Mathematical competence framework : An aid to identifying understanding?

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Research into the teaching of mathematics to engineering students to promote their conceptual understanding (Jaworski and Matthews 2011) has shown the problematic nature of planning for and identifying understanding. I review the project briefly and introduce the idea of competencies from the Danish project, KOM (translated as Competencies and Mathematical Learning). Through the medium of designing inquiry-based tasks for students and use of the competency framework for analysis of tasks, I consider the relevance of such a competency-based analysis and its usefulness (or otherwise) for recognising student understanding. This leads to important questions for further research of a developmental nature.

Keywords: mathematical competency; engineering students, inquiry-based teaching, sociocultural setting, developmental research.

Mathematics for Engineering students:

In this paper I discuss a research project which aimed to study the design and teaching mathematics in ways which enable students' conceptual learning and understanding of mathematics for flexible use in engineering contexts. The project was fundamentally about *teaching:* in particular*, how teaching relates to learning with understanding.*

The project, ESUM, Engineering Students Understanding Mathematics, involved an innovation in teaching and learning. It was a developmental research project; that is, it involved research that both studies development and contributes to that development. It focused centrally on INQUIRY – inquiry in mathematics and learning mathematics and inquiry in the teaching process. It attracted support from the Royal Academy of Engineering through the UK *HE-STEM* programme. Funding supported a researcher to work with the teaching team and paid for a literature review. Research questions included

- How can we enable engineering students' more conceptual understanding of mathematics?
  - What teaching approach (and why)? DESIGN
  - What means of perceiving students' outcomes? APPROACH
  - What outcomes? EVALUATION

A sociocultural frame

In seeking mathematical understanding, we were interested not only in cognitive processes, but in the whole context and culture in which we are active. (Schmittau 2003; Vygotsky 1978; Wertsch 1991). This included the following principles:

- All learning is social; knowledge grows in the social domain within which individual knowledge is formed.

- Learning takes place through participation in social and cultural worlds mediated by social and cultural tools.
- Scientific concepts grow through pedagogical mediation.

Thus we were interested in

- mathematical meanings, relating to an established body of mathematical knowledge;
- perspectives of students and teachers, relating to learning mathematics;
- institutional dynamics and constraints influencing perspectives on learning and teaching;
- worlds (cultures) in and beyond the institutional setting creating parameters and boundaries for engaging in learning and teaching.

The institutional setting (pre-innovation)

A university three-year BSc for first year students in Materials Engineering included a one year module in mathematics. The ESUM project studied the first semester of this module. Students were fresh from school still with perspectives from their school culture. The module was allocated two lectures and one tutorial per week (each 50 minutes). The university encouraged use of a Virtual Learning Environment – LEARN – for communication, holding notes and resources. Assessment was by exam (60%) and 8 computer based tests (40%). Teaching was largely traditional in style with perceived instrumental approaches to mathematics (Artigue, Batanero and Kent 2007; Hiebert 1986; Skemp 1977).

The teaching-research team (co-learners)

The teaching team of three experienced teachers, two having extensive experience of teaching engineering students, had responsibility for interpretation of curriculum, design of innovation and teaching approach, design of questions/tasks/group project (with the help of PhD students). One member (the lecturer) taught the module.

The research team, of four people, included the teaching team plus a research officer (paid for with the *HE-STEM* funding). Together they designed research which included

- Research *in* practice (insider research))
- Research *on* practice (outsider research) (Bassey 1995)

Learning through inquiry – a developmental research methodology & innovation

The inquiry approach aimed to promote learning through an inquiry community in mathematics AND in mathematics teaching). A Community of Inquiry (CoI) was seen to be based on processes of participation and reification as described by Wenger (1998) in a Community of Practice (CoP). It embraced the principles that addressing inquiry-based questions challenges existing ideas and engages students in meaning-making more deeply; it motivates ‘wanting to know’, encouraging asking one’s own questions, and looking critically at outcomes; it enables development of a critical sense through *critical alignment* (Jaworski 2006).

A developmental research process involved linked forms of research:

Insider research – cyclic approach: Insiders, teachers who are also researchers, engage in cycles of activity involving design (of tasks), work in practice (act/teach &

observe), reflect on and analyse what has been done, feedback to further planning, disseminate to others in the field.

Outsider research – data and analysis: Involving research into processes and practices from the outside – taking out data and analysing it through a rigorous research process including design documentation, student surveys, observation of practice (audio recording), interviews; analysis relevant to the kinds of data; overall activity theory analysis; dissemination and publication (Jaworski 2003).

The innovation involved a modification to teaching, with implications for learning mathematics. It included use of inquiry-based questions in lectures and particularly in tutorials; a GeoGebra environment for demonstration in lectures and for student exploratory use in tutorials in relation to inquiry-based questions; small group activity: students in groups of three or four working on tasks in tutorials, discussing solutions together and with the lecturer; a small group (assessed) project: tasks given by the lecturer for exploration by students in a group with requirement to submit a group project for assessment; and changes to assessment to include the assessed project. Figures 1, 2 and 3 show examples of tasks which were designed and used in the ESUM innovation:

Think about what we mean by a *function* and write down two examples. Try to make them different examples.

1. Open question/task in a lecture

In the topic area of real valued functions of one variable

Consider the function $f(x) = x^2 + 2x$ (x is real)

a) Give an equation of a line that intersects the graph of this function:
(i) Twice (ii) Once (iii) Never
(Adapted from Pilzer et al. 2003, 7)

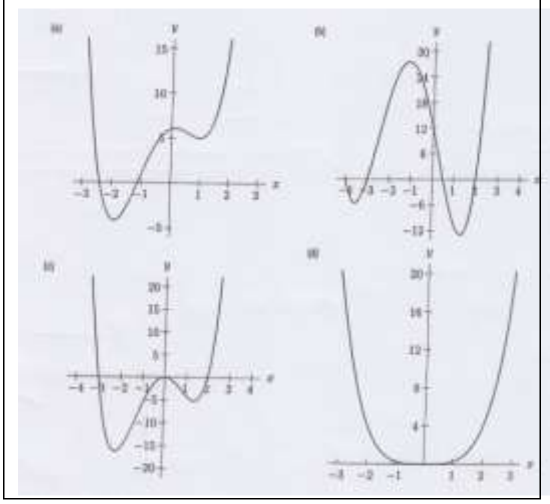
b) If we have the function $f(x) = ax^2+bx+c$ what can you say about lines which intersect this function twice?

c) Write down equations for three straight lines and draw them in GeoGebra

d) Find a (quadratic) function such that the graph of the function cuts one of your lines *twice*, one of them *only once*, and the third *not at all* and show the result in GeoGebra.

e) Repeat for three *different* lines (what does it mean to be different?)

2. A lecture and tutorial task



3. Tutorial task – for small group work

Findings from the ESUM project indicated important differences between perceptions towards mathematical learning, the value of inquiry processes and use of GeoGebra of those designing and delivering teaching (the teaching team) and those experiencing the teaching and learning from it (the students). For details see Jaworski and Matthews (2011), Jaworski, Robinson, Matthews and Croft (2012). Here I focus on our desire to improve students' conceptual understandings of mathematics (compared with previous cohorts) which proved elusive in the ESUM analysis.

What does it mean to understand and how can we recognise understanding?

It was clear in observation of teaching sessions the extent to which students engaged with mathematics and their degrees of conceptualization. This was pleasing in many respects (for those teaching), however, due to being very local and specific, it did not reveal general characteristics or provide objective insight to the nature of conceptual understanding. Examination and test scores showed improvement on previous cohorts, but this was not indicative of the quality of understanding. Thus we sought an alternative approach to discerning understanding.

We became aware that the mathematics working group of the European Society for Engineering Education (SEFI) was promoting a set of competencies deriving from the work of the Danish KOM Project (e.g., Niss 2003; SEFI 2011) for the design of mathematics teaching for engineering students. We decided to look critically at what these might offer. The SEFI document quotes Niss (2003, 183) as follows:

> Possessing mathematical competence means having knowledge of, understanding, doing and using mathematics and having a well-founded opinion about it, in a variety of situations and contexts where mathematics plays or can play a role.
>
> A mathematical competency is a distinct major constituent in mathematical competence

Eight competencies have been identified as follows. See SEFI (2011) and Niss (2003) for a detailed breakdown of what each competency includes.

The ability to ask and answer questions in and with mathematics	The ability to deal with mathematical language and tools
1. Thinking mathematically 2. Reasoning mathematically 3. Posing and solving mathematical problems 4. Modelling mathematically	5. Representing mathematical entities 6. Handling mathematical symbols and formalism 7. Communicating in, with and about mathematics 8. Making use of aids and tools

We began by using these competencies to analyse some of our tasks. For example in Task 2a above, given in a lecture in which students had to work on the task in their seats talking with their neighbours, we analysed as follows:

- The function is easy to sketch – it is easy to see lines which cross it in the three conditions [5]
- Students have to talk to each other [7]
- They have to think about equations for their lines [1] [3] [6]
- They start to reason about the differences between the lines [2]
- They have to give feedback to the lecturer and others in the cohort [2] [7]

We see further analysis of Task 2b-e:

b) generalising from (a) [1, 2, 7]	c) Inventing own mathematical objects and using a technological tool [1, 2, 5, 6, 7, 8]
d) tackling an open-ended problem [1, 2, 3, 7, 8]	e) Addressing mathematical generality [1, 2, 3, 5, 6, 7]

From this example, we believe that our tasks are broadly in line with the stated competencies. There seems to be a region of synergy between the competencies and goals of inquiry- based learning. The tasks designed for the latter seem to fulfil the former. Our next challenge is to try to recognize student understanding in relation to the competencies. The Danish team has suggested three dimensions for specifying and measuring progress:

- *Degree of coverage*: The extent to which the person masters the characteristic aspects of a competency.
- *Radius of action*: The contexts and situations in which a person can activate a competency.
- *Technical level*: How conceptually and technically advanced the entities and tools are with which the person can activate the competence.

The SEFI Mathematics working group is in the process of specifying what such dimensions can look like in relation to the mathematical curriculum for engineering students. With regard to ESUM, we ask how our data might allow us to address the three dimensions in order to discern what competencies students gained/achieved. In fact, existing data is not adequate: it was not collected for this purpose, so we ask what data we would *need* to collect; for example we can record data and analyse it from events such as:

- In lectures: we can ask further questions and encourage students to respond (we recognise that not all can/will do so).
- In tutorials: we can visit groups, talk with them about their current thinking, probe and challenge appropriately (of course, we cannot be with all groups all of the time).
- Assessment: in tests or exams, we can design suitable questions and analyse students' responses (which may or may not reveal understanding).
- Assessment through group project with written report: we can look critically for evidence of understanding (we also need to consider who has done most of the work)

Analysis of the data would allow us to seek evidence of competencies having been addressed. For this to be effective for successive groups of students we need a systematic process which can be achieved quickly and efficiently which requires assessment instruments to be developed to have accord with competency statements. We can see above some of the constraints to this process, and recognise that cultural issues revealed through ESUM will also present challenges (see Jaworski et al. 2012)

Questions for further research using a competency framework

From the above, we see a use of the competencies in evaluating design of tasks and a potential development of instruments for a systematic use of competencies against the three dimensions to measure student progress. The latter needs further consideration. A third potentially valuable use of competencies would be in providing a formative presence, for example, in creating *opportunities* for students to achieve competency and as a tool for teachers in working with students to achieve competency. With respect to this third area of consideration we ask: what are the elements of creating the sociocultural setting in which the desired mathematical practices and ways of being are nurtured as central to participation? ESUM identified students as having an essentially *strategic* focus towards their studies; thus we might also ask: are there ways in which students can develop an awareness of competency as a means of changing the nature of their strategic focus?

Within our sociocultural frame in which we consider teaching for learning mathematics in relation to a cohort of students rather than in terms of individuals – in which we have to take seriously systemic and cultural factors as revealed by ESUM –

we see the above questions as motivating for further research of a developmental nature. The questions are challenging for the design of both teaching and research: teaching is seen as a research process through which teachers and students can come closer in their understandings of what it means to learn mathematics effectively for engineering contexts. We hope to pursue these questions and invite others who are interested to join us in this endeavour.

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