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An Activity Theory Analysis of Teaching Goals versus Student Epistemological Positions

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A teaching innovation for first year engineering students' was designed to involve inquiry-based questions, an electronic graphical medium, small group activity and modifications to assessment. The use of an inquiry approach was intended to encourage students' deeper engagement with mathematics and more conceptual understanding. Data were collected from observations of teaching, ongoing teacher reflections, student surveys, interviews and assessment outcomes. Despite evidence of success in assessments, analyses revealed fundamental differences between students' perceptions of the teaching they experienced and the goals of the teaching team. Activity theory was used to juxtapose contradictory perceptions and highlight issues in the wider sociocultural and institutional settings of the research.

1 THE ESUM PROJECT: APPROACH, METHODOLOGY AND THEORY

Supported by funding from the UK HESTEM programme and the Royal Academy of Engineering, the Project, Engineering Students' Understanding Mathematics (ESUM) set out to develop the teaching of a first year mathematics module for engineering students (one semester of 15 weeks) through an inquiry-based approach (Jaworski and Matthews, 2011). HESTEM is a Government funded Higher Education Programme in the disciplines, Science, Technology, Engineering and Mathematics. (<http://www.hestem.ac.uk>). A team of three experienced teachers and one research officer designed and implemented an innovation in teaching, learning and assessment and studied its progress and outcomes. The team's main goals were to promote more student engagement with, and deeper conceptual learning of mathematics than had been observed in earlier cohorts. The innovation comprised use of inquiry-based questions and tasks and a computer environment (GeoGebra) designed to encourage students' inquiry into mathematical concepts with associated growth of conceptual understanding. GeoGebra is an algebra and graph—drawing package: <http://www.geogebra.org/cms>. Organisation of the cohort ($n = 48$) into small groups of 3 or 4 students and an assessed group project aimed to encourage discussion and collaboration, with related assessment. Traditional assessment comprising a final examination (60%) and 8 computer assisted assessments (CAA tests, 40%) was modified by reducing tests to 4 (20%) to allow 20% for a project report and associated poster. Research was developmental, contributing to the development of practice

as well as studying that development (Jaworski, 2006). Research questions addressed the nature of the innovation and its take-up by students, aspects of students' learning and their perspectives on learning, and teachers' learning about the teaching-learning process (more details below).

The teachers in the team (3) designed the tasks and teaching approach (with the help of two PhD students); one member taught the module, reflecting overtly on the teaching, and two contributed to analysis of data. The research officer observed and recorded lectures (1 hour \times 2 per week) and tutorials (1 hour per week), conducted 2 surveys of student baseline data and perspectives on teaching, conducted individual interviews (2) and focus groups (2 \times four students), and analysed data according to research questions. Project reports, and test and exam scores contributed to findings overall.

Analysis of the two surveys (i) gave baseline information, and (ii) revealed students' first impressions of the module and their participation in it. On-going observation of lectures and tutorials, together with teacher reflections led to a growing awareness (on the research team's part) of the nature and extent of student participation. Students' group project reports contributed written evidence of students' inquiry-based work and associated understandings. Individual and focus group interviews probed students' retrospective perceptions of the module and their evaluation of its contribution to their learning. Students were asked overtly to comment on the elements of the innovation and their responses were triangulated / analysed relative to other data, particularly observation data and the written projects reports.

The institutional setting with established norms and expectations relating to curriculum and assessment, organisation and styles of teaching and associated cultures was hypothesised as a community of practice as articulated by Wenger (1998). The project sought to develop an inquiry community involving teacher and students in which alignment with traditional norms and expectations was challenged through the use of inquiry-based questions and tasks (questions/tasks in which the approach was not immediately obvious – i.e. where the solution is not routine or algorithmic – and which encouraged some level of inquiry or exploration) to promote students' mathematical engagement and thinking supported by collaborative group activity (e.g., Jaworski, 2006). Related assessment emphasised the importance of these aspects of the module by

crediting exploratory processes and ways of thinking in the solutions presented. As well as its aims for students' learning, the ESUM project aimed to advance knowledge of teaching to allow teaching to be linked more overtly to its desired outcomes for students. *Activity theory* was used to explain contradictions emerging from analysis as we discuss further below.

2 ANALYSIS AND FINDINGS

We focus here mainly on analysis of one aspect of the innovation, the use of software, GeoGebra along with inquiry-based questions and tasks. In line with our expressed goals, we asked the following research questions:

1. How do students use GeoGebra, and how do they see it contributing to their learning of mathematics?
2. What do teachers learn from their utilisation of GeoGebra and their observations of students' use and engagement?

GeoGebra had been used particularly in the teaching of functions to offer alternative representations, particularly the juxtapositioning of algebraic and graphical forms. In lectures, both static and dynamic modes were used to illustrate mathematical concepts. Student comments in survey 2 suggested that the dynamic mode was unnecessary as it 'slowed down' the lecture, whereas for the lecturer it allowed students to see how changes in the algebra of a function corresponded with different graphical representations. In tutorials students were asked explicitly to use GeoGebra in working on exploratory tasks. For example one task asked:

"Explore the function $f(x) = ax^2 + bx + c$ using sliders in GeoGebra to vary a , b and c . What can you say about lines which intersect the graph of this function twice?"

The following is typical of responses to such tasks:

"As a group we looked at many different functions using GeoGebra and found that having a visual representation of graphs in front of us gave a better understanding of the functions and how they worked. In this project the ability to be able to see the graphs that were talked about helped us to spot patterns and trends that would have been impossible to spot without the use of GeoGebra." [Group F: project report]

This response seems to fit well with the goals of the teaching team in the use of GeoGebra and an inquiry based approach. However, it must be acknowledged that students were unlikely to express negative opinions in a piece of assessed work. When asked in focus groups about GeoGebra, some students responded rather differently. For example,

"I found GeoGebra almost detrimental because it is akin to getting the question and then looking at the answer in the back of the book. I find I can understand the graph better if I take some values for x and some values for y , plot it, work it out then I understand it...then change the equation. If you just type in some numbers and get a graph then you don't really see where it came from". (Focus Group 1)

Here the student seems to see GeoGebra as drawing the graph for him/her rather than encouraging exploration of how features of the graph can reveal insights to the function. Conversely, he/she opens up the possibility, through 'plotting', that features of the graph (and hence the function) are missed.

"Understanding maths – that was the point of Geogebra wasn't it? Just because I understand maths better doesn't mean I'll do better in the exam. I have done less past paper practice." (Focus Group 2)

This comment from a student emphasises an orientation towards what is required to do well in the end of module examination.

Such quotations showed a student epistemology strategically focused towards achieving the best outcomes in the official assessment and engaging in familiar practices that they perceive as helpful, albeit at the expense of a deeper understanding. While there was evidence that some students understood the purpose of GeoGebra in supporting understanding, nevertheless many saw it as irrelevant or unhelpful in providing what they need to pass the exam. Our analyses showed similar tensions relating to other aspects of the innovation. A curious factor is that exam results were good showing an average at least 10% higher than for previous cohorts. This suggests that despite the critical comments quoted above, students largely achieved well according to their own strategic aims. These findings challenge teachers to find ways of addressing student concerns and ways of thinking while continuing with the innovation. The issue of the examination, and the extent to which its existence leads to contradictions in the activity as a whole, is central to our thinking.

3 ACTIVITY THEORY ANALYSES OF THESE FINDINGS

Activity (in Activity Theory terms) in this research encompasses all of these findings and more. It is the whole with which we work and in which we participate. 'We' are the teachers and researchers in the context of this paper, but in terms of the activity the students are also included as well as other stakeholders, administrators, policy makers and so on. Included also are interlinking and interacting conditions and the issues that are generated through practical interpretation of theoretical goals and their interaction with the cultures involved. Thus the activity is everything, but not just the sum of all the parts. According to Leont'ev (1979), "Activity is the non-additive, molar unit of life ... it is not a

reaction, or aggregate of reactions, but a system with its own structure, its own internal transformations, and its own development” (p. 46). Thus, one reason for employing activity theory is to capture complexity in the wholeness described, as well as to examine specific elements and their contribution to the whole. However, we recognise that different groups within this constituency act in different ways towards the whole: in activity theory terms they have different ‘motives’ for activity or ‘goals’ for their actions (e.g., Leont’ev, 1979). In Engeström’s (1999) terms they have different ‘objects’ within activity. It is here that we recognize the tensions that we have started to discuss above, and here that an activity theory analysis has potential to be of value.

We use activity theory (AT) specifically to address issues that we see *between* the intentions of the approaches to teaching and use of resources (in the innovation) *and* students’ responses, engagement and performance. The institutional context is central to analysis, but hard to factor in. So, one purpose of the use of AT is to try to make sense of the relationship *between* the purposes of the innovation and associated findings *and* the aspects of context in which the innovation is embedded. We see here, therefore, two dimensions to our use of activity theory; we seek to gain insights into and between:

- 1) a) teaching intentions and approaches *and*
 b) students’ engagement, responses and performance;
- 2) a) the purposes of the intervention and associated findings *and*
 b) the context in which the innovation is embedded.

AT is used to help us make sense of relationships between (1) and (2) in the above and between (a) and (b) in each case.

3.1 Using Activity Theory frameworks to make sense of the findings

We express these findings first, using Engeström’s (1999) expanded mediational triangle to explore conflicts and contradictions, and second, using Leont’ev’s three levels of activity: activity ↔ motive, actions ↔ goals, and operations ↔ conditions to aid characterization of activity. In the first of these, due to the obvious differences which have emerged in the ways in which the teaching team and the students perceive the activity as a whole, we hypothesise two activity systems operating side by side - the activity as experienced by the students in contrast with activity as experienced by the teaching team - as shown below. There are apparent areas of overlap between them which we need to explain. This framework emphasizes the differences, tensions or contradictions between the ways in which activity is perceived within the two groups and their differing objects for activity. We start from the triangular representation of Engeström (Figure 1), and use our own tabular form (Table 1) as a more effective way of presenting our data. The central double arrow linking outcomes of activity is of especial interest as we will discuss below.

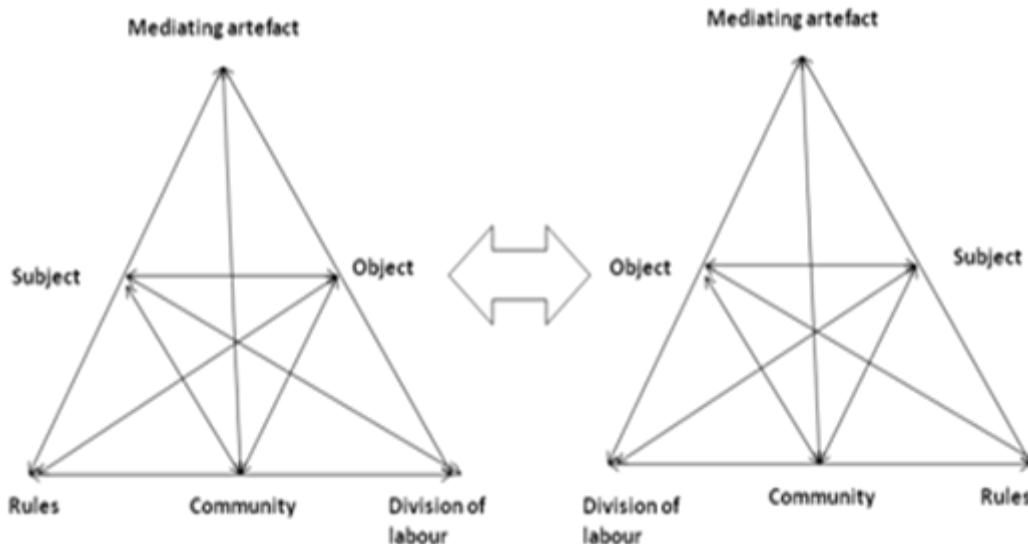


Figure 1 Two versions of Engeström’s *expanded mediational triangle* (EMT) representing teachers’ (on the left) and students’ (on the right) perspectives of the teaching-learning environment as shown in Table 1.

EMT	Teaching Activity	Student Activity
Subject	Teacher or teaching team.	Student or student cohort.
Object	Engaging students conceptually with mathematics so that they learn in a conceptual/relational way rather than an instrumental way. So that they understand the concepts involved in a way that they can use mathematics flexibly in relation to engineering tasks.	To participate in what is offered in the module to some degree and with a range of objectives related to desired outcomes (passing the exam), perceptions of what it means to study and learn (practicing past papers, plotting graphs by hand), and the amount of effort they are prepared to expend.
Mediating artefacts	GeoGebra, inquiry-based questions, small groups, project. Theoretical concepts underpinning the innovation.	The lecturer, GeoGebra, inquiry-based questions, small groups, project, demands of other modules (eg. coursework) which inhibit their devoting time to mathematics, other students, social life.
Rules	Curriculum, assessment, university regulations, university norms/expectations. Nature of discipline and what it means to 'understand' mathematics. Time, particularly in lectures, where concepts sometime have to be rushed.	University programme, curriculum, assessment, university regulations and norms/expectations; expectations of peers, what is needed to be successful (e.g., to pass the exam).
Community	The academic community, the university community, the wider world, and the various cultures that are a part of these communities.	Student, academic, and university communities, the wider world, and the various cultures that are a part of these communities.
Division of labour	There are things that teachers do and that students do, usually different. Teachers have expectations of students' activities and roles.	There are things that teachers do and that students do, usually different. Students have expectations of lecturers' activities and roles.

Table 1 Elements of Engeström's triangle expanded for the two systems

This tabular form emphasises some of the differences (such as the objects of activity of each group) but suggests that certain aspects are in common (such as the academic and university community). Important here is that it is not the objective nature of these communities that is in question but the perceptions of them held within the two groups. Teachers' perceptions of community see relationships within the communities with respect to academic practice, conceptual learning within a discipline, in our case the nature of mathematics, and so on. Students' perceptions of community see relationships in terms of what is required of them, what they are prepared to contribute, and how they discern their position in relation to official authority in contrast with the demands of their own culture. These differences of perception extend to division of labour and how labour within the two groups is perceived very differently, both in terms of own labour and of labour in the

other group. Seen in these terms it is not surprising that outcomes seem quite different in relation to perceptions within the groups, although, in objective terms, measures of achievement have similar value for both groups (i.e. students who get the highest score get the highest grades).

In the second case, we contrast the activity of teaching with the activity of students' learning in Leont'ev's three levels: all *activity* is necessarily *motivated* (level 1) and can be seen in terms of *actions* that are explicitly *goal*-related (level 2). Actions can be seen to be mediated by certain *operations* which are *conditioned* within prevailing circumstances and constraints (level 3). This framework emphasises ways in which the nature of activity is actually different for the two constituencies or cultures involved, that of the teachers and that of the students.

Level	Teaching Team	Students
1	Activity is <i>mathematics teaching-learning</i> . For the teacher(s) it is <i>motivated</i> by the desire for students to gain a deep conceptual-relational understanding of mathematics. We might in this case call it “teaching-for-learning”.	Activity is <i>learning within the teaching environment and with respect to many external factors</i> (youth culture, school-based expectations of university etc.) and is (probably) <i>motivated</i> by the desire to get a degree in the most student-effective way possible with not too much concern with the nature of understanding.
2	Here, <i>actions</i> are design of tasks and inquiry-based questions – with <i>goals</i> of student engagement, exploration and getting beyond a superficial and/or instrumental view of mathematics. <i>Actions</i> include use of GeoGebra with the <i>goal</i> of providing an alternative environment for representation of functions, offering ways of visualizing functions and gaining insights into function properties and relationships. <i>Actions</i> include forming students into small groups and setting group tasks with the <i>goals</i> to provide opportunity for sharing of ideas, learning from each other and articulating mathematical ideas.	For students, <i>actions</i> involve taking part in the module: attending lectures & tutorials; using the LEARN VLE system; using the HELM books; etc. with <i>goals</i> related to student epistemology. HELM – Helping Engineers Learn Mathematics: specially designed workbooks and other materials. http://helm.lboro.ac.uk/ . So <i>goals</i> might include intention to attend lectures & tutorials because this is where you are offered what you need to pass the module; clear views on what ought to be on offer and what you expect from your participation; wanting to know what to do and how to do it; wanting to do the minimum amount of work to succeed; wanting to understand; wanting to pass the year’s work.
3	Here we see <i>operations</i> such as the kinds of interactions used in lectures to get students to engage and respond, the ways in which questions are used, the operation of group work in tutorials and interactions between teachers and students. The conditions include all the factors of the university environment that condition and constrain what is possible – for example, if some tutorials need to be in a computer lab, then they all have to be; lectures in tiered lecture theatres constrain conversations between lecturer and students when tasks are set, limitations on time constrain what can be included.	<i>Operations</i> include degrees of participation – listening in a lecture, talking with other students about mathematics, reading a HELM book to understand some bit of mathematics, using the LEARN page to access lecture notes, Powerpoint etc. The conditions in which this takes place include timetable pressure, fitting in pieces of coursework from different modules around given deadlines, balancing the academic and the social, getting up late and missing a lecture; using social media networks during lectures and tutorials. They also include the organization of lectures and tutorials and participating within modes of activity which do not fit with your own images of what should be on offer.

Table 22 Leont’ev’s levels of activity expanded for the two systems

The above juxtapositioning (Table 2) adds strength to our hypothesis that we have two different activity systems here within (apparently) the same environment with common elements. However, in most cases the common elements are perceived/experienced differently. Perhaps the most important difference is the *object* of activity (Engeström) or *the motivating force* (Leont’ev) for the two systems. Both are valid, but the fact that they are different means that along with other factors – values placed on forms of understanding (the *rules* of the enterprise) or whether GeoGebra is positively helpful in promoting learning (mediating artefacts) – they result in the tensions observed.

4 DISCUSSION

The tensions/contradictions we discern here lie within and between both hypothesised activity systems. Within the students’ system, we see students understanding and valuing the use of GeoGebra in facilitating better mathematical understanding; however, GeoGebra does not seem to aid the determining factor in success, that of passing the exam. The project requires students to develop actions that come in conflict with those they consider necessary to pass the exam.

Hence for students the motivation of the action mediated by Geogebra with respect to the understanding of the knowledge involved in the activity is quite clear, but this is not sufficient to consider Geogebra use as positive (Understanding maths better doesn’t mean I will do better in the exam).

Within the teaching system, teachers seek to create opportunities for deep understanding, while needing to attend to university norms and expectations of fitting in with material constraints (e.g., time and place allocation) and systemic norms (e.g., traditional curricula and assessment modes; demands of the engineering department). In doing so they maintain a process of assessment in conflict with the goals of teaching actions.

Between students and the teaching team we see differences in perception of the value and quality (depth) of understanding in the process of learning. For students, rooted in previous practices from their school experience, and part of university student culture, learning involves knowing what to do and how to do it, in which case the teachers have a responsibility to provide clarity in what should be done and opportunity to gain expertise in doing it. For the teachers,

who want students to achieve deep levels of understanding, theories of engagement and inquiry-based learning make demands on student actions of which students fail to see the value.

Very briefly, we see these tensions and differences in perception aligning themselves with theoretical perspectives expressed in our introductory section. Seeing university mathematics teaching in terms of communities of practice positions established ways of being and doing in the university in terms of accepted norms and expectations. A discussion of whether we see one or two communities in terms of Wenger's (1998) constructs of mutual engagement, joint enterprise and shared repertoire would take us beyond the space we have here to discuss it. However, the transformation of the one or two communities of practice to form communities of inquiry requires new (inquiry)ways of being, doing and thinking and a critical approach to established practices (critical alignment, Jaworski, 2006). Appreciation of the inquiry-based practices introduced in the module is dependent on this more critical approach to learning, and is at odds with established practices. In revealing the contradictions in the practices studied, the activity theory analyses provide insights into conflicts pertaining to the transformative nature of the inquiry-based approaches within established settings.

To overcome the exposed conflicts it is necessary to consider the relationship between the motives that underlie the activity as a whole, both from the perspective of the teacher and of the students, and their relations with systemic norms and traditional practices. The examination is clearly a key factor, motivating, as it does strongly, the students' activity and deriving from the necessity of teachers to conform with university norms. It seems clear that new modes of assessment are needed which are commensurate with the teaching goals and with which students more strategic goals and cultural embeddedness can be compatible. This requires a rethinking of teaching actions to accommodate to students' epistemologies in the early days of university life and nurture approaches to learning and understanding mathematics within an engineering context without losing the goals for deep learning outcomes – a demanding agenda!

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BIOGRAPHICAL NOTES

Barbara Jaworski, Carol Robinson, Janette Matthews, and Tony Croft are all members of the Mathematics Education Centre at Loughborough University. In the ESUM Project Jaworski, Robinson and Croft were the teaching team and Matthews the research officer. Jaworski's research is mainly into the development of mathematics teaching using inquiry approaches and collaboration between teachers and educators. Robinson's research focuses on the teaching and learning of mathematics for engineering students. Matthews' specialism is in design technology, particularly relating to design of textiles in three dimensions; Croft is internationally renowned for the development of mathematics support for students across a wide range of disciplines in higher education.