Mesoscopic Josephson arrays interacting with non-classical electromagnetic fields and their applications

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Abstract: A ring made from a Josephson array in the insulating phase is considered. The ring contains a 'dual Josephson junction' (Josephson junction for vortices). External non-classical electromagnetic fields are coupled to the device and interact with the vortices that circulate the ring. The time evolution of this two-mode fully quantum mechanical system is studied. The effect of the quantum statistics of the photons on the quantum statistics of the vortices is discussed. The entanglement between the two systems is quantified.

1 Introduction

There has been a lot of interest in the interaction of Josephson devices with microwaves for a long time. The new development in the last ten years [1–4] has been the experimental and theoretical study of mesoscopic Josephson devices (with capacitance lower than 10^{-16}F) where quantum phenomena are stronger.

At the same time there have been significant developments in quantum optics. Non-classical electromagnetic fields have been studied extensively both experimentally and theoretically. These fields are carefully prepared in a particular quantum state so that the amount of quantum noise is well defined and the statistics of photons are also well defined.

The purpose of this paper is to present some interdisciplinary work that studies the interaction of mesoscopic Josephson devices interacting with non-classical electromagnetic fields (at gigahertz to terahertz frequencies) [5, 6]. More specifically, we consider a ring made from a Josephson array in the insulating phase. Vortices circulate this ring with high mobility. The ring contains a dual Josephson junction [7–9] through which the vortices tunnel. This is a fully quantum mechanical system and we study explicitly how the quantum noise of the electromagnetic fields affects the quantum noise of the vortex current. Recent work on vortices can be found in [10–28].

Nonlinear systems have been studied extensively in quantum optics in conjuction with nonlinear optical materials. We note that the sinusoidal nonlinearity of dual Josephson junctions in Josephson arrays is much stronger than the polynomial nonlinearity with very small coefficients that describes the nonlinear materials used in quantum optics. The periodicity of the sinusoidal nonlinearity also makes the dual Josephson junctions in Josephson arrays very interesting from a theoretical point of view and requires the development of novel highly nonperturbative methods.

Josephson systems have been used for the production of squeezed microwaves [29, 30] and they are good candidates for the development of devices operating at terahertz frequencies, which is the modern tendency in communications. There is also a lot of work currently, for their use as quantum gates in quantum technology and quantum computing [31].

2 Interaction of Josephson arrays with external non-classical microwaves

We consider a ring made from an array of Josephson junctions with Coulomb coupling constant E_C greater than the Josephson coupling constant E_J. For those values of the parameters the array is in the insulating phase where vortices move with high mobility and charges are confined. Such rings have been considered experimentally in the context of the Aharonov–Casher effect [32–38]. However, our ring has also a 'dual Josephson junction' [7–9]. This plays a similar role for vortices, to the ordinary Josephson junctions for electron pairs. The 'dual phase' of the vortex wavefunction has a discontinuity δ along the dual Josephson junction. This is analogous to the Cooper-pair wavefunction in superconducting rings with Josephson junctions, which has discontinuity ϕ along the junction.

The centre of the ring contains charge $Q(t)$ induced through coupling with an external source of microwaves which are carefully prepared in a particular quantum state (Fig. 1). Microwaves in various quantum states have been produced experimentally in several laboratories (e.g. [29,30]). The system operates at low temperatures ($\hbar\Omega_1 > k_BT$ and $\hbar\Omega_2 > k_BT$), so that the thermal noise is less than the quantum noise in the microwaves and the device. The dissipation in the system is assumed to be negligible.

The Hamiltonian describing this system contains an inductive term $1/2L(I - I_{mw})^2$, a capacitive term $(Q - Q_{mw}/2C)^2$ and a 'dual Josephson' term $E_{dJ}(1 - \cos\delta)$. Here L and

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C are the inductance and capacitance of the system, respectively. For simplicity we assume that the inductance and capacitance of the ring are equal to the inductance and capacitance of the circuit that produces the microwaves. I and I_{mw} are the total and external (microwave) current respectively, flowing in the radial direction of the device. Q and Q_{mw} are the total and external (microwave) charge, in the inner boundary of the ring. E_{dJ} is the dual Josephson coupling constant and $\delta = \phi_0 Q$ the dual phase. $\phi_0 = \pi/e$ is the flux quantum (in units where $\hbar = k_B = c = 1$). The sinusoidal term $E_{dJ}(1 - \cos\delta)$ describes vortex tunnelling through the dual Josephson junction. This is analogous to the well-known term $E_J(1 - \cos\phi)$, which describes the tunnelling of electron pairs in Josephson junctions.

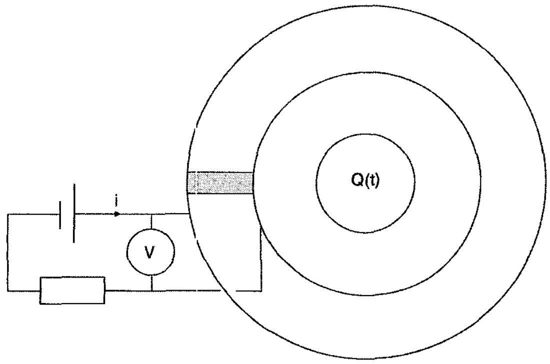

Fig. 1 *Josephson array ring in the insulating phase, coupled to a source of non-classical microwaves*
The ring contains a dual Josephson junction (i.e. a Josephson junction for vortices). The voltmeter measures the vortex current. The current i compensates the dissipation.

Quantisation of the device is done with the creation and annihilation operators

$$a_1 = \left(\frac{1}{2\Omega_1 C}\right)^{1/2} [Q + i\Omega_1^{-1} I] \qquad (1)$$

$$a_1^\dagger = \left(\frac{1}{2\Omega_1 C}\right)^{1/2} [Q - i\Omega_1^{-1} I] \qquad (2)$$

$$[a_1, a_1^\dagger] = 1 \qquad (3)$$

where $\Omega_1 = (LC)^{-1/2}$ is the frequency of the device. We note that this is the frequency of the linear part of the device. The sinusoidal nonlinearity renormalises this frequency (i.e. there is an $a_1^\dagger a_1$ term within the $\cos\delta$ nonlinearity). The electromagnetic field is quantised with the operators:

$$a_2 = \left(\frac{1}{2\Omega_2 C}\right)^{1/2} [Q_{mw} + i\Omega_2^{-1} I_{mw}] \qquad (4)$$

$$a_2^\dagger = \left(\frac{1}{2\Omega_2 C}\right)^{1/2} [Q_{mw} - i\Omega_2^{-1} I_{mw}] \qquad (5)$$

$$[a_2, a_2^\dagger] = 1 \qquad (6)$$

where $\Omega_2 = (LC)^{-1/2}$ is the frequency of the microwaves. We note that for the parameters considered $\Omega_1 = \Omega_2$.

The Hamiltonian can now be written as

$$H = \Omega_1 a_1^\dagger a_1 + \Omega_2 a_2^\dagger a_2 - E_{dJ} \cos[\mu(a_1^\dagger + a_1)] - \Omega_1(a_1^\dagger a_2 + a_1 a_2^\dagger) \qquad (7)$$

where $\mu = \phi_0(\Omega_1 C/2)^{1/2}$.

3 Time evolution

In the absence of dissipation the the density matrix of the system $\rho(t)$ is given by

$$\rho(t) = \exp[iHt]\rho(0)\exp[-iHt] \qquad (8)$$

where $\rho(0)$ is the density matrix at $t = 0$. We have calculated numerically $\rho(t)$ and the reduced density matrices

$$\rho_1(t) = Tr_2\rho(t); \qquad \rho_2(t) = Tr_1\rho(t) \qquad (9)$$

Using the reduced density matrices we calculated the average number of quanta in each mode

$$\langle N_i \rangle = Tr[a^\dagger a \rho_i] \qquad (10)$$

as functions of time.

The infinite dimensional matrix $\langle M_1, M_2|H|N_1, N_2\rangle$ has been truncated for the numerical calculations, with M_1, N_1 taking values from 0 up to K_{1max} and M_2, N_2 taking values from 0 up to K_{2max}. K_{1max} and K_{2max} were taken to be much greater than $\langle N_1\rangle$ and $\langle N_2\rangle$, respectively. As a measure of the accuracy of the approximation we calculated the traces of the truncated matrices. In the limit $K_{1max} \to \infty$ and $K_{2max} \to \infty$ they are equal to 1; and in the truncated case they should be very close to 1. In all our results the above sum was greater than 0.98.

In Fig. 2 we assume that at $t = 0$ the device is in the vacuum state $|0\rangle$ $(a_1|0\rangle = 0)$ and the microwaves are in the number state $|N = 1\rangle$ $(a_2^\dagger a_2|1\rangle = |1\rangle)$. The results presented show an exchange of energy between the microwaves and the device.

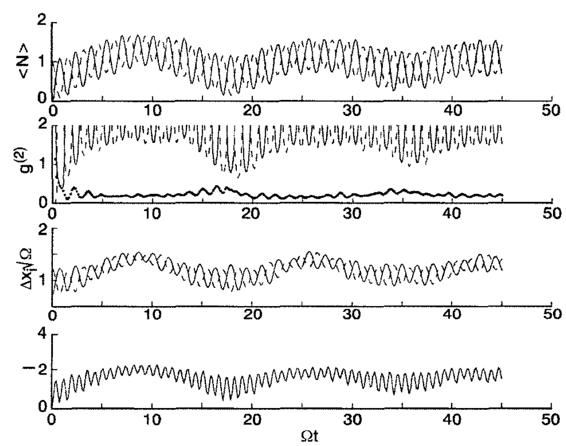

Fig. 2 *Results showing exchange of energy between microwaves and device*
At time $t = 0$ the device is in the vacuum state $|0\rangle$ and the microwaves in the number state $|N = 1\rangle$
$\Omega = \omega = 1.5 \times 10^{-4}$, $E_{dJ} = 10^{-4}$, $\mu = 2.8408$ and truncation $K_{imax} = 10$
First graph: $\langle N_1\rangle$ (solid line) and $\langle N_2\rangle$ (broken line) as functions of time
Second graph: $g_{11}^{(2)}$ (solid line), $g_{22}^{(2)}$ (broken line) and r (dotted line) as functions of time
Third graph: uncertainties $\Delta x_1\sqrt{\Omega}$ (solid line) and $\Delta x_2\sqrt{\Omega}$ (dotted line) as functions of time
Fourth graph: entanglement entropy I (in nats) as a function of time

In Fig. 3 we assume that at $t = 0$ the device is in the vacuum state $|0\rangle$ $(a_1|0\rangle = 0)$ and the microwaves are in the coherent state $|A = 1.5\rangle$ $(a_2|A\rangle = A|A\rangle)$. The results presented show an exchange of energy between the microwaves and the vortices in the device.

The microwaves have been carefully prepared in a quantum state and this implies that the quantum statistics of the photons threading the ring, is known. In our analysis we study a 'quantum Faraday law' and investigate how the quantum statistics and quantum noise of the photons affects the quantum statistics and quantum noise of the tunnelling vortices. To quantify the quantum statistics, we present in figures Figs. 2 and 3 the second order correlations

$$g_{ii}^{(2)} = \frac{\langle N_i^2 \rangle - \langle N_i \rangle}{\langle N_i \rangle^2}; \qquad i = 1, 2 \tag{11}$$

$$g_{12}^{(2)} = \frac{\langle N_1 N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle} \tag{12}$$

and the ratio

$$r = \frac{\left[g_{12}^{(2)}\right]^2}{g_{11}^{(2)} g_{22}^{(2)}} \tag{13}$$

The $g_{11}{}^{(2)}$ describe vortex bunching or antibunching, and the $g_{22}{}^{(2)}$ describe photon bunching or antibunching. The quantum noise is quantified with the uncertainties

$$(\Delta x_i)^2 = Tr(\rho x_i^2) - \left[Tr(\rho x_i)\right]^2; \quad i = 1, 2 \tag{14}$$

which are presented in Figs. 2 and 3.

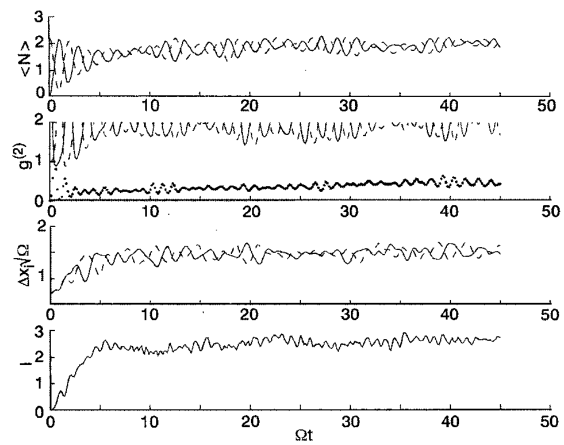

Fig. 3 *Results showing exchange of energy between microwaves and vortices in the device*
At time $t = 0$ the device is in the vacuum state $|0\rangle$ and the microwaves in the coherent state $|A = 1.5\rangle$
$\Omega = \omega = 1.5 \times 10^{-4}$, $E_{dJ} = 10^{-4}$, $\mu = 2.8408$ and truncation $K_{max} = 10$
First graph: $\langle N_1 \rangle$ (solid line) and $\langle N_2 \rangle$ (broken line) as functions of time
Second graph: $g_{11}{}^{(2)}$ (solid line), $g_{22}{}^{(2)}$ (broken line) and r (dotted line) as functions of time
Third graph: uncertainties $\Delta x_1 \sqrt{\Omega}$ (solid line) and $\Delta x_2 \sqrt{\Omega}$ (dotted line) as functions of time
Fourth graph: entanglement entropy I (in nats) as a function of time

At $t = 0$ the system is not entangled, but as time evolves the microwaves entangle with the Josephson array device. As a measure of this entanglement we have calculated the entanglement entropy [39–42]

$$I = S(\rho_1) + S(\rho_2) - S(\rho) \tag{15}$$

where $S = -Tr\rho\ln\rho$ is the von Neumann entropy. The entanglement entropy I is positive according to the subadditivity property. The results show that although originally the system was disentangled, it becomes strongly entangled later.

4 Discussion

We have considered a ring made from a Josephson array in the insulating phase. The ring contains a dual Josephson junction which is described mathematically with a sinusoidal nonlinear term. External non-classical electromagnetic fields are coupled to the device and interact with the vortices that circulate the ring.

We have calculated the time evolution of this fully quantum mechanical two-mode system. The results have shown the exchange of energy between the electromagnetic field and the vortices. We have also shown quantitatively (with the $g^{(2)}$) how the quantum noise of the electromagnetic field affects the quantum noise of the vortices, and how the two modes become entangled. The calculations have ignored dissipation and work is in progress in the direction of assessing the effect of dissipation [43].

The results can be useful in the context of quantum gates based on Josephson technology; and also in the context of terahertz technology.

5 Acknowledgments

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