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PROCEDURES FOR ESTIMATING ENVIRONMENTAL IMPACT FROM RAILWAY INDUCED VIBRATION: A REVIEW

Jorge Avillez
Noise and Vibration, URS
Chetwynd Business Park,
Nottingham, NG9 6RZ, UK
Jorge.DeAvillez@Urs.com

Matthew Frost
Department of Civil Eng.,
Loughborough University,
Loughborough,LE11 3TU, UK
M.W.Frost@lboro.ac.uk

Steve Cawser
Noise and Vibration, URS
Chetwynd Business Park,
Nottingham, NG9 6RZ, UK
Steve.Cawser@Urs.com

Chris Skinner
Noise and Vibration, URS
Chetwynd Business Park,
Nottingham, NG9 6RZ, UK
Chris.Skinner@Urs.com

Ashraf El-Hamalawi
Department of Civil Eng.,
Loughborough University,
Loughborough,LE11 3TU, UK
a.el-hamalawi@lboro.ac.uk

Paul Fleming
Department of Civil Eng.,
Loughborough University,
Loughborough,LE11 3TU, UK
p.r.fleming@lboro.ac.uk

Paul Shields
Noise and Vibration, URS
Chetwynd Business Park,
Nottingham, NG9 6RZ, UK
Paul.Shields@Urs.com

ABSTRACT
Railway induced ground-borne vibration is among the most common and widespread sources of perceptible environmental vibration. It can give rise to discomfort and disturbance, adversely impacting on human activity and the operation of sensitive equipment. The rising demand for building new railway lines or upgrading existing lines in order to meet increasing transit flows has furthered the need for adequate vibration assessment tools during the planning and design stages. In recent years many studies in the fields of rail and ground dynamics have encouraged many prediction techniques giving rise to a wide variety of procedures for estimating vibration on buildings. Each method shows potential for application at different levels of complexity and applicability to varying circumstances. From the perspective of railway environmental impact assessment, this paper reviews some relevant prediction techniques, assessing their degree of suitability for practical engineering application by weighting their methodology (i.e. considerations and requirements) against practicality and precision. The review suggests that not all procedures are practicable (e.g. the attainment of representative parameters needed to run the procedures) whilst others predicate on assumptions which revealed to be too relaxed resulting in insufficient accuracy; however, a combination of methods may provide the necessary balance.

1 INTRODUCTION
Railway induced groundborne vibration may give rise to discomfort, disturbance and interference with specific human activities whenever vibration velocity or acceleration values exceed certain threshold levels. Moreover, vibration-sensitive equipment or its operation may also be adversely affected when subjected to vibration. In recent years, there has been a demand for new railway lines or upgrading existing lines to adjust the train traffic in order to meet demographic flows and commercial-industrial needs. Thus, the demand for adequate vibration assessment tools and the corresponding mitigation measures is growing, not only for the safety of train operation and track stability against deterioration but also for the environmental protection of the alongside built-up area.

Specialist consultants and engineers are often requested to estimate the impacts of vibration from railways in an Environmental Impact Assessment (EIA). This comprises three stages: “scoping” (identifying if there may be a problem and where), “environmental impact assessment” (to quantify the problem and suggest mitigation) and “detailed design” (to aid and decide on mitigation methods). The requirements for a vibration prediction model in terms of complexity, speed of use, and accuracy differ accordingly.

In recently years, several models have been proposed to predict rail induced vibration. Some of which aiming to overcome a particular modelling obstacles focusing on specific aspects such as: geological structure (e.g. type of soil), train characteristic (e.g. speed, geometry), track form, supporting structural system (e.g. tunnel, embankment). For EIA this can be seen as an advantage, allowing the choice of the most convenient method according to the stage being undertaken.

From the perspective of EIA, this review attempts to deepen the understanding of rail-induced groundborne vibration and appraise various prediction methods so as to choose the most appropriate method or combination of methods in accordance to the task at hand. This review will first outline the theory with emphasis on the train track interaction (as the generation mechanisms) and ground (as the medium through which vibration propagates); after which a second section will focus on different modelling techniques appraising their merits in the context of the environmental impact assessment.

2 OUTLINE OF THE THEORY

This section attempts to outline aspects that affect the assessment of rail induced groundborne vibration from the EIA perspective. Firstly, it will give a brief description of the phenomenon followed by two subsections which cover the train-track interaction and propagation path respectively.

Groundborne vibration from railways is a power transmission process where the train pass-by is seen as the primarily source of energy. Vibration is generated by the passage of trains due to the surface irregularities of wheels and rails, the rise and fall of the axle over the periodic rail support such as sleepers, and by propagation of the moving deformation pattern in the track and ground. These sources may excite resonances in the vehicle suspension [1]. The resulting vibration is transmitted through the track structure and propagates as waves through the soil medium where its amplitude and frequency are modified due to reflections and refractions at the interfaces of soil strata, each of which support different shear and compression wave speeds. The vibration is then transmitted to buildings via the foundations and may excite resonance in their structural components. At low frequency (around 6 Hz depending on the layout of the building) the building may rock as a rigid body on its foundation stiffness [2]. At frequencies around 16-200 Hz, lightweight structures (e.g. floor, wall and windows) may be excited into bending resonances [3].

2.1 Generation of vibration: train–track interaction

The power transmitted by the source (track-train interaction) is dependent on the impedance of the system. The rail impedance, which contains a range of eigenfrequencies is determined by the complex stiffness of the whole dynamic system below the rail (e.g. [1, 4]); the wheel impedance is greatly dependent on the mass of the wheel and, to a lesser extent, on the stiffness and damping of the primary suspension (resilient wheel elements). Thus, the nature of induced vibration is determined by the track-form (including rails, ballast, sleepers and embankments), train geometry (car length, bogie span and their arrangement distance between adjacent cars), interaction between the wheels/track, supporting structural system (e.g. the viaduct individual span) and the train speed.

It has been shown that there are two principal mechanisms (e.g. [4, 5, 6]) to be considered in the generation of vibration. The first consists of the time history of the quasi-static deformation pattern produced by a series of momentary impact forces provided by the static weight of the train transferred from the wheels onto rails with specific time delays according to train geometry, sleeper spacing, and speed of motion. A second vibration generation mechanism is caused by the induction of dynamic forces as the unsprung mass of the wheel is excited vertically as it moves over the irregular vertical profile of the track. The first of these tends to be dominant at lower frequencies, although the specific frequency range over which it becomes relevant depends on the soil characteristics, train speed and the condition of the track as well as its design. Both the periodic axle loads and dynamic forces are transmitted from rails to ballast bed via pads and sleepers, and then to the underlying ground.

In an attempt to cast some light on the generation of rail induced ground vibration, Dawn and Stanworth [2] empirically investigate the contribution of both mechanisms mentioned above. They present some measurements from vehicles that appeared to show that at the farfield vibration level below 10 Hz depended more on the total axle load than on the unsprung mass, suggesting that the motion stress field under the train, due to the pattern of axles of the train, was responsible. However, the conclusion was based on a small amount of samples (two vehicles types) at a single site. Predicated on the assumption that, for continuously welded rails and perfect wheels, the most important mechanism of excitation is the quasi-static pressure exerted by the wheel axles onto the track, Krylov ([7, 8]) developed a theoretical model to study rail-induced vibration. However, for train speeds below the speed of surface wave propagation, unlike what is commonly empirical observed, all spectra presented in [7] shows discreet maxima (approximately 60dB higher than adjacent frequencies) at the train passing frequencies and at the frequencies determined by the train geometry. The missing spectral information between these dominant frequencies suggest that the system being modelled is misrepresented.

In order to establish the influence of parameters of track and rolling stock, Jones and Block [5] developed a theoretic model which accounted for both generation mechanisms and layered ground thus incorporated the effect of the low frequency cut-off of the propagation in the top soil – this aspect is further discussed in the next section. By simulating a freight train, where a contribution from axle loads would be expected due to its weight, results demonstrated that at the sleeper the dynamic forces due to the irregular vertical profile of the track dominated over the quasi-static for frequencies above around 15 Hz. A few years later, Jones et al. [9] took this matter further and delivered a paper on a theoretical model deemed adequate for investigating the contribution of each of the two components of actual emissions at both the nearfield and farfield. The study revealed that the contribution from each component is a function of train speed ground properties and the distance between the track and observation point. Similarly, Auersch [4] shows that the deterministic static part rapidly diminishes with distance from the rail line suggesting that it can be negligible at farfield. According to Heckl et al. [10], the quasi-static vibration generated is proportional to the load carried by the train but independent of the dynamics of the vehicle and track quality. This vindicates why freight trains are often observed to yield considerable levels of vibration. On the other hand, vibration caused by the dynamic loading is considered to be independent of train load but not train type. Therefore, increased freight loads will lead to a proportional increase in vibration at low frequencies but not necessarily at higher frequencies. In essence, for conventional operating speed, at low frequencies very close to the track, the vibration is dominated by the quasi-static excitation mechanism, but beyond a quarter wavelength from the track [1] the dynamic excitation mechanism prevails throughout the entire frequency range.

In addition to the irregular vertical profile of the wheels and the track, the generation of ground vibration tends to be of noteworthy amplitude due to wheel defects (such as eccentricity, unbalance and flats) and track features (such as rail joints/welds, points and crossings or changing stiffness of the soil/structure along the track). According to Kurtzweil [11], the presence of wheel flats and loose rail joints can increase vibration levels by 10 to 20 dB. Kazamaki and Watanabe [12] reported a difference of 10 dB between new rails and wheels compared to corrugated rail and wheels with flats from normal service wear. For operational aspects such as doubling of axle loads the tunnel vibration levels will increase by 2 to 4 dB [11]; for conventional operating speeds, the consensus is that overall ground vibration increases by about 4 – 9 dB (typically 6 dB) per doubling of speed [3]. At crossover and turnout an increase of 10 to 15 dB can be expected [11].

It is also noteworthy that track parameters for track on soft ground have greater effect on the response levels for frequencies above 10 Hz [13]. According to [13], the sensitive analysis undertaken showed that the embankment stiffness only affects frequencies above 10 Hz, being proportional at low frequencies (10-16 Hz) and inversely proportional at higher frequencies.

2.2 Propagation Path

2.2.1 Elastic Waves

The stress pattern that the train yields on the track system (rail, sleeper and ballast) is transferred onto the ground beneath and around the train producing both body waves, i.e. shear waves (s-wave) and compression waves (p-wave), and surface waves (e.g. Rayleigh waves (r-wave), which can only travel in the vicinity of the surface). Each of these wave types are characterised by their motion pattern, affecting their strength and speed in accordance to the geological composition. Depending on the medium, combined waves are either 'nondispersive' (where all individual waves travel at the same speed, regardless of their frequency) or 'dispersive' (where propagating speed is frequency dependent). The speed of the propagation wave is a function of the soil's Young's modulus Poisson ratio and density. Table 1 depicts the most relevant aspects for each of the three main types of waves that ground traffic produces.

Table 1: Comparable characteristics of the three main wave types (adapted from [14])

p-wave	s-wave	r-wave
Highest propagation velocity	Intermediate propagation velocity	Lowest propagation velocity
Longitudinal oscillation	Transverse oscillation	Vertical oscillation, but rapidly develops horizontal component with distance
Propagation velocity increased below ground water level.	Propagation velocity decreased by ground water.	Propagation velocity unaffected by ground water but generally lower in moist soil.
'dispersive' Propagation velocity is frequency dependent	'dispersive' Propagation velocity is frequency dependent	'non-dispersive' Propagation velocity is independent of frequency in homogeneous material.
Energy proportional propagation is low	Energy proportional propagation is intermediate	Energy proportional propagation is high

Concerning the receptor (i.e. buildings), when under the influence of surface traffic, r-wave is the most relevant wave types that a passing train induces; it is the type that channels the majority of the induced energy. The diagram of Figure 1 depicts the r-wave behaiviour as a function of depth; here one can detect a rapid decay of vibration amplitude (for both orthogonal directions) with depth.

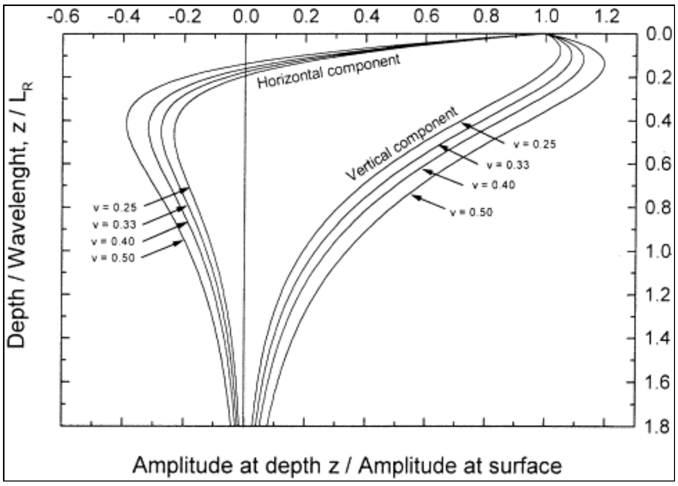

Figure 1. Variation of vibration amplitude with depth of r-wave as a function of Passion ratio (adopted from ref. [15])

2.2.2 Soil Structural Behaviour

The soil through which vibration is transmitted causes the wave amplitude to decrease with distance due to geometrical spreading (also referred to as geometric damping) and also by the loss of energy that the soil offers to the propagating wave, especially if the soil is of granular material due to the friction between grains (referred to as material damping).

Via the analytical approach, Lamb in 1904 (as referred in Hung & Yang 2000 [16] pioneered the classical theory of elastic wave propagation in homogeneous ground. One of the key points that stems out of Lamb's research is the establishment of the amplitude geometric damping rates for each of the wave types (see Table 2) which is widely used by researchers, as a basis for developing empirical prediction models. For instance, at the farfield when considering a homogeneous half-space the geometric spreading can be described by the following equation:

$$A_1 = A_0 \left(\frac{r_0}{r_1} \right)^n \quad (1)$$

Where A_0 and A_1 represents the vibration amplitude at distance from the source r_0 and r_1 respectively, n is Lamb's coefficient.

Table 2: Lamb's predicted geometric attenuation coefficients

	Case a (point source)		Case b (line source)	
	R waves	P&S waves	R waves	P&S waves
At Surface	n= -1/2	n= -2	n= 0	n= -1
Interior		n= -1		n= -1/2

From the table above it can be seen that, in the farfield assumption, the surface response is dominated by the Rayleigh wave; and as shown by Miller and Purvey [17] the Rayleigh waves account for 67.4% of the total energy radiated from the point of excitation.

Material damping, which is related to the material's deformation properties, can also be expected at the interfaces between solids (different types of soil structure) due to air-pumping and friction and also occurs due to radiation of vibration from a finite structure into its surrounding medium [18]. Hence, isolating these effects and measuring their impact is an extremely complex process. Furthermore, as referred in [19] for real soils, there is a variation of material behaviour with depth due to the static stress condition of the soil; the shear modulus increases with the square root of the static stress, and damping decreases with increasing static stress down to a limiting value. Thus, even for homogeneous soil material one would expect, as a function of depth, the shear wave velocity to increase and damping to decrease.

Mintrop (cited by Bornitz 1931 cited in [20]) showed that geometric spreading and material damping attenuation effect can be combined through the expression below.

$$A_1 = A_0 \left(\frac{r_0}{r_1} \right)^n e^{-\alpha (r_1 - r_0)} \quad (2)$$

where α is the attenuation coefficient due to material damping (m^{-1}). For a specific soil α, which is both frequency and soil type dependent [21], is difficult to determine; although there are general guidance for difference soil types (e.g. ref. [22]). However, Attewell and Farmer [23], suggests an α ranging from 0.003 to 0.12 m^{-1} to be used as material damping coefficient.

In reality, the propagating medium is usually stratified, and possesses discontinuities forming layers. In layered ground, some energy is refracted through to adjacent layer(s) and some is reflected. Depending on the density ratio between materials and the angle of incidence at the boundary, the velocity of the reflected and refracted waves can be greater than that of the incident wave. In layered ground additional modes of vibration can propagate along the interfaces of layers, and mode conversion from one type of wave to another may be encouraged. Figure 2 depicts the difference in mode shape in the layer (top) and half-space (bottom similar pattern to Fig. 1).

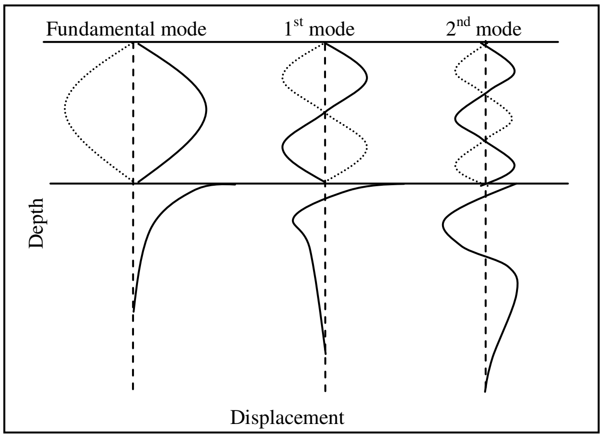

Figure 2. Mode shape sketched for layer (top curves) and half-space (bottom curves)

In general, only the soil material down to half a wavelength of the r-wave has an influence on the response of the surface. As referred in [19], the resonance of the layered soil corresponds with the shear wave speed over half the layer height (see Fig. 2 where half of the fundamental represents highest partial displacement). Auersch [19] presented a study that showed the expected discrepancy, due to resonances induced the upper layer, between homogeneous and layered ground responses. On this study the relationship between the top layer depth and the wave amplitude was observed to be a function of frequency. Furthermore, on layered ground the influence of the underlying half-space on the propagation of high frequencies (i.e. above about 20 Hz) was notable.

When characterising the wave propagation in an inhomogeneous medium (i.e. layered ground) one can expect the r-wave propagation velocity to vary if the sub-soil is composed of soil layers that have different shear velocities [24]; there is a dependency between the frequency of the propagating wave and the depth of the surface layer as demonstrated (e.g. [25]). This is referred to as the cut-off phenomenon, which is a consequence of natural wave impeding effects by shallow layers; shallow surface layers tend to act as a high pass filters. For high speed railways, trains can travel at speeds approaching those of the surface waves. In these situations, modes with wave speeds higher than the r-wave speed but lower than the s-wave speed in the underlying layers are excited on the surface. Thus, the relevance of including the effect of both the railway track structure and the layered structure of the ground has been demonstrated in [6, 25, 26]. For the special case where the train speed is close to the wave speed of the soil, a number of studies (e.g. [8, 25, 27]) suggest that the effect of the moving load may be even more pronounced due to resonance and cut-on frequencies.

2.3 Soil characterization and parameters

There are two common approaches when modelling the soil: half space assumption and layer(s) assumption. In the case of a half space assumption, the wave field is predominantly governed by the r-wave. For the case of a layer(s) assumption, where the ground is assumed homogeneous within individual soil layers, the dispersive nature appears and the wave field is governed by the generalized modal waves that can be characterized by various wave speeds for different frequencies (e.g. [28]). At sufficient depth, the lowest layer can often be represented as a homogeneous half space. As referred in [25] this type of ground modelling has shown adequately to represent the behaviour of the real ground sites over the frequency range of interest. Yet, for relative low amplitudes of vibration many researchers treat the ground as a linear homogeneous elastodynamic material, especially when analysing the relative impact of different rolling stock or track components (e.g. [29]). However, for the prediction of absolute levels the ground needs to be modelled in accordance with the site characteristics.

Especially for layered ground, "dispersion diagrams" (Fig. 3) expressing the propagating wave field are commonly used. The diagram is represented in the frequency–wavenumber domain (which is obtained by taking the Fourier transform from time-space domain) and gives the dependence of propagating wavenumber on frequency facilitating the wavenumber of each mode to be investigated as a function of frequency.

Each line in the diagram (Fig. 3) represents a wave type associated with a cross sectional mode of the layered soil. Here the wave phase velocity (i.e. wave speed at a particular frequency) of each mode at a particular frequency is equal to the inverse slope of an imaginary line drawn from the origin to a point of on the dispersion curve (e.g. the dashed line in the figure represents a specific speed throughout). However, for each mode, the speed at which the energy is channelled (i.e. group velocity) is given by the inverse slope of the dispersion curve representing that mode. f_0, here representing the lower limit of the mode, is referred to as "cut-off frequency" (although some authors refer to it as "cut-on frequency").

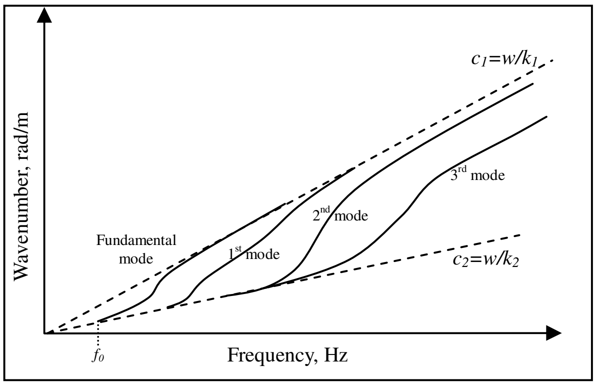

Figure 3. Dispersion diagram dash line represents the shear wave speed (c_1 of upper layer and c_2 of half-space)

Depending on the nature of the investigation there are many seismic techniques that can be used to characterise and extrapolate parameters capable of describing the wave propagation. Within the shallow seismic techniques for the near-surface characterisation of sites, Surface Wave Methods (SWM) is a very powerful technique. It is a non-intrusive method (boring is avoided) where the field data is collected using standard seismic equipment. This technique is capable of obtaining the distribution of soil properties that influence the wave propagation by means of an interpretation procured from wave field observing. The field surveys consist of exciting the ground (e.g. sledgehammer, explosives) and capturing its response using an array of geophones coupled to the ground along a line. Subsequently, collected data undergoes a complex set of analyses in accordance with the chosen procedure, as demonstrated in ref [30]; additionally, judgement and experience are necessary when interpreting plots for an effective analysis. Socco and Strobbia [30] presented a paper giving a general overview of different SWM approaches where many possibilities and limitations were presented and discussed. For an effective ground investigation, acquisition needs to be designed to ensure adequate sampling of the wave field; for example, regions on the dispersion curves plots can be compromised due to aliasing as a consequence of spatial resolution (distance between geophones in the array). For example, in reference [31] for a 1m spatial resolution the dispersion curves above 3 rad/m showed to be difficult to interpret. As demonstrated by Socco and Strobbia [30] in accordance to the array length the analyst is impelled to different modal interpretation. Amongst other identified constrains, the most relevant implications that could be considered as practical limitations for a rail induce vibration environmental assessment is signal-to-noise ratio, considering that environmental impact is mainly undertaken close to residential areas where noise from traffic is to be expected, and explosives as an excitation method would be inappropriate. To illustrate how sensitive this method is to noise Triepaischajonsak et al. [31] reported that the air borne noise from the sledgehammer drops onto an aluminium plate showed up on the transducer readings, even after undertaking active measures such as covering the accelerometers with upturned buckets to reduce acoustic excitation via air.

As a practical ground characterisation method aimed at rail induced vibration, Triepaischajonsak et al. [31] presented a procedure rooted on SWMs in conjunction with theoretical ground model deemed capable of identifying the properties of the material, including its layered structure. The theoretical model that makes part of the process, assumes homogeneous soil layers with their boundaries parallel to the ground surface, is based on expressions of Kausel and Roësset [32]. In this study, site measurements were taken along a line at every 1m over a total length of 42m. Data were analysed to give seismogram (time-spatial domains) plots, as a way of determining the p-waves speeds, and dispersion diagrams. Apart from damping and density, which was assumed to be 2000 kg/m^3, the remaining parameters were derived in terms of the elastic moduli and fundamental wave speeds of the medium. It was found that p-wave speed measured on the top layer can be used on the other layers below without significantly impacting on the results. The narrative suggested that human judgement was of essence in order to select and adjust the information given from SWM that is to be fed into the theoretical model. However, by providing the derived ground parameters to a rail induced vibration models, based on reference [13, 33], good agreement was attained.

FTA [34] proposes a rail induced ground borne vibration prediction methodology where a direct approach based on “Line Source Transfer Mobility” (LSTM) is used to characterise the ground. Based on the assumption that the train can be modelled as an incoherent line source, the ground investigation prescribed in FTA simply assesses the contribution of the intervening ground to the propagation of vibration from a line vibration source (such as a train). This method is efficient in that it holds the capability of describing the line source propagation decay with distance (which as mentioned above is a function of a number of parameters) for a specific site. The method prescribes two different field procedures for obtaining the LSTA: the “Line of Transducers”, which is especially useful for underground testing (avoiding the need for multiple boreholes), and the “Line of Impacts”, which is a more direct approach, requires fewer resources (only 4 to 8 transducers are often needed). “Line of Impacts” consists of measuring the ground transfer mobility (in 1/3 octave-band) at a set of points, evenly spaced (3 to 6 meters) along (or parallel to) the track centre line spanning the train’s length. Transducers that capture the response are combined in an array perpendicular to the line of impacts (ideally 3 to 7, depending on spatial resolution). The point source transfer mobility for each receiver location can then be summed following the trapezoidal rule for numerical integration to directly calculate the line-source transfer mobility.

Both methods described above merit in different ways; the first, based on SWM, is most advantageous when investigating and studying discrete aspects within field of rail induced vibration; whilst the second, based on LSTA, shows adequate for a detailed rail-induce vibration impact assessment.

3 ANALYSIS METHODS AND MODELING APPROACHES

In recent years several models have been proposed to predict vibration propagation into buildings induced by moving trains, each with different degrees of complexity. However, for environmental purpose, ISO 14837-1 [35] suggests breaking the assessment into three stages and recommends that the model used should satisfy each stage accordingly, these stages are: scoping, environmental assessment and detailed design.

Scoping model: to be used at the very early stage of the development of a rail system to identify whether ground borne vibration is an issue for the proposed system and location. This model should predict for the worst case, be simple and quick to use and should rely on generic input parameters, those that will be available at the very early stage of the project’s development. Environmental assessment model: to be used to quantify more accurately the location and severity of groundborne vibration effects for the proposed rail system and the generic form and extent of mitigation required. It will therefore need to consider all the parameters that are critical to determine the absolute levels of groundborne vibration and the benefits of design and mitigation options. The input parameters should be more specific (e.g. vehicle length, axle load, track, speed, geological profile, foundation type etc.). Detailed design model: to be used to support the detailed design and specification of mitigation. This is often used to provide more detailed analysis for one or more components of the system; e.g. source propagation path of receiver.

3.1 Empirical Modelling

Empirical models which rely on extensive and rigorous analysis of collected data provide responses that can be extrapolated and applied on other existing and non-existing installations. Most of the prediction models are composed of several separable independent formulae (empirical laws), each of which serve as a control parameter and can influence, to a certain extent, the final response. The advantage of empirical formulae is that they are usually simple to use.

There are two approaches to consider: using specific measurement results carried out at the relevant site in order to acquire the relevant component constant and adjusting the result to site-specific properties (e.g. soil decay rate for a specific site). The other approach is the use of empirical prediction method derived from statistical consideration of numerous meaningful measurements in a variety of field surveys in order to compile an extensive database allowing statistical analysis to formulate empirical laws from which prediction algorithms can been derived. For such, analysing a set of vibration data in the frequency domain can be very helpful to establish relations and mechanisms that may be involved in vibration excitation caused by trains (e.g. through field measurements at 79 sites Okumura and Kuno [36] set out to establish the influence of parameters such as train type, speed, length, distance to source and background vibration).

Melke and Kraemer [37] used an empirical method called "*diagnostic measurements*" to establish laws that can be used in a prediction model. By observing the train vibration frequency pattern at different train speeds, and analysing the tunnel/soil natural frequency they formulated the expression for the sleeper passing frequencies *fs*:

$$f_s = \frac{c}{l_s} \tag{3}$$

Where c is the train speed in m/s and l_s the space between sleepers in meters. Similarly, reference [38] contains useful remarks and considerations built on the analysis of measurement data.

Models strictly based on empirical laws (e.g. FTA [34] Ch 10: General Vibration Assessment) often do not require detailed knowledge of the site and are not considerer to give accurate predictions. Nevertheless, they are commonly used for scoping and identifying scenarios that require detailed analysis.

An example of such is VIBRA-1 [39] which is a prediction tool for estimating groundborne noise from floor vibration at dwellings adjacent to rail traffic running on both open line and tunnel. The analysis is based on a semi-empirical model that combines the theory of wave propagation (e.g. Eqn. (1)) with data from a number of measurements of ground borne vibration and noise. It uses readily available data (acquired in Switzerland) on train traffic, train type, track sub-soil and on structure of the building. Reference [40] presented the model's validation, which was undertaken in accordance to ISO 14837-1, [35], showing (for open line) very encouraging results for a scoping model; a mean deviation of +3.18 dB and a standard deviation of 6.65 dB. Nevertheless, the data which supported these statistical descriptors ranges around 15 dB. This provides an unacceptable level of uncertainty for the detail design stage.

Another modelling approach commonly used is to estimate the changes caused by different design and operation. Based on this approach, Kurzweil [11] presented a straightforward procedure for estimating the floor vibration and A-weighted noise level in a room of a building in the vicinity of a subway. The method relies on established dynamic properties of the common subway structures and the intervening soil between the tunnel and the dwelling. For the source it relies on measured energy at the wall of a subway tunnel during a pass-by running at 60km/h; reference [11] gives a spectrum (empirically attained) where its upper and lower bound values at each octave band range approximately 10 dB in accordance to the degree of rail and wheels smoothness and substructures (considering both ballasted and direct fixed). For the propagation path the model relies on empirically derived ground vibration attenuation curves (which represents an average soil) given as a function of frequency and distance from the tunnel. The receiver is characterised through both coupling loss at the foundation and the vibration change due to propagation within the building (-3 dB per floor). For heavy masonry either on spread footing or piles the given insertion loss ranges from 10 to 20 dB.

Other similar methods where proposed by other researchers some requiring more parameters. For instance, Melke [41] proposes a similar method based on transmission loss, however it suggests the characterisation of the source by the velocity levels at the rail and then applying coupling losses to the track transition and tunnel transmission and so forth.

There are semi-empirical models which exploit certain wave propagation properties as a way of simplifying the governing mathematical expressions; for instance, by neglecting all wave types except compression waves, Ungar and Bender [42] reduced the elastodynamic complexity to a simple acoustic problem. This allowed them to develop a very simple semi-empirical model for estimating the floor vibration level in a room of a building close to the subway. The model allows for layered ground where the attenuation offered by each layer is given as a function of thickness, loss factor and p-wave speed. It assumed that p-waves travel perpendicular to the layer boundaries and the interface loss is calculated as a function of each layer's density and p-wave speed. For the source, Ungar and Bender provides passby octave-band measurements taken at the tunnel for various subway lines (e.i. NY, Toronto, Paris). For a conservative estimation, the procedure suggests the spectrum resulting from the upper envelope of all the measured data points. The spectrum is then computed to account for the attenuation due to spreading from line source. In order to facilitate the application of the model, Ungar and Bender provide a table which gives the required propagation properties for typical soils type. The input data required are: distance from the surface to the observation point; tunnel radius; thickness and soil class of each layer. This model is limited in that it relies on a short number of measured pass-bys and it does not account for the r-waves that propagate along the surface into the building; thus, for buildings located away from the region above the tunnel the vibration levels are deemed to be under estimated.

For high-speed trains, Rossi and Nicolini [43] proposed a simple-empirical prediction method. Based on the fact that high speed trains run on compressed high-density soil, the soil characteristics where simplified when modelled. This model depends on a few input parameters such as train speed, train mass, rail geometry, soil characteristics and receivers position. It is stated that error is kept below 2.5 dB. However, the output is not frequency dependent, a single value expresses the predicted vibration levels; thus, restricting the usability when assessing human response to both vibration and noise. Nevertheless, it proves adequate for the scoping stage.

For a detailed railway vibration assessment, FTA [34] puts in place a methodology based on the prediction procedure proposed by [44]. The method normalises all the field vibration measurements by removing the soil's contribution from the resulting vibration; thus yielding a quantitative description of the source (as a normalised force density) assumed to be independent of the soil characteristics. This test procedure is based on three quantities: "Line Source Transfer Mobility"

(LSTM), which characterises the transfer of vibration due to a line load; "Force Density Level" (FDL), which represents the power per unit length of an incoherent line source of the dynamic forces induced by the passing train coupled or not (depending on where it was measured) to the track support system; and the train's vibration velocity level (LV), representing the vibration measured during a train passage. The test procedure requires these quantities to be expressed in 1/3 octave-band as the root mean square (RMS) value. Assuming all values are expressed in decibels (logarithm domain), these three quantities relate to each other as such:

$$LV = FDL + LSTM \tag{4}$$

Since FDL cannot be directly measured, FDL is determined by subtracting from the measured LV the computed LSTM (measured at the same site). Finally, by combining FDL with LSTM (measured at the site were predictions are required) it is assumed that the resulting force density can be used to predict the vibration velocity level at other sites with similar train and track characteristics. Predicating on the suspicion that ground characteristics can influence the inferred FDL, the accuracy of the procedure was investigated by means of numerical simulation [45]. It was concluded soil characteristics impact on the FDL. However, if the impacts are performed on the track a good agreement (below 6 dB) can be expected even for extremely different soil types; nevertheless, if the impacts are performed adjacent to the track then the soil will have a significant impact (up to 15 dB for extremely different soil types) on the prediction. Contrary to methods mentioned above, the FTA proposed procedure merits in that it effectively takes into account the ground contribution for each specific site. However, this modelling technique falls short in that it does not provide for an original situation (e.g. new combination of rolling stock and track design).

3.2 Theoretical Modelling

Theoretical models are mainly based on numerical, analytical and semi-analytical methods which rely on complex mathematical formulations and require a significant amount of input parameters if one intends to investigate the entire system solely on numerical solutions. Each method has its own merits and can be used as a prediction tool or just as a mean of investigating a specific components and/or subsystem (e.g. train-track interaction).

Analytical/semi-analytical models are based on algebraic formulations which exploit dynamic law and are typically expressed as a mass spring system. They are seen as a computationally efficient model (in contrast to Finite Element Models) for calculating rail induced vibration. Models such as [13, 46] have been used to study the effect of interaction between the track, the ground and the moving load.

A representative example of these modules is [13] where the prediction of train induced vibration was carried out at three sites. The model requires the knowledge of ground, track, vehicle dynamics and vertical profile of the track. Ledsgard and Burton Joyce sites were modelled according to detailed knowledge of the ground characteristics, track components and vehicle dynamics; Via Tedalda, which lacks specific parameters, the track components (e.g. ballast, sleepers and embankment) were modelled based on typical parameters and the ground parameters were inferred from a figure published elsewhere. Rail vertical profile data was only available for Burton Joyce, as for other two sites typical data was used. At Ledsgard, very good agreement was attained for the displacement along then track. Furthermore, it was also demonstrated that the model has the potential to accurately determine the nonlinear impact that the speed of the moving load has on the track displacement as a function of ground characteristics. Again, this study vindicated that quasi-static response can be neglected at farfield for trains running below the wave speeds in the ground. Conversely, for load speeds exceeding the wave speeds in the ground, since the load speed excites the first mode, the response from quasi-static load dominates. For environmental purposes (i.e. response at farfield) this model showed good agreement in almost all 1/3 octave bands for the Ledsgard site. For the Via Tedalda site, prediction levels are much lower than the measured ones; according to the authors the discrepancies were attributed to the building next to the track (i.e. buildings reflect vibration). For the Burton Joyce site, two sets of measurements 10 m away from the track were available, each measured 20 m apart along the track; it is noteworthy pointing out that measured spectra differ approximately by 10 dB (except in the frequency range 15-40 Hz) which illustrates how sensitive the response is to the precise site location; on the basis that the track conditions were inspected and since it is claimed that its profile was measured it can be deduced that the discrepancy is due to the ground properties. All in all for Burton Joyce, prediction levels best agreed (within approximately 6 dB) at one of the locations.

There are some analytical/semi-analytical models, such as CIVET (Change In Vibration Emitted by Track) for surface rail and PiP (Pipe-in-Pipe) from underground rail, which do not aspire to give absolute vibration levels but simply aim to calculating change in vibration response at an observation point due to changes in the track or vehicle parameters (e.g. prediction of the corrections for the vehicle, track and operating speed).

CIVET [47] is a semi-analytical model based on the same principle as [5]. The track is represented as a 2D, infinite, layered beam resting on a 3D half space. Hysteretic damping is used in the model using a complex stiffness parameter, i.e. a material loss factor. The wheelset (an unsprung mass) acts on the rail via a linearised contact stiffness, while wheel and rail roughness is introduced as a differential displacement function across the contact spring. The vehicle suspension is modelled as a complete one-dimensional system for each wheelset, including primary and secondary elements, bogie and body masses. A half-space foundation model represents the ground as a frequency-dependent support stiffness distribution under the track, and provides a suitable summation of the contributions of vibration from all points along and across the width of the track. CIVET uses only the dynamic forces due to unsprung mass mechanism (not the quasi-static) in its simulation of the excitation and, therefore, is not able fully to simulate all the effects at low frequencies in the near-field. Aspects that would impact on absolute levels such as inhomogeneous ground and not accounting for quasi-static excitation play no relevant part on the model's aim. The lack of quasi-static excitation is justified through the assumption that changes in track design that causes a significant modification of the quasi-static excitation usually do so as a result of some form of load spreading. The accuracy of the differences predicted by the model was validated during the RENVIB project [48].

PiP, first presented by Forrest and Hunt [49, 50], is a semi-analytical model which was developed into software with a user friendly interface by Hussein and Hunt [29] and has been validated against the coupled FE-BE model for the case of a tunnel embedded within a full space [51]. It sets out to evaluate the effectiveness of vibration countermeasures by predicting relative changes in vibration response in accordance to alterations made to specific components of the system, such as slab mass and tunnel width. The tunnel wall and its surrounding infinite soil are modelled as two concentric pipes; where the inner pipe represents the tunnel wall and the outer pipe, with its radius being set to infinity, represents an infinite soil with a cylindrical cavity. Further developments, part of an ongoing process, aim to allow greater modelling flexibility for both computational efficiency and modelling scenarios (e.g. [52]) allowing for tangential forces at the wall making it possible for different arrangements of supports for floating-slab track).

3.3 Numerical Modelling

Numerical Modelling which most frequently take the form of Finite Element Method FEM and Boundary Element Method BEM are capable of a high level of accuracy, limited only by the accuracy of the parameters assumed and computation power. They are mostly recommended when material properties (e.g. arbitrary geometry of structures and ground surface) and geological conditions are too complex for algebraic predictions and comparison with measured data is unavailable. FEM advantageously analyses wave propagation in structures and media with local inhomogeneities and complex material behaviour for analysis of large open domains BEM may be applied. Thus, the entire system, source-path-receive, can be efficiently modelled by combining both methods (i.e. FEM and BEM, referred to as FE-BE), where FE can be applied to the building being modelled and BEM to the layered or half-space ground. For evaluating the effectiveness of mitigation schemes, the FE-BE technique has shown to be very proficient. An example is given by X. Sheng et al. [53] where a wave impeding block (which filters off the low frequency propagating in the same way as a shallow ground layer, does raising the upper bound frequency of the evanescent wave) is simulated.

As with the analytical approach, FE-BE can be used to study the ground vibration generated by the motion of the train axle load on railway track. Auersch [4] presents a hybrid model where each sub-system is modelled accordingly; the vehicle (its multi-body modes) was modelled using the multi-body method, the track was modelled using FEM and the ground was modelled using BEM. Based on specific parameters, Auersch undertook a comprehensive study where specific phenomena (such as the speed at which the sleeper passing frequency meets the vehicle-tack eigenfrequency) could be inspected For instance, the manifestation of Doppler Effect, which the author considers to be due to the sleeper passage excitation when the load moves towards to and away from the observation point, was acknowledged. As suggested by the study, it is this phenomenon that contribute significantly to the observed vibration at farfield within the 80-120 Hz frequencies range. However, when considering the model as an environmental prediction tool, although very good qualitatively, greater discrepancy (within 10 dB over the 4-250 Hz frequency range) was observed even when using specifically measured input data. Again, this proved to be a very efficient tool for studying the phenomena but the accuracy does not outweigh the complexity (both in developing the model and attaining the necessary parameters) to be used as a prediction tool.

A two dimensional (2D) FE-BE model and a three dimensional model have often been proposed for modelling rail induced vibration. 2D models are limited in that they cannot account for wave propagation in the direction of the track nor the passing of the train. On the other hand, 3D requires greater computational resources; it was reported [54] that 3D requires a run time 2000 times longer than for 2D. Furthermore, a comparison between FE-BE 2D and 3D approaches presented by Anderson and Jones [54, 55] revealed that unlike 3D, which has the potential of giving absolute levels when predicting groundborne vibration, 2D models are only capable of giving qualitative results providing a quick tool to assess isolation measures. As a way of overcoming the 3D computational power requirements, researchers like Aubry et al. ([56], referred in [53]) and Papageorgiou and Pei [57], proposed a numerical solution based on the so called 2.5D, or quasi two-dimensional where, as in to the analytical method, the 2D problem is solved for a range of wavenumbers in the third direction. The 3D response is then recovered by using the inverse Fourier transform. This implies that applications concerning moving loads such as trains, the geometry of the structure and subsoil is two-dimensional or periodic and can only be applied to problems with constant geometry along the direction of the track.

Based on the 2.5D coupling FE-BE technique and predicating on the fact that the ground and built up structures can be assumed to be homogeneous in the track direction, Sheg et al. [53] presented a numerical model to predict rail-induced vibration spectra which showed to be proficient as a way of evaluating vibration countermeasures. Computational efficiency was attained by considering the ground and built structures, such as tunnels and tracks, to be homogeneous in the track direction allowing the problem to be modelled using the ‘wavenumber finite’/’boundary element method’ formulated in terms of the wavenumber in that direction. In comparison to conventional, three-dimensional finite/boundary element models, this model revealed to be more computationally efficient since discretisation is only made over the vertical–transverse section of the ground and/or built structures. With this model it is possible to predict complete vibration spectra.

For both underground and surface rail, the main draw back this method presents is that the layers boundaries need to be parallel, along the direction of the track, to the ground surface, and built up areas are restricted to buildings and/or mitigation process (such as trenches) that extent to the infinity in the direction along the track.

4 CONCLUSION

An outline of the theory behind rail induce vibration is presented and the most influential parameters were identified. Representative methods for characterising the behaviour of the ground to the incoming vibration were contrasted; rail induced vibration prediction models have been reviewed and mapped against different stages of the EIA. The review suggests that ground response is highly unpredictable and differs significantly within a short distance; thus, specific ground parameters or response should be collected for both the environmental assessment and the detailed design stages. The review identifies the fact that the near-field effect impacts on the measuring location choice. However, for the scoping stage of the EIA, models based on field observations along with simplified generic governing equations (e.g. VIBRA 1) have proven adequate. Based on the arguments laid throughout it can be envisioned that an ideal modelling technique, for environmental purposes, is to combine the FTA procedure, as a way of estimation the impact from a specific train at a particular site, with a theoretical model (e.g. CIVET) to calculate the change in vibration response in accordance with the proposed design and/or operation (i.e. relative change to the track form and/or vehicle dynamics).

Although numerical models proved to be a very efficient tool for studying the phenomena the accuracy does not seem to outweigh the complexity (both in developing the model and attaining the necessary parameters) to be used solely as a prediction tool. However, due to their geometrical flexibility when representing complex structures, numerical models are justified when predicting the insertion loss offered by a mitigation scheme of a complex nature (due material properties such as the geometry of structures).

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