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Modelling ultrasonic array signals in multilayer anisotropic materials using the angular spectrum decomposition of plane wave responses

Modelling ultrasonic array signals in multilayer anisotropic materials using the angular spectrum decomposition of plane wave responses

Yousif Humeida, Valerie J Pinfield, and Richard E Challis

Electrical Systems and Applied Optics Division, Faculty of Engineering, University of Nottingham, University Park, Nottingham NG7 2RD

Yousif.humeida@nottingham.ac.uk

Abstract. Ultrasonic arrays have seen increasing use for the characterisation of composite materials. In this paper, ultrasonic wave propagation in multilayer anisotropic materials has been modelled using plane wave and angular spectrum decomposition techniques. Different matrix techniques, such as the stiffness matrix method and the transfer matrix method, are used to calculate the reflection and transmission coefficients of ultrasonic plane waves in the considered media. Then, an angular decomposition technique is used to derive the bounded beams from finite-width ultrasonic array elements from the plane wave responses calculated earlier. This model is considered to be an analytical exact solution for the problem; hence the diffraction of waves in such composite materials can be calculated for different incident angles for a very wide range of frequencies. This model is validated against experimental measurements using the Full-Matrix Capture (FMC) of array data in both a homogeneous isotropic material, i.e. aluminium, and an inhomogeneous multilayer anisotropic material, i.e. a carbon fibre reinforced composite.

1. Introduction

Fibre reinforced composites (FRC) are being used extensively for a wide range of applications; that is mainly because of their high strength and stiffness-to-density ratios and their reduced maintenance and life cycle costs. As non-destructive testing (NDT) techniques for examining such materials have been stretched to the limits, a need to develop new techniques to test and examine their integrity and reliability has emerged. Ultrasonic testing is the most commonly used and widely successful technique in the industry for testing composites for reasons of cost, safety, and ease of deployment.

Ultrasonic phased arrays have recently been replacing the use of conventional single element ultrasonic transducers in many applications. That is largely attributed to the advantages that ultrasonic arrays offer such as the potential for better sensitivity and coverage area. Holmes et al [1] and Drinkwater and Wilcox [2] have been working on the development of array techniques for NDE applications. Their philosophy was to record a full set of all time-domain data from all the different elements of the array. This set of data, which is referred to as the Full Matrix Capture (FMC), is utilized for offline post-processing. Furthermore, they demonstrated that, by testing and assessing different types of post-processing techniques, an algorithm called the Total Focusing Method (TFM) produces the highest possible imaging resolution for inclusions and scatterers in some materials, such as metals. It is expected that by using altered versions of the above imaging algorithms for testing thick-section composite materials, significant results will be achieved. However, there is a need for a

forward model to verify the developed algorithms. Modelling ultrasonic signals in composite materials, compared to that in metals, is a very difficult and complicated task. Composite materials, such as FRP, usually have very complex structure and they are generally modelled as inhomogeneous and anisotropic multilayer assemblies.

Plane waves in multilayer media can be simulated using techniques such as the transfer matrix method [3, 4], the global matrix method [5], and the stiffness matrix method [6]. These theories are used to study the propagation of plane ultrasound waves of infinite lateral extent in multilayer materials. They do not accurately reflect real scenarios where the transducers used for transmitting/receiving the ultrasonic waves are of finite dimensions and hence generate bounded beam acoustic signals. Therefore, it is necessary to develop techniques for modelling the response of transducers of finite dimensions. Different methods can be used to model diffraction of bounded beams; the Angular Spectrum method is one of them. It states that the beams at any finite aperture can be described as an infinite set of plane waves propagating in different directions in the spatial frequency domain. These plane waves can be used to evaluate the bounded beams from the finite aperture or transducer at any point [7, 8]. As an example, Rehman et al [9] used the angular decomposition of plane waves to calculate the reflection profile of bounded beams into anisotropic multilayer media.

In this paper, ultrasonic wave propagation in multilayer anisotropic materials has been modelled using plane wave and angular spectrum decomposition techniques. Different matrix techniques, such as the stiffness matrix method and the transfer matrix method, are used to calculate the reflection and transmission coefficients of ultrasonic plane waves in the considered media. Then, the angular decomposition technique is used to derive the diffraction of bounded waves. The current model builds on earlier work by introducing multiple transducer transmitter/receiver elements to simulate a (1D) transducer array, producing predictions of the time-domain signals at receiver elements in various transmit modes. This model is validated against experimental measurements using the Full-Matrix Capture (FMC) of array data in both a homogeneous isotropic material, i.e. aluminium, and an inhomogeneous multilayer anisotropic material, i.e. a carbon fibre reinforced composite.

2. Modelling plane waves in multilayer media

Let us assume that a bulk wave is already excited and travelling indefinitely in an infinite anisotropic material in an arbitrary direction $\vec{n}$. In this case, the equation for elastic motion will be [10]:

$$\rho \frac{\delta^2 u_i}{\delta\, t^2} = c_{ijkl} \frac{\delta^2 u_k}{\delta\, x_j \delta\, x_l} \qquad\qquad 1$$

Where u_i is the displacement vector, ρ is the density, and c_{ijkl} are the components of the stiffness tensor of the anisotropic material. A general solution for this wave equation can be given by the following monochromic plane wave displacements [10]:

$$u_i = P_i e^{\sqrt{-1}k(n_j x_j - vt)} = P_i e^{\sqrt{-1}(k_j x_j - \omega t)} \qquad\qquad 2$$

where P_i is the component of the displacement amplitude vector (the polarization vector), n_j is the component of the propagation direction (unit vector), and k and v are the scalar wavenumber and phase velocity, respectively. $v = \omega/k$, where ω is the circular frequency. $k_j = kn_j$ are the components of the vector wavenumber. The Christoffel equation can be used to calculate the different values of wavenumbers and phase velocities in the above equation and hence the different values of the displacements and stresses can be obtained.

If a bulk wave is travelling in an anisotropic medium (medium 1), interrupted by an interface, and then continues to travel in a different anisotropic medium (medium 2), as can be seen in Figure 1, *Snell's law* and the *continuity conditions* will govern the behaviour of the wave. Three different waves, one

quasi-longitudinal, and two quasi-shear, will refract and start travelling in medium 2 and three similar waves will reflect back and travel in medium 1. The *Snell's law* states that: if a plane wave is travelling from medium 1 to medium 2 through a perfect interface, firstly, the entire incident, refracted, and reflected waves will propagate in the same plane, parallel to the x_1 axis in this case. Secondly, all the projections of the wavenumbers of all these waves on the interface will be identical [10], i.e.

$$\left[k^{(I)} \sin \theta^{(I)}\right]^1 = \left[k^{(R)} \sin \theta^{(R)}\right]^1 = \left[k^{(T)} \sin \theta^{(T)}\right]^2 \qquad 3$$

Generally if the interface or the boundary is perfect and is bounding two solid media 1 and 2, the *continuity condition* states that the displacement vector u_i and the stress vector normal to the interface σ_{i1} have to be equal in the two sides of the interface. Yet for some special cases, some of the continuity conditions might not be applicable anymore. For example if one of the media is a nonviscous fluid, like water, it is found that the shear stresses vanish and the tangential displacements are not necessarily equal anymore. More information about bulk waves in anisotropic materials can be found in [11].
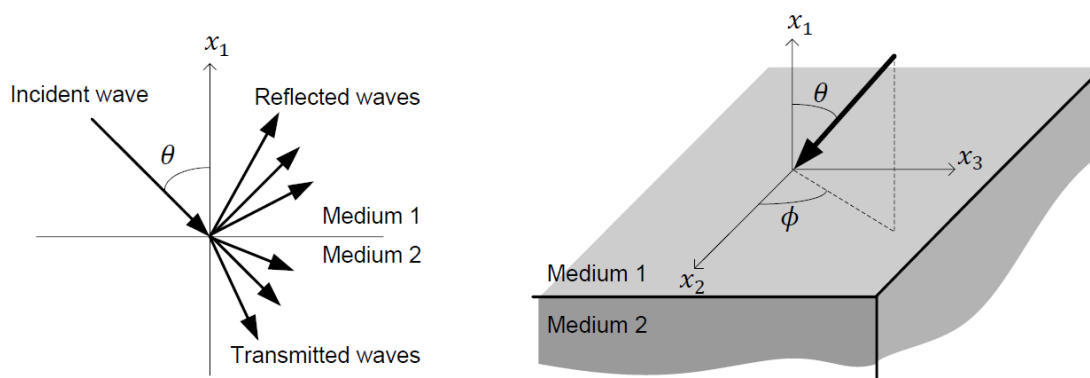

Figure 1. A plane wave in a finite anisotropic material

To study plane waves in multilayer materials, a plate consisting of an arbitrary number $n + 1$ of different layers which are perfectly bonded and stacked normal to the x_1 axis (parallel to the $x_2 - x_3$ plane), as shown in Figure 2, is assumed. This system consists of $(n - 1)$ different layers which are bounded with two semi-infinite layers (layer 0) at the top and (layer n) at the bottom. Each layer is assumed to have infinite length in both x_2 and x_3 directions and a finite thickness of d_k in the x_1 direction. A plane wave is assumed to be travelling in the upper half space towards the multilayer system in a propagation direction $\vec{n}$. This wave is assumed to enter the system at the origin of the global coordinate axes having an incident angle of θ with the x_1 axis and its wave vector projection on the $x_2 - x_3$ plane is assumed to have an arbitrary angle of ϕ with the x_2 axis, all illustrated in Figure 2. In this case, two of the three different k_2 and k_3 wavenumber components will already be known from the semi-infinite material by applying Snell's law and this equation will only be used to get the remaining wavenumber k_1 [12]. The problem of the wave propagation in anisotropic multilayer systems, such as the one shown in Figure 2, will be tackled at this point using two techniques: the transfer matrix method and the stiffness matrix method [6].

In the transfer matrix method, the stress and displacement at the bottom of any arbitrary layer m can be related to the stress and displacements at the top of that layer as follows:

$$\begin{bmatrix} u_i^m(d_m) \\ \sigma_{1i}^m(d_m) \end{bmatrix} = T_m \cdot \begin{bmatrix} u_i^m(0) \\ \sigma_{1i}^m(0) \end{bmatrix} \qquad\qquad 4$$

where T_m is the Transfer matrix for the layer m. Iterative relations can be used to relate the stresses and displacements at the top of the whole plate to those at its bottom.

$$\begin{bmatrix} u_i^{n-1}(d_{n-1}) \\ \sigma_{1i}^{n-1}(d_{n-1}) \end{bmatrix} = T_{n-1} \cdot T_{n-2} \cdot \ldots \cdot T_1 \begin{bmatrix} u_i^1(0) \\ \sigma_{1i}^1(0) \end{bmatrix} \qquad\qquad 5$$

On the other hand, if the stresses at the top and bottom of any specific layer have been related to the displacements at the top and bottom of that layer, a stiffness matrix is derived as follows:

$$\begin{bmatrix} \sigma_{1i}^m(0) \\ \sigma_{1i}^m(d_m) \end{bmatrix} = S_m \cdot \begin{bmatrix} u_i^m(0) \\ u_i^m(d_m) \end{bmatrix} \qquad\qquad 6$$

Where S^m is the Stiffness matrix in layer m.

Now a recursive technique can be used in order to calculate the total stiffness matrix that relates stresses and displacements at the top and bottom of a whole system of multilayer media.

Either of the transfer matrix or the stiffness matrix can be used to calculate the transmission and reflection coefficients for the whole multilayer medium.

The plane wave model has been implemented and provided to us by Prof Michel Castaings (I2M, Universite de Bordeaux I, France).
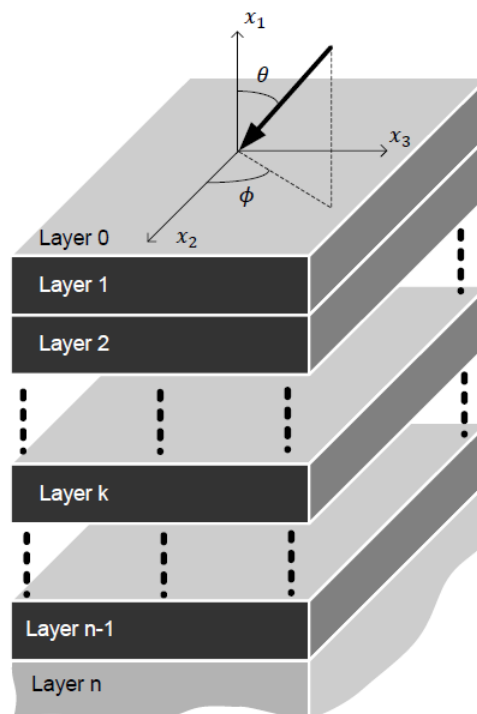

Figure 2. Plane wave with an arbitrary direction incident into the system of $n + 1$ different layers

3. Modelling bounded acoustic beams

A finite dimension transducer is used to transmit monochromatic acoustic waves in an infinite inviscid liquid where the face of the transducer is normal to the x_1 axis as shown in Figure 3. The transducer is assumed to be infinitely long in the x_2 direction while having a finite width of W in the x_3 axis direction. The finite transducer generates a longitudinal acoustic wave with a field $f(x_1 = 0, x_3, \omega)$ at its face that has a propagation vector $\vec{n}$ normal to the face of the transducer.

The angular spectrum method proposes that the Fourier transform can be used to generate the function $F(k_3, \omega)$ as follows [7]:

$$F(k_3, \omega) = \int_{-\infty}^{\infty} f(0, x_3, \omega) \, e^{-jk_3 x_3} . dx_3 \qquad 7$$

$F(k_3, \omega)$ can be interpreted as an infinite number of plane waves with different values of axial wavenumbers k_3 and describes the acoustic signal from the finite transducer in the spatial frequency domain.

Now the inverse Fourier transform can be used to obtain the space varying field at any point as shown by the following equation:

$$f(x_1 = -L, x_3, \omega) = \int_{-\infty}^{\infty} F(k_3, \omega) \, e^{j(-k_1 L + k_3 x_3)} . dk_3 \qquad 8$$
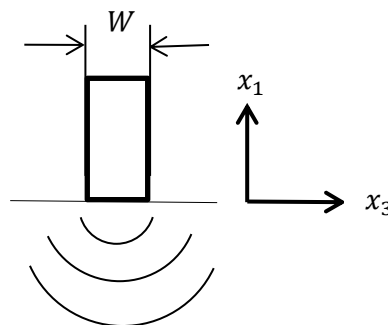

Figure 3. Finite dimension transducer transmitting bounded acoustic waves in an infinite inviscid liquid, the long axis of the transducer element is parallel to the $\boldsymbol{x_2}$ axis

For the purposes of validation, the results of the Angular Spectrum model were compared to the analytical solution of the Rayleigh-Sommerfeld integral which describes the problem of wave propagation of bounded beams in 2-D. More details for the derivation of this integral can be found in [7, 8]. The integral is given as follows:

$$f(x_1, x_3) = \frac{1}{j\lambda} \int_W f(x_1 = 0, x_3) \frac{e^{j\vec{k}.\vec{r}}}{\vec{r}} \cos\theta \, . dx_3 \qquad 9$$

4. Modelling ultrasonic array signals in different materials

The plane wave and the angular spectrum models described in previous sections can be used to model and simulate the time domain pulse echo signal transmitted by transducer 1 into a multilayer material and received by transducer 2 as can be seen in Figure 4

The plane wave model is used to calculate the fields reflected back from the material for a set of different frequencies and wavenumbers. This spectrum is then multiplied by directivity functions and

transducer transfer function in order to account for the beam spread and time domain impulse response for the two transducers, respectively. Then, the 2-D Fourier transform is used to convert these signals from the frequency and wavenumber domains to the time and space domains.

The directivity function D of a transducer of width W can be calculated analytically using equation 10 [1] whereas and for the purposes of this study transducer impulse responses has been determined experimentally.

$$D = sinc\left(\frac{\pi W \sin\theta}{\lambda}\right)$$

10

The same method can be used to simulate the ultrasonic response from the different combinations of transmit and receive elements of a 1-dimensional linear array coupled to a multilayer medium. Since most linear arrays consist of identical elements, a minimal number of calculations are needed to model a whole set of Full Matrix Capture data for these arrays. More details about this model can be found in [13].
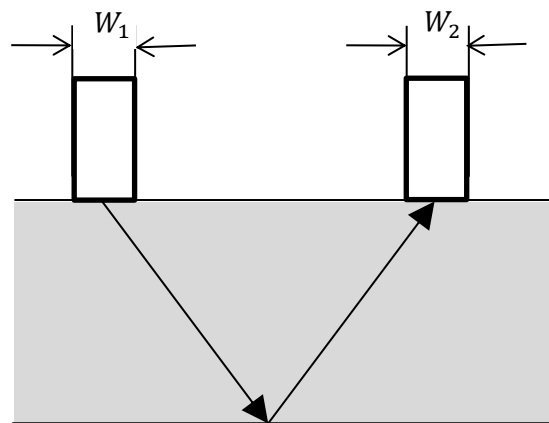

Figure 4. The experimental set up of two different transducers, one transmitting the ultrasonic signal into the material and the other one receiving it.

5. Materials and methods
Two different linear arrays were used in this study. Array (1) is a 64 element array with a central frequency of 5MHz, element width of 0.53mm, and element pitch of 0.63mm. Array (2) is a 64 element array with a central frequency of 2.5MHz, element width of 0.35mm, and element pitch of 0.5mm. Two different types of materials were used. A 30mm aluminium plate with a density of 2780kg/m^3, compressional wave velocity of 6290m/s, and a shear wave velocity of 3000m/s and a 16mm 128 layer carbon fibre reinforced composite with fibre orientations of (0º,45º,90º,-45º), repeated. Properties of individual layers in the composite can be found in [12]. Water has been used throughout all the simulation calculations as a semi-infinite bounding layer with a density of 1000kg/m^3 and compressional wave velocity of 1500m/s.

The model has been programmed using MATLAB. For all simulations presented in this paper sampling rates of 10MHz and 1mm^{-1} in the frequency and wavenumber domains, respectively, have been used.

6. Results
Figure 5 shows results for the calculation of reflection coefficients of a 6 MHz ultrasonic signal on an aluminium plate immersed in water. These results show instabilities in the transfer matrix method for the angles 23 up to 33 degrees whereas stable results are obtained for the stiffness matrix method and

confirming earlier findings, by other authors such as [14] and [15], that the stiffness matrix method is more efficient and stable than the transfer matrix method.
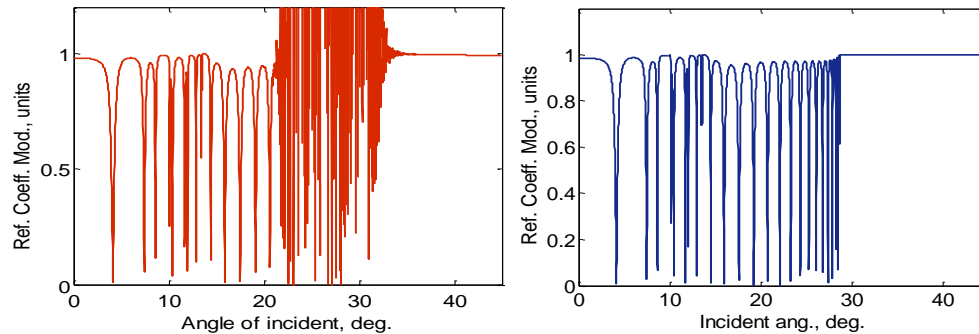

Figure 5. Reflection coefficient of a 6 MHz ultrasonic signal on an aluminium plate immersed in water using the transfer matrix method (top left) and the stiffness matrix method (top right).

Figure 6 compares the pressure profile in water at the x_1 axis (i.e. $x_2 = 0$) calculated by the Angular Spectrum method and the analytical model. The results in Figure 6 suggest that there is excellent agreement between the Angular Spectrum model and the analytical solution and hence the Angular Spectrum method has been chosen as an adequate method to model diffraction of bounded beams in multilayer materials.
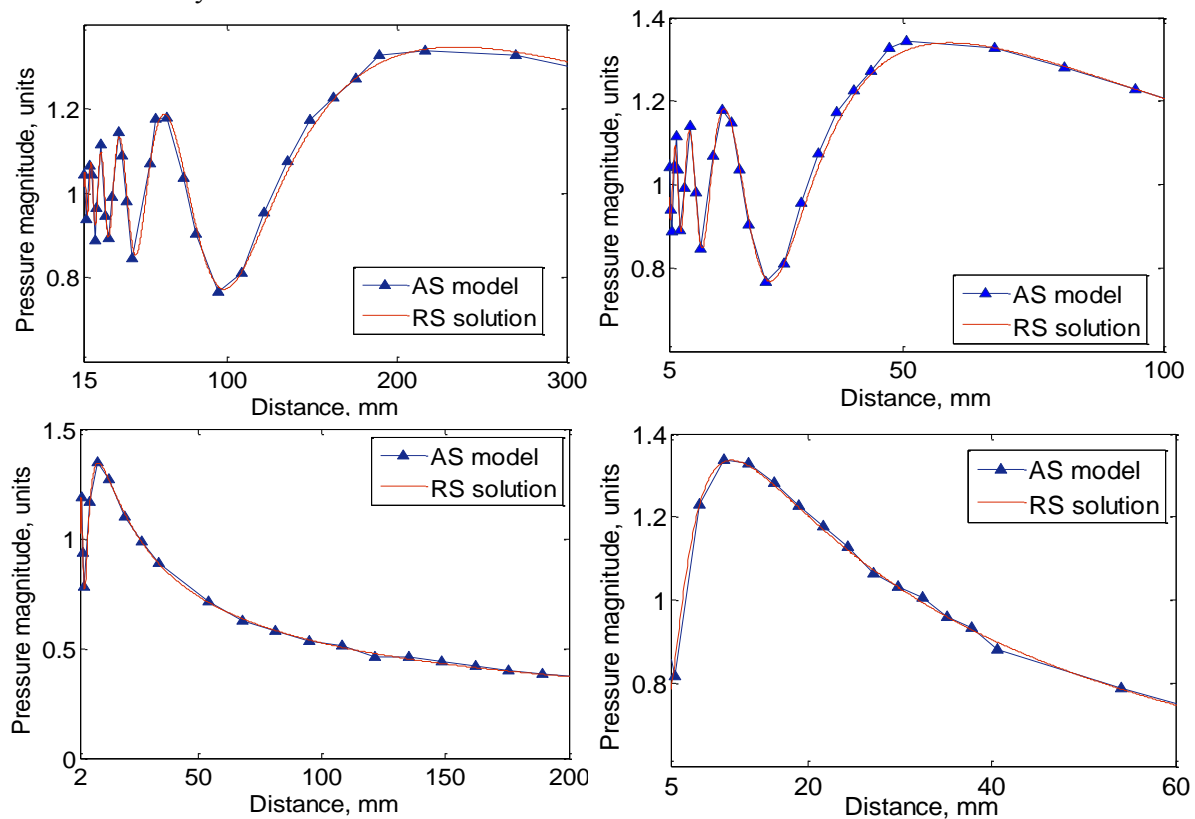

Figure 6. Comparison between the Angular Spectrum (AS) model and the Rayleigh-Sommerfeld (RS) solution for the pressure magnitude in water at the x_1 axis. The pressure was calculated for the following transducer width and frequency values: 10mm, 10MHz (top left), 10mm, 2MHz (top right), 5mm, 10MHz (bottom left), 5mm, 2MHz (bottom right).

The FMC data of array (1) has been calculated for the aluminium plate. Simulations have been compared with experimental measurements as can be seen in Figure 7, which shows signals transmitted by element 1 in the array and received by element 1, 9, 17, 25, 33, 41, 49, and 57. The first pulse that arrives back at each element is the longitudinal wave reflection from the back wall of the aluminium and because the travelled distances increase as we start receiving from the elements further from the transmitting element (element 1), the times of arrival for these pulses increase as well. It is also clear that mode converted signals arrive after the initial back wall echoes, especially for signals received by elements far away from the transmitting transducer. These can be seen from the second set of arrival pulses which are either sent as compressional waves and reflected as shear waves or sent as shear waves and reflected back as compressional waves. Such waves are not present in normal incident signals where mode conversions do not occur.
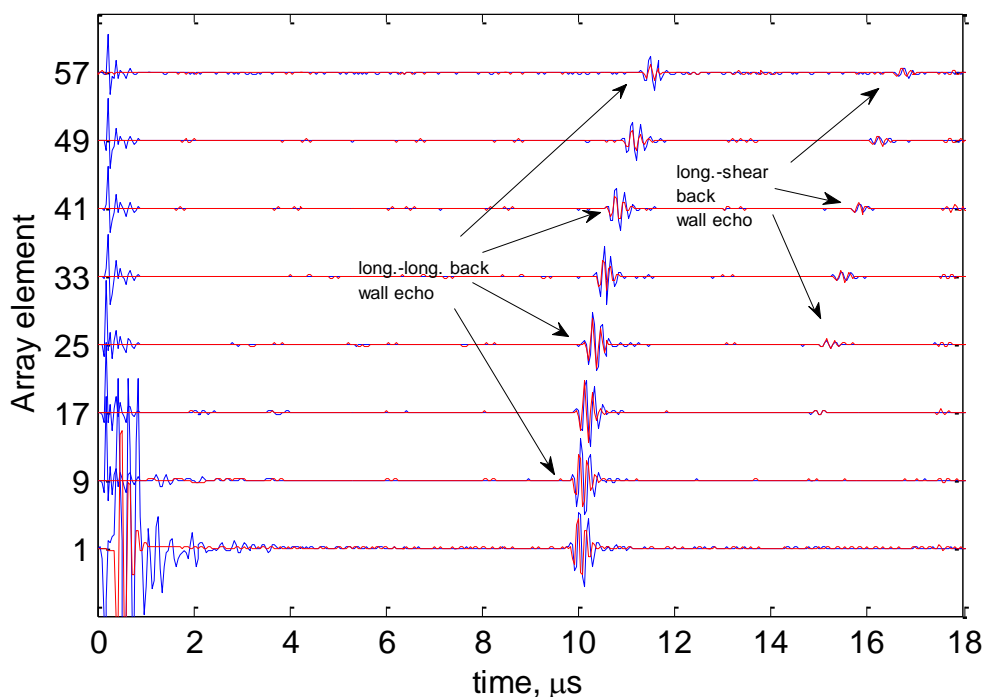

Figure 7. FMC in aluminium calculated using model (red) and experiment (blue)

The FMC data of array (2) has been calculated for the composite plate. This time, signals have been sent from element 1 and received by elements 2, 4, 6, 8, 10, 12, 14, 16, 18, and 20. Simulations have been compared with experimental measurements as can be seen in Figure 8. Two main characteristics in these signals are clear from the figure, namely the back wall echo signals and the back scattered signals from inner interfaces between the top and bottom surfaces. Attenuation in the composite sample is much higher than the aluminium sample; hence multiple reverberations of the back wall echo cannot be detected using these array signals. Both Figure 7 and Figure 8 show good agreement between the simulations and experiments when both pulse amplitudes and times of arrivals are compared. The small discrepancies in both can be attributed to the use of potentially inaccurate material properties for both samples (aluminium and composite) and possibly for bounding materials where gel has been used for the experiments yet water was considered in the simulations. Another source of discrepancies and inconsistency between the experiments and simulations might be the experimental set up itself where very thin gel layers are used to couple the ultrasonic signals into the materials, causing transducer beam deformation and the introduction of some shear waves.
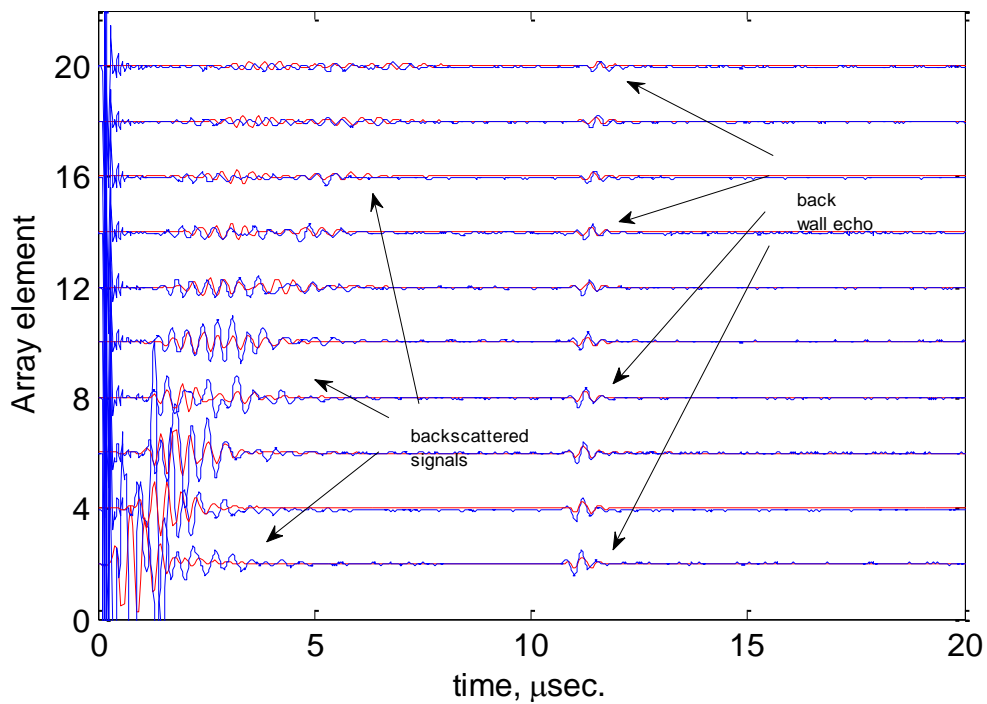
Figure 8. FMC in composite calculated using model (red) and experiment (blue)

The group velocity of the longitudinal wave in the aluminium and composite samples can be calculated, using simulations and/or experiments, from the time measurements of the back wall echo of the FMC signals from Figure 7 and Figure 8 as can be seen in Figure 9.
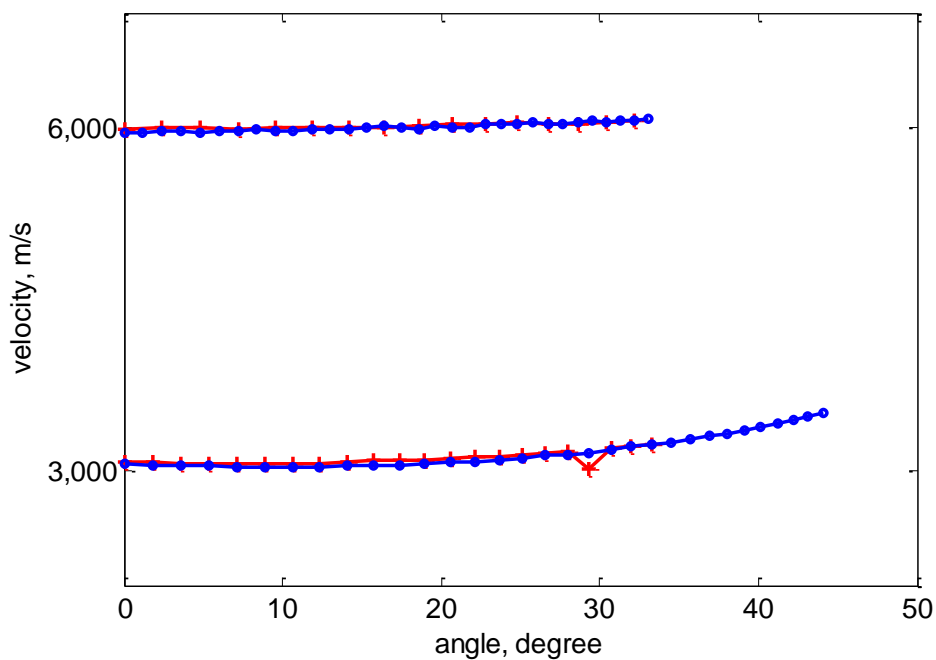

Figure 9. Longitudinal group velocity in aluminium (top curves) and composite (bottom curves) calculated from simulated (red) and experimental (blue) FMC data

7. Conclusion

The motivation behind this work was to develop a model for the propagation of ultrasonic array signals in multilayer anisotropic media. It has been demonstrated in this paper that the plane wave and angular spectrum modelling techniques not only provide a simple way of modelling the problem but also give very accurate results as shown in section 4. When multilayer systems such as FRCs are modelled using these techniques not only back wall echoes are simulated accurately but also backscattered signals from the layers in-between the front and the back wall. Such structural noise has to be considered especially when quantifying and analysing the quality of imaging techniques such as the TFM when they are used for such multilayer materials.

Only simulations in materials with planar surfaces and interfaces have been covered in this work. However, because of the accurate results that have been achieved, it is expected that this model will have significant potential in modelling and quantifying the effect of localised defects embedded within the body of a multilayer material.

8. Acknowledgement

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