Numerical modelling of landfill lining system–waste interaction: implications of parameter variability

A. H. I. Sia[1] and N. Dixon[2]

[1]*Geotechnical Engineer, Atkins, 200 Broomielaw, Glasgow G1 4RU, UK, Telephone: +44 (0)141 220 2314, Telefax: +44 (0)141 220 2001, E-mail: anna.sia@atkinsglobal.com (formerly Doctoral student, Loughborough University, Loughborough, Leicestershire LE11 3TU, UK)*
[2] *Professor of Geotechnical Engineering, School of Civil and Building Engineering, Loughborough University, Loughborough, Leicestershire LE11 3TU, UK, Telephone: +44 (0)1509 228542, Telefax. +44 (0)1509 223981, E-mail: n.dixon@lboro.ac.uk*

Received 4 April 2012, revised 3 August 2012, accepted 4 August 2012

ABSTRACT: Numerical modelling techniques can be used to examine the serviceability limit states of landfill side-slope lining systems in response to waste placement. A study has been conducted in which the variability of significant model input parameters have been investigated within a probabilistic framework using Monte Carlo simulation. Key model parameters are treated as random variables, and the statistical information required to describe their distributions has been derived from a laboratory repeatability testing programme, a literature survey and an expert consultation process. Model outputs include relative shear displacements between lining components, and tensile strains in the geosynthetic layers that occur in response to staged placement of waste against the side slope. It was found that analyses including input parameter variability were able to identify mechanisms influencing liner performance and their probability of occurrence. These mechanisms include large (i.e. $\gg$100 mm) relative displacements at interfaces that can generate post-peak strengths, and mobilised tensile strains in the geomembrane and geotextile layers. Additionally, it was found that relative displacements at the controlling (i.e. weakest) liner interface are greater for landfills with a steep side slope, for stiffer waste and thicker waste lifts, while tensile strains in the geosynthetic elements are greater for steep side slopes, more compressible waste and thinner waste lifts. Outputs from probabilistic analyses such as that used in this study can guide engineers regarding geometries and materials that could produce waste-settlement-generated serviceability limit state failures, and hence can be used to support more reliable designs.

KEYWORDS: Geosynthetics, Landfill, Lining system, Waste settlement, Monte Carlo method, Serviceability limit state, Probability

REFERENCE: Sia, A. H. I. & Dixon, N. (2012). Numerical modelling of landfill lining system–waste interaction: implications of parameter variability. *Geosynthetics International*, **19**, No. 5, 393–408. [http://dx.doi.org/10.1680/gein.12.00025]

1. INTRODUCTION

The design of landfill lining systems includes consideration of both stability (i.e. ultimate limit states, defined as large-scale deformations involving slippage of liner materials or waste) and integrity (i.e. serviceability limit states, defined as loss of function of elements, such as damage to a geomembrane, resulting from small-scale deformations within the lining system). Serviceability limit states are associated with interaction mechanisms between the lining system and the waste. These are a result of the settlements that occur during landfill construction as the waste compresses under its self-weight, and in the longer term as the waste degrades. A design framework for landfill lining systems has been proposed by Fowmes *et al.* (2007) that includes a definition of the limit states that should be considered, and guidance on controlling factors. Typically, a lining system will comprise, from the bottom up, a compacted clay liner, a geomembrane liner, a nonwoven geotextile protection layer, and a gravel drainage layer.

Various researchers (Long *et al.* 1995; Reddy *et al.* 1996; Filz *et al.* 2001; Jones and Dixon 2005; Fowmes *et al.* 2006, 2008) have demonstrated through numerical analyses the complex behaviour of lining–waste interaction and the mechanism of stress transfer in a landfill-lining system. The most advanced analyses (e.g. Fowmes *et al.* 2006, 2008) incorporate a strain-softening interface between each lining component (i.e. multiple interfaces are included), and aim to provide information on the relative displacement at each interface, and hence information on the mobilisation of post-peak shear strengths, and to quantify the tensile strains in the geosynthetic elements (e.g. the geomembrane and geotextile). These analyses were conducted deterministically, although they often included a sensitivity analysis, and therefore they did not consider the significance of uncertainty and variability in the controlling parameters, such as interface strengths and waste stiffness.

This paper describes the development of a probabilistic numerical modelling approach using Monte Carlo simulation to investigate the influence of parameter variability on lining system integrity mechanisms related to waste settlement. The derivation of input parameters and quantification of their variability are detailed. Results are presented from a series of analyses designed to investigate the influence of specific parameters, and these are used to develop guidance for designers on the significance of parameter variability and combinations of conditions that increase the probability of integrity failures. This study will enable designers to consider degrees of uncertainty and variability in analyses linked to a target probability of failure, or to acceptable performance of the lining system.

A major challenge in this type of study is the selection of a suitable probability threshold criterion for acceptable performance. Ideally, this would be agreed by designers, operators and regulators, and be based on prior experience, the costs of repair and the consequences of failure. In the absence of an agreed criterion, this study uses an approach based on the general principle of Eurocode 7 (2004), which states that there should be no more than 5% probability of an adverse condition occurring, based on variability of parameters over the area or volume of material controlling the limit state mechanism. Although the outputs from this study are considered in relation to a 95% probability of occurrence, it is straightforward for alternative thresholds to be applied as appropriate.

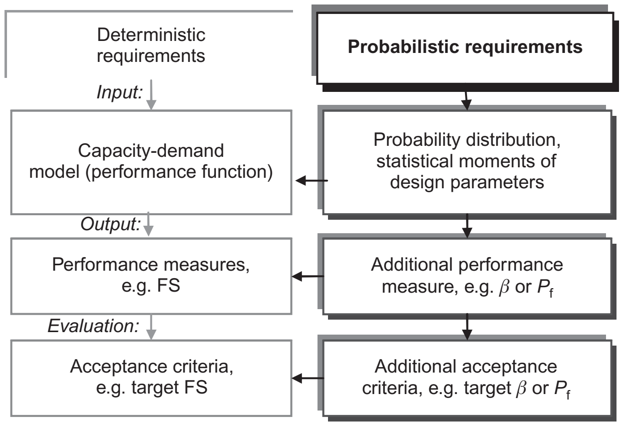

Figure 1. Requirements for a probabilistic approach (FS, factor of safety; β, reliability index; P_f, failure probability)

2. RELIABILITY-BASED DESIGN

Traditional practice requires only the calculation of a global factor of safety to judge the soundness of a design. This means that the engineer's judgement, the stress state and condition of the construction, and the degree of conservatism that was incorporated in the design parameters are combined into a single factor of safety. In this approach, a criterion to achieve a target safety factor does not necessarily warrant a safe design or acceptable performance, because uncertainties are not adequately considered when computing the factor of safety. Reliability-based design has been developed to achieve a satisfactory system performance without ignoring the uncertainties in the design. Figure 1 outlines the fundamentals of a basic reliability analysis. This evaluates the probability that capacity will exceed demand, where either or both capacity and demand are uncertain variables. In a reliability analysis, two additional fundamental requirements are imposed, specifically characterising uncertainty in the design and computing a performance indicator, such as reliability index (β) or failure probability (P_f).

The use of reliability methods in geotechnical engineering is increasing, including application in landfill design. To date, the first-order reliability method as defined by Duncan (2000) has been employed by Koerner and Koerner (2001), Sabbatini *et al.* (2002), Dixon *et al.* (2006) and Sia and Dixon (2008) to assess the probability of failure for landfill stability limit states (i.e. veneer and waste slope stability) using limit equilibrium methods. However, the Monte Carlo simulation approach (Chalermyanont and Benson 2004; El-Ramly *et al.* 2005) is the only versatile reliability technique that can account for implicit functions such as those associated with finite different or finite element methods, and is the approach used in this study. Numerical analysis using a Monte Carlo simulation can solve a problem by generating suitable random parameter values from a postulated input probability distribution (e.g. interface strength parameters), substituting the random values into a specified performance measure (i.e. the landfill construction model), and repeating the analysis for each set of values. An assessment can then be made of the fraction of output numbers (e.g. tensile strength of a geosynthetic) that obey a defined criterion (e.g. 95% probability of occurrence). A plot of the results produces an approximation of the probability distribution, and hence the mean and standard deviation of the performance measure can be calculated if required.

3. LANDFILL NUMERICAL MODEL

The geometry investigated in this study represents the construction of a landfill with waste placed across the full width of the cell (i.e. there is no external waste slope). First, the lining system is constructed on a rigid subgrade, and then the waste is placed in lifts of equal thickness. The two-dimensional model of the landfill cross-section used in this study is shown in Figure 2, with an assumption of plane-strain conditions. The lining system comprises a strong, high-stiffness subgrade, a compacted clay liner, a double-textured geomembrane liner, a nonwoven geotextile protection layer, and a gravel drainage layer.

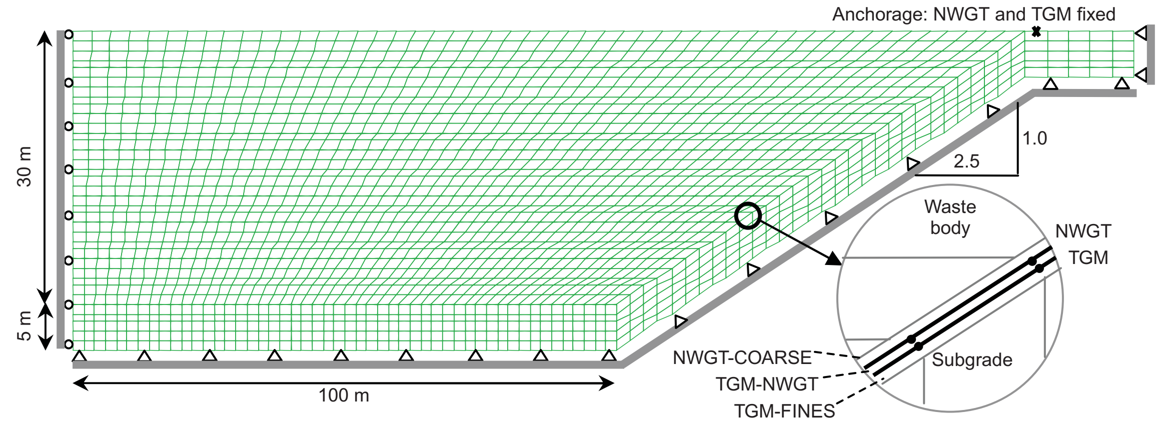

Figure 2. Geometry, lining system and mesh for FLAC landfill model

The waste mechanical properties used in the analyses are representative of municipal solid waste (MSW). Waste settlements generated by compression under self-weight are considered, and these represent short-term conditions. Time-dependent degradation and creep settlements that occur post-closure are not included, and therefore the outputs from the analyses consider only part of the waste-life deformations, and hence lining system response, although the trends will be indicative.

In this study, integrity of the lining system is considered using two primary mechanisms: relative shear displacements between materials at interfaces, and the maximum tensile strains (ε_{tmax}) in the geosynthetic components. To accomplish this, Fast Lagrangian Analysis of Continua (FLAC) software version 4.0.327 was used, because it can accommodate the large displacements and strains that are expected to occur in waste and lining systems. Moreover, FLAC can solve problems that consist of complex geometry and several construction stages, and also allows materials, interfaces and structural elements to be modelled relative to a non-linear or linear stress–strain law in response to applied forces and boundary constraints (Itasca 2000).

A code was written with the FISH programming language embedded in FLAC to automate Monte Carlo simulations and to record the liner responses to respective files. Parameters for a given component (e.g. waste or geosynthetic) were allocated a single value for each analysis, and therefore spatial variability within a material, such as the distribution of stiffness within the waste body, was not considered in this study. All key input parameters are treated as independent variables. Future studies should consider the dependence of groups of parameters such as interface friction and adhesion strength parameters, and unit weight and stiffness of waste.

The landfill model was made up of 85 × 36 zones, in which 30 × 50 zones represent the waste body and the remaining zones constitute the subgrade foundation, and are used to create interfaces between lining components. Interaction between the lining components was modelled using three interfaces: textured geomembrane against compacted clay (TGM-FINES); textured geomembrane against nonwoven needle-punched geotextile (TGM-NWGT); and nonwoven geotextile against gravel (NWGT-COARSE). The nomenclatures in brackets are used to define the three interfaces throughout the rest of the paper. The shear strength–displacement behaviour of each of these interfaces was measured in a laboratory testing programme, which also included repeatability testing to quantify variability, and the measurements are reported by Sia and Dixon (2007).

Each of the interfaces exhibits strain-softening behaviour, which was incorporated in the numerical model using the approach developed by Fowmes *et al.* (2006, 2008). Interfaces on the base and side slope were divided into 10 and 30 segments respectively, to incorporate strain-softening interface behaviour along their lengths. The geomembrane and geotextile were fixed in both the vertical and horizontal directions at the crest of the slope, which models an anchor trench with no slippage allowed, and hence produces maximum possible tensile forces and strains in the geosynthetics.

The waste was placed in six lifts 5 m thick, up to a height of 30 m. Side slope angles of 1:2 (26.6°), 1:2.5 (21.8°) and 1:3 (18.4°) were investigated, which produce slope lengths of 67 m, 80.8 m and 95m, respectively. Generally, slopes of this length would be constructed using benches to aid the deployment of geosynthetic materials and allow staged fabrication of the lining system. The implications of using this simplified slope geometry are discussed in Section 6. A landfill base length of 100 m was used in all analyses, as it has been shown to be sufficient to avoid the generation of interface slippage along the base (Jones and Dixon 2005). Further information on the modelling methodology is provided by Sia (2007).

4. SOURCES OF INFORMATION

4.1. Introduction

Input parameters required for the landfill numerical model are the properties of the waste material, interfaces and geosynthetic components. Four groups of input parameters are considered as random variables in the reliability analyses: the interface shear strength parameters (δ and α), the interface stiffness, the elastic moduli of the geomembrane and geotextile, and the unit weight of the waste. Parameters are represented by probability distribution functions, mean values and the coefficient of variation (i.e. standard deviation/mean) derived using a combination of data collected from a literature survey, the results from laboratory testing, and the subjective opinion of experts from a consultation exercise. Spatial variability of the random variables was not considered. For waste properties this is due to a lack of relevant measurements, and for the strength of interfaces it is a result of test limitations.

Each direct shear test used to obtain interface shear strength behaviour is carried out on a virgin geosynthetic sample, because the surface of a geosynthetic is modified during the shearing process (i.e. the surface of a geomembrane can be polished or roughened, and hence it cannot be reused). This means that the measured variability of interface shear strength is influenced by both material and test factors, and it is not possible to separate them. Exclusion of variability reduction due to spatial averaging over the area of an interface is expected to result in overestimation of failure probability. and hence the outputs of this study are likely to be conservative.

4.2. Subgrade and waste material input parameters

There is growing literature detailing studies of waste mechanical properties, and it at first appears feasible to use this to obtain statistical information on the variability of key waste properties. However, a more detailed analysis of published information demonstrates that the lack of a universal waste classification system to describe tested waste materials, and allow grouping of results from wastes of the same classification, means that it is currently not possible to attempt any meaningful analysis of this information without using significant engineering judgement. The study reported in this paper was carried out before the milestone event the International Symposium on Waste Mechanics, held in New Orleans in March 2008. The proceedings of the symposium have been published as an ASCE Geotechnical Special Publication (Zekkos 2011), which includes specific sections covering the key MSW engineering properties; however, it is still debatable whether there is currently adequate information to provide the statistical measures required.

Although the authors have extensive experience of assessing waste mechanics, it was considered preferable to use a wider group of experts to establish probability distribution functions and statistical descriptions for the key waste properties where there is a dearth of information in the literature. A consultation exercise (also called an expert elicitation) was carried out to inform the selection of waste properties. A total of 13 experts responded to the questions posed on probable uncertainty and distribution in the waste properties, landfill operating conditions and tolerances of construction. The experts were both identified from publications and nominated by their peers. During the collation process, the view of each expert was given equal weight, as they had similar years and areas of experience, and were asked to respond only to questions that were within their field of expertise. Detailed information on this consultation process is provided by Sia (2007).

Table 1 summarises the mean values of the input parameters for the MSW and subgrade materials. The subgrade was assumed to be strong, with a high stiffness, and is denoted as 'rigid'. It was assigned high bulk and shear moduli of 6.7e10 N/m^2 and 6.5e10 N/m^2, respectively, and a Mohr–Coulomb failure criterion was applied. Lining system response due to subgrade deformation can be excluded.

For the waste a modified Mohr–Coulomb model with a volumetric yield criterion (i.e. a double-yield model) was adopted in this study to account for permanent volume changes resulting from the application of stress. This is in contrast to the majority of landfill numerical modelling studies of wastes that have employed a Mohr–Coulomb failure criterion, owing to its simplicity. Waste parameters were obtained from the literature review and in part from the expert consultation, as detailed above. As plastic volumetric strains occur, the tangential bulk modulus (B_c) and shear modulus (G_c) are altered according to a law defined in terms of a constant factor R, where R is defined as a ratio of elastic bulk modulus to plastic bulk modulus with a value greater than unity. Additional material parameters for the MSW model were: the maximum elastic bulk modulus (B) and shear modulus (G), which are taken as upper limits to the tangential bulk and shear moduli; the cap pressure (p_c), which is related to the

Table 1. MSW and subgrade material properties, and their variability

Type	Unit weight (kN/m^3)				ϕ (degrees)	c (Pa)	B (Pa)	G (Pa)	p_c–e_{pv}	R
	Mean	COV (%)	Min	Max						
Subgrade: Mohr–Coulomb material model										
Rigid	26.5	–	–	–	28.0	2.72×10^7	6.70×10^{10}	6.50×10^{10}	–	–
Waste: double-yield model with constant strength										
Intermediate	9.9	10.3	7.6	12.5	31.1	2.5×10^3	5.97×10^5	6.51×10^5	[a]	10
Stiff	9.9	10.3	7.6	12.5	31.1	2.5×10^3	10.63×10^5	11.60×10^5	[a]	10
Compressible	9.9	10.3	7.6	12.5	31.1	2.5×10^3	5.10×10^5	5.56×10^5	[a]	10

[a]Relationships based on the constrained moduli values presented in Figure 3.

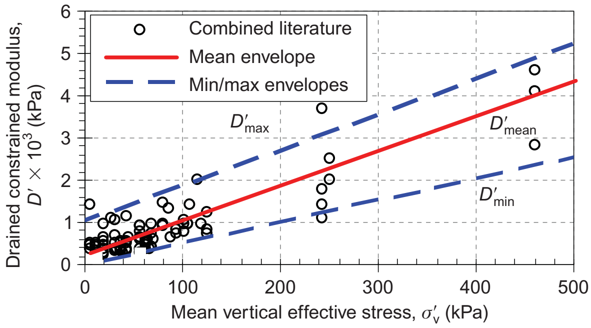

Figure 3. Relationships between constrained modulus and mean applied vertical stress used to derive parameters for waste material model (after Dixon *et al.* 2004)

plastic volume strain (e_{pv}) to represent the 'hardening curve'; and shear strength parameters ϕ_w and c_w. A full description of the double-yield model is provided by Itasca (2000).

Three types of waste with different compressibility were considered in the parametric study, denoted as compressible, intermediate and stiff. These are derived from laboratory and field measurements presented by Dixon *et al.* (2004). Input parameters B, G and $p_c - e_{pv}$ for the waste constitutive model were derived using the maximum, average and minimum drained constrained moduli (D'_{max}, D'_{mean} and D'_{min}, respectively), related to mean vertical stress as shown in Figure 3. The shear modulus of the waste was computed from the constrained modulus by approximating D' to 2.2G, taking the Poisson's ratio of MSW as 0.1 (Dixon and Jones 1998), and the bulk modulus was estimated using the elasticity law. In addition, an R of 10 was assumed, as advised by Itasca (2000), and the $p_c - e_{pv}$ relationship was back-calculated by ensuring that the difference in settlement output from a numerical model of a one-dimensional compression test using the double-yield model in FLAC and a one-dimensional settlement calculation using D' (Figure 3) was less than 5%. Although this approach includes several assumptions, such as the selection of a Poisson's ratio of 0.1, it produces a wide range of waste stiffness values that are consistent with both laboratory and field behaviour.

Based on the back-calculated $e_{pv} - p_c$ relationships, the total self-weight waste settlements obtained from FLAC compression tests for intermediate, stiff and compressible waste during filling to a height of 30 m were 27.9%, 16.1% and 50.0%, respectively. The settlement magnitude of 28% for intermediate waste is consistent with values reported in the literature for short-term compression under self-weight (e.g. Dixon *et al.* 2004; Oweis 2006).

A study by Jones (1999) has shown that waste shear strength does not significantly affect the deformation and stress along the side-slope lining system if the waste shear strength is greater than any exterior waste slope angle. As the model used in this study does not have an external waste slope, the shear strength parameter selection is not critical. Based on the literature and expert consultation, constant values $\phi_w = 31.1°$ and $c_w = 2.5$ kPa were taken, corresponding to high normal stress applications. Waste unit weight (γ_{waste}) was taken as a random variable in the Monte Carlo simulation, since it is known that it can have a significant influence on the interaction between the lining system and the waste (e.g. Jones and Dixon 2005). The γ_{waste} values are related to moderately compacted MSW, with a value for each simulation sampled from a normal probability distribution with a mean of 10 kN/m^3 and coefficient of variation (COV) of 19.4% (Table 1). The normal distribution employed and statistical measures were informed by the expert consultation, which further advised that sampled unit weight values outside the range of 6.9 kN/m^3 and 12.6 kN/m^3 should be discounted by truncating the distribution (Sia 2007).

4.3. Interface properties

Variabilities of shear strength for the three interfaces (TGM-FINES, TGM-NWGT and NWGT-COARSE) were obtained from a repeatability testing programme, which comprised 15 tests at each of four applied normal stresses, ranging from 11 kPa to 201 kPa, using a 305 mm × 305 mm direct shear apparatus (Sia and Dixon 2007). Interface strength parameters were treated as random variables in the Monte Carlo simulations, defined using normal distributions (Sia and Dixon 2007) with adhesion (α) generated from a four-parameter beta distribution to avoid extreme values (e.g. negative values), as detailed by Sia (2007).

Table 2 summarises the interface shear strength behaviour in terms of minimum, mean, maximum and COV values for Coulomb failure envelope parameters adhesion (α) and friction angle (δ). Interface adhesion and friction angle values are presented for specific shear displacements, as this information is required to define the shear strength behaviour of the interface, which limits the shear stress that can be mobilised along a strain-softening interface. The values in Table 2 were generated using numerical simulations of direct shear tests, and were validated to ensure that the shear stress displacement curves could capture the variability exhibited by the repeatability laboratory tests. The variability in the repeatability data sets is small, such that the probability of sampling negative interface shear strength is less than 0.01%, and therefore truncation of the generated parameter distributions was not warranted. However, in cases where greater variability exists, the strength parameter distributions should be truncated (e.g. ±2 standard deviations) to limit the influence of unrealistic extreme values on calculated probabilities of lining behaviour.

Interface shear stiffness was also taken as a random variable, as it is a significant parameter used to compute the mobilised stresses and displacements along the liner. Statistical information for secant interface shear stiffness (k_s), corresponding to a normal stress of about 200 kPa, are presented in Table 3. The parameters in Table 3 were obtained using peak shear strengths and associated displacements derived from the repeatability direct shear testing programme. These measured values include contributions from geosynthetic material shear behaviour, but the influence on interface shear stiffness values used in this study is not considered to be significant, owing to the small shear strains generated in these thin sheet geosynthetic materials. The normal interface stiffness (k_n) was

Table 2. Parameters defining interface shear strength behaviour, and their variability at specified shear displacements

Type	Interface friction angle, δ (degrees)						Interface adhesion, α (Pa)	
TGM-FINES	At 0.5 mm	At 3.5 mm	At 7.5 mm	At 15 mm	At 30 mm	At 60 mm	At 6.6 mm	At 9.8 mm
Mean	14.0	18.9	21.4	19.8	14.0	12.0	3111	2175
COV (%)	0.6	3.6	6.1	7.4	6.4	5.1	16.9	8.7
Min	13.8	17.3	18.7	14.9	11.9	10.4	2126	1699
Max	14.2	21.0	25.6	23.3	16.0	13.4	4491	2647
TGM-NWGT	At 0.8 mm	At 3.0 mm	At 6.5 mm	At 20 mm	At 42 mm	At 80 mm	At 7.3 mm	At 10.9 mm
Mean	11.5	20.0	25.8	17.5	14.0	11.9	4340	3699
COV (%)	0.6	1.9	4.0	4.0	3.4	3.2	5.8	5.1
Min	11.3	19.0	22.8	15.8	12.7	10.8	3748	3211
Max	11.7	20.9	28.6	19.3	15.2	12.9	4950	4165
NWGT-COARSE	At 1.5 mm	At 6.5 mm	At 12 mm	At 30 mm	At 50 mm	At 80 mm	At 28.6 mm	At 43.0 mm
Mean	17.0	31.0	34.0	31.7	30.4	28.9	2430	1663
COV (%)	0.3	1.2	2.3	3.2	4.7	5.8	16.3	26.9
Min	16.9	30.1	31.6	28.6	26.2	24.9	1275	390.8
Max	17.1	31.9	37.1	35.0	34.4	33.0	3359	2715

assigned as 1.1×10^7 N/m, which was 10 times the equivalent shear stiffness based on the recommendation in the FLAC user manual to minimise interpenetration between interfaces without invoking a high number of time steps to reach an equilibrium. The parameter was taken as a constant value throughout the series of simulations, because Jones (1999) demonstrated that different values of k_n have no significant effect on the mobilised stresses and displacements along the interfaces.

4.4. Geosynthetic input parameters

The axial tensile properties of geosynthetic components are required to assess the integrity of liner systems. In the landfill numerical model, the geosynthetic lining components were represented with linear elastic beam elements. A hyperbolic pre-yield stress–strain relationship could be employed to provide a better fit with the measured tensile behaviour, and hence produce less conservative strains, but the added sophistication was not considered warranted, given the relatively small strains (i.e. significantly below yield) anticipated under the conditions modelled. The input parameters required for beam elements include the cross-sectional area, elastic modulus, second moment of area and plastic moment. The second moment of area and plastic moment were set to zero, since sheet geosynthetics used in lining systems are not required to resist moments. For plane-strain conditions, the cross-sectional area per metre width is equivalent to the thickness of the geosynthetic. The tensile elastic properties of the geomembrane and geotextile were obtained from tests carried out by external commercial testing laboratories.

A conservative approach was adopted, using the secant elastic tensile modulus (E_s), defined as peak yield tensile strength over corresponding strain. Elastic tensile moduli of the geomembrane and geotextile were taken as random variables. Statistical measures of the parameters are derived from 49 uniaxial tensile tests for the geomembrane and 35 wide-width tensile tests for the geotextile using log-normal and normal distributions, respectively (Table 4). The compressive behaviour of the geosynthetic materials has not been measured. It is related to confining stress, and is influenced by complex mechanisms such as formation of wrinkles. Therefore compressive stiffness has been set equal to tensile values, and the computed compressive strains are unreliable.

5. ANALYSES AND PROCEDURES FOR MONTE CARLO SIMULATION

The structure for conducting multiple numerical realisations is outlined in Figure 4. Sampling values of the parameters were generated based on the assigned prob-

Table 3. Interface secant shear stiffness parameters, and their variability

Interface type	Secant interface shear stiffness			
	Mean (Pa/m)	COV (%)	Min (Pa/m)	Max (Pa/m)
TGM-FINES	6.84×10^6	11.0	5.23×10^6	9.10×10^6
TGM-NWGT	1.29×10^7	7.2	1.07×10^7	1.59×10^7
NWGT-COARSE	9.60×10^6	8.3	7.84×10^6	1.22×10^7

Table 4. Secant tensile stiffness for the geosynthetics, and their variability

Type	Thickness (m)	Secant elastic modulus, E_s			
		Mean (Pa)	COV (%)	Min (Pa)	Max (Pa)
TGM	0.00200	1.78 × 108	14.8	1.28 × 108	2.85 × 108
NWGT	0.00796	2.83 × 107	13.5	1.42 × 107	3.77 × 107

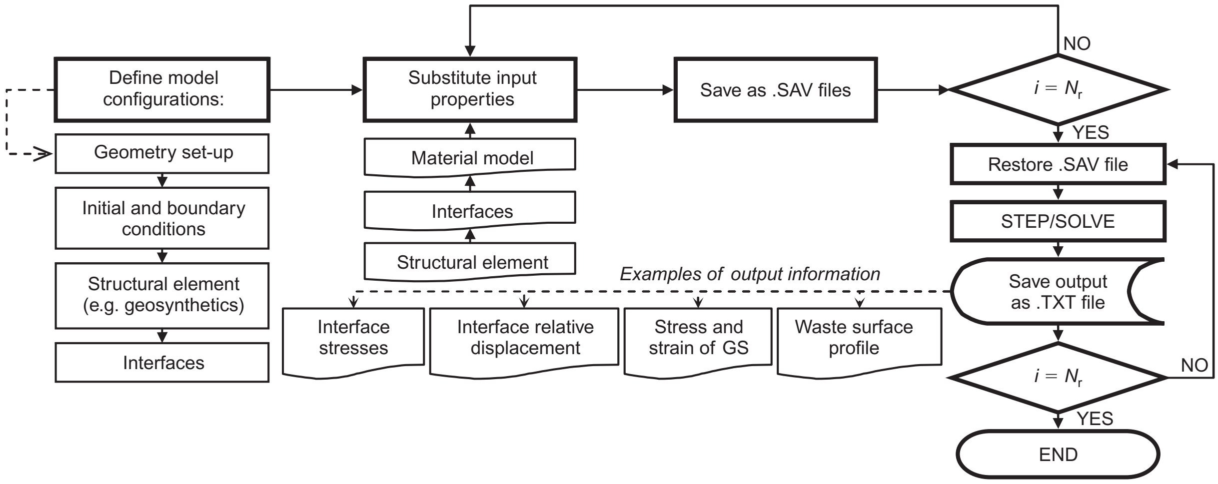

Figure 4. Structure for conducting multiple realisations in FLAC

ability distributions using the Monte Carlo method, with the sampling values generated using BestFit software. This enabled the generated sampling values to be examined for non-plausible values before commencing a series of numerical simulations (i.e. rather than using a FLAC FISH function with a built-in random number generator, where this process could not be followed). These sampling values were then copied in columns to an external input file for FLAC. Each row in an input file represented a material property value for one realisation. Once all the sampling values were assigned to the landfill model into separate FLAC files, each of these files was sequentially solved, the liner responses were recorded into external text files, and were these ordered into different folders. An Excel macro, encoded using the Visual Basic application in the built-in Visual Basic editor of Microsoft® Office Excel 2003 SP2, was used to automate extraction of the required information from all output files in each folder to the respective spreadsheets for assessment and presentation.

Each simulation case contains multiple realisations that allow the liner response to be presented in the form of cumulative distribution plots. In a cumulative distribution chart the horizontal axis represents the range of response values, and the vertical axis states the probability of samples having values less than or equal to the selected threshold response value. From this type of chart, the response value, say *X*, corresponding to a 95% probability of occurrence (*X*@95%) can be read off the chart by interpolation. A key output of the analyses is the relative shear displacement at each interface, as these control the shear strength mobilised, because post-peak displacement generates loss of strength in these strain-softening interfaces and tensile stresses in the geosynthetic components. Consideration of these mechanisms is central to the assessment of liner integrity. To aid the interpretation of relative shear displacement outputs a strength reduction (SR) factor can be used, defined as

$$\mathrm{SR} = \frac{\tau_p - \tau_i}{\tau_p - \tau_{LD}} \tag{1}$$

where τ_p is the interface peak shear strength, τ_{LD} is the interface large displacement shear strength, and τ_i is the post-peak shear strength mobilised between peak and large displacement. An SR value of 0 indicates that an interface segment has not exceeded peak interface shear strengths (τ_p) and an SR close to 1 implies that the interface shear strength of a segment has reduced to the large displacement shear strength (τ_{LD}). The cumulative probability for a strength reduction value relates to the percentage of numerical realisations in which all interface segments have mobilised shear strengths equal to or greater than the post-peak shear strength (τ_i) used to define that strength reduction value.

As the modelled responses of waste and lining system interaction are based on the Monte Carlo approach, the number of realisations can affect the final computed probability of occurrence. Low numbers of realisations are not representative of the probability distribution assigned to the input parameters, and do not cover the different combinations of scenarios that might occur. Conversely, high numbers of realisations require high computational time, are costly, and require large output storage capacity. Nevertheless, even if relatively low sampling values are applied, the results would straddle, or fall in some range, around the exact value (Baecher and Christian 2003). Therefore the outcome probabilities can indicate the adequacy of the number of realisations used, and hence can be used to justify the need for investment in additional analyses to improve the quality of the model outputs.

The effects of realisation numbers on modelled lining responses were investigated through an initial series of three sensitivity analyses in which the same problem was analysed using 250, 500 and 1000 realisations. Note that each realisation uses a different sampled value of the random variable. Full details of this assessment of the number of realisations are given by Sia (2007), and only a summary of the findings is provided here. Consideration of model outputs for strength reduction at the 95% level along the slope interfaces and the cumulative probability distribution of tensile strains in the geosynthetics both show that the outputs are consistent for 250, 500 and 1000 realisations (Figure 5). It can be concluded that 250 realisations of the landfill model used in this study are sufficient to demonstrate the impact of input parameter variability on the response of the lining system, and therefore 250 realisations are used for all analyses presented in this paper.

6. RESULTS AND DISCUSSION

6.1. Simulation cases and method of output presentation

Two types of model simulation have been carried out. First, a series of simulations were conducted to investigate systematically the sensitivity of the model outputs to specific random variables. Table 5 contains information on the simulation case labelling system, the model geometry (i.e. slope angle), and details of the random variables. Simulation cases A to E were all conducted with waste parameters for intermediate stiffness (i.e. intermediate waste settlements of 28%) and six waste lifts of 5 m each. Simulation F is the base case for the 1:2.0 slope, and treats all six listed parameters as random variables; cases B to E treat the parameter(s) in turn as constant values, and the remainder are random variables. A tick in Table 5 indicates that the parameter is a random variable in the simulation case, and a cross indicates that the parameter is a constant. Second, a parametric study was carried out by keeping the sampled values in each numbered realisation of each of the six parameters the same, while varying the model conditions of side slope angle, waste compressibility and thickness of waste lift. Details of parametric simulation cases G to K are presented in Table 6.

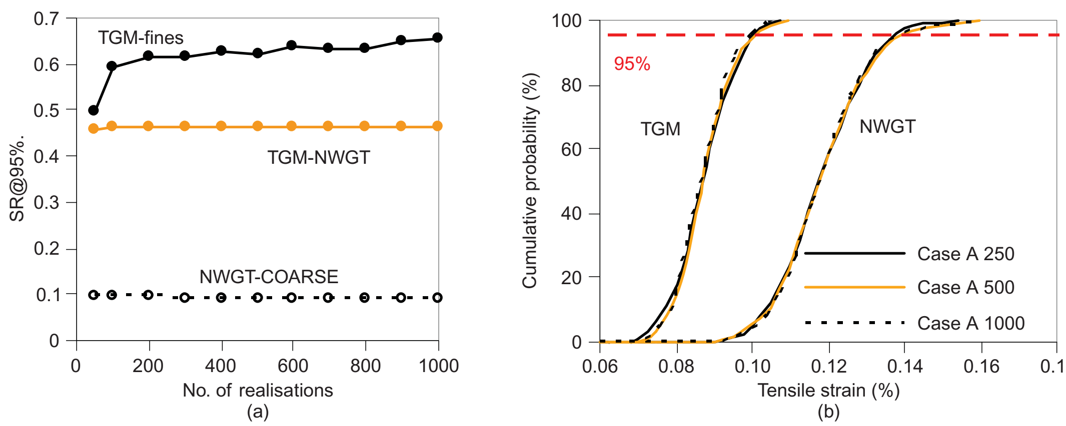

Figure 5. Influence of number of realisations for simulation case A after construction stage 6 on: (a) strength reduction at 95% of occurrence on side slope interface; (b) cumulative probability distribution of strain in geotextile and geomembrane for 250, 500 and 100 realisations

Table 5. Simulation cases: Sensitivity analyses

Simulation case	Random variables						
	Side slope	δ	α	k_s	E_{TGM}	E_{NWGT}	γ_{waste}
A	1:2.5	✓	✓	✓	✓	✓	✓
B	1:2.0	✓	✓	✓	✓	✓	x
C	1:2.0	✓	✓	x	✓	✓	✓
D	1:2.0	x	x	✓	✓	✓	✓
E	1:2.0	✓	✓	✓	x	x	✓
F	1:2.0	✓	✓	✓	✓	✓	✓

Table 6. Simulation cases: parametric analyses

Simulation case	Side slope	Waste stiffness	Waste lift thickness (m)	No. of lifts
A	1:2.5	Intermediate	5	6
F	1:2.0	Intermediate	5	6
G	1:3.0	Intermediate	5	6
H	1:2.5	Stiff	5	6
I	1:2.5	Compressible	5	6
J	1:2.5	Intermediate	10	3
K	1:2.5	Intermediate	2	15

Figures 6a, 6b and 6c show computed relative shear displacements for simulation case A (1:2.5 slope) along the TGM-FINES, TGM-NWGT and NWGT-COARSE interfaces, respectively, after completion of construction stage 6 (i.e. the waste has been placed to the final thickness of 30 m). Each of the three figures contains the outputs from 250 realisations, each realisation using a different set of selected random variables. The relative shear displacements along the base of the landfill are relatively small, being generally less than 5 mm, but along significant lengths of the side slope the computed relative shear displacements are between 10 and 20 mm. The relative shear displacements along the base are not sufficient to generate interface strengths that exceed peak values, and hence strain-softening does not occur. In contrast, the side-slope interface displacements are sufficient to generate post-peak interface strengths.

For each of the three interfaces, Figure 7 shows cumulative distribution curves for the strength reduction factors for the side slope, which are derived from the relative shear displacement data presented in Figure 6. In addition to the end of construction (stage 6), cumulative distribution curves are plotted for the completion of construction stages 4 and 5 (i.e. 20 m and 25 m thick-

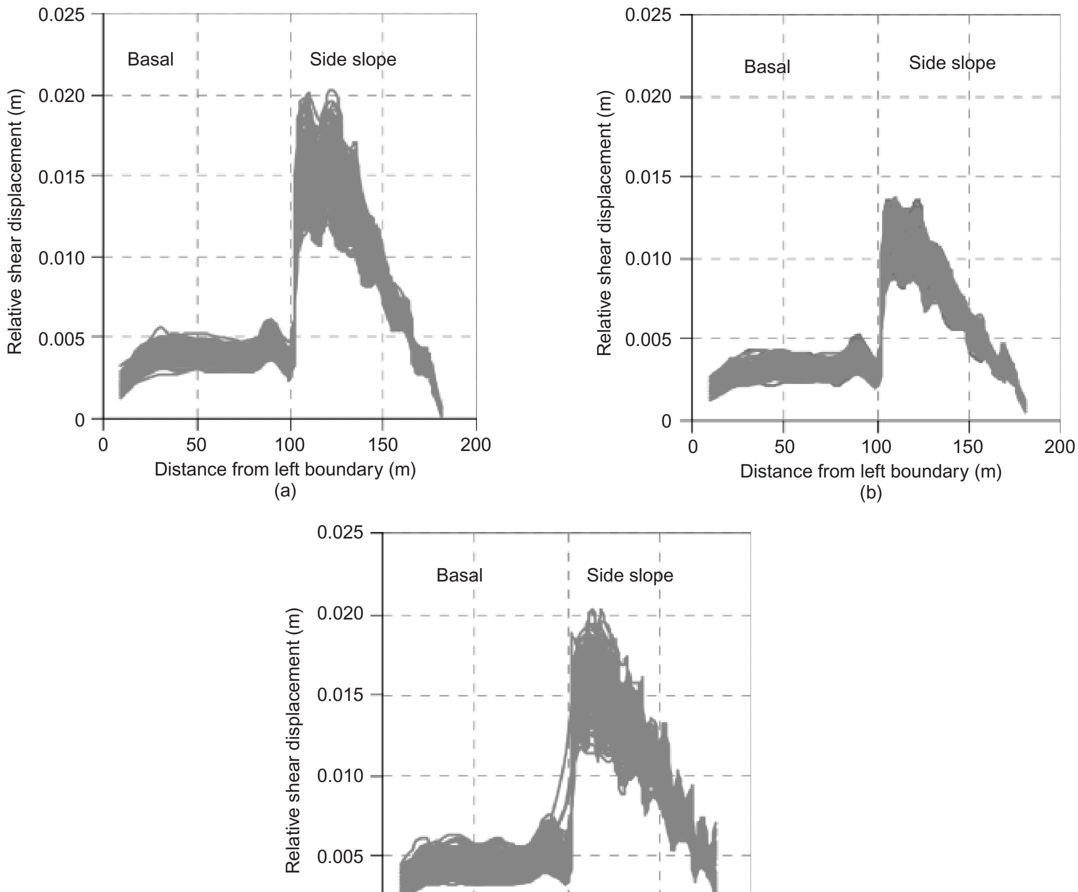

Figure 6. Relative shear displacement distributions for simulation case A after construction stage 6: (a) TGM-FINES; (b) NWGT-TGM; (c) NWGT-COARSE interfaces. Each plot contains results from 250 realisations

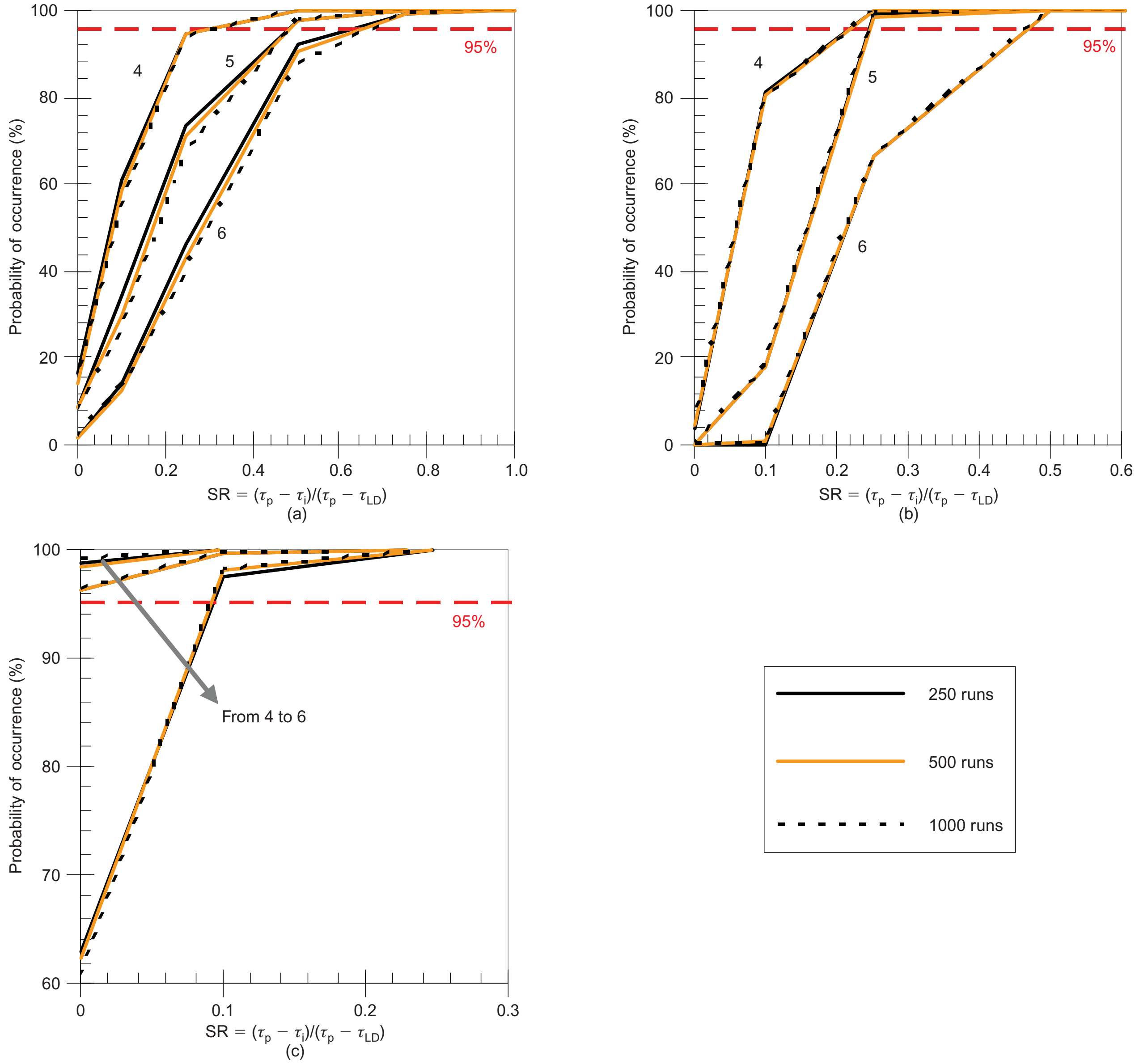

Figure 7. Strength reduction cumulative probability distributions along side slope for simulation case A after construction stages 4, 5 and 6: (a) TGM-FINES; (b) TGM-NWGT; (c) NWGT-COARSE

nesses of waste, respectively) to demonstrate the development of post-peak strength reduction along the side slope as waste is placed. Figure 7a shows that for the TGM-FINES interface, which is the weakest, 95% of the realisations for case A produce strength reduction values (SR@95%) increasing from 0.28 to 0.60 for construction stages 4 to 6, respectively. This means that 12 of 250 realisations have post-peak strengths along the TGM-FINES interface of less than $\tau_p - 0.6(\tau_p - \tau_{LD})$ at the end of construction stage 6. Interface TGM-NWGT has SR@95% values increasing from 0.25 to 0.46 for construction cases 4 to 6, and NWGT-COARSE has SR@95% values of 0 and 0.1. As none of the three interfaces has 100% probability that strength reduction values will be 1.0 after construction stage 6, it can be concluded that for case A the interfaces cannot mobilise large displacement shear strength along the whole length of the side slope. Figure 7 also presents results from analyses with 500 and 1000 realisations. The good agreement with the outputs from 250 realisations justifies the use of this lower number for the main study, as discussed in Section 6.

The tensile strains computed for the geomembrane (TGM) and nonwoven geotextile (NWGT) in simulation case A are shown in Figure 8. In only one of the 250 realisations is the tensile strain calculated to be greater than 1%; the remainder produce low strains in the order of 0.2%. A threshold of 1% for tensile strain was selected in this study, but alternative thresholds could be used in response to region-specific regulations and design practice. Any geosynthetic components that are positioned above the interface with maximum slippage (i.e. the TGM-FINES interface in case A) will display similar strain distributions: that is, the geomembrane (Figure 8b) and geotextile (Figure 8a), strain distributions are comparable. As discussed in Section 4.4, the compression stiffness parameters used are not reliable, and the analysis can identify only the zone of compressive behaviour, not the magnitude. A zone of compression strains is computed

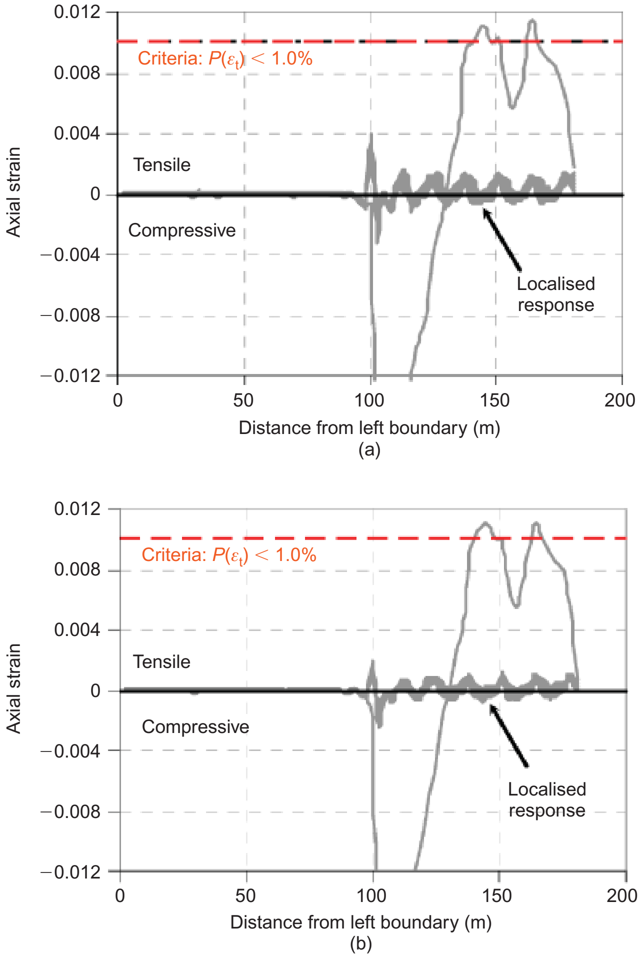

Figure 8. Tensile strain distributions for simulation case A after construction stage 6: (a) nonwoven geotextile; (b) geomembrane

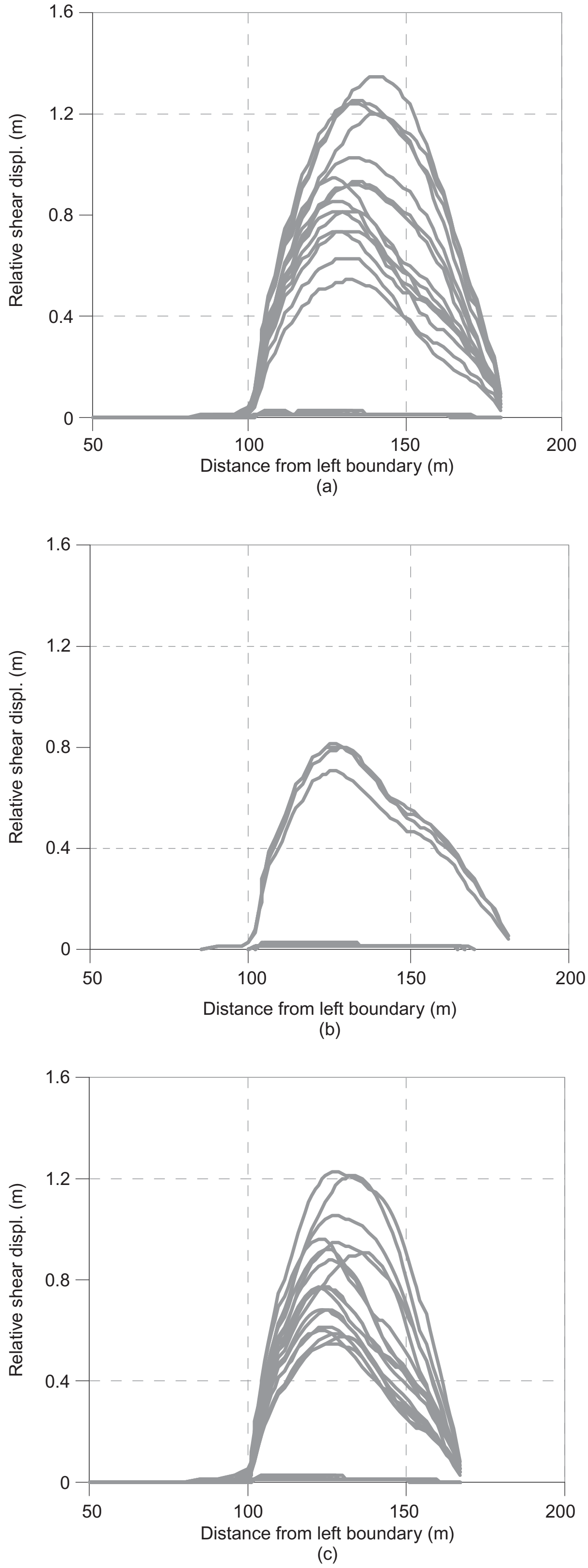

Figure 9. Relative shear displacement distributions along TGM-FINES interface after construction stage 6 for 1:2.0 slope: (a) case C; (b) case D; (c) case E

at the toe of the side slope. This is believed to be a function of the analysis method, linked to waste constraint in the region of the base/slope corner (Jones and Dixon 2005). Given the limitations in these calculated compressive strains, they have been excluded when constructing the cumulative probability distribution plots of maximum strains.

6.2. Sensitivity analyses

A series of four simulation cases (B, C, D and E) were conducted to examine the effect of random variables on the lining responses. In each simulation case, one input parameter of interest was kept constant throughout the 250 realisations, while the others were varied according to their assigned probability distributions (Table 5). The sensitivity analyses were carried out with the steepest side-slope geometry (i.e. 1:2.0), as Jones and Dixon (2005) state that this would be expected to produce the most critical liner response. Relative shear displacements greater than 500 mm occurred along the TGM-FINES interface in 19 of the 250 realisations in case F, which is for all parameters considered as random variables, and forms the base case for this sensitivity analysis.

Figure 9 presents the relative shear displacements for the TGM-FINES interface for cases C, D and E, which produced 15, 4, and 18 realisations, respectively, with side slope displacements greater than 500 mm. Not shown are the results from case B, which produced 13 realisations greater than 500 mm. It can be seen that setting each interface with a constant value of shear strength (case D) reduces significantly the number of realisations with large relative shear displacements. As large relative displacements between liner elements can lead to integrity failure of the system, this result indicates that consideration of the likely variability of interface shear strength is a key factor in landfill design.

Figure 10 depicts the Vern–Euler diagram representation of the realisation numbers that have high relative shear displacement along the TGM-FINES interface after construction stage 6. A set of numbers inside a closed shape are the realisation numbers that have maximum relative shear displacement greater than 100 mm for the simulation cases presented. The overlapping of two or more closed shapes indicates that the realisation numbers inside the overlapped area have failed in each of the simulation cases. Realisation numbers that are not enclosed by a given shape have relative shear displacement less than 100 mm for that specific simulation case. This form of output assessment is possible because each numbered realisation uses the same selected random variable value for each parameter in all of the sensitivity cases.

Figure 10 can be used to examine the factors, including combinations, that lead to large relative shear displacements at interfaces, and hence potentially to integrity failure of the lining system. As examples, for the 1:2 slope and intermediate waste stiffness, realisation number 16 of case C produced large displacements due to low mean shear stiffness, whereas realisations 40 and 57 of case D produced large displacements due to low mean shear strength assigned to the TGM-FINES interface. If the TGM-FINES shear strength increases by a relatively small amount from the mean value, as illustrated in Figure 11a for realisations 40 and 57 of case F, no large relative shear displacements occur at this interface. By inspection, 183 was the only realisation that failed in all simulation cases (i.e. B, C, D, E and F). Review of the input parameter values for this realisation shows that the high value of 12.0 kN/m^3 selected from the distribution for waste unit weight is the significant parameter that produces the large relative shear displacements in this realisation. This result is unlikely to represent landfill practice, as a high waste unit weight should be associated with high waste stiffness, rather than the intermediate waste stiffness used in this sensitivity analysis. This is a limitation of the modelling method, which does not treat stiffness and unit weight as dependent variables.

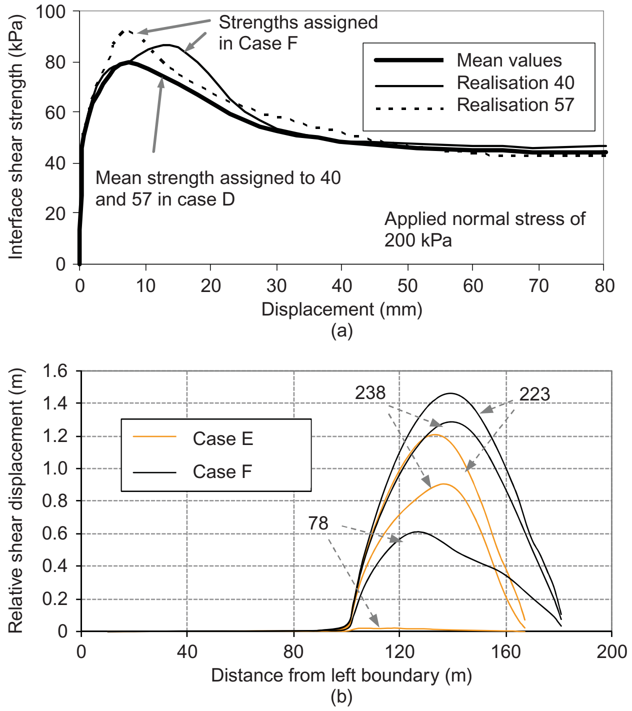

Figure 11. (a) Example variability of selected interface shear strengths in case F compared with mean strength used in realisations 40 and 57, case D; (b) comparison of relative shear displacement distributions for TGM-FINES interface computed in realisations 78, 223 and 238, cases E and F

Figure 11b demonstrates the lesser role of geosynthetic elastic tensile moduli in computed relative shear displacement. Large relative displacements along the TGM-FINES interface are computed in realisation 78 of case F when E_{TGM} and E_{NWGT} have selected values of 1.70×10^8 Pa and 3.05×10^7 Pa, respectively. However, no large displacements are produced in the same-number realisation of case E, even though the selected values of E_{TGM} and E_{NWGT} of 1.79×10^8 Pa and 2.83×10^7 Pa are comparable. Similarly, the relative shear displacement distributions along the TGM-FINES interface after construction stage 6 in realisations 223 and 238 of case F are higher than in case E, even though the only difference in input parameters between the two cases is the elastic tensile moduli selected for the geosynthetics.

Figure 10. Vern–Euler diagram representation of realisation numbers with maximum relative shear displacements greater than 100 mm along TGM-FINES interface for simulation cases B, C, D, E and F

Figure 12 presents plots of geomembrane and geotextile strain for 95% probability of occurrence at each construction stage for all the sensitivity cases. These demonstrate that the maximum strains, and hence a conservative analysis, are achieved by including the variability for all of the significant input parameters (case F). Figure 12 also shows that for the specific slope geometry and materials considered, it is placement of the final 5 m lift of waste, increasing the thickness from 25 m to 30 m, that causes the significant increase in geosynthetic tensile strains.

6.3. Parametric study: slope angle, waste stiffness and lift thickness

Figure 13 shows that relative shear displacements exceed 0.5 m along the TGM-FINES interface when the side slope inclination increases from 21.8° (1:2.5, case A) to 26.6° (1:2.0, case F). The probabilities of occurrence of strength reduction along the TGM-FINES slope interface are presented in Figure 14 for parametric simulation cases

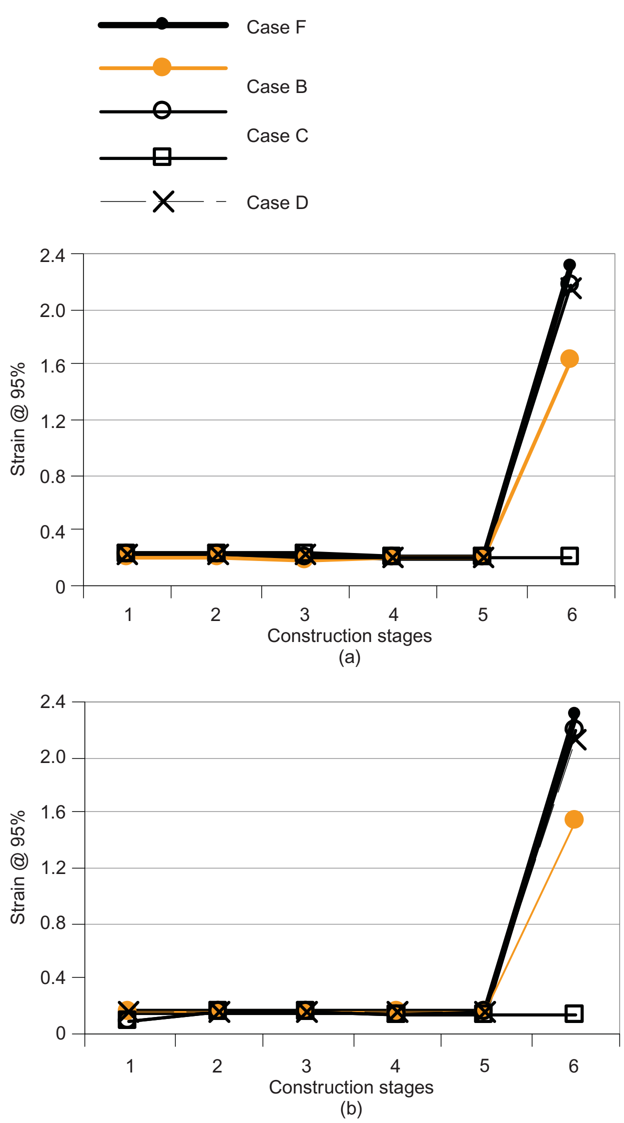

Figure 12. Influence of simulation case on geosynthetic tensile strain at 95% occurrence following completion of waste placement stages 1 to 6: (a) nonwoven geotextile; (b) geomembrane

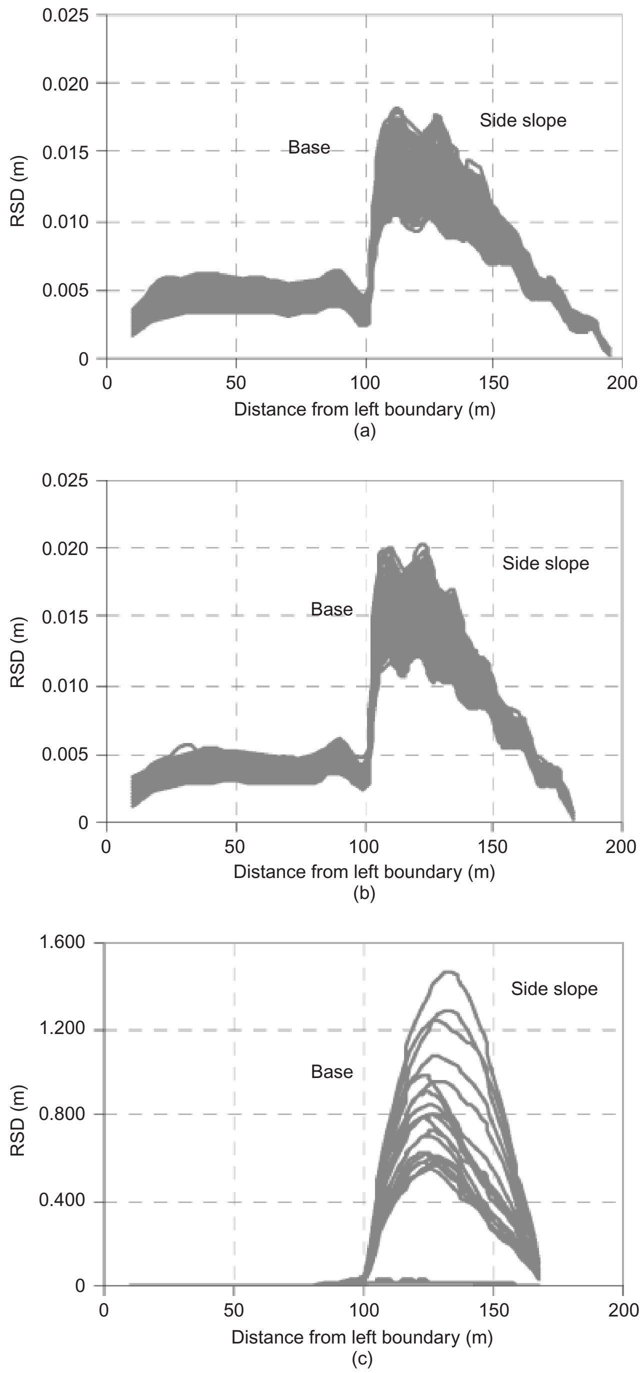

Figure 13. Relative shear displacement distributions along TGM-FINES interface after construction stage 6: (a) case G, 1:3.0 slope; (b) case A, 1:2.5 slope; (c) case F, 1:2.0 slope

A, F, G, H, I, J and K. Curves located to the left of base case A results indicate that lower strengths mobilised, and to the right that higher strengths are mobilised. Table 7 contains the strength reduction values for 95% probability of occurrence (SR@95%) for each simulation case, with values reported following each stage of construction (i.e. waste lift). The data show that higher strength reductions occur for steeper side slopes, stiffer waste and thicker waste lifts.

The phenomenon of larger interface relative shear displacements, and hence higher SR@95% values, associated with stiffer waste was also found by Reddy *et al.*

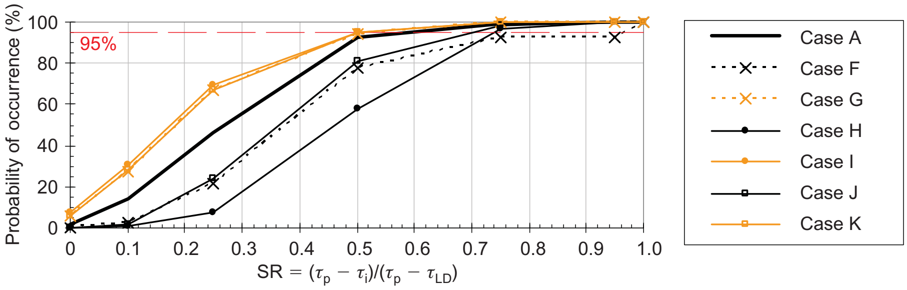

Figure 14. Strength reduction cumulative probability distributions for TGM-FINES interface after completion of construction stage 6: simulation cases A, F, G, H, I, J and K

Table 7. Parametric analyses: strength reduction values for 95% occurrence along weakest side-slope interface for stages of construction

Simulation case	Waste height (m)					
	30	25	20	15	10	5
A	0.60	0.47	0.28	0.10	0.00	0.00
F	0.97	0.63	0.43	0.22	0.00	0.00
G	0.54	0.39	0.21	0.07	0.00	0.00
H	0.74	0.62	0.45	0.20	0.00	0.00
I	0.50	0.34	0.14	0.00	0.00	0.00
J	0.70	–	0.23	–	0.00	–
K	0.50	–	0.00	–	0.00	–

(1996) and Jones (1999). Although no definitive explanation is currently available, it is thought that lower stiffness waste adjacent to the lining system results in preferential deformations in the waste body rather than at interfaces, thus reducing relative shear displacements, while stiffer waste results in lower deformations in the waste body and higher shear displacements on the interfaces. Field measurements are required to substantiate this hypothesis. However, it strengthens the need to consider waste compressibility as a random variable in design, especially since the parameter can have a large range of possible values, with Young's modulus of 500 kPa used by Jones (1999) and 8.0 MPa by Burlingame *et al.* (2007) in numerical modelling studies. Differences in relative shear displacements, and therefore SR@95% values, between simulation case K using 2 m and case J using 10 m waste construction lifts are relatively small. Therefore the selection of waste lift thickness should be based on expected site practice, and can be assigned a fixed value in the numerical model.

In the parametric simulations, only case F, which is the single case with a 1:2.0 slope, generates significant relative shear displacements at interfaces. The remaining cases generate only low values of geosynthetic tensile strains along the side slope. Tensile strains corresponding to 95% of occurrence (strain@95%) after construction stage 6 for the 1:2.5 and 1:3 side slope cases are in the range 0.10–0.24% for the nonwoven geotextile and 0.05–0.16% for the geomembrane, respectively. As highlighted in Section 3, uniform-angled slopes (i.e. without benches) have been modelled, leading to the later stages of waste placement on the 1:2.5 and 1:3.0 side slopes producing long slope lengths that are at the extreme of current practice. However, despite these severe conditions, the relatively low levels of strain are not large enough to raise concerns over the integrity of the lining system.

In contrast, tensile strains corresponding to 95% of occurrence (strain@95%) after construction stage 6 for case F with 1:20 side slope are in the range 0.9–5.5% for the nonwoven geotextile and 0.8–5.5% for the geomembrane, with 95% of all simulations exceeding the selected threshold of 1% tensile strain in a geomembrane. The large relative shear displacements, high strength reductions in mobilised strength at interfaces, and high generated geosynthetic strains for the 1:2.0 slope modelled raise concerns over the integrity of the lining system. The potential for large-scale slippage at interfaces, loss of continuous protection from the geotextile and stress cracking in the geomembrane means that these mechanisms must be considered as part of the design process.

7. CONCLUSIONS

Interaction between the waste body and multi-component side-slope lining systems can generate serviceability limit states. This paper details a study of waste–lining system interaction using a numerical model of a landfill with a 100 m base length and a constant-angle side slope, with waste placed across the full width of the cell in 5 m stages to a final thickness of 30 m. Implications of model parameter variability are investigated using Monte Carlo simulation. Key random variables are the interface shear strength and shear stiffness, geosynthetic tensile stiffness and waste unit weight. A parametric study has been used to investigate slope angle, waste stiffness and thickness of waste lifts. Probability distributions and statistical information on the mean and coefficient of variation of the random variables have been obtained from a combination of a literature review, a laboratory testing programme and an expert consultation exercise. Modelling outputs of liner response include relative shear displacements along the interfaces and tensile strains in the geosynthetic components. Also output are strength reductions along the strain-softening interfaces resulting from the relative displacements. In this study a threshold for acceptable performance of 95% probability has been used, based on Eurocode 7 (2004), which requires that no greater than 5% of adverse conditions should occur, but this can be adapted as required. It has been demonstrated that simulations using 250 realisations produce outputs comparable to those using 500 and 1000 realisations. Using this relatively low number of realisations produces significant time savings for each simulation case.

Sensitivity analyses in which each random variable was kept constant in turn have demonstrated that ignoring the variability of one or more of the significant input parameters would lead to unconservative, and potentially failed, designs. For example, the potential for relative shear displacements greater than 100 mm would be underestimated, and hence the implications of strength reductions at interfaces and tensile stresses in geosynthetics would not be fully considered. For geosynthetic components, the approach presented in this paper enables the probability of exceeding a limiting tensile strain to be evaluated (e.g. 1% for a geomembrane is used in this study) and the role of model input parameter variability to be assessed. It has been shown that, of the parameters investigated in the sensitivity analysis, variability of the interface shear strength has the most significant impact on the computed outputs.

The parametric study has established that relative shear displacements along the weakest interface are larger for landfills with steeper side slopes, stiffer waste, and thicker waste lifts. The variability of tensile strains in the geosynthetic components is more pronounced for steeper side slopes, more compressible waste and thinner waste lifts, although the last is not significant, given that numerical models can use lift thicknesses specified in practice. Outputs for the 1:2.0 side slope and specific lining components considered in this study indicate a high probability that large interface relative shear displacements and high geosynthetic tensile strains will occur. Deformations and strains of the magnitudes calculated can lead to integrity failure of the lining system, after waste placement. This study confirms the findings from previous numerical models of waste–lining system interaction with regard to the magnitude and distribution of interface displacements, and importantly it extends knowledge on the tensile strains mobilised in geosynthetic components. For the first time it provides quantitative information on the impact of parameter variability on the probability of occurrence of mechanisms that can lead to integrity failure of the lining system.

The analyses presented in this paper incorporate several simplifications. Spatial variability of the random variables has not been included, owing to a lack of data for the waste, and limitations and constraints in the direct shear tests used to measure interface shear strength. The side slopes do not include benches, thus forming long lengths with a constant angle that produce lining interaction conditions that are at the extreme, based on current practice. Investigation of side-slope geometries including benches would be a useful addition to the results from this study. In addition, all variables have been treated as independent. Future studies should consider defining the unit weight and stiffness of the waste, and the interface shear strength parameters α and δ, as dependent variables. These simplifications result in conservative outputs (i.e. higher displacements and strains). However, the analyses consider only short-term construction-related waste settlements under self-weight; long-term waste degradation and creep settlements will extend interactions with the lining system, and could lead to integrity failure post landfill closure. The impact of waste degradation should form the focus for a new study.

The model outputs presented in this paper are specific to the problem geometries, materials and properties investigated. The outputs provide insight into waste–lining system interaction relating to integrity, but care should be taken in extrapolating the findings to other systems. The analysis method employed is time consuming, and is unlikely to be appropriate for routine design, but there is merit in using the approach to guide design practice. The Monte Carlo simulation method presented in this paper has been used to investigate the mobilisation of interface post-peak shear strengths for a waste body with an external slope placed against a landfill side slope – a geometry that has been involved in several large-scale waste failures. The probability of generating post-peak strengths is being used to inform the selection of peak, large displacement, or somewhere in-between shear strength parameters for use in limit equilibrium analysis of stability. The outputs of the study will form a companion paper.

REFERENCES

Baecher, G. B. & Christian, J. T. (2003). *Reliability and statistics in geotechnical engineering*, John Wiley, Chichester, UK.

Burlingame, M. J., Egin, D. & Armstrong, W. B. (2007). Unit weight determination of landfill waste using sonic drilling methods. *Journal of Geotechnical and Geoenvironmental Engineering*, **133**, No. 5, 609–612.

Chalermyanont, T. & Benson, C. H. (2004). Reliability-based design for internal stability of mechanically stabilized earth walls. *Journal of Geotechnical and Geoenvironmental Engineering*, **130**, No. 2, 163–173.

Dixon, N. & Jones, D. R. V. (1998). Stress states in, and stiffness of, landfill waste. *Geotechnical Engineering of Landfills*, Dixon, N., Murray, E. J. and Jones, D. R. V., Editors, Thomas Telford, London, UK, pp. 19–34.

Dixon, N., Ng'ambi, S. & Jones, D. R. V. (2004). Structural performance of a steep slope landfill lining system. *Proceedings of the Institution of Civil Engineers – Geotechnical Engineering*, **157**, No. 3, 115–125.

Dixon, N., Jones, D. R. V. & Fowmes, G. J. (2006). Interface shear strength and its use in reliability-based landfill stability analysis. *Geosynthetics International*, **13**, No. 1, 1–14.

Duncan, J. M. (2000). Factors of safety and reliability in geotechnical engineering. *Journal of Geotechnical and Geoenvironmental Engineering*, **126**, No. 4, 307–316.

El-Ramly, H., Morgenstern, N. R. & Cruden, D. M. (2005). Probabilistic assessment of stability of a cut slope in residual soil. *Géotechnique*, **55**, No. 1, 77–84.

Eurocode 7 (2004). BS EN 1997-1:2004: *Geotechnical Design: General Rules*, British Standards Institution, London, UK.

Filz, G. M., Esterhuizen, J. J. B. & Duncan, J. M. (2001). Progressive failure of lined waste impoundments. *Journal of Geotechnical and Geoenvironmental Engineering*, **127**, No. 10, 841–848.

Fowmes, G. J., Dixon, N., Jones, D. R. V. & Cowland, J. (2006). Modelling of lining system integrity. *Proceedings of the 8th International Conference on Geosynthetics*, Yokohama, Japan, CD-ROM.

Fowmes, G., Dixon, N. & Jones, D. R. (2007). Landfill stability and integrity: the UK design approach. *Waste and Resource Management*, **160**, No. WR2, 51–61.

Fowmes, G. J., Dixon, N. & Jones, D. R. V. (2008). Validation of a numerical modelling technique for multilayered geosynthetic landfill lining systems. *Geotextiles and Geomembranes*, **26**, No. 2, 109–121.

Itasca (2000). *Fast Langrangian Analysis of Continua (FLAC) Manuals*, Itasca Consulting Group, Minneapolis, MN, USA.

Jones, D. R. V. (1999). *The Stability of Geosynthetics in Landfill Lining Systems*, PhD thesis, Nottingham Trent University, UK.

Jones, D. R. V. & Dixon, N. (2005). Landfill lining stability and integrity: the role of waste settlement. *Geotextiles and Geomembranes*, **23**, No. 1, 27–53.

Koerner, R. M. & Koerner, G. R. (2001). Geosynthetics design beyond factor of safety; risk assessment using probability of failure analysis. *Proceedings GRI-15, Hot Topics in Geosynthetics-II*, Houston, TX, USA, December 2001, pp. 235–253.

Long, J. H., Gilbert, R. B. & Daly, J. J. (1995). Effect of waste settlement on sloped lining systems. *Proceedings of Geosynthetics '95 Conference*, Nashville, TN, USA, pp. 729–744.

Oweis, I. S. (2006). Estimate of landfill settlements due to mechanical and decompositional processes. *Journal of Geotechnical and Geoenvironmental Engineering*, **132**, No. 5, 644–650.

Reddy, K. R., Kosagi, S. & Motan, E. S. (1996). Interface shear behaviour of landfill composite liner systems: a finite element analysis. *Geosynthetics International*, **3**, No. 2, 247–275.

Sabbatini, P. J., Griffin, M. L., Bonaparte, R., Espinoza, R. D. & Giroud, J. P. (2002). Reliability of state of practice for selection of shear strength parameters for waste containment system stability analyses. *Geotextiles and Geomembranes*, **20**, No. 4, 241–262.

Sia, A. H. I. (2007). *Landfill Lining Engineering Designs: A Probabilistic Approach*, PhD thesis, Loughborough University, UK.

Sia, A. H. I. & Dixon, N. (2007). Distribution and variability of interface shear strength and derived parameters. *Geotextiles and Geomembranes*, **25**, No. 3, 139–154.

Sia, A. H. I. & Dixon, N. (2008). Reliability-based design: veneer cover soil stability. *Geosynthetics International*, **15**, No. 1, 1–13.

Zekkos, D. (2011). *Geotechnical Characterization, Field Measurement and Laboratory Testing of Municipal Solid Waste, Proceedings of the 2008 International Symposium on Waste Mechanics*, Geotechnical Special Publication 209, ASCE, Reston, VA, USA.

The Editor welcomes discussion on all papers published in Geosynthetics International. Please email your contribution to discussion@geosynthetics-international.com by 15 April 2013.