Proceedings of the Institute of Acoustics

RECENT PROGRESS IN THE THEORY OF RAILWAY-GENERATED GROUND VIBRATIONS

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1. INTRODUCTION

In Europe today the dramatic revival of railways to become one of the most advanced and fast developing branches of transportation technology may be compared with the space technology breakthrough of the 1960s [1,2]. The reason is high speeds achievable by the most advanced modern railway trains, e.g. French TGV-trains for which a maximum speed of more than 515 km/h was recorded in May 1990. By the year 2010 the New European Trunk Line will have connected Paris, London, Brussels, Amsterdam, Cologne and Frankfurt by a high-speed railway service that will provide fast and convenient passenger communications between major European centres.

Unfortunately, increased train speeds are likely to raise levels of associated noise and vibration beyond those significant for conventional railways [3-5]. In particular, ground vibrations generated by rail traffic are one of the major sources of noise and vibration pollution in the built environment. They may cause significant nuisance to residents living in nearby properties, and fear of possible property damage both directly and by generating structure-borne noise.

In contrast to the investigations into railway noise radiated directly into the air (see, e.g., the review [3]), very little has been done so far with regard to the theory of generating ground vibrations and associated structure-borne noise. This may be explained partly by the fact that for most above-ground trains direct noise propagating through the air inside buildings is usually more intensive than structure-borne noise generated by building vibrations excited by the interaction of propagating ground vibrations (ground elastic waves) with building structures. However, this is not so for underground rail traffic or for above-ground high-speed trains where ground vibrations may cause the main effect on buildings.

A number of experimental investigations of generated ground vibration have been carried out for conventional passenger and heavy-freight trains travelling both above- and underground (e.g., [6-9]). It has been demonstrated that different excitation mechanisms exist which contribute to the particular frequency bands of railway-induced ground vibration spectra. At low frequencies (below 50 Hz) the most important mechanism is a quasi-static pressure from wheel axles onto the track. For higher frequencies one should mention also the effects of joints in unwelded rails, unevenness of wheels or rails (these mechanisms, as well as a quasi-static pressure, cause vibrations at trainspeed-dependent frequencies), and the effects of carriage- and wheel-axle bending vibrations (which occur at their natural frequencies).

Apart from the paper [10] where qualitative discussion of a quasi-static pressure excitation mechanism has been given, and the works using a simple impedance approach to estimate the energy of generated ground vibrations (see, e.g., [11]), no rigorous theoretical investigations on railway-induced ground vibrations have been carried out. Recently, following growing interest in the theory of railway-induced ground vibrations, such an analysis for above-ground trains has been undertaken at British Rail Research (Derby) [12], where numerical models were developed, and at the Centre for Research into the Built Environment, Nottingham Trent University [13-19], where investigations have been carried out to describe the corresponding phenomena analytically.

In this paper we give a brief review of recent progress in the theory of railway-generated ground vibrations, based mainly on the results of research carried out during the last two years at the Centre for Research into the Built Environment, Nottingham Trent University. The specific problems considered are ground vibrations generated by heavy-freight trains, by high-speed trains, and by underground trains, with emphasis on problems associated with superfast trains, i.e. trains travelling at speeds close to or greater than 300 km/h. In conclusion, we mention some yet unsolved problems in this field and discuss ways in which the theoretical results may be used for working out methods to protect the built environment against railway-induced ground vibrations.

2. OUTLINE OF THE THEORY

Throughout this paper we consider a train having N carriages and moving at speed v on welded track with sleeper periodicity d. Being interested in low-frequency ground vibrations, we take into account only the quasi-static excitation mechanism which results from load forces applied to the track from each wheel axle, causing downward deflection of the track. Even the highest train speeds achievable at present are normally lower than velocities of the so called "track" elastic waves freely propagating in the system comprising track and ground. Therefore, in the majority of practical situations these deflections can be considered as quasi-static, producing a wave-like motion along the track with speed v and resulting in a distribution of the axle load over all the sleepers involved in the deflection distance [13-15]. Thus, each sleeper acts as a vertical force applied to the ground during the time necessary for a deflection curve to pass through the sleeper. This results in generation of elastic ground vibrations. Since, in the relevant frequency band, the characteristic wave-lengths of generated elastic waves are much larger than the sleeper dimensions, each sleeper can be considered as a point-source vertical force. The problem then requires superposition of the elastic fields radiated by all sleepers caused by the passage of all axles.

An important aspect of the above is calculation of the track deflection curve as a function of the elastic properties of track and soil and of the magnitude of the axle load. Since the track deflection distance is greater than the distance between sleepers, one can ignore the influence of rail periodic support by sleepers in the quasi-static problem of track deflection under the impact of a moving load. Instead we treat a track (i.e. two parallel rails with periodically fastened sleepers) as an Euler - Bernoulli elastic beam of uniform weight p lying on an elastic or viscoelastic foundation occupying the half space $z > 0$.

The classical solution for the deflection magnitude w has the form [20,21]

$$w = (T/8EI\beta^3) \exp(-\beta|x|) [\cos(\beta x) + \sin(\beta|x|)] + p/\alpha, \tag{2.1}$$

where E and I are Young's modulus and the cross-sectional momentum of the beam, α is the proportionality coefficient of the elastic foundation, x is the distance along the beam, T is a vertical point force applied at $x=0$, and

$$\beta = (\alpha/4EI)^{1/4}. \tag{2.2}$$

According to eqn (2.1), one can take $x_0 = \pi/\beta$ as the total deflection distance. The constant α in eqn (2.1) depends particularly on the stiffness of the ground and of the rubber pads inserted between rail and sleepers. Calculation of α for typical British Rail tracks [22,23] gives the values $\alpha = 52.6\ MN/m^2$ and $\beta = 1.28\ m^{-1}$. The solution (2.1) is valid for values of the axle load T less than certain critical value [21]:

$$T_{cr} = (2p/\beta)\exp(\pi). \tag{2.3}$$

For $T > T_{cr}$ the behaviour of a deflection curve becomes more complicated and involves peripheral bulges of the track with loss of contact between track and soil.

For calculation of railway-induced ground vibrations, it is customary to use the Green's function formalism [13-16]. The physical meaning of the Green's function for the problem under consideration is that it describes the ground vibrations generated by individual sleepers being regarded as point sources in the low-frequency band. Let us consider each sleeper as a vertical force applied to the surface $z=0$ at $x = 0$ and $y = 0$, with time dependence determined by the passage of the deflection curve through the sleeper:

$$P(t) = T[2w(vt)/w_{max}](d/x_0) , \tag{2.4}$$

where w_{max} is the maximum value of $w(vt)$. Terms on the right of T take into account the distribution of axle load between sleepers within the deflection curve. To derive (2.4), one should use the force balance equation to determine the effective number of sleepers N_{eff} equalising the applied axle load T:

$$\sum_{m=-N_{eff}/2}^{N_{eff}/2} \frac{T}{N_{eff}} \frac{w(md)}{w_{max}} = T. \qquad (2.5)$$

Here m denotes a number of a current sleeper. Numerical solution of eqn (2.5) shows that for β within the range of interest (from 0.2 m^{-1} to 1.3 m^{-1}) the value of N_{eff} may be approximated by a simple analytical formula $N_{eff} = \pi/2\beta d = x_0/2d$ which gives eqn (2.4) after replacing in $w(x)$ the argument x by vt.

The structure of a Green's function depends on the particular problem considered. Thus, for the case of above-ground trains one can make use of results from the well-known axisymmetric problem of excitation of an elastic half space by a vertical point force applied to the surface [24-26]. The solution of this problem with respect to the vertical component of the ground vibration velocity at the surface $v_z(\rho,\omega)$ gives the relevant component of the "Rayleigh wave branch" of the dynamic Green's tensor (or, for simplicity, of the Green's function) G_{zz} for the elastic half space:

$$v_z(\rho,\omega) = P(\omega)G_{zz}(\rho,\omega) = V(\omega)(1/\sqrt{\rho})exp(ik_R\rho), \qquad (2.6)$$

and

$$V(\omega) = (\pi/2)^{1/2}P(\omega)(-i\omega)q(k_R)^{1/2}k_t^2 \, exp(-i3\pi/4)/\mu F'(k_R). \qquad (2.7)$$

Here $\rho = [(x-x')^2 + (y-y')^2]^{1/2}$ is the distance between the source (with current coordinates x', y') and the point of observation (with coordinates x, y), $\omega = 2\pi f$ is a circular frequency, $k_R = \omega/c_R$ is the wavenumber of a Rayleigh surface wave where c_R is the Rayleigh wave propagation velocity, $k_l = \omega/c_l$ and $k_t = \omega/c_t$ are the wavenumbers of longitudinal and shear bulk elastic waves, where $c_l = [(\lambda + 2\mu)/\rho_0]^{1/2}$ and $c_t = (\mu/\rho_0)^{1/2}$ are longitudinal and shear propagation velocities, and $q = (k_R^2 - k_l^2)^{1/2}$. The factor $F'(k_R)$ is a derivative of the Rayleigh determinant

$$F(k) = (2k^2 - k_t^2)^2 - 4k^2(k^2 - k_t^2)^{1/2}(k^2 - k_l^2)^{1/2} \tag{2.8}$$

taken at $k = k_R$, and $P(\omega) = (1/2\pi)\int_{-\infty}^{\infty} P(t)\, exp(i\omega t)dt$ is a Fourier transform of $P(t)$. The factor $1/\sqrt{\rho}$ in eqn (2.6) describes the cylindrical spreading of Rayleigh waves with propagation distance. Note, that for above-ground trains, it is sufficient to take account of Rayleigh waves only since they make the main contribution to the ground vibration field at the surface. It is seen from (2.4) and (2.6) that the Fourier transform $P(\omega)$ plays an important role in determining the spectra of radiated waves. In the case under consideration, $P(\omega)$ should be determined separately for $T \le T_{cr}$ and for $T > T_{cr}$.

To calculate the vibration field radiated by a complete moving train requires the superposition of fields generated by each sleeper activated by all axles of all carriages, with the time and space differences between sources (sleepers) being taken into account. Using the Green's function formalism, this may be written in the form [13-16]

$$v_z(x,y,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(x',y',\omega)G_{zz}(\rho,\omega)dx'dy' , \tag{2.9}$$

where $P(x',y',\omega)$ describes the total distribution of forces along the track. This distribution is found by taking a Fourier transform of the time- and space-dependent track deflection function $P(t, x',y'=0)$. To account for all axles and carriages one may write this function in the form

$$P(t, x',y'=0)= \sum_{m=-\infty}^{\infty}\sum_{n=0}^{N-1} A_n[P(t - (x'+nL)/v) + P(t - (x' + M +nL)/v)]\, \delta(x'-md)\delta(y'). \tag{2.10}$$

Here N is the number of carriages, M is the distance between bogies in each carriage and L is the total carriage length. Dimensionless quantity A_n is an amplitude weight-factor to account for different carriage masses (for simplicity we suppose all carriage masses to be equal: $A_n=1$).

Taking the Fourier transform of (2.10) and substituting the result into (2.9), one obtains the following expression for the frequency spectra of vertical vibrations at $z=0$ generated by a moving train:

$$v_z\,(x=0, y=y_0, \omega) = V(\omega) \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} [exp(-\gamma\omega\rho_m/c_R)/\sqrt{\rho_m}][1+ exp(iM\omega/v)]\cdot$$

$$exp(i(\omega/v)(md + nL) + i(\omega/c_R)\rho_m) \; . \qquad (2.11)$$

In writing eqn (2.11) we account for attenuation in soil by replacing $1/c_R$ in the exponentials by the complex value $1/c_R + i\gamma/c_R$, where $\gamma << 1$ is a constant describing the "strength" of dissipation of Rayleigh waves in soil [27-29].

3. GROUND VIBRATIONS FROM CONVENTIONAL AND HEAVY-FREIGHT TRAINS

According to the formula (2.11), the spectrum of train-induced vibrations is quasi-discrete, with the maxima at frequencies determined by the condition $(\omega/v)(md + nL)= 2\pi l$, where $l =1,2,3,...$ Obviously, $n=0$ corresponds to the passage frequencies $f_ps = (v/d)s$ determined by the train speed v and the sleeper period d where $s = 1,2,3,...$. Other, more frequent maxima, are determined either by the carriage length L $(m=0)$ or by a combination of both parameters (for $n\neq 0$, $m\neq 0$). There are also many zeros present in the train vibration spectra. The frequencies of these zeros f_z may be used in practice for suppressing vibrations at chosen frequencies [13-15]. If, for instance, we want to suppress one of the train passage frequencies f_ps, we should choose f_z to be equal to f_ps.

Numerical calculations carried out for typical parameters of track and train show that theoretically calculated ground vibration spectra depend strongly on the axle loads of the carriages: if the axle load exceeds a critical value beyond which peripheral bulges appear in the track, the vibration level increases significantly, especially at higher frequencies. By proper selection of the distance between wheel axles in a bogie, and between bogies in a carriage (or between sleepers in a track) it is possible effectively to suppress vibration levels at the train passage frequencies [13-15].

4. GROUND VIBRATIONS FROM HIGH-SPEED PASSENGER TRAINS

In this section we discuss excitation of ground vibrations by high-speed trains, including superfast trains that travel at speeds close to or greater than *300 km/h* [16-18]. From the point of view of generating ground vibrations, it is particularly important to study the effect of trains approaching the "sound barrier" with regard to the velocity of Rayleigh surface waves propagating in the ground. In May 1990, nine runs of TGV trains at over 500 km/h (i.e., over 138.8 m/s) were made by the French Railway Company (SNCF) on the section of the track between Courtalain and Tours. These speeds are greater than the velocities of Rayleigh waves in soft sandy soils (*90 - 130 m/s*). Hence, significant radiation effects for ground vibrations might be expected in these areas, similar to those of Mach radiation of shock waves by supersonic jets or Cherenkov radiation of light by electrons moving with speeds exceeding the velocities of light in the media.

The theory described in section 2 is also applicable to trains moving at very high speeds. One should take into account that for high-speed passenger trains the track deflection function for one axle load $w(x)$ usually has the "classical" form (2.1). For the specific case of "trans-Rayleigh trains", i.e. trains travelling at speeds larger than Rayleigh wave velocity, an additional analytical treatment is useful to elucidate the special features of the problem and to clarify the time and space distributions of radiated waves [17].

It is convenient first to consider the vibration field generated by a single load moving at speed v along a part of a track having a small number of sleepers $2Q+1$. Let the point of observation be arbitrary located on the ground surface, i.e. $\rho_m = [y^2 + (x-md)^2]^{1/2}$. Then, for far-field distances ($R >> Qd$, where $R = [y^2+x^2]^{1/2}$) the expression for ρ_m can be simplified as follows

$$\rho_m \approx R - md\cos\Theta , \tag{4.1}$$

where $\cos\Theta = x/R$. Substitution of (4.1) into (2.11), with just one axle load (for simplicity) and with a limited number of sleepers being taken into account, gives the following expression for the vertical component of ground vibration velocity:

$$v_z(x, y, \omega) = \frac{V(\omega)}{\sqrt{R}} \exp[i(\omega/c_R)R] \sum_{m=-Q}^{Q} \exp[i(\omega/v)md - i(\omega/c_R)md\cos\Theta] , \tag{4.2}$$

where we have neglected the term $md\cos\Theta$ in the denominator.

It is seen from (4.2) that maximum radiation takes place if all the exponentials in the sum are equal to one, i.e. the expressions in the square brackets of all exponentials are zeros. This is possible if the train speed v and the Rayleigh wave velocity c_R satisfy the condition $\cos\Theta = c_R/v$, which is similar to the conditions for Mach or Cherenkov radiation. Since the observation angle Θ should be real ($\cos\Theta \le 1$), this implies that v should be larger than c_R. In this case the ground vibrations are generated as cylindrically attenuated surface waves (factor $\sqrt{R}$ in the denominator) symmetrically propagating at angles Θ with respect to the track, and with amplitudes larger than for "sub-Rayleigh trains" [16,17].

All principal features of the above mentioned analysis remain valid for tracks with an infinite number of sleepers. Dissipation of Rayleigh waves in the ground and their geometrical attenuation (factor $\rho_m^{-1/2}$ in (2.11)) mean that only about 200 sleepers need to be considered. However, since in this case we deal with the near-field of radiating track, the analytical description is very bulky (like in the Fresnel zone of classical flat radiators), and it is preferable to use direct numerical calculations of formula (2.11) with the exact expression for the distances ρ_m .

The significant increase in amplitudes of vibrations for $v > c_R$ is explained by two features. The first is the obvious fact that the fields radiated by different sleepers are combined in phase. Therefore, an increase by the number of effectively radiating sleepers of the track, i.e. about 200 times, can be expected compared with the average vibration level for conventional trains. The second feature is the dependence of functions $P(\omega)$ or $V(\omega)$ on train speed v . The numerical calculation shows an average increase in radiation of an individual sleeper of about 10 times for $v = 138.8$ *m/*s (*500 km/h*) compared with $v = 13.88$ *m/s* (*50 km/h*) [17]. Thus, a total increase of ground vibration amplitudes by 1000-2000 times (60-66 dB) can be expected for the case of trans-Rayleigh trains.

It is interesting to note that according to (4.2), the amplitudes of the vibration field radiated by parts of the track at angles $\Theta = arccos(c_R/v)$ depend neither on the periodicity of sleepers d nor on their number $2Q+1$. They are determined only by the track distance considered [17]. This means that radiation of ground vibrations by trans-Rayleigh trains can also take place on tracks without sleepers. Note however that, for conventional low-speed trains ($v << c_R$), the ground vibrations in the form of waves are not generated in the framework of the mechanism considered. This agrees with the well known result of elasticity theory [30] that, for loads moving along the free surface of an elastic half space at speed $v < c_R$, radiated wave-fields do not exist (only localised quasistatic fields can accompany the moving load). Thus, the presence of sleepers is responsible for generating ground vibrations by conventional trains due to the mechanism of quasi-static wheel pressure considered here.

Numerical calculations undertaken for a train consisting of N=5 equal carriages show that the averaged ground vibration level from a train moving with trans-Rayleigh speed *138.8 m/s (500 km/h)* is approximately 70 dB higher than from a train travelling at speed *13.8 m/s (50 km/h)* [17,18]. This predicted very large increase in ground vibrations agrees well with the above mentioned general analytical estimates. The absolute level of ground vibration velocities generated by trans-Rayleigh trains might be as high as 10 mm/s (140 dB re 10^{-9} m/s). Vibrations of such high level may even cause damage of nearby properties. Fortunately, soils with very low Rayleigh wave velocity (around 100 m/s) are uncommon, the most typical range of c_R values being *250-500 m/s*. Nevertheless, the designers and builders of tracks for superfast trains should be aware of the potential risk of excessive ground vibrations. One has either to avoid areas such as soft sandy soils with low Rayleigh wave velocity, or to undertake special mitigation measures to protect the built environment from the expected severe ground vibrations.

Measures to reduce ground vibrations from trans-Rayleigh trains need to take into account the peculiarities of ground vibration fields generated by such trains. One such measure might be based on smallness of the radiation angle Θ for trans-Rayleigh trains in most of the situations. Therefore, if the track is placed inside an open waveguide for surface waves, then most of the radiated energy will be trapped and dissipated in the waveguide without damaging the area outside. Specially modified embankments or trenches can be suggested as possible waveguides, where the total reflection of surface waves incident at sliding angles on the boundary of their top or bottom flat areas would provide the waveguide effect. Waveguides of this kind are similar to topographic waveguides that have been studied intensively for ultrasonic applications over the last 20 years [31,32]. There is little doubt that similar ideas could be realised for embankments and trenches carrying tracks for superfast trains.

5. UNDERGROUND TRAINS

Compared with above-ground trains, the case of underground trains is more complicated for theoretical description, mainly because of the influence of tunnel geometry making the problem of constructing the corresponding Green's function extremely difficult. For not very shallow tunnels, bulk acoustic waves usually make a major contribution to the ground vibration field near the surface, in contrast to the case of above-ground trains where Rayleigh surface acoustic waves almost always prevail. Here we briefly describe the results of the approximate approach that considers the problem in the low-frequency approximation [19], i.e., the characteristic wave-lengths of generated bulk acoustic waves in the ground are supposed to be essentially larger than the diameter of the tunnel.

In such an approximation the analytical expression for the vertical component of the particle velocity of ground vibrations generated on the ground surface by trains travelling underground may be written in the following form:

$$v_Z(x,y,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(x',y',z',\omega)G_{ZZ}(r,\omega)dx'dy'dz' , \qquad (5.1)$$

where $G_{ZZ}(r,\omega)$ is the correspondent component of the half-space Green's function describing the field of a vertical point source located in the depth of the elastic medium, $r = [(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2}$ is the distance from the current elementary source to the observation point, and $P(x',y,'z',\omega)$ describes the total distribution of vertical load forces along the underground track.

For long distances r (in comparison with wave-lengths of radiated waves) the Green's function $G_{ZZ}(r,\omega)$ at $z = 0$ has the form (only bulk elastic waves are being taken into account)

$$G_{ZZ}(r,\omega)|_{z=0} = (i\omega/4\pi\rho_0 r)[e^{i(\omega/c_l)r}(1 + R_l(\omega,\varphi))\cos^2(\varphi) - e^{i(\omega/c_t)r}(1 + R_t(\omega,\varphi))\sin^2(\varphi)]. \qquad (5.2)$$

Here terms with the exponents $e^{i(\omega/c_l)r}$ and $e^{i(\omega/c_t)r}$ describe the contributions of radiated longitudinal and shear bulk waves, $R_l(\omega,\varphi)$ and $R_t(\omega,\varphi)$ are the corresponding reflection coefficients from the surface for the incident longitudinal and shear waves respectively (note that each of these coefficients takes account of both longitudinal and shear waves reflected from the surface), and φ is the observation angle relative to the vertical direction ($\cos\varphi = (z-z')/r$). The dependence on frequency in $R_l(\omega,\varphi)$ and $R_t(\omega,\varphi)$ takes account of impedance load resulting from the influence of buildings or other engineering structures on the surface. In what follows we consider, without loss of generality, that all the energy of radiated waves is absorbed by the structure, i.e., $R_l(\omega,\varphi) = 0$ and $R_t(\omega,\varphi) = 0$.

The load-force distributions along underground tracks may be written in the form similar to that for the above-ground tracks (see Section 2). The time-dependent load function taking account of all axles and carriages has the following form:

$$P(t, x',y',z') = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} A_n [P(t - (x'+nL)/v) + P(t - (x' + M +nL)/v)]\,\delta(x'-md)\delta(y')\delta(z'-H), \tag{5.3}$$

where all notations are the same as before.

Taking the Fourier transform of (5.3), then substituting it into (5.1) and making transformations similar to that considered in Section 2, one can obtain the following expression for the frequency spectra of vertical vibrations at $z=0$ generated by an underground train:

$$v_z(x=0, y=y_0, \omega) = [i\omega P(\omega)/4\pi\rho_0] \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} [1+ \exp(iM\omega/v)]\, \exp[i(\omega/v)(md + nL)] \times$$
$$(1/r_m)\{\exp[(-\gamma_l \omega r_m/c_l) + i(\omega/c_l) r_m] \cos^2(\varphi) -$$
$$\exp[(-\gamma_t \omega r_m/c_t) + i(\omega/c_t) r_m] \sin^2(\varphi)\}. \tag{5.4}$$

Here $r_m = [y_0^2 + (md)^2 + H^2]^{1/2}$, and γ_l and γ_t are the attenuation constants respectively for longitudinal and shear waves.

Numerical calculations of the ground vibration velocity according to eqn (5.4) have been carried out as functions of different train, track and soil parameters [19]. In particular, it has been shown that for most practical values of tunnel depth H (H less than *150-160 m)* the main contribution to the vertical component v_z of the total ground vibration field at the ground surface is due to the radiated shear bulk waves rather than to the longitudinal bulk waves. For larger depths, the contributions of shear and longitudinal waves become comparable with each other and the resulting field shows an oscillatory behaviour versus H.

Calculation of generated ground vibrations as functions of the observation distance from the track y_0 shows that the total field and the contributions of longitudinal and shear waves separately decrease with the distance y_0, especially the field of longitudinal waves. Comparison of the vibration spectra generated by underground trains with the corresponding spectrum of ground vibrations from above-ground trains with the same parameters shows that, although lower in amplitude, the shapes of the ground vibration spectra for underground trains are very similar to those generated by trains travelling above the ground. This implies that one can use the same methods of suppression of ground vibrations at selected frequencies that have been considered for above-ground trains, i.e., choosing special relations between the track and train parameters.

6. CONCLUDING REMARKS

The results of theoretical investigations briefly reviewed in this paper show that ground vibration spectra generated by railway trains depend strongly on the axle loads of the carriages: if the axle load exceeds a critical value beyond which peripheral bulges appear in the track, the vibration level increases significantly, especially at higher frequencies. By proper selection of the distance between wheel axles in a bogie, and between bogies in a carriage (or between sleepers in a track) it is possible effectively to suppress vibration levels at the train passage frequencies.

Superfast trains moving at speeds approaching or exceeding the Rayleigh wave velocity in the ground can cause very large increases in ground vibration level relative to conventional trains. Fortunately, soils with very low Rayleigh wave velocity (around 100 m/s) are uncommon, the most typical range of c_R values being 250-500 m/s. Nevertheless, the designers and builders of tracks for superfast trains should be aware of the potential risk of excessive ground vibrations. One has either to avoid areas such as soft sandy soils with low Rayleigh wave velocity, or to undertake special mitigation measures to protect the built environment from the expected severe ground vibrations.

In the case of underground trains, it has been shown that for most practical values of tunnel depth, the main contribution to the vertical component of the total low-frequency ground vibration field at the ground surface is due to radiated shear bulk waves rather than to longitudinal bulk waves. The shapes of the ground vibration spectra for underground trains are very similar to those generated by trains travelling above the ground. This implies that one can use for underground trains the same methods of suppression of ground vibrations at selected frequencies that have been suggested for above-ground trains, i.e., choosing special relations between the track and train parameters.

In spite of the promising theoretical results reported here, much work remains to be done. Among the most important problems related to railway-induced ground vibrations is the expansion of the approach developed for calculation of ground vibrations from underground trains beyond the low-frequency approximation. Another important task is to consider other mechanisms of generating ground vibrations by both above-ground and underground trains. Other unexplored areas for theoretical research are associated wave propagation and scattering problems. To the best of our knowledge no directly relevant theoretical investigations into the propagation of railway-induced ground vibrations (i.e. surface and bulk acoustic waves in the ground) from the vicinity of the track or road to rather distant buildings have been carried out so far. The phrase "directly relevant" means that, first, the complex structure of the initial wave field of ground vibrations generated by such complicated time- and space-distributed sources like trains and road vehicles should be taken into account in solving propagation problems. And, second, one should take into account that real ground has vertical and horizontal inhomogeneities of soil elastic properties and mass density that may cause significant dispersion and scattering of elastic waves. Besides this, the surface of the ground may be rough, may have grooves, steps, hills, etc. (including those artificially created) that may also cause reflection and scattering of waves as well as deflection of their propagation trajectories. All these problems need to be properly investigated.

To conclude this discussion, we consider it important to point out ways in which the above described theoretical results and results of future investigations could lead to deriving effective protection methods against railway-generated ground vibrations. We suggest three possible ways of decreasing railway-induced ground vibrations.

The first is based on understanding the physical nature of the excitation mechanisms. Using the analytical or numerical results obtained, the parameters of the track-train system may be chosen to minimise the vibration radiation intensity directly. This is the most economic way of reducing generated ground vibrations.

The second way is to use the theoretical results on the influence of natural properties of the ground, including its topography, on the formation of quiet (shadowed) areas for the radiated vibration wave-field. This way of protection is possible for future developments.

The third way is based on artificial methods of protection, e.g., by considering wave-scatterers or absorbers placed in the path of wave propagation to the building.

Further investigations of all related theoretical problems will generate new ideas for all these groups of mitigation methods that, appropriately combined, will be able to provide reliable protection of the built environment against railway-induced ground vibrations.

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