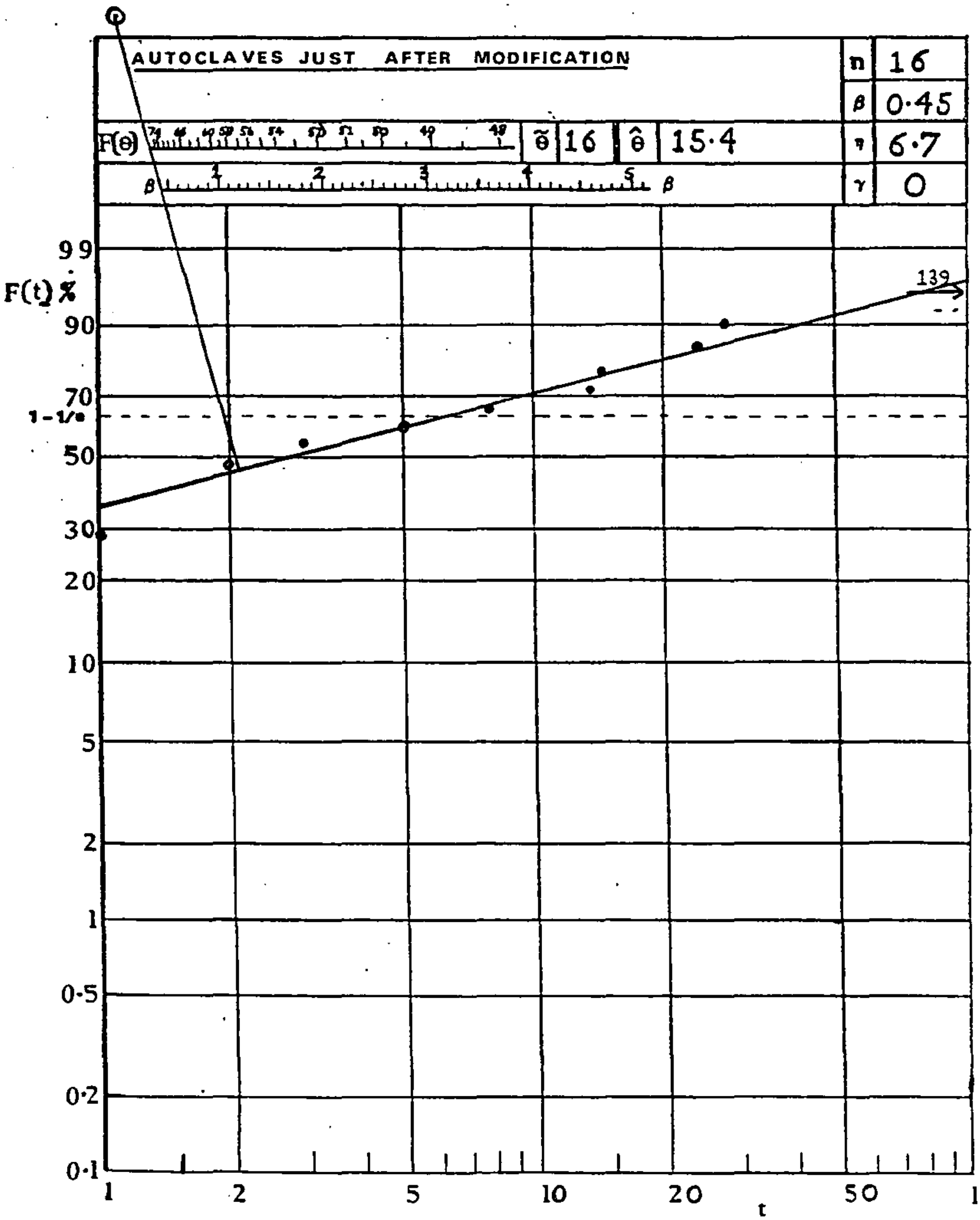

AUTOCLAVES JUST AFTER MODIFICATION
n 16
β 0·45
η 6·7
γ 0
F(θ)
θ̃ 16
θ̂ 15·4
β
F(t) %
99
90
70
1-1/e
50
30
20
10
5
2
1
0·5
0·2
0·1
1
2
5
10
20
50
t
139

Figure 11.1.5

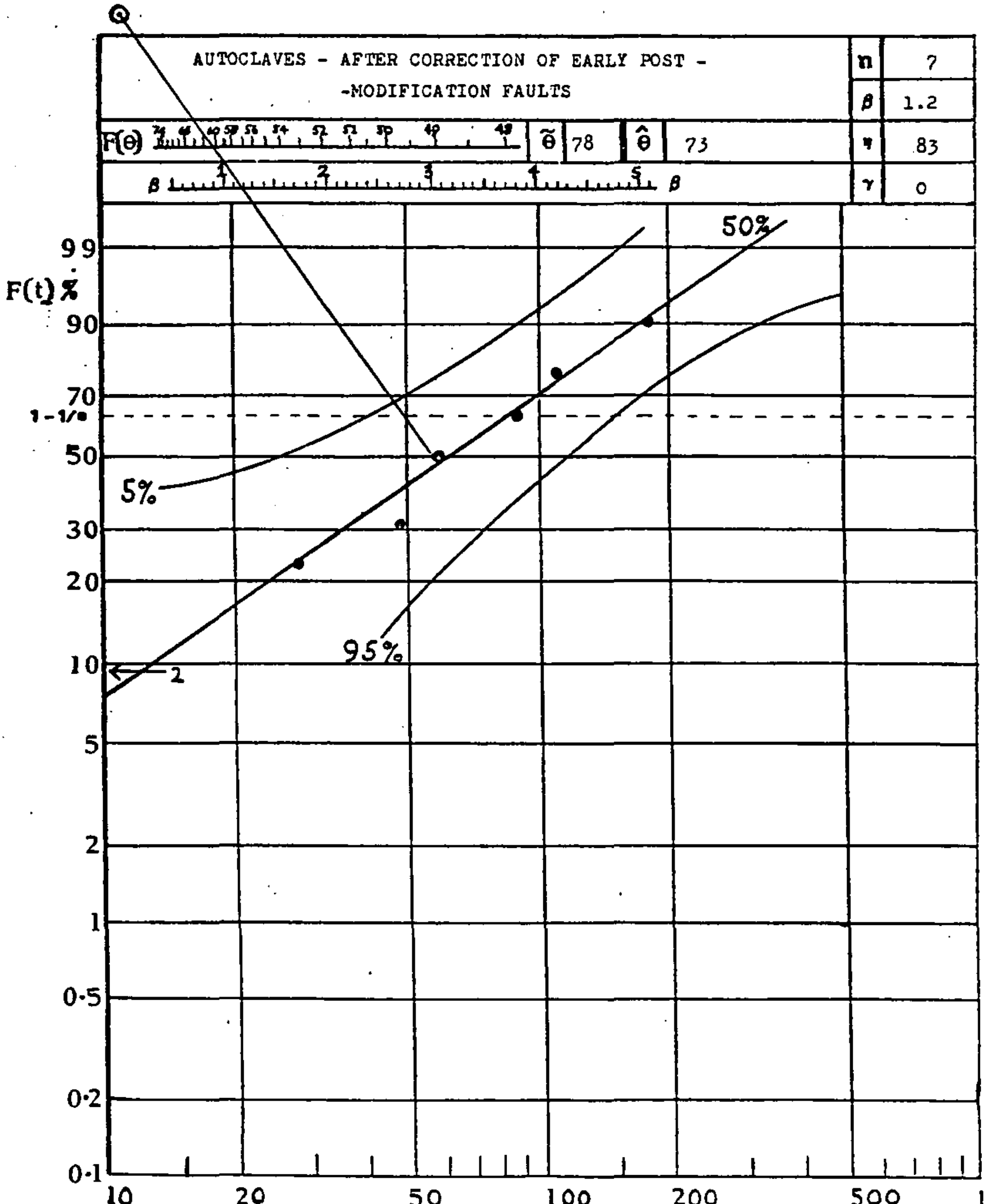

AUTOCLAVES - AFTER CORRECTION OF EARLY POST -
-MODIFICATION FAULTS
n 7
β 1.2
η 83
γ 0
θ̃ 78
θ̂ 73
F(θ)
β
F(t) %
99
90
70
1-1/e
50
30
20
10
5
2
1
0·5
0·2
0·1
50%
5%
95%
2
10
20
50
100
200
500
t

Figure 11.1.6

11.2 The Petrochemical Plant Study

11.2.1. Introduction. In this company data had been collected for about three years and stored on computer. Briefly, the records gave details of the date, the work done, the labour, stores and contract costs and the production lost for every failure and other shutdown for each of the many hundreds of equipments on the site. The maintenance data system was able to exchange information with pre-existing computer systems for work measurement and recording (labour costs) stores ordering and accounting, contract costing and main financial accountancy (profit and loss account) The data system was therefore almost 'ideal' in the sense discussed in Section 9 above, but with important omissions discussed below in detail which made some desirable analyses impossible.

As with the main study reported in Section 10, a portion of the complex was brought under particular study, and this portion was, as before, that which had the highest rates of failure and maintenance expenditure. In this case the plant chosen produced styrene monomer with ethyl-benzene as an intermediate product. Within the ethyl-benzene/styrene monomer (eb-sm) plant attention was focussed on the pumps. First, though, a study of an average section of the petrochemical complex, an ethylene plant was made in order to provide a perspective for judging the eb-sm results.

At one time it had been hoped to make this the main study or as large a study as that reported at Section 10 but administrative details took longer than expected and other work intervened. Also relevant in this connection is the much longer time-scale for failures. In a petrochemical plant times between failures (tbf's) are measured in months and years rather than the days and months found at the heavy chemicals plant described at Section 10. Again, with a pre-existing computer system and good facilities for

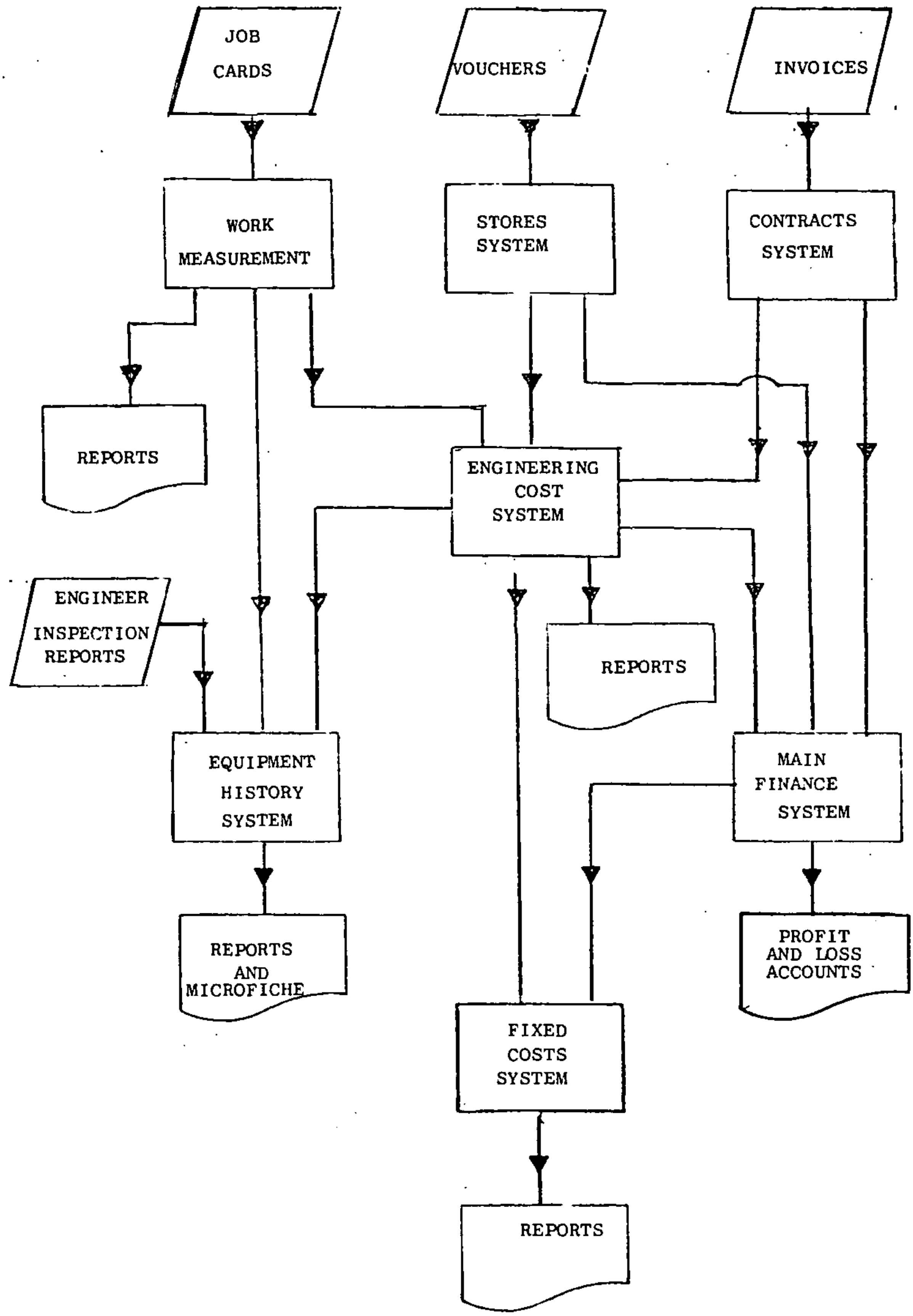


Figure.11.2.1 Integrated Management Information System

developing further software it was felt that the company would prefer to have suggestions as to how to proceed rather than a lot of specific analyses. However, a demonstration of the benefits of failure data analysis was specifically desired and has been provided.

The ethylene plant pump data analysis below confirms the prevalence of the hyper-exponential distribution found in other studies. Whilst early failures are present, it is not usually possible to discern inherent equipment reliability weaknesses amongst the results of maintenance shortcomings. The analysis of eb-sm plant pump data is a demonstration of what can be done by Pareto analysis alone to isolate the principal causes of downtime in a plant. The failures in this case were heavily concentrated in a few equipments most of which were 1-out-of-2 or 2-out-of-3 redundant. Without specific running times therefore, accurate frequency analysis was not possible.

Finally, the data collection and processing system is discussed in the light of the exercises carried out and the difficulties experienced, and suggestions made for its improvement.

11.2.2. Reliability Information System. The data is extracted from parts of the total management information system having the following features.

a) Equipment Inventory File. This file records basic engineering information such as manufacturer, model number, capacity, energy consumption, rating, dimensions drawing numbers etc. It corresponds in most respects to the Configuration Control File proposed in Section 9.

b) Maintenance History File. In this file up to three lines of 120 characters can be used to record each job. This record

MASTER

EXTRACT

*Delete not applicable

CC	[illegible] Code	[illegible] Grp	Job No.	Suffix	Grp	[illegible]
1 2 A 1	3 5	6	7 10			11 No. of men / Est. man Hrs

78

Priority	Job to be completed by – Date:	EQUIPMENT NUMBER 12 13 18	MAINTENANCE COST CENTRE 19	Cat	Work Ord No. 26	SS 27	Internal Job No.

Originators Job Request (printed)

Enter Prefix letter here, i.e. P or V or E etc.

PROJECTS: Prefix | Project Number | Sub | SS

Plant and Unit Location:

Plant/ Equipment Line – Available	on Request/By Arrangement
	From
	To

Fire Permit Safety Cert. Required.	YES	Obtain Clearance from:
	NO	

Drawings/Sketch/MESC/Ref:

Deliver Completed job to: Delv Point

JOB METHOD (For Engineering use only)

Originator: (print name)

Date | Tel:

Authorised by: | Date: | Tel:

Extract Cards	01	02	03	04	05	06	07	08	09	10	11	12	13	14

Special tools/Material/Preparation

*Clear up work area on completion of Job

Changes to work:

*This card must NOT be removed from Planning Office

1st Copy

P.0014

B/M Code or Description	Time in Dec. Hrs.		Time and Date	Elapsed Hrs.
(BACK)		OFF		
		ON		
		OFF		
		ON		
Add P.S.H. for change of work		OFF		
TOTAL P.S. HOURS	28 31	ON		
Interference Allowance		OFF		
Job & Travel Allowance		ON		
J & T Allow. Addit. Jobs		OFF		
Special Allowance		ON		
No. of Men / Standard Hours	32 35		Total Elapsed Hours	36 39

40–51 Job Activity Report: Ring up to a maximum of 6 codes

				E,A Pipeline Change	F,A Burners Cleaned
A,A Planned Maint.	B,A Considered a frequent fail	C,A Align & Adjust	D,A Piston ring Change	E,B Gasket Change	F,B Burner tip Change
A,B Breakdown	B,B Considered a temp. repair	C,B Oil seal Change	D,B Valve Change	E,C Basket Change	F,C Furnace Decoke
A,C Partial O/Haul	B,C Spares N.A.	C,C Wear ring Change	D,C Fan Change	E,D Blade Change	F,D Blockage cleared
A,D Complete O/Haul	B,D	C,D Coupling Service	D,D Sprocket Change	E,E Barrel Change	F,E Steam lance Change
A,E Fail after Maint.	B,E Bearing Change	C,E Coupling Change	D,E Belt Change	E,F Screw Change	F,F Exchanger Clean
A,F Fail on start	B,F Mech.seal Change	C,F Pump casing Change	D,F Roller Change	E,G Screen Change	F,G Exchange Float head joint Change
A,G Corrosion fail	B,G Shaft Change	C,G Pump Backplate Change	D,G Gear Change	E,H Slats Change	F,H Leak Repaired
A,H Lubrication fail	B,H Shaft Sleeve Change	C,H Lubricator Service	D,H Gearbox Change	E,I Steam Nozzle Change	F,I
A,I Blockage fail	B,I Impeller Change	C,I Packing Change	D,I Chain Change	E,J Agitator Service	F,J

52 54 Cause of failure on Extra information 55 80

Figure. 11.2.2 Petrochemical Plant Job Card also handles renewals and modifications. Input comes from Job Cards for smaller jobs and Engineer's/Supervisor's Reports for larger inspections and major overhauls etc. The engineer's reports are filed separately and referenced by the computer which also holds a 93-character summary. Information from Job Cards is recorded against up to 6 from a total of 53 codes which are simply ringed by the supervisor. On output the computer prints the codes in full and the design is such that the combination pretty well tells the story of the incident. Of its type, this coding system is considered very good, the codes being particularly well-chosen. A Job Card is reproduced at figure 11.2.2. The codes remind the supervisor of what is considered important and ensure that most common operations are reported in the same phraseology which helps computer sorting for Pareto analyses. The additional 24-character space for special comments, is a useful feature, safety regulations are provided for on the Job Card.

c) Lost Production Record. This file records the date the quantity of product lost, whether a full stoppage or under utilisation (slowdown) whether planned maintenance or break-down and brief details of the cause and repair. This system would not record an incident in which a standby took over the duty during repairs, unless the standby failed before the original failure was repaired.

11.2.3. Ethylene Plant Data Analysis. The data relates to 85 assorted pumps used in the ethylene plant over a 19-month period (October 1975 to April 1977 inclusive). It includes records of overhauls and repeat overhauls as well as specific classification of failure. The decision to overhaul a pump anywhere in the petrochemicals complex is customarily taken on the basis of current, pressure and vibration readings or else on the condition revealed by opening up to repair a failure. Overhauls are intended to restore a pump to good-as-new. Repeat overhauls are defined as occurring within two months of previous overhaul. This analysis was undertaken because it was desired to obtain a set of results which might be regarded as typical of petrochemical plant pumps generally against which to judge the results from the eb-sm plant

a) Pareto Analysis. Table 11.6 below is a breakdown of the principal causes of failure and the incidence of overhauls and repeat overhauls.

TABLE 11.6. ETHYLENE PLANT PUMP DATA PARETO ANALYSIS

Description	No.	%	mtbfs (in months)
Seals/Glands	119	49.0	13.6
Overhauls	62	25.5	26.0
Cleaning	14	5.8	115.4
Repeat Overhauls	7	2.9	230.7
Leaks	7	2.9	230.7
Motor Failures	5	2.0	323.0
Couplings	5	2.0	323.0
Bearings	2	0.8	807.5
Others	22	9.1	73.4
Total	243	100	6.65

Mechanical seals and packed glands were clearly the most prevalent cause of failure, particularly as they are also commonly renewed or refurbished at overhauls. 11% of overhauls had to be repeated within 2 months. These repeat overhauls are clear evidence of serious errors in maintenance work on about that scale. Specific enquiries led to a feeling that the most usual cause of repeat overhauls could be failure to align the machine properly at the first overhaul leading to early bearing, seal and wear-ring failures, but the sample is too small to be certain of this. If jobs could be traced back to individual fitters, then training could be given where required, otherwise a programme of refresher courses for fitters in seal renewal and pump alignment is indicated. The importance of independent inspection of at least the larger jobs before closing up is emphasised. With mechanical seals the problem is to make fitters aware of the consequences of even the tiniest amounts of dirt between the faces and to provide working conditions which allow them to achieve cleanliness. There is considerable standardisation of pumps on the site and it would probably be worthwhile using complete spare pumps for repair or refit by replacement (RXR) to reduce downtime and allow fitting work to occur in a cleaner place and under less pressure to finish quickly.

b) Frequency Analysis. Strictly, the analysis of this data for distribution cannot be completely valid without knowledge of the starting times of redundant equipments. Pumps which did not fail at all were assumed not to have run and omitted from the analysis. One objective of the analysis was to discover whether the failures and particularly the mechanical seal failures, were distributed hyper-exponentially, ($\beta < 1$). The other objective was to obtain a typical value for the mtbf to compare with the eb-sm plant values. About the same degree of redundancy exists in both plants so figures over calendar time (ignoring the effect of redundancy upon running time) would give a fair comparison. A total sample of 243 failures (191 tbf's and 52 censored) is summarised at Table 11.7 and Figure 11.2.3

TABLE 11.7 ETHYLENE PLANT PUMP FAILURE FREQUENCY ANALYSIS

TIME	FAILURES	SURVIVORS	MN.ORD	MEDIAN RANK
1 Month	46	187	47.9658	.195833
2	85	95	140.026	.57406
3	15	75	157.165	.644473
4	12	45	175.131	.718285
5	14	30	196.557	.806313
6	2	27	199.719	.819308
7	5	20	208.235	.854293
8	3	16	213.6	.876334
9	4	10	221.706	.90964
10	3	4	230.067	.943987
11	2	0	239.356	.982151

The Weibull analysis shows an overall β-value of about 1 which would be expected in a well-maintained plant. There are indications though of some early failures in the first 'dog-leg' and of wear-out failures starting at about 10 months. Another sign that early failures are occurring indicating short-comings in maintenance practice is that the mean as calculated from the graph does not agree with the maximum likelihood estimate (from the number of failures,

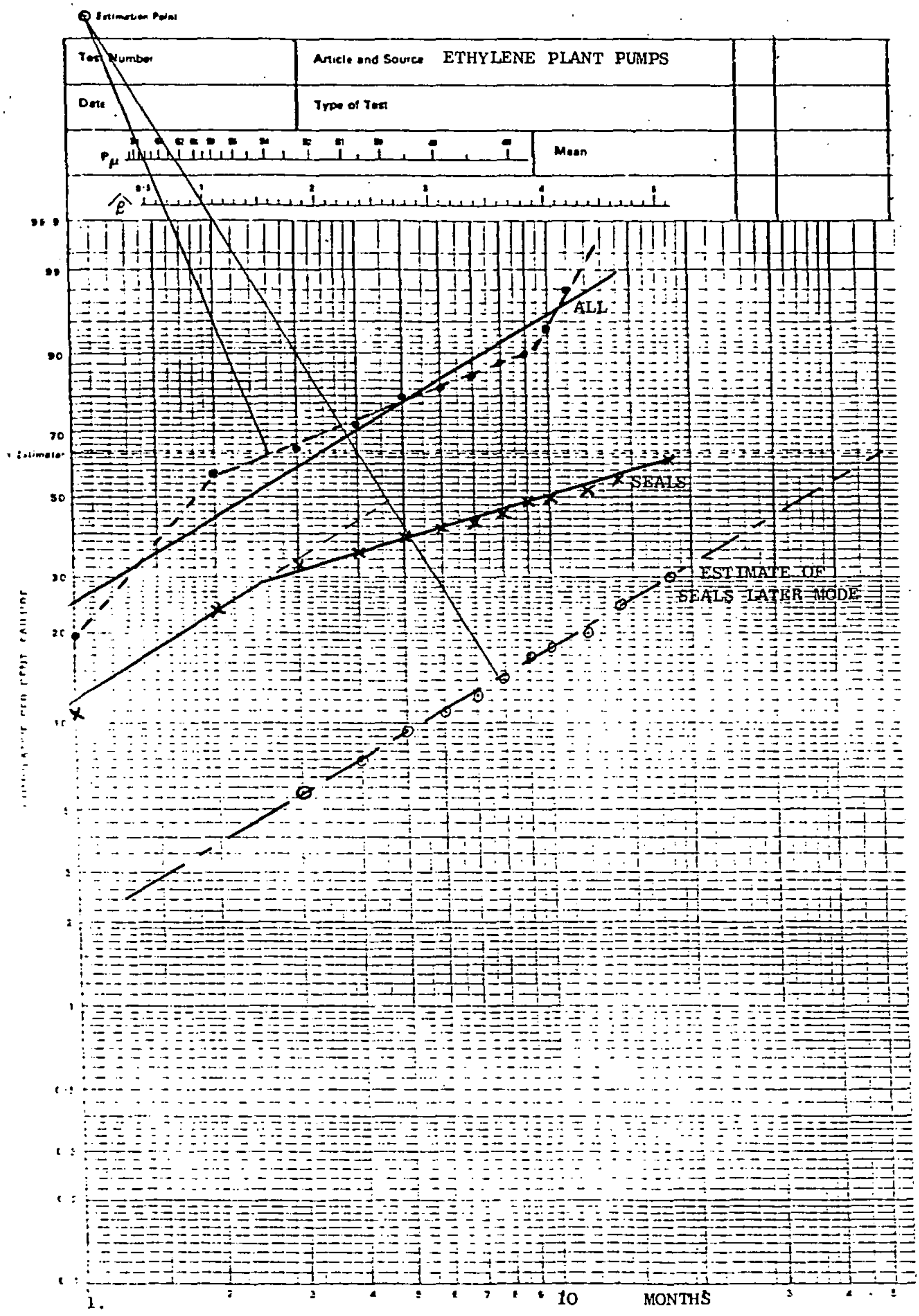


Figure 11.2.3 Ethylene Plant Pumps Weibull Analysis

the time and the number of pumps).

The mtbf figures are

From η , β	3.1 months
Maxlik	6.50 months

"Omitting pumps with no failures from the reckoning, the maxlik estimate came down to just over 4 months, which agrees well with the estimate from the second part of the composite plot of slope $\beta = 0.69$.

i.e. $\theta = \eta\Gamma(1 + 1/\beta) = 3.1\ \Gamma\ 2.45 \approx 4.05$

It is not considered likely that the shape of the plot would have been so well defined had cumulative hazard plotting been employed.

Table 11.8 and Figure 11.2.3 show an analysis of the mechanical seal failures extracted from the data above. The plot shows a definite hyper-exponential dog-leg with a break-point at about 27%. 24% of failures occurred within two months of the previous failure and 11% within one month. This indicates carelessness in fitting the seals or in re-aligning and balancing the pumps on about this scale. It should also be borne in mind that seals are usually changed at overhauls. Even without taking this factor into account a replot of the data with the early failures (before two months) treated as censored data showed that well-fitted seals should have a mean life of about 4 years. Note that the slope of the graph is then $\beta = 1$ which would indicate that mechanical seals even if well fitted fail for a variety of reasons in which wear is included but is overshadowed by external causes and obscured by renewals at overhauls.

TABLE 11.8 ETHYLENE PLANT PUMPS SEAL FAILURES

TIME	FAILURES	SURVIVORS	MN.ORD	MEDIAN RANK
1	18	144	18.5521	.109033
2	21	119	40.8103	.241997
3	13	102	55.0643	.327147
4	4	86	60.0286	.356801
5	5	80	66.306	.394301
6	3	76	70.1195	.417082
7	2	73	72.6953	.432469
8	3	68	76.6663	.456191
9	4	63	82.0389	.488285
10	1	60	83.4254	.496567
12	3	53	87.8767	.523158
14	3	46	92.6841	.551876
18	2	44	95.889	.571022

11.2.4 Ethyl Benzene - Styrene Monomer Plant Data Analysis. A preliminary analysis was made of the 10 months data provided covering all items in the eb-sm plant. This is shown in Table 11.8

TABLE 11.8 EB-SM Plant Preliminary Analysis

DESCRIPTION	No.FITTED	INCIDENTS	mtbm MONTHS
Gas Compressors	4	31	1.29
Screw Conveyors	3	19	1.58
Pumps	121	312	3.88
Others	235	287	8.19
Total	363	649	5.59

Others included heat exchangers, columns, fans, tanks, valves etc, i.e. plant with few or no moving parts. It is quite usual for most failures to occur to moving machinery. The mean times between maintenance actions (mtbm's) are instructive. Those for the gas compressors and screw conveyors definitely invite further investigation and the figure for pumps is markedly worse than in the ethylene plant. (see Table 11.6).

No other class of item was below the average mtbm of 5.59 months. Further Pareto analyses were conducted on the gas compressor,screw conveyor and pump data with the following results.

a) Pump Data. It was quickly seen by glancing through the pump data records that 15 pumps out of the 121 were failing much more frequently than the other 106. These fell into three classes, caustic pumps, ethyl-benzene pumps, and complex re-cycle pumps. The analysis of data by fluid pumpedis at Table 11.9

TABLE 11.9 EB-SM PUMP DATA BY SERVICE

DESCRIPTION	NO.FITTED	INCIDENTS	mtbm(MONTHS)
Complex Recycle	3	47	0.64
Caustic	4	43	0.93
Ethyl-Benzene	8	78	1.03
Others	106	144	7.36
Total	121	312	3.88

Thus, 15 pumps accounted for nearly half the incidents. The mtbm's given actually flatter the pumps because the complex recycle pumps are in 2 out of 3 standby redundancy, the caustic pumps consist of two sets of two pumps each in 1 out of 2 standby and of the ethyl-

-benzene pumps only one has no standby. There is also an unfitted spare for the complex recycle pumps. The reasons for failure of these 15 pumps were next examined. An incident often covers the maintenance of several parts of a pump and the reports do not always make clear the reason for the original failure. All the reasons are recorded in the table below so the numbers considerably exceed the number of incidents.

TABLE 11.10 EB-SM PLANT PUMP PARETO ANALYSIS

CAUSE OR PART AFFECTED	COMPLEX RECYCLE PUMPS	CAUSTIC PUMPS	ETHYL BENZENE PUMPS	TOTAL
Mechanical Seals	17	26	44	87
Vibration/Alignment	6	17	18	35
Overhauls	5	9	16	30
Gasket/Blockage	11	7	8	26
Bearings	3	3	17	23
Casing/Backplate	8	-	10	18
Repeat Overhauls	2	6	6	14
Shaft/Shaft Sleeve	2	-	11	13
Impeller/Wear rings	6	-	7	13
Pipework	5	5	2	12
Corrosion/Erosions	8	-	1	9
Valves	2	2	4	8
Oil Seal	-	-	7	7
Others	2	2	1	5
Total No. of Incidents	47	43	78	168

Mechanical seals are obviously a major source of trouble. Scanning the data it was noted that as often as not the seal failed within a month of an overhaul. Nearly half of the 30 overhauls had to be repeated within two months, usually because vibration levels had become excessive. This vibration was attributed to a number of causes including unbalanced impellers, failure to secure pipe brackets and holding down bolts. Incorrect alignment caused many of the early seal failures, but not all, so some were probably caused by minute dirt particles introduced at the time of fitting. A frequency analysis of the seal failures was undertaken but has not been included because its hyper-exponential shape ($\beta<1$) could be attributed at least in part to the redundancy i.e. long times to failure would be recorded when a pump had been on standby and much shorter ones when it was running giving a hyper-exponential characteristic. If it were known that when standbys were brought into use by a failure or overhaul they stayed in use until the next failure then this question could be resolved, but enquiries were inconclusive on this point; it seems that policy varies even within a single plant.

b) Gas Compressors. Table 11.11 gives a Pareto analysis of the incidents recorded against the gas compressors.

TABLE 11.11 EB-SM PLANT GAS COMPRESSOR PARETO ANALYSIS

Description	No. of incidents
Lubrication Failure	12
Overhauls	5
Valves	4
Repeat overhauls	2
Others	8

The biggest problem is obviously lubrication failure. One compressor failed twice in 700 hours requiring very extensive overhaul in one instance by the manufacturer.

This problem should be investigated. It is likely to be caused either by failure to perform simple servicing operations or by a design fault. It was also noted that the maintenance costs of these compressors were comparatively high.

c) Screw Conveyors. The failure modes analysis for the screw conveyors is at Table 11.12.

TABLE 11.12 EB-SM PLANT SCREW CONVEYORS PARETO ANALYSIS

Description	No.
Gland Packing	9
Blockages	5
Gearbox	2
Alignment	2
Overhauls	2
Bearings	1
Others	1

The most common cause of maintenance is the glands which continually need re-packing. This is a nuisance but it does not cause much loss of product or expense. The blockages are possibly an operational problem which could be avoided by closer process control. One conveyor needed two gearboxes in quick succession which may indicate carelessness in alignment, but the data are too few for firm conclusions.

11.2.5. Data System Critique. Comparing this data system with the 'ideal' data system described in Section 9, the differences are few and small. Further, most of the data requirements to extend the system to the 'ideal' can be met without alteration to input simply by modifying the internal links with other parts of the management information system. The largest remaining gap in the data base is that equipment downtimes and running times are not recorded as such, and because of the presence of full and partial redundancy they cannot be deduced from the product loss file. Equipment downtimes, which should be broken down into waiting times and active repair times, are needed for calculations of the inherent availability of individual equipments and to draw attention to long waiting times often caused by spares shortages and slow supply of parts by manufacturers. Equipment availability figures are required for system availability analysis (to suggest plant layout modifications) and to eliminate inherently unreliable or unsuitable items from present and future production plants. The importance of feed-back of equipment R & M information to manufacturers is again emphasised.

11.3 Paper Mill Study

11.3.1 Introduction - This study consists of a single analysis. There was no change of policy followed by a second analysis. The mill processes native hard wood to pulp with waste paper and cardboard to produce a strong rough brown material, known as fluting paper which is the raw material for the manufacture of the type of corrugated cardboard used for packing cases. Although strictly classed as a process industry, mechanical handling and of course sawing and shredding of raw materials play a larger part than in most process plants and as much as in some industries normally classed as manufacturing. Alone among the studies reported in this thesis the management of this plant has always conscientiously operated a preventive maintenance schedule. The maintenance manager is fully convinced that pm is efficacious in maintaining production at the highest possible rate. The plant runs smoothly and generally meets its production targets.

11.3.2 Maintenance Policy - There is a schedule of monthly, bi-monthly and annual pm routines, based upon a mixture of experience and makers' advice. Records are kept and the periodicity of the routines is adjusted in the light of experience. Most of the bi-monthly routines actually started as monthly, but were extended in this way. Most of the non-annual routines call for inspection and ocpm as required. The annual routines on the other hand mostly require the machinery to be opened for the renewal of wearing parts or adjustment of clearances.

Production plans imply an availability target for the plant of about 90%. This allows for about 12-14 hours downtime per 7-day week plus a week for the annual shutdown. Over 10 years an average of 88% has been achieved. The usual procedure is to employ maintenance staff on building and structural maintenance, checks which can be performed with machinery running,and on the maintenance of standby machinery until either a failure occurs or the build-up of impending failures predicted by instrumentation and the five senses becomes such that a worthwhile amount of pm can be achieved in a short shut-down period. However, when the plant has been working well it is sometimes stopped anyway after about a fortnight to allow the maintainers access. In this way the pm is kept up to schedule. The maintenance staff is not put under pressure to resume production and is usually able to get up-to-date on pm at each stoppage. This is partly because the achievement of the required product quality depends upon well-maintained machinery, but also because the maintenance manager's conviction that pm pays is shared by the rest of the management team.

About 15% of jobs are caused by failures,25% are defects deferred to the next stoppage and 60% pm routines. This is close to the empirically-determined optimum corrective/preventive maintenance ratio recommended in most of the maintenance organisation texts see for example Priel(2.41)(1974).

11.3.3. Redundancy. There is very little redundancy in the plant apart from the two-out-of-three arrangement for the boilers which produce process steam and also, through turbo generators, part of the electrical power requirement. This means that the stoppages for maintenance are very important to the condition of the plant.

11.3.4 Instrument Maintenance. Although maintenance on the rest of the plant was continuous, the pm for instruments and controls was concentrated into the annual shutdown period. Automation had increased over the years and it was becoming difficult to get everything checked and refitted in the week allowed even with contract labour and a lot of refit by replacement (R x R). Another consequence of this policy was that problems, particularly with the automatic control systems were experienced on restarting. The staff were considering extending the pm system to cover instruments in the same way as other parts of the plant. This suggestion was supported by the author (whose advice was sought) on the following grounds.

a) Post-maintenance and post-modification early failures (teething problems) would not be so complicated or so serious if only part of the control system was disturbed at one time. Problems with controls tend to take a long time to diagnose relative to the time taken to actually repair the defect, and this time increases rapidly as the number of possible causes increases. Possible causes increase faster than the number of items disturbed because of problems involving more than one item.

b) Failure rate could be reduced by altering the periodicity of routines.

c) Holdings of spare instruments could be reduced because items exchanged for one routine could be refurbished and used in another place as part of another routine, a procedure not possible if all work is done at once. Also, holding against failures could be less if the failure rate could be reduced by better maintenance.

11.3.5. Data Record System. The data system records brief details of each job performed on each machine going back 8 to 10 years in some cases. Maintenance schedule achievement is not recorded but is known to be high. Where scheduled inspections led to corrective action this is recorded by the system. Work done to correct failures is, of course, also so recorded. It is unfortunate that it is not possible from the records to tell to which of the two categories a particular maintenance action belongs. The analyses below are therefore not strictly of failures but of failures plus times to on-condition preventive maintenance (ocpm). They therefore approximate the base, (underlying or maintenance-free) distribution functions more closely than the corresponding distributions under maintenance. An advantage of this is that early failures, perhaps induced by careless work at ocpm actions, will show up as short times between 'failures' (tbf's), whereas if only true failures were analysed the tbf would go back to the previous true failure and the fault would not be seen as maintenance-induced. There is a very large amount of data in these records which would repay further analysis. It is to be hoped that the University will be permitted to render further assistance by sending students to perform further analyses. The data analysed below concerns items of plant which have been less satisfactory than most and therefore not typical of the general state of affairs at the plant which has been described above.

11.3.6. Data Analyses. were conducted on items of plant selected by the plant management as having been more troublesome than most. Apart from the boilers which were three identical units all the items were single items of mechanical plant. They were

a) Boilers (3 in number) water tube, oil or gas fired

b) Digester Bottom Scraper This unit consists of a motor which drives a scraper through a complicated transmission involving hydraulics, belts and gears. The scraper is fitted at the bottom of a large, heated, pressurised vessel containing dilute semi-processed pulp, the drive entering through a gland at the bottom of the vessel. Its purpose is to prevent coagulation at the bottom of the vessel.

c) New Primary Refiner This unit separates the fibres without reducing their length by rubbing them between closely set plates revolving eccentrically.

d) Secondary Refiner Chest Pump which pumps semi-refined pulp.

e) The Fourth Dryer Section is one of a series of sections through which the made paper passes on its way from the paper-making machine to the final winding and roll-slitting operations. It consists of steam-heated rollers, tensioners, chain drives, and other machinery.

f) Rewinder This is the last stage of the process in which the paper is made into rolls and slit to width. This is a heavy machine, the rolls weigh about 10 tonnes.

- - - - -

The one, two or three-line work descriptions provided a better base for Pareto (failure mode) analysis than the coded data of the acid plant at Section 10 above. It was also possible to see which items or failure syndromes were giving rise to early failures, simply by inspection of the records. This was done by hand as a computer programme would not have put the nuances of phraseology satisfactorily in the correct categories.

After the experience of the study at Section 10 it was considered that despite the large data-sets it would be preferable to have cumulative distribution analyses by median ranks, F(t), rather than cumulative hazard, H(t), plots for the distributions of times between maintenance actions (tbm's). A computer programme was therefore written. This consisted of a bubble sort to put the tbm's and any censored times in ascending order and then find mean order numbers, F(t) and H(t). F(t) was preferred to H(t) because it is easier to pick up changes of distribution from F(t) graphs.

11.3.7. Boiler Data Analysis. The data covers a period of 9 years from mid 1969 to mid 1978.

a) Pareto Analysis. The three main causes of maintenance work were found to be steam leaks, gauge glasses and the combustion system including atomising steam. The last category would have been higher probably if the boilers had not been operated on gas for several years.

TABLE 11.3.1.a. BOILER PARETO ANALYSIS

Description.	No.	%
Steam Leaks	340	49.5
Gauge Glasses	172	25.1
Combustion System	72	10.5
Others	102	14.9
Total	686	100

mtbm- 686 incidents to 3 boilers in 3496 days = 15.3 days.

TABLE 11.3.1b. BOILERS.

TIME. T	FAILURES.	SURVRS. S	MN.ORD. M (I)	MED.RNK F(T)	CUM.HAZ. H (T)
1	65	621	65	.942599E-1	.994703E-1
2	55	566	120	.174388	.192129
3	37	529	157	.228293	.259673
4	39	490	196	.285111	.336181
5	42	448	238	.3463	.425697
6	37	411	275	.400204	.511797
7	38	373	313	.455565	.608688
8	31	342	344	.500728	.695334
9	16	326	360	.524038	.743176
10	16	310	376	.547349	.793422
11	24	286	400	.582314	.873867
12	22	264	422	.614365	.953764
13	20	244	442	.643502	1.03239
14	26	218	468	.681381	1.14482
15	14	204	482	.701777	1.21104
16	14	190	496	.722174	1.28195
17	14	176	510	.74257	1.35828
18	7	169	517	.752768	1.39875
19	14	155	531	.773164	1.48496
20	8	147	539	.784819	1.53778
21	13	134	552	.803759	1.63004
22	11	123	563	.819784	1.71536
23	4	119	567	.825612	1.74829
24	11	108	578	.841638	1.84485
25	5	103	583	.848922	1.89203
26	9	94	592	.862034	1.983
27	4	90	596	.867861	2.02625
28	2	88	598	.870775	2.0486
29	6	82	604	.879516	2.1188
30	2	80	606	.88243	2.14334
31	3	77	609	.886801	2.18132
32	6	71	615	.895542	2.2619
33	1	70	616	.896999	2.27599
34	4	66	620	.902826	2.3344
35	3	63	623	.907197	2.38056
36	6	57	629	.915938	2.47981
37	4	53	633	.921766	2.55191
38	2	51	635	.92468	2.59001
39	1	50	636	.926136	2.60962
40	4	46	640	.931964	2.69213
41	1	45	641	.933421	2.71387
42	1	44	642	.934878	2.7361
43	2	42	644	.937791	2.78208
44	1	41	645	.939248	2.80589
45	1	40	646	.940705	2.83028
46	2	38	648	.943619	2.88092
47	1	37	649	.945076	2.90724
52	2	35	651	.94799	2.96204
54	1	34	652	.949446	2.99061
55	4	30	656	.955274	3.11383
56	1	29	657	.956731	3.14717
57	2	27	659	.959645	3.21737
58	2	25	661	.962558	3.29286
62	1	24	662	.964015	3.33286
63	2	22	664	.966929	3.41801
64	1	21	665	.968386	3.46346
66	1	20	666	.969843	3.51108
68	2	18	668	.972756	3.61371
69	1	17	669	.974213	3.66927
70	2	15	671	.977127	3.79059
73	1	14	672	.978584	3.85726
74	2	12	674	.981498	4.00561
75	1	11	675	.982955	4.08895
80	3	8	678	.987325	4.39097
91	1	7	679	.988782	4.51597
101	1	6	680	.990239	4.65882
102	1	5	681	.991696	4.82549
105	1	4	682	.993153	5.02549
116	1	3	683	.99461	5.27549
125	1	2	684	.996066	5.60882
131	1	1	685	.997523	6.10882
145	1	0	686	.99898	7.10882

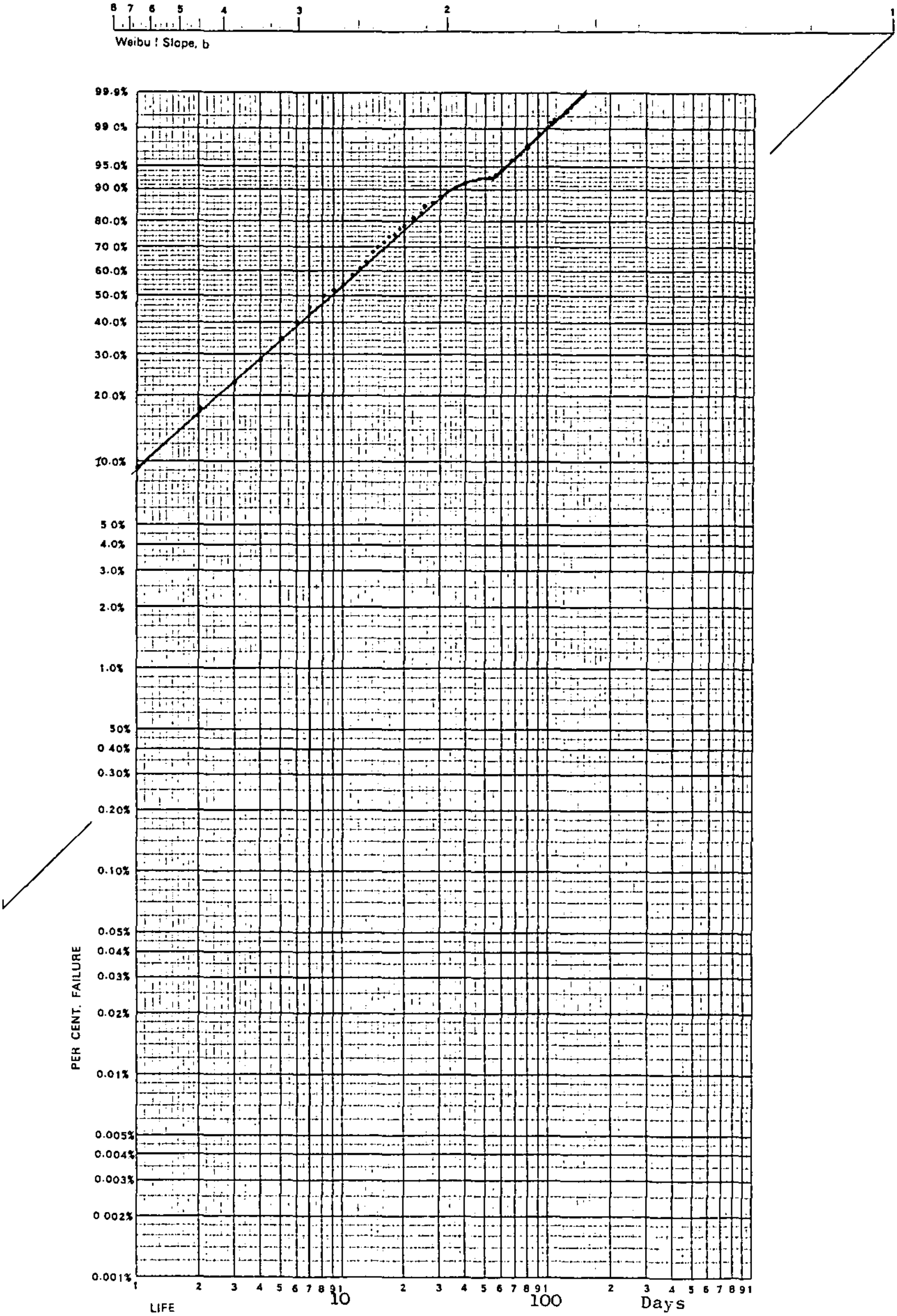


Figure 11.3.1a Weibull Plot of Paper Mill Boilers Times between Maintenance Actions

TABLE 11.3.1 c BOILER GAUGE CLASSES DATA.

TIME T	FAILURES	SURVRS S	MN.ORD. M (I)	MED.RNK. F (T)	CUM.HAZ. H (T)
1	4	168	4	.214617E-1	.234614E-1
2	1	167	5	.272622E-1	.294138E-1
3	4	163	9	.050464	.535841E-1
5	1	162	10	.562645E-1	.597191E-1
6	2	160	12	.678654E-1	.721031E-1
7	2	158	14	.794664E-1	.846424E-1
8	1	157	15	.852668E-1	.909715E-1
9	5	152	20	.114269	.123232
10	6	146	26	.149072	.163371
11	4	142	30	.172274	.191055
12	3	139	33	.189675	.212332
13	4	135	37	.212877	.241425
14	8	127	45	.259281	.30228
15	5	122	50	.288283	.342285
16	2	120	52	.299884	.358746
17	2	118	54	.311485	.375483
18	1	117	55	.317285	.383957
19	3	114	58	.334687	.409821
20	11	103	69	.398492	.510823
21	2	101	71	.410093	.530336
22	3	98	74	.427494	.560338
23	2	96	76	.439095	.580851
25	5	91	81	.468097	.634055
26	4	87	85	.491299	.678754
27	2	85	87	.5029	.701877
28	2	83	89	.514501	.725546
29	5	78	94	.543503	.787293
30	3	75	97	.560905	.826259
31	2	73	99	.572506	.853106
33	1	72	100	.578306	.866804
34	3	69	103	.595708	.909063
36	2	67	105	.607309	.938262
37	1	66	106	.613109	.953187
38	4	62	110	.636311	1.01522
39	3	59	113	.653712	1.06441
40	2	57	115	.665313	1.0986
42	1	56	116	.671114	1.11615
43	1	55	117	.676914	1.134
45	2	53	119	.688515	1.1707
46	1	52	120	.694316	1.18957
47	4	48	124	.717517	1.26882
48	1	47	125	.723318	1.28965
49	2	45	127	.734919	1.33267
50	1	44	128	.740719	1.35489
52	1	43	129	.74652	1.37762
53	1	42	130	.75232	1.40087
54	1	41	131	.758121	1.42468
55	3	38	134	.775522	1.49971

TIME T	FAILURES	SURVRS S	MN.ORD. M (I)	MED.RNK. F (T)	CUM.HAZ. H (T)
56	1	37	135	.781323	1.52603
59	1	36	136	.787123	1.55306
63	2	34	138	.798724	1.6094
65	1	33	139	.804524	1.63882
68	2	31	141	.816125	1.70037
70	1	30	142	.821926	1.73263
71	1	29	143	.827726	1.76596
76	1	28	144	.833527	1.80044
77	2	26	146	.845128	1.87319
78	1	25	147	.850928	1.91166
80	2	23	149	.86529	1.99332
82	1	22	150	.86833	2.0368
83	1	21	151	.87413	2.08226
86	1	20	152	.87993	2.12988
87	1	19	153	.885731	2.17988
90	1	18	154	.891531	2.23251
92	1	17	155	.897332	2.28806
102	1	16	156	.903132	2.34689
104	1	15	157	.908933	2.40939
108	1	14	158	.914733	2.47605
110	1	13	159	.920534	2.54748
113	1	12	160	.926334	2.6244
115	1	11	161	.932135	2.70774
124	2	9	163	.943736	2.89865
140	2	7	165	.955337	3.13476
164	1	6	166	.961137	3.27761
208	1	5	167	.966937	3.44428
216	1	4	168	.972738	3.64428
223	2	2	170	.984339	4.22761
366	1	1	171	.990139	4.72761
387	1	0	172	.99594	5.72761

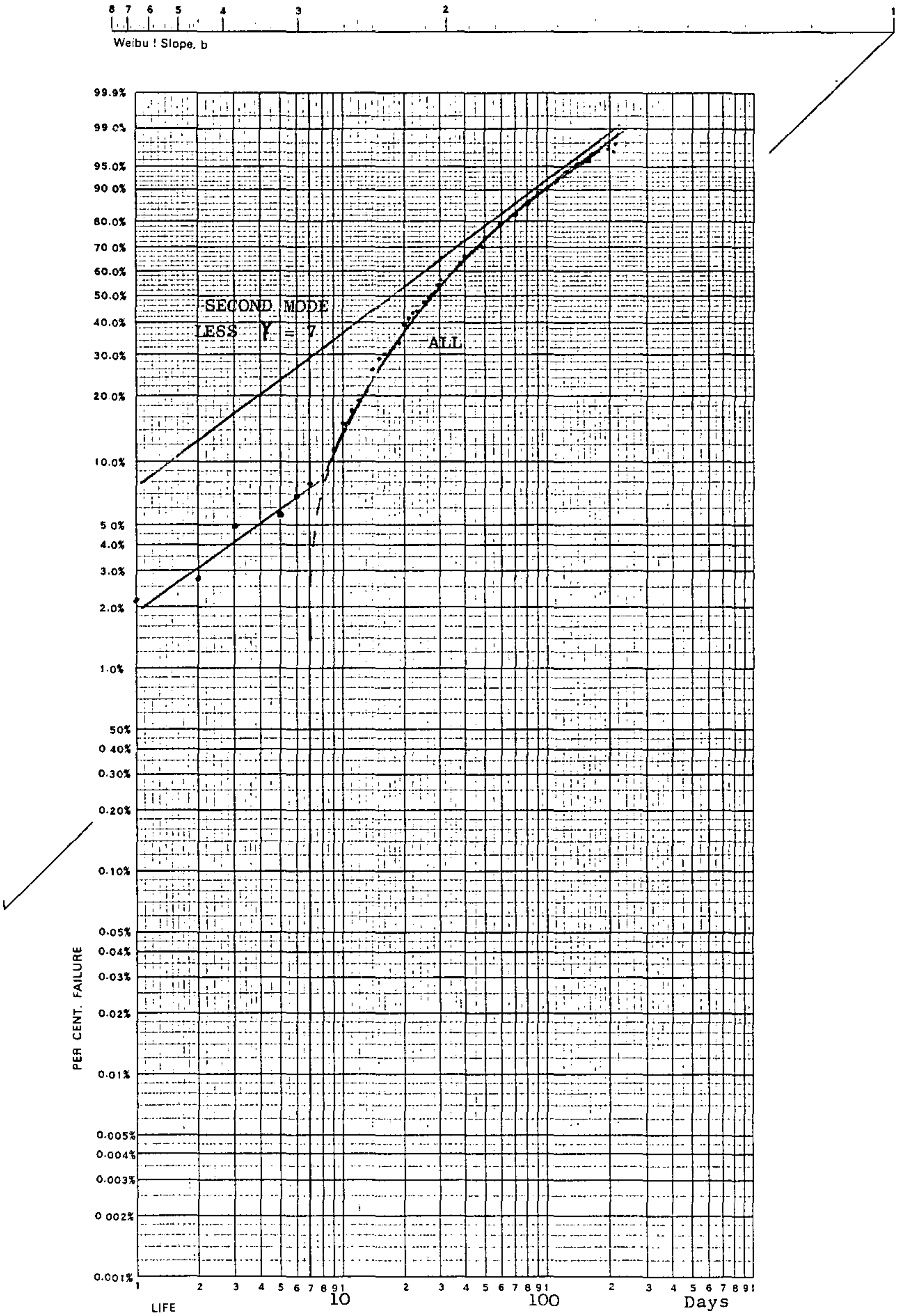


Figure 11.3.1b. Weibull Plot Paper Mill
Boiler Gauge Glasses.

TABLE 11.3.2 b DIGESTER BOTTOM SCRAPER.

TIME T	FAILURES	SURVRS. S	MN.ORD. M (I)	MED.RNK. F (T)	CUM.HAZ. H (T)
1	56	211	56	.208302	.234894
2	56	155	112	.417726	.542473
3	30	125	142	.529918	.756812
4	17	108	159	.593493	.902366
5	14	94	173	.645849	1.04052
6	9	85	182	.679506	1.1406
7	4	81	186	.694465	1.18851
8	7	74	193	.720643	1.27831
9	0	64	203	.75804	1.42245
10	2	62	205	.76552	1.45394
11	4	58	209	.780479	1.52008
12	8	50	217	.810396	1.66713
13	1	49	218	.814136	1.68713
14	6	43	224	.836574	1.81634
15	2	41	226	.844054	1.8634
16	1	40	227	.847794	1.88779
17	2	38	229	.855273	1.93843
18	2	36	231	.862752	1.99178
19	2	34	233	.870232	2.04813
20	1	33	234	.873972	2.07754
21	2	31	236	.881451	2.13909
23	1	30	237	.885191	2.17135
25	2	28	239	.89267	2.23916
27	1	27	240	.89641	2.27488
29	2	25	242	.903889	2.35038
30	2	23	244	.911369	2.43204
31	1	22	245	.915108	2.47552
35	2	20	247	.922588	2.5686
36	1	19	248	.926328	2.6186
37	1	18	249	.930067	2.67123
39	1	17	250	.933807	2.72678
44	1	16	251	.937547	2.78561
45	2	14	253	.945026	2.91477
46	1	13	254	.948766	2.9862
48	1	12	255	.952506	3.06313
49	1	11	256	.956245	3.14646
68	1	10	257	.959985	3.23737
70	1	9	258	.963725	3.33737
75	1	8	259	.967465	3.44848
98	1	7	260	.971204	3.57348
101	1	6	261	.974944	3.71634
124	1	5	262	.978684	3.883
131	1	4	263	.982423	4.083
150	1	3	264	.986163	4.333
175	1	2	265	.989903	4.66634
235	1	1	266	.993643	5.16634
252	1	0	267	.997382	6.16634

b) Frequency Analysis. The Weibull plot at Figure 11.3.1a consists of two distinct parts, each with $0.9 < \beta < 1$ and an overall mean of about 14 days. The actual figures in this plot are not so important as its shape which suggests two dominant types of failure. This reinforces the Pareto analysis indications. The 'kink' in the curve did not show up on a H(t) plot which gave $\beta = 0.83$, $\eta = 13.8$. With so large a data-set, confidence limits were omitted.

The steam leaks when plotted separately had $\beta = 1$ as expected from their diverse causes. The plot of the gauge glasses is more interesting see Figure 11.3.1b . The graph is bimodal, one mode of very early failures, before 8 days accounting for about 8%, whilst the other mode has γ 7.0 days. Both modes have $\beta < 1$. On investigation it was found that there were two gauge glasses per boiler and the data did not always permit separation. The analysis is therefore of pairs of gauge glasses and could possibly indicate that when one shatters, both should be renewed.

11.3.8. Digester Bottom Scraper Data Analysis

TABLE 11.3.2a. DIGESTER BOTTOM SCRAPER PARETO ANALYSIS

Description.	No.	%
Glands and Cooling Water	168	62.9
Bearings and Gears	45	16.9
Belts and Pulleys	27	10.1
Others.	27	10.1
Total	267	100%

mtbm - 267 incidents in 3324 days = 12.45 days

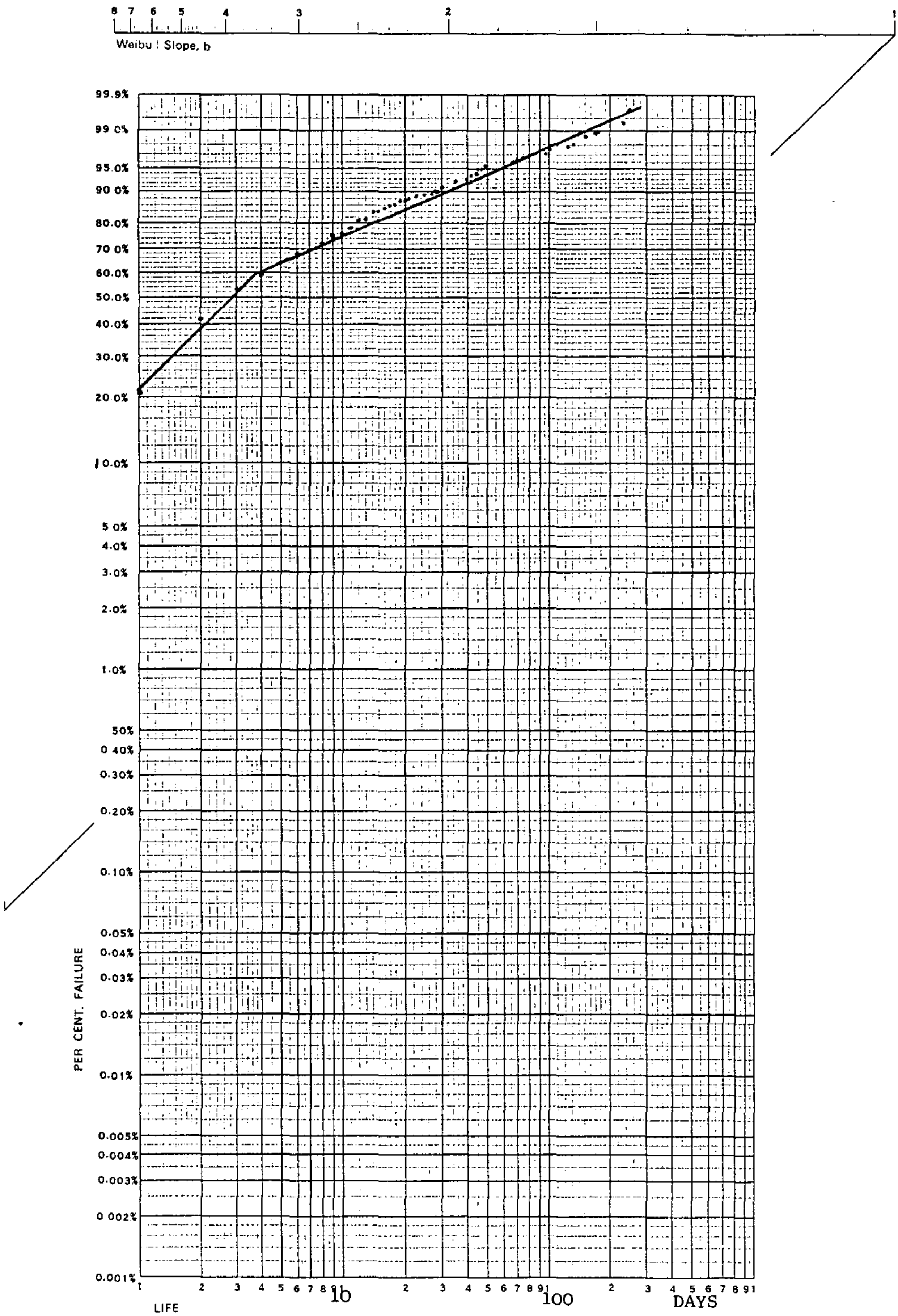


Figure 11.3.2 Weibull Plot Paper Mill Digester Bottom Scraper

Investigations showed that not only did the gland fail unacceptably often but that when it did it tended to allow hot pulp to cause damage to the transmission and on occasions even the motor. It was suggested that it would be better if the gland had to leak, and it would have been difficult to make the final vertical drive shaft to the scraper run absolutely true, then it would be better if it leaked plain water. Some dilution of the contents of the digester could be accepted so a modification was proposed whereby the gland was fed with water about halfway up its length the two portions of packing rings being separated by a lantern ring. Results of this modification are not known, but the point is not whether or not this particular idea works but to recognise that the problem is one which can only be solved by modifying the plant.

b) Frequency Analysis. 60% of incidents occur within 4 days of the previous incident and the dog-leg curve is typical of the hyper-exponential distribution. The overall Weibull β value is 0.52. See Figure 11.3.2

11.3.9 New Primary Refiner. As its name implies this is a unit installed since the plant was opened, actually in 1974. The data is therefore not so extensive, but nevertheless 76 incidents were recorded. There is standby redundancy with the older unit but the new one is preferred.

a) Pareto Analysis. The plates referred to below are those between which the pulp material is rubbed; one moves, the other is static. The clearances are very fine and the plates are usually exchanged as a pair for refacing and balancing. Some of the problems arise from foreign bodies in the material, but there is no cause recorded against others.

Foreign bodies could come from the raw material, from other machines or even from the refiner itself e.g. from the balance weights which have several times been found missing.

TABLE 11.3.3a. NEW PRIMARY REFINER PARETO ANALYSIS

Description.	Number	%
Plates	46	60.5
Balance Weights	5	6.6
Bearings	4	5.3
Others	21	27.6
Total	76	100%

b) Frequency Analysis. The Weibull plot shows a mildy hyper-exponential (β = 0.82) mode followed by a wear-out mode (β = 1.68). Further investigations showed that there were 18 incidents in the first 75 days of operation i.e. an average of 4.2 days between incidents. The remaining 58 failures occurred over 1417 days, an average of 24.4 days between incidents. Clearly this is a case of true early failures rather than maintenance-induced faults.

The overall mtbm was 1492 /76 = 19.6 days. A separate analysis of 43 times between plate changes after the initial 75 day period gave the following result β =1.85, η= 37.

The mtbm calculated from 1417/43 or from β and η is 32.9 in each case. The data also fits reasonably well to a lognormal distribution suggesting that the cause of failure could be a form of fatigue. 86% of plate failures could be prevented by a

TABLE 11.3.3 b NEW PRIMARY REFINER ALL DATA.

TIME T	FAILURES	SURVRS. S	MN.ORD. M (I)	MED.RNK. F (T)	CUM.HAZ. H (T)
1	3	73	3	.353403E-1	.400047E-1
2	5	68	8	.100785	.110455
3	4	64	12	.153141	.170623
5	1	63	13	.16623	.186248
7	4	59	17	.218586	.25131
8	4	55	21	.270942	.320901
9	1	54	22	.284031	.339083
10	1	53	23	.29712	.357602
11	3	50	26	.336387	.415308
13	2	48	28	.362565	.445716
14	5	43	33	.42801	.564515
15	2	41	35	.454189	.61158
16	1	40	36	.467278	.635971
18	3	37	39	.506545	.712927
19	3	34	42	.545812	.796304
20	5	29	47	.611257	.95286
21	1	28	48	.624346	.987343
23	1	27	49	.637435	1.02306
25	4	23	53	.689791	1.18022
26	1	22	54	.70288	1.2237
27	2	20	56	.729058	1.31677
29	3	17	59	.768325	1.47496
30	1	16	60	.781414	1.53378
31	2	14	62	.807592	1.66295
32	1	13	63	.820681	1.73438
33	3	10	66	.859948	1.98555
34	1	9	67	.873037	2.08555
35	1	8	68	.886126	2.19666
36	1	7	69	.899215	2.32166
37	2	5	71	.925393	2.63118
38	1	4	72	.938482	2.83118
39	1	3	73	.951571	3.08118
41	1	2	74	.96466	3.41451
51	1	1	75	.977749	3.91451
53	1	0	76	.990838	4.91451

TABLE 11.3.3 c PLATES ONLY.

TIME T	FAILURES.	SURVRS. S	MN.ORD. M (I)	MED.RNK. F (T)	CUM.HAZ. H (T)
7	1	41	1	.165094E-1	.238095E-1
8	3	38	4	.872641E-1	.988408E-1
11	1	37	5	.110849	.125157
13	1	36	6	.134434	.152184
14	1	35	7	.158019	.179961
18	1	34	8	.181604	.208533
20	2	32	10	.228774	.268248
24	1	31	11	.252358	.299498
25	1	30	12	.275943	.331756
27	3	27	15	.346698	.435286
30	1	26	16	.370283	.472323
31	2	24	18	.417453	.550785
32	2	22	20	.464623	.63593
33	2	20	22	.511793	.729003
34	1	19	23	.535377	.779003
35	2	17	25	.582547	.88719
37	2	15	27	.629717	1.00851
39	1	14	28	.653302	1.07518
40	4	10	32	.747641	1.39777
41	1	9	33	.771226	1.49777
43	1	8	34	.794811	1.60889
44	1	7	35	.818396	1.73389
47	1	6	36	.841981	1.87674
48	2	4	38	.889151	2.24341
52	1	3	39	.912736	2.49341
55	1	2	40	.936321	2.82674
62	1	1	41	.959906	3.32674
76	1	0	42	.983491	4.32674

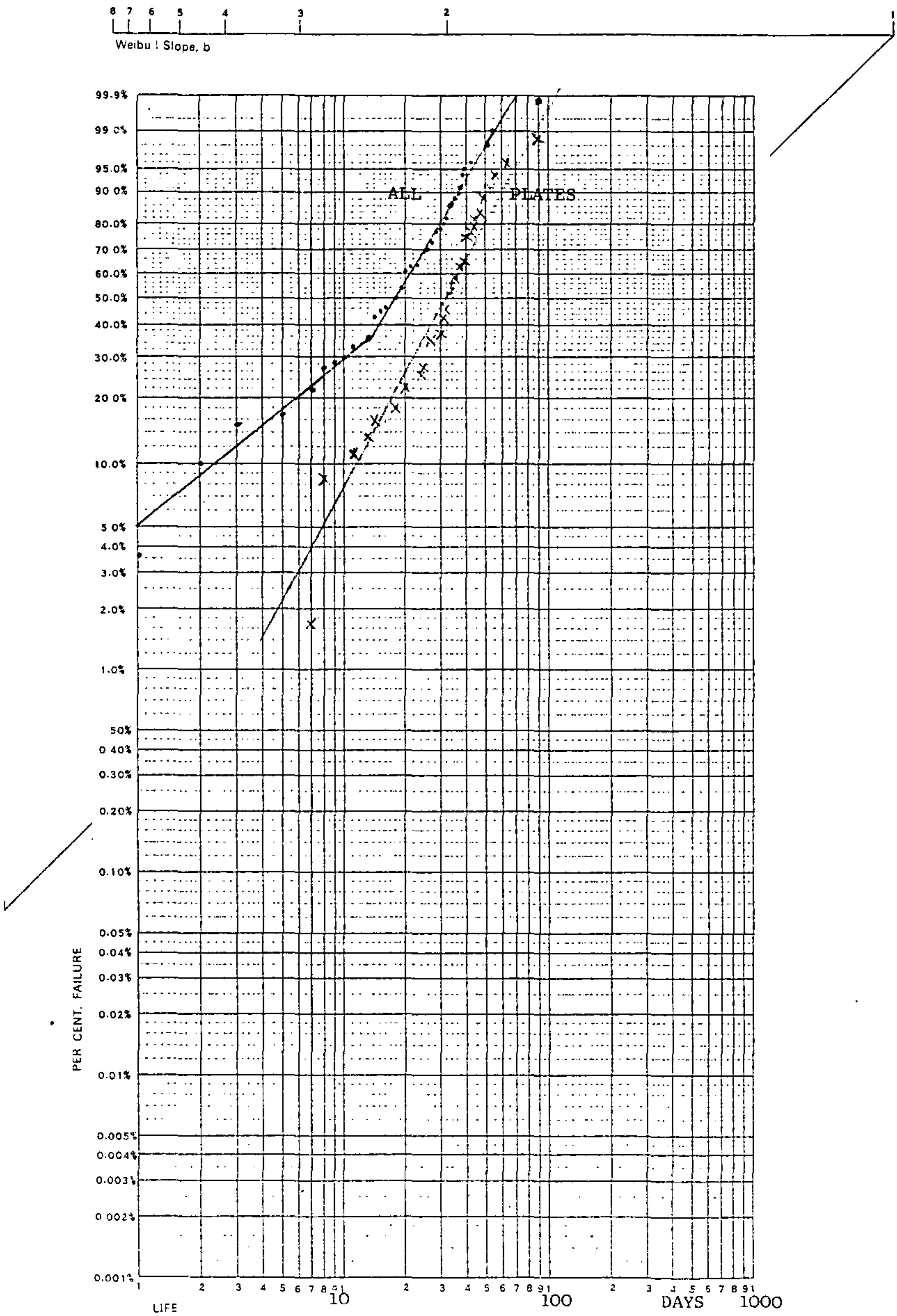


Figure 11.3.3 Weibull Plot Paper Mill
New Primary Refiner

TABLE 11.3.4 a SECONDARY REFINER CHEST PUMP ALL DATA.

TIME. T	FAILURES	SURVRS. S	MN.ORD. M (I)	MED.RNK. F (T)	CUM.HAZ. H (T)
1	1	16	1	.402299E-1	.588235E-1
3	1	15	2	.977012E-1	.121324
5	1	14	3	.155172	.18799
9	1	13	4	.212644	.259419
12	1	12	5	.270115	.336342
16	1	11	6	.327586	.419675
19	1	10	7	.385057	.510584
31	1	9	8	.442529	.610584
56	1	8	9	.5	.721695
58	1	7	10	.557471	.846695
62	1	6	11	.614943	.989553
83	1	5	12	.672414	1.15622
122	1	4	13	.729885	1.35622
158	1	3	14	.787356	1.60622
192	1	2	15	.844828	1.93955
216	1	1	16	.902299	2.43955
229	1	0	17	.95977	3.43955

TABLE 11.3.4 b SECONDARY REFINER CHEST PUMP OMITTING FIRST YEAR'S DATA.

TIME T	FAILURES	SURVRS. S	MN.ORD. M (I)	MED.RNK. F (T)	CUM.HAZ. H (T)
12	1	6	1	0.095	0.1429
56	1	5	2	0.229	0.3095
58	1	4	3	0.365	0.5095
83	1	3	4	0.500	0.7595
122	1	2	5	0.635	1.0929
158	1	1	6	0.770	1.5929
216	1	0	7	0.905	2.5929

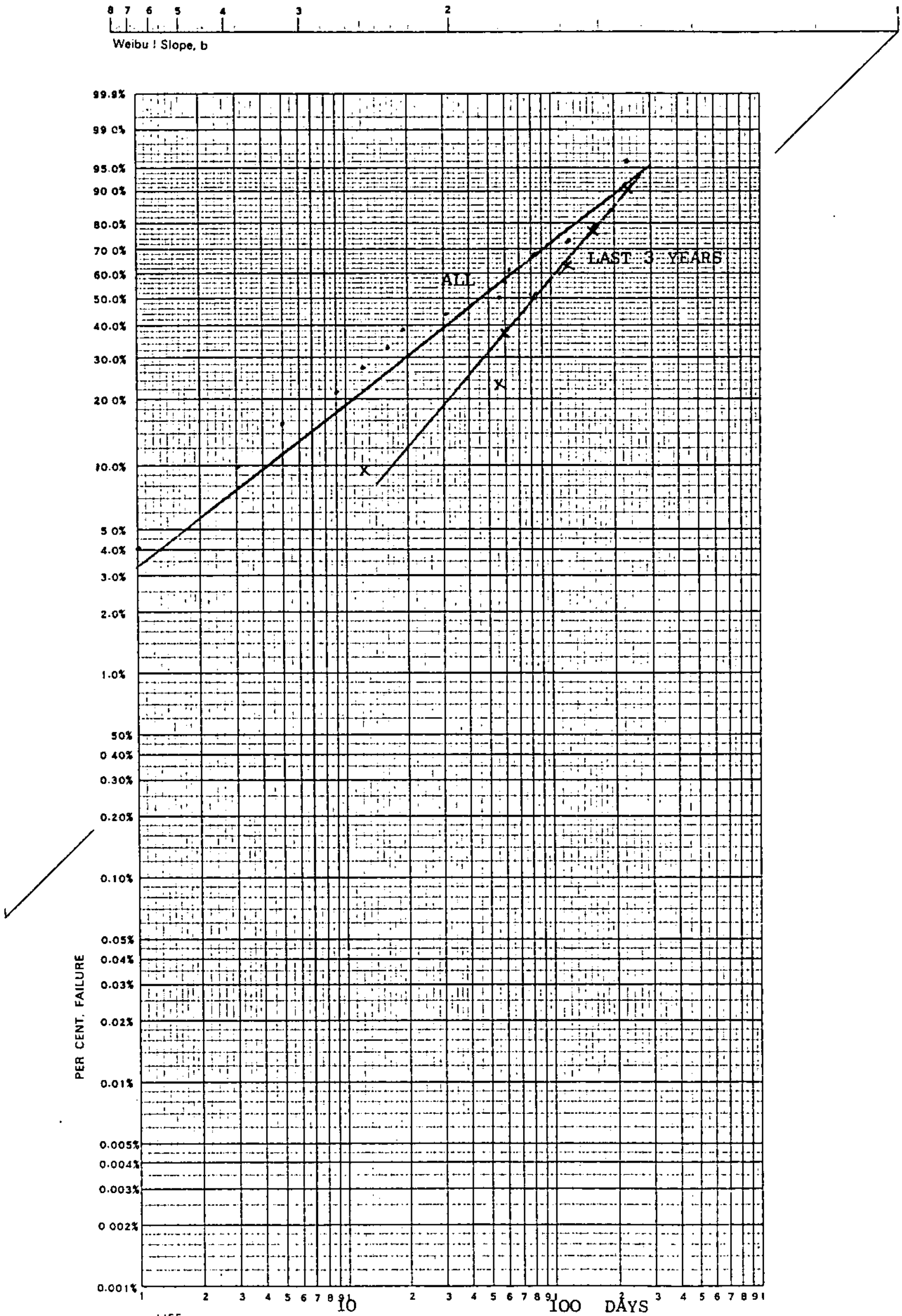


Figure 11.3.4 Weibull Plot Paper Mill
Secondary Refiner Chest Pump

fortnightly routine change of plates, but whether this would be worthwhile would depend upon the cost ratio C_F/C_M between a forced and a planned change. See Glasser (3.90). $\sigma/\eta \simeq 0.51$ and $\theta/\sigma = 1.75$ for $\beta = 1.85$. Optimum plate change intervals t* for various values of C_F/C_M and $\theta/\sigma = 1.75$ are given below from Glasser's chart, with the proportionate cost-rate advantage over fm.

C_F/C_M	t*	$1-c_{ppm}/c_{fm}$
2	45.5	0.07
5	21.8	0.18
10	14.4	0.38

11.3.10 Secondary Refiner Chest Pumps There was no pattern discernible in the data. Pareto analysis did not show any dominant mode. The mildy hyper-exponential Weibull plot (β=0.81) is due to true early failures, 10 out of 17 failures occurred in the first year and only 7 in the next 3 years.

mtbf 74.3 days including all failures
147.8 days in the latest 3 years
36.5 days over the first year

Replotting the last 7 failures gave β= 1.27

This is a thoroughly satisfactory equipment now that the early failure period is over.

11.3.11 Fourth Dryer Section.

a) Pareto Analysis Being a conglomerate of machinery which happens to be co-located, rather than a single entity, it was not expected that there would be any predominant mode of failure. From the analysis three types of incident would repay further investigation, namely the Roto-chambers, the steam nozzles and the sight glasses. However, it is not

TABLE 11.3.5 b PAPER MILL FOURTH DRYER SECTION DATA.

TIME T	FAILURES	SURVRS. S	MN.ORD. M (I)	MED.RNK. F (T)	CUM.HAZ. H (T)
1	8	173	8	.424476E-1	.450779E-1
2	8	165	16	.865491E-1	.922842E-1
3	7	158	23	.125138	.135501
4	4	154	27	.147189	.161061
5	9	145	36	.196803	.221079
6	8	137	44	.240904	.277631
7	7	130	51	.279493	.329881
8	11	119	62	.340132	.417938
9	9	110	71	.389746	.496238
10	5	105	76	.41731	.542542
11	2	103	78	.428335	.561681
12	9	94	87	.477949	.652652
13	9	85	96	.527563	.752735
14	7	78	103	.566152	.838152
15	5	73	108	.593716	.903964
16	6	67	114	.626792	.98912
17	6	61	120	.659868	1.08221
18	2	59	122	.670893	1.11527
19	2	57	124	.681918	1.14946
20	6	51	130	.714995	1.25966
21	2	49	132	.72602	1.29927
22	3	46	135	.742558	1.36179
23	2	44	137	.753583	1.40575
24	2	42	139	.764609	1.45173
25	1	41	140	.770121	1.47554
26	2	39	142	.781147	1.52493
27	2	37	144	.792172	1.57689
29	4	33	148	.814223	1.68968
30	2	31	150	.825248	1.75123
31	1	30	151	.830761	1.78349
33	1	29	152	.836273	1.81682
34	4	25	156	.858324	1.96251
35	1	24	157	.863837	2.00251
36	1	23	158	.86935	2.04418
37	1	22	159	.874862	2.08766
40	1	21	160	.880375	2.13311
41	2	19	162	.8914	2.23073
42	2	17	164	.902426	2.33892
44	2	15	166	.913451	2.46024
46	2	13	168	.924476	2.59834
47	1	12	169	.929989	2.67526
49	4	8	173	.95204	3.06062
53	1	7	174	.957552	3.18562
56	1	6	175	.963065	3.32847
65	2	4	177	.97409	3.69514
67	1	3	178	.979603	3.94514
68	1	2	179	.985116	4.27847
72	1	1	180	.990629	4.77847
171	1	0	181	.996141	5.77847

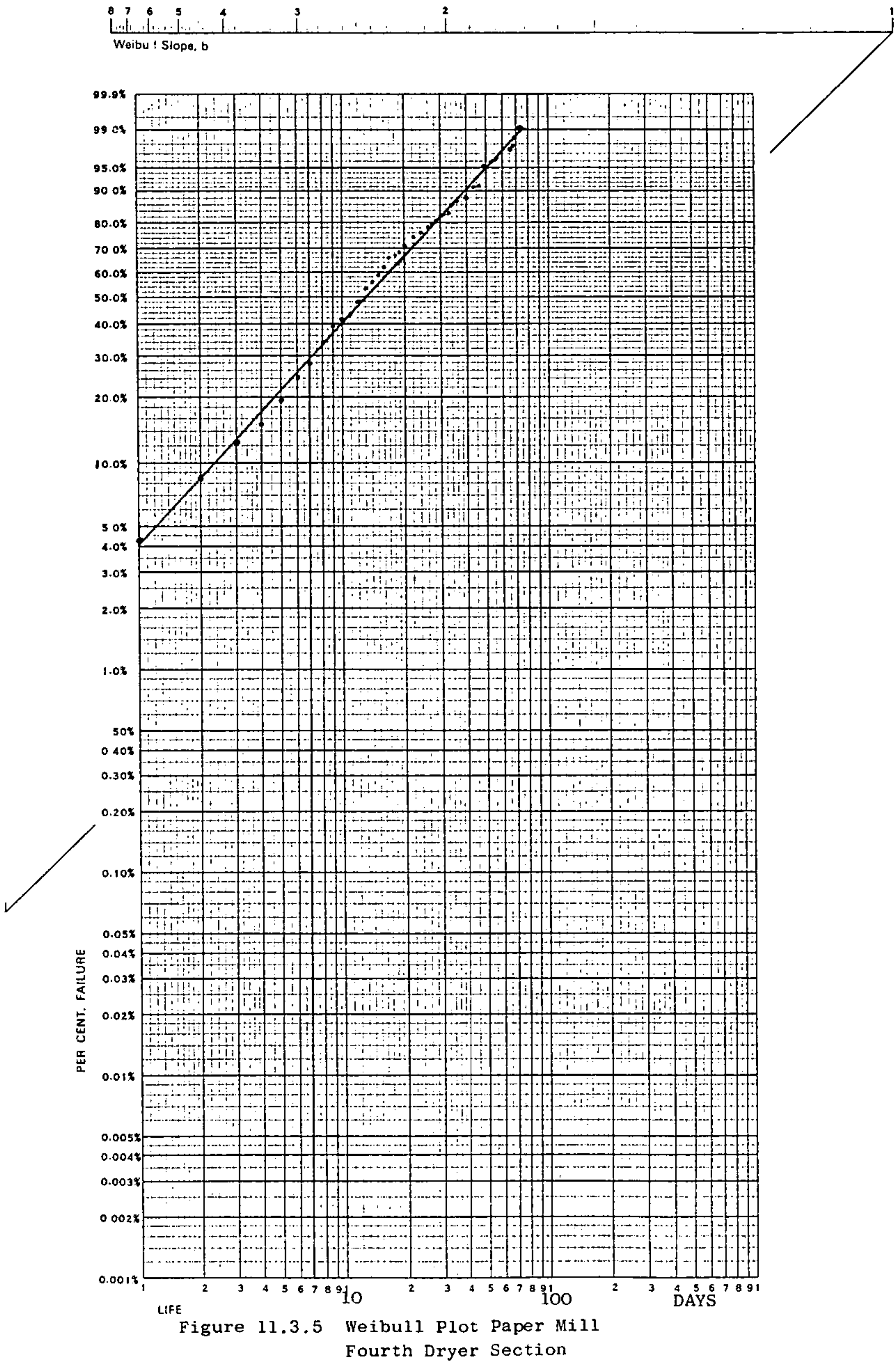


Figure 11.3.5 Weibull Plot Paper Mill
Fourth Dryer Section

known how many of the incidents recorded against these categories are pm routines. There is a disparity between the number of stoppages (181) and the total number of maintenance incidents (309) because in many cases more than one job was done whilst the line was stopped.

TABLE 11.3.5a. FOURTH DRYER SECTION PARETO ANALYSIS

Description	No.	%
Roto-chambers	84	27.2
Steam Nozzles	59	19.1
Sight Glasses	44	14.2
Bearings	23	7.4
Chain Tension	25	8.1
Seals(Oil and Steam)	9	2.9
Others	65	21.1
Total	309	100%

b) Frequency Analysis. The Weibull plot gives $\beta = 1.1$ $\eta = 18.5$. The mtbm calculated from 182 incidents in 3413 days is 18.75 which tallies fairly well with the value calculated from $\eta\Gamma(1+1/\beta)$ of 18.0. With such a mixture of failure modes $\beta = 1$ was to be expected. It would be easier to obtain useful statistical information from this data if the records were kept of individual machines rather than a whole section.

11.3.12. Rewinder. This was the largest data-set in this study with 903 incidents in 2987 days giving a mean time between

TABLE 11.3.6 b PAPERMILL REWINDER DATA.

TIME T	FAILURES	SURVRS. S	MN.ORD. M (I)	MED.RNK. F (T)	CUM.HAZ. H (T)
1	312	591	312	.34503	.423614
2	188	402	500.317	.553484	.806889
3	98	304	598.485	.662148	1.08591
4	80	224	678.621	.750854	1.39071
5	56	168	734.716	.812947	1.67765
6	35	133	769.775	.851755	1.91048
7	33	100	802.831	.888345	2.19442
8	22	78	824.867	.912738	2.44148
9	11	67	835.886	.924935	2.59245
10	8	59	843.899	.933805	2.7186
11	4	55	847.906	.93824	2.78819
12	8	47	855.92	.94711	2.94384
13	11	36	866.938	.959307	3.20724
14	4	32	870.945	.963742	3.32331
15	6	26	876.955	.970395	3.52738
16	8	18	884.968	.979265	3.88669
17	2	16	886.972	.981483	4.00107
18	1	15	887.973	.982591	4.06357
20	1	14	888.975	.9837	4.13024
21	2	12	890.978	.985918	4.27859
22	2	10	892.982	.988136	4.45283
24	2	8	894.985	.990353	4.66394
25	4	4	898.992	.994788	5.29847
26	1	3	899.993	.995897	5.54847
27	1	2	900.995	.997006	5.8818
29	1	1	901.997	.998115	6.3818
35	1	0	902.998	.999223	7.3818

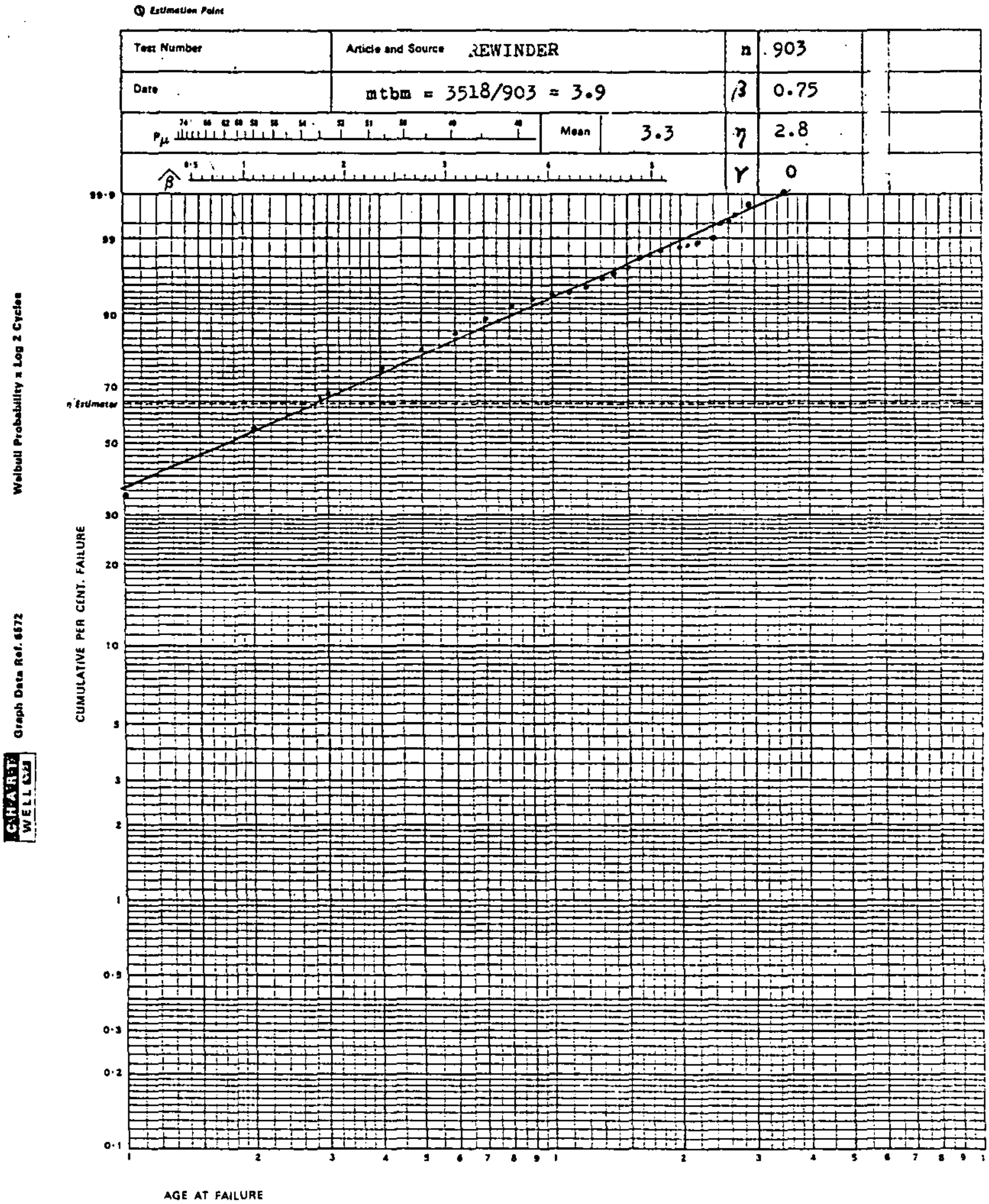


Figure 11.3.6. Weibull Plot for Rewinder

maintenance incidents (mtbm) of 3.3 days. This is the last machine in the production line.

a) Pareto Analysis. By far the most troublesome item is the brakes. The table shows other modes with > 5% of incidents.

TABLE 11.3.6a REWINDER PARETO ANALYSIS

Description	No.	%
Brakes	480	53.1
Hydraulics	172	19.0
Ejector	81	9.0
Trim Fan & Ducts	47	5.2
Others	123	13.7
Total	903	100%

b) Frequency Analysis. The Weibull β - value of 0.75 confirms that early failures are occurring. Without making a separate plot, it is obvious that this is due to the very frequent necessity to adjust the brakes.

The matter of the brakes should be investigated. It is possible that the operators are abusing the machinery or that the fitters are doing something wrongly when they adjust the brakes but it is perhaps more likely that the brakes are under-designed. Consideration should be given to the following.

1) enlarging the brakes

2) automating their adjustment

3) ensuring that the drum is never driven against the applied brakes.

4) a more complex system of brake application controls and cut-outs.

11.3.13. Footnote; The analyses above demonstrate again the power of combined failure-modes (Pareto) and frequency (dist--ribution) analysis. The more detailed work descriptions in these data increase the power of the Pareto analysis so that once a troublesome item had been identified, problem solutions could be tentatively suggested even by an engineer who was not very familiar with the plant.

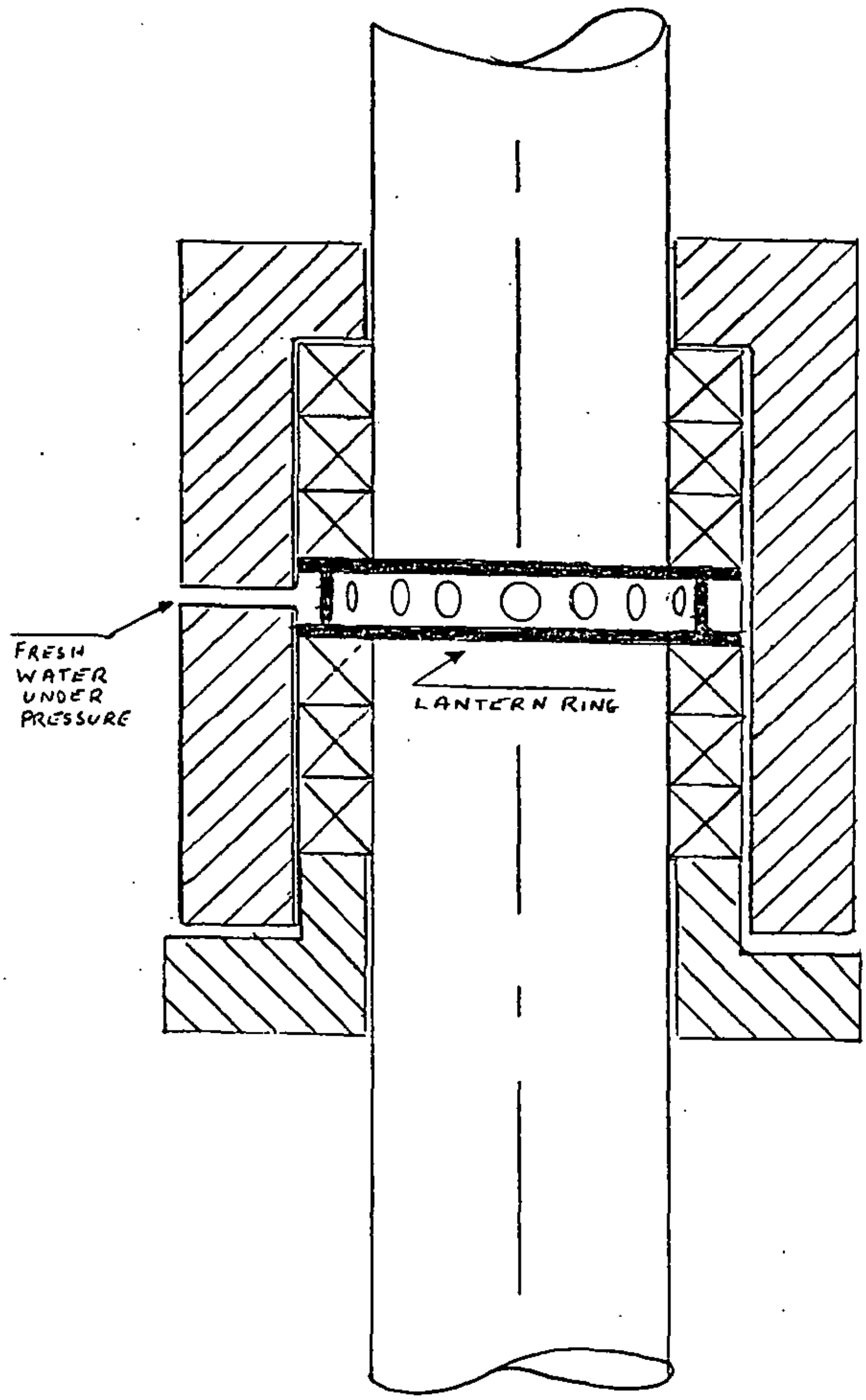


Figure 11.3.6 Digester Bottom Scraper - Suggested Gland Modification showing Lantern Ring and Fresh Water Feed

12. DISCUSSION AND LITERATURE

12.1 General

This section contains a discussion of the special features and results of the experiments described in Sections 10 and 11. In this way similarities and divergences of interpretation and in the results themselves can be juxtaposed. The literature discussed is that concerned with issues raised by sections 10 and 11 and does not purport to be comprehensive. References concerned with the theory of data analysis are given elsewhere in the General References Section 1. In Section 3 of the references, books and papers are listed which are concerned with the mechanics of data collection and the results of particular analysis. Not all of these references are discussed here, but all those that are cited appear in Section 3 of the References.

12.2 Scientific Method

The experiments described in Sections 10 and 11.1 had one important feature which sets them apart from most other time-limited data collection and analysis experiments. They were conducted as far as possible by the time-honoured scientific method. That is, the system was observed, and the results analysed, a theory was formed and then tested by perturbing the system prior to a further period of observation and analysis. On a continuous basis, this type of analysis is performed by organisations having full data collection systems, such as the Fighting Services, aero engine manufacturers, nuclear power authorities and major public undertakings.
Most 'one-off' experiments run out of time or money before the stage of reappraisal of conclusions in the light of changes is reached. Consequently, conclusions which should really have been tentative and subjected to objective tests are handed down based usually upon a

somewhat naive interpretation of maintenance based upon renewal theory, namely that if $\beta \leq 1$ pm is not worthwhile. (see Chapter III).

A full experiment takes a very long time if the results are to be statistically convincing, and time being money it is necessary first to convince a management that significant improvements can be expected from data collection and direct observation of maintenance methods, followed by modifications to the maintenance system. If it is assumed that the maintenance policy is already optimal or there is no prospect of getting it changed even if evidence were produced, then it is possible to produce recommendations based upon one data-set only. Many investigations for example Berg (1977) (2.8) and Basker et al (1977) (2.7) make this convenient assumption in order to be able to 'optimise' some other factor (in Berg's case the number of times a pump should be repaired before renewal and in Basker's the number of repair staff). The assumption should always be justified, but seldom is.

12.3 Quality of Data

12.3.1 Major Experiment. As a matter of deliberate policy the major experiment (Section 10) concentrated upon quantity rather than quality of data. It was known beforehand that the Trade Unions involved would not sanction the filling in of detailed pro-formas, and the plant management was not prepared to press the matter. Several other points need to be made about the data.

a) Repair times were not recorded

b) Details of failures were not recorded

c) Although true pm work was not included, deferred defects were recorded as failures, and there was no way to identify them separately.

d) For standby redundant pumps there was no way of telling which item was in use. It was assumed for calculation that the standby was started when the first one failed and was not changed back until the next failure, and similarly for one-out-of-three and two-out-of-three systems, this being the nearest and simplest policy to what was actually done. Some other items are used only intermittently; these it was impossible to cater for - they are recorded as running continuously.

e) The modes of failure were arbitrary and it is more than likely that some failures were not put in the correct categories e.g. the subtle distinction between 'holes and breaks' and 'leaks' was probably lost on at least one of the three data recorders involved in the exercise.

f) Some early failures were not counted because they occurred the same or the next day and were, as a matter of policy, analysed as continuation of the same failure. Because it was not recorded whether they were one failure or several the tendency to early failure which was observed in the data was therefore probably more pronounced than the calculation would suggest.

g) The initial list of equipments contained 17 pumps which did not in fact exist. These were discovered by making specific enquiries about all items which had apparently never failed. The data analysis were adjusted for these and no error resulted, but several analyses had to be reworked. The effect on the scale of the distribution estimate (η or θ) is much greater than the effect upon its shape (β) when a few (relative to a number of data) non-existent items are counted as suspensions (censoring) at the total period of data collection.

h) There were three recorders involved in the exercise. All these relied upon verbal and written reports of the operating and maintenance staff rather than direct observation. The first recorder was a sick man, the gap in recording in 1977 was caused by his sickness and eventual death. It is possible that the younger recorder who eventually took over the task was able to record a higher percentage of the actual failures. The third man was employed throughout on the Lost Time Sheets for this and other plants in the complex. In this field, researchers have to accept what data they can get. The possibility of uneven recording does not alter the general picture that failure rates fell after maintenance was tightened up, but if recording had been more accurate the effect might have been seen to be more dramatic and a few anomalies such as the recorded fall in Flash Vessel reliability between 1976 and 1978 which is apparently at variance with the rise in availability of the evaporators as a whole recorded in the Lost Time Sheets. The partial redundancy in the evaporators. meant that the policy changes were particularly effective so that far fewer recorded 'failures' actually caused disruption to production. Many of those recorded were minor defects found during or deferred until planned preventive maintenance periods. Taken over the whole plant this means that the improvement recorded between 1976 and 1978 was possibly more dramatic than the 'failure' figures can show.

i) Perhaps the most important point of all is that the choice of classes of equipment was somewhat arbitrary. 'Acid Pumps' for example, included equipment for two different acids, one of which was pumped at several different strengths. A hyper-exponential distribution (see Appendix B) is the weighted sum of two exponentials distributions. This implies that care is needed in the choice of classes of equipments not to impute an early failure pattern to what is really two sets of data arbitrarily combined. This danger was partially offset by two considerations. First, the persistence of $\beta < 1$ in the plots for separate modes of failure, while not conclusive, does suggest that the early and late modes are caused otherwise than by disparate populations. (In the autoclave experiment $\beta < 1$ was found for the tbf's of a single equipment). Second, many of the early failures were very early, there being only one, two or three days of operation between successive failures.

Anyakora Engel and Lees (2.5), Moss (2.36) and Lees (2.34) as well as the theoretical explanation of Carter (5.8) all provide evidence of the failure rate being generally sensitive to the conditions under which an item is used. Many various wear-out conditions considered as one data-set would be expected to produce a delayed-start rectangular distribution, which would have a rising hazard rate. To produce a falling overall $z(t)$ at least some must have individually falling $z(t)$'s if wear-out modes are present at all to any significant extent in the data. A strong tendency to falling $z(t)$ is therefore not masked by combination of many data-sets. An apparently falling z(t) can arise in the inadvertent combination of two data-sets of exponential form , because such is a hyper-exponential distribution.

12.3.2. Autoclave Experiment. In the autoclave experiment (Section 11.1) the quality of data was much higher. Although some of the descriptions were a bit vague the device of asking recorders to state which previous failure the present one resembled, if any, had the desired effect of producing a true Pareto analysis. The classification of failures under pre-determined headings as in the major experiment is not a true Pareto analysis. A true Pareto analysis requires fairly precise information on each failure or a definite statement that two failures were identical or at least very similar. The proper use of Pareto analyses is the justification of re-design for the next generation, the modification of present items and the amendment of the instructions for their operation and maintenance. For this it is not enough, in general, to know only the parts affected or the symptoms; specific treatment requires specific diagnosis. In other aspects the data is known to be complete and reliable because it could be checked against other records, and was recorded by operators trained to appreciate the need for precise records. All cycles were the same, there is no question about the real amount of use the autoclave sustained, c.f. the case of the redundant and partially redundant pumps in the major study. A greater volume of data would have been preferred for this experiment, particularly in the later stages, because the system behaved exactly as theory (see Appendix B) predicted and it is felt that in this case the conclusions can only be seriously doubted on grounds of insufficient data.

Because of this doubt, confidence limits at 5% and 95% have been drawn on the relevant Weibull plots.

12.4 Uses of Failure Data from Maintained Systems

12.4.1 To Improve Maintenance Performance. The studies described above show that Pareto analysis combined with distribution analysis and observation of the working practices of fitters etc. is a powerful method of identifying deficiencies of the maintenance system. Where there are deficiencies in standards of supervision and craftsmanship so that early failures occur, it becomes impossible to discern the pattern of failures due to wear-out modes. As the proportion of early failures is reduced the distribution becomes discernibly bi-modal. If the second mode is a wear-out mode then the Pareto analysis can be used to identify it and the methods of the next chapter applied to modify the preventive maintenance schedule so as to reduce its frequency.

12.4.2. To Predict Spare Parts Usage. Failure modes analysis is frequently by the parts consumed, and even when not specifically in this form it is often possible to find out what parts are used in the most common jobs. Downtime can then be reduced or costs optimised by adjusting the stores holdings. Again, the true position can be masked by the occurrence of early failures which lead to the early renewal of parts. However, the early failures could be due to poor quality in the spares so this possibility will always need to be checked out.

12.4.3. To Improve Equipment Design. An item with a persistent frequent failure mode and a relatively long life ahead of it will cost its owners a great deal of money in lost production and repairs. The sort of procedure followed in the four studies described leads to knowledge of such modes. Knowledge is usually sufficient to suggest a modification to the design to reduce or eliminate the failure mode. All too often such work is not followed to its logical conclusion; the problems are identified but nothing is done. Evidence of frequency and costs can be used to combat this management inertia and to concentrate the limited capacity for design detailing and parts manufacturing where it will produce the maximum saving. Manufacturers of machinery are usually glad to help if they know of a problem with their products. There is no excuse for collating and analysing data without reporting the results to the equipment manufacturer so that he can improve his designs in the next generation and coöperate with users in the modification of existing equipment.

(As in the consumer field, the reluctance of the British customer to complain of unsatisfactory products induces complacency, leads to design stagnation, loss of orders at home and abroad and so to Carey Street or Nationalisation. A good example of this sort of decline is the British Motorcycle industry. From the number of foreign machines seen in British process plants recently it would seem that it may be happening there also).

12.4.4. To Optimize Maintenance Schedules. This is a tall order and requires skill and care. The basic problem is that optimization models usually require an estimate of the base failure distribution (for examples see Chapter III), while maintenance modifies the base distribution so that the observed distribution is different. The operation of the system under fm for a period is the most obvious way of obtaining an estimate of a base distribution, but such a policy may well be dangerous or expensive and in any case the distribution estimate probably would not remain valid. A better method is to record on-condition actions as well as actual failures and to estimate the distribution as if the on-condition actions were failures at a time between ocpm and the next scheduled inspection. It is not correct as was shown in (2.4 6) to count ocpm actions as censored data because the method of dealing with such data (Appendix A) assumes that the action was random and that the item concerned was of average condition for its age. An item given ocpm is of course, by definition, near to failure. The most logical assumption that can be made of its condition is that it would have failed before the next inspection because otherwise it should have been left until at least then. It follows that the estimate of would-be failure time is the conditional mean given failure occurs in the inspection interval immediately following ocpm. For equal intervals this is half-way, i.e. $\tau/2$ should be added to all ocpm times to obtain estimated tbf's. In the major study (Section 10 above) it was not possible to distinguish clearly between on-condition maintenance and failures. In the early data this hardly mattered as there was virtually no ocpm. When pm was re-introduced it was not possible to change the data collection arrangements. Again, the effect is to lower the estimate of the mtbf's in the post-action (1978) period. In the autoclave study, there was no pm to confuse the issue except for the overhaul which accompanied modification. Inept actions as part of the overhauls probably contributed to the very low β-value just after modification.

12.5. Published Studies by Others

12.5.1. General Remarks. In the rest of this paragraph are reviewed some papers which are of interest because they provide either:

a) further evidence of the prevalence of $\beta < 1$ in maintained plant.

or

b) evidence of $\beta \geq 1$ in maintained plant

or

c) other interpretations of results having $\beta < 1$.

or

d) practical rather than theoretical evidence of the benefits or otherwise of pm.

12.5.2. Other Evidence of $\beta < 1$. Berg (1977) (2.8) investigating chemical process pumps and valves calculated the failure characteristics (β, θ) by failure number up to tenth failures from new. i.e. a data set contained only times from the ith to the i + 1st failure for i = 0 to 9. He found $\beta < 1$ in almost every data-set.

Basker et al (1977) (2.7) gave data for automatic lathes which when analysed by median rank plot to a β-value of 0.78.

Carter 1979 (2.15) gives examples of $\beta < 1$ in military equipment, although care is needed because in his plots the location parameter γ is not zero. On the whole it might be considered more appropriate to regard these as lognormal rather than Weibull distributions because a Weibull $\beta < 1$ with positive γ makes little physical sense generally. There is another possible explanation, though. The components (fan belts, radiators, andwater pumps of road vehicles) quite possibly survive with very few or no failures until the first occasion that a mechanic takes a spanner to them, after which about 5% fail quickly due to maintenance errors. It seems reasonable that the first service of fan belts should be at 3000 miles and of radiators and waterpumps at 12,000 miles and these approximate the γ values given very closely. These comments have been passed to Professor Carter as part of the discussion of the paper but at the time of writing he has not replied.

12.5.3. Analyses with $\beta \geq 1$. Bott and Haas (1978) (2.11) report on the failure rate and cumulative hazard functions of nuclear sodium circuit components. On the whole these show $\beta > 1$. Two reasons can be advanced for this which show that these results confirm rather than deny the proposition that $\beta < 1$ can be caused by poor maintenance or alternatively by two modes of failure. First, the dire consequences of carelessness in the nuclear field lead to an expectation of high standards of maintenance, workmanship and design of the components such that it is difficult to make fitting mistakes and rare to be given inadequate spares. Secondly, the data is for components, the lowest level of subdivision possible , so the probability of two modes of failure is reduced.

Kamath et al (1978) (2.30) found falling failure rates amongst transistors but were able to discern two failure modes each having lognormal form. This again is a component study and the bimodality could be due either to a proportion of substandard transistors which fail lognormally but early or to difference of treatment in service or pre-service storage.

Keller and Stipho (1979) (2.31) report an exercise in data collection and analysis at a Chlorine plant not unlike those described in Sections 10 and 11, but they fitted their data to normal, lognormal gamma and exponential distributions as well as the Weibull. In spite of hints of conditionslikely to lead to inadequate maintenance,all the β values in the data reported were greater than unity, some much greater, except one which was 0.9. The classification of failures in this paper is unusual, being based upon types of failure rather than types of equipment for distribution analysis. Thus, for example all accidents and maloperation failures are analysed as a group having

$\beta \approx 3$. No satisfactory reconciliation of these results with Sections 10 and 11 has been found.

Jardine and Kirkham (1973) (2.28) in their analysis of data from sugar refinery centrifuges found in the main that $\beta \approx 1$. In discussion Venton pointed out that they had not accounted for censored data. When this was done, it was interesting to note that most of the β values were reduced and one became definitely less than 1. (In reworking the 1975 data from Section 10 above to account for 17 pumps which were on the forms but did not in fact exist and which were analysed previously as censorings at the total data collection period, it was found that β was unaffected. Theoretical investigation showed that the omission of this type of censoring would not affect β, but only η whereas progressive censoring would affect both β and η). Although the authors play down the effect it might have, there is a statement that some routine maintenance is performed. Also, the product requires cleanliness and is in itself chemically benign. Failures are therefore mostly due to mechanical overload or wear, rather than chemical action and dirty conditions. One would expect filter-cloths to fail because of wear and the value of $\beta \approx 1$ must surely indicate that other causes are present as well as the wearout mode which should predominate. Those causes should have been sought. $\beta = 1$ is only the expected value for an item which has many modes of failure. If, as in the case of these sugar filter cloths many modes cannot arise from the presence of many components, then it must be because of many different causes of a single mode, in this case a hole in the cloth. The expected distribution would be extreme value or log extreme value (see Appendix A). So $\beta = 1$ begs for explanation and does not preclude the possibility of some early failures due to manufacturing flaws in the cloths or carelessness in fitting them to the centrifuges.

12.5.4. Other Interpretations of $\beta < 1$. Many authors with perhaps more knowledge of mathematics than experience of maintenance supervision aver that $\beta < 1$ means that no maintenance is optimal. These do not actually provide a physical interpretation of the data in the maintained case at all, but rely upon renewal theory, applied without regard to engineering factors. (see Appendix B). Berg (1977) (2.8), found for centrifugal and vacuum pumps and for valves that θ_i the mean time from the i - 1st to the ith failure increased in i whilst the frequency analysis of all ith failures gave $\beta_i < 1$ for most i $<$ 10,(i = 0 implies new item). On the assumption that the pm schedules were optimal Berg proceeded to optimize the failure number i for planned renewal. He interprets $\beta < 1$ as showing that the items are not repaired to good-as-new and shows that where the average number of parts renewed is higher then the subsequent θ_i is also higher. So Berg is taking the view that the maintenance standard is immutable and making the best of what is left. Quite possibly this view can be justified in terms of management obstinacy or the high cost of repairs relative to renewal, but this is not the way the argument is developed in the paper. In the present work there is a hitherto unstated assumption that the mean failure cost or downtime costrate includes allowance for renewals when it is judged that a renewal would be cheaper in the long run than a repair to good-as-new or to some standard condition. Berg's discovery that in items produced in quantity to a standard design under quality control there is no period of falling failure rate from new is not surprising to the writer and accords with the theory of early failures advanced in Appendix B. Whilst it is appreciated that not all the evidence could be condensed into one paper, Berg's figure 3 does not suggest immediately a rising failure rate with failure number if first failures are ignored. Rather, it confirms that given particular maintenance standards and policies the failure rate settles down to a constant average value until, in the region of i = 15 to 20 where Berg admits his data are thin, there is evidence of rising failure rate. Such a failure rate would be sensitive to pm intensity as to level and to pm depth as to duration. The reversed shape of the initial transient possibly indicates uneconomically low standards of maintenance. Rather than the optimality of throwing away pumps aged only 2 years and after only 8 failures, it possibly shows how quickly an expensive asset can be depreciated by lack of proper maintenance. The very different conclusions reached by Berg and the writer indicate how important it is not to lose sight of the experience of generations of engineers who have insisted upon high standards of maintenance amongst a plethora of mathematical analysis.

CHAPTER III MODELS FOR MAINTENANCE OPTIMIZATION

13 INTRODUCTION

Note: 'Renew' is used here to avoid ambiguity between the sense of 'put back' and that of 'substitute' which is inherent in 'replace' and its derivatives, and which can cause much confusion in maintenance instructions and reports.

13.1. Basic Procedure

A maintenance optimisation consists of first constructing a mathematical model of the relationships between the measurable problem parameters and then finding those values of such of the parameters as may be voluntarily varied which best satisfy a chosen criterion of optimality whilst not transgressing any absolute bounds which may be imposed by special requirements or by physical limitations unconnected with optimality. In general the criterion of optimality will include cost, costrate (cost per unit time) or another cost-related factor such as downtime. Voluntary actions such as inspections, renewals and overhauls involve cost (or etc.) as do involuntary events such as failures or deteriorations of performance. The frequency of the involuntary events will, however, be affected by the schedule or circumstances of voluntary actions. Herein lies the scope for optimization. It is usually, but not always, advantageous to model the system as a single cycle or a series of identical or mathematically-related cycles involving costs (or etc.) and times each multiplied by its expectation, or probability of occurrence.

In application it is most advisable to check that when the optimised policy is applied the expected reaction occurs in the system. If it does not, then something is wrong, either with the model structure or with the parameter values.

13.2 Types of Model

Several types of model have been proposed in the literature which is reviewed below for the behaviour of items subject to failure and subjected to preventive maintenance. The main consideration in choosing a model is the form of the failure time distribution. Different models are applicable for decreasing, constant and increasing hazard rate (failure rate). In some cases there may be a choice available. Some ancillary conditions which also affect the choice are whether or not the failures are self-announcing, and the time-horizon of the whole problem which may be finite or virtually infinite. Other factors are the effect of limited manpower, the criteria for optimality and the accuracy or confidence limits of the problem parameters. Seven species of model have been identified, the classification being more a matter of convenience than historical order or derivation. A model may simultaneously be of more than one type.

a) Renewal Models or Periodic Preventive Replacement(ppm) are perhaps the simplest of all. Items fail according to a known distribution function unless they are renewed or restored to good-as-new or a standard condition. Optimisation consists in finding the unique renewal interval which satisfies the criteria. Items fitted in large numbers may be renewed all at once regardless of intermediate failures (Block Renewal)

b) Inspection/on condition Maintenance Models (ocpm). The item is inspected at intervals determined from the results of previous inspections or from the failure time distribution under failure maintenance, either to observe whether it has failed or to judge whether it will fail before the next inspection. Unfavourable inspection results lead to positive maintenance action to restore the item to a pristine or known condition. Optimisation consists in finding the best inspection schedule against the criteria of optimality which are usually either cost or availability.

c) Continuous Condition Monitoring(ccm) either by a human watch-keeper or a specialised instrument incorporating automatic shut down or warning devices is conceptually a limiting form of ocpm where the inspection intervals have become infinitesimally short. The deterioration of a gradually failing item can be watched to obtain the maximum life, or alternatively action may be taken when a measured operating parameter reaches a prescribed value. Optimisation is not an applicable concept here, because ccm is a limiting version of ocpm, so ccm is compared with other policies. Continuous monitoring is restricted to what can be observed without stopping the item, whereas ocpm is not necessarily so limited.

d) Repair/Overhaul/Renewal Models envisage two or three levels of maintenance. Repairs may deal only with the immediate cause of failure or incorporate overhauls. Overhauls which may be triggered by a failure or by some means of scheduling or monitoring as discussed in (a)(b),(c) above do not restore the item to as-new condition but to a progressively worse condition at successive overhauls. Renewal similarly may be triggered by the condition found at the start of an overhaul or by some other method of scheduling. The Repair Limit Method is included in this category.

e) Models Involving Manning or Gang Size In the types of model discussed above it is generally assumed that labour is available as required to do the repairs and pm. Such a policy involves having staff idle for some of the time or else employed on pm work which they can leave at once when a failure occurs and return to later. Optimisation is usually to find the best gang size against the criteria and limiting conditions. Jardine (3.112)Chap.7 shows that under fm men must be idle for part of their time if the cost of repairs plus lost time is minimised.

One of the strongest arguments for pm as against fm is that it employs expensive and scarce craftsmen when they would otherwise be idle awaiting failures to repair. As the failure rate is usually and intentionally reduced by pm the ratio pm/(pm + fm) in terms of manhours or other criteria becomes a factor which invites optimisation. Priel (2.41) asserts from experience that the ratio should be about 0.6 at which point the total work load (repairs + pm) in terms of manhours is ~25% less than under fm. The average remaining fm load is covered $2\frac{2}{3}$ times by the original work-force at this value of the ratio.

f) Markov Models are such that the item exists in one only at a time of several states and passes between the states at constant rates. The probabilities of moving from one state (e.g. Failed, operating, under pm etc) to another in unit time are constant and may be represented by a matrix. Matrix algebra is used to find the proportions of total time spent in each state and the means and variances of individual sojourns. In this type of model distributions of time intervals are all of negative exponential form whether they be failures, delays, repairs or inspections. This means that strictly periodic inspections cannot be represented accurately. The truth in most cases probably lies somewhere between the strictly periodic interval and the total randomness of the Markov model; and the practical difference in cost rates etc. is not usually very much. It is possible also to synthesise other distributions by introducing dummy states. This is the method of stages as expounded by Singh and Billinton (4.69) and exemplified recently by Allan and Antonopoulos (4.1).

g) Adaptive Models contain within their structure the means of self-correction. For example, a model may be based upon, but at the same time be re-estimating the parameters of the failure time distribution. This is a most useful property for any model which works through a computerised data collection system. It makes it possible to start with schedules based on parameter estimates which are little better than guesses and allow the system to self-adjust to the right policy, and to follow changes in the values of distribution and cost parameters due to external conditions and plant ageing.

h) Conglomerate Models. Items and systems rarely suffer from one mode of failure only. Each mode of failure can be modelled and a schedule of inspections or renewals produced for each mode. However, if there are any savings to be made against the criterion of optimality by carrying out routines for two or more modes simultaneously then the sum of the mode schedules will be sub-optimal. A routine action that is performed early is eventually performed more often, and one that is left until later than the mode-optimal time carries a greater risk of failure. These factors must be set against the savings. Conglomerate models take factors of this kind into consideration. Included in this category are Opportunistic Models where advantage is taken of a stoppage for a failure or compulsory routine elsewhere in the item or system to perform pm with no or only marginal effect upon the factor to be optimised.

i) Limited Time Horizon Models. Where the plant has a fixed life, renewal, overhaul and repair decisions will be influenced by the impending end of the life, more and more as it gets closer. In such circumstances discounted cash flow and dynamic programming are often useful.

j) Simulation Models. The relationships which must be contained in the model may be too complicated or mathematically ill-conditioned for analytical solution. In this situation the only course open to the researcher is a random number simulation model. Typically the parts of a conglomerate model might be analytically soluble but optimisation over the whole system require the use of simulation. A loose term for simulation models is Monte Carlo Techniques because early sources of random numbers were associated with gambling. Unless great care is taken with the random number streams and the elimination of transients, simulation models can indeed be something of a gamble.

13.3 Criteria for Optimality

It is most important that the most appropriate criterion for optimality is chosen before starting to construct a model, and that any other limiting factors such as statutory maximum inspection or renewal intervals or a limited time horizon after which the plant is to be dismantled are known as they will affect the choice of modelling method. This will involve assessment of management aims and detailed knowledge of limitations on plant performance. Criteria which have been used for models are as follows:

Maximum Availability

Minimum Downtime

Minimum Costrate (cost/unit time on average)

Minimum (Present Value of) Total Costs

Maximum (Present Value of) Total Benefit or Profit.

Maximum Readiness for Occasional Use.

Maximum Production to a fixed time Horizon

Maximum Average Production Rate

Maximum Plant Operating Life

A most important decision is whether or not to discount costs where cost is the criterion for optimality. Jardine (3.112 p 68) shows that if the time horizon is limited minimising

$$c = E(C)/(E(T)$$

where E(C) and(E(T) are the expected cycle cost and time respectively is not optimal if costs are discounted to present value. However, he also asserts, reasonably, that if the time horizon is long relative to the average cycle time then discounting the costs to present value will not alter the optimal intervals for renewal or inspection. From another viewpoint, fixed time horizon problems in which the cycle time is long relative to the total time under consideration are usually investment rather than maintenance problems.

In practice, the maintenance manager rarely knows when the items in his charge will be renewed as a whole, he has learned to distrust forecasts of renewal and is not often fully consulted about the need for such renewal. He must therefore work on assumptions involving long time horizons relative to the component renewal and inspection schedules which are his responsibility. His actions are bound to affect the costs upon which capital plant renewal decisions must be based though.

Derman and Sacks(3.61) postulate but do not pursue optimisation by minimum cycle costrate i.e. Min $E(C/T)$ rather than Min $E(C)/E(T)$. The two criteria are not identical in outcome for ocpm unless the base failure distribution is exponential (constant failure rate). The alternative criterion is appropriate to cases where the cycle is long relative to the total life, (or indeed, is the total life), and so is suitable for the optimisation of equipment investment and maintenance costs discounted to present value.

A distinction should also be made between the minimum cost over all time and the minimum cost per unit running time or productive time. The second criterion is appropriate where time is not at a premium or unit costs are more important than delivery.

13.4 Effect of Form of Failure Distribution

The form of the underlying or base time distribution of failures in modes against which the inspections, overhauls or renewals are effective is vital information for most optimisation models. Where maintenance is already imposed some failures are already prevented and an analysis of tbf's will not accurately estimate the underlying or base distribution either as to scale or shape. It may be necessary to estimate the base distribution from a combination of actual failure times and synthesised failure times consisting of a renewal interval with a bit added, reflecting the assessed condition at renewal. Models using such loosely assessed distribution functions should for preference be adaptive so that all the relevant data available are used to improve the distribution estimate. The effect of maintenance and poor maintenance upon the observed distribution is discussed below and in Appendix B.

Unless the hazard rate function (failure rate) is increasing with time, renewal models cannot be applied. It is possible, as will be shown below, to find optimal inspection/ocpm schedules for items with non-increasing hazard rates, provided that some observable change takes place giving warning of impending failure. Inspections may alternatively be needed to reveal that failure or unacceptable departure from normal conditions has occurred, this being a good model for Quality Control drift problems but not so useful in the Maintenance field where failure is usually considered catastrophic and self-announcing.

13.5 Sensitivity Analysis

It may be that some of the parameters of a problem are not accurately known. This may not be a problem if the effect of the parameter within the possible range of variation is small. In other cases it may be essential to know a value within very close limits. Having found an optimum through a model calculation it is advisable to check the sensitivity of the results to parameter variation. This may lead the researcher to simplify his model by eliminating parameters which have little effect upon results. Models which are highly sensitive to an uncertain parameter can sometimes be avoided by re-casting the model. A model is only as good as the data processed through it.

14. LITERATURE OF MAINTENANCE OPTIMIZATION MODELS

14.1 General Remarks

14.1.1. Quantity, Theory and Practice. There is a very large number of books and theoretical papers on maintenance optimization modelling. This is in contrast to the much smaller number of reports of applications of the theories . Several reasons may be advanced for this and it is likely that all of them contribute to the contrast in numbers .

a) Maintenance managers who would apply the techniques if they were brought to their attention do not read the journals and books in which the models are published.

b) Maintenance managers in industry are unable to understand some of the mathematics upon which the models are based.

c) The models' conditions and assumptions are so idealised that they seriously restrict applicability.

d) Applications are more common in practice than the literature would suggest. Much of the research of the theory having been done under military auspices, it is likely that many applications have not been publicly reported. Also, potential authors may be restrained by a secretive attitude on the part of their employers or a feeling that a report of a successful application would not be of much interest.

e) The less complex models require fairly accurate R & M data on the items to be maintained and the more complex adaptive models require that such data be collected continuously. Industrial management information systems are not readily changed by the financial managers, who are usually in charge, to meet the needs of the engineering staff (see Chapter II)

14.1.2. Dispersion. Another problem in reviewing the literature of maintenance models is that it is diffused widely through the Management, Applied Maths, Statistical, Logistic, Reliability, OR, Quality and Maintenance Engineering journals. Occasionally models of general applicability are reported in journals specialising in their authors' own application field. The Maintenance Engineering press on both sides of the Atlantic tends to be pragmatic and simplistic, to cater for the average rather than the advanced Plant Engineer. The present dispersion makes for much unconscious repetition of virtually the same model. The case for a specialised journal of terotechnology theory and application is quite strong.

14.1.3. Choice of Papers for Review. With the above considerations in mind the material reviewed in the next few paragraphs has been chosen on one or more of the following grounds.

a) It is relevant to the two models developed by the writer, described in Sections 15 and 16.

b) Major milestones in the development of a class of model, and major review papers.

c) It is not discussed in any of the review papers referenced and is considered worthy of inclusion.

d) It is the clearest of a number of papers on similar models.

e) The models have actually been applied and their success measured in service.

14.2 Books

14.2.1 Early Queueing Models. Morse(1958) (3.168) and Cox and Smith (1961) (3.52) following Benson and Cox (1951) (3.33) dealt with queues in a manner applicable to manpower- limited maintenance modelling. A maintenance workshop is a multi-server queue, but may have the additional constraint of machine tool availability. Cox's (1962) book (3.51) on Renewal Theory is the classic work to which others have later returned for inspiration. It lays the foundation upon which all subsequent theories for periodic renewal (ppm) have been built, but it does not deal with the alternative inspect/ocpm situation.

14.2.2. Barlow Proschan & Hunter(1965) (3.18) devote nearly 30% of their book, to the operating characteristics and optimization of maintenance. Much of this had appeared in 1962 in a book edited by Arrow, Karlin and Scarf (3.10) and was later revised in presentation for their other book on the Statistical Theory of Reliability (3.19). Block and age renewal policies are compared and the generalised condition for age replacement optimality derived i.e.

$$c^* = \min_t \left[C(t)/T(t) \right] = \min_t \left[\frac{C_F F(t) + C_M R(t)}{\int_o^t uf(u)du + t\ R(t)} \right] \quad (14.1)$$

where C(t) is the cycle cost for a ppm policy of renewal at failure and at time t since last failure or renewal, and T(t) is the resulting average cycle time. When $t = t^*$, $c = c^*$. They also deal with block renewal optimization and with ppm with minimal repair at intermediate failures. This is the 'bad-as-old' alternative to the usual 'good-as--new' assumption about the condition of the item after repair. If the cost of a renewal is C_R and the renewal period t it follows that

$$c^*_{bao} = \min_t \left[C_F \int_o^t z\ (u)\ du + C_R/t \right] \quad (14.2)$$

The integral being the expected number of failures in (0,t).

For a Weibull distribution

$$t^*_{bao} = \eta \left[C_R/C_F \ (\beta-1) \right]^{1/\beta} + \gamma \ , \ \beta > 1 \qquad (14.3)$$

Barlow and Proschan then derive the cost rate equation for a ppm model in which the time horizon for optimization is finite. They point out that if the time horizon is infinite the renewal policy remains unaltered, but that if it is not then the optimum policy must be affected more and more by the approaching end as time goes on. The policy for the remaining time depends upon the actual time of the latest renewal so that it is not possible to calculate more than one planned renewal time ahead. They develop a method for calculating the next planned renewal time given the remaining time to the end of the project and the fact that a renewal has just occurred.

Turning to inspection policies Barlow & Proschan give a solution to the problem of scheduling inspections, the principle of which can be generally applied. This is that all the partial derivatives of the expected cycle cost-rate with respect to each and every inspection time should be zero.

i.e. $$\sum_{i=1}^{n} \partial c/\partial t_i = 0 \qquad (14.4)$$

where n, the number of inspections to preemptive renewal, is to be found and may be infinite.

The presentation of this work is clearer in their second book (3.19). Calculations based on this principle can be extremely tedious. If $z(t)$ is decreasing simplification is possible by finding α such that

$$C(X) - \alpha T(X) = (0) \qquad (14.5)$$

where X is the vector of inspection times $(x_1, x_2, x_3, \ldots)$ and $C(\cdot)$, $T(\cdot)$ are the expected cycle cost and time respectively. (0) is a zero vector in (α, x). For $z(t)$ constant, use can be made of the fact that the optimal inspection schedule has constant intervals.

However, in neither of their books do Barlow and Proschan suggest using inspections to anticipate failure rather than to reveal it.

14.2.3. Jorgenson, McCall & Radner (1967) (3.114) review the field up to that time and present models of various kinds, many of them based upon their own various individual and combined papers of the 1950's and 1960's (3.114-6) Preventive (ppm), preparedness, inspection, opportunistic and adaptive models are included but perhaps the most valuable parts of the book are the examples and the extensive bibliography. Although by now itself a little dated this is an important source. They say in Chapter 2, p 50 of the book " We regard a complete solution of a problem as a description of the physical situation a definition and characterization of an optimal maintenance policy . . . and a derivation of the operating characteristics of this policy. Our discussion centres on problems that have a complete solution". This deliberate (and defensible) attitude probably explains why they did not attempt a model in which inspections are carried out to see whether a failure is imminent rather than whether it has already occurred.

14.2.4. Jardine (1973) (3.112) is a model of simplicity and clarity. It is ideal for the beginner in this field and should be on the shelf of every maintenance manager. He gives models for ppm against several criteria of optimality, inspection models in which the failure rate is a function of the inspection frequency and models for deciding whether to repair overhaul or replace. The question of whether or not to discount costs to present value is clearly put and answered. He shows that it is only necessary to use DCF methods if the time horizon is finite. He also considers queueing models for number of staff required and a simulation for the situation where both men and machines are in limited supply. Jardine clearly states on p 22 of the book that constant z(t) is likely in complex equipment when all failure modes are considered together and that this does not preclude the possibility of an optimal maintenance policy other than fm, but he does not give an inspection/ocpm model in which inspection anticipates rather than discovers failure.

14.3. Review Papers

14.3.1. McCall (1965) (3.152) (88 references). This is a comprehensive review of the early work on maintenance models and includes both ppm and inspection/ocpm models.

14.3.2. Pierskalla and Voelker (1976) (3.190)(259 references) When this painstaking and comprehensive review, mainly of papers between 1965 and 1975, was published, the writer was in the middle of his own survey. Papers examined in detail in this paper have not been specifically reviewed below, unless they are considered mile-stones or of unusual clarity. The net was cast rather wider by Pierskalla and Voelker than would be appropriate here. They include combined maintenance and logistic models: burn-in programmes and a number of other marginal matters.

Their conclusions that maintenance models should find increasing application in high technology and military circles and that the most appropriate area for further research is into conglomerate models producing overall optima by modifying a set of sub-optimisations are agreed. However, there is surely also a need for more simple models that can be applied in the field by the less able Plant Engineer and for solutions to inspect/ocpm problems where inspection anticipates failure rather than reports that it has already occurred.

14.4 Some Important and Recent Papers.

14.4.1 General. The papers are presented in chronological order rather than by types of model involved. In this way it is hoped to achieve an historical as well as a technical perspective.

14.4.2. Benson and Cox (1951)(3.33)This is an early statement although they reference even earlier work of the so-called 'machine interference problem' in which m repairmen service n fallible machines, $m < n$. Expressions are developed for the machine availability A, and operator utilisation, U (i.e. the proportion of the repairmen's total time spent actually repairing a machine. For m = 1 it is shown that

$$A = B_1(\lambda/\mu, \ n - 1)/B_1(\lambda/\mu, n)$$

where

$$B_1(x,n) = \sum_{i=0}^{n} \left[xn!/(n-i)! \right] \qquad (14.6)$$

and

$$U = nA\lambda/\mu \quad \text{for } n\lambda < \mu \qquad (14.7)$$

For general $m < n$

$$A = B_m(\lambda/\mu, n-1)/B_m(\lambda/\mu, n)$$

where

$$B_m(x,n) = \sum_{i=0}^{m-1} \left[x^i n!/i!(n-i)! + x^m n! B_1(x/m, n-m)/m!(n-m)! \right]$$

and $U_m = nA\lambda/\mu m$ (14.8)

These results are basic to any model involving the best use of limited manpower. For example if it is known how the average failure rate λ varies with pm intensity it should be possible to calculate the ideal maintenance periodicity for various gang sizes to maximise availability, or minimise combined maintenance and downtime costs.

14.4.3. Koenigsberg (1958) (3.132) developed a general theory of closed-loop queues which he applied to the operation and maintenance

of mining machinery. The paper contains several ideas useful in queueing and manpower problems.

14.4.4. Derman (1961) (3.62) introduces the minimax principle for decisions under uncertainty into an inspection model. The distribution of tbf's is assumed to be unknown and the objective is to find the inspection schedule with the smallest maximum loss. Under Dermans model the inspections discover that failure has occurred and loss is proportional to the time from failure to discovery. However this method appears to be adaptable to the more realistic model in which inspections anticipate failure with an efficiency which is functionally dependent upon their frequency. Minimax inspection models such as this one can be used to obtain an initial schedule, which can be adaptively improved as data is gathered. This paper and Derman's later work (3.58 to .3.64) are important contributions to the subject, see Pierskalla & Voelker (3.190).

14.4.5. Drinkwater and Hastings (1967) (3.72) is the first paper on the well-known repair limit method which was developed and exemplified by Hastings (1969) (3.96) and (1970) (3.93), Hastings and Thomas (1971) (2.23), Nakagawa and Osaki(1974)(3.177), Jardine et al (1976) (2.29). See figure 14.1.

The novelty of the method is demonstrated by the absence of references to other work in the original paper. Under the repair limit method, as each breakdown occurs, an estimate of the repair cost is made and compared with a limit which varies with the age of the item. If the estimate exceeds the limit the item is renewed, if not then it is repaired. Under the model the graph of the sum of acquisition costs and average total base maintenance costs including repairs is drawn (see Figure 14. 1) against age. The renewal time t* after

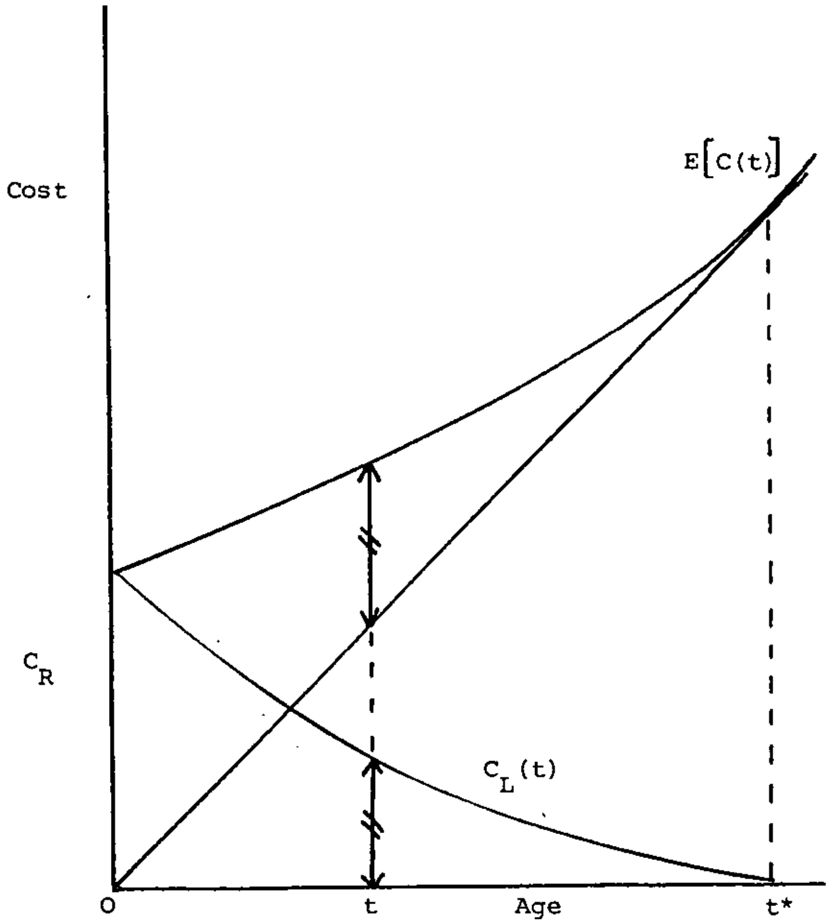


Figure 14.1 Repair Limit Method

<u>Procedure</u> when item fails aged t and is estimated to cost C_F to repair

1. If $t > t^*$ then renew at cost C_R
2. If $t \leq t^*$ and $C_F < C_L(t)$ then repair
3. If $t \leq t^*$ and $C_F > C_L(t)$ then renew

Explanation: $E[C(t)]$ is the expected total cost to time t including purchase, maintenance and repairs.
C_R is the purchase or renewal cost
$C_L(t)$ is called the 'Repair Limit' and is the function formed by the difference between the $E[C(t)]$ curve and its tangent at the origin.

which the item is renewed at the next failure is given by the point at which the curve subtends a tangent at the time of purchase and zero on the cost scale, t*. If a failure occurs then it may be repaired provided that the cost does not exceed the vertical interval from the tangent to the graph.

The policy is adaptive; the authors show how to re-estimate the parameters from operational data. The method has been used by the British Army for vehicles and other equipment for over 10 years with considerable success, see the first major reappraisal by Mahon & Bailey (1975) (2.35). The policy is usually operated in respect of large repairs and scheduled major overhauls arising against a background which includes regular ppm and inspection /ocpm schedules, which the policy uses to calculate the maintenance costs but does not specifically challenge. The philosophy is that if the repair limit is exceeded the cheaper option is to get a new item now rather than later because repair of the present one plus average future costs would exceed the price of a new item plus its future costs to any time horizon at or beyond t*. The later papers are in terms of block times (usually years) and the repair limits are optimised using discrete dynamic programming. Nakagawa and Osaki (3.177) examine a modified model in which the repair is started and the decision to scrap deferred until work to extent of the repair limit has been done. This restriction adds to cost without simplifying calculations much.

Repair Limit policies base decisions on the most up-to-date information on item condition. Against this there is the tacit assumption that post-repair value or life and repair cost cannot be varied. In fact the quality of repairs can be anywhere in the range from bao to gan.

14.4.6. Derman & Klein (1966) (3.63) provide a solution of a model in which costs depend on the order in which tasks are undertaken. In the paper transfer costs are represented as distances in a 'travelling-salesman-type' problem but this could be adapted to the problem where savings are available by doing maintenance jobs together rather than separately. The model could lead to an overall optimization based upon sub-optimizations, of ppm and inspect/ocpm elements. The realisation is in terms of average rates of failure repair and inspection so that Markov matrix methods can be applied.

14.4.7. Vergin (1968) (3.249) examines the same type of conglomerate problem as Derman & Klein (ibid) but here the approach is through dynamic programming. Machine interference is also considered and suggestions made for a multi-component, multi-machine model under conditions of limited manpower.

14.4.8. Glasser(1968)(3.90, 3.91) is not the earliest but the most clear and useful exposition of the theory of age, (ppm) and block renewal. The basic equation for ppm is

$$c^* = \min_t \left[\left\{ C_M F(t) + C_F R(t) \right\} \Big/ \left\{ \int_0^t u\, f(u)du + tR(t) \right\} \right] \tag{14.9}$$

t*, the optimal renewal interval for an infinite time horizon is the value of t which realises c*. In his two papers Glasser gives charts for finding t* in terms of the mean and variance of f(t) and the cost ratio C_F/C_M. The charts differ slightly for truncated Normal, Weibull and Gamma distributions. Any of the charts can be used to get a rough solution to the problem even if f(t) does not fit any of the above distribution forms. There is much to be said for getting only a rough point solution, because it is always good practice to examine the sensitivity of the solution to small changes in the parameters which may not be known with certainty.

14.4.9 Fox (1966b) (3.87) considers age renewal (ppm) under discounting. The longer a renewal is delayed the less its cost discounted to present value but the greater the probability that a failure intervenes. However, since both C_F and C_M are discounted by the same percentage for a given periodicity there is no change in t* unless the time horizon is limited. Fox also shows that the minimax ppm solution for f(t) unknown is fm, which suggests that data should be collected under fm to determine f(t) rather than under some intermediate schedule. However, this is also conditional upon an infinite time horizon, against which the fm period becomes insignificant. In his second paper Fox (1968) (3.88) develops an adaptive policy using Bayesian techniques and DCF for a Weibull distribution of known shape (β) but unknown scale (η) . He assumes that $1/\eta^{\beta}$ has Gamma likelihood in order to have a conjugate prior. The optimisation is by D.P.

14.4.10. Eckles (1968) (3.74) also optimizes by D.P. from a discrete time model. He assumes that repairs and inspections were operations with varying degrees of certainty of successful outcome, whilst renewal has a certain effect. Taking an empirical bath-tub curve of z(t) under fm he shows that inspect/ocpm with preemptive renewal according to schedule optimized with respect to total cost rate generally saves over straight ppm. This was the only paper found which regards inspection as an action to be taken to prevent failure.

14.4.11. Kent (1970) (3.129). In this paper the effect of discounting future costs to present value is dealt with by modifying the parameters of the corresponding undiscounted problem.

<u>14.4.12. Kamien and Schwartz (1971)</u> (3.119) develop a model due originally to Thompson (1968) (3.245), which leads to a combined optimization of <u>pm</u> to reduce wear and age to exchange for a new item. Machine value falls as output rate declines and resale price reduces with time, but the former can be offset by <u>pm</u>. Thompson had shown that in these circumstances a period of maximum effective <u>pm</u> should be followed by one of none at all.

The later paper assumes no decline in output rate but increasing failure rate with age. Using optimal control theory they show that maintenance intensity should be reduced with time as the failure rate increases and that the optimal policy may be very sensitive to the sale date. A reasonable conclusion from this work which is not actually drawn by the authors would be that it is likely to be worthwhile applying a full schedule of <u>pm</u> to a manufacturer's recommendations at least until output declines or failure rate increases despite sustained maintenance effort. Also, the paper provides further theoretical evidence in support of the 'bath-tub curve' model for observed z(t).

Di Palo (1971) (3.188), considers the availability of a standby system as it varies with the number of repairs which can be simultaneously undertaken. The paper illustrates the futility of elaborate redundancy if the maintenance staff level is inadequate.

14.4.13. Munford & Shahani(1972,1973) (3.169 ,3.170) and Shahani & Newbold(1972) (3.233) make a simplification to the inspection model of Barlow & Proschan (1973) (ibid) which is slightly sub-optimal but much quicker to calculate. Instead of finding the vector t_i, $i = 1, 2 \ldots n$ of inspection times such that $\partial c/\partial t_i = 0$ for all i they assume that constant base risk between inspections is nearly optimal and then show by example that it is i.e.

$$\left\{ R(t_i) 1 R(t_{i+1}) \right\} / R(t_i) = p - \text{(constant over i.)} \quad (14.10)$$

The model envisages failure discovered by inspection, fixed inspection costs, C_I and failure costs proportional to undiscovered time. In the next section of this thesis a model is developed using this constant risk idea, but for inspections which lead to ocpm at cost C_M. Failure at cost C_F occurs with probability (1-r) where r represents the proportion of imminent failures which are missed by the inspections. The Shahani model is useful as such for quality control, for settling the intervals between serial samples taken to see whether the machinery has 'failed' and is producing unacceptable goods. Two of the papers include Newbold's nomograms which allow p*, the best value of p, and the corresponding minimum cost rate c* to be found by two alignment operations.

<u>14.4.14. Kao E. (1973)</u> (3.124) gives a model which envisages deterioration through a number of states, sojourns in each state being random variables. The model is discrete in time and semi-Markov. The basic model uses control theory to optimize the average cost rate, first against renewal when a certain state (to be determined) is reached, and then more generally in terms of renewal after an optimized sojourn in that state unless failure intervenes.

<u>14.4.15. Sethi (1973),</u> (3.228) Machine renewal periodicity and pm schedule are optimized simultaneously in P.V. terms by Pontryagin's maximum principle.

<u>14.4.16. Kander & Raviv (1974)</u> (3.123) This is another D.P. model for <u>ppm</u> of the minimax type but in this case the base distribution function F(t) is known for one value only of time t. This information is shown to affect the optimum schedule only up to the time at which F(t) is known.

<u>14.4.17. Keller(1974)</u> (3.128)in an inspection model treats the inspection frequency as a continuous <u>pdf</u> and optimizes by the calculus of variations. His approach is most useful where inspection is very frequent relative to the renewal frequency. Although inspection is presented as an operation to discover that failure has occurred, and cost is proportional to the time it remains undiscovered, this model could be adapted to deal with the case where degree of deterioration is detectable by inspection and subsequent maintenance costs rise steadily the longer the deterioration is allowed to continue. A <u>minimax</u> solution is also given and it is shown that inspection intervals should be equal when the base distribution of <u>tbf's</u> is exponential.

14.4.18 Jardine et al (1975) (2.29) extend the Hastings repair limit method to consider opportunistic pm concurrent with major repairs. Tax allowances and the time value of money are taken into account in this D.P. model with a fixed time horizon. Tax allowances cause earlier renewal; it would be more logical if early disposal was adversely taxed because the machines concerned are either imports the country can ill afford or are diverting production from possible exports to re-equipment of home factories, and transport fleets. Tax allowances also encourage sloppy maintenance of equipment with a shorter planned life-cycle and the production of shoddy machinery.

14.4.19 Ran and Roselund (1976) (3.195) envisage maintenance costs as a continuous and increasing function of time $c_M(t)$ and renewal costs at failure and before failure $C_F > C_R$ as fixed in a ppm model. They show that the overall discounted cost-rate of a policy of renewal at T or prior failure is

$$c(T) = \left\{ \int_0^T \left[\exp(-rt)\ f(t)\ (C_F - C_R) + c_M(t) \right] dt + C_R \right\} \Big/ \left[\int_0^T \exp(-rt)\ R(t) dt \right] \qquad (14.11)$$

where r is the discount rate. They show that T* increases with r under z(t) increasing and linear $c_M(t)$. They also consider $c_M(t)$ as an oscillating function, as well it might be under a schedule of maintenance requiring more and less expensive routines at varying intervals making up a maintenance cycle culminating, say in a major overhaul.

14.4.20 Bellingham and Lees (1976) (3.29) examine continuous condition monitoring (ccm) using Bayesian logic to develop generalised equations of the conditional distribution of tbf's given the monitor signal. The model of Section 16 of this thesis is a particular realisation where the changes of state occur at constant rates making it possible to use matrix algebra to obtain the state probabilities and hence the overall costs.

14.4. 21. Bosselaar (1976) (3.36) testifies to the efficacy of predictive maintenance (inspect/ocpm) even when no real attempt has been made to optimise the intervals between inspections. Savings over ppm and fm are recorded.

14.4. 22. Alam and Sarma (1977) (3.4) This model considers machine interference using control theory methods.

14.4. 23. Berg and Epstein (1978) (3.34) A tutorial paper comparing the merits of age, block and failure-only maintenance or renewals. The paper defines regions, in terms of the basic parameters of a ppm problem, where each is optimal over the others.

Schneeweiss(1977) (3.221) derives the pdf of times that faults remain hidden before they are revealed by a scheduled inspection in terms of the pdf of tbf's f(t) and inspection frequency pdf g(t).

14.4. 24. Nakagawa (1977) (3.178) reviews his own earlier work which is not referenced because of this review. Single, standby repairable and standby unrepairable systems are considered under various criteria for optimality. The treatment is very general and it is doubtful whether some of the integral functions could be realised in every case.

14.4.25 Basker, Manan & Husband(1977))(3.26) is a clear example of the empirical simulation model. It is empirical in the sense that the tbf and ttr distributions are determined by data collection and that thereafter the optimum maintenance policy and gang-size is found by organised trial and error through simulation according to these pdf's for separate sections and for an entire factory. Simulation in such circumstances serves as a quicker and cheaper substitute for learning by experience. These authors chose to use empirical distributions but it is arguable that the distributions should be fitted to the most suitable mathematical form especially when the sample sizes were as low as 26.

14.4.26. Basker and Husband (1978) (3.27) describe a case study involving a slight modification to a model due to Jardine in which n overhauls are scheduled in a time T and the cost of operation rises between overhauls according to

$$c_o(t) = A - B \exp(-qt) \qquad (14.12)$$

where t is the time since last overhaul, and there are in addition charges for downtime which vary in a similar manner and a fixed price for each overhaul. The optimum overhaul interval is found by differentiation of the combined cost rate equation. In the particular case examined it was found that the intervals should be halved for a saving of some 5% which is a remarkable lack of sensitivity.

They also apply another Jardine model $\lambda/(\mu_I)$ in which the major breakdown frequency is a function of the inspection frequency μ_I.. Optimization is in terms of maximum value of output over an extended period. In this case the savings are not recorded.

14.4.27 Chan & Downs(1978) (3.46) develop a first-order Markov model with three states viz. up, under maintenance and failed. They acknowledge that not all maintenance work is beneficial by assigning a non-zero transition rate from the maintenance to the failed state. Optimization is alternatively for availability or costrate. This is a useful simplification of the common phenomenon of early failure due to unsound maintenance discussed in Appendix B and exemplified in Chapter II.

14.4.28. Mine & Nakagawa (1978)(3.161) This is an age renewal ppm model for a mixed tbf distribution consisting of the weighted sum of several distributions

i.e. $$F(t) = \sum_{i=1}^{n} a_i F_i(t) , \quad \sum_{i=1}^{n} a_i = 1 \qquad (14.13)$$

The policy is to renew at t at cost C_R, or repair on failure before t in mode i at cost C_{Fi}. It is shown that the cost rate overall is

$$c(t) = \left\{ C_R \sum_{i=1}^{n} a_i R_i(t) + \sum_{i=1}^{n} C_{Fi} a_i F_i(t) \right\} \Big/ \left\{ \sum_{i=1}^{n} a_i \int_0^t R_i(u)du \right\} \qquad (14.14)$$

It is shown that fm may be optimal over this policy and the conditions for this to be so are delineated.

15. AN INSPECTION MODEL BASED ON CONSTANT INTERVAL RISK

15.1 Introduction

This model is the subject of a paper published by the Institute of Electrical and Electronic Engineers in the U.S.A (3.230)

In the course of plant visits connected with the data collection and analysis experiments described in Chapter II one reason given for the current maintenance policy of on-failure corrective maintenance (fm) or 'laissez-fail' by chemical plant maintenance managers was that they had collected some sample data and found it to be exponentially or hyper-exponentially (Weibull $\beta \leq 1$) distributed. Consulting the nearest handbook of Operational Research they soon concluded that pm would be counter-productive. As discussed in the previous paragraphs, this is true of ppm but not necessarily of ocpm. It is also fundamentally unsound to conclude that a good fit to a Weibull distribution with shape parameter $\beta \approx 1$ must mean that the failures are truly random and unpredictable in nature. As explained in Appendix B , the expected tbf distribution for a complex and/or maintained item is exponential because of the randomisation of component ages by previous failures and pm(3.143)Furthermore, $\beta < 1$ in such a case possibly indicates poor maintenance and $\beta > 1$ a dominant mode of failure not yet catered for in the pm schedule, or that the item as a whole is nearing the end of its life. The level and length of the flat portion of the well-known 'bathtub curve' of failure rate over item life can be adjusted, as to level by altering the frequency of pm, and as to length by adjusting the depth of maintenance or number of components in the pm schedule.

Models for inspection and maintenance should be able to accomodate hyper-exponential ($\beta < 1$), exponential ($\beta = 1$) and wearout ($\beta > 1$) distributions of tbf's, and as an item moves through the 'bathtub curve' from 'teething troubles' through 'useful life' to 'senility' it should be possible to adjust the ocpm schedule to suit.

A major difficulty in the construction of any model in which inspection anticipates rather than merely reports failure is that failures will occur between inspections with probabilities which must be known to complete the model. The longer the inspection intervals the greater the probability that there will be a failure before the next inspection despite all appearing to be well at the present one. The uncertainty which must necessarily surround the mathematical relationship between this probability and the schedule of inspecting can be resolved by assuming a form of relationship and finding the parameters by trial and error. That is, the model must be adaptive in at least that respect.

In the model described below the risk of failure between inspection times under fm is constant i.e. the base or underlying risk, or the would-be risk if the inspections were totally useless and no on-condition renewals were made, does not vary from one inspection interval to the next. This approximation has been used previously by Munford and Shahani (3.169) in a model where failure was discovered by inspection and the goods produced between failure and discovery had to be rejected. The novel feature of this model is that it is assumed that there is a unique relationship between the base interval risk and the observed proportion of cycles ending in failure.

The model provides a method of adaptively optimising inspection schedules where failures are self-announcing but inspection can detect signs of pending failure so saving the cost-difference between 'failure' and 'ocpm plus inspections'.

15.2 Description

Renewal theory, see for example Glasser (3.91), assumes all items are from one population. Inspection can divide a population into subpopulations according to assessed condition. Maintenance action can shift items from one subpopulation ('failed" or 'failure probably pending') to another 'good'. The weakness of renewal optimisation is that it does not allow items to be other than 'good' or 'failed'.

Indication of pending failure is often available at small cost C_I per inspection before important loss of performance. Whether an inspect/ ocpm policy saves over fm or ppm depends upon the base failure distribution f(t) and the mean total failure and ocpm costs C_F, C_M. To simplify the general treatment of Barlow & Proschan (3.18) it is assumed that equal base interval risks p are nearly optimal.

The problem is to find the least costly inspection schedule and compare its long-term cost-rate with f m or the best ppm policy. Any pm policy implies more frequent maintenance actions than f m. Therefore total long-term cost-rate is minimised, not average cycle cost.

Models describing the effect of the value of p on inspection effectiveness are discussed.

15.3 Assumptions

1) Inspection occurs at cycle start ($t_o = 0$) and then such that the base interval risk is a constant p for each interval t_{i-1} to t_i. This results in a mear optimal policy at an ideal value p* of p.

2) Inspection assessments can be wrong in two ways

   a) Assessed as failing in next interval when it would not

   b) Assessed as surviving next interval and it then fails.

   Only type b errors occur.

3) Adverse inspection results lead at once to ocpm, favourable inspection results to operation for another interval, except that

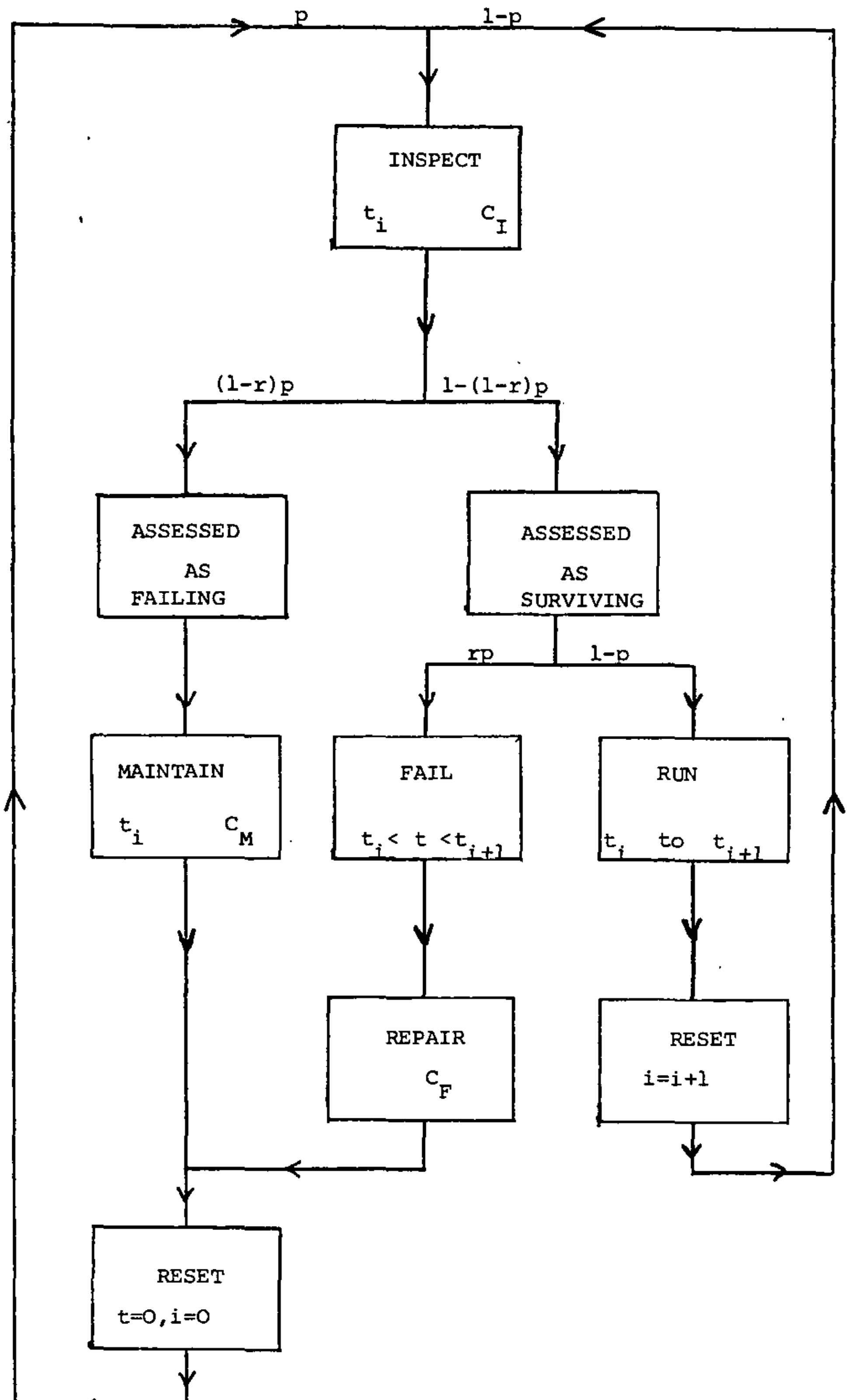


Figure 15.1 Flow Diagram of Constant Risk Model

after n favourable inspections preemptive pm is performed after one more interval (if the item survives) to prevent inspections at uneconomically short intervals. (The limit n is to be found.)

4) Inspection error is such that the actual interval risk of failure is rp where $r < 1$; r depends only upon p, i.e. $r \equiv g(p)$ and in particular is independent of the number of inspections since the latest maintenance action.

5) Items revert to standard condition after any maintenance action, i.e. f(t) is independent of the number of cycles the item has undergone.

15.4 General Model

15.4.1 Risk and Inspect Times

$$p = \left\{R(t_{i-1}) - R(t_i)\right\} / R(t_{i-1}); \; i=1,2\dots n \qquad (15.1)$$

But $t_o = 0$ so

$$R(t_i) = (1-p)^i \qquad (15.2)$$

Expected Number of Inspections per cycle

The unconditional total probability of "ocpm at t_{i-1} or failure in the following interval" is $p(1-p)^{i-1}$. This is the same as the probability that the number of inspections (including the initial inspection at $t_o = 0$) is i.

i.e. $Pr(I=i) = p(1-p)^{i-1}$

By moments, the expected number of inspections is

$$E(I) = \sum_{i=0}^{n} i \, p \, (1-p)^{i-1} \qquad (15.3)$$

15.4.2. Expected Cycle Cost

This consists of the inspection, ocpm, and failure costs, each multiplied by its expectation in a single cycle

$$E(C) = E(I)C_I + (1-r)C_M + rC_F. \qquad (15.4)$$

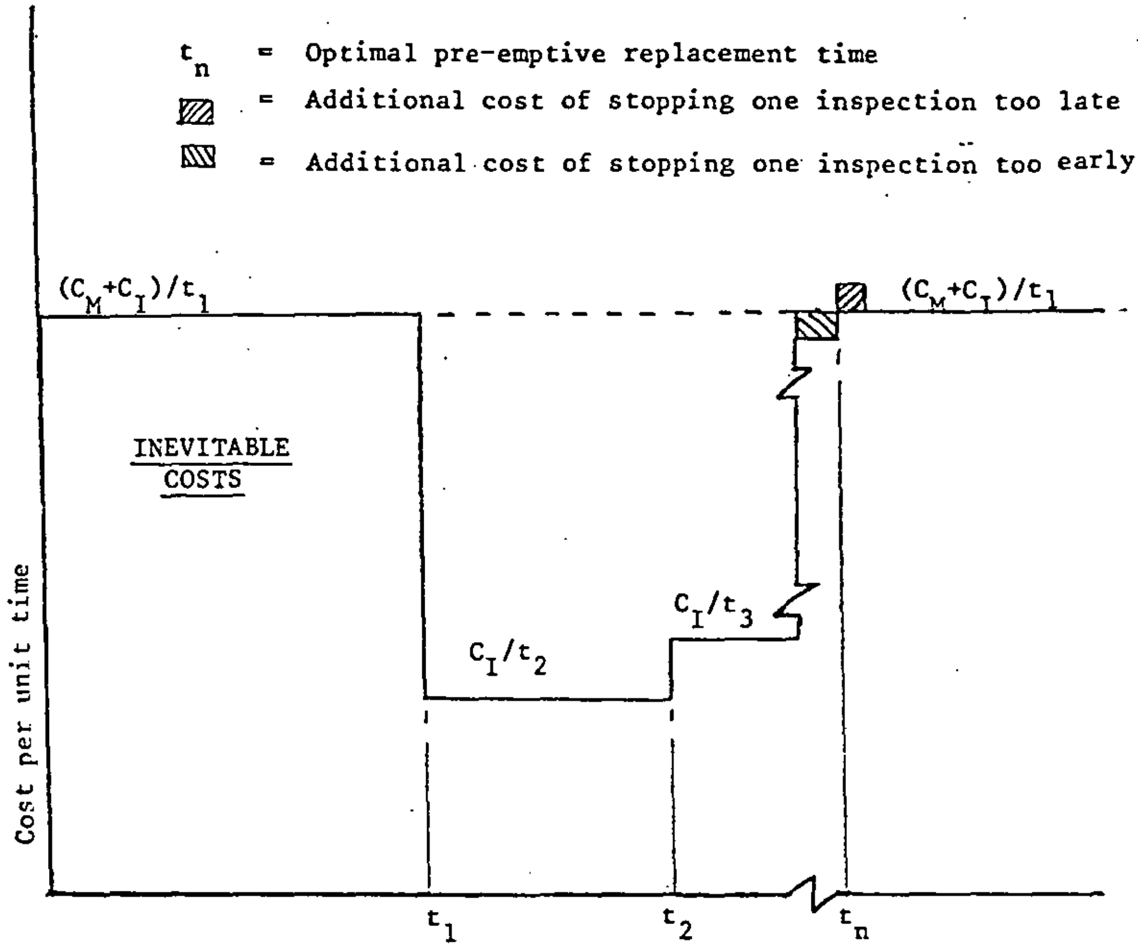


Figure 15.2 Stopping Rule for Increasing Hazard Rate

15.4.3 Expected Cycle Time

This consists of the expected times of failure and ocpm, each multiplied by its probability in a single cycle

$$E(T) = (1-r)E(T_{ocpm}) + r\int_0^{t_n} t\ f(t)\ dt/(1-(1-p)^n) \qquad (15.5)$$

$$= (1-r)\ E(T_{ocpm}) + r\theta \text{ for large } n, \qquad (15.6)$$

where $E(T_{ocpm})$ is the expected running time to ocpm given that the cycle ends in ocpm:

$$E(T_{ocpm}) = \sum_{i=0}^{n-1}\left[t_i p(1-p)^i\right] + t_n\ (1-p)^n \qquad (15.7)$$

15.4.4. Stopping Rule to find n

For increasing base hazard rate, a stopping rule is required to find n such that at t_n pre-emptive pm at cost C_M becomes cheaper than further inspections at shortening intervals. It will be economic to perform pm if the cost rate, given successful inspection, for a further interval t_n to t_{n+1} is greater than $C_M + C_I$ over a first interval t_1. Therefore,

$$n = \max\left(i \ \middle|\ C_I/(t_i - t_{i-1}) < (C_M + C_I)/t_1\right) \qquad (15.8)$$

15.4.5 Optimisation

To optimise the cost rate many values of p are tried. For each p the corresponding n must be found. If $t_i - t_{i-1}$ is increasing or constant in i, then $n \to \infty$.

Let $c = E(C)/E(T)$

If the costs C_F, C_M, C_I are fixed, all else can be expressed in terms of p and optimum schedule will be obtained at $p = p^*$ when

$$c^* = \min_p \left(E(C)/E(T)\right). \qquad (15.9)$$

15.4.6 Constant Base Hazard Rate

Consider the case where

$$R(t) = \exp(-t/\theta).$$

Substitute in and invert (15.2)

$$t_i = -\theta i \log (1-p) = i\tau \qquad (15.2c)$$

where τ is the constant inspection interval

From (15.8) $n \rightarrow \infty$, and from (15.3)

$$E(I) = 1/p \qquad (15.3c)$$

Similarly

$$E(C) = C_I/p + (1-r)C_M + rC_F; \qquad (15.4c)$$

$$E(T_{ocpm}) = \tau(1-p)/p, \qquad (15.7c)$$

$$E(T) = \tau(1-p)(1-r)/p + r\theta. \qquad (15.5c)$$

15.4.7 Returnable Defectives

A refund may be payable on spare parts and/or workmanship found to be defective at initial inspections. But under (15.4) another C_M would be spent on repeat ocpm. Let the refund reduce the repeat ocpm cost C_M to dC_M, $d \leq 1$. Rejection at initial inspection occurs with probability $p(1-r)$ so

$$E(C)_d = E(I)\, C_I + (1-r)(1-pd)C_M + rC_F. \qquad (15.4b)$$

If spare parts can be inspected prior to installation and a full refund is payable on defectives, d may be close to 1 provided that workmanship is not a problem. On the other hand, if inspection must follow installation and nonrefundable labour and lost production costs are a high proportion of C_M then d may be close to zero.

15.4.8 Comments on the Model

1. The interval risk p and the base failure pdf f(t) determine the inspection times, t_i and the maximum number of inspections n. As $p \rightarrow 0$ inspections are more frequent. $p = 1$ implies f m.

2. Constant p is optimal if the base hazard rate is constant. Munford and Shahani (3.169) showed near-optimality in an apparently less favourable case but I have not been able to prove that constant p is optimal in general. However, constant interval risk or hazard might be required for assurance of availability.

3. r is a measure of inspection effectiveness. $r = 0$ implies perfect, and $r = 1$ worthless, inspection. Because type a inspection errors are forbidden, type b must increase with p. Inspections seek signs of pending failure, less likely to be present or seen if failure is remote. The inspector permits any item not showing signs of pending failure, to continue to avoid a type a error, even if $p > 0.5$. Therefore r is an increasing function of p, and r,p coincide at 0, 1.

4. For computer optimisation special software is required to avoid the problem of cusps in the c versus p curve where the value of n changes. Fibonacci search is usually but not always successful. Gradient methods suffice only if z(t) is nonincreasing.

15.5 Inspection Effectiveness Models

15.5.1. General

It is necessary in application to choose a model for $r = g(p)$, collect data to see if it gives the anticipated result for optimised p, and try again if it does not fit. Linear or even a constant r model can fit over a limited range of p but for computer application, a model covering the whole range of p, r from 0 to 1 is needed. The initial estimate of f(t) can also be incorrect or shift over time. f(t) can be re-estimated by adding assessed residual lives to ocpm times and analysing these data with actual failure times by usual methods as a complete (uncensored) sample, (Methods described in Appendix A).

15.5.2. Markov Model

Miller & Braff's analysis (3.158) for constant z(t) is a Markov model. The interval between inspections is exponentially distributed rather than strictly periodic. It was shown by simulation that this made negligible difference to the reduction in observed failure rate. To use this model it is necessary to assess how long (on average) indication of pending failure would be present before actual failure occurred, and to assume that this time also is exponentially distributed. The base mtbf is the sum of two components : θ_1 = mean operating time before failure indication becomes available, and θ_2 = mean time for which warning would remain before failure. Inspection is necessary to observe the indication of pending failure. The state transition rate matrix with failure as an absorbing state is

		0	1	2
Up	0	$1-1/\theta_1$	$1/\theta_1$	0
Failure Pending	1	$1/\tau$	$1-1/\tau-1/\theta_2$	$1/\theta_2$
Down	2	0	0	1

The model is

$$r = \theta/(\theta+\theta_1\theta_2/\tau) = 1/\left\{1-b/\log(1-p)\right\} \qquad (15.10)$$

$$b = \theta_1\ \theta_2/\theta^2.$$

Equation 15.10 is derived by Miller and Braff in terms of failure and inspection rates. What they call 'benefit' is 1-r. This model has physical basis only for constant hazard rate but can be used empirically in other cases. The assumptions implied by this model for r when the hazard rate is not constant are that z(t) may be assumed constant over any single inspection interval and that the local mtbf is related to p in the same way in every interval. Let

$$\theta_i = \left[\theta \mid (I=i)\right] = (t_i-t_{i-1})\Big/\int_{t_{i-1}}^{t_i} z(t)dt$$

and similarly for θ_{1i} and θ_{2i}, then the assumption is that

$$b = \theta_{1i}\ \theta_{2i}/\theta_i^2 \text{ is constant in } i \qquad (15.11)$$

15.5.3. Beta Model

Let r be the ordinate for which the cdf of a Beta distribution with parameters (u,v) is p:

$$p = \text{betf}\ (r;u,v) \qquad (15.12)$$

where betf implies the Beta distribution function. r is therefore the inverse beta distribution function. The parameters u, v can be

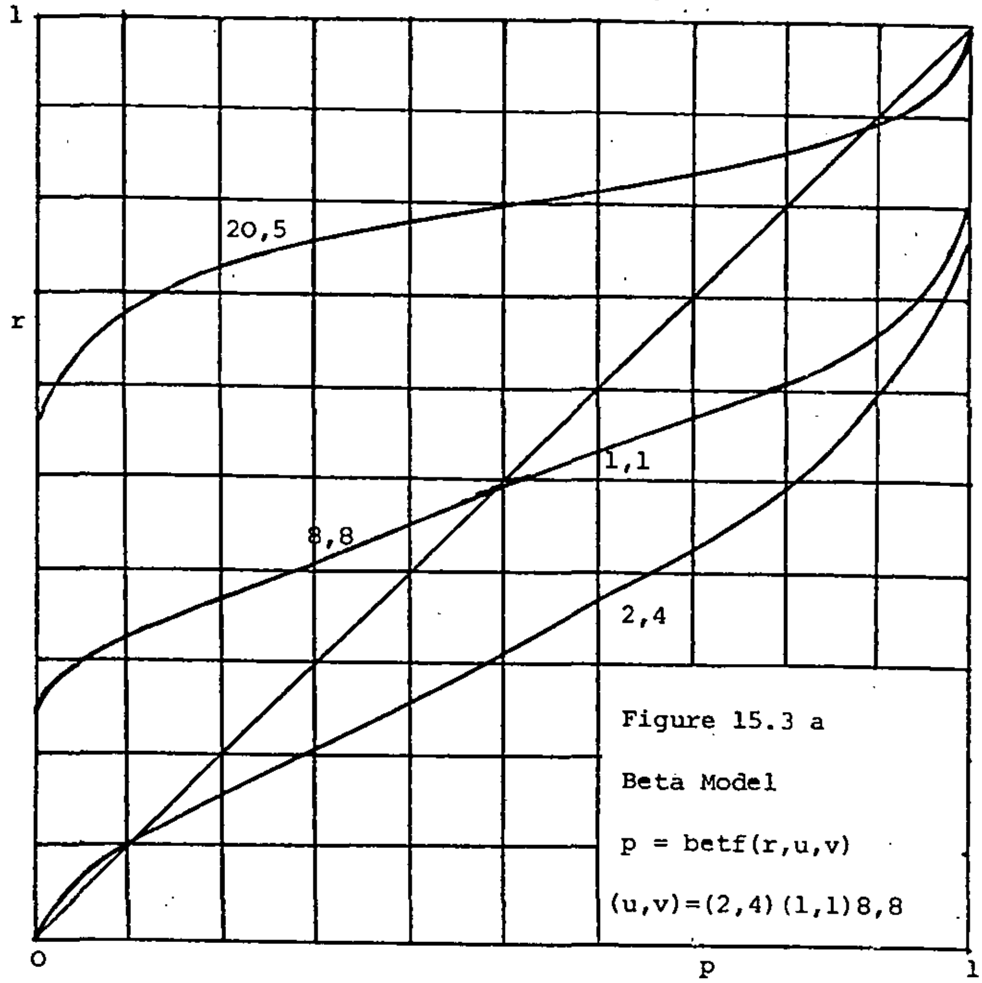


Figure 15.3 a

Beta Model

p = betf(r,u,v)

(u,v)=(2,4)(1,1)8,8

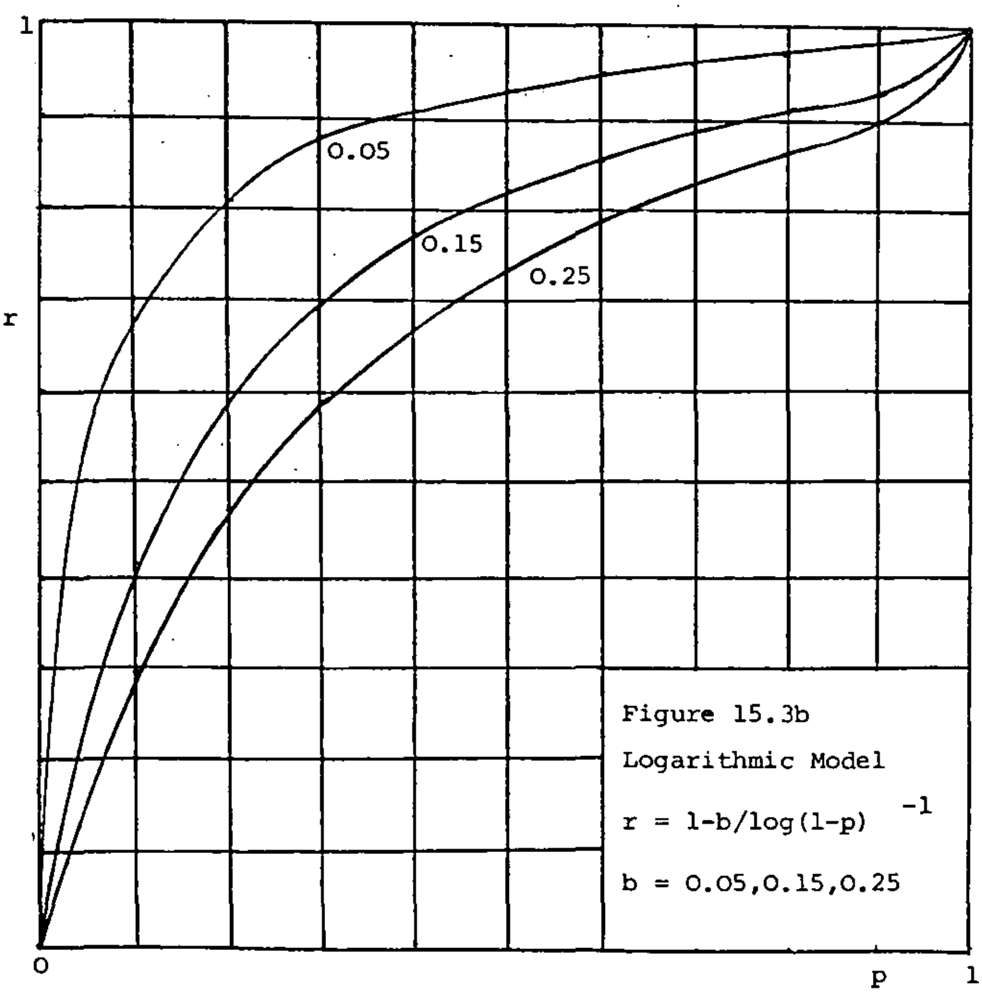
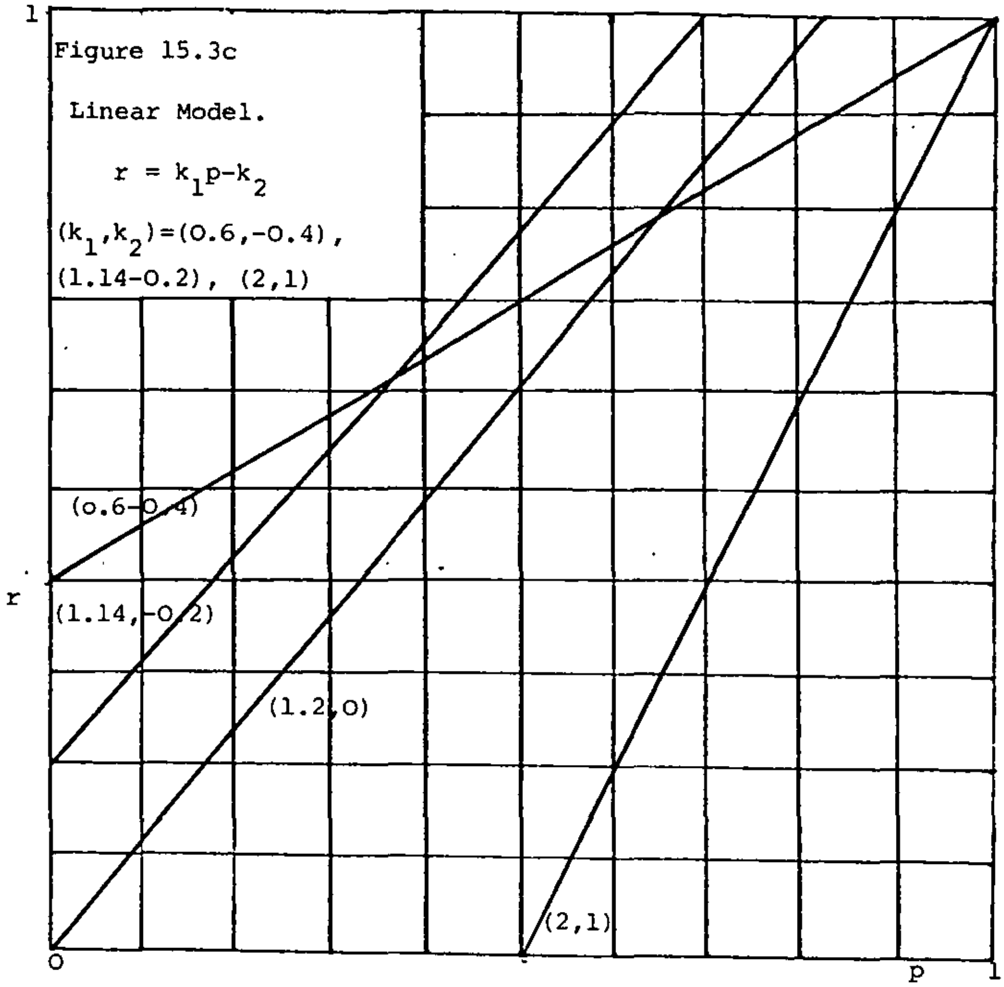


Figure 15.3b

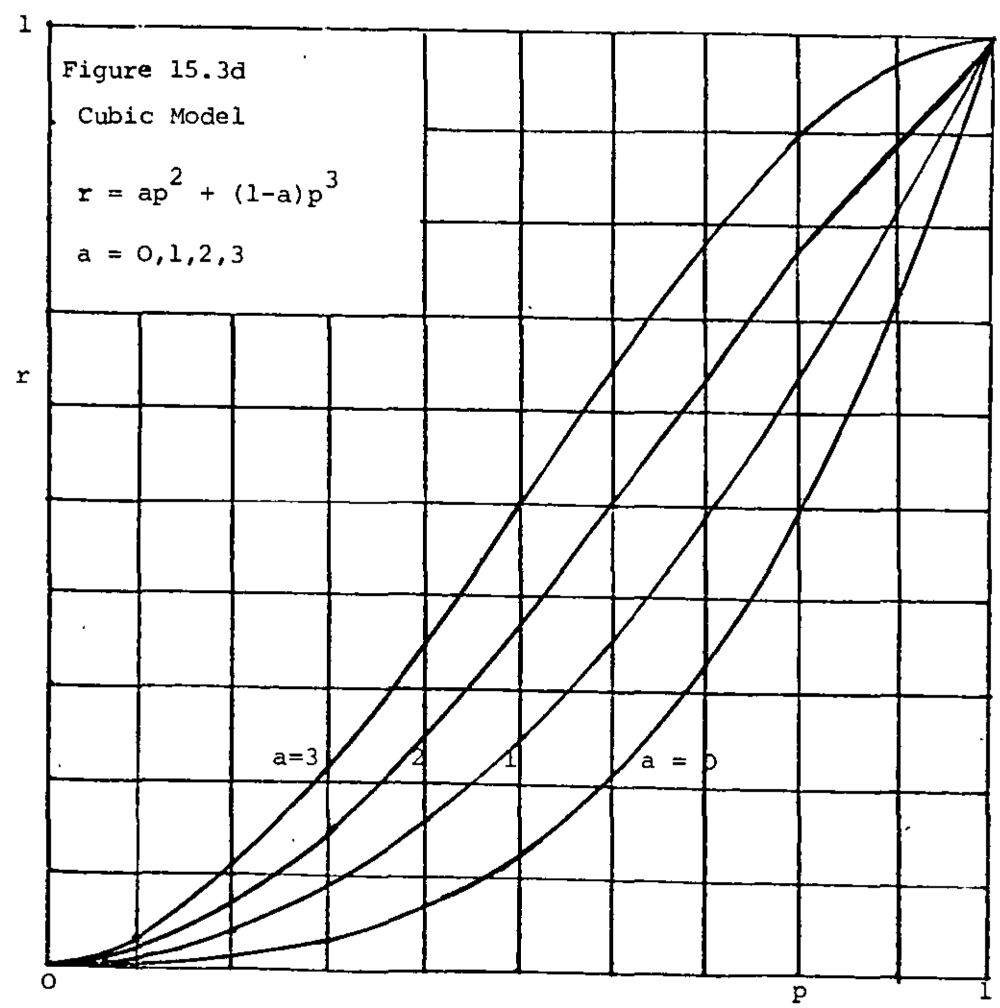
Logarithmic Model

$r = 1-b/\log(1-p)^{-1}$

b = 0.05,0.15,0.25



1
Figure 15.3c
Linear Model.
r = k1p-k2
(k1,k2)=(0.6,-0.4),
(1.14-0.2), (2,1)
(o.6-0.4)
r
(1.14,-0.2)
(1.2,0)
(2,1)
0
p
1



1
Figure 15.3d
Cubic Model
r = ap2 + (1-a)p3
a = 0,1,2,3
r
a=3
2
1
a = 0
0
p
1

found from two (p,r) pairs derived from operation of different schedules. The basis for this model lies in the binomial nature of the inspection and cycle outcomes. It is however extremely tedious for calculation; even when coded for computer it involves long trial-and-error calculations which use a lot of computer time.

15.5.4. Cubic Model

Three points define a cubic. If two of them are (0,0) and (1,1) there is no constant term and if additionally the gradient must not be negative in the 0 to 1 range of either variable, a single parameter curve is the outcome i.e.

$$r = ap^2 + (1-a)p^3 \; ; \; 0 \leq a \leq 3 \qquad (15.13)$$

This model covers the case where $r < p$ in the 0 to 1 range of interest. It could be inverted, writing p for r and vice-versa but there is no real advantage over the logarithmic model. There is no physical basis for this model; it is simply a convenient curve which sometimes may fit the observed data.

15.5.5. Linear Models

A general linear model is

$$\left.\begin{array}{l} r = k_1 p - k_2 \quad k_1 > 0; \quad 0 < p < 1; \; 0 < r < 1 \\ r = 0, \; p < k_2/k_1 \\ r = 1 \text{ elsewhere} \end{array}\right\} \qquad (15.14)$$

This model is able to represent the following cases.

a) some fixed proportion of failures is inevitable; even if $p = 0$ r has a positive value. ($k_2 < 0$)

b) Up to a certain value of p all failures will be prevented. ($k_2 > 0$)

c) Above a certain value of p no failures are detected $\{(k_1 + k_2/k_1) < 1\}$

d) (a) or (b), but not both, with (c).

The disadvantages of two-parameter linear models are that there is no physical basis and that two determinations, that is two data-sets with different schedules of inspection, are necessary for a complete determination.

If only one condition from (a),(b),(c) above applies then the line can be assumed to pass through either (0,0) or (1,1) and one parameter disappears. If the line passes through (0,0) then

$$r = k_3 p \qquad 0 < p,\ r < 1$$
$$r = 1 \text{ elsewhere} \qquad (15.15)$$

If the line passes through (1,1) then

$$\left.\begin{array}{l} r = (p-k_4)/(1-k_4) \quad 0 < p, r < 1,\ k_4 < 1 \\ r = 0 \quad p < k_4 \end{array}\right\} \qquad (15.16)$$

One of these models (15.15) (15.16) might be applied when only one data-set exists in order to provide a first attempt at optimisation as the conditions under which the second set is collected.

<u>15.5.6 General Power Model</u>

This model has three parameters and so needs three realisations of r to determine the parameters.

$$\left.\begin{array}{ll} r = a(p-k_5)^b & k_5 < p < 1 \\ r = 0 & p \leq k_5 \\ r = 1 & p \geq k_5 + a^{-1/b} \end{array}\right\} \qquad (15.17)$$

A simplified version is:

$$r = p^b \qquad b > 0 \qquad (15.18)$$

which is an alternative to the logarithmic model when only one data-set is available.

15.6 Results

15.6.1 Computer Programmes. A programme was written for the Beta model, but it proved so clumsy and time-consuming in operation that it will not be discussed further. A simple programme for the constant hazard rate case ($\beta = 1$) using the Miller and Braff's parameterization of r was written which includes a graphical routine to examine the sensitivity of c to t. This programme was named CONHAZ and is listed at Appendix C. A more general programme which also uses the logarithmic model is INSPEF. INSPEF compares the best ocpm schedule with fm and also, for $\beta > 1$, with the best ppm interval on the basis of overall average costrate to an infinite time horizon. INSPEF is also listed at Appendix C. Both programmes have a maximum coresize of 19K and take $\sim$15 mill units to compile. A single solution for CONHAZ takes $\sim$12 mill units including the plot routine. For INSPEF, with two optimizations to be performed and more complex algorithms the solution time is $\sim$ 20 mill units. Both programmes use Fibonacci search for the routines which search for the lowest costrate. For CONHAZ this was not strictly necessary but was done to save time in programme development. Gradient methods would be satisfactory because there are no cusps in the c versus τ curve unless the number of inspections before pre-emptive renewal, n, is finite and varies with p. The author recognises that these programmes are not as efficient as they might have been if expert help had been sought.

15.6.2. Presentation. An effort was made to find a simple way of presenting the information contained in the computer results. Graphs do not really help because only three parameters can be covered at a time i.e. by plotting one against another for a series of lines for which the third is constant. Nomograms suffer from the same disability. It seemed likely that the costs could be combined into one dimensionless ratio viz $(C_F - C_M)/C_I$ which could be shown to be a parameter of the generalised problem but this is not so although $(C_F-C_M)/C_I$ is a parameter of the problem when $\beta = 1$. In arriving at this position a very large number of results were printed out from the programmes. Those which follow are intended to illustrate the general comments of the next paragraph rather than provide in themselves a complete picture.

In the printouts

N stands for n printed as 9999 when infinite

TN/MTBF stands for t_n/θ printed as $> 10^5$ when infinite

C*OCPM/CFM stands for $c^*_{ocpm}\theta/C_F$

C*PPM/CFM stands for $c^*_{ppm}\theta/C_F$

T*PPM/MTBF stands for t^*_{ppm}/θ printed as $.999 \times 10^{34}$ when infinite

where

c^*_{ocpm} is the inspection schedule best costrate

c^*_{ppm} is the ppm schedule best costrate

t^*_{ppm} is the optimum ppm period between renewals.

The last column indicates the cheapest policy from ocpm, ppm and fm. ppm is not applicable for $\beta \leq 1$.

The total range examined was as follows:

$C_F = 2\ (\times 2) 1024$

$C_M = 1\ (\times 2)\ 512 \qquad C_M < C_F$

$\beta = 0.5, 1, 2, 3, 5$

$b = 0.05, 0.15, 0.25$

$d = 0, 1$

with $C_I = 1$, $\eta = 1$, $\gamma = 0$, normalised throughout.

Values of β up to 19 were also investigated in one run. Values of $b > 0.25$ were not investigated because it was envisaged that b would be estimated initially from $\theta_1 \theta_2/\theta^2$ even though β was not always 1. This fraction cannot be more than 0.25 if $\theta = \theta_1 + \theta_2$.

15.6.2. Comments on Results.

a) Whether or not an initial inspection is performed at $t_o=0$ can make up to 3% difference in the overall costrate. When $\beta \leq 1$ it is always cheaper to have the initial inspection. For $\beta > 1$ it depends on other factors as well, particularly relative cost values. For $1 < \beta < 2$ there is little difference and for $\beta \geq 2$ it may be cheaper to omit the initial inspection; as $\eta \to 1$ it becomes a handicap.

b) For $\beta < 1$ a schedule with lower cost rate than fm is possible. This remains true even when $C_M = C_I$ because the value of an inspection may be greater than that of renewal/ maintenance when z(t) is falling.

c) For constant costs C_F, C_M, C_I, savings of ocpm over ppm get smaller as β increases and finally reverse so that ppm is cheaper.Omitting the initial inspection merely delays this effect a little. However, if C_F and C_M are relatively close and C_I small relative to both, even values of $\beta > 15$ are not sufficiently peaky to make ppm cheaper than ocpm. Also, an ocpm schedule would be less sensitive than ppm to inaccuracy in η at high β values.

d) The sensitivity of costrate to p and r is very often not great for reasonable parameter values. As regards C_F,C_M, C_I relative values are important. It seems to be helpful when assessing the cost values to think about how many inspections can be bought for the saving to be made from on-condition as opposed to failure renewal. i.e. $(C_F - C_M)/C_I$.

e) The expected cycle time E(T) is relatively insensitive to both p and r, particularly when $\beta \leq 1$. The restricted range of b in the logarithmic model for g(p) gave rise to a situation where if $E(T_{ocpm})$ was small then r was large so that the proportionate effect upon E(T) was small.

f) Worthwhile savings over ppm or fm are obtainable over a large range of parameter values. It is always worth checking the costrate of an ocpm schedule before deciding upon fm or ppm as a maintenance policy, provided of course that the item concerned does give some warning of impending failure.

15.6.3. Examples from Results of Computer Programme INSPEF

For Programme see Appendix C.

R= 0.050 BETA= 0.500 D= 0.000

CT	CM	P*	R	N	TM/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
2	1	0.99835	0.99225	9999	0.50000E 06	1.50824	1.00000	0.99999E 34	FM
4	1	0.99835	0.99225	9999	0.50000E 06	1.25399	1.00000	0.99999E 34	FM
4	2	0.99835	0.99225	9999	0.50000E 06	1.25594	1.00000	0.99999E 34	FM
8	1	0.99835	0.99225	9999	0.50000E 06	1.12686	1.00000	0.99999E 34	FM
8	2	0.99835	0.99225	9999	0.50000E 06	1.12784	1.00000	0.99999E 34	FM
8	4	0.99835	0.99225	9999	0.50000E 06	1.12979	1.00000	0.99999E 34	FM
16	1	0.97515	0.98665	9999	0.50000E 06	1.06318	1.00000	0.99999E 34	FM
16	2	0.99835	0.99225	9999	0.50000E 06	1.06378	1.00000	0.99999E 34	FM
16	4	0.99835	0.99225	9999	0.50000E 06	1.06476	1.00000	0.99999E 34	FM
16	8	0.99835	0.99225	9999	0.50000E 06	1.06671	1.00000	0.99999E 34	FM
32	1	0.17693	0.79568	9999	0.50000E 06	0.99731	1.00000	0.99999E 34	OCPM
32	2	0.19062	0.80896	9999	0.50000E 06	1.00362	1.00000	0.99999E 34	FM
32	4	0.22717	0.83756	9999	0.50000E 06	1.01492	1.00000	0.99999E 34	FM
32	8	0.41007	0.91346	9999	0.50000E 06	1.03122	1.00000	0.99999E 34	FM
32	16	0.99835	0.99225	9999	0.50000E 06	1.03517	1.00000	0.99999E 34	FM
64	1	0.06131	0.55859	9999	0.50000E 06	0.83181	1.00000	0.99999E 34	OCPM
64	2	0.06222	0.56233	9999	0.50000E 06	0.83878	1.00000	0.99999E 34	OCPM
64	4	0.06477	0.57250	9999	0.50000E 06	0.85250	1.00000	0.99999E 34	OCPM
64	8	0.07082	0.59213	9999	0.50000E 06	0.87907	1.00000	0.99999E 34	OCPM
64	16	0.08435	0.63801	9999	0.50000E 06	0.92818	1.00000	0.99999E 34	OCPM
64	32	0.15963	0.77670	9999	0.50000E 06	1.00467	1.00000	0.99999E 34	FM
128	1	0.03211	0.39494	9999	0.50000E 06	0.64957	1.00000	0.99999E 34	OCPM
128	2	0.03232	0.39655	9999	0.50000E 06	0.65435	1.00000	0.99999E 34	OCPM
128	4	0.03246	0.39755	9999	0.50000E 06	0.66387	1.00000	0.99999E 34	OCPM
128	8	0.03336	0.40427	9999	0.50000E 06	0.68276	1.00000	0.99999E 34	OCPM
128	16	0.03549	0.41955	9999	0.50000E 06	0.71992	1.00000	0.99999E 34	OCPM
128	32	0.04030	0.45139	9999	0.50000E 06	0.79140	1.00000	0.99999E 34	OCPM
128	64	0.05894	0.54854	9999	0.50000E 06	0.91929	1.00000	0.99999E 34	OCPM

B= 0.050 BETA= 1.000 D= 0.000

CF	CM	P*	R	N	TN/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
2	1	0.99835	0.99225	9999	0.10000E 07	42.46194	1.00000	0.99999E 34	FM
4	1	0.00010	0.00200	9999	0.10000E 07	20.82275	1.00000	0.99999E 34	FM
4	2	0.00010	0.00200	9999	0.10000E 07	21.07226	1.00000	0.99999E 34	FM
8	1	0.00010	0.00200	9999	0.10000E 07	10.41237	1.00000	0.99999E 34	FM
8	2	0.00010	0.00200	9999	0.10000E 07	10.53713	1.00000	0.99999E 34	FM
8	4	0.00010	0.00200	9999	0.10000E 07	10.78664	1.00000	0.99999E 34	FM
16	1	0.00010	0.00200	9999	0.10000E 07	5.20719	1.00000	0.99999E 34	FM
16	2	0.00010	0.00200	9999	0.10000E 07	5.26956	1.00000	0.99999E 34	FM
16	4	0.00010	0.00200	9999	0.10000E 07	5.39432	1.00000	0.99999E 34	FM
16	8	0.00010	0.00200	9999	0.10000E 07	5.64383	1.00000	0.99999E 34	FM
32	1	0.00010	0.00200	9999	0.10000E 07	2.60459	1.00000	0.99999E 34	FM
32	2	0.00010	0.00200	9999	0.10000E 07	2.63578	1.00000	0.99999E 34	FM
32	4	0.00010	0.00200	9999	0.10000E 07	2.69816	1.00000	0.99999E 34	FM
32	8	0.00010	0.00200	9999	0.10000E 07	2.82292	1.00000	0.99999E 34	FM
32	16	0.00010	0.00200	9999	0.10000E 07	3.07243	1.00000	0.99999E 34	FM
64	1	0.00010	0.00200	9999	0.10000E 07	1.30330	1.00000	0.99999E 34	FM
64	2	0.00010	0.00200	9999	0.10000E 07	1.31689	1.00000	0.99999E 34	FM
64	4	0.00010	0.00200	9999	0.10000E 07	1.35008	1.00000	0.99999E 34	FM
64	8	0.00010	0.00200	9999	0.10000E 07	1.41246	1.00000	0.99999E 34	FM
64	16	0.00010	0.00200	9999	0.10000E 07	1.53721	1.00000	0.99999E 34	FM
64	32	0.00010	0.00200	9999	0.10000E 07	1.78673	1.00000	0.99999E 34	FM
128	1	0.00010	0.00200	9999	0.10000E 07	0.65265	1.00000	0.99999E 34	OCPM
128	2	0.00010	0.00200	9999	0.10000E 07	0.66045	1.00000	0.99999E 34	OCPM
128	4	0.00010	0.00200	9999	0.10000E 07	0.67604	1.00000	0.99999E 34	OCPM
128	8	0.00010	0.00200	9999	0.10000E 07	0.70723	1.00000	0.99999E 34	OCPM
128	16	0.00010	0.00200	9999	0.10000E 07	0.76961	1.00000	0.99999E 34	OCPM
128	32	0.00010	0.00200	9999	0.10000E 07	0.89436	1.00000	0.99999E 34	OCPM
128	64	0.00010	0.00200	9999	0.10000E 07	1.14388	1.00000	0.99999E 34	FM

B= 0.050 BETA= 3.000 D= 0.000

CF	CM	P*	R	N	TN/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
2	1	0.35938	0.89906	1	0.05516E 00	1.48787	0.87957	0.90747E 00	PPM
4	1	0.14839	0.76262	1	0.60875E 00	1.18588	0.61700	0.62053E 00	PPM
4	2	0.30750	0.88623	1	0.60209E 00	1.23046	0.87957	0.90747E 00	PPM
8	1	0.03961	0.44701	1	0.38431E 00	0.97260	0.40595	0.46598E 00	PPM
8	2	0.10016	0.67854	1	0.52921E 00	1.04563	0.61700	0.62053E 00	PPM
8	4	0.99835	0.99225	2	0.26203E 01	1.13006	0.87957	0.90747E 00	PPM
16	1	0.01043	0.17338	1	0.24512E 00	0.76479	0.26085	0.36089E 00	PPM
16	2	0.02379	0.32500	1	0.32335E 00	0.86879	0.40595	0.46598E 00	PPM
16	4	0.06313	0.56600	2	0.56789E 00	0.98514	0.61700	0.62053E 00	PPM
16	8	0.99835	0.99225	5	0.35563E 01	1.06697	0.87957	0.90747E 00	PPM
32	1	0.00332	0.06231	1	0.16710E 00	0.56102	0.16589	0.28314E 00	PPM
32	2	0.00650	0.11531	1	0.20917E 00	0.67205	0.26085	0.36089E 00	PPM
32	4	0.01639	0.24842	2	0.35939E 00	0.78138	0.40595	0.46598E 00	PPM
32	8	0.04041	0.45206	5	0.06165E 00	0.91695	0.61700	0.62053E 00	PPM
32	16	0.99835	0.99225	13	0.48902E 01	1.03542	0.87957	0.90747E 00	PPM
64	1	0.00143	0.02775	1	0.12608E 00	0.38958	0.10499	0.22345E 00	PPM
64	2	0.00225	0.04303	1	0.14670E 00	0.48283	0.16589	0.28314E 00	PPM
64	4	0.00513	0.09330	2	0.24357E 00	0.57705	0.26085	0.36089E 00	PPM
64	8	0.01243	0.20015	5	0.44455E 00	0.68234	0.40595	0.46598E 00	PPM
64	16	0.02855	0.36662	13	0.00865E 00	0.83252	0.61700	0.62053E 00	PPM
64	32	0.13880	0.74928	36	0.19622E 01	1.01018	0.87957	0.90747E 00	PPM
128	1	0.00061	0.01199	1	0.94800E-01	0.26047	0.06629	0.17687E 00	PPM
128	2	0.00092	0.01807	1	0.10891E 00	0.32974	0.10499	0.22345E 00	PPM
128	4	0.00225	0.04303	2	0.18483E 00	0.40278	0.16589	0.28314E 00	PPM
128	8	0.00449	0.08257	5	0.31615E 00	0.48093	0.26085	0.36089E 00	PPM
128	16	0.01032	0.17177	13	0.57420E 00	0.58556	0.40595	0.46598E 00	PPM
128	32	0.02243	0.31208	36	0.10467E 01	0.74326	0.61700	0.62053E 00	PPM
128	64	0.05646	0.53755	101	0.20201E 01	0.92176	0.87957	0.90747E 00	PPM

R= 0.050 BETA= 3.000 D= 0.000

CF	C_p	P*	R	N	T_N/MTBF	C*OCP_M/CF_M	C*PP_M/CF_M	T*PPM/MTBF	BEST
256	1	0.00022	0.00437	1	0.67561E-01	0.16985	0.04181	0.14016E 00	PPM
256	2	0.00041	0.00819	1	0.83403E-01	0.21633	0.06629	0.17687E 00	PPM
256	4	0.00092	0.01807	2	0.13722E 00	0.26978	0.10499	0.22345E 00	PPM
256	8	0.00225	0.04303	5	0.25085E 00	0.32680	0.16589	0.28314E 00	PPM
256	16	0.00421	0.07761	13	0.42548E 00	0.39740	0.26085	0.36089E 00	PPM
256	32	0.00921	0.15621	36	0.77638E 00	0.50071	0.40595	0.46598E 00	PPM
256	64	0.01866	0.27173	101	0.13832E 01	0.65010	0.61700	0.62053E 00	PPM
256	128	0.03075	0.38449	249	0.22187E 01	0.82864	0.87957	0.90747E 00	OCPM
512	1	0.00011	0.00221	1	0.53784E-01	0.10940	0.02635	0.11118E 00	PPM
512	2	0.00017	0.00347	1	0.62508E-01	0.14161	0.04181	0.14016E 00	PPM
512	4	0.00041	0.00819	2	0.10508E 00	0.17658	0.06629	0.17687E 00	PPM
512	8	0.00092	0.01807	5	0.18624E 00	0.21494	0.10499	0.22345E 00	PPM
512	16	0.00225	0.04303	13	0.34494E 00	0.26411	0.16589	0.28314E 00	PPM
512	32	0.00416	0.07772	36	0.59583E 00	0.32951	0.26085	0.36089E 00	PPM
512	64	0.00846	0.14517	101	0.10640E 01	0.42835	0.40595	0.46598E 00	PPM
512	128	0.01359	0.21492	249	0.16853E 01	0.55522	0.61700	0.62053E 00	OCPM
512	256	0.01624	0.26914	249	0.18604E 01	0.74682	0.87957	0.90747E 00	OCPM
1024	1	0.00010	0.00200	1	0.52008E-01	0.07329	0.01661	0.88215E-01	PPM
1024	2	0.00010	0.00200	1	0.52008E-01	0.09137	0.02635	0.11118E 00	PPM
1024	4	0.00017	0.00347	2	0.78755E-01	0.11363	0.04181	0.14016E 00	PPM
1024	8	0.00041	0.00819	5	0.14262E 00	0.13919	0.06629	0.17687E 00	PPM
1024	16	0.00092	0.01807	13	0.25609E 00	0.17122	0.10499	0.22345E 00	PPM
1024	32	0.00225	0.04303	36	0.48439E 00	0.21444	0.16589	0.28314E 00	PPM
1024	64	0.00420	0.07765	101	0.84205E 00	0.27621	0.26085	0.36089E 00	PPM
1024	128	0.00715	0.12546	249	0.13587E 01	0.35970	0.40595	0.46598E 00	OCPM
1024	256	0.00851	0.14592	249	0.14402E 01	0.47059	0.61700	0.62053E 00	OCPM
1024	512	0.01178	0.19162	249	0.16063E 01	0.68193	0.87957	0.90747E 00	OCPM

B= 0.250 BETA= 0.500 D= 0.000

CF	CM	P*	R	N	TN/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
2	1	0.99953	0.96844	9999	0.50000E 06	1.53215	1.00000	0.99999E 34	FM
4	1	0.99835	0.96244	9999	0.50000E 06	1.26826	1.00000	0.99999E 34	FM
4	2	0.99931	0.96678	9999	0.50000E 06	1.27515	1.00000	0.99999E 34	FM
8	1	0.48936	0.72888	9999	0.50000E 06	1.09943	1.00000	0.99999E 34	FM
8	2	0.63715	0.80218	9999	0.50000E 06	1.13155	1.00000	0.99999E 34	FM
8	4	0.99894	0.96479	9999	0.50000E 06	1.14690	1.00000	0.99999E 34	FM
16	1	0.21639	0.49377	9999	0.50000E 06	0.86348	1.00000	0.99999E 34	OCPM
16	2	0.22808	0.50872	9999	0.50000E 06	0.89658	1.00000	0.99999E 34	OCPM
16	4	0.25850	0.54470	9999	0.50000E 06	0.95961	1.00000	0.99999E 34	OCPM
16	8	0.38258	0.65857	9999	0.50000E 06	1.06806	1.00000	0.99999E 34	FM
32	1	0.12345	0.34514	9999	0.50000E 06	0.64545	1.00000	0.99999E 34	OCPM
32	2	0.12655	0.35117	9999	0.50000E 06	0.66669	1.00000	0.99999E 34	OCPM
32	4	0.13158	0.36074	9999	0.50000E 06	0.70870	1.00000	0.99999E 34	OCPM
32	8	0.14574	0.38654	9999	0.50000E 06	0.79064	1.00000	0.99999E 34	OCPM
32	16	0.19009	0.45750	9999	0.50000E 06	0.94354	1.00000	0.99999E 34	OCPM
64	1	0.07814	0.24555	9999	0.50000E 06	0.47145	1.00000	0.99999E 34	OCPM
64	2	0.07888	0.24736	9999	0.50000E 06	0.48359	1.00000	0.99999E 34	OCPM
64	4	0.08017	0.25053	9999	0.50000E 06	0.50781	1.00000	0.99999E 34	OCPM
64	8	0.08300	0.25737	9999	0.50000E 06	0.55592	1.00000	0.99999E 34	OCPM
64	16	0.09084	0.27587	9999	0.50000E 06	0.65072	1.00000	0.99999E 34	OCPM
64	32	0.11340	0.32498	9999	0.50000E 06	0.83287	1.00000	0.99999E 34	OCPM
128	1	0.05127	0.17390	9999	0.50000E 06	0.34011	1.00000	0.99999E 34	OCPM
128	2	0.05161	0.17490	9999	0.50000E 06	0.34671	1.00000	0.99999E 34	OCPM
128	4	0.05189	0.17570	9999	0.50000E 06	0.35988	1.00000	0.99999E 34	OCPM
128	8	0.05280	0.17829	9999	0.50000E 06	0.38618	1.00000	0.99999E 34	OCPM
128	16	0.05472	0.18373	9999	0.50000E 06	0.43855	1.00000	0.99999E 34	OCPM
128	32	0.05894	0.19550	9999	0.50000E 06	0.54229	1.00000	0.99999E 34	OCPM
128	64	0.07221	0.23066	9999	0.50000E 06	0.74459	1.00000	0.99999E 34	OCPM

B= 0.250 BETA= 1.000 D= 0.000

CF	CM	P*	R	N	TN/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
2	1	0.00010	0.00040	9999	0.10000E 07	22.92709	1.00000	0.99999E 34	FM
4	1	0.00010	0.00040	9999	0.10000E 07	11.46375	1.00000	0.99999E 34	FM
4	2	0.00010	0.00040	9999	0.10000E 07	11.71366	1.00000	0.99999E 34	FM
8	1	0.00010	0.00040	9999	0.10000E 07	5.73207	1.00000	0.99999E 34	FM
8	2	0.00010	0.00040	9999	0.10000E 07	5.85703	1.00000	0.99999E 34	FM
8	4	0.00010	0.00040	9999	0.10000E 07	6.10694	1.00000	0.99999E 34	FM
16	1	0.00010	0.00040	9999	0.10000E 07	2.86624	1.00000	0.99999E 34	FM
16	2	0.00010	0.00040	9999	0.10000E 07	2.92872	1.00000	0.99999E 34	FM
16	4	0.00010	0.00040	9999	0.10000E 07	3.05367	1.00000	0.99999E 34	FM
16	8	0.00010	0.00040	9999	0.10000E 07	3.30358	1.00000	0.99999E 34	FM
32	1	0.00010	0.00040	9999	0.10000E 07	1.43332	1.00000	0.99999E 34	FM
32	2	0.00010	0.00040	9999	0.10000E 07	1.46456	1.00000	0.99999E 34	FM
32	4	0.00010	0.00040	9999	0.10000E 07	1.52704	1.00000	0.99999E 34	FM
32	8	0.00010	0.00040	9999	0.10000E 07	1.65199	1.00000	0.99999E 34	FM
32	16	0.00010	0.00040	9999	0.10000E 07	1.90190	1.00000	0.99999E 34	FM
64	1	0.00010	0.00040	9999	0.10000E 07	0.71686	1.00000	0.99999E 34	OCPM
64	2	0.00010	0.00040	9999	0.10000E 07	0.73248	1.00000	0.99999E 34	OCPM
64	4	0.00010	0.00040	9999	0.10000E 07	0.76372	1.00000	0.99999E 34	OCPM
64	8	0.00010	0.00040	9999	0.10000E 07	0.82620	1.00000	0.99999E 34	OCPM
64	16	0.00010	0.00040	9999	0.10000E 07	0.95115	1.00000	0.99999E 34	OCPM
64	32	0.00010	0.00040	9999	0.10000E 07	1.20106	1.00000	0.99999E 34	FM
128	1	0.00010	0.00040	9999	0.10000E 07	0.35863	1.00000	0.99999E 34	OCPM
128	2	0.00010	0.00040	9999	0.10000E 07	0.36644	1.00000	0.99999E 34	OCPM
128	4	0.00010	0.00040	9999	0.10000E 07	0.38206	1.00000	0.99999E 34	OCPM
128	8	0.00010	0.00040	9999	0.10000E 07	0.41330	1.00000	0.99999E 34	OCPM
128	16	0.00010	0.00040	9999	0.10000E 07	0.47578	1.00000	0.99999E 34	OCPM
128	32	0.00010	0.00040	9999	0.10000E 07	0.60073	1.00000	0.99999E 34	OCPM
128	64	0.00010	0.00040	9999	0.10000E 07	0.85065	1.00000	0.99999E 34	OCPM

B= 0.250   BETA= 3.000   D= 0.000

CF	CM	P*	R	N	TN/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
2	1	0.37253	0.65087	1	0.86824E 00	1.45344	0.87957	0.90747E 00	PPM
4	1	0.19556	0.46537	1	0.67358E 00	1.04597	0.61700	0.62053E 00	PPM
4	2	0.32997	0.61564	1	0.82541E 00	1.17751	0.87957	0.90747E 00	PPM
8	1	0.07842	0.24624	1	0.48586E 00	0.76278	0.40595	0.46598E 00	PPM
8	2	0.14930	0.39276	1	0.61009E 00	0.88688	0.61700	0.62053E 00	PPM
8	4	0.24936	0.53430	2	0.93049E 00	1.09102	0.87957	0.90747E 00	PPM
16	1	0.02922	0.10604	1	0.34662E 00	0.53992	0.26085	0.36089E 00	PPM
16	2	0.05426	0.18245	1	0.42791E 00	0.65344	0.40595	0.46598E 00	PPM
16	4	0.10894	0.31570	2	0.68679E 00	0.79007	0.61700	0.62053E 00	PPM
16	8	0.17620	0.43672	5	0.11082E 01	1.01770	0.87957	0.90747E 00	PPM
32	1	0.01170	0.04496	1	0.25473E 00	0.36799	0.16589	0.28314E 00	PPM
32	2	0.02019	0.07543	1	0.30596E 00	0.45914	0.26085	0.36089E 00	PPM
32	4	0.04331	0.15046	2	0.49914E 00	0.55820	0.40595	0.46598E 00	PPM
32	8	0.08006	0.25026	5	0.83681E 00	0.69494	0.61700	0.62053E 00	PPM
32	16	0.13022	0.35817	13	0.13657E 01	0.93270	0.87957	0.90747E 00	PPM
64	1	0.00505	0.01985	1	0.19229E 00	0.24381	0.10499	0.22345E 00	PPM
64	2	0.00827	0.03214	1	0.22675E 00	0.31007	0.16589	0.28314E 00	PPM
64	4	0.01762	0.06638	2	0.36822E 00	0.38096	0.26085	0.36089E 00	PPM
64	8	0.03647	0.12937	5	0.63894E 00	0.46738	0.40595	0.46598E 00	PPM
64	16	0.06103	0.20122	13	0.10476E 01	0.60765	0.61700	0.62053E 00	OCPM
64	32	0.10016	0.29684	36	0.17474E 01	0.83889	0.87957	0.90747E 00	OCPM
128	1	0.00225	0.00891	1	0.14670E 00	0.15857	0.06629	0.17687E 00	PPM
128	2	0.00364	0.01437	1	0.17233E 00	0.20400	0.10499	0.22345E 00	PPM
128	4	0.00762	0.02967	2	0.27794E 00	0.25278	0.16589	0.28314E 00	PPM
128	8	0.01646	0.06224	5	0.48843E 00	0.30924	0.26085	0.36089E 00	PPM
128	16	0.03114	0.11232	13	0.83275E 00	0.39182	0.40595	0.46598E 00	OCPM
128	32	0.04840	0.16558	36	0.13587E 01	0.53132	0.61700	0.62053E 00	OCPM
128	64	0.06956	0.22385	101	0.21706E 01	0.74979	0.87957	0.90747E 00	OCPM

B= 0.250 BETA= 3.000 D= 0.000

CF	CM	P*	R	N	TN/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
256	1	0.00143	0.00568	1	0.12608E 00	0.10277	0.04181	0.14016E 00	PPM
256	2	0.00225	0.00891	1	0.14670E 00	0.13325	0.06629	0.17687E 00	PPM
256	4	0.00346	0.01367	2	0.21349E 00	0.16457	0.10499	0.22345E 00	PPM
256	8	0.00757	0.02952	5	0.37655E 00	0.20155	0.16589	0.28314E 00	PPM
256	16	0.01563	0.05929	13	0.66012E 00	0.25194	0.26085	0.36089E 00	OCPM
256	32	0.02663	0.09745	36	0.11092E 01	0.33234	0.40595	0.46598E 00	OCPM
256	64	0.03763	0.13301	101	0.17587E 01	0.46108	0.61700	0.62053E 00	OCPM
256	128	0.04659	0.16025	249	0.25552E 01	0.68036	0.87957	0.90747E 00	OCPM
512	1	0.00061	0.00242	1	0.94800E-01	0.06518	0.02635	0.11118E 00	PPM
512	2	0.00092	0.00367	1	0.10891E 00	0.08479	0.04181	0.14016E 00	PPM
512	4	0.00225	0.00891	2	0.18483E 00	0.10706	0.06629	0.17687E 00	PPM
512	8	0.00357	0.01409	5	0.29272E 00	0.12987	0.10499	0.22345E 00	PPM
512	16	0.00770	0.02998	13	0.52057E 00	0.16141	0.16589	0.28314E 00	OCPM
512	32	0.01472	0.05601	36	0.90853E 00	0.20835	0.26085	0.36089E 00	OCPM
512	64	0.02232	0.08282	101	0.14739E 01	0.28318	0.40595	0.46598E 00	OCPM
512	128	0.02674	0.09781	249	0.21162E 01	0.40210	0.61700	0.62053E 00	OCPM
512	256	0.03172	0.11422	249	0.22422E 01	0.62929	0.87957	0.90747E 00	OCPM
1024	1	0.00029	0.00117	1	0.74421E-01	0.04138	0.01661	0.88215E-01	PPM
1024	2	0.00041	0.00165	1	0.83403E-01	0.05389	0.02635	0.11118E 00	PPM
1024	4	0.00092	0.00367	2	0.13722E 00	0.06776	0.04181	0.14016E 00	PPM
1024	8	0.00225	0.00891	5	0.25085E 00	0.08371	0.06629	0.17687E 00	PPM
1024	16	0.00377	0.01489	13	0.41017E 00	0.10299	0.10499	0.22345E 00	OCPM
1024	32	0.00788	0.03067	36	0.73678E 00	0.13116	0.16589	0.28314E 00	OCPM
1024	64	0.01330	0.05083	101	0.12383E 01	0.17484	0.26085	0.36089E 00	OCPM
1024	128	0.01695	0.06401	249	0.18149E 01	0.24032	0.40595	0.46598E 00	OCPM
1024	256	0.01831	0.06883	249	0.18626E 01	0.35827	0.61700	0.62053E 00	OCPM
1024	512	0.02187	0.08126	249	0.19774E 01	0.59225	0.87957	0.90747E 00	OCPM

B= 0.250 BETA= 5.000 D= 0.000

CF	CM	P*	R	N	TN/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
256	1	0.00061	0.00242	1	0.24755E 00	0.04100	0.01792	0.27250E 00	PPM
256	2	0.00092	0.00367	1	0.26905E 00	0.05651	0.03118	0.31326E 00	PPM
256	4	0.00143	0.00568	1	0.29374E 00	0.08436	0.05421	0.36043E 00	PPM
256	8	0.00276	0.01095	2	0.38519E 00	0.12674	0.09410	0.41535E 00	PPM
256	16	0.00608	0.02383	5	0.54196E 00	0.18902	0.16286	0.48039E 00	PPM
256	32	0.01282	0.04908	11	0.73705E 00	0.28317	0.28003	0.55960E 00	PPM
256	64	0.02424	0.08938	25	0.98772E 00	0.43047	0.47425	0.66353E 00	OCPM
256	128	0.04030	0.14130	58	0.12960E 01	0.67684	0.76975	0.82884E 00	OCPM
512	1	0.00029	0.00117	1	0.21409E 00	0.02361	0.01030	0.23715E 00	PPM
512	2	0.00041	0.00165	1	0.22924E 00	0.03255	0.01792	0.27250E 00	PPM
512	4	0.00061	0.00242	1	0.24755E 00	0.04879	0.03118	0.31326E 00	PPM
512	8	0.00143	0.00568	2	0.33742E 00	0.07363	0.05421	0.36043E 00	PPM
512	16	0.00283	0.01122	5	0.46498E 00	0.11023	0.09410	0.41535E 00	PPM
512	32	0.00594	0.02328	11	0.63151E 00	0.16582	0.16286	0.48039E 00	PPM
512	64	0.01183	0.04545	25	0.85467E 00	0.25131	0.28003	0.55960E 00	OCPM
512	128	0.02019	0.07543	58	0.11263E 01	0.39032	0.47425	0.66353E 00	OCPM
512	256	0.03036	0.10979	138	0.14550E 01	0.63020	0.76975	0.82884E 00	OCPM
1024	1	0.00013	0.00051	1	0.18142E 00	0.01356	0.00591	0.20639E 00	PPM
1024	2	0.00022	0.00088	1	0.20202E 00	0.01877	0.01030	0.23715E 00	PPM
1024	4	0.00029	0.00117	1	0.21409E 00	0.02814	0.01792	0.27250E 00	PPM
1024	8	0.00061	0.00242	2	0.28436E 00	0.04253	0.03118	0.31326E 00	PPM
1024	16	0.00143	0.00568	5	0.40528E 00	0.06387	0.05421	0.36043E 00	PPM
1024	32	0.00283	0.01120	11	0.54425E 00	0.09634	0.09410	0.41535E 00	PPM
1024	64	0.00578	0.02267	25	0.74020E 00	0.14594	0.16286	0.48039E 00	OCPM
1024	128	0.01066	0.04112	58	0.99037E 00	0.22444	0.28003	0.55960E 00	OCPM
1024	256	0.01639	0.06201	138	0.12844E 01	0.35593	0.47425	0.66353E 00	OCPM
1024	512	0.02139	0.07960	249	0.15251E 01	0.59321	0.76975	0.82884E 00	OCPM

F= 0.150 BETA= 0.500 D= 0.000 NO INITIAL INSPECTION

CF	CI	P*	P	N	TM/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
2	1	0.99835	0.97712	9999	0.50000E 06	1.00085	1.00000	0.99999E 34	FM
4	1	0.99835	0.97712	9999	0.50000E 06	1.00042	1.00000	0.99999E 34	FM
4	2	0.99835	0.97712	9999	0.50000E 06	1.00043	1.00000	0.99999E 34	FM
8	1	0.99835	0.97712	9999	0.50000E 06	1.00021	1.00000	0.99999E 34	FM
8	2	0.99835	0.97712	9999	0.50000E 06	1.00022	1.00000	0.99999E 34	FM
8	4	0.99835	0.97712	9999	0.50000E 06	1.00022	1.00000	0.99999E 34	FM
16	1	0.23192	0.63756	9999	0.50000E 06	0.97901	1.00000	0.99999E 34	OCPM
16	2	0.25941	0.66680	9999	0.50000E 06	0.99605	1.00000	0.99999E 34	OCPM
16	4	0.99835	0.97712	9999	0.50000E 06	1.00011	1.00000	0.99999E 34	FM
16	8	0.99835	0.97712	9999	0.50000E 06	1.00012	1.00000	0.99999E 34	FM
32	1	0.10648	0.62875	9999	0.50000E 06	0.78968	1.00000	0.99999E 34	OCPM
32	2	0.10967	0.63643	9999	0.50000E 06	0.80596	1.00000	0.99999E 34	OCPM
32	4	0.11633	0.65190	9999	0.50000E 06	0.83772	1.00000	0.99999E 34	OCPM
32	8	0.13304	0.68764	9999	0.50000E 06	0.89760	1.00000	0.99999E 34	OCPM
32	16	0.20272	0.80942	9999	0.50000E 06	0.99740	1.00000	0.99999E 34	OCPM
64	1	[illegible]	0.30083	9999	0.50000E 06	0.60171	1.00000	0.99999E 34	OCPM
64	2	[illegible]	0.30763	9999	0.50000E 06	0.61216	1.00000	0.99999E 34	OCPM
64	4	0.06433	0.30708	9999	0.50000E 06	0.63294	1.00000	0.99999E 34	OCPM
64	8	[illegible]	0.31684	9999	0.50000E 06	0.67400	1.00000	0.99999E 34	OCPM
64	16	[illegible]	0.3421[illegible]	9999	0.50000E 06	0.75375	1.00000	0.99999E 34	OCPM
64	32	[illegible]	0.41443	9999	0.50000E 06	0.90059	1.00000	0.99999E 34	OCPM
128	1	0.03357	0.21208	9999	0.50000E 06	0.44568	1.00000	0.99999E 34	OCPM
128	2	[illegible]	[illegible]	9999	0.50000E 06	0.45168	1.00000	0.99999E 34	OCPM
128	4	0.04013	0.21449	9999	0.50000E 06	0.46367	1.00000	0.99999E 34	OCPM
128	8	0.04086	0.21762	9999	0.50000E 06	0.48757	1.00000	0.99999E 34	OCPM
128	16	0.04208	0.22530	9999	0.50000E 06	0.53490	1.00000	0.99999E 34	OCPM
128	32	0.04668	0.24088	9999	0.50000E 06	0.62823	1.00000	0.99999E 34	OCPM
128	64	[illegible]	0.28988	9999	0.50000E 06	0.80606	1.00000	0.99999E 34	OCPM

T= 0.350 BETA= 1.00 D= 0.000 NO INITIAL INSPECTION

CF	CD	R*	K	N	TH/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
2	1	0.99835	0.97712	9900	0.10000E 07	29.44990	1.00000	0.99999E 34	FM
4	1	0.99835	0.97712	9900	0.10000E 07	15.22490	1.00000	0.99999E 34	FM
4	2	0.99835	0.97712	9900	0.10000E 07	15.22500	1.00000	0.99999E 34	FM
8	1	0.99835	0.97712	9900	0.10000E 07	8.11250	1.00000	0.99999E 34	FM
8	2	0.99835	0.97712	9900	0.10000E 07	8.11250	1.00000	0.99999E 34	FM
8	4	0.99835	0.97712	9900	0.10000E 07	8.11251	1.00000	0.99999E 34	FM
16	1	0.00010	0.00067	9900	0.10000E 07	3.61951	1.00000	0.99999E 34	FM
16	2	0.00010	0.00067	9900	0.10000E 07	3.68197	1.00000	0.99999E 34	FM
16	4	0.00010	0.00067	9900	0.10000E 07	3.80688	1.00000	0.99999E 34	FM
16	8	0.99835	0.97712	9900	0.10000E 07	4.55626	1.00000	0.99999E 34	FM
32	1	0.00010	0.00067	9900	0.10000E 07	1.81014	1.00000	0.99999E 34	FM
32	2	0.00010	0.00067	9900	0.10000E 07	1.84137	1.00000	0.99999E 34	FM
32	4	0.00010	0.00067	9900	0.10000E 07	1.90382	1.00000	0.99999E 34	FM
32	8	0.00010	0.00067	9900	0.10000E 07	2.02873	1.00000	0.99999E 34	FM
32	16	0.00010	0.00067	9900	0.10000E 07	2.27855	1.00000	0.99999E 34	FM
64	1	0.00010	0.00067	9900	0.10000E 07	0.20545	1.00000	0.99999E 34	OCPM
64	2	0.00010	0.00067	9900	0.10000E 07	0.22107	1.00000	0.99999E 34	OCPM
64	4	0.00010	0.00067	9900	0.10000E 07	0.25230	1.00000	0.99999E 34	OCPM
64	8	0.00010	0.00067	9900	0.10000E 07	1.01475	1.00000	0.99999E 34	FM
64	16	0.00010	0.00067	9900	0.10000E 07	1.13964	1.00000	0.99999E 34	FM
64	32	0.00010	0.00067	9900	0.10000E 07	1.38948	1.00000	0.99999E 34	FM
128	1	0.00010	0.00067	9900	0.10000E 07	0.45311	1.00000	0.99999E 34	OCPM
128	2	0.00010	0.00067	9900	0.10000E 07	0.46092	1.00000	0.99999E 34	OCPM
128	4	0.00010	0.00067	9900	0.10000E 07	0.47653	1.00000	0.99999E 34	OCPM
128	8	0.00010	0.00067	9900	0.10000E 07	0.50776	1.00000	0.99999E 34	OCPM
128	16	0.00010	0.00067	9900	0.10000E 07	0.57021	1.00000	0.99999E 34	OCPM
128	32	0.00010	0.00067	9900	0.10000E 07	0.69512	1.00000	0.99999E 34	OCPM
128	64	0.00010	0.00067	9900	0.10000E 07	0.94495	1.00000	0.99999E 34	OCPM

R= 0.150 BETA= 3.000 D= 0.000 NO INITIAL INSPECTION

CF	CH	P*	R	N	TN/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
2	1	0.21329	0.61528	1	0.69582E 00	0.94469	0.87957	0.90747E 00	PPM
4	1	0.06442	0.30744	1	0.45390E 00	0.79704	0.61700	0.68053E 00	PPM
4	2	0.21329	0.61528	1	0.69582E 00	0.94469	0.87957	0.90747E 00	PPM
8	1	0.01974	0.11730	1	0.30363E 00	0.61126	0.40595	0.46598E 00	PPM
8	2	0.06442	0.30744	1	0.45390E 00	0.79704	0.61700	0.62053E 00	PPM
8	4	0.19274	0.58803	2	0.84407E 00	1.01558	0.87957	0.90747E 00	PPM
16	1	0.00706	0.04508	1	0.21504E 00	0.43440	0.26085	0.36089E 00	PPM
16	2	0.01974	0.11730	1	0.30363E 00	0.61126	0.40595	0.46598E 00	PPM
16	4	0.06177	0.29827	2	0.56365E 00	0.80060	0.61700	0.62053E 00	PPM
16	8	0.99835	0.97712	5	0.35563E 01	1.00012	0.87957	0.90747E 00	PPM
32	1	0.00286	0.01876	1	0.15910E 00	0.29421	0.16589	0.28314E 00	PPM
32	2	0.00706	0.04508	1	0.21504E 00	0.43440	0.26085	0.36089E 00	PPM
32	4	0.02135	0.12577	2	0.39281E 00	0.58895	0.40595	0.46598E 00	PPM
32	8	0.05116	0.25931	5	0.71709E 00	0.75285	0.61700	0.62053E 00	PPM
32	16	0.11368	0.44583	13	0.13012E 01	0.98219	0.87957	0.90747E 00	PPM
64	1	0.00143	0.00942	1	0.12608E 00	0.19397	0.10499	0.22345E 00	PPM
64	2	0.00286	0.01876	1	0.15910E 00	0.29421	0.16589	0.28314E 00	PPM
64	4	0.00818	0.05189	2	0.28462E 00	0.40975	0.26085	0.36089E 00	PPM
64	8	0.02047	0.12117	5	0.52562E 00	0.52541	0.40595	0.46598E 00	PPM
64	16	0.04139	0.21986	13	0.91726E 00	0.68002	0.61700	0.62053E 00	PPM
64	32	0.08582	0.37429	36	0.16554E 01	0.90173	0.87957	0.90747E 00	PPM
128	1	0.00061	0.00403	1	0.94800E-01	0.12535	0.06629	0.17687E 00	PPM
128	2	0.00143	0.00942	1	0.12608E 00	0.19397	0.10499	0.22345E 00	PPM
128	4	0.00343	0.02241	2	0.21297E 00	0.27456	0.16589	0.28314E 00	PPM
128	8	0.00863	0.05462	5	0.39333E 00	0.35462	0.26085	0.36089E 00	PPM
128	16	0.01876	0.11212	13	0.70190E 00	0.45344	0.40595	0.46598E 00	PPM
128	32	0.03409	0.18783	36	0.12059E 01	0.60280	0.61700	0.62053E 00	OCPM
128	64	0.05754	0.28326	101	0.20333E 01	0.80948	0.87957	0.90747E 00	OCPM

H= 0.150 BETA= 0.500 D= 1.000

CF	CM	P*	K	N	TN/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
2	1	0,99835	0.97712	9999	0.50000E 06	1.51137	1,00000	0.99999E 34	FM
4	1	0,99835	0.97712	9999	0.50000E 06	1.25520	1,00000	0.99999E 34	FM
4	2	0,99835	0.97712	9999	0.50000E 06	1.25530	1,00000	0,99999E 34	FM
8	1	0,84937	0.92658	9999	0.50000E 06	1.12205	1,00000	0.99999E 34	FM
8	2	0,89002	0.93637	9999	0.50000E 06	1.12320	1,00000	0,99999E 34	FM
8	4	0,92948	0.94646	9999	0.50000E 06	1.12454	1,00000	0.99999E 34	FM
16	1	0,24270	0.64953	9999	0.50000E 06	0.96667	1,00000	0,99999E 34	OCPM
16	2	0,26590	0.67526	9999	0.50000E 06	0.98324	1,00000	0,99999E 34	OCPM
16	4	0,32997	0.72749	9999	0.50000E 06	1.01106	1,00000	0.99999E 34	FM
16	8	0,57832	0.85203	9999	0.50000E 06	1.04246	1,00000	0,99999E 34	FM
32	1	0,11678	0.45293	9999	0.50000E 06	0.76046	1,00000	0.99999E 34	OCPM
32	2	0,12034	0.46086	9999	0.50000E 06	0.77594	1,00000	0.99999E 34	OCPM
32	4	0,12830	0.47791	9999	0.50000E 06	0.80603	1,00000	0,99999E 34	OCPM
32	8	0,14857	0.51743	9999	0.50000E 06	0.86226	1,00000	0.99999E 34	OCPM
32	16	0,23265	0.63839	9999	0.50000E 06	0.95334	1,00000	0.99999E 34	OCPM
64	1	0,06810	0.31981	9999	0.50000E 06	0.57268	1,00000	0,99999E 34	OCPM
64	2	0,06883	0.32223	9999	0.50000E 06	0.58282	1,00000	0,99999E 34	OCPM
64	4	0,07012	0.32646	9999	0.50000E 06	0.60293	1,00000	0.99999E 34	OCPM
64	8	0,07403	0.33895	9999	0.50000E 06	0.64255	1,00000	0,99999E 34	OCPM
64	16	0,08373	0.36827	9999	0.50000E 06	0.71897	1,00000	0.99999E 34	OCPM
64	32	0,11916	0.45824	9999	0.50000E 06	0.85625	1,00000	0,99999E 34	OCPM
128	1	0,04268	0.22530	9999	0.50000E 06	0.42128	1,00000	0.99999E 34	OCPM
128	2	0,04286	0.22602	9999	0.50000E 06	0.42717	1,00000	0.99999E 34	OCPM
128	4	0,04331	0.22790	9999	0.50000E 06	0.43892	1,00000	0,99999E 34	OCPM
128	8	0,04456	0.23307	9999	0.50000E 06	0.46230	1,00000	0.99999E 34	OCPM
128	16	0,04648	0.24088	9999	0.50000E 06	0.50864	1,00000	0.99999E 34	OCPM
128	32	0,05189	0.26213	9999	0.50000E 06	0.59931	1,00000	0.99999E 34	OCPM
128	64	0,07012	0.32646	9999	0.50000E 06	0.76954	1,00000	0,99999E 34	OCPM

B= 0.150 BETA= 1.000 D= 1.000

LF	CM	P*	R	U	TN/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
2	1	0.00010	0.00067	9999	0.10000E 07	25.48265	1.00000	0.99999E 34	FM
4	1	0.00010	0.00067	9999	0.10000E 07	12.74166	1.00000	0.99999E 34	FM
4	2	0.00010	0.00067	9999	0.10000E 07	12.99148	1.00000	0.99999E 34	FM
8	1	0.00010	0.00067	9999	0.10000E 07	6.37116	1.00000	0.99999E 34	FM
8	2	0.00010	0.00067	9999	0.10000E 07	6.49607	1.00000	0.99999E 34	FM
8	4	0.00010	0.00067	9999	0.10000E 07	6.74589	1.00000	0.99999E 34	FM
16	1	0.00010	0.00067	9999	0.10000E 07	3.18592	1.00000	0.99999E 34	FM
16	2	0.00010	0.00067	9999	0.10000E 07	3.24837	1.00000	0.99999E 34	FM
16	4	0.00010	0.00067	9999	0.10000E 07	3.37328	1.00000	0.99999E 34	FM
16	8	0.00010	0.00067	9999	0.10000E 07	3.62310	1.00000	0.99999E 34	FM
32	1	0.00010	0.00067	9999	0.10000E 07	1.59329	1.00000	0.99999E 34	FM
32	2	0.00010	0.00067	9999	0.10000E 07	1.62452	1.00000	0.99999E 34	FM
32	4	0.00010	0.00067	9999	0.10000E 07	1.68697	1.00000	0.99999E 34	FM
32	8	0.00010	0.00067	9999	0.10000E 07	1.81188	1.00000	0.99999E 34	FM
32	16	0.00010	0.00067	9999	0.10000E 07	2.00171	1.00000	0.99999E 34	FM
64	1	0.00010	0.00067	9999	0.10000E 07	0.79698	1.00000	0.99999E 34	OCPM
64	2	0.00010	0.00067	9999	0.10000E 07	0.81259	1.00000	0.99999E 34	OCPM
64	4	0.00010	0.00067	9999	0.10000E 07	0.84382	1.00000	0.99999E 34	OCPM
64	8	0.00010	0.00067	9999	0.10000E 07	0.90628	1.00000	0.99999E 34	OCPM
64	16	0.00010	0.00067	9999	0.10000E 07	1.03119	1.00000	0.99999E 34	FM
64	32	0.00010	0.00067	9999	0.10000E 07	1.28101	1.00000	0.99999E 34	FM
128	1	0.00010	0.00067	9999	0.10000E 07	0.39882	1.00000	0.99999E 34	OCPM
128	2	0.00010	0.00067	9999	0.10000E 07	0.40663	1.00000	0.99999E 34	OCPM
128	4	0.00010	0.00067	9999	0.10000E 07	0.42224	1.00000	0.99999E 34	OCPM
128	8	0.00010	0.00067	9999	0.10000E 07	0.45347	1.00000	0.99999E 34	OCPM
128	16	0.00010	0.00067	9999	0.10000E 07	0.51593	1.00000	0.99999E 34	OCPM
128	32	0.00010	0.00067	9999	0.10000E 07	0.64084	1.00000	0.99999E 34	OCPM
128	64	0.00010	0.00067	9999	0.10000E 07	0.89066	1.00000	0.99999E 34	OCPM

B= 0.150 BETA= 3.000 D= 1.000

CF	CM	P*	R	N	TN/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
2	1	0.36395	0.75103	1	0.85972E 00	1.42007	0.87957	0.90747E 00	PPM
4	1	0.18004	0.56958	1	0.65324E 00	1.07863	0.61700	0.62053E 00	PPM
4	2	0.32139	0.72104	1	0.81657E 00	1.15172	0.87957	0.90747E 00	PPM
8	1	0.06257	0.30106	1	0.44936E 00	0.82743	0.40595	0.46598E 00	PPM
8	2	0.13332	0.48821	1	0.58572E 00	0.92519	0.61700	0.62053E 00	PPM
8	4	0.25658	0.66405	2	0.94083E 00	1.08101	0.87957	0.90747E 00	PPM
16	1	0.02072	0.12250	1	0.30867E 00	0.60703	0.26085	0.36089E 00	PPM
16	2	0.04146	0.22014	1	0.39031E 00	0.71770	0.40595	0.46598E 00	PPM
16	4	0.09350	0.39555	2	0.65085E 00	0.84033	0.61700	0.62053E 00	PPM
16	8	0.19054	0.58495	5	0.11407E 01	1.02497	0.87957	0.90747E 00	PPM
32	1	0.00772	0.04914	1	0.22164E 00	0.42281	0.16589	0.28314E 00	PPM
32	2	0.01390	0.08535	1	0.26988E 00	0.52076	0.26085	0.36089E 00	PPM
32	4	0.03183	0.17739	2	0.44955E 00	0.62203	0.40595	0.46598E 00	PPM
32	8	0.06663	0.31493	5	0.78524E 00	0.75148	0.61700	0.62053E 00	PPM
32	16	0.14354	0.50812	13	0.14143E 01	0.95093	0.87957	0.90747E 00	PPM
64	1	0.00321	0.02096	1	0.16522E 00	0.28369	0.10499	0.22345E 00	PPM
64	2	0.00537	0.03466	1	0.19629E 00	0.35821	0.16589	0.28314E 00	PPM
64	4	0.01183	0.0735[illegible]	2	0.32215E 00	0.43592	0.26085	0.36089E 00	PPM
64	8	0.02625	0.15060	5	0.57159E 00	0.52623	0.40595	0.46598E 00	PPM
64	16	0.05008	0.25513	13	0.97889E 00	0.66277	0.61700	0.62053E 00	PPM
64	32	0.10518	0.42559	36	0.17778E 01	0.86201	0.87957	0.90747E 00	OCPM
128	1	0.00143	0.00942	1	0.12608E 00	0.18587	0.06629	0.17687E 00	PPM
128	2	0.00225	0.01477	1	0.14670E 00	0.23816	0.10499	0.22345E 00	PPM
128	4	0.00488	0.03156	2	0.23946E 00	0.29354	0.16589	0.28314E 00	PPM
128	8	0.01093	0.06825	5	0.42570E 00	0.35603	0.26085	0.36089E 00	PPM
128	16	0.02253	0.13191	13	0.74657E 00	0.44285	0.40595	0.46598E 00	PPM
128	32	0.03972	0.2127[illegible]	36	0.12701E 01	0.58125	0.61700	0.62053E 00	OCPM
128	64	0.06792	0.31924	101	0.21528E 01	0.77405	0.87957	0.90747E 00	OCPM

B= 0.150 BETA= 3.000 D= 1.000

CF	CM	P*	R	N	TN/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
256	1	0.00061	0.00403	1	0.94800E-01	0.12009	0.04181	0.14016E 00	PPM
256	2	0.00092	0.00610	1	0.10891E 00	0.15532	0.06629	0.17687E 00	PPM
256	4	0.00225	0.01477	2	0.18483E 00	0.19275	0.10499	0.22345E 00	PPM
256	8	0.00481	0.03115	5	0.32353E 00	0.23491	0.16589	0.28314E 00	PPM
256	16	0.01043	0.06535	13	0.57637E 00	0.29065	0.26085	0.36089E 00	PPM
256	32	0.01971	0.11716	36	0.10021E 01	0.37527	0.40595	0.46598E 00	OCPM
256	64	0.03120	0.17447	101	0.16505E 01	0.50387	0.61700	0.62053E 00	OCPM
256	128	0.04366	0.22934	249	0.24992E 01	0.70180	0.87957	0.90747E 00	OCPM
512	1	0.00029	0.00195	1	0.74421E-01	0.07680	0.02635	0.11118E 00	PPM
512	2	0.00061	0.00403	1	0.94800E-01	0.10025	0.04181	0.14016E 00	PPM
512	4	0.00092	0.00610	2	0.13722E 00	0.12466	0.06629	0.17687E 00	PPM
512	8	0.00225	0.01477	5	0.25085E 00	0.15243	0.10499	0.22345E 00	PPM
512	16	0.00491	0.03170	13	0.44786E 00	0.18839	0.16589	0.28314E 00	PPM
512	32	0.01013	0.06355	36	0.80137E 00	0.23991	0.26085	0.36089E 00	OCPM
512	64	0.01702	0.10266	101	0.13452E 01	0.31923	0.40595	0.46598E 00	OCPM
512	128	0.02221	0.13026	249	0.19879E 01	0.43550	0.61700	0.62053E 00	OCPM
512	256	0.02883	0.16320	249	0.21708E 01	0.64657	0.87957	0.90747E 00	OCPM
1024	1	0.00017	0.00116	1	0.62508E-01	0.04890	0.01661	0.88215E-01	PPM
1024	2	0.00022	0.00146	1	0.67561E-01	0.06367	0.02635	0.11118E 00	PPM
1024	4	0.00061	0.00403	2	0.11244E 00	0.08025	0.04181	0.14016E 00	PPM
1024	8	0.00143	0.00942	5	0.21559E 00	0.09888	0.06629	0.17687E 00	PPM
1024	16	0.00225	0.01477	13	0.34494E 00	0.12102	0.10499	0.22345E 00	PPM
1024	32	0.00514	0.03322	36	0.63874E 00	0.15266	0.16589	0.28314E 00	OCPM

R= 0.250 LETA=15.000 D= 0.00000 INITIAL INSPECTION

LF	CH	P*	E	N	TN/MTBF	C*OCPM/CFM	C*PPM/CFM	T*PPM/MTBF	BEST
256	8	0.00041	0.00165	1	0.66256E 00	0.04867	0.04474	0.73730E 00	PPM
256	16	0.00092	0.00367	1	0.71191E 00	0.09365	0.08613	0.76608E 00	PPM
256	32	0.00225	0.00891	2	0.77392E 00	0.17858	0.16550	0.79734E 00	PPM
256	64	0.00422	0.01663	4	0.82281E 00	0.33283	0.31664	0.83377E 00	PPM
256	128	0.01265	0.04845	8	0.91203E 00	0.60918	0.59822	0.88351E 00	PPM
512	1	0.00010	0.00040	1	0.63349E 00	0.00387	0.00324	0.63624E 00	PPM
512	2	0.00010	0.00040	1	0.63349E 00	0.00695	0.00625	0.65988E 00	PPM
512	4	0.00010	0.00040	1	0.63349E 00	0.01311	0.01205	0.68461E 00	PPM
512	8	0.00017	0.00070	1	0.65216E 00	0.02526	0.02322	0.71034E 00	PPM
512	16	0.00041	0.00165	1	0.68254E 00	0.04867	0.04474	0.73730E 00	PPM
512	32	0.00092	0.00367	2	0.73836E 00	0.09296	0.08613	0.76608E 00	PPM
512	64	0.00225	0.00891	4	0.80267E 00	0.17471	0.16550	0.79734E 00	PPM
512	128	0.00414	0.01654	8	0.85982E 00	0.32557	0.31664	0.83377E 00	PPM
512	256	0.01146	0.04408	16	0.94100E 00	0.59226	0.59822	0.88351E 00	OCPM
1024	1	0.00010	0.00040	1	0.63349E 00	0.00233	0.00168	0.61341E 00	PPM
1024	2	0.00010	0.00040	1	0.63349E 00	0.00387	0.00324	0.63624E 00	PPM
1024	4	0.00010	0.00040	1	0.63349E 00	0.00695	0.00625	0.65988E 00	PPM
1024	8	0.00010	0.00040	1	0.63349E 00	0.01311	0.01205	0.68461E 00	PPM
1024	16	0.00017	0.00070	1	0.65216E 00	0.02526	0.02322	0.71034E 00	PPM
1024	32	0.00041	0.00165	2	0.70790E 00	0.04831	0.04474	0.73730E 00	PPM
1024	64	0.00092	0.00367	4	0.76586E 00	0.09093	0.08613	0.76608E 00	PPM
1024	128	0.00225	0.00891	8	0.83249E 00	0.16974	0.16550	0.79734E 00	PPM
1024	256	0.00412	0.01586	16	0.89036E 00	0.31357	0.31664	0.83377E 00	OCPM
1024	512	0.01010	0.03905	32	0.96247E 00	0.57568	0.59822	0.88351E 00	OCPM

16. A MARKOV MODEL COMPARING CONTINUOUS MONITORING WITH INTERVAL INSPECTION

16.1 INTRODUCTION

This model is the subject of a paper read at the Second National Reliability Conference(3.231). It developed out of contact with the Central Electricity Generating Board who needed a method of deciding what form of monitoring should be fitted at manned and especially remote and unmanned generating stations. Monitors for generating and other plant are nowadays becoming quite sophisticated, using microprocessors and mini-computers which are supplied with many operating parameters such as temperatures and pressures and which produce analyses of faults as they occur, offer advice or initiate action to minimise the effect of faults rather than simply raising an alarm.

Continuous condition monitoring,(ccm), of vital and expensive equipment is gaining popularity over planned and on-condition preventive maintenance policies(ppm,ocpm). These models provide a basis for cost comparison between continuous monitoring and inspections at constant rate. Frequently, but not always, the total cost of a failure or maintenance stoppage is proportional to the downtime of the equipment, because the cost of lost production plus maintenance labour is much greater than the fixed costs of failure repairs and maintenance actions. A monitor which gives warning of impending failure allows work to be planned and so reduces downtime, especially if several jobs can be done during one shutdown. There is always a risk though, in leaving an impending failure to a convenient time that a full failure will intervene at higher cost.

Markov state transition rate matrices form the basis of analytical calculations of steady state availability, mean and variance of operating times between periods of downtime and mean downtime for generalised ccm and ocpm systems which include random delay in starting maintenance known to be required and monitors subject to failure and repair. Costs are compared firstly assuming them proportional to downtime and secondly for fixed mean failure and maintenance costs. Methods of optimising the inspection interval are discussed for the ocpm case. Special and limiting cases which are separately treated include perfect monitors and zero delay in beginning maintenance. An interactive BASIC computer program was written. During debugging the special cases detailed arose as those which produced incorrect answers when certain transition rates reached limiting values. Although the models detailed have all constant transition rates, the versatility of the modelling method could be extended by the use of dummy states.

The matrix algebra has been kept simple. Alternative methods of analysis involving Laplace Transforms and the Gauss-Jordan method of matrix inversion were rejected because failure rates tend to be very small both absolutely and in relation to repair rates. In these circumstances standard computer subroutines tend to give inaccurate results, this being particularly true of the matrix inversion and determinant routine provided with BASIC-PLUS. Algebraically the matrix methods are not new, see Sandler(3.208) or Singh and Billinton (4.69) but the application and the optimisation are novel.

16.2 Definitions and Assumptions

16.2.1 Definition

Under a ccm policy the system consists of an equipment to be monitored and a monitor which gives continuous assessment of the equipment's condition. Both equipment and monitor are subject to failure and repair. The system is always in one of the six states S_0 to S_5 defined as follows

S_0 - Equipment and monitor both operating, no indication of impending failure.

S_1 -As S_0 but impending failure indicated. S_1 is entered from S_0 at rate λ_1.

S_2 - Monitor failed, equipment operating. S_2 is entered from S_0 at rate λ_c.

S_3 - Equipment under ocpm. Monitor available but switched off. S_3 is entered from S_1 at rate λ_2 and left for S_0 at rate μ_1.

S_4 - Equipment failed Monitor switched off. S_4 is entered from S_1 at rate λ_3 and left for S_0 at rate μ_2.

S_5 - Equipment and Monitor both failed. S_5 is entered from S_2 at rate $\lambda_u = \lambda_1\lambda_3/(\lambda_1+\lambda_3)$.

Under the alternative ocpm policy there is no permanent monitor but the condition of the equipment is assessed at random intervals and if maintenance is seen to be required this is performed after a random delay (for planning or until the next scheduled shutdown). During this delay failure may occur. States for the ocpm policy are defined as follows.

S_6 - Equipment operating, no impending failure indication. S_6 to S_7 at rate λ_1.

S_7 - Equipment operating, indication of impending failure available on inspection. S_7 to S_8 at rate μ_3 and S_{10} at λ_3.

S_8 - Equipment operating. Impending failure detected at last inspection but ocpm not yet started. S_8 to S_9 at a rate λ_2 or to S_{10} at λ_3.

S_9 - Equipment stopped for ocpm. S_9 to S_6 at μ_1 (maintenance rate)

S_{10}- Equipment stopped for failure repair. S_{10} to S_6 at μ_2 (repair rate)

A cycle runs from one failure or maintenance action to the next.

16.2.2. Assumptions

(1) All transition rates between states are small and constant i.e. all time distributions for failures, repairs, delays and inspections are of negative exponential form. This allows the use of Markov theory in the models and implies that the probability of two state transitions in unit time is negligible. (If transition rates are too large the unit of time can be reduced).

(2) There is always indication of impending failure available at the equipment for a period before it fails. (The warning may not be detected or heeded but it is always present for a time before failure).

(3) The system spends most of its time in S_0 (or S_6) and operates unless under maintenance or repair.

(4) If the monitor indicates impending failure but itself fails before maintenance starts, the indication is ignored as probably due to monitor malfunction.

(5) The monitor cannot fail in S_3 or S_4.

(6) The monitor under ccm has a constant average total running cost rate c_c which includes its direct repair and maintenance costs.

(7) Inspections under the alternative, ocpm policy each incur a fixed cost C_I and do not affect availability.

(8) Failure costs more on average than ocpm.

6.2.3. Description of Failure, Repair and Inspection Rates

The physical meanings of the transition rates are as follows.

λ_u is the overall unmaintained (base,underlying) failure rate.

λ_1 is the reciprocal of the mean time for indication of impending failure to become available if an inspection is made or the monitor is operating.

λ_2 is the reciprocal of the mean administrative or planning delay in taking maintenance action measured from the time of detection by monitor or inspection.

λ_3 is the reciprocal of the mean time from indication of impending failure becoming available to failure assuming no preventive action.

μ_1 Reciprocal of mean time to maintain.

μ_2 Reciprocal of mean time to repair a failure.

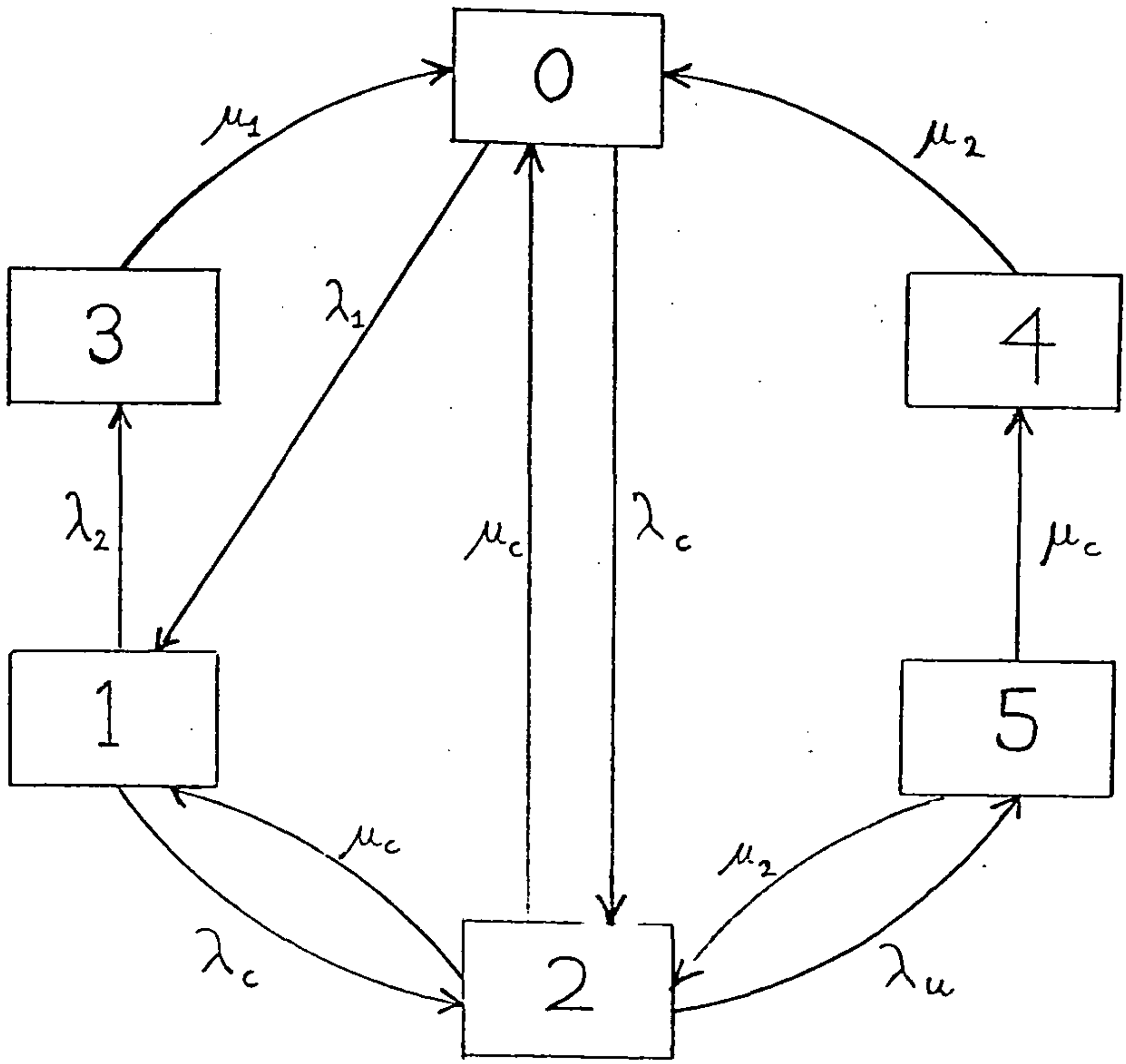


Figure 16.1 ccm Model State Transition Rate Diagram

μ_3 Reciprocal of mean time between inspections.

Now $1/\lambda_u$ is the base mtbf therefore

$$1/\lambda_u = 1/\lambda_1 + 1/\lambda_3$$

$$\lambda_u = \lambda_1\lambda_3/(\lambda_1 + \lambda_3) \qquad (16.0)$$

16.3 Continuous Monitoring Model (ccm)

16 3.1 Transition Rate Matrix

The transition rates between the 6 states S_0 - S_5 are shown in Equation (1). The matrix Q_{ccm} consists of elements which are the

conditional mean transition rates from the row to the column state given that the system is in the row state.

$$Q_{ccm} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{cccccc} 1-\lambda_1-\lambda_c & \lambda_1 & \lambda_c & 0 & 0 & 0 \\ 0 & 1-\lambda_2-\lambda_3-\lambda_c & \lambda_c & \lambda_2 & \lambda_3 & 0 \\ \mu_c & 0 & 1-\mu_c-\lambda_u & 0 & 0 & \lambda_u \\ \mu_1 & 0 & 0 & 1-\mu_1 & 0 & 0 \\ \mu_2 & 0 & 0 & 0 & 1-\mu_2 & 0 \\ 0 & 0 & \mu_2 & 0 & \mu_c & 1-\mu_2-\mu_c \end{array}\right] \end{array}$$

(16.1)

16.3.2. Steady State Availability

To find the long-term cost-rate of the ccm policy it is first necessary to find the fractions of time (P_i, i = 0...5) spent in each state. P(t) is the vector of state probabilities at time t and $\dot{P}(t)$ its time derivative.

$$\dot{P}(t) = Q^T_{ccm} \cdot P(t) \qquad (16.2)$$

For the steady state $\dot{P}(t) = (0)$ and simultaneous differential equations may be formed as follows. Consider a small time increment δt. The probability of S_0 at $(t + \delta t)$ consists of the probability of S_0 at t, and no change, plus the probability of entering S_0 from another state in δt. i.e.

$P_o(t + \delta t) = P_o(t).(1-\lambda_1\delta t-\lambda_c\delta t) + P_2(t)\mu_c\delta t+P_3(t)\mu_1\delta t+P_4(t)\mu_2\delta t.$

But

$\dot{P}_o(t)=P_o(t)\ (-\lambda_1-\lambda_c)+P_2(t)\mu_c+P_3(t)\mu_1+P_4(t)\mu_2$

because as $t\to 0, \{P_o(t+\delta t)-P_o(t)\}/\delta t\to \dot{P}_o(t)$ (16.3)

In the steady state as $t\to\infty$, $\dot{P}_o(t)$ and all other $\dot{P}_i(t)$ tend to zero. For this condition there is a system of equations which in matrix form may be written

$$\left.\begin{array}{l} (Q_{ccm} - I)^T.\ P = (0) \\ \sum_{i=0}^{5} P_i = 1 \end{array}\right\} \qquad (16.4)$$

The full array of equation 16.4 is

$$\left.\begin{array}{l} -(\lambda_1+\lambda_c)P_o + \mu_c P_2+ \mu_1 P_3+ \mu_2 P_4 = 0 \\ \lambda_1 P_o-(\lambda_2+\lambda_3 +\lambda_c)P_1 = 0 \\ \lambda_c P_o + \lambda_c P_1 - (\mu_c+\lambda_u)P_2+\lambda_2 P_5 = 0 \\ \lambda_2 P_1-\mu_1 P_3 = 0 \\ \lambda_3 P_1-\mu_2 P_4+\mu_c P_5 = 0 \\ \lambda_u P_2- (\mu_2 + \mu_c)P_5 = 0 \\ P_o+ P_1 + P_2+ P_3 + P_4 + P_5 = 1 \end{array}\right] \qquad (16.4a)$$

These equations can be solved by substituting ratios of the form P_i/P_3 into the last equation of 16.4a divided by P_3.

To shortcut this procedure let

$q=\lambda_u\lambda_c\lambda_1(\lambda_c+\lambda_1+\lambda_2+\lambda_3)/\lambda_1\lambda_2\ (\mu_c+\lambda_u).(\mu_c+\mu_2)\ -\mu_2\lambda_u$ (16.5)

$v = \mu_1(\lambda_c+\lambda_2+\lambda_3)/\lambda_1\lambda_2) + \mu_1/\lambda_2 + (\mu_c+\mu_2)q/\lambda_u +1+ (\lambda_3\mu_1+q\lambda_2)/\mu_2\lambda_2 +q$

(16.6)

then

$$\left.\begin{array}{l} P_o = \mu_1(\lambda_c+\lambda_2+\lambda_3)/v\lambda_1\lambda_2,\ P_1 = \mu_1/v\lambda_2 \\ P_2 = q(\mu_c+\mu_2)/v\lambda_u,\quad P_3 = 1/v \\ P_4 = \lambda_3\mu_1/v\lambda_2\mu_2 + \mu_2 q/\mu_2 v,\quad P_5 = q/v \end{array}\right\} \qquad (16.7)$$

S_0, S_1 S_2 are 'up' states and S_3, S_4, S_5 are 'down' states. The long-term running availability of the equipment is

$$A_{ccm} = P_o + P_1 + P_2 = 1 - (P_3 + P_4 + P_5) \quad (16.8)$$

16.3.3. Mean Up and Down Time (General)

Rau (3.197), (1970) gives the following analysis based on the canonical form of the matrix Q with the down states made absorbing. If necessary, the matrix is rearranged so that transitions between up states appear top left, transitions from up to down states top right, a zero matrix bottom left and an identity matrix bottom right, thus:

$$\begin{array}{c|c} & \text{up} \quad \text{down} \\ \begin{array}{c}\text{up}\\ \text{down}\end{array} & \left[\begin{array}{c|c} Z & X \\ \hline 0 & I \end{array}\right] \end{array}$$

For the ccm model the matrix is

$$Q'_{ccm} = \left[\begin{array}{ccc|ccc} 1-\lambda_1-\lambda_c & \lambda_1 & \lambda_c & 0 & 0 & 0 \\ 0 & 1-\lambda_2-\lambda_3-\lambda_c & \lambda_c & \lambda_2 & \lambda_3 & 0 \\ \mu_c & 0 & 1-\mu_c-\lambda_u & 0 & 0 & \lambda_u \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \quad (16.9)$$

One of the few conditions on the validity of this method is that all the states should communicate.

The fundamental matrix 'N' is defined by

$$N = I + Z N$$

$$\text{giving} \quad N = (I-Z)^{-1} \quad (16.10)$$

The element $n_{i,j}$ of N is the mean time spent in S_j before absorption given that the cycle starts in S_i. The sum of the top row is therefore the mean time to first failure from S_o at t = 0. As an estimate of the mtbf in the ccm case this sum is only an approximation because in a minority of cases repair will be to S_2. It is possible to allow for this by calculating the proportion of such repairs and allowing for the change in mean up time but for credible values of the failures and repair rates the errors are relatively small.

The conditional probabilities of a particular cycle ending in downstate j having begun in upstate i are given by the matrix:

$$B = NX \tag{16.11}$$

Thus mtbf and mttr are given by:

$$\theta \approx \sum_{j=0}^{s} n_{o,i} \tag{16.12}$$

$$\phi \approx \sum_{j=s+1}^{s+r} b_{o,j}/\mu_j \tag{16.13}$$

where μ_j is the repair rate from state j to an upstate.

For the ccm model

$$I\text{-}Z_{ccm} = \begin{bmatrix} \lambda_1+\lambda_c & -\lambda_c & -\lambda_c \\ 0 & \lambda_2+\lambda_3+\lambda_c & -\lambda_c \\ -\mu_c & 0 & \mu_c+\lambda\mu \end{bmatrix} \tag{16.14}$$

In general N may be found from

$$N = (I\text{-}Z)^{-1} = \text{adj}\,(I\text{-}Z)/\left|I\text{-}Z\right|$$

where adj (I-Z) is the transposed matrix of signed co-factors such that

$$\alpha_{i,j} = (-1)^{i+j+2} \left| M_{j,i} \right|$$

where $\left| M_{j,i} \right|$ is the minor determinant of (I-Z) with row j and column i removed.

The variances of up and down times can be found by an extension of the above methods in which $(I-Z)^2$ is substituted for (I-Z) in otherwise similar calculations. These variances will not be worked out as they are not needed in the development of cost equations but it is interesting to note that in general the variance is less than or equal to the square of the mean. When it is less the system as a whole has in a sense acquired an increasing hazard rate by virtue of the surveillance of inspections.

It is possible to find mean times to a full failure (S_4, S_5, or S_{10}) by regarding S_3 and S_9 as states.

16.3.4. Mean Up and Down Times for ccm Model

Applying this to the ccm model

$$\left| I-Z_{ccm} \right| = z = wxy + \lambda_c\mu_c(\lambda_1+x)$$

where

$$w = \lambda_1+\lambda_c, \quad x = \lambda_2+\lambda_3+\lambda_c, \quad y = \mu_c+\lambda_u \tag{16.15}$$

then

$$z.N_{ccm} = \begin{bmatrix} xy & \lambda_1 y & \lambda_c(\lambda_1+x) \\ \mu_c\lambda_c & wy-\mu_c\lambda_c & w\lambda_c \\ x\mu_c & \lambda_1\mu_c & wx \end{bmatrix}$$

The mttf from S_o and approximate system mtbf is given by

$$\theta_{ccm} \approx (\mu_c+y)(\lambda_1+x) \Big/ \left[wxy + \lambda_c\mu_c(\lambda_1+x) \right] \tag{16.16}$$

$$z.B_{ccm} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 3 & 4 & 5 \end{array} \\ \begin{bmatrix} \lambda_1\lambda_2 y & \lambda_1\lambda_3 y & \lambda_\mu\lambda_c(\lambda_1+x) \\ \lambda_2(wy-\mu_c\lambda_c) & \lambda_3(wy-\mu_c\lambda_c) & w\lambda c\ \lambda_u \\ \lambda_1\lambda_2\mu_c & \lambda_1\lambda_3\mu_c & wx\lambda_u \end{bmatrix} \end{array} \tag{16.17}$$

An approximate system mttr is found by adding the terms in the top row of matrix B (i.e. 16.17 divided by the determinant z) weighted in proportion to the appropriate mean repair times $1/\mu_1$, $1/\mu_2$. This is so because matrix B consists of the conditional probabilities of failure in S_3, S_4, S_5 given a start in S_0, S_1, S_2. A cycle cannot start in S_1 but it can occasionally start in S_2 if the equipment is repaired before the monitor in the previous cycle. The probability of a cycle starting in S_2 is small and the difference this makes to the repair time in the next cycle is also very small. The approximation on the assumption of all cycles starting in S_0 is therefore a good one.

$$\emptyset_{ccm} \approx \lambda_1\lambda_2 y/Z\mu_1 + \lambda_1\lambda_3 y/Z\mu_2 + \lambda_u\lambda_c(\lambda_1+x)/Z\mu_2$$
$$\approx \left(\lambda_1\lambda_2 y\mu_2 + \lambda_1\lambda_3 y\mu_1 + \lambda_u\lambda_c(\lambda_1+x)\mu_1\right)\Big/ Z\,\mu_1\mu_2 \qquad (16.18)$$

16.4 Inspection /On-Condition Maintenance Model (ocpm)

16.4.1 Transition Rate Matrix

The transition rate matrix is as follows, where $\lambda_1,\lambda_2,\lambda_3,\ \mu_1,\mu_2$ are as in the ccm model and $1/\mu_3$ is the mean time between inspections.

$$Q_{ocpm} = \begin{array}{c} \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} \begin{array}{c} \begin{array}{ccccc} 6 & 7 & 8 & 9 & 10 \end{array} \\ \begin{bmatrix} 1-\lambda_1 & \lambda_1 & 0 & 0 & 0 \\ 0 & 1-\mu_3-\lambda_3 & \mu_3 & 0 & \lambda_3 \\ 0 & 0 & 1-\lambda_2-\lambda_3 & \lambda_2 & \lambda_3 \\ \mu_1 & 0 & 0 & 1-\mu_1 & 0 \\ \mu_2 & 0 & 0 & 0 & 1-\mu_2 \end{bmatrix} \end{array} \qquad (16.19)$$

The matrix form of the steady state probability equation is

$$(Q_{ocpm}-1)^T . P = (0) \qquad (16.20)$$

From the full array of equation (16.20) the following ratios arise

$$P_7/P_6 = \lambda_1/(\mu_3+\lambda_3), \quad P_7/P_8 = (\lambda_2+\lambda_3)/\mu_3 \quad P_9/P_8 = \lambda_2/\mu_1$$

$$P_6/P_8 = (\mu_3+\lambda_3),\ (\lambda_2+\lambda_3)/\lambda_1\mu_3,\ P_{10}/P_8 = (\lambda_3/\mu_2)\left[1+(\lambda_2+\lambda_3)/\mu_3\right]$$

$$1/P_8 = (\mu_3+\lambda_3).(\lambda_2+\lambda_3)/\lambda_1\mu_3 + (\lambda_2+\lambda_3)/\mu_3+\lambda_2/\mu_1+(\lambda_3/\mu_2)\left[1+(\lambda_2+\lambda_3)/\mu_3\right]$$

(16.21)

S_6,S_7,S_8 are 'up' states, S_9,S_{10} are 'down' states. The steady state system availability is

$$A_{ocpm} = P_6+P_7+P_8 = 1-(P_9+P_{10}) \qquad (16.22)$$

which may be evaluated from equations (16.21).

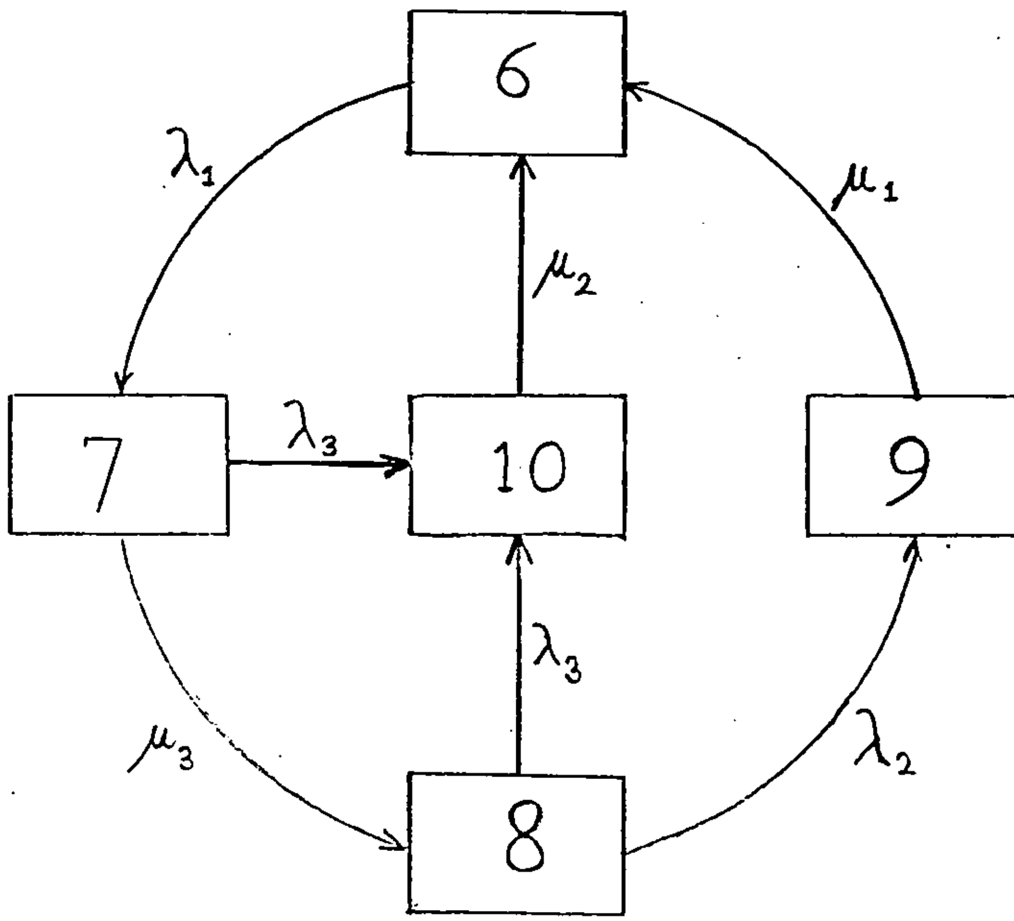


Figure 16.2 ocpm Model State Transition Rate Diagram

16.4.2. Mean Up and Down Times for ocpm Model

$$I-Z_{ocpm} = \begin{bmatrix} \lambda_1 & -\lambda_1 & 0 \\ 0 & \mu_3+\lambda_3 & -\mu_3 \\ 0 & 0 & \lambda_2+\lambda_3 \end{bmatrix} \qquad (16.23)$$

In this case

$$z_{ocpm} = \left| I-Z_{ocpm} \right| = \lambda_1(\mu_3+\lambda_3)(\lambda_2+\lambda_3)$$

$$z_{ocpm} \cdot N_{ocpm} = \begin{bmatrix} (\mu_3+\lambda_3)(\lambda_2+\lambda_3) & \lambda_1(\lambda_2+\lambda_3) & \lambda_1\mu_3 \\ 0 & \lambda_1(\lambda_2+\lambda_3) & \lambda_1\mu_3 \\ 0 & 0 & \lambda_1\mu_3 \end{bmatrix} \qquad (16.24)$$

It follows that

$$\theta_{ocpm} = 1/\lambda_1 + 1/(\mu_3+\lambda_3) + \mu_3/(\mu_3+\lambda_3)(\lambda_2+\lambda_3) \qquad (16.25)$$

Because repair is always to S_6 the calculation is precise.

The mtbf can be approached in another way. All cycles start in S_6 and proceed to S_7 in average time $1/\lambda_1$. Similarly all cycles contain an average sojourn in S_7 of $1/(\mu_3+\lambda_3)$. From there, a proportion of cycles proceed direct to a down state but a complementary proportion $\mu_3/(\mu_3+\lambda_3)$ spends an average time $1/(\lambda_2+\lambda_3)$ in S_8. The ccm model mttf can be worked out in much the same manner.

$$B_{ocpm} = \begin{array}{c|cc} & 9 & 10 \\ \hline 6 & \lambda_2\mu_3/(\mu_3+\lambda_3)(\lambda_2+\lambda_3) & \lambda_3/(\mu_3+\lambda_3)+\lambda_3\mu_3/(\mu_3+\lambda_3)(\lambda_2+\lambda_3) \\ 7 & \lambda_2\mu_3/(\mu_3+\lambda_3)(\lambda_2+\lambda_3) & \lambda_3/(\mu_3+\lambda_3)+\lambda_3\mu_3/(\mu_3+\lambda_3)(\lambda_2+\lambda_3) \\ 8 & \lambda_2\mu_3/(\mu_3+\lambda_3)(\lambda_2+\lambda_3) & \lambda_3\mu_3/(\mu_3+\lambda_3)(\lambda_2+\lambda_3) \end{array} \qquad (16.26)$$

Therefore

$$\emptyset_{ocpm} = \lambda_2\mu_3/(\mu_3+\lambda_3)(\lambda_2+\lambda_3)\mu_1$$

$$+ \lambda_3 \left[1 + 1/(\lambda_2+\lambda_3)\right] \Big/ \mu_2(\mu_3+\lambda_3) \qquad (16.27)$$

$\emptyset_{ocpm}$ is also precise because repair is always to S_6.

16.5 Cost Comparison

16.5.1 Cost Proportional to Downtime

If costs are proportional to downtime there will be a cost per unit downtime c_d which includes both lost production and maintenance or repair costs.

a) Continuous Monitoring. The overall rate of expenditure for a ccm policy is

$$c_{ccm} = (1-A_{ccm})\, c_d + c_c \qquad (16.28)$$

Where c_c is the total ownership cost-rate of the monitor including its maintenance and repair costs.

b) Inspection/ocpm Policy. The cost-rate of an ocpm policy varies with the inspection rate μ_3. For any set of values for the other μ,λ parameters of the problem there will be an optimum value μ_3*of μ_3. For a fair comparison with ccm it will usually be necessary to find μ_3*. The lowest cost-rate possible for an ocpm policy, c^*_{ocpm} can then be found. Remembering that A_{ocpm} is a function of μ_3.

$$c_{ocpm} = (1-A_{ocpm})c_d+\mu_3 C_I \qquad (16.29)$$

where C_I is the cost of a single inspection

Substituting from equation (16.21)

$$c_{ocpm}=c_d(a\mu_3+b)/(k\mu_3+h) +C_I\ \mu_3 \qquad (16.30)$$

where $a = \lambda_2/\mu_1+\lambda_3/\mu_2$, $b= (\lambda_3/\mu_2).(\lambda_2+\lambda_3)$, $k=(\lambda_2+\lambda_3)/\lambda_1+1+a$,

$h = b(1+\mu_2/\lambda_1)+\lambda_2+\lambda_3$

Then differentiate, equate to zero to find

$$\mu^*_3 = \left[-F\pm\sqrt{(F^2 - 4EG)}\right] \Big/ 2E \qquad (16.31)$$

where $E = C_I\ k^2/c_d$, $F = 2C_I\ k/c_d$, $G = C_I h^2/c_d + ah-bk$

Then substitute μ^*_3 in Equation (16.30) to find c^*_{ocpm}.

16.5.2. Fixed Failure and Maintenance Costs

If the costs are not proportional to downtime then availability is not the prime consideration and a different approach is required see §15 above and Jardine (3.122). Let the average total cost of a single failure, C_F, and of a single maintenance action, C_M, be invariable. Equations can then be formed for the average cycle cost C and time T of each policy. Policy cost-rates can be found by dividing C by T.

For either policy the cycle time is either

$$T = \theta + \phi \quad \text{or} \quad T = \theta \qquad (16.32)$$

The choice depends on whether optimisation is more appropriately over total time or over running time.

a) Continuous Monitoring Policy. Let the relative frequencies of maintenance and failure repair actions be p_3 and $1-p_3$.

Then

$$\phi_{ccm} = p_3/\mu_1 + (1-p_3)/\mu_2$$

$$p_3 = (\phi_{ccm}\mu_1\mu_2 - \mu_1)/(\mu_2 - \mu_1) \qquad (16.33)$$

$$C_{ccm} = p_3 C_M + (1-p_3)C_F + \theta_{ccm} c_c \qquad (16.34)$$

$$c_{ccm} = C_{ccm}/T_{ccm} \qquad (16.35)$$

b) Inspection/ocpm Policy. Let the relative frequencies of maintenance and failure repair actions be p_9 and $1-p_9$.

Then

$$p_9 = (\phi_{ocpm}\mu_1\mu_2 - \mu_1)/(\mu_2 - \mu_1) \qquad (16.36)$$

$$C_{ocpm} = p_9 C_M + (1-p_9)C_F + C_I\ \theta_{ocpm}\mu_3 \qquad (16.37)$$

$$c_{ocpm} = C_{ocpm}/T_{ocpm} \qquad (16.38)$$

But ϕ_{ocpm} is a function of μ_3 through equations (16.24) and (16.27). To find the optimum $\mu_3{}^*$ of μ_3 and the corresponding minimum cost rate c^*_{ocpm} it is necessary to work through equations (16.25) and (16.27) and

(16.36) to (16.38) for a number of values. Some organised form of trial and error is called for such as Fibonacci search if using a computer, otherwise plot c_{ocpm} versus μ_3. A graph shows sensitivity of c to μ_3.

16.6 Special Cases - No Planning Delay

If there is no planning delay a special case arises for both ccm and ocpm. For ccm the situation represents an automatic trip. In the ocpm case the machine is shut down immediately an incipient failure is detected. It is worthwhile to compare the latter policy with one involving planning delay. It may pay to shut down at once if $C_F \gg C_M$ or $\mu_1 \gg \mu_2$ despite increase in C_M or c_d arising from taking immediate action.

16.6.1 Continuous Monitoring

S_1 disappears and the matrix becomes

$$Q_{ccmz} = \begin{array}{c} \\ 0 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{ccccc} 1-\lambda_1-\lambda_c & \lambda_c & \lambda_1 & 0 & 0 \\ \mu_c & 1-\mu_c-\lambda_u & 0 & 0 & \lambda_u \\ \mu_1 & 0 & 1-\mu_1 & 0 & 0 \\ \mu_2 & 0 & 0 & 1-\mu_2 & 0 \\ 0 & \mu_2 & 0 & \mu_c & 1-\mu_2-\mu_c \end{array}\right] \end{array} \qquad (16.39)$$

By the same methods the availability, mean up time, mean down time and variance of up times are found

$$\text{Let } z_1 = \lambda_c\mu_1 \Big/ \left[\lambda_1 \mu_c+\lambda_u-\mu_2\lambda_u/ (\mu_2+\mu_c)\right]$$

$$z_5 = \lambda_1+ \lambda_c,\ z_6= \mu_c+\lambda_u,\ z_4= z_5+z_6,\ z_7=\mu_c\lambda_c. \qquad (16.40)$$

$$z_2= (z_5-\mu_c z_1-\mu_3)/\mu_2,\ z_3= \mu_1/\lambda_1+z_1+1+z_2+\lambda_u+z_1/(\mu_2+\mu_c)$$

Then

$$A_{ccmz} = z_3(\mu_1/\lambda_1+z_1) \qquad (16.41)$$

$$\theta_{ccmz} = 1/z_5 +\lambda_c/z_5 z_6 \qquad (16.42)$$

$$\phi_{ccmz} = \lambda_u\ \lambda_c/z_5 z_6 \mu_2 +\lambda_1/z_5 u_1 \qquad (16.43)$$

$$\sigma^2_{ccmz} = 1/z_5^2 +\lambda_c/z_5 z_6^2. \qquad (16.44)$$

Costs are as for the general case but using ccmz availability, mtbf, and mttr.

16.6.2. Inspection/On-Condition Maintenance

S_8 disappears and the matrix becomes

$$Q_{ocpmz} = \begin{array}{c} \\ 6 \\ 7 \\ 9 \\ 10 \end{array} \begin{array}{c} \begin{array}{cccc} 6 & 7 & 9 & 10 \end{array} \\ \begin{bmatrix} 1-\lambda_1 & \lambda_1 & 0 & 0 \\ 0 & 1-\mu_3-\lambda_3 & \mu_3 & \lambda_3 \\ \mu_1 & 0 & 1-\mu_1 & 0 \\ \mu_2 & 0 & 0 & 1-\mu_2 \end{bmatrix} \end{array} \qquad (16.45)$$

$$P_6/P_7=(\mu_3+\lambda_3)/\lambda_1, \qquad P_9/P_7 = \mu_3/\mu_1,$$

$$P_{10}/P_7=\lambda_3/\mu_2, \quad 1/P_7 = (\mu_3+\lambda_3)/\lambda_1+1+\mu_3/\mu_1+\lambda_3/\mu_2 \qquad (16.46)$$

$$A_{ocpmz} = \frac{(\mu_3+\lambda_3)/\lambda_1 + 1}{(\mu_3+\lambda_3)/\lambda_1+1+\mu_3/\mu_1+\lambda_3/\mu_2} \qquad (16.47)$$

$$\theta_{ocpmz} = 1/\lambda_1+1/(\mu_3+\lambda_3) \qquad (16.48)$$

$$\sigma^2_{ocpmz} = 1/\lambda_1^{\,2}+ 1/(\mu_3+\lambda_3)^2 \qquad (16.49)$$

$$\phi_{ocpmz} = (\mu_1\lambda_3+\mu_2\mu_3)/(\mu_3+\lambda_3)\mu_1\mu_2 \qquad (16.50)$$

a) Costs Proportional to Downtime

$$c_{ocpmz} = c_d(a_z\mu_3+b)/(k_z\mu_3+h) + C_I\mu_3$$

where

$$a_z = \lambda_1/\mu_1 \qquad b_z = \lambda_1\lambda_3/\mu_2$$

$$k_z = 1 + a_z, h_z = \lambda_1+\lambda_3+b_z \qquad (16.51)$$

Then use equation (16.31) to find μ_3^* and substitute in (16.51) to find c*.

b) Fixed Failure and Maintenance Costs

The procedure is unaltered except for the suffices of A., θ and ϕ.

16.6.3 Perfectly Reliable Monitor (ccm)

A special case arises if the monitor has negligible failure rate. This might be achieved by better equipment or by redundancy; it will be usually more expensive. Calculations are considerably simplified.

The results represent an upper bound on the effectiveness of a ccm policy. If $\lambda_c = 0$ then S_2 and S_5 are eliminated and the transition rate matrix becomes

$$Q_{ccms}= \begin{array}{c} \\ 0 \\ 1 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 3 & 4 \end{array} \\ \begin{bmatrix} 1-\lambda_1 & \lambda_1 & 0 & 0 \\ 0 & 1-\lambda_2+\lambda_3 & \lambda_2 & \lambda_3 \\ \mu_1 & 0 & 1-\mu_1 & 0 \\ \mu_2 & 0 & 0 & 1-\mu_2 \end{bmatrix} \end{array} \qquad (16.52)$$

By the same method as used above

$$A_{ccms}= 1/\left[1+\lambda_1 . \ (\lambda_2/\mu_1+\lambda_3/\mu_2)/(\lambda_1+\lambda_2+\lambda_3)\right] \qquad (16.53)$$

$$\theta_{ccms}= 1/\lambda_1+ 1/(\lambda_2+\lambda_3) \qquad (16.54)$$

$$\sigma^2_{ccms}= 1/\lambda_1^{\,2}+ 1/(\lambda_2+\lambda_3)^2 \qquad (16.55)$$

$$\phi_{ccms}= (\lambda_2/\mu_1+\lambda_3/\mu_2) \ / \ (\lambda_2+\lambda_3) \qquad (16.56)$$

16.6.4 Zero Delay and Perfect Monitoring

This case is computationally trivial but important conceptually. It represents a bound point in the area of possible policies. S_1, S_2, S_4, S_5 all disappear and the matrix becomes.

$$\begin{array}{c} \\ 0 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 3 \end{array} \\ \begin{bmatrix} 1-\lambda_1 & \lambda_1 \\ \mu_1 & 1-\mu_1 \end{bmatrix} \end{array} \qquad (16.57)$$

Failures do not occur.

$$A_{zp} = \mu_1/(\mu_1 + \lambda_1) = P_o = 1 - P_3 \quad (16.58)$$

$$\theta_{zp} = 1/\lambda_1, \quad \phi_{zp} = 1/\mu_1 \quad (16.59)$$

$$\sigma^2_{zp} = 1/\lambda_1{}^2 \quad (16.60)$$

$$c_{zp} = c_d\lambda_1/(\mu_1+\lambda_1) + c_c \quad (16.61)$$

$$\text{or } c_{zpf} = \lambda_1 C_M + c_c \quad (16.62)$$

16.6.5. Perfect Monitor Zero Repair and Maintenance Times

This model is useful when the cost is to be optimised over operating time only and the equipment is shut down when the monitor is failed or under maintenance. This might apply if the monitor is required for assurance of safety in a dangerous operation or when it is desired to calculate unit costs for an equipment producing goods or a service. The matrix becomes

$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 3,4 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 3,4 \end{array} \\ \begin{bmatrix} 1-\lambda_1 & \lambda_1 & 0 \\ 0 & 1-\lambda_2-\lambda_3 & \lambda_2+\lambda_3 \\ 1 & 0 & 0 \end{bmatrix} \end{array} \quad (16.63)$$

$A = 1$

$$\theta = 1/\lambda_1 + 1/(\lambda_2+\lambda_3) \quad (16.64)$$

$$\sigma^2 = 1/\lambda_1{}^2 + 1/(\lambda_2+\lambda_3)^2 \quad (16.65)$$

It is necessary to postulate a failed state even though the equipment spends no time in it, in order to calculate the mean and variance. These are as for the more general case, with repair rates; times to failure would not be expected to be affected by repair times. There is no downtime, therefore no ϕ and so a new method of calculating the proportion of maintenance as opposed to failure cycles must be found. In this case it is obvious by inspection that

$$p_3 = \lambda_2/(\lambda_2 + \lambda_3) \quad (16.66)$$

Regarding costrate, the case where cost is proportioned to downtime is trivial and that where costs are fixed can be solved through equations (16.34) and (13.35) putting $T = \theta$.

16.6.6. Inspect/ocpm with Negligible Repair and Maintenance Times

This is the ocpm case corresponding to the ccm case considered in 16.6.5 above. Similar strictures and methods apply. The matrix is

$$Q = \begin{array}{c} \\ 6 \\ 7 \\ 8 \\ 9,10 \end{array} \begin{array}{c} \begin{array}{cccc} 6 & 7 & 8 & 9,10 \end{array} \\ \left[\begin{array}{cccc} 1-\lambda_1 & \lambda_1 & 0 & 0 \\ 0 & 1-\mu_3-\lambda_3 & \mu_3 & \lambda_3 \\ 0 & 0 & 1-\lambda_2-\lambda_3 & \lambda_2+\lambda_3 \\ 1 & 0 & 0 & 0 \end{array}\right] \end{array} \qquad (16.67)$$

This leads to Q indentical with (16.23) and so

θ and σ^2 are as shown in the more general case.

$A = 1$, and of course $\phi = 0$

The mean θ and variance σ^2 are given by equations (16.24) and (16.26) as before. The proportion of maintenance as opposed to failure cycles p_9 can be fixed by a double application of the rule developed in 16.6.4 above.

$$p_9 = p_8(p_9 S_8)$$

$$p_9 = \mu_3\lambda_3/(\lambda_2+\lambda_3)\ (\mu_3+\lambda_3) \qquad (16.68)$$

16.6.7 Zero Delay Zero Repair and Maintenance Times

This is the case considered by Miller and Braff (3.158) who derived the reduction in failure rate in terms of the inspection rate but did not pursue the argument to a discussion of costs. Only S_6 and S_7 exist but S_{10} is reinstated to calculate the mtbf.

$$Q = \begin{array}{c} \\ 6 \\ 7 \\ 9,10 \end{array} \begin{array}{c} \begin{array}{ccc} 6 & 7 & 9,10 \end{array} \\ \left[\begin{array}{ccc} 1-\lambda_1 & \lambda_1 & 0 \\ 0 & 1-\mu_3-\lambda_3 & \mu_3+\lambda_3 \\ 1 & 0 & 0 \end{array}\right] \end{array} \qquad (16.69)$$

$$T = \theta = 1/\lambda_1+1/(\mu_3+\lambda_3) \qquad (16.70)$$

$$p_9 = \mu_3/(\mu_3+\lambda_3) \qquad (16.71)$$

The case of costs proportional to downtime is trivial. For fixed costs, finding μ_3^* involves solving a cubic equation or else using variational methods, (preferable if the computer code already exists for the more general case). These remarks also apply to the model at 16.6.6.

16.7. Examples

16.7.1 Comment

The worked examples below demonstrate the application of the theory. They also illustrate some additional points which would have been more difficult to present analytically. The examples show that it is unwise to decide the policy without calculation. The numbers are fictional but the situations are real.

16.7.2. Example 1 Given warning, a steel mill roller can be renewed in about 2 hours. If the stoppage follows a failure repairs take on average 12 hours. The mean life of rollers is 500 hours. The cost of production lost during downtime is £5000/hr and greatly exceeds the direct costs of maintenance and repair. Alternative policies are ccm at £0.4 per hour or inspections costing £1.50 each. Warning of imminent failure appears about 10 hours before complete failure. It is convenient to change the roller at the start of a shift of 8 hours.

$\lambda_u = 1/500 = 0.0020$

$\lambda_2 = 1/(8/2) = 0.250$

$\lambda_3 = 1/10 = 0.100$

so $\lambda_1 = 1/490 = 0.002041$ from equation (2)

$\mu_1 = 1/2 = 0.500$

$\mu_2 = 1/12 = 0.0833$

$c_d = £5000/\text{hr}$ $\quad c_c = £0.4/\text{hr}$ $\quad C_I = £1.50$

The monitor is assumed to be perfectly reliable.

For the ccms policy

$A_{ccms} = 0.99024$

$c_{ccms} = £49.20/\text{hr}$

For the ocpm policy

a = 1.7000	b = 0.42000	
k = 174.200	h = 17.92	
E = 9.1037	F = 1.873	G = -42.6037

$\mu_3^* \;=\; 2.063$ inspection/hr from equation(31)

$c^*_{ocpm} \;=\;$ £55.14/hr from equation (23)

For comparison a fm policy would cost

$$c_{fm} = c_d\,(1-A_{fm}) = \frac{12 \times 5000}{500 + 12} = £117.19\ /hr.$$

This is a close decision but the monitor could be duplicated or even triplicated without bringing c_{ccms} within 10% of c^*_{ocpm}, with three fitted, monitor failure could certainly be neglected in the calculation. So in this instance the decision would be to order inspections at 30 minute intervals to confirm the ocpm cost estimate. A monitor would be installed if still reckoned to be cheaper. If the monitor proved unreliable then it might be duplicated or triplicated rather than reverting to inspection, depending on the measured costs of both policies.

16.7.3 Example 2 A peak-lopping unattended electricity generating station consists of 4 gas-turbine alternator sets whose condition is to be monitored from a remote station.

A decision has to be made whether to fit dedicated monitors for each set or have only one monitor which switches between the 4 sets.

The monitor works on a cycle. It reads a number of turbine parameters and through logic circuits, and by searching its memory, prints either a reassurance or gives a warning of impending failure. A monitor cycle takes 40 seconds after which it starts again if dedicated or switches to the next turbine. Other data are as follows

$\lambda_u = 1.0000 \times 10^{-7}$/sec. $\lambda_3 = 5.00 \times 10^{-3}$/sec, $\lambda_2 = 0.02$/sec

$\lambda_1 = 1.00002 \times 10^{-7}$/sec $\mu_1 = 1.0000 \times 10^{-5}$/sec $\mu_2 = 2.5000 \times 10^{-6}$/sec

$\lambda_c = 3.00000 \times 10^{-7}$/sec $\mu_c = 4.0000 \times 10^{-4}$/sec

$C_F =$ £220,000. $C_M =$ £30,000. $c_c =$ £0.0005 /sec

The monitor, produces an assessment only at intervals, however short. This is therefore ocpm not ccm. Miller and Braff(3.158) in a similar situation showed by simulation that regularly timed inspections made little difference to availability.

Monitor availability $A_c = \mu_c/(\mu_c + \lambda_c) = 0.99925$

For a dedicated monitor and for each turbine set.

$\mu_3 = .02498/\text{sec}$, $P_8 = 3.267735 \times 10^{-6}$, $P_9 = 0.0065355$, P_{10}=0.0130758
$A = 0.980389$, $\theta = 2777.74$ hrs   = 55.565 hrs.

$C_I = £0.02$, $p_9 = 0.666555$, $C = £98354.49$ per cycle

$c = £0.009643$ /second = £34.71/hr.

For a shared monitor and for each turbine set.

$\mu_3 = 6.24530 \times 10^{-3}/\text{sec}$. $P_8 = 2.163792 \times 10^{-6}$, P_9=4.327584×10^{-3}

$P_{10} = 0.0173234$, $A = 0.978349$, $\theta = 9999928.96$ seconds,   = 2212199.66 secs

$C_I = £.02$, $p_9 = 0.595668$, $C = £111823.0825$ per cycle

$c = £0.00916$/second = £32.96 hr

So the shared monitor is cheaper overall, but only by a small margin, easily upset by inaccurate estimates of the parameters.

In either case less than two thirds of failures are prevented by the monitor. This unsatisfactory performance is not due primarily to the monitor, a 4 : 1 variation in μ_3 produces only an 11% shift in p_9. The main cause is the relative closeness of λ_2 and λ_3. The probability that the warning time is less than the shut-down delay is given by $\lambda_3/(\lambda_2 + \lambda_3)$ which in the example is 0.2. So even under <u>ccm</u> the proportion of potential failures prevented by the monitoring system can be no more than 0.8. Longer average warning or quicker shutdowns or both are required for significantly improved performance.

<u>16.7.4. Example 3</u> In this example there is zero planning delay and also negligible repair and maintenance times. This is the case considered by Miller and Braff (3.158). The object of this example is to demonstrate their contention that whether the inspections are randomly distributed or strictly periodic makes little difference to the results (costrate and periodicity of optimal schedule). The figures chosen for the examples are therefore calculated for both models.

A power station boiler feed pump has <u>base</u> <u>mtbf</u> θ = 10 000 hours

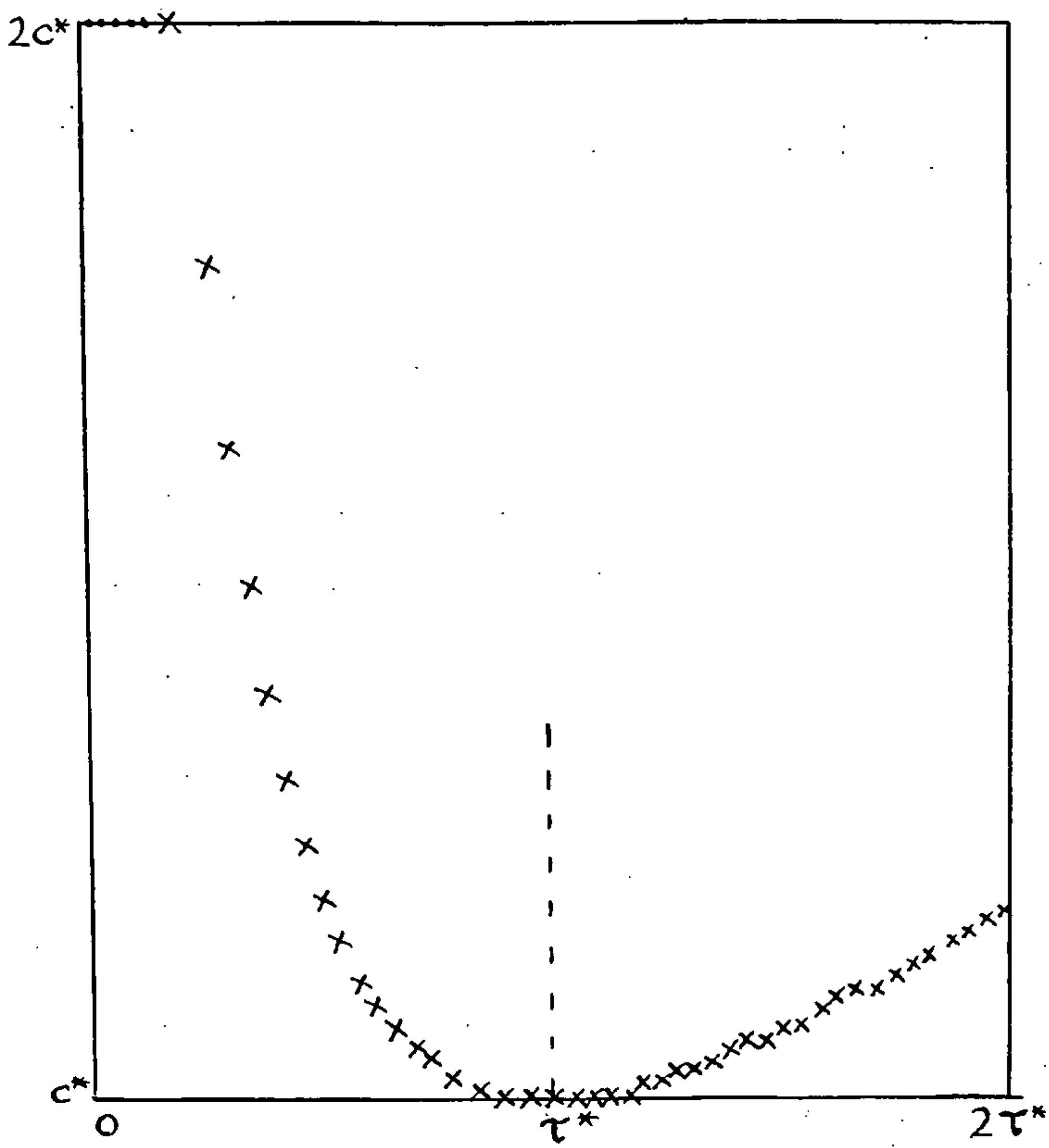


Figure 16.3 Sensitivity of Costrate to Inspection Interval for Example 3(a)

Note: This plot was traced from a reduced copy of computer output using program CONHAZ. The plot is 50 x 50 spaces or lines so the accuracy is 2%.

and shows signs of impending failure at about $\theta_2 = 200$ hours before actual failure. A failure costs $C_F = £500,000$ including downtime costs. If the pump is stopped before failure the total cost is only $C_M = £8,500$. Impending failure is detected by inspections which cost $C_I = £100$ each. The company is fully insured for initially faulty equipment (d = 1).

a) Periodic Inspections Computer results assuming constant hazard rate using methods of § 15 above were

Optimum inspection interval	τ^*	=	22.2 hours
Cost rate of optimum schedule	c*	=	£10.37 hours
Mean cycle time	E(T)	=	9990 hours
Proportion of failure cycles	r	=	0.102
Cost rate relative to fm	$c^*\theta/C_F$	=	0.207

Figure 16.3 shows that τ can be varied ± 10% less than 2% variation in c. This range includes the convenient interval of 24 hours between inspections.

There is no provision for refunds on initially faulty equipment in the Markov model. Recalculating the problem putting d = 0 gives almost identical answers. The costrate is the same to the nearest penny per hour and the optimum interval is within 5 minutes.

b) For the Markov model (16.6.7) try first $\mu_3 = 1/22.2 = 0.0\dot{4}\dot{5}$ giving

$\theta = 9800 + 1/(0.05 + .0\dot{4}\dot{5}) = 9820$

$p_9 = .0\dot{4}\dot{5}/(.0\dot{4}\dot{5} + .005) = 0.9001$

$r = 1 - p_9 = 0.0999$

$c = (500,000 \times .0999 + 8500 \times .9001)/9820 + 100 \times .0\dot{4}\dot{5} = £10.37/\text{hr}.$

For 20 hrs and 25 hrs c is £10.42 /hr in both cases, and 22 hrs and 22.5 hrs both give very slightly greater cost rates than 22.2 hrs so it seems the optimum interval is little altered.

The only parameter which has changed substantially from (a) above is the mean up time before a maintenance or repair action. So Miller and Braff's contention that there is little practical difference between periodic and randomly distributed inspection intervals is supported at least in this example.

16.8. Computer Programme

A computer programme titled MONIT. BAS written for interactive use in BASIC-PLUS is described and listed in Appendix C.

17. DISCUSSION OF MAINTENANCE MODELS

17.1 Models for Real Problems

It is a feature of nearly all the models examined in Section 14 that they fit only idealised situations, which probably never actually exist in all particulars. The researcher must decide beforehand, or by trial and error, which features of the real situation must be modelled and which can be omitted or approximated. A reasonable procedure would be to calculate optima for several policies and models and see whether the answers differed greatly. The great proliferation of models probably arises because investigators feel dissatisfied with existing models and so end up by making new ones. They would then come up against difficulties in representing some features in their model and end up with more approximations of which the only redeeming aspect is that they can be solved analytically, albeit with some rather obscure mathematics.

The most usual practical problems in maintenance are:

a) whether to use inspect/ocpm or ppm or variants of these policies.

b) To balance the advantages of renewal or maintenance before failure against the loss of utility caused by such early action such that a defined objective function is optimized in a defined manner.

The results of Section 15 show that even when the choice is restricted to two policies an immediate decision is not possible as regards (a) above unless z(t) is non-increasing. This is of course well known from the literature (3.112), however it was a little surprising to find that provided inspections are cheap and C_F and C_M not too far separated it is quite possible that ocpm on the basis of inspections is cheaper than ppm for $\beta > 1$ and even $\beta \gg 1$.

The problems arising from (b) are generally avoided in the literature by modelling inspection as an operation to discover whether failure has taken place. If measurements are needed to discover failure then the problem is really (or at least analagous to) one of sampling inspection for quality control. The usual maintenance problem is to anticipate failure which is obvious when it occurs. Engineering failures are often classified as catastrophic or gradual. In the second case it is often stated that failure is a matter of opinion or standards. This ignores a large class of failures which are, finally, sudden or catastrophic but which exhibit some portents for a period before failure. For example bearing failure in an engine may have a random external cause but it is often attributable to fair wear and tear, in which case audible and visible signs are available to diagnose the imminent failure (big-end knock, oil pressure). Even when the root cause is an initiating random event, failure need not be immediate, although no less inevitable.
There is then very often scope for considerable savings if repairs are made before this inevitable catastrophic event. Inspection against such events, to detect the signs before the full failure can be cheap or expensive, and most importantly more or less subject to errors of the two kinds, detailed at 15.3 above. In the most general case, the initiating event is not detected or is in fact not one event but a cumulative effect of many occasional over-stressings or mild abuses, as, for example, fatigue failure of items whose normal load is within the fatigue limit. Where this is so, the inspection routine must cover the whole life; if the event is self-announcing, inspections need, in theory, start only afterwards. At each inspection a judgement must be made as to whether the item will last

until the next inspection. The strictly periodic model at Section 15 does not allow for an intermediate decision, namely to inspect at a shorter interval, to reduce p on the basis of the inspection result. It is hoped to develop this idea into a model at some later date, but the model for ocpm at Section 16, does allow for particular variation in the inspection intervals. Kander (3.259) envisages different inspect frequencies for each of several states in a Markov model, but a model free from Markovian restraints is really needed to meet the case. Kander (ibid) also suggests continuous monitoring during the last stage, in order to extract the maximum life but he envisages continuing to failure which is unnecessary; warning time should be used to plan as smooth a renewal or repair as possible, so saving money.

17.2. Need for Simple Models

A model which is workable only with the aid of a main-frame computer and comprehensible only to an honours mathematics graduate is unlikely to be applied by practising maintenance engineers. This is not to say that models should be simplified in concept but that they must be simple to operate. Ideally the optimization calculation should be reduced to entering graphs or nomograms having calculated dimensionless and/or normalised parameters of the problem. Good examples of this approach are Shahani and Newbold (3.223) and Glasser (3.91). In contrast, the models proposed in Sections 15 and 16 above both rely upon computer programmes for optimization. This is the inevitable result of taking into account more of the variables which exist in real maintenance situations. They are therefore more likely to be taken up by maintenance consultants, large firms of suppliers wishing to give advice to purchasers on maintenance intervals, and large operators with computer data collection systems. The last-named would be able to take advantage of the suggestions for adaptive operation.

17.3. Nomograms and Graphs

A nomogram or graph can, by its nature, only allow the determination of one quantity given the value of two others. Using more than one nomogram or graph, possibly with one or more common scales it is possible sometimes to use the first answer or the same two quantities to determine another required output, and so on. The problem must be divisible into parts in which two parameters or previously-determined quantities are combined in a single equation which determines another quantity. If there are three or more parameters or etc. in an equation, nomograms and graphs are not useful. In problems involving successive determinations working towards an optimum, the technique is less effective because the whole procedure must be iterated in order to build up a graph of the objective function against the major variable e.g. c versus p in the model of Section 15, or c versus μ_3 in Section 16. It was found that in the absence of a computer both the models could be operated more quickly using a hand-held calculator and working through the equations than by a succession of graphs which were less accurate and only applicable to the case of constant z(t).

For the case of z(t) constant only, a graphical solution for E(T) in the ocpm model at Section 15 is provided at Figure 17.1. The equation for E(C) is better worked on a calculator. It is noted that E(T) does not vary very much and Table 17.1 below gives a reasonable approximation for $0.05 \leq b \leq 0.25$ i.e. $0.05 \leq t/\theta \leq 3$.

Values outside the upper limit of p given above are hardly likely to be required in practical problems. For $\tau < 0.05$,

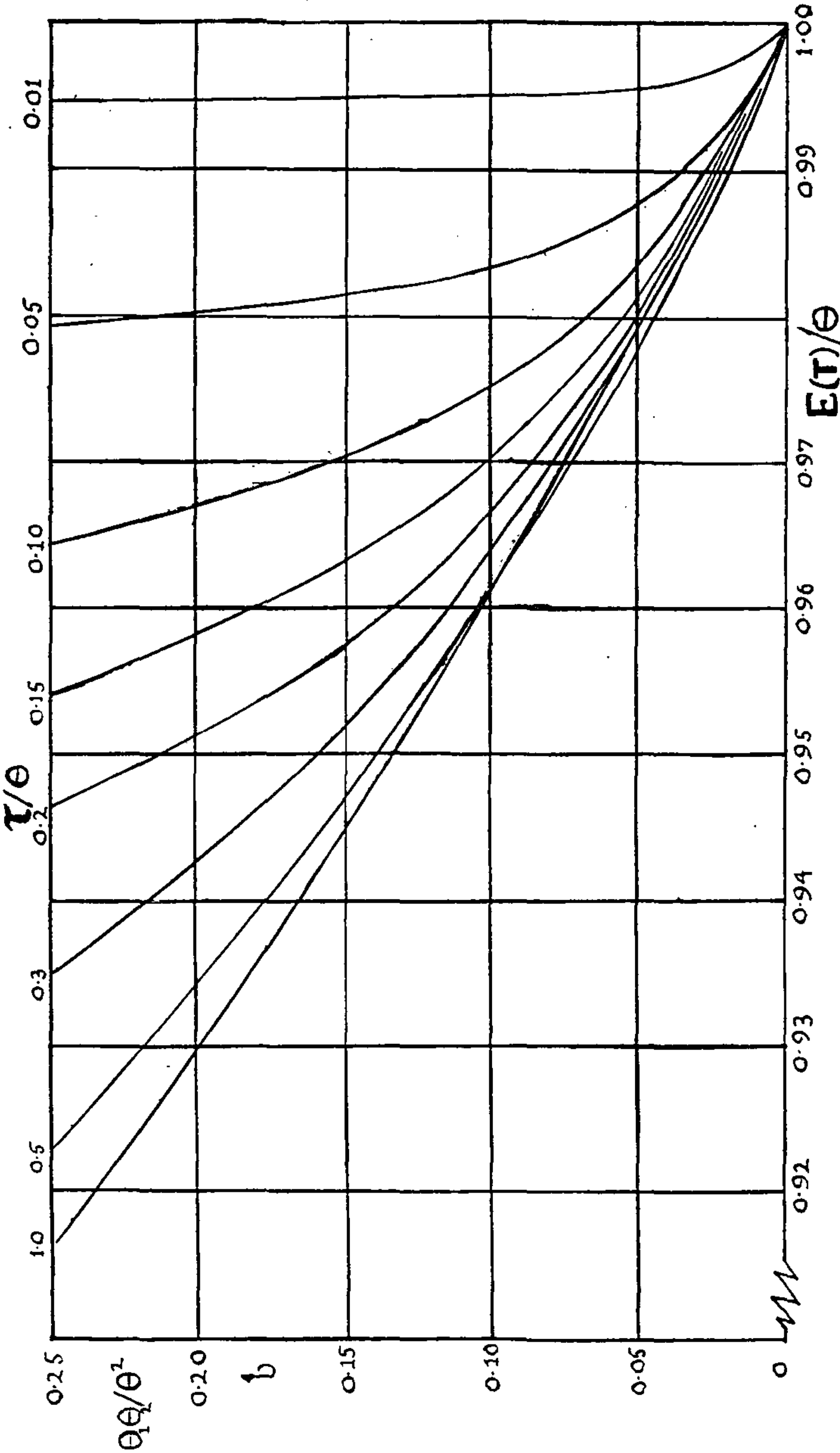


Figure 17.1 Graph for finding Expected Cycle time given constant hazard rate, b and τ/θ

$E(T)/\theta$ may be taken as 0.98 - 1.00 on a scale which gives a value of 0.995 at $\tau = 0.01$.

TABLE 17.1 AVERAGE VALUES OF NORMALISED CYCLE TIME FOR z(t) CONSTANT

b	$E(T)/\theta$
0.05	0.982
0.10	0.966
0.15	0.952
0.20	0.939
0.25	0.926

Although it is not worthwhile to provide graphs or nomograms for the equation for E(C) it is worth noting that the inspection costrate, C_I/τ for $\beta = 1$ can be taken out of the equation for c and the remainder normalised in C_I/θ, thus

$$c = C_I/\tau + \{C_I/\theta\}\{B + D\,r\}/\{E(T)/\theta\} \qquad (17.1)$$

where $B = C_M/C_I$

and $D = (C_F - C_M)/C_I$ are dimensionless

parameters of the problem, and r is of course a function of p which is a function of τ.

Even in the general case where z(t) is not constant, the only obstacle to a graphical solution in four stages is the finding of n, the stopping number of inspections. If this requirement is dropped the answers will be suboptimal, but not seriously so because E(T) remains insensitive to p for reasonable values of β. The equations become

$$E(C) = C_I\left\{1/p + C_M/C_I + r\,(C_F - C_M)/C_I\right\} \qquad (17.2)$$

and

$$E(T) = \eta\left\{(1-r)\left\{(1/p)\log(1-p)\right\}^{1/\beta} + r\,\Gamma(1+1/\beta)\right\} \qquad (17.3)$$

Given p, r can be evaluated from the p, r graph of the appropriate model. Figure 17.2 evaluates $q = \left\{(1/p)\log(1-p)\right\}^{1/\beta}$ for various values of β, and Figure 17.3 can be used to find $\theta/\eta = \Gamma(1+1/\beta)$. (The normalised standard deviation σ/η of the Weibull is also shown upon Figures 17.3). If the distribution is of another form then if R^{-1} denotes the inverse Survival function

$$q = (1/p)\, R^{-1}\left\{ (1-p)^{i}\right\}$$

The second term of E(T) is $r\theta$ where θ is the distribution mean which is very easy to evaluate in most other distribution forms.

None of these graphs in themselves provide optimization, they merely aid the calculation of c given a value of p, the base distribution of tbf's, the details of r = g(p) and the three costs C_F, C_M, C_I. However, multiple evaluations lead to a graph of c versus p which is useful both for optimization and the examination of sensitivity of c to p.

17.4 Generality of the Matrix Methods

Using the methods developed in Section 16, no insuperable difficulty is foreseen in extending to more complicated models. Non-constant transition rates can be dealt with by the method of dummy intermediate states, see Singh and Billinton (4.69). Large matrices in themselves are no great problem provided that they can be inverted. Alternatively, varying rates can be dealt with directly by putting the function into the matrix, evaluating the state probabilities at unit time intervals and so building up state likelihood functions with respect to time. The transition rates can be regarded as constant for unit time (or for longer periods if more convenient but with some loss of accuracy) so that Markov methods can be applied to a changing matrix. In this way the mean time to a down state θ and the relative probabilities of the various down states p_j can be found. From the

Figure 17.2 Weibull Conditional Mean Time to ocpm

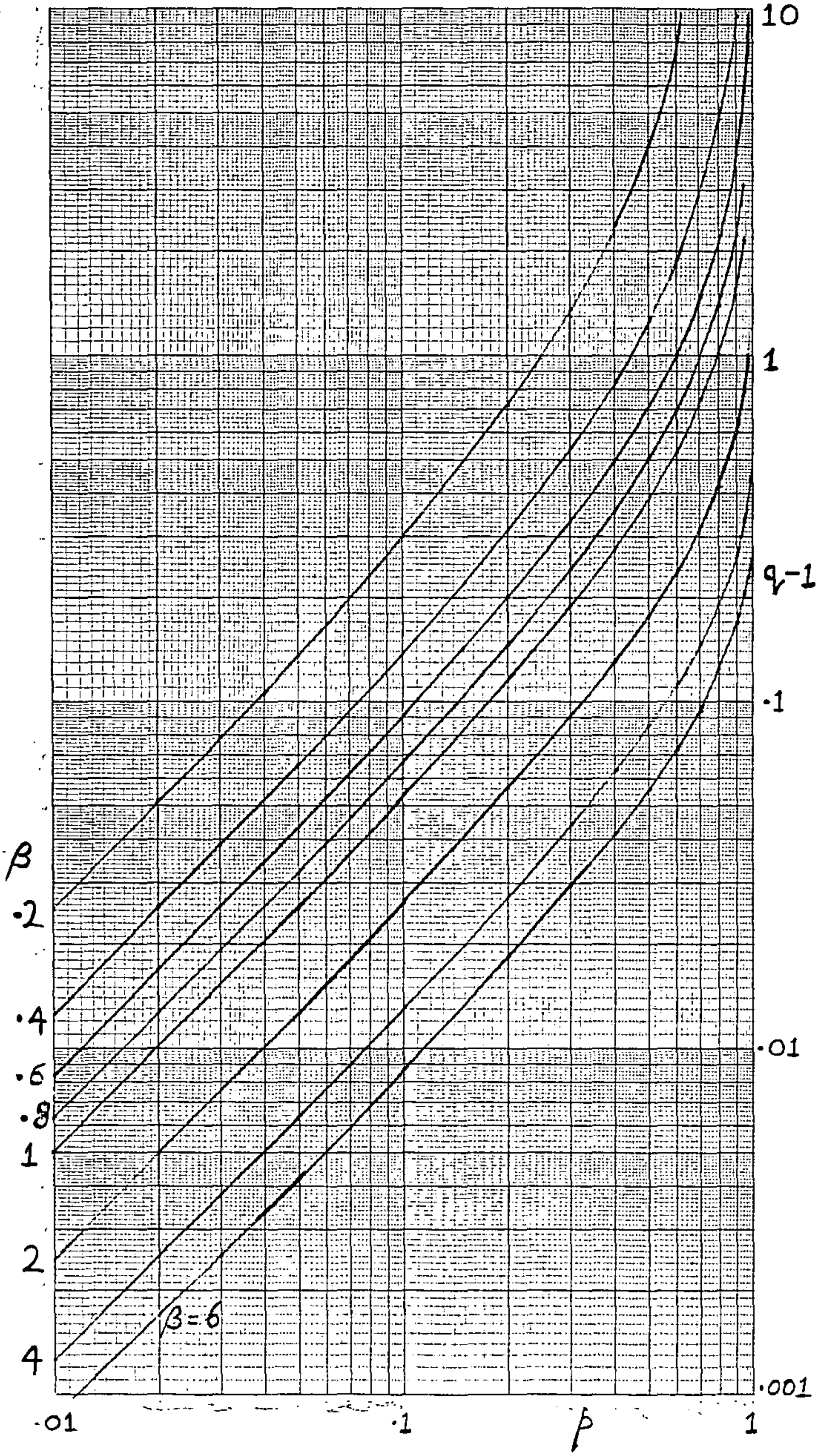
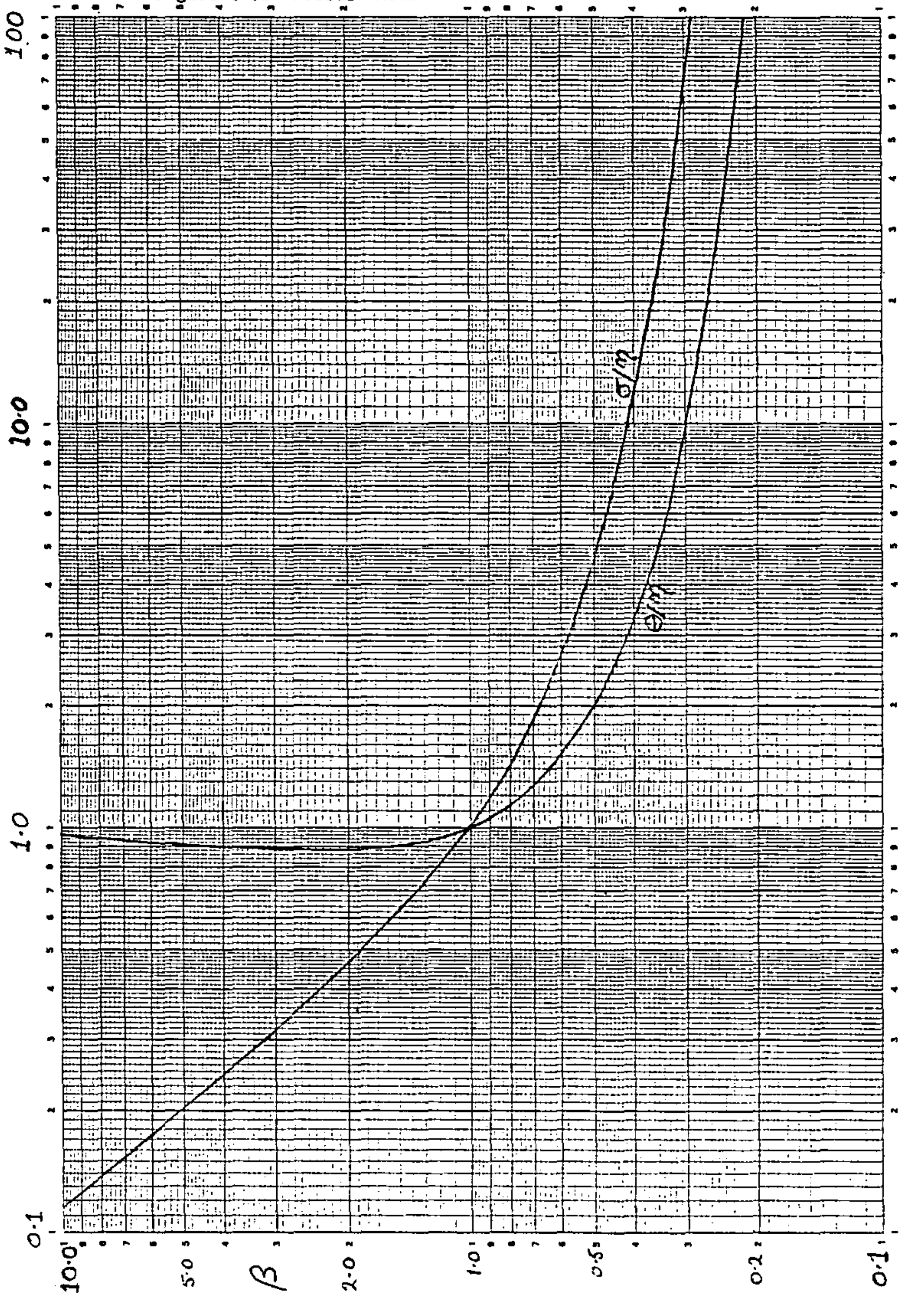


Figure 17.3 Weibull Mean and Standard Deviation



downtime distributions and the p_j one could calculate ϕ and hence A. The probability of state i after time t starting in state O at time O is, for constant transition rates $^{t}q_{o,i}$, i.e. the element (O,i) of the matrix Q multiplied by itself t times. If the matrix itself changes with time the procedure is unchanged. A computer could quite easily be programmed to take on the drudgery of such repetitive calculations. To deal with varying repair rates the matrix could be redrawn with the upstates combined and the downstate probabilities initially p_j, but it would usually be just as quick to calculate the means of the repair time distributions ϕ_j and find ϕ from

$$\phi = \sum_j p_j \phi_j$$

It is hoped to write a paper based upon the method outlined above at a later date.

17.5 Unequal Inspection Intervals

It is doubtful whether in most cases unequal inspection intervals, which are theoretically required under optimal scheduling unless $\beta = 1$, are a practical proposition. To operate them on a large scale would require a complicated system to remind the maintenance staff to perform the inspections. A more practical approach would be to regard $z(t)$ as constant over finite periods and schedule accordingly, taking values of failure rate which minimise the cost-rate over the finite period. The exact formulation of the optimum interval under these conditions is a subject for further research.

The main impetus of the work reported at Section 16 came from the CEGB, as already discussed, but liason with a major oil company also produced a request for consideration of inspection intervals, in this case for pressure vessels. These pressure vessels are presently inspected according to legal requirements at biennial intervals. The company was interested in a method of determining whether this interval, which they suspected was arbitrary, was anywhere near optimal. To conduct an experiment would have taken about 20 years as the vessels have long lives, so method was more important than practical demonstration in this instance. Although there was a considerable number of histories of previous vessels available, there were too few failures for statistical analysis of failure rate versus age. It was evident, though, that previous vessels followed a bathtub curve but were usually renewed before $z(t)$ increased after the constant period. The reasons for such renewals before failure were of interest, being mainly the onset of such rapid destroyers of confidence as scab pitting and fatigue cracks. If these are regarded as failures at the conditional mean time in the next inspection interval the number of data increases, and a base failure rate could be estimated for each combination of material of construction, temperature and fluid contained, The model of Section 15 or the ocpm model of Section 16 could then be used for optimisation of the inspection interval, noting that this will not be optimal with respect to the prediction of the final up-turn in $z(t)$ or the increased hazard in early life due to design faults, manufacturing deficiencies, overloading during the operators' learning period etc. At first appreciation it would seem that pressure vessels should be inspected at installation and again following the commissioning period. If all is well then the interval can probably be increased until $z(t)$ starts to rise. It then becomes a question of cost whether to persist with the now ageing vessel with increasingly frequent inspections involving heavy shutdown costs or to get a new vessel. Because the life of pressure vessels may determine the lifespan of the whole plant this is an important question. At the two ends of the lifespan the model at Section 15 is applicable. At all stages consideration should be given to imposing a maximum value of risk between inspections p, rather than relying on costs alone. Alternatively, the cost of failure C_F or the mean failure downtime could be made larger to include notional probability-costs of hazards to personnel, as well as the repair and lost production costs. Pressure vessel inspections are fairly costly because they generally involve a shutdown. Shutdowns are usually annual events so the inspection interval has only to be optimised to the nearest year. The real question then becomes whether the statutory interval is more than a year in error on reasonable criteria of safety and cost. If it is, then a case should be made out, based upon data analysis and modelling for changing it. This reasoning applies to the constant failure period.

If the inspections are relatively expensive and safety is involved it may be advisable to follow the bathtub curve by varying the inspection frequency using the methods of Section 15, but only if the arithmetic leads to variations of more than 6 months from the ideal interval, otherwise an overall mean failure rate should be used (β=1 assumed). Ultimate life should be determined as the point when the annual risk becomes unacceptable using the full curve. i.e. when annual inspections no longer give adequate assurance. Before this there may be a change from the usual periodicity to annual inspections based on similar criteria of acceptable risk of failures between inspections.

This exercise shows how the theory developed in this chapter could be used to justify changes in policy and legal requirements in the difficult area where both safety and reliability are involved.

CHAPTER IV. REDUNDANCY, SYSTEM RELIABILITY AND INTERSTAGE STORAGE

18. REDUNDANCY

18.1 Basic Theory - Open and Short Circuit Failures

18.1.1 Basic Theory - The basic theory of redundancy is discussed in Appendix A. The literature will not be reviewed but a number of the more important and relevant books and papers appear in Section 4 of the References. Redundancy theory developed mainly to meet the problems of the electronics, electric power and telecommunication industries; complex redundant systems are less common in industrial plant, although the value of standby equipment for important manufacturing functions is beginning to be appreciated.

18.1.2 Open and Short Circuit Failures - Most failures to process and manufacturing plant are open-circuit in the sense that the failure causes a loss of transmission of function or material to the next stage of manufacture. However, short-circuit failures are not unknown. For example, a set of reaction vessels may normally process material in series, but it may be possible to by-pass any one of them without serious loss. If the by-pass is used this may be considered a short-circuit failure; if the by-pass is absent or fails shut then the failure is open-circuit. If the by-pass fails open (cannot be shut) this is also a short-circuit failure of the vessel + by-pass. Attention is therefore drawn to the analysis due to Jenney (4.48) at Appendix A.

18.2 Partial Redundancy

18.2.1 Definition - Partial redundancy occurs where all or most items of a manufacturing stage are required for full output but where a limited service can be provided by fewer than the number required for full output. Partial redundancy is fairly common in the process industries and possibly more so in manufacturing.

18.2.2. Debottlenecking and Partial Redundancy - A frequent occurrence in chemical plant is that plant designed to have full redundancy is later uprated with respect to output so that the redundancy becomes partial or even disappears. A debottlenecking operation consists in finding the stage having the lowest maximum output rate and making engineering changes so that the rating of the whole plant may be increased. If such schemes involve the disappearance or degrading of redundancy it is quite possible for the loss of availability to more than cancel out the gain in output rate. A frequent casualty of this situation is pm which can no longer be conveniently done without reducing output in the face of increased demand for product. A vicious circle is then set up in which availability deteriorates because pm is neglected which in turn makes it even less likely that planned downtime will be permitted in the future. This is what is believed to have happened at the plant described in Section 10. There was a considerable improvement in mean output rate from the plant as a whole when pm was reintroduced using schedules based upon the inherent redundancy and partial redundancy.

18.3 Active and Standby Redundancy - Data Problems

18.3.1 Definitions - The meanings of active and standby redundancy are explained at Appendix A and in BS4778.

18.3.2. Data Problems - To calculate the reliability or availability of an item which forms part of a standby redundant or partially redundant stage of a system it is necessary to know running times between failures. It frequently happens that this data is not recorded and all that is available is calendar time information about the occurrence of failures and perhaps the repair times. Failure and repair rates or distribution functions calculated from such information may be thought of as considering a standby system as if it were an equivalent active parallel system. This is not exactly so but it is a fair approximation in the circumstances and can lead to rather simpler calculations for throughput availability, see 19.2 below.

19 SYSTEM AVAILABILITY

19.1 Definition Problems

To the Reliability specialist, Availability is usually defined as a probability, based on a defined timescale, that an item is operating or available to operate. It is well-known that given that the item is 'up' at time zero the availability will after a short transient tend to a steady state value which can be expressed in the form

i.e. $A(t) \to \underline{mtbf}/(\underline{mtbf} + \underline{mttr})$ as $t \to \infty$
in the usual notation

or

$$A(\infty) = \theta/(\theta + \phi) = \mu/(\mu + \lambda) \qquad (19.1)$$

To the plant manager, availability is often the ratio of actual or possible output to rated or expected output over a long period. It is unfortunate that this confusion exists, but it is so widespread that it must be accomodated rather than denied. The plant manager's definition will be called 'Throughput Availability'. Pearson (4.61) whose thesis was concerned with the evaluation of throughout availability used the term 'throughput capability' which is possibly more precise but concedes nothing to the plant manager's concept of availability.

19.2 Throughput Availability

19.2.1 Literature - Whilst other authors consider calculating availability or reliability at reduced outputs (examples are 4.2, 4.47, 4.73,) of plant containing stages with full or partial standby redundancy Pearson, (1975)(4.61) has produced algorithms for finding system throughput availability from previously-calculated stage availabilities at all possible outputs. He envisages different rated outputs at each stage so that the line is unbalanced. One or more stages with minimum rated output constitute a bottleneck to increased production. The system throughput availability is then the sum of the

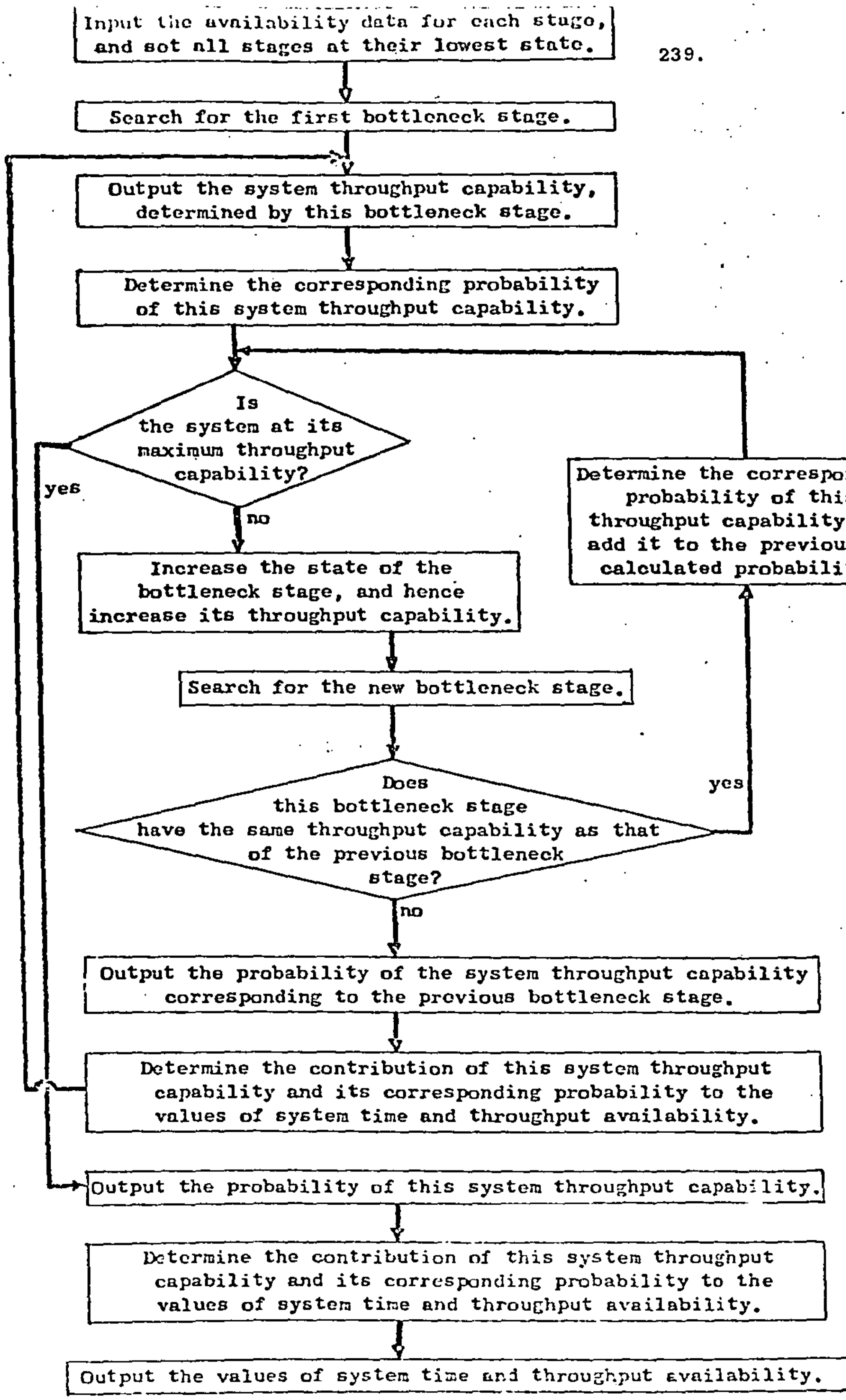


FIGURE 19.1 FLOW CHART FOR SYSTEM THROUGHPUT AVAILABILITY (AFTER PEARSON)

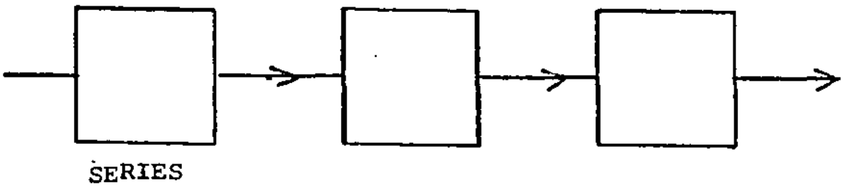
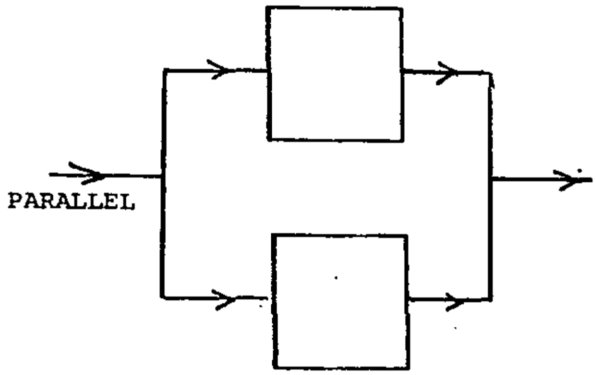
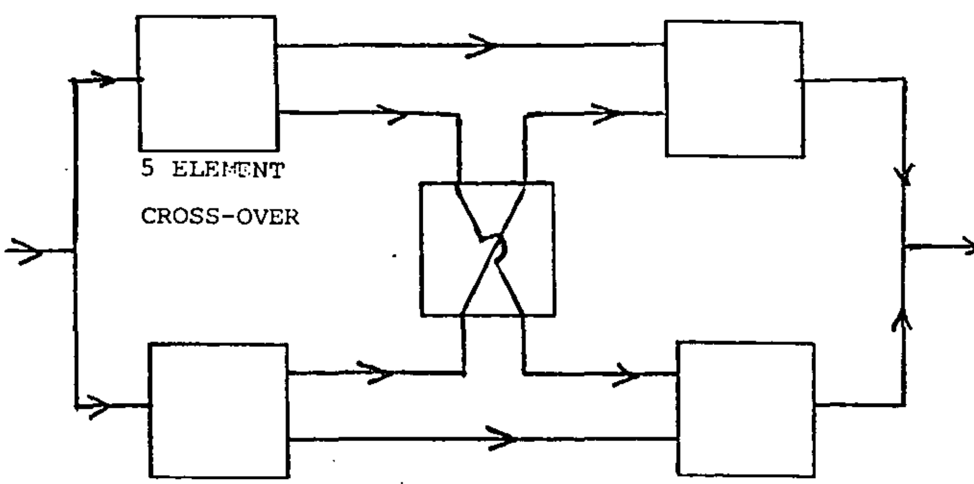
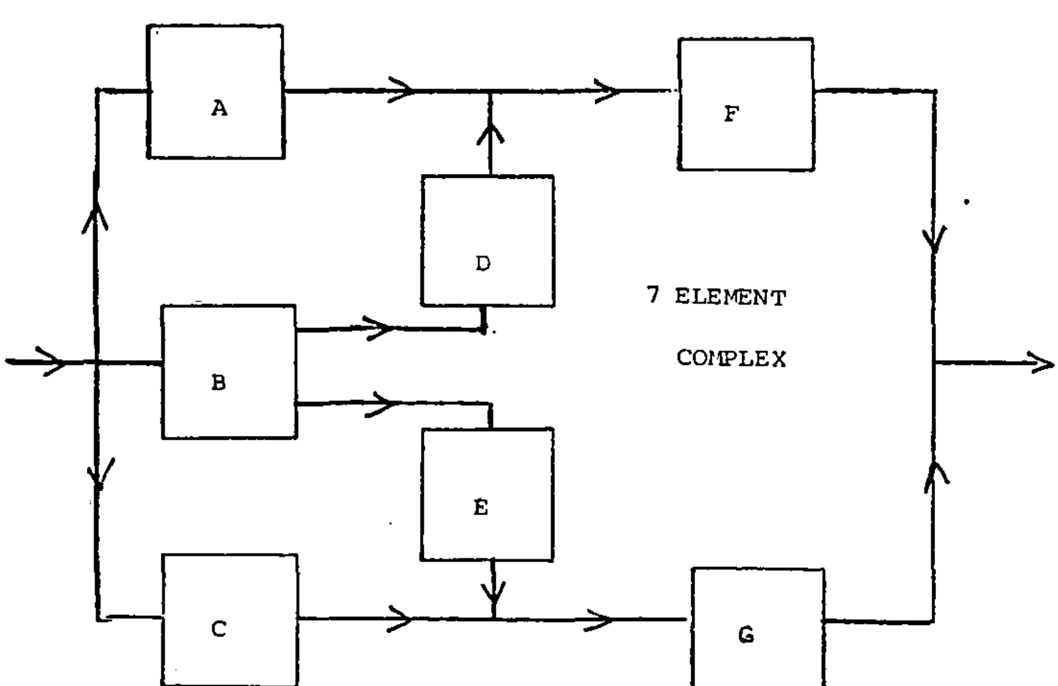


FIGURE 19.2

PEARSON'S BASIC CONFIGURATIONS







possible throughputs each multiplied by the proportion of total time that it is expected that the system will spend in the states (of items 'up' and 'down') corresponding to that throughput. Pearson envisaged four types of system and subsystem ,namely series, parallel, a five-element cross-over system, and a seven-element system, see Figure 19.2. (Pearson makes much of the last two configurations, but actually they can be solved quite easily by Bayes theorem. They are also much less common in practice than the standby and active parallel configurations) Pearson's flow chart is reproduced as Figure 19.1

19.2.2. Suggested Procedure - To evaluate system throughput availability the following procedure is proposed.

1) Divide the system into a series of stages which have potentially different throughput probability distributions. Usually this will mean dividing the system into functional stages. In some plants it will be necessary to divide the system first into streams, evaluate these as systems and then combine the stream results.

2) Find the throughput availability probability distribution of each stage. This will require data on the failure rate and repair rate of each item in the stage. For redundant and semi-redundant stages it will be necessary to make separate calculations for each of the possible states. A convenient method is to form state transition rate matrices, leading to the steady state probability of each state, see Section 16 and Appendix A. In matrix notation if the transition matrix is Q and the vector of steady state probability P then

$$\left.\begin{aligned} &(Q-I)^T . P = (0) \\ \text{and} \quad &\sum_i P_i = 1 \end{aligned}\right\} \qquad (19.2)$$

gives a set of simultaneous equations which can be solved to find the state probabilities P_i. This method is the most flexible because it is able to account for different repair policies, simplifications and approximations are however discussed below.

3) Estimate, using figure 19.1 the throughput availability of the complete system.

4) Calculate or optimize the effects of interstage storage and maintenance. These are discussed below at Sections 20 and 21. For a system without storage, the individual probabilities of various throughput capabilities at each stage will be required, but if storage is fitted it may be useful to calculate the stage total throughput availability (capability).

19.2.3. Special Cases and Approximations

1) Matrix Methods as outlined above, and as detailed in a different context at Section 16, may be used for both standby and active parallel redundant systems with any number of repairmen, see Rau (3.197.) and can even be extended to Pearson's 5 and 7 - item configurations. The size of the matrix for n different items in any configuration each having r states is r^{n+1} square. If, however, the elements are in m out of n standby or parallel and are all identical, the size of the matrix is only $(n + 1)^{r-1}$ square. Furthermore, the matrix will be relatively sparse, having three elements per inner row (column) and only two in the first and last rows (columns). Call such a matrix of state transition rates Q and let the elements of $(Q-I)^T$ be $q_{i,j}$. $i,j = 0,1,2, \ldots n$, where the state numbers represent the number of items failed. Let the state probabilities P_i $i = 0,1,2 \ldots n$ and let $p_i = P_i/P_o$. Then it is easy to show using equation (19.2) that

$$\left.\begin{aligned}
p_o &= 1\\
p_1 &= -q_{o,o}/q_{o,1}\\
p_i &= -(p_{i-2}\, q_{i-1,i-2} + p_{i-1} q_{i-1,i-1}) \big/ q_{i-1,i}\\
P_o &= 1 \Big/ \sum_{i=0}^{n} p_i\\
P_i &= P_o\, p_i \qquad\qquad A_T = \sum_{i=0}^{n} P_i\, u_i
\end{aligned}\right] \quad (19.3)$$

where u_i are the state relative throughput rates and A_T the throughput availability.

2) Active Parallel configurations of identical items can be calculated as follows. Sandler (3.208) and Singh and Billinton (1977) (4.69) give the following formula for the availability of at least m out of n items in active parallel.

$$A = \sum_{i=m}^{n} \left[\binom{n}{i} \mu^{i} \lambda^{n-i} \right] \Big/ (\mu + \lambda)^{n}$$

$$= \sum_{i=m}^{n} \left[\binom{n}{i} a^{i} (1-a)^{n-i} \right]$$

where a is the item availability

$$a = \mu/(\mu + \lambda) = \theta/(\theta + \phi) \qquad (19.4)$$

The individual terms of the summation give the state probabilities. If m items give exactly rated output and all items give the same output under all circumstances then the throughput availability of the stage is seen to be

$$A_T = \sum_{i=1}^{n} u_i \binom{n}{i} a^{i} (1-a)^{n-i} \qquad (19.5)$$

where $u_i = i/m$ for $i < m$ and 1 elsewhere.

Equation (19.5) may also be used to approximate an m out of n standby system where the failure data is in terms of calendar rather than running time.

3) Standby configurations of identical items can be approximated by calculating as in equation (19.5) but substituting α for a where

$$\alpha = \mu/(\mu + m\lambda/n) \qquad (19.6)$$

This approximation always overestimates A_T and using a rather than α always underestimates A_T. Bounds for A_T can therefore be found quite easily.

For the case where m = 1 Sandler (ibid) gives

$$A = 1 - 1 \Big/ \sum_{i=0}^{n} \left[n\, \rho^{\,n-i} / i! \right] \qquad (19.7)$$

where $\rho = \mu/\lambda$

For a 2 out of 3 system with 3 repairmen giving 50% output on one item

$$
\left.
\begin{aligned}
P_o &= \mu^3/Z, \qquad P_1 = 2\lambda\mu^2/Z \\
P_2 &= 2\lambda^2\mu/Z \qquad P_3 = \lambda^3/Z \\
A_T &= (\mu^3 + 2\lambda\mu^2 + \lambda^2\mu)/Z \\
&= (\rho^3 + 2\rho^2 + \rho)/(\rho^3 + 2\rho^2 + 2\rho + 1) \\
&\text{where } Z = \mu^3 + 2\lambda\mu^2 + 2\lambda^2\mu + \lambda^3 \\
&\text{and } \rho = \mu/\lambda
\end{aligned}
\right] \qquad (19.8)
$$

Comparing results for $\mu = 0.2$, $\lambda = 0.001$, $m = 2$, $n = 3$

From (19.5) $A_{T1} = 0.999963$

From (19.6) $A_{T2} = 0.999983$

$(A_{T1}+A_{T2})/2 = 0.999972$

$\left\{A_{T1}(n-m) + A_{T2}m\right\}/n = 0.999976$

c.f from (19.8) $A_T = 0.999975$

The weighted average was also found to be the best approximation in other examples. When the data is in the form discussed at paragraph 18.3.2 above then the average item calendar time failure rate can be used in conjunction with equation 19.5 to obtain approximate answers.

20 INTERSTAGE STORAGE

20.1 Introduction

Intermediate storage between production stages is used in both manufacturing and process industries. Its purpose may be to iron out variation in stage process time or to hold the product of a batch stage which precedes a continuous stage or to decouple series stages which could otherwise be subject to interruption of production for every failure anywhere in the line. This section is concerned mainly with the last of these purposes, firstly because of its more general applicability and secondly because a storage facility which provides effective decoupling against stage unavailability will probably be more than adequate for the other purposes mentioned, see Buzacott(1967)(4.20)

The provision of interstage storage permits production to continue behind a failure until the preceding stores are full and ahead of it until the following stores are empty. In the limit as the stores become infinite the availability of the series is governed by the minimum of the stage availabilities i.e.

$$A = \min_j \left(A_j \right) \qquad (20.1)$$

At the other limit with no storage

$$A_o = \prod_j \left[A_j \right] \qquad (20.2)$$

For any interstage storage capacity vector $S = s_{1,2}, s_{2,3} \ldots s_{j,j+1}$

$$A_o < (A_s) < A \qquad (20.3)$$

where A_s is the availability given S. It is obviously of interest to find the values of vector S which maximise A_s or which minimise a cost function based upon the balance between the increased profit from better availability and the increased initial working capital plus inventory cost. An interesting sub-problem would involve an upper limit on total storage capacity or its cost.

In the context of this thesis, buffer storage can also allow maintenance work to be done without interruption to production. However, stopping stages for maintenance causes an increase in downtime and may have an adverse effect upon effective A_j unless there is a reduction in total downtime as a result of reductions in overall mttm and failure rate.

Whether or not there is a maintenance dimension to any problem, a first requirement is a model connecting S and A_s.

Any solution for A_s which does not take account of preceding and succeeding stages' R & M characteristics and storage facilities is only approximate because it denies interactions that are certainly present. The analytical models found in the literature omit either this factor

or else fail to account for unbalance in the rated or maximum output of stages by means of which it may be possible to fill some of the stores whilst the line is operating at the normal rated output of the slowest stage. It is likely that it is impossible to find an analytical solution to the generalised problem. The last resort of the operational researcher, namely simulation must then be invoked.

20.2 Literature and Taxonomy of Storage Models

20.2.1 General Remarks. The literature on this subject is known to be extensive. The present review and list of papers is not as comprehensive as that devoted to the maintenance models (Chapter III) because less time was spent on it. The literature search was conducted by Mr. R.M. Patel for his M.Sc project (1978)(4.60). The comments on the papers reviewed are however mainly the present writer's.

Three main streams of research may be discerned namely:

1) Analytical Models using simplifying assumptions which allow answers to be obtained quickly but with some loss of accuracy. A useful property of many of these models is that an equivalent availability for the stage plus its downstream store is found. The leader of this school is E.J. Henley.

2) Discrete Analytical Models in which the work in hand is considered to be in pieces rather than a continuous flow. Such models were developed for use in manufacturing as opposed to process industry, but can be adapted for process work by considering the output in unit time as discrete. By changing the unit of time the accuracy can be adjusted to particular requirements. The pioneer in this field, whose analysis has apparently not been bettered is A.J. Buzacott, (4.19-4.22)

3). Analytical Models in which the probability of discrete levels in the store after discrete times given the discrete probability distributions of throughput for the plant preceding and succeeding the store. The research in this direction has been led by D.H.Allen, until lately of Nottingham and now of Stirling University.

4) Queueing Models have been used by Koenigsberg (4.49) and others rather to find the required store capacity than to calculate the effect of any particular store size. Typically, standard or modified queueing theory is used to find the maximum queue lengths between stages with known R & M characteristics. A principal result in queueing theory is that the mean service rate must exceed the mean arrival rate if the queue is not to grow without bound. In terms of the present application this means that the throughput availability of a stage should exceed the combined throughput availability of all the prior stages.

5) Simulation Models Random number simulation has been used to solve complex problems with many storages. Most of these cannot be specifically reviewed because their authors usually fail to describe precisely how they went about the simulation. From experience of the simulation languages CSL (4.29) and ECSL(4.28) in other applications it would seem *prima facie*, that they would be suitable for finding the maximum storage required for full decoupling or the effects of particular storage capacities. Both languages deal in discrete entities so answers for continuous systems would be approximate.

20.2.2 Buzacott (1967) (4.20) proposed a model in which stores are filled only during the downtime of stages upstream. Buzacott's model is also discrete; that is the plant output is considered as a series of distinct items as in an assembly line rather than a continuous stream as in a process plant. Other restrictions in Buzacott's model are that the line is balanced, i.e. the stage outputs are all equal to the rated output, and that a stage either produces at that rate or at rate zero (when blocked or failed). The failure probability function is assumed to be geometric (i.e. discrete constant failure rate). It is tacitly assumed that there is never a raw material shortage and that output is never restricted by lack of storage for finished product. The reasoning, which will not be reproduced here, starts by considering two stages with a store between. It is shown that whatever the store capacity, the maximum proportionate gain in availability for a given total failure rate $\lambda = \lambda_1 + \lambda_2$ results from placing the store such that $r = \lambda_1/\lambda_2 = 1$. This is so whether the repair time distributions are random or fixed time. This axiom is proved for the case where the repair rates μ_1 μ_2 are equal but it is probably approximately true for unequal μ_1, μ_2. The proportionate variation in μ_j is from experience, likely to be small for stages having approximately equal failure rate and in the absence of stage redundancy. The smaller the variance in repair times the greater the gain in availability for a given store capacity. The gain g defined by:

$$g(s) = (A_s - A_o)/(1-A_o) \tag{20.1}$$

is given by:

$$g(s) = h(s)/\left[1 + r\mu_1/\mu_2\right] \tag{20.2}$$

where h(s) is a function of the storage capacity s which represents the conditional probability that the line is producing given that the first stage is down.

For random repair time distributions let

$$q = (1+r-\mu_2)/(1+r - r\mu_1)$$

then for random repair distributions

$$h(s) = (r\mu_1/\mu_2)\,(1-q^s)/(1-q^s r\mu_1/\mu_2)$$

except in the special case where $r = 1$ when

$$h(s)/r{=}1 = s\mu_1\mu_2/\left[\mu_1+\mu_2+(s-1)\mu_1\mu_2\right]$$

or for $\mu_1=\mu_2=\mu$ $\qquad h(s) = s\mu/\left[2+\mu(s-1)\right]$

For a three stage line with two stores Buzacott then shows that g is maximised for equal stores with the line divided into stages such that $\lambda_1 = \lambda_3$ and $\lambda_2 = \lambda_1$ h(s) where h(s) is determined for the two-stage line at r = 1. The method extends to the general case of N stages. Again, ideally $s_1 = s_2 = s_3 \ . \ . \ . = s_{N-1} = s$

$$\lambda_1 = \lambda_N \text{ and } (\lambda_i \quad i = 1, N) = \lambda_1 h(s)$$

$$g(s_N) = (N-1)\, h(s) \,/ \left\{ 2 + (N-2)\, h(s) \right\}$$

For more than two stages it is necessary to assume equal repair rates and r $\simeq$ 1 to obtain an analytical answer.

Buzacott advocates the following policy as being probably optimal with respect to throughput availability. "When no stage is under repair all stages in the line operate and there is no change in store levels. When a stage breaks down the stages after the stopped stage continue to operate until the stocks between them and the stopped stage are exhausted the stock in the buffer following the stopped stage being used first. The stages before the stopped stage continue to operate until the buffers between them and the stopped stage are full, the buffer before the stopped stage being filled first. As soon as repair of the broken down stage is completed all stopped stages begin operating again. Thus the levels only change suring a breakdown".

Buzacott's analysis has been placed first because it is considered to be probably the most useful, combining relative accuracy with fairly simple calculation. The restrictions in the model seem to be less probelmatic than others.

<u>20.2.3 Koenisberg (1959) (4.49)</u> reviewed early work in the subject and identified three main streams of approach.

a) loss transfer in which the fractions of losses due to stage downtimes are transferred to succeeding stages.

b) stochastic, in which the state probabilities are determined from the uptime and downtime distributions of stages and the store capacities.

c) queueing models with maximum queue lengths.

20.2.4 The Loss Transfer Method due originally to Vladziyevsky(1952), (4.71) transfers the production loss in one stage to the succeeding stages. It is possible to find the optimum number of sections into which the system should be divided for a given total storage capacity or cost. Buffer store 'failure' is also included. The losses are transferred forwards only; a major omission in this model is that the effect of filling up the stores behind a failure is not included. Also, the losses are considered additive whereas in fact stages may be down simultaneously.

20.2.5 The Stochastic Method attributed by Koenigsberg to P.C. Finch is not restricted to balanced lines. The stage rated output rates are considered constant at u_i such that

$$u_{i-1} \geqslant u_i \quad \text{for all } i$$

and the failure and repair rates λ_i, μ_i of the ith stage are constant for all i but may be different for each stage.

For m stages there are 2^{m+1} states of stages up or down and associated with each state is a probability that this is the state of the system and also a conditional probability that the system is producing, which is determined by the storage capacity as well as by the R & M characteristics.

Consider a system with two stages and a store of capacity s and a balanced line $u_1 = u_2 = u$.

The gain in availability due to the store is shown to be

$$g(s) = \Psi_j \left\{ 1 - (\Psi_i - \Psi_j)/(\Psi_i/z - \Psi_j) \right\}$$

where

$$\Psi = \lambda/\mu \;, \quad \Psi_j = \min(\Psi_1, \Psi_2)$$
$$\Psi_i = \max(\Psi_1, \Psi_2)$$

and

$$z = \exp\left\{ N \sigma \; (\Psi_i - \Psi_j) \right\}$$

$$\sigma = (\lambda_1 + \mu_2)(\lambda_2 + \mu_1) \;/\; u \left\{ \Psi_1(\mu_1 + \lambda_2) + \Psi_2(\lambda_1 + \mu_2) \right\} \qquad (20.4)$$

In the special case where $\psi_1 = \psi_2 = \psi$ which from Buzacott is known to be optimal

$$g(s) = \psi\left\{1 - u(1+\psi)(\mu_1+\mu_2) \;/\; u(1+\psi)(\mu_1+\mu_2) + s(\mu_1+\gamma\mu_2)(\mu_2+\psi\mu_1)\right\} \qquad (20.5)$$

As with Buzacott's method, there seems to be no inherent difficulty in expanding to a system with three or more stages.

<u>20.2.6 Queueing Models</u> abound, but most are based on the analysis of Hunt(1956),(4.45) Hunt's third case in which finite queues are in front of each stage except the first where the bunker is infinite is appropriate. Most queuing models assume random arrival of items for processing, an assumption which may be regarded as unwarranted and likely to lead to the provision of more storage than is actually necessary.

<u>20.2.7 Simulation</u> Freeman (1964),(4.35) lists the following rules based upon his experience with random number simulation of systems with storage.

a) Avoid extreme allocations but see (b) below

b) The worse a bad stage is the more storage should be placed after it.

c) More storage is needed between two bad stages than between a bad and a good stage. The worse the two bad stages are the greater the sensitivity of the system to the allocation.

d) The optimum relative allocation is substantially independent of the total storage available.

e) The end of the line is more sensitive to changes of allocation than the front . Thus if a bad stage appears late in the line it should be allocated a large share of the total available storage.

Wood et al (1974) (4.74) outline but do not detail a model using linear programming (LP) suitable for multiple-product plants. In this model, for each operating unit, the failure and repair distributions, pm schedules and rated output are known and for each storage unit the capacity and current level. A set of linear inequalities describes the interdependencies between operating and storage units as resource balances and logical conditions. At the start of each simulated day the model determines the starting conditions by sampling the distributions and consulting the pm schedule. The LP algorithm (a standard computer package) then determines the optimal operating rates for the units for that day against a previously assigned objective function. The authors state that the model cannot look ahead to change the daily objective function to suit a long-term aim but there would appear to be no inherent difficulty in making this extension other than the possibility that the LP problem will become unsolvable. It would seem at first blush that daily objective functions could be obtained by feeding the previous days achievements into an algorithm representing the long-term objective. For finite time horizons dynamic programming (DP) might be suitable for this long-term model. Alternatively, the production objectives over a limited time horizon could be represented by notional stores for finished and sold products. (See also 20.3.3 below).

Masso and Smith (1974) (4.54) describe the results of a simulation exercise which assumed different exponential failure and repair distributions in a three-stage two-store line in which all the rated outputs were equal, and there was no redundancy. 27 different configurations were examined, the only common factor being a universal mttr of ϕ = 110 times the time to produce a unit at rated output, applicable to all three stages. mtbf's

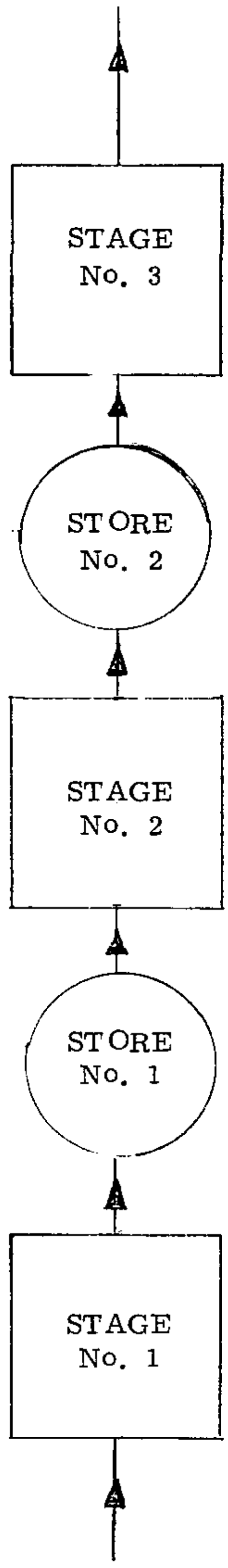


Figure 20 Three-stage, two-store system discussed by Buzacott

were 165, 330 or 990 time units. The important conclusions were

a) the availability increased with storage showing diminishing returns.

b) the ideal storage configuration was taken to be the lowest total allocation that consistently gave an availability response within 5% of A_∞. This was found to be linearly related to the shortfall in availability

i.e. $$s^* = k\,(A_\infty - A_{s^*}) \qquad (20.6)$$

Where s* is the optimum total storage. k is the constant, A_∞ the system availability at infinite storage. The authors give k = 2429.

c) s* was also found to be linearly related to the stage unavailability $(1-A_i)$

$$s^* = k_1(1-A_1) + k_2(1-A_2) + k_3(1-A_3) \qquad (20.7)$$

The authors give $k_1 = 462$, $k_2 = 758$, $k_3 = 943$. Freeman's conclusion that the end of the line is more critical than the start is confirmed by the finding that $k_3 > k_2 > k_1$.

d) With regard to the allocation of s* between s_1 and s_2 another simple relation was found. Let

$$\Delta_{i,j} = \min(A_i, A_j) - A_i A_j$$

for (i,j) = (1,2) and (2,3)

then

$$s_i^* = s^* \Delta_{i,j} / (\Delta_{1,2} + \Delta_{2,3}) \qquad (20.8)$$

<u>20.2.8 Simplified Models</u>. For many purposes it is only necessary to know that a certain store capacity will give at least a certain equivalent stage-plus-store reliability or availability. The exact figures do not matter, perhaps because the stores are available only in fixed sizes or because other data have large potential errors. It is also

convenient for further calculations if a stage with its following store can be considered as a unit with its own mtbf and mttr independent of the rest of the system. It has been demonstrated that this independence is in fact a false concept but the loss of accuracy need not be great in practice. Other uses for such models are to provide starting conditions for an iterative procedure aimed at optimising the vector S of storage capacities and as to check the reasonableness of results obtained by more complicated methods.

Rosen and Henley (4.64), (1974) treat the store as an externally-refilled reserve. They assume that it is always full when called upon to supply the line. They show that in these circumstances the mtbf of the stage + store is given by

$$\theta_s = \left[\mu + \lambda \left\{ 1-\exp(-\mu s)\right\}\right] \Big/ \mu\lambda \exp(-\mu s) \qquad (20.9)$$

for exponential distributions of tbf's and ttr's. R.M. Patel (4.60), (1977) (under the author's supervision) extended this model from Reliability to Availability by finding the equivalent stage + store mttr.

$$\begin{aligned} \phi_s &= \int_s^{\infty} \mu\,(t-s) \exp(-\mu t)\,dt \\ &= (1/\mu) \exp(-\mu s) \qquad (20.10) \end{aligned}$$

It follows that an approximate availability for stage + store is:

$$A_s = \left[\mu + \lambda \left\{1-\exp(-\mu s)\right\}\right] \Big/ \left[\mu + \lambda\left\{1-\exp(-\mu s) + \lambda \exp(-2\mu s)\right\}\right] \qquad (20.11)$$

and that this approximation will be an overestimate for truly exponential distributions. If the repair time distribution variance is less than ϕ^2 (it usually is in practice) then the approximation is better. It should be noted that both mtbf and mttr are modified by this procedure.

Allen and Coker (4.60), (1979) describe a Markov model in which the stores can exist only at certain particular levels after a time interval, having started the interval in another such position. The stage R & M characteristics have constant failure rate and point (constant) repair time. The model is described as a random walk between the reflecting barriers of the store full and empty conditions.

Q is the matrix of state transition probabilities . The states refer to store contents but these are altered by the current states of the upstream and downstream stages which have discrete throughput distributions, thus the store contents change by discrete amounts in unit time. If p_i, i = 1, 2, n are the probabilities of the store being in states 1, 2 . . . n corresponding to levels $\Psi_1, \Psi_2, \ldots {}_n$ then the vector P can be determined according to the authors from

$$Q.P. = P$$

although a more useful formulation for solution would be

$$(Q-I)^T . P = (0) \qquad (20.12)$$

As the repair time is fixed there is a limit to the useful extension of the storage beyond which no increase in throughput availability takes place.

20.3 Suggested Procedure for Interstage Storage

20.3.1 Initial Remarks. The procedures suggested below have not actually been tried, paper exercises excepted, so they must be regarded as tentative and theoretical. They aim to be logical extensions from papers reviewed above which a practising plant operator can use to maximise or raise to a required level the availability of his plant. Interstage storage is regarded as an alternative to redundancy rather than an additional feature, that is a stage may have redundancy or storage but not both, but there may be both storage and redundancy in a plant taken as a whole. It is also assumed that failure and repair time distributions are exponential. From the literature this appears to be a conservative assumption which it is hoped will balance some of the other approximations which tend to optimism. The stage boundaries of a plant are usually dictated mainly by the process itself; it may not be possible to divide the line into many stages of equal failure rate as suggested by the literature results. No model has been found which takes account of the in-process inventory (part of which may be able to act as buffer storage for the previous stage) and no suggestion for doing so appears below. Finally, it is assumed that a prior requirement for a cost minimisation exercise is a model of the effect of storage and other factors upon throughput availability. Methods for cost minimisation are not described as they are likely to be both simple and of only local applicability, when factors such as cost/hazard trade-offs, site restrictions and the market forecast are included. Suggestions are made though about the effect of pm in a line with storage.

20.3.2 Preventive Maintenance Effects. The desired effect of pm is taken to be to increase the throughput availability. To be effective, the reduction in unscheduled downtime afforded by pm must exceed the increase in scheduled downtime. In a system fitted with interstage storage, pm can be done on a stage whilst the following store feeds the line downstream, and, possibly, the previous store is filled up by the previous stages. If both can be achieved then the possibility of complete decoupling with respect to pm exists; the variances of pm times are likely to be very small because the work is fully planned and performed by persons who have done exactly the same job before. The size of stores for facilitating pm without loss of output from other stages is likely to be smaller than those required under an fm policy. In a system under pm the maximum availability will result from sizing the store to accomodate the sum of the maximum requirement calculated on the basis of the observed (as opposed to the base) failure and repair characteristics and the maximum expected in flow or out flow (whichever is greater) during pm routines. This is so that whatever the contents of the stores when scheduled maintenance is started in reverse sequence starting with the last stage, no stage will need to stop because another is under maintenance. Where the failures and repair times are distributed some limit must be placed on the store size such as that which will accomodate say 90% of failure repairs plus that for the maximum pm. Whilst the last stage (n) is being maintained customers are fed from a stock of finished goods from store (n) whilst the rest of the line fills store n-1. Whilst stage n-1 is next maintained, store n-1 feeds stage n whilst the rest of the line fills up store n-2 . . . and so on. Because of the reductions in total stage downtimes and the greater predictability of requirements the total storage capacity required under a pm scheme should be less for the same total system throughput availability than under fm.

20.3.3 Raw Materials and Customers. It is possible to treat the supply of raw materials and the delivery of goods to customers as the first and last stages of the chain and to determine the required size of raw materials and finished goods stocks on the same basis as

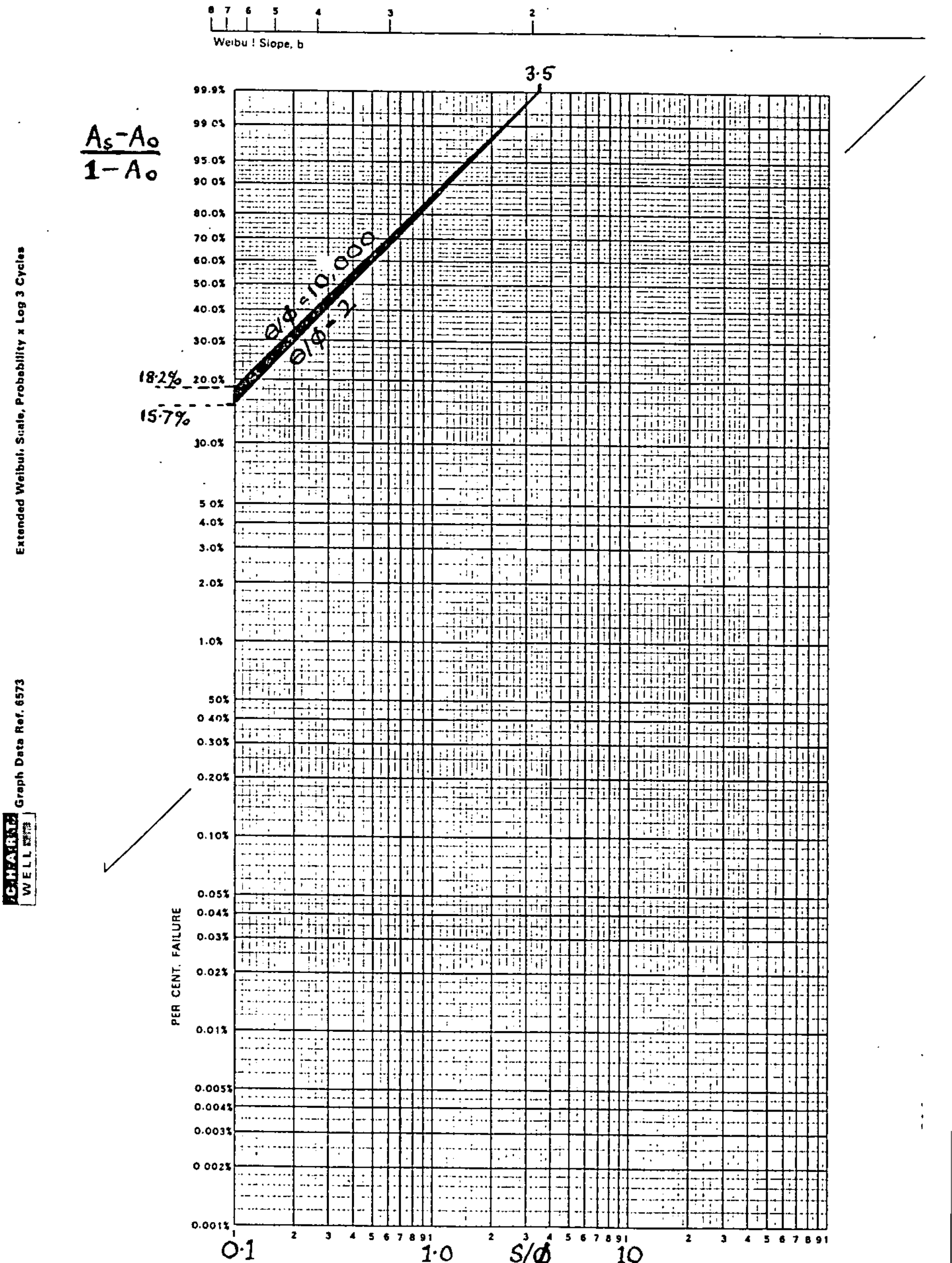


FIGURE 20.3 AVAILABILITY IMPROVEMENT BASED ON EQUATION 20.11

the process interstage stores. Perhaps a more useful exercise would be to regard the whole production line as a single stage with a store and pseudo-stages on either side of it representing raw materials supply and customers. (More useful because the fluctuations in demand and raw material bulk supply may be on a much greater time-scale than those in the manufacturing line.) With regard to raw materials the store capacity calculated would be that required over and above the simple calculation of consumption between deliveries to cover lateness in deliveries statistically distributed as to both frequency of occurrence and duration (delay) in the same sort of way as failures to a previous stage.

20.3.4 Properties of Equation 20.11. An investigation (by the author) of the properties of equation 20.11 led to the dimensionless relationship shown in Figure 20.3. It was found that for any particular value of $\rho = \theta/\phi = \mu/\lambda$ a straight line was obtained on Weibull paper and that 99.9% of the possible availability improvement $(1-A_0)$ was always obtained for a store which would take 3.5 times the mttr to empty at rated output. Values of ρ from 2 to 10,000 were investigated with the rather simple result shown on the figure. An interesting feature is that the percentage improvement in the availability of the stage-plus-store is significant even for $S/\phi = 0.1$. Note also that ~85% of the possible improvement results from a store which is able to cover repair times up to the mttr. In practice, chemical plant mean repair times are about 6 to 8 hours on moving machines such as pumps which fail frequently relative to static plant such as pressure vessels, and the variance of ttr's is usually smaller than ϕ^2. From this it is concluded that stores holding about 1 day's supply would usually be adequate.

20.3.5 Preliminary Storage Allocation. When a production line of several stages is to be upgraded in throughput availability by means of interstage storage it is suggested that the preliminary allocations, to be improved as necessary by techniques discussed in later paragraphs, are made as follows:

a) Decide between which processes storage is technically feasible.

b) Establish the cost or quantity boundaries upon total storage and any limitations at particular stages. e.g. restrictions on inventory because of poisonous or combustible substances.

c) Establish the R & M characteristics of the stages without storage. A stage is all the processes between adjacent possible or actual storage locations.

d) Use Figure 20.3 to find preliminary storage capacities such that the stage-plus-store throughput availabilities As_i are roughly equalised subject to any constraints imposed by (b) above, and so that their product is about equal to the target system total throughput availability.

e) Compare the costs of these stage allocations with those of redundant and partially redundant configurations.

f) Decide preliminary redundancy and storage allocations.

20.3.6. Improvement of Storage Allocations. If it is important to have a more precise prediction of the system throughput availability than could be provided by the procedures outlined in the previous paragraph then it will be necessary, for more than 3 stages and two stores, to use iterative procedures in conjunction with a simulation or analytical model.

In a simulation model the store capacities should be systematically varied starting with the last store and working back through the line finding the minimum cost or maximum availability configuration at each stage. This procedure should be repeated until there is no further change in the capacities recommended by the successive optimisations.

In analytical models such as those of Buzacott (4.20) or Finch (4.49) only two stages and a store or three stages and two stores can be dealt with at a time. Starting with the last stage it should be possible to treat the whole of the rest of the plant as a single stage and find the store capacity required between it and the last stage (or the penultimate stage in a three stage model). Calculations would then move one stage upstream and the last two stages would then be lumped. At each pass up the line the estimate of the lumped stages' total throughput availability would be improved and this would bring the calculated required storages S_i closer to optimality.

For the first two or three passes the model of Masso and Smith (4.54) might be used in the same way, working back through the line against the direction of production and treating all the stages in each direction beyond the two stores currently under review as single stages having R & M characteristics as calculated in the previous pass or derived from Figure 20.3.

CHAPTER V CONCLUSIONS

21. CONCLUSIONS FROM CHAPTER II - DATA SYSTEMS AND ANALYSIS

21.1 Data System Structure and Capabilities

From the writer's experience in the Royal Navy and the results of the studies reported in Sections 10 and 11 it is concluded that there are financial and other benefits to be gained from applying the information gathered by a maintenance data system. Furthermore, the system should be as comprehensive as possible. The mounting of special detailed data collection exercises to investigate cases where a primary statistic such as mtbf is outside the normal or acceptable range is the alternative, but it is doubtful whether the marginal cost of extra data collection outweighs the losses which would occur during the longer diagnosis period.

The cost of maintenance and reliability data collection should be booked as marginal to the cost of the essential plant management information system.

Plants with good data systems were seen to have higher availability than those with poor systems and those with any system were better than those with none.

Once it has been decided to computerise the system, the arguments for making it as comprehensive as possible become stronger. The initial investment, even when costed as marginal is greater and so must be justified by greater capability in automatic data analysis. The only computerised system studied (see Section 11.2) was still under development and although able to produce the data for Pareto analyses, could not give tbf's and ttr's for frequency analyses. A computerised system, it is concluded, is an expensive toy unless it is both comprehensive as to recording and fully automated for analyses, both Pareto and frequency, both of failures and repairs (but see 21.3.2 below). The Petrochemical plant system could and should be developed to include these missing capabilities.

Where data systems existed, it was noted that the data were often not formally analysed. Although the existence of the records often meant that the major problems were recognised, none of the plants studied took full advantage of the opportunities for analysis to solve them. This was mainly because the responsible engineers had little or no training in Reliability or Operational Research.

Good morale is vital to accurate data form - filling and can be achieved by feeding back helpful information from the computer outputs to the maintenance staff to help them recognise their strengths and weaknesses and give them a chance to contribute to the process of improvement.

A principle of the design of computerised data systems should be that wherever possible the burden of filling in forms should be eased by programming the computer to perform the required calculations using past data in store as well as the current failure or other report. e.g. it is in order to ask for the time the failure was discovered, to be filled in on a form but not how many hours since the previous failure; the latter information should be obtained within the computer by subtraction.

Trade Unions were observed to be suspicious of the form-filling involved. It is therefore vital that the purposes of the data collection are fully and simply explained first to shop stewards and later directly to the workers. Assurances should be given that no job will be lost and that the purpose is to help everyone do a better job creating more wealth for distribution to themselves as well as shareholders.

Efficient maintenance was observed to require usually more rather than fewer staff so this is not an empty promise.

21.2 Ratio of Preventive to Corrective Maintenance - How far to take pm.

In a capital-intensive industry such as heavy chemicals or petrochemicals, plant availability in older plant was judged to be vital to survival in a competitive world where if a plant was seen not to be giving as profitable a return on investment as a new plant might, then it would be shut down. Profit for a plant with constant output rate and a ready market for its products rises linearly with availability above the break-even point. Greater and more predictable overall availability results in better delivery and leads to better customer relations. There is a chronic shortage of good maintenance labour, but if they can be recruited, extra maintenance staff are a good investment as long as they can raise availability.

Results tend to support the widespread opinion that for minimum overall cost about 60% of the maintenance workload should be planned. In individual plants the optimum proportion may well be higher. Whilst a preventive routine is inevitably performed more often than the corresponding failure repair under fm, the planned shut down usually results in less overall loss of production than unscheduled failures.

In simple systems such as those discussed below (Section 22) and in Chapter III analytical optimisation techniques may be used but in a system as complex as a whole plant it was concluded that after a bold start from a fm situation, further increases in the pm/fm ratio, and increases in maintenance staff needed to cover them should be small and their effects measured against an objective criterion such as the increase in availability, or more broadly financial return, before deciding whether a further small increase might be advantageous

21.3 Methods of Analysis

21.3.1 Pareto Analysis was employed in all the studies. If the term is interpreted fairly loosely. A full Pareto analysis is a staged operation as was carried out for the petrochemical plant equipment (Section 11.2). It consists of narrowing the area of search progressively by identifying the classes of items of plant then specific items and finally specific failure modes which cause the majority of failures. The Pareto principle is often stated in terms of the great preponderance (sometimes 80% or 90% is mentioned) of failures occurring to a very small number (sometimes 10% or 20% is mentioned) of items. On the evidence of the studies it is clear that this is sometimes but certainly not always the case under fm. Under any sort of preventive maintenance the principle does not appear to apply as starkly as it is often stated. Nevertheless it is possible to pick out a few modes of failure which are significantly more frequent than the others. This softening of the principle is hardly surprising because the usual objective of pm is to reduce failure rates. It follows that pm routines will be directed against the most common failure modes. Ideally, the maintenance schedule should aim to reduce failures such that the total cost, including lost production or downtime cost, of preventive routines and residual failures is minimised. To achieve this aim it is necessary to analyse the data for distribution of times between failures as well as by modes.

21.3.2. Frequency Analysis Pareto or failure modes analysis is necessary to identify the principal causes of failure but analysis for distribution or hazard rate is needed to find the most advantageous schedule of maintenance to limit the frequency of such failures. A pre-requisite of the methods of optimisation discussed in Chapter III is knowledge of the failure pdf. Because maintenance must be specific to particular features of a complex equipment it follows that both failure modes and frequency analyses are necessary in general.

On the whole, graphical methods of frequency analysis were considered preferable to computer-based analyses, because they allow engineering judgement to be exercised by the analyst. However, an exception is made in the case of interactive computer suites which present graphs to the analyst.

For the large data sets involved in the reported analyses, cumulative hazard analysis was first tried. In some cases this method was not able to discern a bi-modal situation as well as a cumulative frrequency plot. The method adopted in the later analysis at Section 11.2 and 11.3 was to find the mean order numbers and median ranks by computer and then plot the results by hand. This is now considered to be the best method. The data should be held as calendar failure times and identified by equipment type and mode of failure. From this data tbf's for any combination of equipments and modes can be calculated by the computer which then goes on to find mean order numbers and median ranks.

Small data-sets should always be plotted with confidence limits. The Weibull or other plot otherwise gives a false impression of accuracy.

Excessive extrapolation of plots should be avoided. A car which has reached Staines from London is not necessarily proceeding to Plymouth. In the context of the data analysed in Chapter II, this means that in general the plots should be mistrusted unless the data collection period considerably exceeds the sample <u>mtbf</u>.

21.4 Maintenance-Induced Failures

The evidence of the major and minor studies taken together with the theoretical considerations of Appendix B2 and B3 is that provided the data collection period considerably exceeds the mtbf, a tbf distribution analysis with hyper-exponential characteristics (Weibull $\beta < 1$) is prima facie evidence of room for improvement in maintenance methods. It is not proof that the maintenance is being done incompetently, but it suggests that the maintainers may require some help in the form of supervision or training.

Extension of the theory in Appendix B would suggest some danger of maintenance induced secondary failures following preventive routines. This danger would increase as the maintenance routines increased in complexity from servicing through parts renewed to complete overhauls. There was some direct evidence of this in the petrochemical plant results (Section 11.2) where repeat overhauls (less than two months after the first) were not as uncommon as they should be in a careful maintenance organisation. The author's experience of ships in the immediate post-refit period suggests that the phenomenon is by no means confined to the plants or industries investigated. Many motorists have experienced the car which goes wrong just after the garage has had it for major or even minor preventive maintenance.

To cure the problem of secondary failures it must first be recognised for what it is. Distribution analysis of data will show that $\beta < 1$ and this should be coupled with observation of maintenance methods, technical investigation of failures or both. Technical investigation is in the nature of forensic science and it is not easy to obtain irrefutable results. This was known to the author from previous experience and it was for this reason that direct observation of fitters at work was preferred despite the danger that the results would be affected (for better or worse) by the presence of the investigator. It is concluded that an effective means of showing that failures are being induced by unsatisfactory maintenance is this quite powerful combination of indication from $\beta < 1$ followed by direct observation. Independent observation should reveal the specific causes or else shame the fitters into doing better work, the effect in either case being beneficial. It can also reveal the need for training or re-training inadequate personnel, both tradesmen and supervisors.

It was noted where preventive maintenance was introduced but little was done to improve standards of work (as in the chemical plant study of Section 10) that there was sometimes an increase in mtbf without significant change in the Weibull β-value. A theoretical explanation of this observation, which needs to be confirmed practically in a few more cases, is that there is a reduction in primary failures but that roughly the same proportion of primary repairs result in secondary, i.e. maintenance-induced, failures. In these circumstances it is actually possible for failures induced by poor preventive maintenance to have the effect of raising β. Where pm events are not considered as failures in the analysis, any secondary failures which regularly follow a periodic routine would, by themselves form a peaky ($\beta > 1$) distribution with a mean at approximately the pm interval. The practice at the petrochemical (Section 11.2) plant of recording overhauls on the same print-out as the failures is therefore to be applauded. A further implication is that although an increase in both β and mtbf is prima facie evidence of improved standards of maintenance and the efficacy of pm, it is not proof thereof. However, pm work is less hurried, better planned, more familiar and generally less exacting for the fitter. There should be therefore less likelihood that pm will lead to a secondary failure than is the case for failure repair which may be carried out under conditions not conducive to good workmanship.

To help to ensure that pm routines are well done and so do not lead to early failures two methods suggest themselves as worth trying.

a) Pre-closing inspections by another fitter or a supervisor. From the author's experience it is known that this is effective for marine machinery. The majority of marine engineers would probably consider it a necessity.

b) Provide clear and precise instructions as to how the pm routine should be done, by what grade of labour and using what tools. Time allowances should err on the side of generosity to ensure against haste. This has also been tried at sea but with what effect on the early failure rate is not known. Its proportionate effect would probably be greater in the chemical plant environment where the present standards of fitting are comparatively low and where supervision is less strict. Incentive schemes for maintenance should reward good rather than fast work, as measured by early failure rate.

Some kinds of maintenance induced failure can be prevented by design or re-design. Any reduction in primary failure rates of components will of course remove the trigger for secondary failures but it is also possible to make misassembly impossible, provide easy means and checks for shaft alignment. Efforts to improve Maintainability can therefore be doubly rewarding given that it is unlikely that standards of workmanship can be improved other than over a very long period.

The presence of maintenance-induced failures jeopardises attempts to seek out the causes of other failures. These unnecessary events distort both Pareto and frequency analyses. It is concluded that maintenance and reliability improvement efforts at existing works would be directed first towards eliminating early failures by the methods outlined above. When data substantially free from early failures have been obtained it will be possible to progress to the optimisation of the maintenance schedule. Whilst it is possible to separate the early from the ordinary failures on a probabilistic basis, existing procedures for this are by no means precise. The resulting estimates of the distribution parameters of the ordinary failures must be regarded with considerable suspicion.

21.5 General Conclusions

There is little doubt that fm is seldom if ever the optimum policy for a repairable system subject to several failure modes. The introduction of pm to the system operated under fm was shown to be financially worthwhile and to produce an increase in availability.

Overall mtbf and availability were shown to be sensitive to changes in the <u>pm</u> policy and schedule.

By themselves, Pareto analysis and distribution analysis can be misleading. In combination they become more powerful but where maintenance is concerned the picture is not complete without direct observation of methods and organisation. As always, statistical analysis is a tool which can be mis-used. Its results should be questioned when they offend against common sense and experience. Alternative explanations should be sought involving engineering as well as mathematical reasoning.

21.6 Conclusions Drawn in Joint Paper Submitted for Publication.

The following conclusions are taken from the paper by Sherwin and Lees (2.48) which covers the work reported in Sections 10 and 11.1.

1). Maintenance investigations which are based both on collection and analysis of data and on observation of maintenance practices can yield worthwhile information on which to base modifications of maintenance policies and practices.

2) The quality of the data collected should be as high as practicable. Both the design and the operation of the data collection system are therefore important.

3). The starting point of the analysis is normally the determination of the overall failure rates of the equipments and of the failure rates in particular failure modes, but it is very desirable to extend the analysis to the determination of the failure regimes (variation of hazard rate with time) both for the equipment overall and for the individual failure modes. The determination of a failure regime requires data on times between failures.

4) Information on the failure regime can be determined by Weibull analysis and is particularly useful in formulating maintenance

modifications.

5) The cumulative hazard method of determining Weibull parameters provides a convenient alternative to the conventional method, particularly for large data sets, but appears to be less effective in identifying variations such as bimodal distributions.

6) The hyper-exponential distribution is another useful tool for the analysis of the failure regime.

7) The early failure regime (shape parameter $\beta < 1$) appears to be particularly prevalent in process plant equipment. The determination of an early failure regime may be regarded as prima facie evidence of maintenance deficiencies, but it may be partly an artefact of the analysis and, if real, partly due to other causes.

9) The β-value is inaccurate if the ratio of the observation period to the mtbf is low.

10) There is a large number of potential causes of early failure, involving both maintenance and non-maintenance features. The maintenance features include incorrect fault identification, incorrect repair technique, incorrect replacement parts, incorrect assembly and dirty working conditions.

11) If the existing maintenance policy is mainly breakdown maintenance and if there is significant plant downtime, it may be possible to obtain significant reductions in downtime by greater use of preventive maintenance.

12) Preventive maintenance policies should be an appropriate mix of periodic preventive maintenance (ppm) and on-condition preventive maintenance (ocpm).

13) In some cases preventive maintenance policies may involve more frequent stoppages, but may still reduce downtime because the average

downtime period is much less.

14) Improvements in preventive maintenance may lead to reduction in the number both of early failures and of later failures.

15) Improvements in maintenance practices may lead particularly to reduction of early failures.

16) If the mtbf is low, improvement in maintenance will increase it. But if the β-value is less than unity, improvement in maintenance will raise it towards unity if there is a more than proportional reduction in early failures. If the β-value remains much the same, this is likely to mean that any reduction in early failures has been due to the reduction in breakdown maintenance demands rather than to improvement in maintenance practices.

17) Even if a real differiential reduction of early failures relative to later failures and thus an increase in the true β-value, has been achieved, this may be difficult to determine. In particular the increase in mtbf requires a proportional increase in the observation period if the β-value is to be estimated to the same accuracy.

22. CONCLUSIONS FROM CHAPTER III - MAINTENANCE OPTIMISATION MODELS

22.1 Theory and Application

A great number of models exists but reports of successful applications are few. It is concluded that the models are not reaching or are not understood by many who should be using them in their daily work. It is possible, but less likely, (from admittedly limited observation) that application outside large organisations is more widespread than the amount of literature would suggest. It is known that even specialist maintenance consultants do not recommend or even comprehend the full variety of techniques available. Money could certainly be saved by wider application of existing techniques.

22.2 The Constant Hazard Rate - No Maintenance Fallacy

. Where the message is simple and advises less immediate work it is understood and applied without question. In some cases this leads to misapplication of theory. Typical of this phenomenon is the widespread belief that constant failure rate implies the optimality of fm. As explained in Appendix B, the expected distribution of times between failures to a maintained complex item, particularly under scheduled maintenance is exponential. i.e. the hazard rate, superficially is constant. However, this failure pattern is composite and is sensitive to changes in pm schedules directed against individual failure modes.

There probably occur some failures which are the direct result of truly random events with Poisson characteristics. If the failure is immediate ot its imminence following the random initiating event is undetectable then and only then is there no possibility of effective pm. In such cases (thought to be fairly rare) ppm, ocpm and ccm are all unable to prevent failures. All other cases are amenable to relaibility and availability improvement by one kind of pm or another. Whether higher availability resulting from pm is financially worthwhile is another question dealt with separately below.

22.3 Constant Interval-Risk Model

Models in the literature which are closest to that described in Section 15 differ from it in one essential way. They conceive of inspection as an operation to discover failure rather than to prevent it. The novelty in Section 15 lies in facing squarely the maintenance manager's problem of how to schedule inspections designed to prevent failures. It is emphasised that inspections of this kind rely upon engineering judgement and so some assumptions must be made in order to obtain answers. In this case the enabling assumption extra to the failure-discovery model requirements is that when the inspector decides to maintain rather than continue to the next inspection he is always right. This makes for a simple model which could be modified later to account for an overall or individual inspector's correctness of judgement measured objectively by allowing some failures to occur.

Having made this assumption modelling methods can be as for the failure-discovery case. The further simplification of equal-risk inspection intervals was made because it was observed from comparison of the work of Barlow and Proschan(3.19) and of Munford and Shahani (3.169,3.170) that the loss of optimality was small. It was further felt that the inaccuracy of this assumption was likely to be less than that arising in practice from inspectors error.

In practice the p versus r relation must be found by trial and error. It is only a model to allow an answer to be calculated and later refined and needs to be accurately known only in the region of optimality.

Referring to the comments at 15.6.2. the following conclusions are drawn.

a) From experience it is thought that the practical savings arising from initial inspection would be greater because a unimodal distribution like the Weibull cannot represent the presence of initial faults in materials and workmanship. That there is usually a saving in theory as well is a bonus. Note also that the comparison is between two optimum schedules not of the same schedule with and without initial inspection which would have given a greater saving.

b) In the $\beta < 1$ case initial and early inspections mitigate the cost-effects of early failures by effectively altering the distributions of failures. In this and other cases it is most important to keep the estimate of the underlying or base distribution under constant review.

c) It has always been assumed that where $\beta > 1$ the optimum <u>ppm</u> schedule should be found and used. The results of the present work show that one cannot be sure without calculation that ocpm is not cheaper. At the outset, with new equipment, f(t) will not be known. An ocpm schedule is at first safer and will lead to an earlier and more accurate estimate of f(t) upon which the decision between ocpm and ppm can be made and the schedule based.

d) The computed results show that the sensitivity of cost rate to p and r is often not great. This is important because these two quantities must at the outset be subjectively estimated to provide an initial schedule. The costs C_F, C_M, C_I, need be known only in terms of their relative values; $(C_F - C_M)/C_I$ and C_F/C_M are ratios which are based upon readily-understood concepts. Also, the relative values are likely to change less (and less often) than the absolute.

e) The additional work in Section 17 makes approximate calculation easier in the absence of a computer. This is based upon the stability of E(T) over a wide range of base distribution parameter values. The implication is that within the bounds of the model as an upkeep policy, minimising cycle costs is almost as effective as minimising cost-rate.

22.4 Markov Models

In theory first order Markov models based on transition rate matrices are useful only when the failure, repair and other rates involved are constant. However, most of the required results depend largely upon ratios in the form $\mu/(\mu+\lambda)$ or similar. It is well known that such ratios tend to constant values with time and that they usually settle very quickly. The form of the distributions of failure, repair and other times therefore matters less in practice than is immediately apparent. First order Markov models provide simple if approximate solutions to otherwise tedious problems.

The frequency enhances the significance of comparative results based upon Markov models such as those developed in Section 16. It is unlikely that a decision between ccm and ocpm based on constant μ's and λ's would be changed by more accurate modelling of non-constant failure rates etc. unless that decision were very close. Where results are anyway fairly accurate rankings would also tend to be accurate.

In any case non-constant failure rates were shown in Section 15 to require non-periodic inspections for minimum cost-rate schedules. This is considered impractical unless the inspection rates are changed only by relatively infrequent stages. This in turn is equivalent to an assumption of locally-constant inspection rate. It was also shown by worked example that whether the inspections were Poisson or periodic made little difference to the optimum frequency or the resulting cost-rate. The practical position is usually somewhere between random and periodic inspections. It follows that a Markov model which is able to represent other features such as delay in starting maintenance is probably more useful in practical problems than the model in Section 15.

It is concluded that the models of Section 16 would be applicable to a large number of practical problems in industry.

23. CONCLUSIONS FROM CHAPTER IV

23.1 Large Single Stream Plants - Redundancy

The arguments in favour of large single-stream plant are mainly concerned with initial cost rather than running cost. There is, however, less technical risk associated with smaller and multi-stream plants. The specialised nature of the plants often means that reliability testing of new designs for large plant must be omitted and this can and has led to the building of plants in which so much commissioning trouble was experienced that they can never make a profit overall for their owners. From a reliability viewpoint, the steps in size increase should be smaller. It is quite possible that multi-streamed plants could be built a stream at a time using profits from one stage to finance the next and advancing reliability on the basis of operating experience. The other advantage of the partial redundancy which can be provided in a multi-stream plant at little extra cost. Large machines in single stream plants are likely to be considered. Nevertheless, standby plant for a single stream plant should always be considered at the planning stage. It is unwise to decide upon a particular plant layout without first considering the expected availability and the confidence to be placed in the estimate.

23.2 Intermediate Storage

An alternative to redundancy is the provision of intermediate storage. Whilst this increases the inventory costs it may be cheaper than redundancy for the same degree of protection against the effects of failure. The matter should be settled on the basis of through-life costs and the hazards associated with large inventories.

First-order calculations of the effects on availability and costs of intermediate storage are relatively easy but in cases where the decision between redundancy and storage is close, more precise methods may be needed. Where storage is available in fixed capacities only, rough methods will usually suffice to reach a decision, but in a free-ranging study accurate methods are required.

Attention is drawn to Freeman's (4.35) conclusions listed at 20.2.7 which are agreed.

In a simplified analysis based upon the availability of a single stage and its externally-filled downstream store, it was shown that returns in Availability from increased storage diminished, and that a store of sufficient capacity to cover a repair taking 3.5 times the mttr (Figure 20.3) would cover 99.9% of repairs whatever the ratio mtbf/mttr, provided that the distributions of tbf's and ttr's were not hyper-exponential. Approximately 85% of the inherent unavailability of the stage could be removed by having a store able to cover an average repair time. As industrial repair times are usually of the order of 6 to 8 hours it was concluded that stores holding more than about 1 days supply would not be needed and that a great deal of unavailability can be prevented by relatively small stores, provided that the refill time is reasonably short.

Preventive maintenance routines usually take on average less time than failure repairs. A storage facility designed to cover failures should normally be more than adequate for pm. Regular pm would afford periodic opportunities for refilling stores depleted by failures.

Unless stores are externally filled, the throughput availability of a line is limited to no more than that of the weakest stage. In the limit, the stores are of infinite capacity for total decoupling.

Although analytically exact methods are available, their conditions are not always practical. Simulation is needed to solve the general case. However, initial allocations can be made by quite simple methods and may be of comparable accuracy to the basic data in the planning phase of a new plant. A suggested procedure was given at paragraph 20.3.5.

24. CONCLUSIONS BASED ON MORE THAN ONE CHAPTER

24.1 Efficacy of Preventive Maintenance

It was shown both theoretically and practically that pm was able to reduce the operating costs and enhance the availability of various process plants. This was the case even where failure rates were apparently constant and especially where they were apparently falling. Far from being a firm indication that no maintenance was optimal, which from the start appeared contrary to engineering experience and common-sense, Weibull $\beta < 1$ was shown to be prima facie evidence that maintenance was being poorly done. It was not possible to confirm this conclusion without both the practical evidence of Chapter II and the theoretical evidence of Chapter III and Appendix B.

24.2 Need for Data Collection

It is very difficult to prove that data collection is worthwhile. There is always the chance that the collection cost exceeds the cost difference between a schedule based upon experience, guesswork, makers schedules etc. and that of an optimised schedule based upon data collection analysis and modelling. What is more certain is that it is virtually impossible to find the best schedule without keeping formal records. Optimal schedules can be devised for all failure distributions with a rising failure rate and for others where some warning of impending failure was given or where the individual modes in a composite distribution had rising failure rates. It is concluded that the need is to prove that fm is optimal in the rare cases where that is the case rather than to justify pm which in one form or another is almost always the cheapest policy. Justification, either way, requires the collection of data both qualitative and quantitative.

24.3 Why is Preventive Maintenance Rejected?

It seems odd that pm should be so little applied in the chemical industry when it is seen to be efficacious where it has been used. It is appropriate at this stage to seek reasons and to speculate a little in the absence of hard evidence.

The misconception about the 'bath-tub' curve and the O.R. solution to the renewal problem are undoubtedly partly responsible for some firms abandoning pm. In two cases the author was so informed. There is unwillingness amongst maintenance engineers and even some academics to accept that the usual agrument ignores some of the facts.

This era has been called with some justice the 'throwaway age' and the chemical industry must bear some of the responsibility for that attitude which presently permeates our lives. So it is partly habit. It is a habit which we will have to change both in the general and the particular very soon. Shortage of energy will mean making useful objects last longer and no doubt this will include even chemical plant which currently suffers much neglect.

The ratio of value added to process costs in the chemical industry is obviously high in general. In some products it is difficult not to make a handsome profit. This engenders carelessness of the machinery which can be renewed relatively cheaply - or the whole plant can be scrapped and a more efficient one built.

In other cases high demand has led to abandonment of pm, initially as a short-term measure to meet a contracted date; but this is followed typically by other 'emergencies' until failure rates rise to meet the capacity of the repair staff to make repairs and there is no spare labour for pm.

Another factor may be that government aid and tax concessions are generally available to build new or replace used plant but not for making one last a few years longer by extra maintenance expenditure.

24.4 Overall Conclusion

Reliability and Operations Research theory can be applied to maximise availability or minimise costs at process and other plants, but it is most necessary to have a sound data collection and analysis system upon which to base the optimisation.

APPENDIX A

RELIABILITY THEORY AND TECHNIQUES

A1 INTRODUCTION

A1.1 Origin - BS 5760

This appendix arises from the author's draft for the forthcoming British Standard Guide on Reliability BS 5760 Part 2. 'Reliability Methodology'. The full draft runs to about 100 pages so only sections which are useful in the present context are reproduced, and these have been somewhat abridged. Where matter has been covered elsewhere, as for example the 'bathtub curve' in Appendix B, the relevant section from the draft has been omitted.

A1.2 Purpose

The purpose of the Appendix is two-fold:

a) To provide a handy reference to established theory and techniques used in the thesis. In drawing up its plans for new standard, BSI committee QMS 2/3 of which the writer is a member, recognised that some extant reliability texts were partly unsound and that others were getting out-of-date. The purpose of BS 5760 Part 2 is to provide an up-to-date if conservative statement of the 'state of the art'.

b) To illustrate the extent to which the BSI has moved towards recognition of the importance of Maintenance in Reliability since the publication of BS Drafts for Development Nos 10-16 which were the starting point for the new BS.

A1.3 BS 5760 Part 1

The first part of the new BS will probably be published by the time this thesis is examined. It is concerned with Reliability Management. Here also there has been a shift of emphasis to include more consideration of the maintained situation than was included in DD's 10-16.

A2 STATISTICAL TERMS IN RELIABILITY

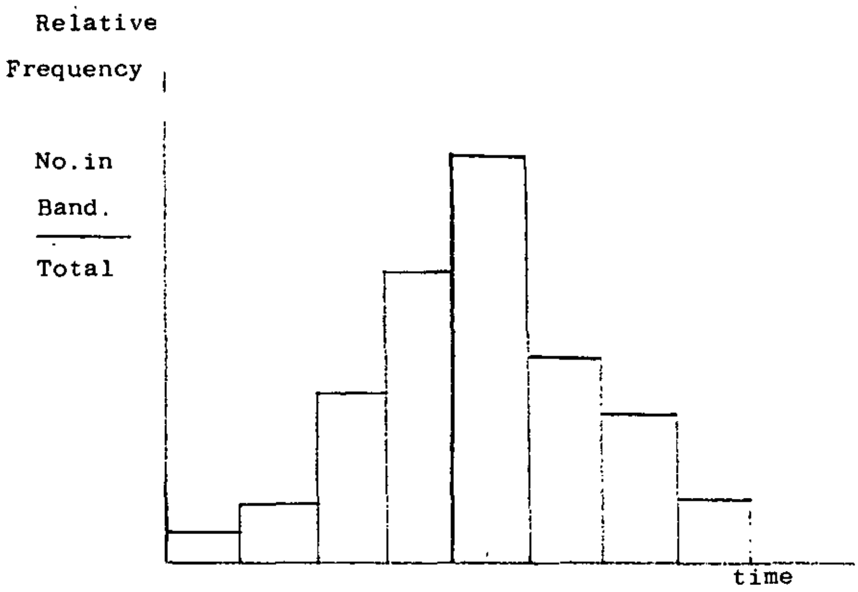
A2.1 Distribution (See BS 5532(2.17,2.18,1.30-1.48)

Reliability work is generally concerned with the distribution of times to failure, or in the case of repairable (maintained) systems, times between failures. Apart from one-shot devices such as ammunition, engineering items are designed to endure for a period of time (or a number of cycles or a distance run or some other appropriate variable).

Due to variations in the construction of a series of nominally like items and further variability of treatment in use, times to or between failures also exhibit variation. Frequency analysis shows some sections of the possible range of time to be more popular than others. For convenience of calculation the frequency analysis is often fitted to curves described by mathematical functions called probability density functions (pdf's). A pdf is the curve of the probability that a random item from a large population of like items will have a certain life or time between failures. A pdf can be estimated from a random sample of times to failure of individual items by arranging the times in bands of equal width. This histogram is then smoothed to estimate the pdf. Conceptually the pdf is the theoretical shape of the curve as the sample becomes infinite and the time bands infinitesimal. The total area under a pdf is always unity and is represented by $\int_{-\infty}^{+\infty} f(t)\,dt$

$$f(t) = f(a,b,c....t) \text{ where } a,b,c \text{ are constants called parameters} \quad \text{(A2.1)}$$

The integral of the pdf f(t) is the proportion of an infinite population which fail before t. This is called the (Cumulative)Distribution Function or Cdf. Reliability is the probability that an item does not fail. In the case of one-shot device it is simply the proportion which operate successfully, and in the distributed, time related, case reliability is the complement of the Cdf.



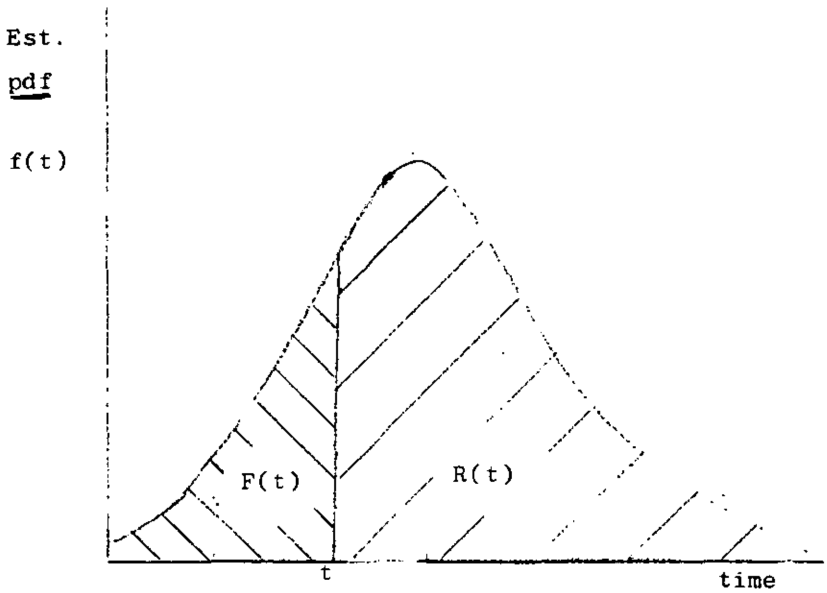


Figure A2.1 Histogram of Relative Frequencies and estimate of probability density function showing cumulative distribution function and reliability

Thus:

$$\text{Cdf} \quad F(t) = \int_0^t f(x)dx \qquad (A2.2)$$

Reliability function $R(t) = 1 - F(t)$ (A2.3)

Reliability of a one-shot device $R = (1-p)/p$ (A2.4)

Where p is the proportion which fail to operate.

A2.2 Moments of a Distribution (See BS 5532 1.20-1.22, 2.36,2.37)

If the values of a pdf are each multiplied by a power r of the distance from some fixed point a on the time scale

thus:

$$\mu_{a,r} = \int_{-\infty}^{+\infty} (t-a)^r f(t)dt \qquad (A2.5)$$

then $\mu_{a,r}$ is the rth moment of f(t) about a. Usually the moments are about the mean (central moments) or the origin. The mean itself the first moment about the origin. The second moment about the mean is the variance, and its positive square root is called the standard deviation.

$$\theta = \mu_{1,0} = \int_{-\infty}^{+\infty} t\ f(t)dt \qquad (A2.6)$$

$$\sigma^2 = \mu_{2,0} = \int_{-\infty}^{+\infty} (t-\theta)^2 f(t)dt \qquad (A2.7)$$

Dimensionsless coefficients without reference to scale can be used to compare the shapes of different distributions, viz.

σ/θ is the coefficient of Variation

$\mu_{3,\theta}/\sigma^3$ is the coefficient of Skewness

$\mu_{4,\theta}/\sigma^4$ is the coefficient of Kurtosis(peakedness)

A2.3 Hazard

Statistical hazard is a fundamental reliability concept. If n items start to operate at $t = 0$ and r_1 of them fail and w_1 are withdrawn from service for other reasons before t_1, and r_2, w_2 fail and are withdrawn before t_2 then the hazard $Z_{1,2}$ for the interval t_1 to t_2 is defined as the number failing divided by the number at risk. Strictly, the number at risk used in a calculation of $Z_{1,2}$ should be the average number for the interval.

i.e.

$$S_{1,2} = \tfrac{1}{2} \left((n-r_1-w_1) + (n-r_2-w_2) \right) \quad (A2.8)$$

but a frequent convention is to use the number of survivors from the previous interval

$$S_1 = n - r_1 - w_1 \quad (A2.9)$$

The hazard in either case is given by

$$Z_{1,2} = (r_2-r_1)/S \quad (A2.10)$$

The mean hazard or observed failure rate for the interval t_1 to t_2 is given by

$$Z_{1,2} / (t_2-t_1) \quad (A2.11)$$

A2.4 Hazard Rate Function or Instantaneous Failure Rate

The hazard rate or instantaneous failure rate, referring to A2.11 above, is the limit as t_2-t_1 becomes infinitesimal of the observed mean failure rate for an infinite population at time t_1.

$$z(t_1) = \lim_{\substack{(t_2-t_1) \to 0 \\ n \to \infty}} \left[Z_{1,2}/(t_2-t_1) \right] \quad (A2.12)$$

An equivalent way of defining z(t) is in terms of f(t) and R(t).

$$z(t) = f(t) / R(t) \quad (A2.13)$$

In words this definition means that z(t) is the conditional probability of failure in the interval t to t + 1 given survival to t in the limit as the time units become infinitesimal.

A2.5 Likelihood (See BS 5532 (2.49)

If n independent times to failure of like items are observed to be x_i, i = 1, 2,..n then the likelihood that they all come from a pdf f(t) is given by the product rule of probability i.e.

$$L = \prod_{i=1}^{n} \left[f(x_i) \right] \quad (A2.14)$$

A pdf is a likelihood function for times to failure.

If all the parameters of f(t) are known or assumed except a then a likelihood

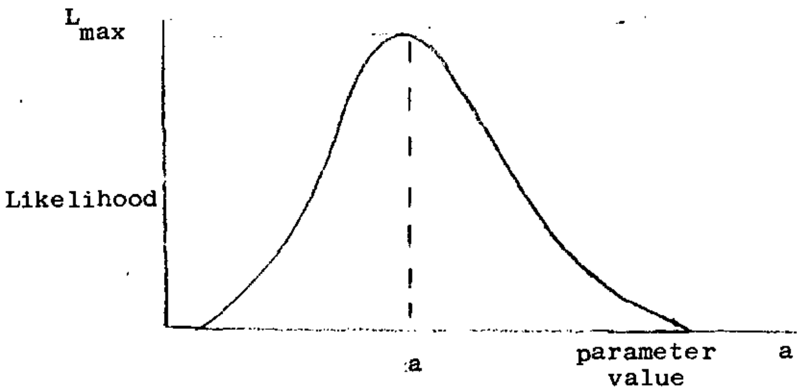


Figure A2.2 Likelihood Function of Parameter

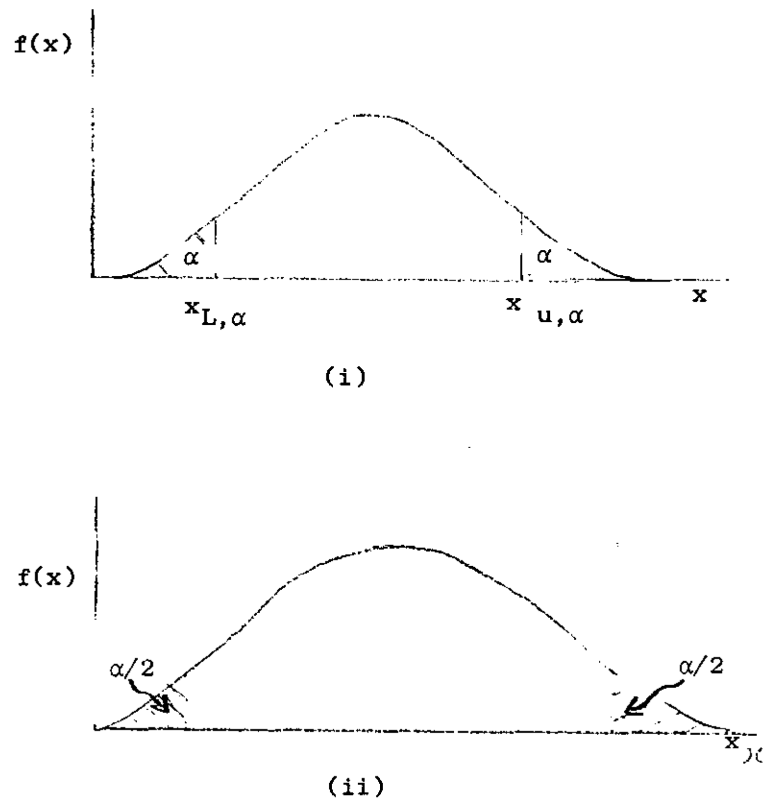


Figure A2.3 (i) Single sided confidence limits of probability α

(ii) Double sided confidence limits of probability α

function for a can be synthesized from the sample values x_i by calculating L for various trial values of a.

The value $\hat{a}$ which corresponds to L_{max} is the maximum likelihood (maxlik) estimate of a given the other parameters.

A2.6 Confidence Limits (See BS 5532 (2.59-2.69)

A confidence limit is a point x_α upon the likelihood function of any parameter or characteristic such that the area under the function up to that point is α. This means that the probability that the value of the characteristic is less than x_α is α. x_α is called the lower one-sided confidence limit of x of probability $1-\alpha$. Confidence limits are often two-sided. The two-sided confidence limits of x of probability are $\alpha = x_{\alpha/2}$ and $x_{1-\alpha/2}$. There is a probability $1 - \alpha$ that x lies between the limits and equal probabilities $\alpha/2$ that x is above or below the confidence band. The level of confidence in each case is $(1-\alpha)$. 100%.

A2.7 Measures of Central Tendency, Mean, Median, Mode

Any likelihood function, including pdf, has a mean, a median and a mode. In various applications all these measures of typicality or central tendency are used in reliability. They do not coincide except in special cases such as the Normal pdf.

For example, a frequently required estimate of central tendency is the 'best' value of F(t) to assign to the ith failure out of a sample of n, the objective being to estimate F(t) and its parameters from the sample data. In this case the likelihood function for $F(t_{i,n})$, the 'true' value of F(t) for the ith failure out of a sample of n is of Beta form. The Beta is an inversion of the Binomial distribution. viz:

$$L_{F(t)} = F(t)^{i-1}(1-F(t))^{n-1} \Big/ \int_0^i u^{i-1}(1-u)^{n-i}du$$

$$= n!\, F(t)^{i-1}(1-F(t)^{i-1}(1-F(t))^{n-i} \Big/ (i-1)!.(n-1)! \qquad \text{(A2.15)}$$

for i integer.

By taking moments and numerical integration it can be shown that

Mean Rank $\bar{L}_{F(t_{i,n})} = i/(n + 1)$

Median Rank $L_{0.5,F_{(t_{i,n})}} \simeq (i-0.3)/(n + 0.4)$ (Bénard's Approximation)

Mode (Maxlik) $L_{F(t_{i,n})} = (i-1)/(n-1)$, $i > 1$ (A2.16)

The meanings are as follows. If the true F(t) were known and N values of $t_{i,n}$ sampled, then as $N \to \infty$ the average of the values of F(t) corresponding to the N times of the ith failure out of n would then be the Mean Rank.

$$\text{ie.} \quad \underset{N \to \infty}{\text{LIMIT}} \left[\sum_{j=i}^{N} F(t_{i,n})_j / N \right] = i/(n+1)$$

(A2.17)

In the same experiment 50% of the $F(t_{i,n})$ would lie below the Median Rank value of $F(t_{i,n})$ and 50% above. The Median Rank is the 50% confidence limit of the Beta likelihood function $L_{F(t)}$. Also in the same experiment the value that appeared most often would be the Mode or Maxlik estimate $(i-1)/(n-1)$.

In distribution and parameter estimation these values are used in reverse as estimators of $F(t_{i,n})$. Different estimates can thus be obtained from the same sample data. Which measure to use is a matter of judgement, custom and circumstances. If confidence limits are required as well then the median is often used and if not then the mean is more useful. Maxlik estimators are commonly used for parameter estimation.

A2.8 Types of Value for Reliability Characteristics

Reliability or the characteristics of reliability such as mean time to failure, moments of the distribution, can be known or estimated in several ways which give rise to different values, as follows.

a) Population Values are based upon complete data for an entire finite population. By their nature they cannot be known during the life of the items and they are degraded to sample values as soon as another item is put into service. They are simple to calculate, for example the Reliability (to time t if appropriate) is simply the number surviving (to time t) divided by the number the population.

$$R = (n - r)/n \qquad \text{(A2.18)}$$

Any population, particularly a small one, is really only a sample, and statistically it is often more appropriate to treat population data as for sample data.

b) Sample or Observed Values are calculated from sample data, treating the data as if they were the entire population. Such values are easy to calculate but may be biased as estimators of the true value. For example the observed mean and variance of a sample of n items are:

$$\bar{x} = \sum_{i-1}^{n} x_i/n \qquad \text{which is unbiased}$$

(A2.19)

and

$$s^2 = \sum_{i=1}^{n} (x_i-\bar{x})^2/n \qquad \text{which is biased}$$

(See BS 5532. (2.58)

c) Estimated and Assessed Values are derived from data, usually by statistical methods. They are usually and most correctly quoted as a measure of central tendency with confidence limits.

d) True Values are conceptual only and can never be known precisely from sample data. They are population values for an infinite population. They are commonly estimated from sample data by a measure of central tendency and confidence limits.

A2.9 Sample Data - Censoring

It is often required to estimate F(t) from a sample of times to failure some of which may be incomplete; that is observation ceased before failure took place. It is not correct to ignore these incomplete lives; knowledge that an item operated for a certain time without failure is useful and must not be discarded.

Again, failures may occur by a mode which is not under investigation as well as by that mode which is being studied. The former are incomplete lives in the context of the estimation of $F(t)$ for the particular mode of failure. Data with incomplete lives as well as failure times is called censored. There are three types of censoring as follows:

Type 1 Time terminated. The test is continued until all items have either run for a fixed time or failed prior to this time.

Type II Failure terminated. The test is stopped as soon as r out of n of the items have failed.

Type III Items are withdrawn at random from the test. This type of censoring, also called progressive censoring, is typical of field data (from actual service) and of tests in which failures are replaced on the testbed by new or repaired items.

Types I and II are particular cases of Type III. Notice that withdrawing items on condition after inspection is not censoring at all but preventive maintenance. In censoring, all items have an equal chance of failing up to the withdrawal time.

A3. ESTIMATION OF FAILURE DISTRIBUTION

A3.1 Introduction

It is the aim of this Guide to give only an outline of the procedure which should be followed in estimating failure and repair time distributions and to provide some warnings about common errors. Any procedure should consist of statistical analysis to sufficient depth to elicit the required information as can be obtained from the sometimes limited data. The limitations of data should be recognised by calculating confidence limits as well as mean or median readings and it is advisable to plot the results on suitable graph paper to obtain a visual impression of the data limitations.

The exponential model(constant failure rate) is so simple to use that it is tempting to omit the procedures involved in distribution parameter estimation and simply take the observed failure rate. Also, the collection of suitable data for full distribution analysis costs more than for observed failure rate calculation, so justification is required for the increased effort. The following two advantages are the basic ones from which others follow :

a) The form of the distribution gives clues as to the nature of the failures. For example, an exponential distribution (demonstrated not just assumed) may well mean that an item is subject to several failure modes but that none of these dominate. This might give confidence that the design or the maintenance schedule is well-balanced. A hyper-exponential distribution indicates the possibility of ineffective maintenance and a Normal distribution the presence of a dominant wearout failure mode. A lognormal distribution of failures to identical parts would lead one to suspect fatigue as the cause.

b) More accurate prediction of reliability and reliability confidence limits is possible. This is essential knowledge for setting economic maintenance and replacement intervals and for judging how many machines may be required to sustain a required rate of output. For example, it is possible to modify an equipment in such a way that although its mean failure rate is reduced, the reliability over a vital mission time has actually fallen. If the development engineers fail to monitor distribution as well as mean they could well miss the point.

A3.2 Suggested Outline Procedure

More details of distributions are given in A3.3 and for more detailed analysis of any particular step the standard reference books should be consulted. This outline is designed so that analysis may be taken to the

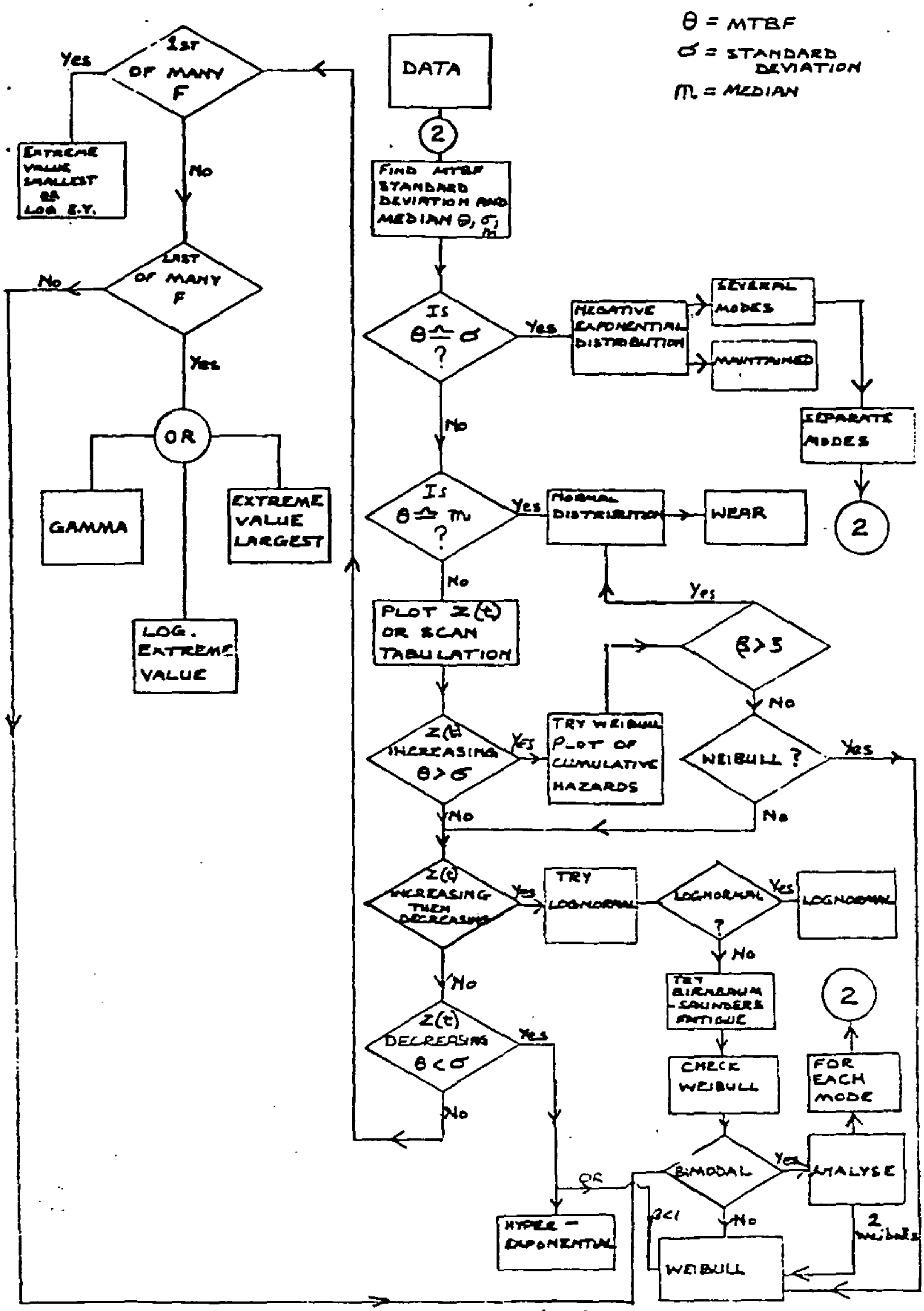


Figure A3.1 Flow Chart for Failure data Frequency Analysis desired, or necessary depth with minimum wasted effort.

i) Calculate the sample mean, $\sum_{i=1}^{n} x_i/n = \bar{x}$

ii) Estimate the standard deviation from the sample data but if the sample is heavily censored go straight to step (v) below.

$$s = \left[\sum_{i=1}^{n} (x_i - \bar{x})^2\right]^{\frac{1}{2}} \Big/ (n-1)$$

iii) Estimate the sample median value as the middle failure time or the; mean of the two middle failure times if the sample number n is even.

iv) Figure A3.1 states that if $\bar{x}$ =s then it is likely that z(t) is constant i.e. a negative exponential distribution. Constant z(t) may indicate the need for deeper analysis of the causes of the failure because it frequently arises as the sum of several failure modes, none being dominant. Qualitative information will be needed for this analysis. Data from maintained systems also often has this form of distribution. The next most common distribution form is the Normal, evidence of which is mean and median approximately equal in the sample data.

v) The next step is to place the complete and censored times (unknown finishes) in ascending order of time. Divide the time between the first and last failures into about 10 equal intervals. Then construct a table as shown A3.1 of the hazard, survivors, cumulative hazard and hazard rate. In the table x_j represent individual times to failure, t_i are the times at which determinations of hazard are made. If there are only a few actual failure times they may be used as such. The interval hazard is taken to be the number of failures in the interval divided by the number at risk at its start. The first calculation

TABLE A3.1 CUMULATIVE HAZARD CALCULATIONS.

Interval	F Failures	C Censored	S Survivors	Z Hazard	z(t) Hazard Rate	H Cumulative Hazard
0 to $x_1 = t_1$	0	C_1	$N-C_1 = S_1$	0	0	0
x_1 to t_2	F_1	C_2	$N-F_1-C_1-C_2$	$Z_1 = \frac{F_1}{(N-C_1)}$	$\frac{F_1}{(N-C_1)(t_2-x_1)}$	$\frac{F_1}{(N-C_1)}$
t_2 to t_3	F_2	C_3	$S_2-F_2-C_3$	$Z_2 = \frac{F_2}{S_2}$	$\frac{F_2}{S_2(t_3-t_2)}$	$H_1 + Z_2$
t_i to t_{i+1}	F_i	C_{i+1}	S_i	$\frac{F_i}{S_i}$	$\frac{Z_i}{(t_{i+1}-t_i)}$	$H_{i-1} + Z_i$

is usually made at the time of the earliest failure so that $t_1 = x_1$.
Plot z(t) against t. If z(t) is increasing then the distribution is probably either Weibull or Normal. If z(t) increases and then either levels off or falls the lognormal or Gamma distributions are possible . If z(t) is falling the distribution is hyper-exponential, and either the true hyper-exponential or Weibull model should be used.

vi) By plotting the cumulative hazard H versus time on log v log paper it is possible to get a rough estimate of the Weibull parameters.

vii) If item failure results from the first or last failure of many components (e.g. the first blade to fail brings the helicopter down) then extreme values , log extreme value or Gamma distribution may give the best fit.

viii) Having chosen a distribution form it will usually be required to estimate the Cdf F(t) using graphical methods or other methods. It is strongly advised that confidence limits be calculated or plotted as well as central estimates.

ix) The Kolmogorov-Smirnov goodness of fit test is applicable to discover which of a number of possible distributions gives the best model. In general the fit will be very good indeed by the usual standards, due to the false impression of accuracy given by graphical methods. A graphical alternative to judge goodness of fit is to draw confidence limits at (say) 5% and 95% about the distribution form and line which looks the best fit and then see how well alternatives fit within these limits.

It is strongly emphasised that finding the most suitable distribution model and its parameters is as much a matter of judgement, experience and the actual failure mechanisms as it is of the statistics and mathematics. For this reason computer programs to find parameters may not be satisfactory unless they produce graphical representations of trial distributions for interactive consideration. by the reliability engineer.

A3.3 Distribution Forms

A3.3.1 Negative Exponential

$$f(t) = \frac{1}{\theta} \exp\left(-\frac{t}{\theta}\right) ;R(t) = \exp\left(-\frac{t}{\theta}\right);\ z(t) = \frac{1}{\theta} \cdot \sigma^2 = \theta^2$$

θ is the mean. Alternative parameter failure rate $\lambda = \frac{1}{\theta}$.,

The negative exponential distribution is the simplest possible form, having only one parameter. It is therefore commonly assumed to hold in the absence of better information. It represents the distribution of purely random events. Because z(t) in constant, failure in any unit interval, given survival to the start of the interval, is the same. There is no dependence on past history. An item just renewed is no more or less likely to fail in the next hour than one which has been in service for a long time.

This is the expected overall failure distribution for an equipment consisting of many diverse components each with different reliability characteristics none of which are dominant. This may be termed a pseudo-random situation. The distribution is useful in this situation but its origin should never be forgotten because unlike a true random distribution found for example, in some electronic parts, the value of θ is sensitive to the intensity of maintenance.

Note: The two parameter exponential distribution is occasionally used for reliability work. It represents random events starting to occur after a fixed period and so has found application in maintained systems where inspection grants immunity from failure for an interval. Substitute u=t- for t above.

A3.3.3. Normal or Gaussian

$$f(t) = (1/\sigma\sqrt{2\pi}) \exp\{-(t-\theta)^2/2\sigma^2\} ;\ R(t) = \int_t^{+\infty} f(t)\,dt;\ z(t)=f(t)/R(t)$$

σ is the true value of the standard deviation.

The integrals are undefined but are tabulated for reference.

The Normal distribution typifies failures due to wear. Note that the function exists to -oo and so may have significant value at t=0. Since negative time is usually meaningless in reliability the model should be used with care unless $\theta > 3\sigma$ when the intercept at t = 0 becomes negligible

A3.3.3. Weibull Distribution

This is perhaps the most useful distribution. It can take many shapes by variation of the parameters.

β is a shape parameter $\beta > 1$ corresponds to wear out/increasing z(t)

$\beta = 1$ corresponds to random(exponential)

$\beta < 1$ corresponds to decreasing z(t)

η is a scale parameter or characteristic life

γ is a location parameter before which failure do not occur.

$$f(t) = \beta(t-\gamma)^{\beta-1} \exp\left[-\{(t-\gamma)/\eta\}^{\beta}\right] \qquad \theta = \eta\Gamma(1+\tfrac{1}{\beta}) + \gamma$$

$$R(t) = \exp\left[-\{(t-\gamma)/\eta\}^{\beta}\right]$$

$$z(t) = \beta(t-\gamma)^{\beta-1}/\eta^{\beta} \qquad \sigma^2 = \eta^2\Gamma(1+2/\beta) - (\theta-\gamma)^2$$

(A2.20)

Sometimes the substitution $\alpha = \eta^{\beta}$ or $\delta = \eta^{1/\beta}$ is made.

The Weibull has wide applicability (not only in Reliability) but should not be used automatically without considering other distributions. If $\beta \geq 3{\cdot}5$ the Normal is often a better model because it is able to represent the data using two parameters rather than three.

A3.3.4. Gamma Distribution.

A two-parameter version is given here but a location parameter is possible.

$$f(t) = \lambda^{c}t^{c-1}\exp(-\lambda t)/\Gamma c \qquad \text{where } \Gamma c = \int_{o}^{\infty} \exp(-u)u^{c-1}du$$

$$R(t) = \int_{t}^{oo} f(t)$$

is the complete gamma function. For c integer, $\Gamma c = (c-1)!$; $c > o$

$$\theta = c/\lambda \; ; \quad \sigma^2 = c/\lambda^2$$

(A2.21)

If c is integer:

$$R(t) = \exp(-\lambda t) \sum_{i=0}^{c-1} (\lambda t)^i / i!$$

$$z(t) = f(t)/R(t)$$

Shape is determined by c

c > 1 a unimodal distribution

$c \to +\infty$; $f(t) \to$ Normal

c = 1 Exponential

c 1 Hyper-exponential, decreasing failure rate

It is noteworthy that $z(t) \to 1$ as $t \to \infty$

The Reliability function is intractable unless c is integer. The Gamma distribution has the most useful property that the combined effect of many Gamma distributions is another Gamma distribution. The parameters are additive thus

$$(1/\lambda_f) = \Sigma (1/\lambda_i) \text{ and } c_F = \Sigma c_i$$

If c is an integer this distribution is often called the Erlang Distribution.

A3.3..5 Lognormal Distribution

In this case the logarithims of the times to failure are normally distributed

$$f(t) = (1/t\sigma \sqrt{2\pi}) \exp\left[-\frac{\{\log(t/m)\}^2}{2\sigma^2}\right]$$

$$R(t) = \int_t^\infty f(t)dt$$

$$Z(t) = f(t)/R(t)$$

m = exp(μ) where μ is the mean of the logarithms of the times to failure.

$\theta = \exp(\sigma^2/2)$

σ is the standard deviation of the logs of the failure times

Median = m ; Variance $m^2 \exp(\sigma^2)\{\exp(\sigma^2)-1\}$

This and the Birnbaum and Saunders fatigue distribution given below are the only common forms with z(t) at first increasing then decreasing.

This is typical of some fatigue failure distributions although others have been fitted to a Weibull distribution. There is physical evidence for such a distribution for fatigue failures and philosophical and empirical evidence for the lognormal being the appropriate distribution for repair times and other service time, involving partitioning e.g. finding a book in a library.

A3.3.6. Birnbaum-Saunders Fatigue Distribution

This distribution which is based upon a physical model of fatigue failures is somewhat complicated.

Define $x = (1/\alpha)\ \{ (t/\beta)^{\frac{1}{2}} - (t/\beta)^{-\frac{1}{2}} \}$

where α and β are the distribution parameters. Then the distribution of failure times taken to be a standard Normal distribution (Mean zero, variance unity) in x.

i.e. $R(t) = \frac{1}{2\pi} \int_x^{\infty} \exp(-v^2/2)\, dv$ (v is a dummy variable of integration)

$\theta = \beta(i + \frac{\alpha^2}{2})$; $\sigma^2 = (\alpha\beta)^2(1 + \frac{5\alpha^2}{4})$

The parameters may be estimated roughly from n failure times t_i, i=1,.... n, as follows:

$\alpha = (S/\hat{\beta} + \hat{\beta}/R - 2)^{\frac{1}{2}}$; $\hat{\beta} = (SR)^{\frac{1}{2}}$

where $S = (i/n) \sum_{i=1}^{n} t_i$ and $1/R = (1/n)\sum_{i=i}^{n} (1/t_i)$

which are the mean and harmonic mean values of the sample.

A better estimator of β is the positive solution of $g(x) = 0$ where

$$g(x) = x^2 - x(2R + K(x)) + R\ (S + K(x))$$

$$K(x) = i/\left[(i/n) \sum_{i=i}^{n} (x + t_i)^{-1} \right]$$

These are the Maximum Likelihood estimators.

A3.3.7 Hyper-Exponential Distribution

$$f(t) = 2k^2\lambda \exp(-2k\lambda t) + 2\lambda(1-k)^2 \exp\left[-2\lambda t(1-k)\right] ;$$

$$R(t) = K \exp(-2k\lambda t) + (1-k) \exp\left[-2\lambda t(1-k)\right] ;$$

$$z(t) = \frac{2\lambda\left[K^2 + (1-k)^2\right] \exp\left[-2\lambda t\ (1-2k)\right]}{\left[k + (1-k) \exp\{-2\lambda t(1-2k)\}\right]} \quad ; \ 0 < k \leq 0.5$$

$\theta = 1/\lambda$; $\sigma^2 = (\frac{1}{2}\lambda^2)\ (\frac{1}{K} + \frac{1}{1-K})^2$

The hyper-exponential represents a concentration of failure times at each end of the scale. i.e. a lot of short times and a lot of long times, but few of medium length.

The hyper-exponential is an alternative to the Weibull with $\beta < 1$ as a model for early failure. It is more realistic because f(t) has a finite value at t = 0, but the Weibull model is usually preferred because it is easier to manipulate. The parameters may be estimated from data by first finding the data mean; the reciprocal of which estimates λ and then finding the best value of k by trial and error or maximum likelihood. Early failures are usually caused by shortcomings in maintenance practice or poor quality control of spare parts.

When k = 0.5 the hyper-exponential becomes an exponential distribution with $\theta = 1/\lambda$.

A3.3.8 Extreme value Distribution

The distribution of the smallest extreme is given. To obtain the distribution of the largest extreme put (a - t) for (t - a).

There are two parameters

Location parameter a, The Mode

Scale parameter b.

$$f(t) = (1/b) \exp \left[(t-a)/b\right] \{\exp \left[(t-a)/b)\right] \}$$

$$R(t) = \exp \{- \exp \left[(t-a)/b\right] \qquad \theta = a-0.57721\, b$$

$$z(t) = (1/b) \exp \left[(t-a)/b\right] \qquad \sigma^2 = b^2 \pi^2/b$$

The form gives a distribution with exponential hazard which for the smallest extreme is positively skewed (mode and median greater than mean) The logarithms of the times in a two parameter ($\gamma = 0$) Weibull distribution have an extreme value distribution such that $b = 1/\beta$ and $a = \log_e \eta$. The Reliability at a is 1/e. The pdf does not start at zero, in general there is a finite probability of immediate failure. This will not be a problem if $a > 3\sigma$.

A3.3.9. The Log-Extreme Value Distribution

In this distribution the logarithms of the times have an Extreme Value Distribution

$$f(t) = \delta \alpha^{\delta} / (t^{\delta} + 1) \quad \exp\left[- (\alpha/t)^{\delta}\right]$$

$$R(t) = 1 - \exp\left[- (\alpha/t)^{\delta}\right]$$

$$z(t) = f(t)/R(t).\text{(No simplification).}$$

Parts having extreme value strength distributions may have failure distributions of this type. For example in a separately bladed turbine the strength of the wheel is the strength of the weakest blade fixing. If this strength decreased logarithmically with time the failure distribution will have the log-extreme value form.

A3.4. Weibull Analysis by Cumulative Hazard

The Cumulative Hazard calculation tabulated at A3.2 above may be used to estimate the Weibull parameters of the distribution.

For the Weibull distribution.

$$z(t) = f(t)/R(t) = \beta\ (t-\gamma)^{\beta-1}/\eta^{\beta}$$

The integral of $z(t)$ is called the cumulative hazard function.

Thus $H(t) = \int_0^t z(x)\,dx = \left[(t-\gamma)/\eta\right]^{\beta}$

Taking logarithms $\log H(t) = \beta \log (t-\gamma) - \beta \log \eta$

Thus if $(t-\gamma)$ v $H(t)$ were plotted on log v log paper the result would be a straight line of slope which intercepts $H(t) = 1$ at $(t-\gamma) = \eta$ The cumulative hazards calculated at 9.2 above are estimates of $H(t)$ at various values of t. Conventionally the points are plotted at the start of each interval. The value of γ is found by first plotting t v $H(t)$. If the plot is straight $\gamma = 0$. If not subtract trial values of γ from each t_i and re-plot until the best value is found when the plot will be straight. It is not as accurate as a cumulative distribution plot. This method is only useful for the Weibull distribution but has the advantage that many data can be reduced quickly to a distribution estimate by manual calculation and plotting.

A3.5 Parameter Estimation from Cumulative Distribution Function Using Scaled Papers

Special graph papers are available upon which the Weibull, Normal Lognormal and Extreme Value distributions can be plotted as straight lines. The parameters can then be measured from the graph in various ways. The function plotted is the cumulative distribution function cdf $F(t) = 1-R(t)$ $= \int_0^t f(x)\,dx$.

Weibull paper can be used to plot $R(t)$ for the Log Extreme Value distribution, and ordinary log v log paper for the Negative Exponential distribution. $F(t)$ is the proportion failing in time t, but when

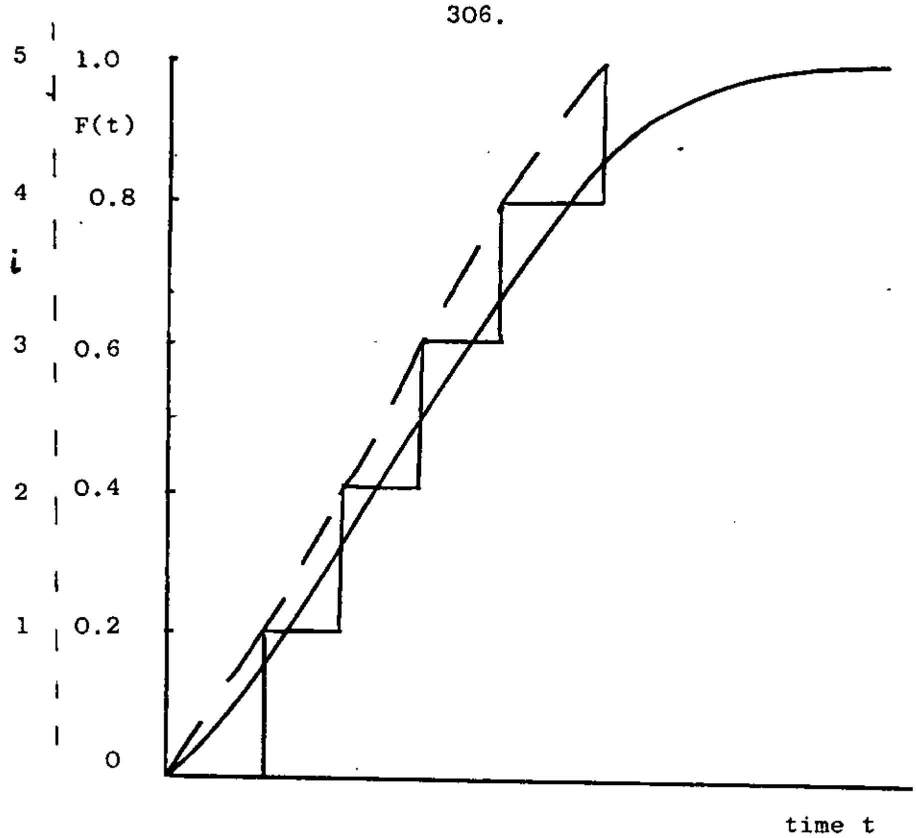


Figure A3.2 Sample Proportion Overestimates Distribution

analysing data it is not sufficient simply to use the sample proportion. This is best illustrated by considering Figure A3.2, which shows a stepped empirical Cdf for a sample of 5 compared with the true distribution. Clearly, a line through the plotted points would consistently over-estimate the proportion failing.

A3.5.1. Mean Rank

The easiest adjustment is to use mean ranks. For a complete set of n failures with no censorings the mean rank of the i^{th} failure is i/(n+1).

A3.5.2. Median Rank

The mean rank has statistical drawbacks, particularly if confidence limits are to be calculated about the estimated line. A better estimate is the Median Rank (MR). The Median Rank is the 50% point of the assumed Beta distribution of the ith event out of n. They can be found from tables of the Beta distribution using parameters i, n - i + 1 or approximated by the formula due to Bénard.

$$MR \simeq (i-0.3)/(n + 0.4)$$

Bénards approximation is accurate to 1% for $n > 5$ and 0.1% for $n > 50$.

A3.5.3. Censored Samples - Mean Order Number

In the general case the samples may be progressively censored, that is, items are removed from service at haphazard times before failure, or the data collection period ends without their failing (i.e. unknown finishes). They cannot be ignored because that is throwing away the information that they did not fail up to so many hours. The assumption is usually made that they might with equal probability have failed in any of the intervals between events (event = failure or further censoring) or after all of them. Analysts should be aware of this assumption because it is not always valid. For example, items removed on condition following critical inspection are near to failure and should not be counted as censored . A convenient formula for calculating the mean order number MO_i of the ith failure out of a total sample of n.

$$\text{is } MO_i = MO_{i-1} + \frac{n + 1 - MO_{i-1}}{1 + S_i}$$

where S_i is the number of survivors (items that remain at the instant of the ith failure) including any censored simultaneously with the ith failure.

This formula incorporates the assumption of equal probability. For censored samples MO_i is substituted for i in Bénard's approximation or an interpolation is made between the tabulated Beta values.

Note: The Binomial and Beta are mutual inverses but only the Beta can legitimately be interpolated; the Binomial is discrete. Hence the recommendation to use the Beta to find Median Ranks rather than the Binomial.

A3.5.4. Confidence Limits

The fit of points on the special graph papers is often deceptively good, giving a false impression of accuracy. It is therefore strongly recommended that confidence limits be plotted in addition to the median line. It is usual to plot the 5% and 95% lines unless there is a specific requirement for another limit to be plotted perhaps to show with confidence 100 $(1 - \alpha)$% that reliability R at time t will be achieved.

Percentage ranks can be obtained from tables of the Beta distribution using parameters i, n-i+1, or in the case of censored data MO_i, $n-MO_i+1$ and interpolating.

A3.5.5. Making the Plot

Plot the median ranks against the failure times on the graph paper which seems appropriate from examination of the elementary statistics (mean, standard deviation and median), and the failure mechansism(s). Try other possible papers and choose the distribution form giving the best fit.

Draw in the confidence limits and examine graphically whether a simpler distribution would be appropriate. In drawing the best line account should be taken of the scaling of the paper and of the cumulative nature of F(t) plots. The fit should be better where the graph lines are close and with increasing F(t). Outliers may need separate consideration; first failures in a data set are frequently found to be due to other causes when they do not fit an otherwise good line.

Scatter in the statistics causes 'snaking' of the points about the cumulative line.

A3.6 Computed Estimation of Parameters

A3.6.1. Introduction

It is strongly recommended that a graphical plot with confidence limits be made in addition to any computed parameter estimates. It is possible with some interactive program suites to output a display or even a permanent record which shows the points as well as the fitted line. The most popular method for calculating parameters is Maximum Likelihood which is usually biased e.g. it is biased by a factor n/(n-1) for standard deviation of the Normal. However, the bias tends to zero as sample size increases. Best linear invariant estimators which are simply n factors by which the n failure times are multiplied to give an estimate of one or other of the parameters are computationally efficient where computer core is not at a premium. It is also possible to deal with certain types of censoring by this method. Parameter estimates may be obtained from uncensored data by the Method of Moments. Where least squares or other line fitting procedures are used account should be taken of the heteroscedasticity introduced by the nature of the function. This is equivalent to giving more emphasis to later points when making a graphical plot. Both the Method of Moments and Maximum Likelihood assume the <u>form</u> of the distribution before proceeding to estimate the parameters.

It may be possible to resolve the question of distributional form by means of Goodness-of-fit tests after estimating parameters according to several different forms. e.g. The final test could be to choose that distribution which minimises the Kolmogorov- Smirnov statistic, which is the maximum absolute deviation of any data point from the estimate of F(t).

$$d = \max_i \left[F(t_i) - MR_i \right]$$

A3.6.2. Method of Moments

As discussed in A.2 above the moments of a distribution describe both its shape and its scale. Distribution parameters can be estimated by equating the sums of moments of observations in a sample to the moments of a distribution form. The parameter estimates are not in general unbiased unless steps are taken to unbias the moment estimates. The most common example of the method is in the estimation of the parameters of the Normal distribution from the sample mean and variance.

Given that three or fewer parameters, perhaps representing shape, scale and location, are sufficient for all common distribution forms, three moments are usually enough to form equations from the sample data which can be simultaneously solved to find the parameter estimates. However, mean and variance are only sufficent for a two-parameter distribution form if symmetry about the mean is assumed, as in the Normal. Otherwise there will be two solutions for one of the parameters. This can be resolved usually by calculating the coefficient of Skewness. The method is not suitable for censored samples.

A3.6.3. Maximum Likelihood

Let the form of the distribution be

$f(t) = f(t, \alpha, \beta, \gamma \ldots\ldots)$ with reliability function R(t) where α, β, γ etc are parameters and t the time variable. For a set of data (times to or between failures) t_i, $i = 1,2, . . . n$

and a set of survivors of ages x_j, $j = 1,2, . . . m$.

The combined probability (likelihood) of all the n + m events is:

$$L = \prod_{i-1}^{n} f(t_i) \cdot \prod_{j=1}^{m} R(x_j)$$

Taking logs $\mathcal{L} = \text{Log } L = \sum_{i-1}^{n} \log f(t_i) + \sum_{j=1}^{m} \log R(x_j)$

If the likelihood is a maximum so is its logarithm. Form partial differential equations putting

$$\partial\mathcal{L}/\partial\alpha, \quad \partial\mathcal{L}/\partial\beta \text{ etc } . . . = 0$$

in rotation to obtain successively closer estimates of α, β , etc which maximise $\mathcal{L}$. Note that if the sample is heavily censored the effect of the $R(x_j)$ terms is likely to swamp the smaller $f(t_i)$ terms. The estimates will then be poor under usual criteria for convergence and very sensitive to small changes. It is necessary therefore to continue to the limit of discrimination to get best estimates (i.e. use all the significant figures available in the chosen method of computation.)

The base of logarithms is immaterial in theory but usually 'e' in practice for convenience. Exact analytical solutions are unusual. Generally, one must proceed by successive approximation.

The procedure is less complex if m=o and analytical solutions are then also more likely.

Note that $d \log f(x)/dx = f'(x)/f(x)$. This substitution often simplifies intermediate expressions arising in this method.

A3.6.4. Least Squares

If the cumulative distribution function can be reduced to a straight line by transformation of variables, a least squares fit may be made to estimate the parameters from data. As this is a cumulative plot account must be taken of the expectation of less variability in later readings.

A4. REDUNDANCY AND SYSTEM RELIABILITY

A4.1 Lüsser's Rule for Series Systems

In a series system, failure of any component constitutes system failure.

$$R_F = \prod_{i=1}^{n} R_i$$

i.e. the reliability of the system is the product of the reliabilities of the components. In alternative form we may write that the failure probabilities are additive.

$$P_F = \sum_{i=1}^{n} P_i \qquad \text{provided } P_i \ll 1 \qquad P = 1-R$$

Availabilities - particularly steady state availabilities - can be dealt with in the same way.

$$A_F = \prod_{i=1}^{n} A_i$$

A4.2. Active Parallel Redundancy

In this case n identical items share provision of the function, which can be sustained by as few as m. System failure occurs with the $(m+1)^{th}$ overlapping failure. Clearly, we have a Binomial situation.

Probability of all n remaining Available A^n

Probability of exactly one failure $nA^{n-1}(1-A)$

Probability of exactly two failures $\frac{n(n-1)A^{n-2}(1-A)^2}{2!}$

etc..

Probability of exactly n-m failures (m survivors) $\frac{n!}{(n-m)!\,m!} A^m(1-A)^{n-m}$

A_F is then the sum of these terms, the coefficients being most easily evaluated from Pascal's triangle.

i.e.

$$A_F \quad \sum_{j=0}^{m} \left[\frac{n!}{j!\,(n-j)!} A^{n-j}(1-A)^j \right]$$

$$R_F(t) = \sum_{j=0}^{m} \left[\frac{n!}{j!\,(n-j)!} R(t)^{n-j}(1-R(t))^j \right], \quad (0!=1)$$

if m = 1, this expression reduces to

$$A_F = 1 - (1-A)^n$$

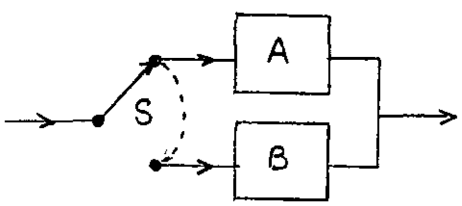
Similarly,

$$R_F(t) = 1 - (1-R(t))^n$$

In the case of Availability it is assumed that repairs to redundant failed items are started immediately following failure and without rendering the system unavailable. The maximum availability calculated above will not be realised unless m + 1 repair teams can be mustered as required.

A4.3. Switched Standby Systems

In this case the redundant items are not 'switched on' until required. One only is required to operate.



Four types of failure are possible.

1) Failure to switch when required.

2) Spurious switching to a failed unit

3) Failure of switch to transmit

4) Failure of both (all) elements

Note that the 'switch' may be manual or auto and need not be electrical - it could be a pneumatic or hydraulic valve system. The simple case of the perfectly reliable switch is examined first since it represents an upper limit of system reliability. With the switch initially as shown in the figure the system cperates if either A operates or A having failed, B operates. Assuming no failures to non-operating units.

$$R_F = R_A + R_B (1-R_A)$$

If there were a third element C

$$R_F = R_A + R_B(1-R_A) + R_C(1-R_A)(1-R_B)$$

or in general

$$R_F = R_1 + \sum_{1=2}^{n} \left[R_i \prod_{j=1}^{i=1} (1-R_j) \right]$$

Where reliability of units is a function of time. i.e. $R_i = R_i(t)$ it is necessary to be more circumspect since the operating time of later units depends upon the earlier failure times. If there are more than two units, usually all units will be identical.

For the mean time to failure we may write immediately

$$\theta_F = \sum_{i=1}^{n} \theta_i$$

since it is obvious that the expected time to failure of the system must be the sum of the expected times to failure of the units. For the Gamma distribution the pdf may be written

$$f(t) = (\lambda t)^{c-1}. \ \lambda \exp(-\lambda t)/\Gamma(c)$$

where $\Gamma(c) = \int_0^{\infty} u^{c-1} \exp(-u)du$ - the well known Gamma function

$\Gamma(c) = (c-1)!$ for c integer.

The distribution of the sum of n Gamma variates is another Gamma variate with parameters

$$1/\lambda_F = \sum_{i=1}^{n} 1/\lambda i, \qquad c_F = \sum_{i=1}^{n} c_i$$

of which the standby redundancy of n elements each with failure rate λ is a special case. The failure time distribution is of Gamma form with

$$\theta_F = 1/\lambda_F = n/\lambda = n\theta$$

$$c_F = n$$

i.e.

$$f(t) = (\lambda t)^{n-1} \exp(-\lambda t)/(n-1)!$$

The corresponding Reliability Function is:

$$R_F(t) = \sum_{i=0}^{n-1} \left[(\lambda t)^i/i!\right] .\exp(-\lambda t)$$

In the Normal case the means and variances may be added together to find the mean and variance of the combined distribution.

The Weibull case may also be dealt with approximately by adding means and variances, The accuracy of this approximation to a combined Normal distribution increases with the number of units involved, (Central Limit Theorem) but it is poor for small numbers of units and shape parameter $\beta < 1$.

For identical Weibull units the combined reliability function can be obtained in Gamma form by transformation of variables

$$\text{i.e.}\quad R_F = \left[\sum_{i=1}^{n-1} (u/\alpha)^i/i!\right] .\exp(-u/\alpha)$$

where $u = (t-\gamma)^\beta$ and $\alpha = n^\beta$

Returning to the case where the switch is not perfectly reliable consider now a two unit system A,B, S. Let R_A, R_B be the unit reliabilities R_d be the probability that the switch operates when required, R_e be the probability of no switching when not required, and R_c be the switch reliability with respect to transmission.

Then the system succeeds if either

1) A and B succeed

or 2) A succeeds, B fails

or 3) A fails, B succeeds.

These states are mutually exclusive so probabilities may be added i.e.

$$R = R_A R_B R_c + R_A R_c R_e (1-R_B) + R_B R_c R_d (1-R_A)$$

If $R_c = R_d = R_e = 1$, this result is consistent with the first case considered above.

A4.4 Availability

(a) The active parallel case has been considered already

(b) Maintained Standby Systems - one only to run out of n.

Assuming perfect switching, that repairs to failed items are undertaken without rendering the system unavailable, that the items a re identical and that their R and M functions are exponential, it has been shown that if only one repair can be undertaken at one time

$$A_F = 1 - 1 \bigg/ \sum_{i=0}^{n} \left[(\mu/\lambda)^i\right]$$

For other combinations of items and requirements see A4.6.

(c) Unmaintained Systems - one only to run out of n.

The title is applied to systems in which no repairs take place until system failure. Redundancy serves mainly to increase the system mtbf. In standby unmaintained system mtbf, θ_F is simply the sum of the individual mtbf's.

$$\theta_F = \sum_{i=1}^{n} \left[\theta_i\right]$$

In an active parallel system in which one item only is required to operate, and all n items are identical

$$\theta_F = \theta \sum_{i=1}^{n} \left[1/i\right]$$

The mtbf and hence the Availability of unmaintained systems depend on the number of repairs which can be progressed simultaneously. See A4.6, Transition rate matrices. It is important to note that an unmaintained system with only one repair team offers no increase in Availability over a single item. . The advantage in such cases lies entirely in the increased system mtbf. Full exploitation of redundancy depends on the provision of extra manpower to carry out repairs.

A4.5 Complex Arrangements - Bayes' Theorem

The majority of practical systems are amenable to analysis in terms of the general formulae in the above paragraphs. However, systems which are not so amenable do exist. The exceptions can usually be dealt with by the application of Bayes Theorem of Probability. In Availability (or Reliability) terms this may be stated thus:

$1 - A_F$ = (Prob. system is down if item X never fails). (Prob.X is available) + (Prob. system is down if X is down). (Prob X is down)

or A_F = (Prob.system available given X available)(Pr.X is available) + (Prob. system available given X down) (Prob. X is down).

For example full production at a certain plant depends in part on the availability of at least 2 of three feed pumps. The three feed pumps constitute the sub-system under examination. They are assumed to be identical for Reliability and Maintainability (R & M) purposes. Normally Nos 1 and 2 are used. The centre pump, C, is started following the first failure. The probability A_F that production will not be limited by feed pump unavailability is found as follows from Bayes Theorem. System failure will occur when 1 and 2 are down together, or when 1 or 2 having already failed, C also fails. Call pump No 1,X.

then $1 - A_F$ = (1- Availability of 2 and C in standby).A_1

+ (1- Availability of 2 and C in series). $(1-A_1)$

let $A_1 = A_2 = A_C = A$

therefore $1-A_F = AZ + (1-A)(1-A^2)$

where $Z = (\lambda^2/(2\mu^2+2\mu\lambda+\lambda^2)$ (From standby case with two repairmen, calculated using methods of A4.6 below).

and $A = (\mu/\mu+\lambda)$

If $\mu = 0.2$ repairs/hr and $\lambda = 0.001$ failures/hr

$A = 0.995$ and $z = 0.0000124$

whence $A_F = 0.99993775$

The availabilities of some alternatives to the two-out-of-three arrangement are (given same μ & λ), 2 full-duty pumps $1-Z = 0.9999876$ and the most flexible arrangement of 4 half duty pumps. 0.999999689 (any two out of 4).

Bayes Theorem is quite general, if a problem cannot be solved by one application, 'Bayes within Bayes ' should be tried until the analysis is complete.

A4.6 State Transition Rate Matrices

Another approach to complex systems is to form state transition rate matrices, i.e. to present the instantaneous probabilities of passing from one condition of so many items up and so many down to another such condition in the form of a matrix, which after transformation yields the system Availability or Reliability. The method is used to obtain the availability relationships for standby redundancy quoted above. Such matrices quickly assume very large proportions as the systems get larger, and the increased accuracy afforded is seldom useful when related to that of the data and the inevitable assumptions in a practical study. Indeed it is very often the case that the unavailability of a system is predominantly due to the state of the art limitations in a few items, the availability of the remainder being some orders of magnitude greater. In such cases the application of redundancy to these few items usually raises the system figures to a point well within the specification and no further action is necessary apart from raising spares allowances, manpower and space to cover the redundancy.

The matrix approach assumes that no more than one event can take place in unit time, a condition which becomes less tenable as the system grows larger. The effect can be delayed by using shorter time units. See also 4.7 below.

A4.7 Reliability of Redundant Maintained Systems

Although the main concern is often with availability it is still frequently necessary to estimate the reliability characteristics of maintained systems. For example in transport applications the reliability over various voyage or mission times might be of vital importance.

a) Transition Matrix Method. This method is applicable to systems with exponential component failure and repair distributions. When used for calculating availability the matrix consists of probabilities of transition from the row to the column state in unit time. All are either zero or less than unity. For calculating reliability the final or 'system failed' state is made an absorbing state, that is, if state n is such a state then element (n,n) of the matrix is 1 and (n,i), i=0 to n-i are all 0 for a reliability calculation. R(t) or A(t) may be found by raising the matrix to the t th power or by Laplace transforms. As an example to illustrate the principles take the case of a single stand-by identical with the preferred equipment. The matrix for

Reliability is

$$P_R = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 1-\lambda & \lambda & 0 \\ \mu & 1-\mu-\lambda & \lambda \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

For an availability calculation assuming two simultaneous repairs are possible

$$P_A = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 1-\lambda & \lambda & 0 \\ \mu & 1-\mu-\lambda & \lambda \\ 0 & 2\mu & 1-2\mu \end{bmatrix} \end{array}$$

λ, μ are the unit failure and repair rates.

Differentiating the equations corresponding to the matrix columns of P_R. 0,1,2 refer to state numbers (of units failed)

$$P_o'(t) = -\lambda P_o(t) + \mu P_1(t)$$

$$P_1'(t) = \lambda P_o(t) - (\mu + \lambda) P_1(t)$$

$$P_2'(t) = \lambda P_1(t)$$

and initially (Time Zero)

$$P_o(0) = 1 \text{ and } P_1(0),\ P_2(0) = 0$$

Take Laplace Tranforms

$$(s+\lambda)P_o(s) - \mu P_1(s) = 1$$

$$\lambda P_o(s) - (\mu + \lambda + s) P_1(s) = 0$$

$$\lambda P_1(s) - sP_2(s) = 0$$

By Cramers Rule

$$P_1(s) = \det \begin{vmatrix} s+\lambda & 1 \\ \lambda & 0 \end{vmatrix} \Big/ \det \begin{vmatrix} s+\lambda & -\mu \\ \lambda & -\lambda-\mu-S \end{vmatrix}$$

$$P_1(s) = \lambda / \{s^2 + s(\mu + 2\lambda) + \lambda^2\}$$

Take the roots of the denominator of this expression equated to zero

$$S_1 = \tfrac{1}{2}\{-(\mu + 2\lambda) + (\mu^2 + 4\mu\lambda)^{\frac{1}{2}}\}$$

$$S_2 = \tfrac{1}{2}\{-(\mu + 2\lambda) - (\mu^2 + 4\mu\lambda)^{\frac{1}{2}}\}$$

$$P_1(t) = \lambda\{\exp(S_1 t) - \exp(S_2 t)\} / (S_1 - S_2)$$

Similarly

$$P_o(s) = -(\mu + \lambda + s) / \{ s^2 + s\ (\mu + 2\lambda) + \lambda^2 \}$$

$$P_o(t) = \{(\mu + \lambda - S_1) \exp (S_1 t) - (\mu + \lambda - S_2) \exp(S_2 t)\} / (S_1 - S_2)$$

$$R(t) = P_o(t) + P_1(t)$$

$$R(t) = \{S_2 \exp (S_1 t) - S_1 \exp (S_2 t)\} / (S_1 - S_2)$$

Now $P_2'(t)$ is the pdf of the system failure distribution

$$f(t) = P_2'(t) = \lambda^2 \{ \exp (S_1 t) - \exp (S_2 t)\} / (S_1 - S_2)$$

This provides a check on R(t) since by definition

$$R(t) = 1 - \int_o^t f(t)dt$$

The meantime to system failure is $\theta = \int_o^\infty t\ f(t)dt$

and the variance is

$$\sigma^2 = \int_o^\infty t^2\ f(t)dt = \theta^2$$

θ and σ^2 may be found directly.

Alternatively, assuming the failed state occupies the final row and column of P_R, a new matrix Q can be defined as the Identity Matrix I less P_R with this final row and column removed.

The mean first passage time $\overline{\theta}_i$ from the initial state i to the failed state is then given by the matrix equation

$$\overline{\theta}_i = [\ p_i\ ]\ [\ N\ ]\ [\ U\ ]$$

where p_i is the vector of initial state probabilities

U is a unit column vector

and N is Q^{-1}

The θ of concern is usually that from the initially successful state 0 to the failed state and for this case

$$\bar{\theta}_0 = \sum_{i=1}^{N-1} (-1)^{i+1} \; D_{i1} / \text{Det } Q$$

Where D_{i1} is the determinant formed by deleting the i^{th} row and 1^{st} column of Q.

In the example

$$Q = \begin{pmatrix} \lambda & -\lambda \\ -\mu & \mu+\lambda \end{pmatrix}$$

and hence

$$\bar{\theta}_0 = \frac{\mu + 2\lambda}{\lambda^2}$$

$$[\, p_i \,] = [\, 1, 0, 0, \ldots\ldots \,]$$

and the above matrix equation reduces to

$$\bar{\theta}_0 = \sum_{K=1}^{N-1} N_{1k}$$

ie the sum of the elements of the first row of N.

By the process of matrix inversion this can be written

For a 1 out of n system with n repairmen

$$\theta_n = \left(\frac{1}{\lambda}\right) \sum_{i=0}^{n-1} \left[m! \left(\frac{\mu}{\lambda}\right)^i \Big/ (i+1)\ (n-i+1)! \right]$$

and for n active equipment parallel system with n repairmen

$$\theta = \frac{1}{\lambda} \sum_{i=0}^{n-1} \left\{ i + \left(\frac{\mu}{\lambda}\right)^i \Big/ (i+1) \right\}$$

The variance column vector can be found from the matrix equation

$$\sigma^2 = 2 [N] [\overline{\theta}] - [\overline{\theta}_s]$$

where $N = Q^{-1}$ as previously

$\overline{\theta}$ = column vector of mean times to first failures as defined above

θ_s is a column vector with each element given by $\overline{\theta}_{si} = (\overline{\theta}_i)^2$

b) Failure and Repair Time Distributions are not always exponential. An extension matrix method allows for the use of Gamma (Erlang) distributions by defining c states for each unit where c is the shape parameter of the Erlang distribution but for other distributions other methods must be found.

c) Single Standby - general distribution. In this case the failure and repair pdf's are f(t), g(t) for the preferred unit and $f_2(t)$ is the failure pdf for the standby, the pdf of the system failure time is

$$f_{1,2}(t) = \{1 - A_1(t)\} \quad f_2(t)$$

where $A_1(t)$ is the availability function of the first unit

$$A_1(\underline{L}) = 1 - \int_0^t f_1(t) \ \{1-g(t)\} \ dt$$

For large t, A(t) may be estimated by the steady state availability, $A = \Theta / (\Theta + \emptyset)$, where Θ, $\emptyset$ are the means of $f_1(t)$, g(t).

d) Active Parallel Systems. Each element has availability, A(t) and failure pdf f(t). If there are n identical elements then there are n ways in which n-1 may be failed when the last unit fails.

The probability of each of these ways occurring at time t is

$1 - A(t)^{\,n-1} f(t).$

The failure time pdf for the system is therefore

$$f_n(t) = n \{ 1-A(t) \}^{n-1} f(t)$$

substituting

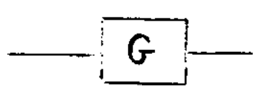
$$f_n(t) = n f(t) \left[\int_o^t f(t)\{1 - g(t)\} dt \right]^{n-1}$$

The integrals concerned may be intractable in which case the required reliability must be found by numerical integration e.g. by Simpson's Rule.

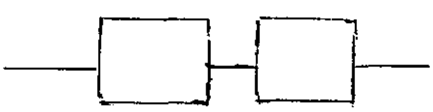
A4.8 Open and Short Circuit Failures

Many systems, particularly but not exclusively electronic systems, consist of units subject to both short and open circuit failure.

Let q = Pr (Open circuit failure), s = Pr (short circuit failure)
The reliability of various configurations is then as follows



$R_G = 1 - q - s$

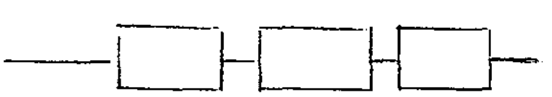


E

open circuit : $1-R_o=2q-q^2$

short circuit : $1-R_s= s^2$

therefore $R_E = (1-q)^2-s^2$



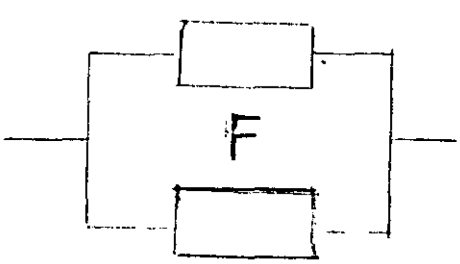
A

similarly for three units in series

$R_A = (1-q)^3-s^3$

and for n units in series

$R_n = (1-q)^n-s^n$



open circuit : $1-R_o= q^2$

short circuit : $1-R_s=2s-s^2$

therefore $R_F = (1-s)^2-q^2$

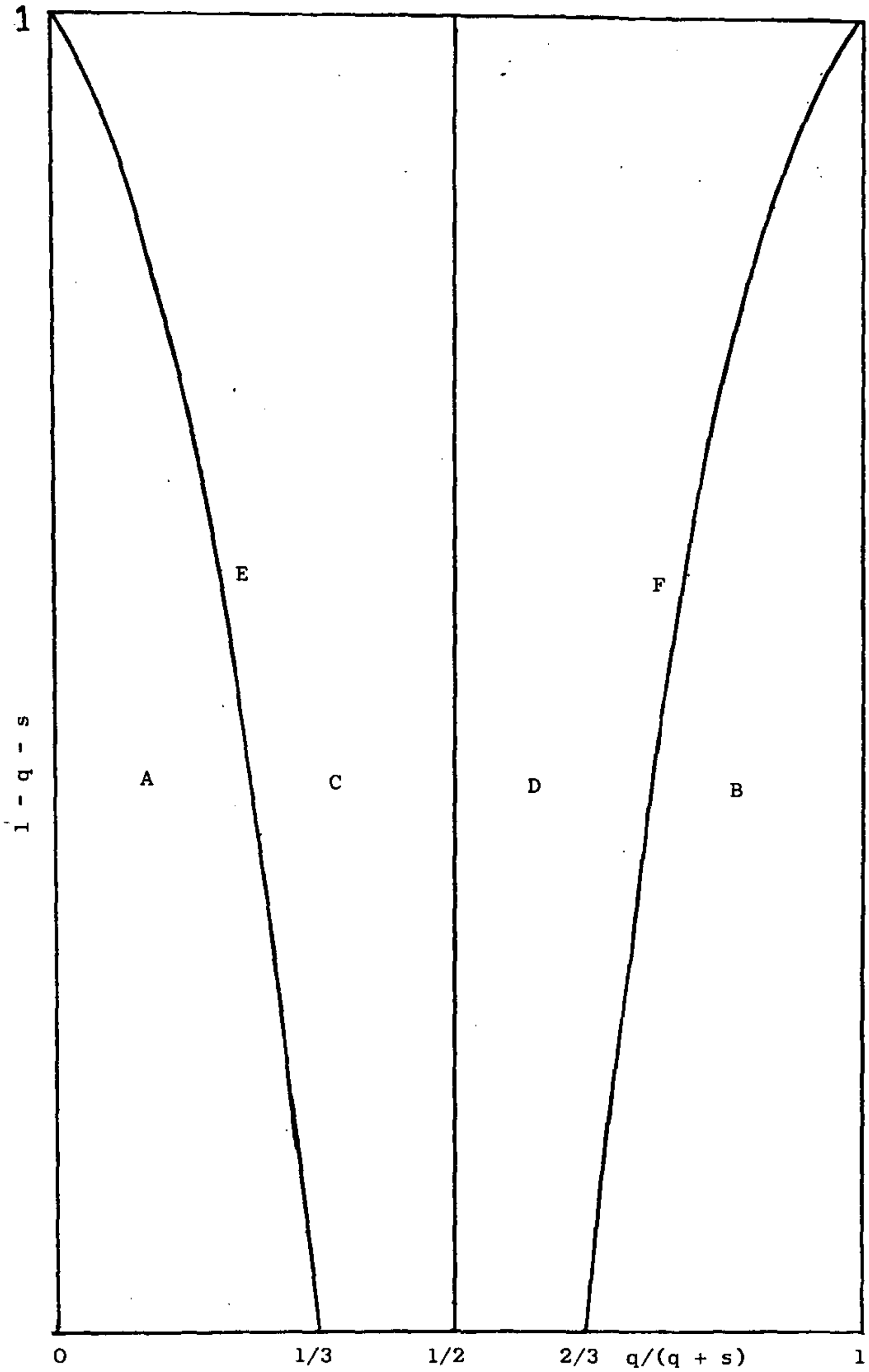
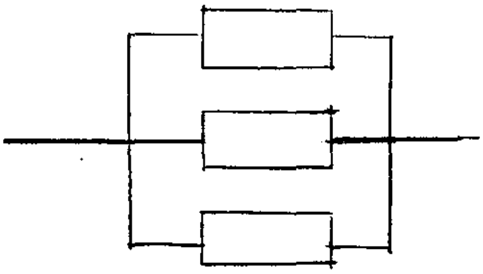


Figure A4.1 Open and Short Circuit Redundancy
(After Jenney)

The figure shows which of the arrangements illustrated in paragraph 4.8 is most reliable. On the curved lines two items are as reliable as three.



B

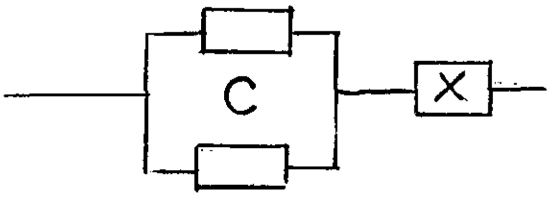
similarly for three units in parallel

$$R_B = (1-s)^3 - q^3$$

and for n units in parallel

$$R_n = (1-s)^n - q^n$$

By Bayes Theorem

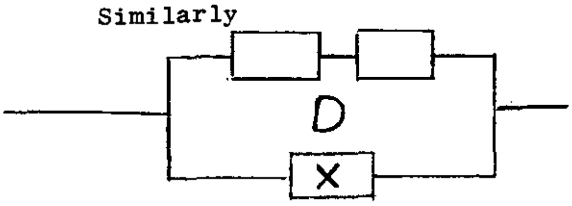


open circuit: $1-R_o = q^2(1-q)+q$

short circuit: $1-R_s = 0+(2s-s^2)S$

therefore $R_c = 1-q-q^2+q^3-2s^2+s^3$

Similarly



$$R_D = 1-s-s^2+s^3-2q^2+q^3$$

Note the symmetry with R_C

Where both short and open circuit failures are possible it is usually not cost-effective to use more than three units. Depending upon the unit reliability $V = 1-q-s$ and the proportion of open circuit failures $P = q/(q+s)$, A,B,C or D may be the most reliable arrangement. In limiting cases two units are as good as three.

viz:

when $R_A = R_C$ also $= R_E$

and when $R_B = R_D$ also $= R_F$

When $P = 0.5$. $P_c = R_D$. If V is in addition either very low or very high there is little advantage over a single unit.

Open and short circuit redundancy may be summarized by plotting the lines where $R_A = R_c = R_E$, $R_E = R_D = R_F$ and $P = 0.5$ on a scale of $V=1-q-s$ versus $P = q/(q+s)$. Showing the regions where A,B,C, D are the most reliable arrangements. This analysis of open and short circuit redundancy is due to B.W. Jenney

A5. DEVELOPMENT OF RELIABILITY THROUGH SERVICE OR TESTING-DUANE MODEL

The empirical relationship described below enables the effect of a continued effort at improvement of MTBF θ to be predicted from early results at the same rate of effort. The 'same effort' rate implies the employment in germane positions of people of the same calibre and resourced at the same level throughout the programme of development even though the effort will be subject to a progressive diminishing rate of return in terms of increased MTBF. J.T. Duane's model has been found to be applicable to many diverse systems from computers to jet engines. It says simply that

$$\bar{\theta} = KT^{\alpha} \quad \text{and} \quad \hat{\theta} = \bar{\theta} / (1-\alpha)$$

$\hat{\theta}$ is the best estimate of the reciprical of the current hazard rate and $\bar{\theta}$ is the ratio of the total running time T tototal number of failures since the start of the development programme. Clearly, if θ is improving $\hat{\theta} > \bar{\theta}$ the growth rate can be estimated by making spot estimates of θ during the early stages of the programme K is a constant.

The measure of total effort and at the same time a factor in the calculation of $\bar{\theta}$ is the total test time. T. By Duane's model the plot of $\bar{\theta}$ versus T on log versus log paper is a straight slope α and that for $\hat{\theta}$ is a parallel straight line separated by the factor $1/(1-\alpha)$.

The result follows because by definition $\theta = T/F$ where F is the total number of failures to time T.

Hence $F = (1/K)\ T^{1-\alpha}$

The current value of failure rate is obtained by differentiation.

$$\hat{\lambda} = (1/\hat{\theta}) = (dF/dT) = (1-\alpha)/KT^{\alpha}$$

therefore

$$\hat{\theta} = KT^{\alpha}/(1-\alpha)$$

The model is empirical because there is no theoretical reason to expect a power function for $\bar{\theta}$.

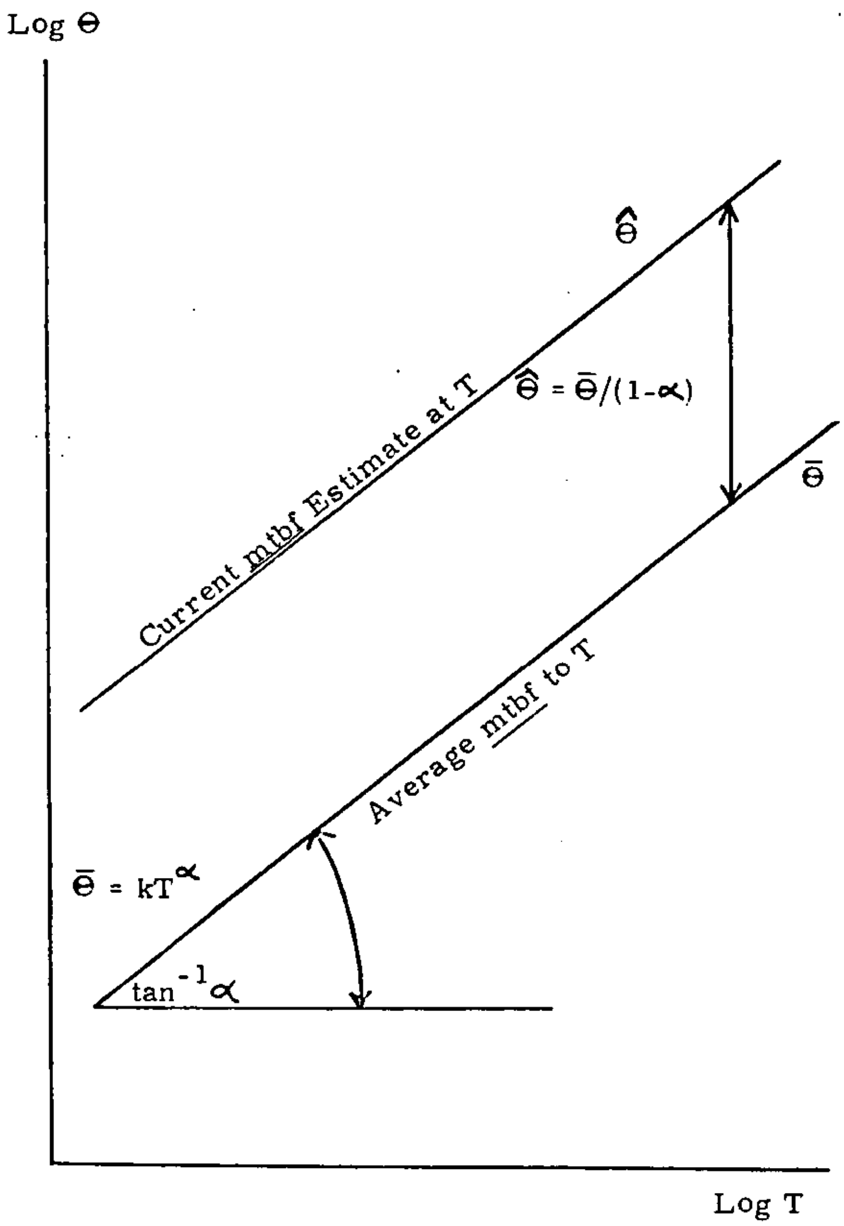


Figure A5.1. Duane Model for Growth of mtbf under constant resource allocation

A6 PHYSICS OF FAILURE

A6.1 INTRODUCTION

The Physics of Failures approach to reliability assessment consists in calculating or assessing the failure distribution from consideration of the physical properties and dimensions and the possible modes of failure of the components of the item. In the most general sense the strength and stress distributions are juxtaposed to estimate the probability of failure.

A6.2. When to Use the Physics of Failure Approach

Designers almost always use an approach of this kind initially (or a code of practice which is based usually upon a combination of Physics of Failure and safety factors derived from experience of the reliability of like items), under various conditions of service. It will be shown that the reliability assurance given by Physics of Failure alone is usually low and for this reason, its sole use is recommended only faute de mieux. Sometimes though, the functions to technology can be advanced in no other way. It is always safe to measure reliability directly than to rely on such indirect calculation alone.

A6.3. Stress and Strength Distributions

The life of a component may be considered as a series of applications of varying stress. A population will have distributed strengths. Due to material and dimensional variations the strength may also be subject to attrition over time in which case the failure rate will increase with time, and stress/ strength are here used in the most general sense.

A frequent criterion in design is the Factor of Safety, which is simply the ratio of the means of the distribution

A6.4 Margin of Safety

A better measure which takes account of variability as well as mean value, and which therefore relates more directly to the failure

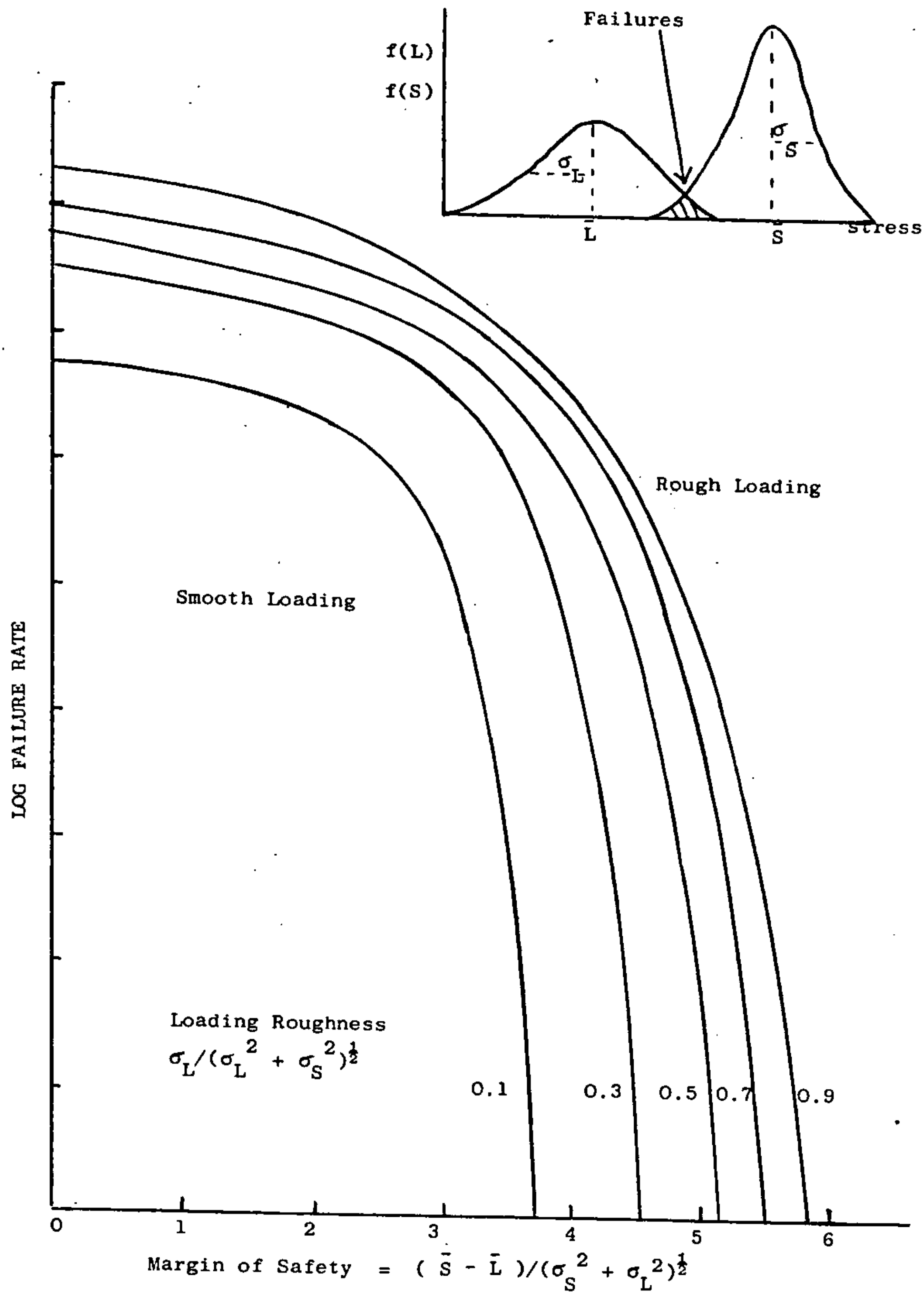


Figure A6.1 Sensitivity of Failure Rate to Margin of Safety and Loading Roughness (After Carter) distribution is Carter's Margin of Safety which is the reciprocal of coefficient of variation of the distribution of the difference between strength and stress .(2.15)

Where σ_s^2 σ_L^2 are the variances of the strength and stress distributions. Another dimensional quantity is needed in the analysis before generalisation about failure rate can be made. This is the relative roughness of loading $\sigma_L/(\sigma_L^2 + \sigma_S^2)^{\frac{1}{2}}$. Given no attrition of strength with time, the failure rate per application of stress is then determined for a given number of stress applications. It is found Fig.A6.1 that particularly in the high reliability region Failure rate is extremely sensitive to small changes or miscalculations of MOS. This is why Physics of Failure is not an accurate prediction of the scale of the failure time distribution. It is lucky if the order of magnitude turns out to be correct.

A6.5 Distribution Form and Physics of Failure

A different aspect of the approach through fundamental mechanisms of failure is that the form (if not as shown above the scale) of the distribution of times to failure is predictable from the mechanisms of failure. This feature can also be used in reverse to find a clue to the primary cause of failures when the physical evidence has been destroyed by secondary events.

Wear or attrition	Normal
Fatigue, repair times	Log normal, Birnbaum Saunders
Random causes	Exponential
Maintenance Deficiencies	Hyperexponential, Weibull ($\beta < 1$)
First or last of many	Gamma, Extreme Value, Log Extreme value

The method of modelling is typified by the following argument for the Normal distribution as a model for failures due to wear.

Wear may be considered as a succession of removals of very small particles from a surface, each removal exposing another particle to risk. If the time from exposure to removal of each particle is identically exponentially distributed with failure rate λ, and c particles must be removed for failure then the failure time distribution is of Gamma form with parameters c, λ. As $c \to \infty$ the Gamma tends to the Normal form. Actually the form of the particle removal time distributions is irrelevant, because the Central Limit Theorem states that a Normal form is general for such a convolution of identical distributions.

APPENDIX B

THEORETICAL CONSIDERATIONS CONCERNING THE EFFECT OF MAINTENANCE UPON OBSERVED FAILURE AND REPAIR TIME DISTRIBUTIONS

B1. INTRODUCTION

In this appendix arguments taken from various books and papers are combined into a discussion of how maintenance affects the distribution of times between failures (tbf's) and repair times (ttr's).

It is relegated to an Appendix because there is little new about the material, which is however somewhat diffused through the literature, so that the complete sequence of arguments has apparently not been presented previously in a single document. Another reason for placing this material in an Appendix is to avoid making, even by implication, any claim to primacy in connecting hyper-exponentially distributed failures with inadequate maintenance practices.

The most commonly-invoked model for both maintained and unmaintained equipment consisting of many components is the bath-tub curve. Some would claim that it applies to components as well as to more complex equipments. This model is discussed in some detail in order to dispel some of the confusion which presently surrounds it due to unthinking mis-application of the exponential distribution.

Next, a theory is developed as to why the hyper-exponential distribution or other models having standard deviation greater than mean such as the Weibull with fractional shape parameter ($\beta < 1$) have so often to be invoked in studies of maintained equipment.

Finally, the adequacy of the exponential model for repair times is challenged and the alternatives, the lognormal and gamma distributions, discussed.

References to the literature are given as the material in them is used to develop the themes of the Appendix.

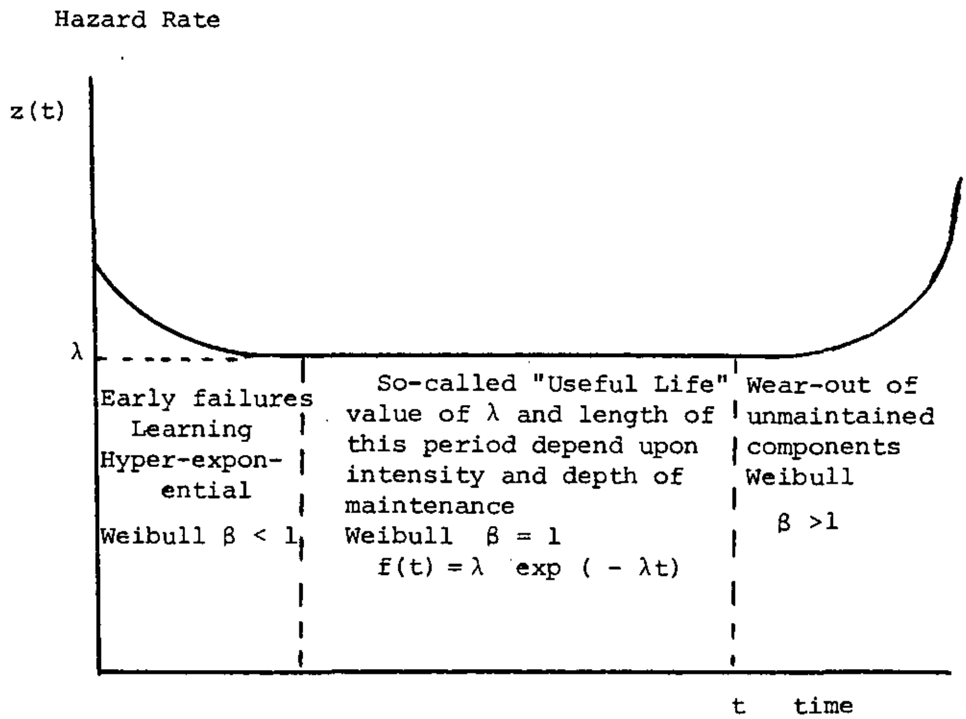


Figure B2.1 'BATHTUB CURVE' FOR A COMPLEX MAINTAINED ITEM

B2. THE BATH TUB CURVE

B2.1 Description

The origins of the bath tub curve are by now obscured. It appears in most texts on Reliability without attribution.

The description refers to the shape of the curve of instantaneous (conditional) failure rate versus time, which is observed, or said to be observed, over the lifetimes of maintained and unmaintained equipments and components, Figure B2.1. The instantaneous failure rate or hazard rate function is related to the probability density function as follows.

$$\text{pdf} = f(t)$$

$$\text{cdf} = F(t) = \int_0^t f(t)\,dx$$

$$\text{Reliability} \quad R(t) = 1 - F(t)$$

$$\text{Hazard Rate} \quad z(t) = f(t)/R(t)$$

Conceptually it is the conditional probability of failure in the interval t to t + 1 given survival to t. The model postulates from experience that all complex equipments and systems and some components have empirical z(t) curves consisting of three portions.

a) a period of falling z(t), variously called the infant-mortality period, or early failure period

b) a period of roughly constant z(t) often called the 'useful life' followed by

c) a period of increasing z(t) referred to as the 'wear out' period.

B2.3 Applicability to Components

Whether the curve applies to components as opposed to complex equipments has been questioned by Carter(5.9) by Talbot (5.42) and by Kamath et al (2.30). Whilst it is fairly easy to imagine a period during which early failures take place due to original quality faults (e.g. porosity in castings, dry joints in electronics, poor fit and surface finish in mechanical components), and even easier to conceive of an increasing failure rate towards the end of life due to some form of gradual attrition finally leading to inability to withstand working stress, (e.g. wear, fatigue, oxidation etc.), it is less easy to justify the middle, constant failure rate portion of the curve except as the fortuitous sum of two 'tails', one increasing the other decreasing. If indeed there are really only two modes and not three as the model postulates then it should show in a Weibull or other frequency analysis. Highly reliable components would be expected to show virtually zero failure rate during the period from the end of early failures to the start of wear-out. Apart from a few failures due to <u>external</u> conditions, such as maloperation, overloads imposed by failures elsewhere in the system and so on this is what is to be expected. If the central period of roughly constant failure rate is due not entirely to random occurrences but also to the tails of early and wear-out periods, it is wrong, as Talbot (5.42) has pointed out, to conclude from the observed fact of roughly constant z(t) that the underlying f(t) being exponential in form, z(t) represents an irreducible minimum failure rate. In fact z(t) can be improved by redesign or derating, by protection from overstress, by quality control in manufacture, by lubrication to reduce wear etc.

Now $f(t) \equiv -d\,R(t)/dt$ by definition

so $$\int_0^\infty z(t)dt \equiv \int_0^\infty -d\,R(t)/R(t) = -\log R(t)$$

$$R(t) \equiv \exp\left(-\int_0^\infty z(t)dt\right)$$

If $z(t) = \lambda$(constant)

$$R(t) = \exp(-\lambda t)$$

and so $$f(t) = \lambda \exp(-\lambda t)$$

which is the exponential distribution <u>pdf</u>.

This is a <u>mathematical model</u> of the central period perhaps, but it is not necessarily, in fact it is probably not, a <u>description</u> of what is happening to the population of like components from which the failure rate curve is derived.

Carter's (ibid) challenge is more fundamental, based as it is on the physics of failure of mechanical components. He has shown that for failures which occur because of the interaction of strength and stress distributions, with possible attrition of mean strength over time that the hazard rate curve for successive stress applications can take many shapes and that the bath-tub is the exception rather than the rule. However, his theoretical approach does lead generally to an expectation of higher $z(t)$ for t small and this is often followed by a period of sensibly constant $z(t)$. $z(t)$ cannot increase again unless there is attrition of strength. Under steady strength $z(t)$ is steady also after the early failures have occurred. However, monotonically increasing and decreasing hazard rates are possible as are U-shapes with no constant portion.

B2.4 Applicability to Systems

'Systems' in this context means any collection of components organised into an equipment, or several equipments, or indeed a whole plant. A mathematical treatment of the argument presented below showing that a constant, or rather apparently constant, failure rate is to be expected for a maintained system is given by Lloyd and Lipow(3.143 App 9B)

During the early life of a system or plant, the operators are learning by trial and error how to avoid overstressing components and the maintainers are also learning to avoid repetition of failures. There will, perhaps, be some design faults to be put right, and almost certainly some faulty components to be renewed after early failure. A falling failure rate is therefore to be expected.

After this shake-down period the starting times of components subject to failure will have become randomized by previous failures and renewal. In any case the time from the failure of one component to the next failure of the same or any other component in the system seems likely to be randomly distributed i.e. Poisson events with an exponential distribution of tbf's. For the moment consider only repair of failures i.e. fm and no pm. Lloyd and Lipow's (ibid) reasoning is a mathematical expression of this thought. Consider a system of N components each with the same failure distribution f(t) where t is reckoned from the last renewal and all renewals are due to failures. Then, if at a point t_o in <u>calendar</u> time the ages of the components are x_j; j = 1 . . .n then the reliability of the series system from t_o to $t_o + t$ is

$$R_{N,t} \Big| t_o = \prod_{j=1}^{N} \left[R\ (x_j + t) \Big/ R\ (x_j) \right] \qquad (B2.1)$$

This is a conditional reliability given survival of all the components to x_j. The unconditional reliability is given by

$$R_{N,t} = \left[\int_o^{+\infty} (R(x+t)/R(x))\ g\ (x_j t_o) dx \right]^N \qquad (B2.2)$$

where $g\ (x_j t_o)$ is the likelihood function of the ages of the N components at t_o

Now

$$\lim_{t_o \to \infty} \left[g\ (\ x_j t_o) \right] = R(x)\ /\theta \qquad (B2.3)$$

where θ is the individual components' common mtbf. The integral of (B2.3)over o to+∞ is 1. Formal proof of B2.3 will be omitted.

$$R_{N,t,} = \left[\int_o^{+\infty} (x + t)\ dx\ \ /\ \theta \right]^N$$

$$= \left[1 - t/\theta + \int_o^t F(y)\ dy\ /\theta \right]^N \qquad (B2.4)$$

where y = x + t.

Lloyd and Lipow then spend some time showing that the last term of B4 is small so that

$$\lim_{N \to \infty} \left[R_{N,t} \right] = \exp\ (-tN/\theta\ ) \qquad (B2.5)$$

Then if the system actually consists of N_1 components of mtbf θ_1, N_i with θ_i. . ., N_n with θ_n

$$R_N \simeq \exp\left(-t \sum_{i=1}^{n} (N_i/\theta)\right) \qquad (B2.6)$$

The result is independent of the individual distributions and shows that, whatever form these take, the expected observed combined distribution tends to the exponential. It also suggests that the combined failure rate is the sum of the individual component failure rates. Talbot (ibid) has challenged this extension of the theory as follows. A tacit assumption in the calculation above has been that failures are independent, that is that failure of one component does not affect the propensity for failure of any other component. In practice this is not so; a failure causes extraordinary stress elsewhere in the system which may not cause failure at once but rather shortens the remaining life of other components. This effect is probably the source of at least part of the constant failure rate for individual components, and explains why sometimes the overall failure rate of a system during the second phase of the bath-tub is greater than the sum of the components' failure rates. It also implies that efforts to improve reliability by redesign or de-rating may be rewarded more highly than would be expected from computation on the assumption of independence, and explains why component test results are usually better than failure rates from field data.

B2.4 The Effect of Maintenance

If parts are renewed before entering the wearout period, the flat portion of the observed component hazard rate curve can be extended indefinitely. With each renewal there is a small probability that it is defective and will fail early. However, this probability can be reduced by inspecting all spares before fitting. Nevertheless there is no point in renewing components if the failure rate is going to be the same or worse after the renewal. This means that components should not be renewed until their failure rate is increasing, under any policy where renewal is at fixed intervals from previous failure or scheduled renewal (ppm). However, when the maintenance schedule includes inspections and on-condition renewals (ocpm) as well as ppm then individual failures can be prevented on the basis of particular rather than general (distributional) knowledge of the condition of fallible components. Thus if an event occurs in operation which weakens a component in a detectable fashion and such that failure is not immediate but still inevitable then an inspection can prevent that failure from occurring by triggering an on-condition renewal.

No maintenance schedule embraces all the components in an equipment and eventually those components not subject to renewals will start to fail. Also, unrenewed parts which are adjusted in the course of routine maintenance will run out of adjustment (e.g. an engine can only be rebored so many times because matching pistons are not made above a certain size).

Maintenance in the forms of ppm and ocpm is therefore able to influence the scale and the shape of the bath-tub curve. The frequency of pm affects the level of the constant failure rate portion and the depth of pm (number of components included in the schedule) affects the length. The failure rate of any component or system can be reduced to any required level at a price by a combination of preventive maintenance, redesign and de-rating. If downtime rather than cost is the criterion of performance then there will be a limit to the amount of scheduled downtime which can be accepted for each hour of failure (unscheduled) downtime that is on average saved.

The commonly-held belief that when the failure rate is falling or constant the best maintenance policy is to wait until it fails is now seen to be facile and not applicable to complex equipment in general.

The expected observed overall distribution of tbf's for a complex equipment suffering several modes of failure is exponential. Departures from this norm require explanation. If the failure rate is increasing then either it is entering the wear out phase or one or two failure modes with individually increasing hazard functions are dominant. In either event maintenance action rather than inaction is usually required either to renew the whole equipment or to introduce pm to deal with the dominant failure modes. Periodic renewal (ppm) is often preferable to inspection/on-condition maintenance schedules (ocpm) in these circumstances. The case of decreasing observed hazard rate implying a hyper-exponential or Weibull ($\beta < 1$) distribution is discussed in detail in the next section.

Berg (2.8), analysing tbf's of process pumps and valves under fm found that average failure rate increased generally with failure number, the mean time from the ninth repair to the tenth failure being about half the mean time from new to first failure. Berg reported this as evidence against the generality of the bath-tub model and explained the perverse effect as being due to the policy of minimal repair; only the immediate cause of failure was repaired, no attempt was made to restore the item to good-as-new. The tabulated data produced by Berg would actually be inconclusive as to whether the failure rate was rising or falling, if the data from new to first failure were omitted. The failure rate goes up and down by about 1/3 of its mean value in each direction with varying failure numbers and appears, towards the end of the data at failure numbers > 8, to be settling down. A certain amount of 'noise' would be expected and until the component ages become randomised thoroughly by the early failures, misleading results are likely. In a well tried item like a pump, possibly installed by its makers and guaranteed for a period, the low failure rate to first failure shows that early failures can be eliminated by taking care with design and installation. It is the level portion of the curve which is inevitable for a complex equipment - eventually and for a large enough sample the average failure rate must settle down to a constant value which depends upon the maintenance policy and will remain at that value until failures occur to parts which cannot readily be renewed or until scope for adjustments is all taken up.

An interesting side issue to the discussion above was raised by Aird in written discussion of the paper by Talbot (ibid). It is often given as an example of the bath-tub curve that such a shape is obtained in the case of human mortality. It would be a good example because the data-set is large and its accuracy high. Aird showed by means of Weibull plots that there were only two 'modes of failure' in the death statistics, one with $\beta < 1$ and the other with $\beta > 1$. The human body, it can be argued, is a system under fm since very few people visit a doctor unless they feel ill. It also contains several components which are vital and virtually irreplaceable (spare-part surgery is discounted) and which have increasing failure rates. The falling failure rate at the beginning is due to the early deaths of the congenitally weak. Accidents, which one might initially suppose would cause a random mode, in fact occur with increased frequency to the very young and the old, those in their twenties and thirties being less prone. Aird's analysis demonstrates that single Weibull modes do not necessarily imply unique causes; both congenital weaknesses and failures of vital organs are essentially diverse.

B3. MAINTENANCE AND HYPER-EXPONENTIAL FAILURE TIME DISTRIBUTIONS

B3.1 Introduction

This final part of the Appendix deals with the theory connecting an observed hyper-exponential or Weibull ($\beta < 1$) distribution of tbf's with deficiencies in maintenance practice. The text is based upon relevant parts of a paper read at the 5th Symposium on Reliability Technology, Bradford, September 1978. (2.47).

B3.2 Literature and Instances of Falling Hazard Rate

The earliest reference found to falling failure rate in maintained equipment is Waddington (1.1)(1942) referring to the maintenance of Coastal Command aircraft of the R.A.F. It was noticed that failure rate increased immediately after scheduled pm had been performed. However, the failures which occurred in immediate post-pm periods were not usually serious, but rather instances of inattention to detail and the results of hurry and inadequate supervision. An examination of serious failures showed that although they did increase slightly after pm they were also showing statistically inconclusive indications of increasing again after a more or less level period.

Weibull β-values less than unity were recorded by Berg (2.8). The analyses were Weibull distribution estimates for sets of tbf's for each failure number. That is each data-set contained only times from the ith repair to the i + 1st failure. The series ran from new to the tenth breakdown. Almost all the β-values were less than unity and those that were approximately equal to 1 ($0.9 < \beta < 1.1$) occurred where the number of parts replaced at previous failures was relatively high.

During service at the (Royal Navy) Ship Maintenance Authority the writer became aware of $\beta < 1$ in Weibull analyses of tbf's for equipment operated under pm schedules which included both ppm and ocpm elements. Such analyses were not commonly found in Naval equipment, but were more frequent amongst equipment maintained by the Weapon and Electrical Engineering Department, than by the Marine Engineering Department (Propulsion, Refrigeration, piped services). The failure-response policies of the two branches of the Service were different. The Weapon and Electrical Branch by and large sent the most junior available rating to investigate. He was trained to send for a more senior rating if he considered it necessary, and so on up the line of technical responsibility. The Marine Engineering Branch started by sending an artificer (a senior rating who has served an apprenticeship) or, if it sounded really serious, an officer, to investigate. He detailed what work was to be done and supervised its progress, as often as not participating himself. The work was always independently inspected by an officer or another senior rating on completion or before 'closing up'. Another relevant factor was that the Weapon and Electrical Branch do not usually act as operators for the machinery they maintain whereas the Marine Engineering Branch are mainly user-maintainers. The Official Secrets Act forbids the publication of detailed examples. The relevance of these observations on Naval practice will later become clear; suffice it to state now that the Fighting Services do a lot of maintenance in peace-time, some of it probably beyond the level that could be justified economically in commercial plant.

Many further instances of Weibull $\beta < 1$ have been reported privately to the writer, it is probably much more common than the short list of papers and reports on the subject would suggest. Data analyses reported in Chapter II of the thesis suggest that $\beta < 1$ is the rule rather than the exception in process plant.

Vesely (5.44) and Aird (5.1) have independently suggested that the phenomenon of $\beta < 1$ in tbf analyses may be due to poor maintenance practices. The rest of this section is concerned with an examination from theory of how poor maintenance practices might lead to hyper-exponentially distributed tbf's in an equipment which contains several parts subject to failure.

B3.3 Hyper-Exponential Distribution Forms

The usual model for a hyper-exponential distribution is the two-parameter Weibull with shape parameter $\beta < 1$. The cumulative distribution function is

$$F(t)_w = 1 - \exp\{-(t/\eta)^\beta\} \qquad (B3.1)$$

where η is a scale parameter known as the Characteristic Life. The mean time between failures (mtbf) is given by

$$\theta_w = \eta\ \Gamma(1 + 1/\beta) \qquad (B3.2)$$

This model was used in Section II, but there is an alternative model given by Jardine (3.112)

$$F(t)_H = 1-k \exp(-2k\lambda t)-(1-k)\exp\{-2(1-k)\lambda t\}\ 0\ k\ 0.5 \qquad (B3.3)$$

In both cases the hazard rate function or instantaneous failure rate decreases with time.

$$z(t) \equiv \{dF(t)/dt\} \ / \ \{1-F(t)\} \qquad (B3.4)$$

Equation (4) means that hazard rate is the conditional probability of failure in the unit interval following t given survival to t.

$$z(t)_w \equiv \beta t^{\beta-1}/\eta^\beta \qquad (B3.5)$$

and for true hyper-exponential

$$z(t)_H = 2\lambda\{k^2+(1-k)^2\}\exp\{-2\lambda t(1-2k\}\ \{k+(1-k)\}\ \exp\{-2\lambda t(1-2k)\} \qquad (B3.6)$$

(Note : 'Hyper-exponential' is used here as an adjective. Equation B3.3 is often called 'The Hyper-Exponential Distribution'.)

At this point the general preference for the Weibull will be obvious, but the other model illustrates better how a hyper-exponential distribution arises. It is a combination of two types of failure. The first mode is early failure and the other is the usual pseudo-random negative exponential distribution which results from lumping together all the failures which occur in a complex equipment (see B2.3 above)

The two models do not coincide precisely but a rough equivalence may be obtained by the method of moments, that is by equating means and variances. The mean of the hyper-exponential is simply $1/\lambda$

The variance of the Weibull is

$$\sigma_w^2 = \eta^2 \, \Gamma(1+2/\beta) - \theta_w^2 \qquad \text{(B3.7)}$$

The variance of the hyperexponential follows from consideration of the distribution as the weighted sum of two exponential distributions.

$\theta_1 = 1/2k\lambda$ occuring with probability k

plus $\theta_2 = 1/2\,(1-k)\lambda$ occuring with probability (1-k)

The variance of a combined variate is the sum of the component variances and the variance of an exponential distribution is equal to the square of the mean.

The variance of the hyperexponential is therefore

$$\sigma_H^2 = (1/4\lambda^2) \; \{1/k + 1/(1-k)\} \qquad \text{(B3.8)}$$

for both distributions $\sigma > \theta$

B3.4 Conditions for the Hyper-Exponential Failure Distribution

B3.4.1. The Hyper-Exponential Distribution - can and does arise in complex equipment whether or not it is regularly maintained. Reference has already been made to the expectation of a random distribution for unmaintained equipments with many modes of failure. Put another way, the base failure distribution for a complex equipment is negative exponential. If the observed distribution is not of this form then external factors are operating to change it. Some possible external factors leading to a hyper-exponential distribution are now examined.

B3.4.2. Incomplete Maintenance - consider an equipment which has run without failure for a relatively long time. It then fails. In order to get back on stream quickly, only the immediate cause of the failure is repaired. As it goes back into service a number of other failures are more or less imminent. In some cases it may simply be that the design is such that a number of parts have wearout distributions (Weibull $\beta > 1$) with about the same MTBF. This after all is an aim of a good design - that all wearing parts should require overhaul at the same time. In other cases the side effects of the failure that has been repaired are not corrected and again early failure results. After a few early failures all the immediate wearout problems are solved and all the consequential damage repaired and so more by good luck than good management the equipment again runs without failure for a relatively long time. The cycle is then repeated. Clearly it would be more economic to restore equipments to reasonably good condition when they fail instead of suffering several equipment shutdowns which are bound to add up to more lost time. From another viewpoint the bath tub theory depends upon replacement or restoration to good-as-new when failures occur. A frequently observed instance of incomplete maintenance was fitting new mechanical seals to eroded pump shafts.

B3.4.3. Incorrect or Incompetent Maintenance - In this case the fitter, due to pressure from Production, inadequate supervision or lack of training, sows the seeds of the next failure whilst repairing the first. He may do any of a number of things such as

(i) Allowing ingress of dirt

(ii) Missassembly

(iii) Fitting the wrong part

(iv) Re-using consumables that should be renewed such as split pins and loose packing

(v) Failing to check alignment properly

(vi) Failing to adjust clearances etc correctly.

Notice that these are ultimately organisational or management faults. It is not fair to blame the fitter or his Trade Union for all that is wrong in maintenance practice. Prevention in this case requires changes in management practice. With modern equipment specific training will be necessary, work should never be hurried and should always be checked independently by the foreman or, in the case of important repairs, by the plant paintenance engineer himself. Whenever possible, machines should be removed to a clean workshop or a clean area created around the work before opening. Common fit items are a help in reducing plant downtime because it becomes economic to provide an unfitted standby equipment if such can cover for several identical installed equipments. (It may even be economic to provide a fitted switched stadby specifically to cover for an item with high failure rate, but that is another subject altogether).

B3.4.4. Poor Quality Spare Parts - Where quality control is not applied and to a lesser extent where poor sampling procedures are used the strength distribution of spare parts is likely to be bimodal. The left-hand mode will lead to early failures. Prevention consists in buying good quality spares from reputable sources, insisting upon supplier quality assurance. Spare parts should be inspected before fitting and repetitive early failures of the same part investigated in conjunction with the supplier.

B3.4.5. Over-Maintenance - If scheduled maintenance is carried out too frequently, opportunities for incompetent work and faulty spares increase, and can produce a hyper-exponential pattern. This has been observed in computers and seems to apply to most electro-mechanical systems. In chemical plant it may be expected to apply to safety and control equipment. Again the solution follows from the problem - better quality control of spare parts and inspection of workmanship, which having been applied successfully can be followed by a reduction in the frequency of preventive maintenance.

B3.4.6. Transient Conditions - Overloading - When plant is being started up after a failure there is a transient period during which extraordinary stresses, currents temperatures and other conditions may exist. e.g. starting torques in pumps and mixers, electric motor starting current surges, boiler superheater tube metal temperatures under low steam take-off rates. In such cases the failure may occur immediately or there may be a cumulative weakening effect leading to failure at a later start-up. Such failures are fairly rare because their causes should be taken into account in design and operating instructions. Their incidence, may be greatly increased by the chemical plant flow-rate development procedure known as de-bottlenecking. If machinery is run at speeds for which it was not designed, the extreme sensitivity of failure rate to stress discussed by Carter (5.9) becomes painfully obvious. This type of failure is excluded from the data on which Chapter II is based because failures on start-up were counted as continuations of the previous failure.

B3.4.7. Bedding-in Of New Parts. Extraordinary stresses leading to increased failure rate for a period after repair may occur due to new parts bedding-in. This slightly begs the question of incomplete or incompetent maintenance or poor spares, because if failure occurs before the new part has settled then perhaps the mating part should have been renewed also, or the quality of the fitting work or the spare was questionable. Whilst the existence of borderline cases is not denied it is asserted from observation that it is usually quite easy in practice to classify a particular failure. The advent of dimensional quality control procedures in the manufacture of machinery and spares has eliminated most of the potential causes of bedding-in failures. True bedding-in failures which are 'nobody's fault' are rare and often reflect poor or outdated design. An example is the scraping of the large end bearing shells for a reciprocating steam engine. Although the fitters were first-rate and knew precisely what was required, hand scraping tools could not produce a new surface on the white metal which carried no risk of a bedding-in failure.

B3.4.8 New Plant and Post-modification Failures. The classic bath-tub curve shows falling failure rate at the beginning of equipment life. As already discussed, this is due partly to teething troubles some of which are design faults while others relate to the 'learning curves' of both operators and maintenaners. The same sort of effect, perhaps on a smaller scale both as to failure rate elevation and time duration can occur when plant is modified. These transient teething troubles should not however be confused with permanent rises in the average failure rate due to increased stresses arising from de-bottle-necking modifications.

B4. THE DISTRIBUTION OF REPAIR TIMES

B4.1 Distributions

B4.1.1 The Lognormal Distribution - is described in Appendix A and it is there stated that it is generally applicable to repair times. A complete account of the distribution and its statistical and philosophical implication and uses is the book of Aitchison and Brown (5.2) which is the standard work on the subject. A useful shorter discussion appears in Goldman and Slattery's book (5.22) pp 45-62. Goldman and Slattery also state that the lognormal is frequently found to give the best fit amongst the alternatives of exponential, Weibull, largest extreme value and Gamma distributions. Horvath quoted by Goldman and Slattery (ibid) found that store service times and the times to find a book in a library were lognormally distributed. When the practical repair time data is added to these observations and many others the common factor is that all these lognormally distributed activities consist of partitioning or systematic categorisation. Repair times typically consist of a variable time for diagnosis depending upon the familiarity of the technician with the equipment and the frequency of particular classes of failure, followed by a less variable time for actual repair. Most fitters will be able to diagnose the more common faults at once and repair them in about the same time, but less frequently a new man or a new fault will cause a longer repair time. A distribution skewed to the right is therefore to be expected. There are other distribution forms with this type of skew and sometimes advantage can be taken of their generally easier mathematics but the lognormal is the only one whose hazard rate can be finally decreasing after starting as an increasing function at time zero. For these circumstances only the lognormal will do - any other distribution form will introduce unnecessary inaccuracy. (This observation applies to the use of the lognormal for tbf's and well as ttr's).

B4.1.2. Exponential Distribution - As with ttf's and tbf's there is great pressure from those who would simplify an essentially complex matter to use this single-parameter distribution. The model can never be correct because a set of repair times with a modal value of zero is an obvious impossibility. However, if $\sigma/m < 1$ in the lognormal model, the slope of the falling portion of the $z(t)$ curve is not great and above a certain value of time a constant approximation would be acceptable. An exponential displaced forward in time by a fixed minimum repair time is a convenient compromise sometimes employed in modelling.

B4.1.3. Gamma Distribution - The hazard rate of the Gamma distribution either rises or falls with time at an ever-decreasing rate towards an asymptotic value of $1/\lambda\Gamma c$. It cannot therefore fully represent a true lognormal distribution. A special case where the shape parameter c is an integer is often called the Erlang distribution. A Gamma distribution can be represented in a first order Markov model as c states with constant and equal transition rates. The convenience of this parameterisation makes it attractive for modelling purposes, and it is better than an exponential assumption made without adequate evidence.

B4.1.4. Hyper-Exponential Distribution or Weibull ($\beta < 1$). Occasionally the distribution may be so skewed that the hyper-exponential or a Weibull model ($\beta < 1$) is adequate. As both are two-parameter distributions as is the more 'correct' lognormal the only excuse for such a model is convenience in computation.

B4.2. The Effect of Maintenance on Repair Time Distributions

B4.2.1 Effect of Familiarity. As maintenance personnel become better acquainted with a particular piece of plant, diagnosis of common faults may be expected to become quicker. The likely effect is to move the mode of the repair time distribution to an earlier time and to increase the randomness of the longer repair times. The distribution is likely to move towards but not to the exponential.

B4.2.2. Effect of Preventive Maintenance. The more common repair times around the mode of the base distribution are removed by pm. The mean of the remainder is likely to be greater and their distribution more random. Again the effect is a move towards the exponential.

B4.2.3. The Combined Effects of familiarity and preventive maintenance are likely to be towards the exponential. It is unfortunate that the major data experiment could not include repair times as this is the only way that convincing evidence that the exponential model is satisfactory can ever be produced.

B5. AVAILABILITY

B5.1 Types of Availability

Definitions of availability fall into three types as follows:

a) Point Time Availability. A(t). This is the probability of the up state at time t having started in the up state at time zero. As with all definitions the 'up' and 'down' states and the time scale must be defined. Its value depends upon the tbf and ttr distributions especially upon their means. Green and Bourne(5.24 Chap.10) show that for exponential distributions of tbf's and ttr's.

$$A_E(t) = \mu/(\mu+\lambda) + \{\lambda/(\mu+\lambda)\} \exp\{-t(\mu+\lambda)\} \qquad (B5.1)$$

where λ = failure rate, and μ repair rate

For most other distribution forms it can be shown that A(t) consists of a constant term equal to mtbf /(mtbf + mttr) plus a decaying time-dependent term. Green and Bourne's (ibid) distinction between failure and repair distribution which are functions of real time and those which are zeroed by each repair should be noted.

b) Average or Steady State Availability over a long period is the same as point availability as time tends to infinity. One can arrive at the same type of definition involving the ratio average failure rate to failure rate plus repair rate or mtbf/(mtbf+mttr) or similar ratios depending on the time-scale either by extending definition (a) to $t \to \infty$ or similarly by considering the average complete cycle from the completion of one repair to the completion of the next. The second route is totally independent of the distribution function of tbf's and ttr's, it depends only upon their means. In Markov models such as those described in III§16, ratios of transition rates having the same $\mu/(\mu+\lambda)$ format as availability appear frequently as the average probabilities of being in certain states. Markov models based upon average transition rates tend to give fairly accurate results even when the distributions are known not to be exponential. This is because the answers usually required are averages over long time periods and the distribution effects are smothered in the same way as above.

c) Managerial Availability. The ratio of actual to rated, expected, or required plant output is often called 'availability' in management information documents. This usage is confusing but widespread, and so must be accepted as an alternative definition. If the plant or item has only one output rate, there is no storage of intermediate products between stages of production, the time-scale is calendar; and all downtime, including that associated with pm is counted, then managerial and probabilistic availabilities are equivalent. It is possible to arrange matters so that the difference shows the effect of inter-stage storage.

B6. BIMODAL DISTRIBUTIONS

B6.1 Applicability to Context

Bimodal distributions, their separation and the calculation of the proprtions of events in each mode is placed here principally because bimodality in observed distributions of both tbf's and ttr's may be the upshot of maintenance. It is possible as shown above to have early failures due to poor maintenance and later failures due to other causes. It is also possible to separate a lognormal or similar repair time distribution into a familiar mode with a low mean and a more random mode of unfamiliar repairs. Goldman and Slattery(5.22) demonstrate that any number of exponential distributions may be subtracted from a lognormal leaving another lognormal which becomes increasingly exponential-like as the process continues. It is important to be able to do so because it is likely that the familiar mode will be the result of a frequent failure mode and so be dealt with by pm. Estimates of the parameters of both modes and the proportionality will be required for planning purposes.

B6.2 Separating the Modes

B6.2.1 Graphical Methods of Separation are described by Bompas-Smith(2.10) for the four types of mixed distribution shown in figures B6.1 to B6.4. Bompas-Smith shows that it is possible, when the scales of two distributions are well-separated in time to estimate the two sets of parameters graphically and without reference to the qualitative aspects of the data. The methods described in B6.2 to B6.4 are slightly better. Figure B6.3 may be a true hyper-exponential or a less severe version of B6.2. If the first part has a Weibull β value of about 0.8 - 0.9 then it is likely to be a true hyper-exponential. However, when the distribution is as in Figure B6.4 in a Weibull or Cumulative Hazard plot it is possible to interpret the results as possibly lognormal or Gamma in form. In this case the doubt can only be resolved by the qualitative data. In the case of repair times the bimodal interpretation is possibly as valid as the lognormal.

In all cases of bimodal separation, unless one mode has a delayed start ($\gamma > 0$ in the Weibull model) it must be remembered that the line represents the sum of the two distributions functions at all points - it is only if one distribution dominates for part of the time span that they can be separated.

B6.2.2. Analytical Methods of Separation are described by Mann et al (5.38) and by Kamath et al (2.30). The techniques rely upon maximum likelikhood (maxlik) or least squares. In both cases a distribution form, such as the Weibull must be assumed for each distribution. The number of data points must exceed the number of parameters to be estimated. These methods will not be detailed as they have not been used. Optimization in 5,6 or 7 dimensions, which is what is involved here, is a time-consuming procedure even when a computer is employed. However, one of the parameters found by the optimization is the proportion of early failures, based upon the same criteria of maximum likelihood or least squares.

B6.3 Finding the Proportion in each Distribution

B6.3.1 Given two sets of Parameters. Kamath et al (ibid) describe two methods based respectively upon maximum likelihood and Bayesian inference. The first method is not really available to practising

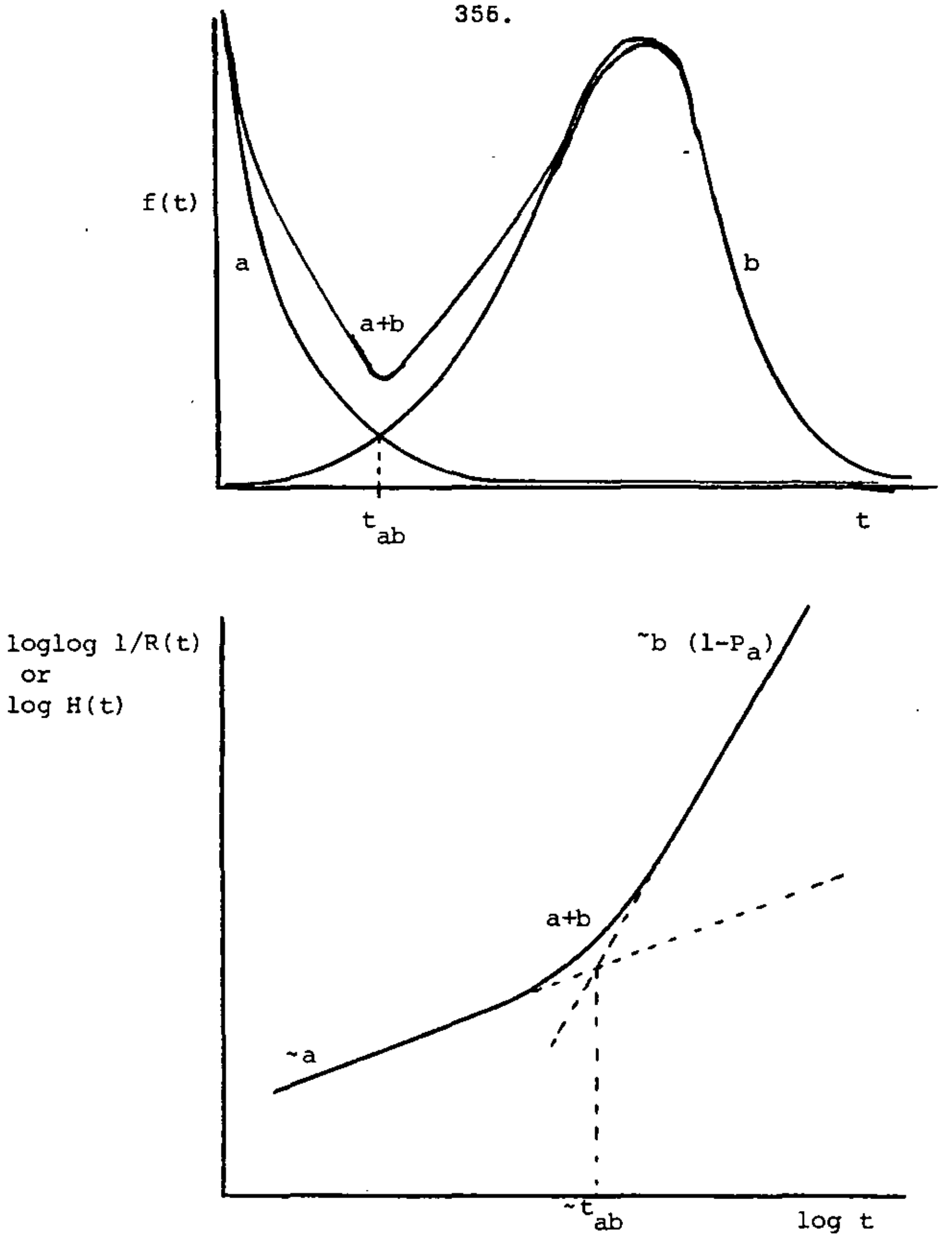


Figure B6.1 Graphical separation of two modes of failure with significant overlap.

Procedure:

1. Estimate mode of a from early part of graph
2. Estimate proportion of early failures P_a as F(t) at intersection of tangents t_{ab}
3. Estimate mode b as tangent to later part of graph with F(t) divided by factor (1-P_a)

Assumptions:

1. Distribution tails either side of t_{ab} are of about equal area
2. Tangents in log graph intersect at about t_{ab}

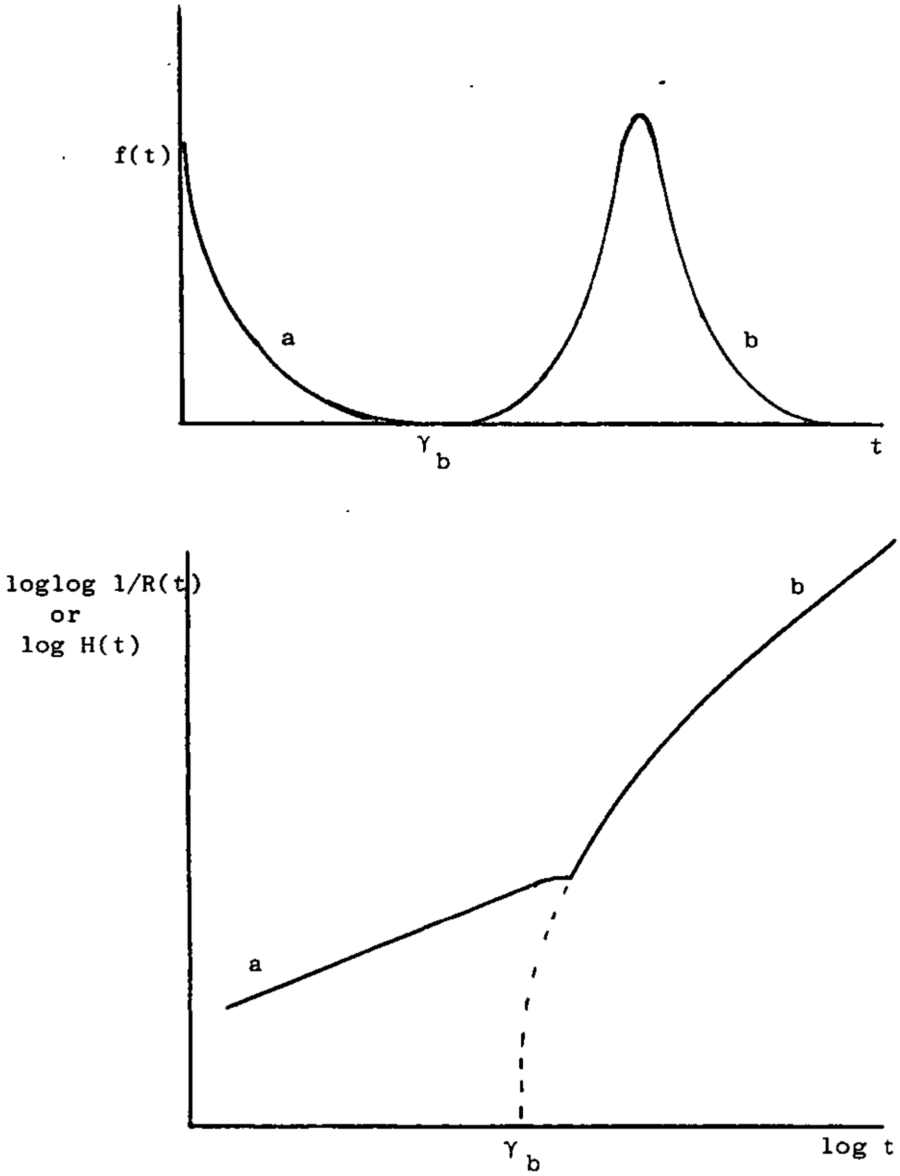


Figure B6.2 Graphical Separation of two modes of failure without significant overlap with delayed start of second mode

Procedure: As for Figure B6.1 but put $\gamma_b = t_{ab}$

Assumption: Failure by mode 'a' after γ_b is most unlikely

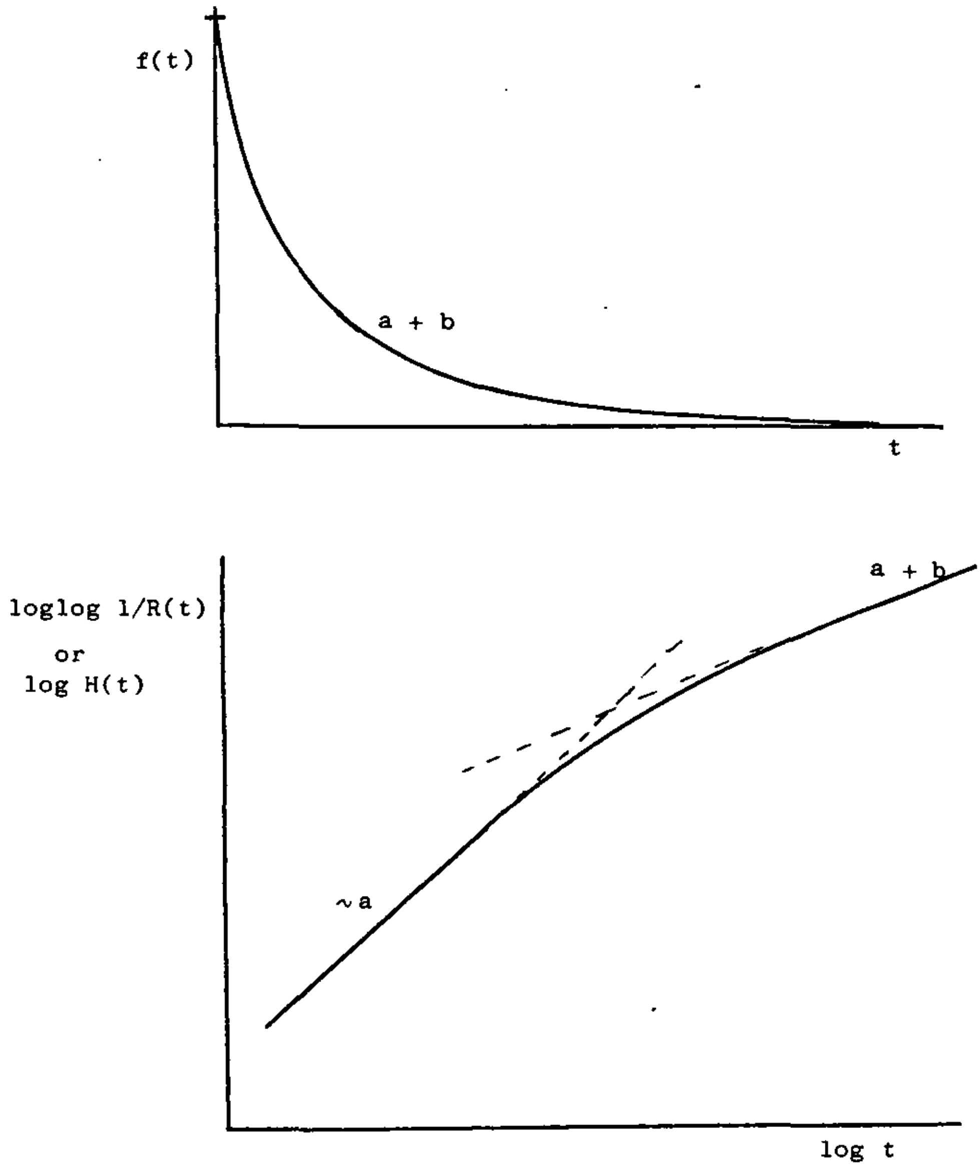


Figure B6.3 Graphical separation of two modes of failure - weak second mode or true hyper-exponential

Procedure:

1. If $0.8 < \beta_a < 1.0$ then it is probably hyper-exponential proceed to find λ and k analytically e.g. by maximum likelihood
2. If not then see Figure B6.4

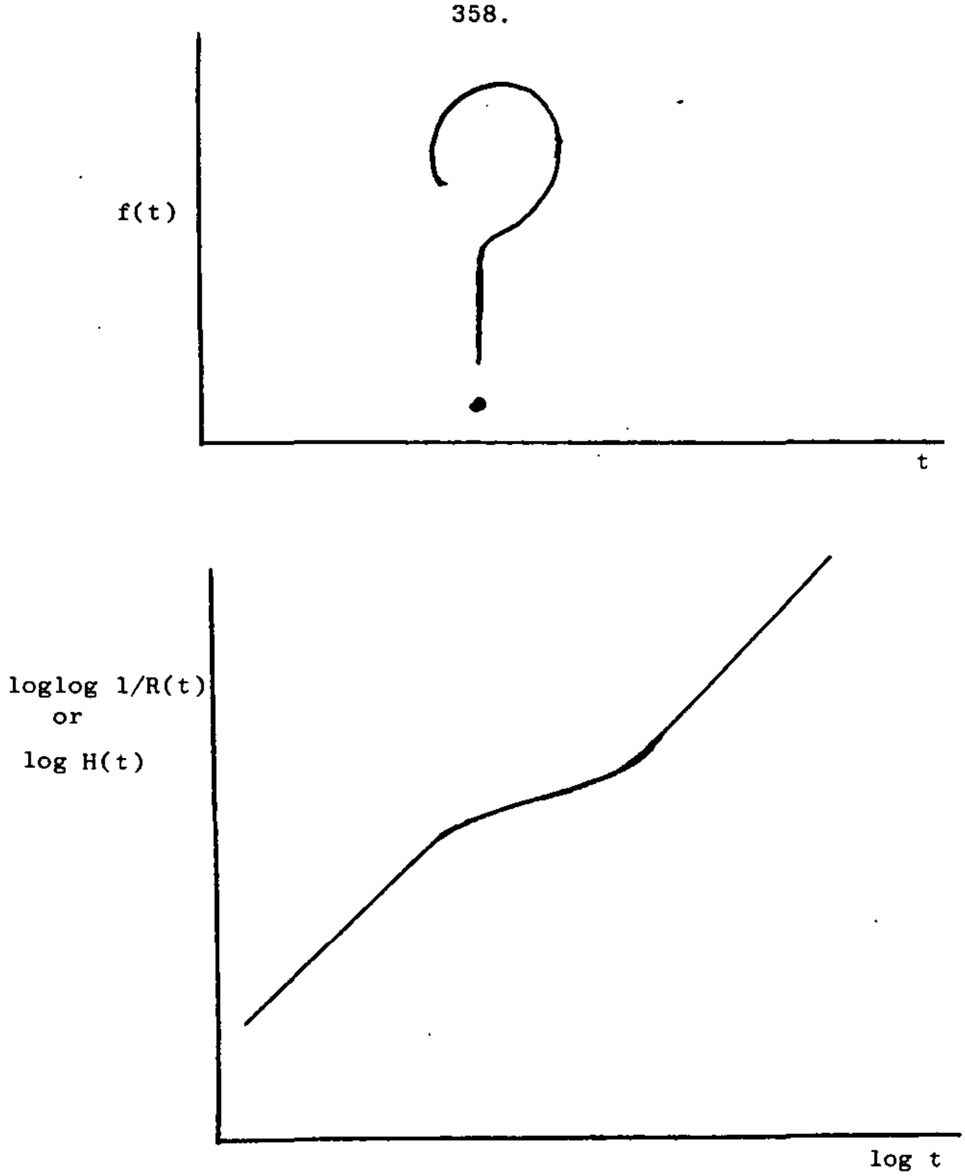


Figure B6.4 Graphical Separation of two modes of failure - cranked curve

Interpretation:

1. This curve could be interpreted as bimodal with more overlap than Figure B6.1. Graphical methods fail unless the ends are very well defined by numerous data, and one mode has $\beta > 1$, in which case the method of B6.1 can be used. Otherwise qualitative data are needed to separate into modes.
2. Alternatively replot on lognormal paper.

maintenance engineers unless they have access to a computer. The Bayesian inference method is much simpler but it was found through experiments with simulated data of known distribution that it was inaccurate without the extensions which are now described, and sometimes became unacceptable even with the extensions. Kamath et al proposed the following calculation for P, the proportion of early failures given a data set t_i, i = 1, . . . n of tbf's or ttf's and two distribution pdf's $f_a(t)$ and $f_b(t)$.

$$P_o = \sum_{i=1}^{n} \left[f_a(t_i) \;/\; \{ f_a(t_i) + f_b(t_i) \} \right] \Big/ n \qquad \text{(B6.1)}$$

For a censored data-set n in equation B6.1 should be replaced by M_n, the mean order number of the last failure. The prior probability that t_i belongs to $f_a(t)$ in equation B6.1 is 0.5. When the method was tried using 400 data simulated by inverting the mean ranks, and, in a second run, the median ranks of appropriate numbers of failures from two Weibull distributions it was found that the value of P_o was a most inaccurate estimate of P. So P_o was fed back to equation B6.2 below to obtain P_1.

$$P_1 = \sum_{i=1}^{n} \left[P_o f_a(t_i) \;/\; \{ P_o f_a(t_i) + (1-P_o) f_b(t_i) \} \right] / M_n \qquad \text{(B6.2)}$$

Sometimes further iteration produced convergence to a near-correct answer, sometimes oscillation and sometimes a degeneration ($P_\infty \rightarrow 1$ or 0) Convergence tended to be slow in large samples ($\sim$ 1000) and oscillation was more frequent in smaller samples ($\sim$100)

B6.3.2. From a Two-Parameter Single Weibull with $\beta < 1$. The alternative model for hyper-exponential distributions given at B3.3 above is the weighted sum of two exponentials. The weights are (1-k) and k and the failure rates 2 (1-k)λ and 2kλ respectively. Since $k \leq 0.5$ the former failure rate $1/\theta_a = 2(1-k)\lambda$ represents the early failures.

While $1/\theta_B = 2\,k\lambda$ is the failure rate of the later failures.

$$4k\,(1-k) = \theta^2/\sigma^2 = \{\Gamma(1+1/\beta)\}^2 \Big/ \left[\{\Gamma(1+2/\beta) - (1+1/\beta\}^2\right] \qquad \text{(B6.3)}$$

This equation results in the table below

TABLE B6.1 - WEIBULL /HYPEREXPONENTIAL EQUIVALENCE

β	θ/σ	θ^2/σ^2	1-k	θ_a/θ_b	$\frac{1}{2}$ - k
0.3	0.185	0.34	0.99	0.010	0.49
0.4	0.318	0.101	0.974	0.027	0.474
0.5	0.447	0.200	0.947	0.056	0.447
0.6	0.569	0.323	0.911	0.098	0.411
0.7	0.683	0.467	0.865	0.156	0.365
0.8	0.793	0.629	0.804	0.244	0.304
0.9	0.898	0.807	0.720	0.389	0.220
1.0	1.000	1.000	0.500	1.000	0.000

B6.3.3. Combined Distributions. If the early mode of a bimodal distribution separated graphically is hyper-exponential then the proportion of early failures may be calculated as P(1-k).

B6.3.4. Discussion. The estimator (1-k) clearly over-estimates the proportion of avoidable early failures due to maintenance mistakes since it is the early distribution which becomes the only distribution as $\beta \to 1$. A better estimate would seem to be ($\frac{1}{2}$ - k) but theoretical justification is lacking. Either measure can of course be used comparatively to judge improvement after a change in maintenance policy, but then the raw value of β is almost as good for that purpose.

The difficulty with the Bayesian inference method in convergence led to preliminary investigation of maximum likelihood methods for progressively censored samples. Even if the Weibull distributions are restricted to two parameters ($\gamma = 0$), with P, this means a 5-parameter optimization program which Kamath in discussion over the same paper declared to be too formidable in the cases of Type I and Type II censoring. With progressive censoring it would be even more difficult, and so this line of inquiry was abandoned.

Better approximations using simulated data were obtained if P_o was chosen close to P. In practice it will be possible to count up early and late failures and so find a better initial estimate than $\frac{1}{2}$.

A quick estimate for P_o is to take the value of F(t) corresponding to the intersection of the tangents of the two lines or the point of discontinuity in the plot on Weibull paper of the combined distribution.

Both the Bayesian inference method and the method of moments applied to the hyper-exponential model lack accuracy. They have been used in default of anything better rather than on their inherent merits which are few if simplicity is discounted.

B6.4 CONCLUDING REMARKS

The difficulty experienced with theoretical methods of separating just two distributions in a set of combined data highlights the value of qualitative and descriptive elements in failure data. It will always be easier, more certain and more accurate to classify failures by different modes from descriptions of the damage found and the repair work. Simple classification of failures under a number of arbitrary coded headings as in Section 10 contains less information to assist the task of separating the avoidable early failures than a full description. However, such classification is helpful in eliminating or confirming as a possible cause of $\beta < 1$ in combined data, the existence of two dominant modes of failure.

If the ratio of the means is high the distributions can be separated and analysed graphically. A more likely result when the ratio is smaller is that the plot will appear to have one mode with $\beta < 1$.

Referring to 6.3.4. above, a rough justification for the estimate $P \simeq \frac{1}{2} - k$ is that for an early failure caused by a maintenance error to be possible a 'normal' failure must first have occurred. If this is so, then $0 \geqslant P \geqslant \frac{1}{2}$. However it is possible to make another maintenance error leading to another early failure whilst repairing the first early failure.

APPENDIX C

COMPUTER LISTINGS

C.1. PROGRAMMES LISTED

C1. 1 INSPEF

This programme is in ICL 1900 FORTRAN. It finds the best constant interval risk inspection /ocpm schedule for a normalised Weibull distribution ($\eta = 1$, $\gamma = 0$) with costs normalised to $C_I = 1$, $C_F = C_F/C_I$, $C_M = C_M/C_I$. It also calculates the best ppm policy if $\beta > 1$ and compares fm, ppm, ocpm. The inspection effectiveness model is $r = 1/[1-b/\log(1-p)]$.

C1. 2. CONHAZ

CONHAZ is also in 1900 FORTRAN. It finds the best inspect/ocpm schedule for Z(t) constant $r = 1/[1-b/\log(1-p)]$ where $b = \theta_1\theta_2/\theta_2$, $\theta_1 + \theta_2 = \theta$ and θ_1 is the mean time to the availability of warning of impending failure when inspection is made. A plot routine to examine the sensitivity of c to t is incorporated.

C1. 3 MONIT

MONIT is in BASIC-PLUS. It follows the Markov models of Section 16. BASIC-PLUS was employed so that the programme could be used interactively.

C2. LISTING OF FORTRAN PROGRAMME "INSPEF"

```
      TRACE 0
      MASTER INSPEF
C     FINDS BEST CONST INTERVAL HAZARD INSPECT/REPLACE SCHEDULE FOR
C     NORMALISED WEIBULL DISTN (GAMMA=0,ETA=1)-COMPARES TO AGE REPLT
      INTEGER     CF,CM,CI,POLICY(3)
      COMMON T(250),Y(250),W(40),N,C,    CF,CM,B,PBEST,NBEST,CBEST,
     1 CSTAR,TSTAR,CC,ITER,ABEST,RWT,     TFAIL,RBEST,R,TN,BB
      DATA POLICY(1)/'OCPM PPM  FM'/
      CI=1
      LC=0
      D=0.0
      JRUN=0
      JRUN=JRUN+1
   80 BB=0.0
      DO 68 J=1,5
      BB=BB+0.05
      IF(BB.GT.0.05) BB=BB+0.05
      B=0.0
      DO 69 JJ=1,5
      B=B+0.5
      IF(B.GT.1.0) B=B+0.5
      IF(B.GT.3.0) B=B+1.0
      IF(LC.GT.25) WRITE(2,75)
      IF(LC.GT.25) LC=0
      WRITE(2,74) BB,B,D
      WRITE(2,72)
      LC=LC+2
      CF=1
      DO 70 IA=1,10
      CF=2*CF
```

```
      IF(JRUN .GT.0) CF=CF+2
      DO 71 JA=1,10
      CM=(2**JA)/2
      IF(JRUN .GT.0)  CM=CM+2
      IF(CM.GE.CF) GO TO 71
       CFAIL=0.
       CSTAR=0.
       TSTAR=0.
       CBEST=0.
       PBEST=0.
       NBEST=0
      C=0.
            BPLUS=1.0+1.0/B
            IFAIL=1
      TFAIL=S14AAF(BPLUS,IFAIL)
      CFAIL=CF/TFAIL
            CALL PEOPT1
      IF(B.GT.1.0) CALL AGEREP
      IF(B.LE.1.0) TSTAR=TFAIL
      IF(B.LE.1.0) TSTAR=99999E29*TFAIL
      IF(B.LE.1.0) CSTAR=CFAIL
      JK=3
      IF(CSTAR.LT.CFAIL) JK=2
      IF((CBEST.LT.CSTAR).AND.(CSTAR.LE.CFAIL)) JK=1
      TN=TN/TFAIL
      CSTAR=CSTAR/CFAIL
      CBEST=CBEST/CFAIL
      TSTAR=TSTAR/TFAIL
      WRITE(2,73)CF,CM,PBEST,RBEST,NBEST,TN,CBEST,CSTAR,TSTAR,POLICY(JK)
      LC=LC+1
      IF(LC.GE.50) WRITE(2,75)
      IF(LC.GE.50) WRITE(2,74) BB,B,D
      IF(LC.GE.50) WRITE(2,72)
      IF(LC.GE.50) LC=0
   71 CONTINUE
      IF(JRUN.GT.0) CF=CF-2
   70 CONTINUE
   69 CONTINUE
   68 CONTINUE
      STOP
   72 FORMAT(1H ,                    '    CF    CM       P*          R       N
     3TN/MTBF C*OCPM/CFM C*PPM/CFM  T*PPM/MTBF     BEST')
   73 FORMAT(1H ,2I5,2X,2F9.5,I5,2X,E12.5,2F10.5,2X,E12.5,2X,A4)
   74 FORMAT(1H ,/////////,6X,'B=',F6.3,6X,'BETA=',F6.3,6X,'D=',F6.3)
   75 FORMAT(1H1)
      END

LENGTH   412, NAME  INSPEF


      TRACE 0
      SUBROUTINE PEOPT1
C     SEEKS BEST CONST.HAZ POLICY OUTPUTS INSP/REPL SCHED,HAZ,COST/I
      INTEGER       CF,CM,CI
      COMMON T(250),Y(250),W(40),N,C,      CF,CM,B,PBEST,NBEST,CBEST,
     1 CSTAR,TSTAR,CC,ITER,ABEST,RWT,          TFAIL,RBEST,R,TN,BB
      ITER=0
       N=1
       C=0.
       NBEST=0
```

```
      PA=0.0001
      PB=0.9999
      PC=(PB-PA)*0.381966+PA
      PD=(PB-PA)*0.618034+PA
      CALL HAZC(PC)
      CA=C
      CALL HAZC(PD)
      CB=C
   51 IF(ABS(CA-CB).LT.0.001*CA.AND.PD-PC.LT.0.001*PC) GO TO 50
      ITER=ITER+1
      IF(CA.GT.CB) GO TO 48
      PB=PD
      CB=CA
      PD=PC
      PC=(PB-PA)*0.381966+PA
      IF(PC.GT..9975) GO TO 50
      CALL HAZC(PC)
      CA=C
      GO TO 51
   48 PA=PC
      CA=CB
      PC=PD
      PD=(PB-PA)*0.618034+PA
      IF(PD.LT..0025) GO TO 50
      CALL HAZC(PD)
      CB=C
      GO TO 51
   50 PBEST=(PC+PD)/2.0
      CALL HAZC(PBEST)
      PBEST=P
      CBEST=C
      TN=T(N)
      IF(B.LE.1.0) TN=1000000.0
      IF(B.LE.1.0) N=10000
      NBEST=N-1
      RETURN
      END

LENGTH   180, NAME   PEOPT1



      TRACE 0
      SUBROUTINE HAZC(P)
C     FINDS COSTRATE ,R AND N GIVEN P
      INTEGER      CF,CM,CI
      COMMON T(250),Y(250),W(40),N,C,      CF,CM,B,PBEST,NBEST,CBEST,
     1 CSTAR,TSTAR,CC,ITER,ABEST,RWT,        TFAIL,RBEST,R,TN,BB
       QA=0.
       TA=0.
       CD=0.
       CC=0.
       NA=0
      PZ=1.0-P
      TB=ALOG(1.-P)
      TB=-TB
      R=1.0/(1.0+BB/TB)
      BA=1./B
      T(1)=0.
      IF(B.LT.1.01.AND.B.GT.0.99) GO TO 22
      IF(B.LE.1.0) GO TO 23
```

```
      N=(CF-CM)/B
 9    N=N+2
    N=N+N/10
    IF(N.GT.250) N=250
    DO 8 I=2,N
      T(I)=(I-1)*TB
      T(I)=T(I)**BA
 8    CONTINUE
       S=T(2)/(1.0+CM)
      IF(T(N)-T(N-1).GT.S.AND.N.LT.250) GO TO 9
      DO 10 I=2,N
        IF(T(I)-T(I-1).LT.S.AND.NA.EQ.0) NA=I-1
10    CONTINUE
    IF(NA.EQ.0) NA=250
      N=NA
    TEMP=PZ**(N-1)
    TBAR=T(N)*TEMP
    AIBAR=TEMP*(N-1)
      DO 12 I=2,N
       TBAR=TBAR+T(I)*P*(PZ**(I-1))
       AIBAR=AIBAR+(I-1)*P*(PZ**(I-2))
12    CONTINUE
    GO TO 25
23  AIBAR=1.0/P
    IF(AIBAR.GT.100) GO TO 28
    TBAR=0.0
    I=0
26  I=I+1
    TERM=((I*TB)**BA)*(PZ**I)*P
    TBAR=TBAR+TERM
    IF(TERM.GT.0.0001*TBAR.AND.I.LT.1000) GO TO 26
    TBAR=(((I+1)*TB)**BA)*(PZ**(I+1))+TBAR
25  TA=(1.0-R)*TBAR+R*TFAIL
    CC=AIBAR+(1.0-R)*CM+R*CF
    C=CC/TA
    RETURN
28  IF(B.LT.1.01.AND.B.GT.0.99) GO TO 29
    TBAR=TFAIL
    GO TO 25
29  TBAR=TB*PZ/P
    GO TO 25
    END
```

LENGTH   403, NAME   HAZC

```
      TRACE 0
      SUBROUTINE AGEREP
C     CALCULATES OPTIMUM TIME T* AND COST/TIME FOR AGE REPLACEMENT
C     SEE GJ GLASSER JNL QUAL TECH 1,2 APR 1969
      INTEGER       CF,CM,CI
      COMMON T(250),Y(250),W(40),N,C,      CF,CM,B,PBEST,NBEST,CBEST,
     1 CSTAR,TSTAR,CC,ITER,ABEST,RWT,          TFAIL,RBEST,R,TN,BB
        TD=0
        TF=9.21**(1/B)
      TG=(TF-TD)*0.381966+TD
      TH=(TF-TD)*0.618034+TD
      CALL GLASSR(TG)
      RG=RWT
      CALL GLASSR(TH)
```

```
      BH=RWT
   22 IF(TH-TG.LT.0.0001*TG) GO TO 20
         IF(BG.GT.BH) GO TO 21
         TF=TH
         BH=BG
         TH=TG
      TG=(TF-TD)*0.381966+TD
      CALL GLASSR(TG)
      BG=RWT
         GO TO 22
   21    TD=TG
         BG=BH
         TG=TH
      TH=(TF-TD)*0.618034+TD
        CALL GLASSR(TH)
      BH=RWT
         GO TO 22
   20     TSTAR=(TH+TG)/2.0
      CALL GLASSR(TSTAR)
      CSTAR=RWT
      RETURN
      END

LENGTH   104, NAME  AGEREP


      TRACE 0
      SUBROUTINE GLASSR(TD)
C     CALCULATES GLASSER'S RW(T) FOR SUBROUTINE AGEREP
      INTEGER      CF,CM,CI
      COMMON T(250),Y(250),W(40),N,C,     CF,CM,B,PBEST,NBEST,CBEST,
     1 CSTAR,TSTAR,CC,ITER,ABEST,RWT,          TFAIL,RBEST,R,TN,BB
          RWT=CM*EXP(-TD**B)
          RWT=RWT+CF*(1-EXP(-TD**B) )
          ABEST=A(TD,0.,10)
          RWT=RWT/ABEST
      RETURN
      END

LENGTH    64, NAME  GLASSR


      TRACE 0
      FUNCTION A(TD,TE,M)
C     FINDS INTEGRAL EXP(-T**B) FROM TE TO TD BY SIMPSON'S RULE
      INTEGER      CF,CM,CI
      COMMON T(250),Y(250),W(40),N,C,     CF,CM,B,PBEST,NBEST,CBEST,
     1 CSTAR,TSTAR,CC,ITER,ABEST,RWT,          TFAIL,RBEST,R,TN,BB
              A=0
              AA=(TD-TE)/2./M
              AB=TE
      MA=1+2*M
      MB=2*M
      MC=MB-1
              W(1)=EXP(-TE**B)
      DO 15 I=2,MA
                AB=AB+AA
               AC=-AB**B
```

```
                  U(I)=0.
                  IF(AC.GT.-50) W(I)=EXP(AC)
   15           CONTINUE
      DO 16 I=2,MB,2
                  A=A+4.*W(I)
   16           CONTINUE
      DO 17 I=3,MC,2
                  A=A+2.*W(I)
   17           CONTINUE
                A=(A+W(1)+W(2*N+1))*AA/3.
      RETURN
      END

LENGTH   144, NAME   A


      FINISH
```

C3 LISTING OF FORTRAN PROGRAMME "CONHAZ"

MASTER CONHAZ

```
C     FINDS BEST INSPECT INTERVAL FOR Z(T) CONSTANT
C     PLOTS COST RATE V TAU TO SHOW SENSITIVITY
      COMMON CF, CM, A, Y, TAU, R, ITER, CBEST, PBEST, TIME, C, D, AK,
     1B, Z, T2
      COMMON/A/ JBLANK, JDOT, JCROSS, JSTAR
      READ(1. 19) JBLANK, JDOT, JCROSS, JSTAR
    4 READ(1. 11) THETA, CI, CM, CF, D, A, AK, T2
C       A IS FOR R=A*P*P+(1-A)*P**3
C       A MUST LIE BETWEEN 0 & 3
C       AK IS FOR R=(P-AK)/(1-AK)
C       T2 IS THETA2 IN MILLER & BRAFF'S MODEL
C       ESTIMATE ONE & PUT OTHERS NEGATIVE ON DATA CARD
      IF(THETA. LE. O. O) STOP
      IF(CM. GE. CF) GO TO 4
      T2=T2/THETA
      CM=CM/CI
      CF=CF/CI
      CALL PEOPT2
      CALL PLOTCT
      CBEST=CBEST*CI/THETA
      TAU=TAU*THETA
      T2=T2*THETA
      TIME=TIME*THETA
      CF=CF*CI
      CM=CM*CI
      CFAIL=CF/THETA
      COVC=CBEST/CFAIL
      WRITE(2. 5)THETA, T2, AK, A, D, CF, CM, CI, TAU, R, CBEST, TIME,
     1ITER, PBEST
      GO TO 4
    5 FORMAT(1H, CONDITIONS'/' MTBF THETA=', E14. 6, 10X, 'WARNING
     1TIME(THETA2)='E14. 6/' OR K=', E14. 6, 10X, 'OR A=', F10. 6, ' IGNORE
     2NEGATIVES      PROPN REFUNDABLE (D)=', F9. 6/' FAILCOST (CF)
     3=', E14. 6, 10X, 'MAINT COST (CM)=', E14. 6, 10X, 'INSPECTCOST (CI)
     4=', E14. 6/' RESULTS'/' OPTIMUM INSPECT INTERVAL (TAU*)=',
     5E14. 6/' PROPN OF FAILURE CYCLES (R)=', F9. 6, 10X, 'MINIMUM
     6COSTRATE(C*)=', E14. 6/' MEAN CYCLE TIME E(T)=', E14. 6, 5X,
     7'ITERATIONS', I4. 5X, 'INTERVAL RISK', F12. 9)
   11 FORMAT(8G9. 4)
   19 FORMAT(4A1)
      END
```

```
      SUBROUTINE PEOPT2
      CONTROLS FIBONACCI SEARCH, CHANGES VALUE OF P
      COMMON CF,CM,A,Y,TAU,R,ITER,CBEST,PBEST,TIME,C,D,AK,
     1B,Z,T2
      ITER=0
      PF=1.0
      IF(T2.GT.0.0) PF=T2
      PA=PF/CF/100.0
      PA=PA+AK
      PB=0.99999
      PC=(PB-PA)*0.381966+PA
      PD=(PB-PA)*0.618034+PA
      CALL COSTRT(PC)
      CA=C
      CALL COSTRT(PD)
      CB=C
   51 IF(ABS(CA-CB).LE.0.00001*CA) GO TO 50
      IF((PD-PC).LE.0.0001*PC) GO TO 50
      ITER=ITER+1
      IF(ITER.GT.100) GO TO 50
      IF(CA.GT.CB) GO TO 48
      PB=PD
      CB=CA
      PD=PC
      PC=(PB-PA)*0.381966+PA
      CALL COSTRT(PC)
      CA=C
      GO TO 51
   48 PA=PC
      CA=CB
      PC=PD
      PD=(PB-PA)*0.618034+PA
      CALL COSTRT(PD)
      CB=C
      GO TO 51
   50 PBEST=(PC+PD)/2.0
      CALL COSTRT(PBEST)
      CBEST=C
      IF(CBEST.GT.(CF+1.0)) GO TO 52
      RETURN
   52 PBEST=1.0
      R=1.0
      TAU=1.0
      Y=1.0
      TIME=1.0
      CBEST=1.0+CF
      RETURN
      END
```

```
      SUBROUTINE COSTRT(P)
C     FINDS COST RATE GIVEN P & PARAMETER FOR R MODEL
      COMMON CF, CM, A, Y, TAU, R, ITER, CBEST, PBEST, TIME, C, D, AK,
     1B, Z, T2
      TAU=-ALOG(1.0-P)
      IF(T2.LE.0.0) GO TO 60
      R=T2*(1.0-T2)/TAU
      R=1.0/(1.0+R)
      GO TO 62
   60 IF(A.IT.0.0.OR.A.GT.3.0) GO TO 61
      R=A*P*P+(1.0-A)*P*P*P
      GO TO 62
   61 R=0.0
      IF(AK.GT.P) R=(P-AK)/(1.0-AK)
   62 RA=1.0-R
      C=1.0/P+RA*CM+R*CF
      C=C-P*RA*CM*D
      ETI=TAU*(1.0-P)/P
      TIME=R+RA*ETI
      C=C/TIME
      RETURN
      END

      SUBROUTINE PLOTCT
C     PLOTS COSTRATE VERSUS INSPECT INTERVAL FROM TAU/25 TO
     12TAU (50 PTS)
      DIMENSION LINE(101)
      COMMON CF, CM, A, Y, TAU, R, ITER, CBEST, PBEST, TIME, C, D, AK,
     1B, Z, T2
      COMMON/A/ JBLANK, JDOT, JCROSS, JSTAR
      TG=TAU
       RBEST=R
       RTIME=TIME
      TL=TG/25.0
      T=0.0
      DO 71 J=1, 101
         LINE(J)=JDOT
   71 CONTINUE
      WRITE(2.72) LINE
      LINE(1)=JDOT
      DO 73 J=2.101
          LINE(J) = JBLANK
   73 CONTINUE
      DO 74 I=1,50
          IF(I.NE.25) GO TO 78
          DO 77 J=1,101
              LINE(J)=JDOT
   77     CONTINUE
   78 T=T+TL
          P=1.0-EXP(-T)
          CALL COSTRT(P)
          JC=IFIX(100.0*C/CBEST)-99
```

```
   IF(JC.LE.0) JC=1
     IF(JC.GT.101)JC=101
     LINE(JC)=JCROSS
     IF(JC.EQ.101.OR.JC.EQ.1) LINE(JC)=JSTAR
     WRITE(2.75) LINE
     DO 79 J=2,101
        LINE(J)=JBLANK
79   CONTINUE
     LINE(1)=JDOT
74 CONTINUE
   TAU=TG
    R=RBEST
    TIME=BTIME
   WRITE(2.76)
   RETURN
72 FORMAT(1H1.'C1',101A1,'2C* COSTRATE')
75 FORMAT(1H .101A1)
76 FORMAT(1H .'2TAU* INSPECT INTERVAL')
   END

   FINISH
```

C4. LISTING OF BASIC PLUS PROGRAMME "MONIT.BAS"

MONIT.BAS

```
10 DIM R(3,3) , N(3,3)
20 PRINT "COMPARES COST RATES OF CONTINUOUS MONITORING"
30 PRINT "WITH SPOT CHECKS AT CONSTANT MEAN RATE"
40 PRINT "COPYRIGHT D.J. SHERWIN 1978":PRINT:PRINT
50 PRINT "ALL TRANSITION RATES MUST BE  1, ALL MEAN TIMES   1"
60 PRINT:PRINT "CONDITIONS"
70 PRINT "------------"
80 INPUT "UNMONITORED MTBF",L
90 INPUT "MEAN MAINTENANCE TIME",U1
100 INPUT "MEAN WARNING TIME",L3
110 INPUT "MAINTENANCE PLANNING DELAY",L2
120 INPUT "MTTR (FAILURE)",U2
130 INPUT "TOTAL COST RATE OF MONITOR",C5
140 INPUT "COST OF ONE INSPECTION",C4
150 INPUT "ARE COSTS PROP'L TO DOWNTIME",Z$
160 IF Z$="NO" GO TO 190
170 INPUT "COST OF UNIT DOWNTIME",C1
180 GO TO 210
190 INPUT "MEAN COST OF ONE FAILURE",C2
200 INPUT "MEAN COST OF ONE MAINTENANCE",C3
210 L=1/L:L2=1/L2:L3=1/L3:U1=1/U1:U2=1/U2
220 L1=L*L3/(L3-L)
230 INPUT "IS MONITOR PERFECTLY RELIABLE",A$
240 IF A$="YES" THEN U4=0 : L4=0 : GO TO 280
250 INPUT "MONITOR MTBF",L4
260 INPUT "MONITOR MTTR",U4
270 U4=1/U4 : L4=1/L4
280 J%=0
290 IF U1+U2+U4  0.5 OR J%=2 GO TO 360
300 L=L/60 : L1=L1/60 : L2=L2/60 : L3=L3/60 : L4=L4/60
310 U1=U1/60 : U2=U2/60 : U4=U4/60 : C1=C1/60 : C1=C1/60 : C5=C5/60
320 J%=J%+1
330 PRINT "TIME UNITS HAVE BEEN DIVIDED BY 60";
340 IF J%=2 THEN PRINT "TWICE"
350 GO TO 290
360 PRINT:PRINT:PRINT "CONTINUOUS MONITORING"
370 PRINT "-------------------------"
380 IF A$="N" GO TO 410
390 IF L2=0 THEN GO SUB 1200 ELSE GO SUB 1522
400 GO TO 420
410 IF L2=0 THEN GO SUB 1800 ELSE GO SUB 1970
420 PRINT "STATE","PROBABILITY","DEFINITION OF STATE"
430 PRINT   "S0"   ,   P0   ,   "EQT AND MONITOR UP"
440 PRINT   "S1"   ,   P1   ,   "EQT UP MONITOR WARNING"
450 PRINT   "S2"   ,   P2   ,   "EQT UP MONITOR FAILED"
460 PRINT   "S3"   ,   P3   ,   "EQT UNDER MTCE MONITOR OFF"
470 PRINT   "S4"   ,   P4   ,   "EQT FAILED MONITOR OFF"
480 PRINT   "S5"   ,   P5   ,   "EQT AND MONITOR FAILED"
490 PRINT:PRINT:PRINT"AVAILABILITY(P0+P1+P2)",A1
500 PRINT"MEAN DURATION OF UP TIMES=";T1
505 PRINT "STD. DEV'N. OF UP TIMES=";T2
510 PRINT "MEAN DURATION OF DOWN TIMES=";T3
```

MONIT.BAS (continued)

```
520 PRINT:PRINT:PRINT "INSPECTION /ON CONDITION MAINTENANCE"
530 PRINT"--------------------------------------"
540 IF Z$="NO" GO TO 570
550 IF L2=0 THEN GO SUB 1310 ELSE GO SUB 1670
560 GO TO 580
570 GO SUB 800
580 PRINT"STATE","PROBABILITY","DEFINITION OF STATE"
590 PRINT"S6",P6,"OPERATING, NO WARNING TO BE SEEN"
600 PRINT"S7",P7,"WARNING THERE - NOT YET SEEN"
610 PRINT "S8",P8,"WARNING SEEN PLANNING DELAY"
620 PRINT"S9",P9,"FAILED, UNDER REPAIR"
640 PRINT:PRINT:"AVAILABILITY (P6+P7+P8)",A2
650 PRINT"OPTIMUM INSPECT INTERVAL",1/U3
660 PRINT "MEAN DURATION OF UPTIMES",T4
670 PRINT"STD.DEV. OF UPTIMES",T5
680 PRINT"MEAN DURATION OF DOWNTIMES",T6
690 PRINT:PRINT"COST RATE COMPARISON"
700 PRINT"-------------------------"
710 PRINT"COST RATE OF CONTINUOUS MONITORING",C6
715 IF C7=0 GO TO 730
720 PRINT "COST RATE OF BEST INSPECT/OCPM POLICY", C7
730 IF Z$="NO" THEN C8=C2/(1/L+1/U2) ELSE C8=C1*L/(L+U2)
740 PRINT"COST RATE OF FAILURE MAINTENANCE",C8
745 PRINT:PRINT
750 PRINT "IF INPUTS WERE HOURS THEN OUTPUTS ARE   ";
755 IF J%=0 THEN PRINT"HOURS"
760 IF J%=1 THEN PRINT"MINUTES"
770 IF J%=2 THEN PRINT"SECONDS"
775 IF C7=0 THEN STOP
780 INPUT "IS GRAPH OF COSTRATE V INSPECT INTERVAL WANTED"Q$
785 IF Q$="YES" GO TO 2390
790 STOP
800 !SUBROUTINE FOR INSPECT/OCPM FIXED COSTS
810 !------------------------------------------
820 U3=L
830 IF L2=0 THEN GO SUB 2250 ELSE GO SUB 1010
840 C=D : 1%=0
850 FOR U3=2*L STEP L WHILE D  =C
860 IF D  C THEN C=D
870 1%=1%+1
880 IF L2=0 THEN GO SUB 2250 ELSE GO SUB 1010
890 NEXT U3
900 U9=U3
910 1%=1%-3
920 IF 1%  1 THEN 1%=1
930 FOR U3=1%*L TO (1%+3)*L STEP L/100
940 IF L2=0 THEN GO SUB 2250 ELSE GO SUB 1010
950 IF C  D THEN U9=U3
960 NEXT U3
970 U3=U9
980 IF L2=0 THEN GO SUB 2250 ELSE GO SUB 1010
990 C7=D
1000 RETURN
1010 !SUB-SUBROUTINE TO FIND U3*
1020 !----------------------------
```

MONIT.BAS (continued)

```
1030 L9=(L2+L3)/U3
1040 P8=(U3+L3)*L9/L1+L9+1+L2/U1+(1+L9)*L3/U2
1050 P8=1/P8
1060 P6=P8*(U3+L3)*L9/L1
1070 P7=P8*L9
1080 P9=P8*L2/U1
1090 P=P8*L3*(1+L9)/U2
1100 A2=1-P-P9
1110 T4=1/L1+1/(U3+L3)+U3/(L2+L3)(U3+L3)
1120 W=U3+L3 : Y=L2+L3
1130 T5=SQR(1/L1/L1+1/W/W+U3/W/Y/Y
1140 T6=L2+U3/(U3+L3)/L2+L2)/U1+L3(1+1/(L2+L3))/U2/(U3+L3)
1150 IF Z$="YES" THEN D=(1-A2)*C1+U3*C4 : RETURN
1160 Q=T6*U1*U2-U1
1170 Q=Q/(U2-U1)
1180 D=A2*((Q*C3+(1-Q)*C2/T4+C4*U3)
1190 RETURN
1200 !SUBROUTINE FOR CCM PERFECT MONITOR, NO DELAY
1210 !-----------------------------------------------------
1220 A1=U1/(U1+L1)
1230 P0=A1
1240 P3=1-A1
1250 P1=0 : P2=0 : ⅓4=0 : P5=0
1260 T1=1/L1
1270 T2=T1
1280 T3=1/U1
1290 IF Z$="YES" THEN C6=P3*C1+C5+C9/T1 ELSE C6=L1*C3+C5
1300 RETURN
1310 !SUBROUTINE FOR OCPM PROPNL COSTS NO DELAY
1320 !-----------------------------------------------
1330 A=L1/U1
1340 B=L1*L3/U2
1350 K=1+A
1360 J=L1+L3+B
1370 E=C4*K*K/C1
1380 F=2*E*J/K
1390 G=C4*J*J/C1+A*J-B*K
1400 U3=(-F+SQR(F*F-4*E*G))/2/E
1410 P7=1/((U3+L3)/L1+1+U3/U1+L3/U2)
1420 P6=P7*(U3+L3)/L1
1430 P8=0
1440 P9=P7*U3/U1
1450 P=P7*L3/U2
1460 A2=P6+P7
1470 T4=1/L1+1/(U3+L3)
1480 T5=SQR(1/L1/L1+1/(U3+L3)/U3+L3)
1490 T6=(U1*L3+U2+U3)/(U3+L3)/U1/U2
1500 C7=(1-A2)*C1+U3*C4
1510 RETURN
1520 !SUBROUTINE FOR CCM REL. MONITOR WITH DELAY
1530 !-----------------------------------------------
1540 P2=0:P5=0
1550 P1=1/((L2+L3)/L1+1+L2/U1+L3/U2)
```

MONIT.BAS (continued)

```
1560 P0=P1*(L2+L3)/L1
1570 P3=P1*L2/U1
1580 P4=P1*L3/U2
1590 T1=1/L1+1/(L2+L3)
1595 T2=SQR(1/L1/L1+1/(L2+L3)/(L2+L3))
1600 T3=(L2/U1+L3/U2)/(L2+L3)
1610 A1=1-P3-P4
1620 IF Z$="YES" THEN C6=(1-A1*C1+C9/T1 : RETURN
1630 C6=(T3*U1*U2-U1)/(U2-U1)
1640 C6=C6*C3+(1-C6)*C2+T1*C5
1650 C6=C6/(T1+T3)
1660 RETURN
1670 !SUBROUTINE FOR OCPM PROPNL COSTS WITH DELAY
1680 !------------------------------------------------
1690 A=L2/U1+L3/U2
1700 B=L3*(L2+L3)/U2
1710 K=(L2+L3)/L1+A+1
1720 J=L3*(L2+L3)/L1+(L2+L3)*(1+L3/U2)
1730 E=C4*K*K/C1
1740 F=2*E*J/K
1750 G=C4*J*J/C1+A*J-B*K
1760 U3=(-F+SQR(F*F-4*E*G))/2/E
1770 GO SUB 1010
1780 C7=D
1790 RETURN
1800 !SUBROUTINE FOR CCM FAILLIBLE MONITOR NO DELAY
1810 !------------------------------------------------
1820 Z1=L4*U1/(L1*(U4+L-U2*L/(U2+U4)))
1830 Z5=L1+L4 : Z6=U4+L : Z7=U4*L4
1840 Z2=(Z5-U4*Z1-U3)/U2 : Z4=Z5+Z6
1850 Z3=U1/L1+Z1+1+Z2_L*Z1/(U2+U4)
1860 A1=Z3*(U1/L1+Z1)
1870 T1=Z4/(Z5+Z6-Z7)
1880 T2=SQR((Z5*Z6+Z6*Z6+2+Z7)/((Z5*Z5+Z7)*(Z6*Z6+Z7)-Z4*Z4*Z7))
1890 T3=T1(1-A1)/A1
1900 P1=0 : P0=Z3*U1/L1
1910 P2=Z1*Z3 : P3=1/Z3
1920 P4=Z2*Z3 : P5=Z3*L*Z1/(U2+U4)
1930 IF Z$="YES" THEN C6=(1-A1)*C1+C5+C9/T1 : RETURN
1940 Q=(T3*U1*U2-U1)/(U2-U1)
1950 C6=(Q*C3+(1-Q)*C2+T1*C5)/(T1+T3)
1960 RETURN
1970 !SUBROUTINE FOR CCM FALLIBLE MONITOR WITH DELAY
1980 !------------------------------------------------
1990 Q=L*L4*U1*(L1+L2+L3+L4)
2000 Q=Q/(L1*L2*((U4+L)*U4+U2)-U2*L))
2010 V=U1*(L2+L3+L4)/L1/L2+U1/L2+1+Q
2020 V=V+(U2+U4)*Q/L+(L3*U1+Q*L2)/U2/L2
2030 P0=U1*(L2+L3+L4)/V/L1/L2
2040 P1=U1/V/L2
2050 P2=Q*(U2+U4)/V/L
2060 P3=1/V
2070 P4=L3*U1/V/L2/U2+Q*U4/V/U2
```

MONIT.BAS (continued)

```
2080 P5=Q/V
2090 A1=1-P3-P4-P5
2100 R(1,1)=L1+L4:R(1,2)=-L1:R(1,3)=-L4
2110 R(2,1)=0:R(2,2)=L2+L3+L4:R(2,3)=-L4
2120 R(3,1)=- U 4:R(3,2)=0:R(3,3)=U4+L
2130 MATN=R
2140 MATN=INV
2150 T1=N(1,1)+N(1,2)+N(1,3)
2160 MATR=R*R
2170 MATN=INV
2180 T2=SQR(N(1,1)+N(1,2)+N(1,3))
2240 T3=T1*(1-A1)/A1
2242 GO SUB 1620
2245 RETURN
2250 !SUB-SUBROUTINE FOR U3* FIXED COSTS NO DELAY
2260 !-----------------------------------------------
2270 L9=U3+L32280
2280 P7=1/(L9/L1+1+U3/U1+L3/U2)
2290 P8=0 : P9=P7*U3/U1
2300 P6=P7*L9/L1 : P=P7*L3/U2
2310 A2=P6+P7
2320 T4=1/L1+1/L9
2330 T5=SQR(1/L1/L1+1/L9/L9)
2340 T6=(U1*L3+U2*U3)/(U3+L3)/U1/U2
2350 Q=(T6*U1*U2-U1)/(U2-U1)
2360 D=(Q*C3+(1-Q)*C2+C4*T4*U3)/(T4+T5)
2370 RETURN
2390 !SUBROUTINE PLOTS COSTRATE V INSPECT INTERVAL
2395 !-----------------------------------------------
2400 PRINT:PRINT "SENSITIVITY OF C TO U3"
2410 PRINT"------------------------"
2420 PRINT:PRINT"IC*-------------------------------------------------
2430 U9=U3
2440 FOR U3=U9/25 TO 2*U9 STEP U9/25
2450 IF Z$="NO" AND L2=0 THEN GO SUB 2252
2455 IF Z$="YES" AND L2=0 THEN GO SUB 1410
2460 IF L2  0 THEN GO SUB 1010
2470 D=INT((D-C7)*50/C7)+1
2480 IF D  50 THEN D=50
2490 PRINT "I";TAB(D);"X" UNLESS U3==U9
2495 IF U3==U9 THEN PRINT "IX--U3* MINIMUM"
2510 NEXT U3
2520 PRINT "2XU3*---INSPECTION FREQUENCY"
2540 END
```

APPENDIX D

SYMBOLS ABBREVIATIONS AND SPECIAL TERMS

D.1. INTRODUCTION

Symbols, abbreviations and terms used in a special sense in this thesis are explained in context on the first occasion of use. For the reader's convenience and reference, they are defined also in this appendix. The usage generally represents a consensus of standard texts on Reliability and is sometimes at variance with Operational Research conventions. The aim has been consistency and the avoidance of confusion. The same symbol has been used for more than one purpose where this is the convention and no confusion is likely.

D.2. SYMBOLS

D.2.1 ROMAN LETTERS

a	constant, location parameter of extreme value distribution
A,A(t)	Availability, steady state or Average, to time t
b	constant, shape parameter of extreme value distribution
B	Function defined in context at para.14.4.7.
c	costrate (cost per unit time), shape parameter of Gamma distribution
C	Cost (fixed), see suffices for details)
d	Proportion of C_M repayable in respect of initially defective items
e	base of natural logarithms ,2.7183

E, $E\{x\}$	constant, Expected value of x (mean)
f(t)	probability density function of failure or repair time distribution
F(t)	Integral of f(t) to time t, cumulative distribution function
g(x)	a function of x and constants only
$G_a(x)$	Inverse distribution function of x of probability a
H(t)	cumulative hazard function $\int_0^t \{f(u)/(1-F(u))\}du$
i	generalised index number
I	Identity (unit diagonal)matrix,No of inspections in a cycle.
J	second generalised index number
k	constant defined in context,shape parameter of hyper exponential distribution
K	constant defined in context
L, $\mathcal{L}$	Likelihood, Log likelihood
m	median of a distribution,especially log normal parameter
M(t)	Maintainability - <u>cdf</u> of the distribution of <u>ttr's</u>
n	number of items or failures etc. of a particular kind in a data-set
N,N(t)	number of items or failures etc. in a data-set, renewal function(expected no. of renewals in t.)
p	conditional risk of failure in an interval, element of P.
P(a)	Probability of event a

q	Element of Q, constant defined in context
Q,Q',Q"	Matrix of state transition rates in Markov Maintenance model, Q with downstates amalgamated and made absorbing, I-Q'.
r	proportion of failure cycles in a maintenance model, discount rate in a DCF problem, failures in interval.
R,R(t)	Reliability, to time t, 1-F(t), survival function
s	Transformed variable in the Laplace transform, estimate of standard deviation.
S	No. of survivors
t	time variable
T	a fixed time or mean cycle time
u	parameter of Beta distribution, dummy variable of integration
v	parameter of Beta distribution, dummy variable, constant.
w	constant
W	
x	a variable, used as second time variable when required to avoid confusion
y	a variable, conditional mean time to failure in a specified interval
z(t) = f(t)/R(t)	Hazard Rate function (instantaneous failure rate)
Z	Hazard = Failures/Starters in an interval.

D.2.2. GREEK LETTERS

α	probability in a confidence limit, alternative Weibull parameter equal to $1/_{\eta}{}^{\beta}$, modified availability
β	shape parameter of Weibull distribution $F(t) = 1 - \exp\left[-\left\{(t-\gamma)/\eta\right\}^{\beta}\right]$
γ	location parameter of Weibull distribution
δ (1)	small increment of
δ (2)	Alternative Weibull parameter equal to η^{β}
μ	repair rate especially when constant, maintenance rate, inspection rate
λ	failure rate, especially when constant
σ^2, σ	Variance, Standard deviation of distribution
Θ	Mean time between failures, mean of a distribution
$\emptyset$	Mean Repair time or mean down time mttr
∇	('nabla') Pharmocopoeal measure of sterility, the temperature - time integral in an autoclave cycle above 80°C.
η	Weibull scale parameter
υ	λ/μ
ρ	μ/λ

D.2.3. SUFFICES

A	pertaining to acquisition or purchase. e.g. C_A purchase price.
a,b,	pertaining to item, a,b,
c	pertaining to the monitor in a ccm maintenance model
d	pertaining to downtime
f,F.	pertaining to failures
I.	pertaining to inspections
J	pertaining to scrap(Junk) e.g. C_J
i,j,k,	generalised member of a series. 2 or 3, suffices may be used together to indicate generalised location of an element of a matrix
m	last of a series where a second symbol is required, n, is preferred
M	pertaining to maintenance (ocpm) action
n,N	last number of a series
R	pertaining to renewal e.g.$C_R = C_A - C_J$ for an unfailed item.
r	pertaining to repairs
S	pertaining to sale in a serviceable condition
u	pertaining to up time
w	pertaining to waiting time
α	of probability α
ν	with ν degrees of freedom

ccm, ocpm, see D2.4 Special Terms

D.2.4. SPECIAL TERMS

Availability	- unless otherwise stated = mtbf/(mtbf + mttr) q.v.
Base	- referring to conditions where only failures are repaired (fm) e.g. the base or underlying failure distribution as opposed to the observed distribution under pm
bao	bad-as-old, minimal repair maintenance policy
ccm	continuous condition monitoring, policy involving ccm, as suffix pertaining to ccm
DCF, DP	Discounted Cash Flow, Dynamic Programming
fm	- failure only maintenance - no preventive maintenance
gan	good-as-new after repair or maintenance
observed	see also base. the value of a statistic etc as actually recorded, including the effect of maintenance where applicable
ocpm	- on condition preventive maintenance, policy involving inspections at scheduled intervals or constant average rate and maintenance only if impending failure is detected at such inspections
O.R.	Operational Research
mtbf	mean time between failures, ratio of total running time to total failures in a data-set
mttf	mean time to failure - analagous to mtbf but referring to items which are not repaired
mttr	mean time to repair-ratio of total repair time to total repairs in a data-set
mttm	mean time to maintain analagous to mttr but pertaining to preventive maintenance actions and failure repairs considered as a single set of data
maxlik	maximum likelihood
minimax	giving the minimum value of maximum loss.

pm	preventive maintenance - covers ocpm and ppm
ppm	periodic preventive maintenance, ppm policy maintenance actions are performed at fixed intervals regardless of condition of item
PV	Present Value
R.&.M.	Reliability and Maintainability
R.X.R.	Repair or/ refit by replacement. A policy of renewing an item entirely to save downtime. The defective or time expired item may be repaired at leisure and fitted at a subsequent R.X.R operation. Applies to both fm (repair) and pm (refit)
tbr	time between failures (for repairable item) see mtbf
ttf	time to failure (for items renewed on failure).see mttf
ttr	time to repair, see mttr
ttm	time to maintain, see mttm
LP	Linear Programming.

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