Assumptions and Approximations Typically Applied in Modelling LV Networks with High Penetrations of Low Carbon Technologies

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***Abstract*— Uncertainties in the assessment of LV network capacity to accommodate PV and other low-carbon technologies can lead to installation constraints or costly network reinforcements that may not be entirely necessary. This paper reviews the numerous assumptions often used in such assessments and highlights those relating to time resolution of demand models, harmonics, network grounding and impedance modelling as being particularly questionable. In many cases, the individual assumptions may be low risk, but there is greater uncertainty when assumptions are applied in combination.**

Keywords-component; network simulation, harmonics, photovoltaics, reinforcement planning

I. Introduction

This paper considers the assumptions often used in low voltage (LV) network modelling, particularly power flow analysis or 'load flow' where the objective is to calculate voltages and currents within the network in response to a specified set of connected loads and generators. Other operational parameters determined can include unbalance, harmonic distortion, losses and thermal impacts.

Such modelling is often used in the assessment of networks to accommodate proposed PV and other low-carbon technologies and plays an important role in ensuring that network operational parameters are maintained within suitable limits. On the other hand, over-cautious assessment can lead to constraints being applied to the installation or operation of low-carbon technologies or to costly network reinforcement that may not be entirely necessary. The accuracy of such models is therefore an important matter.

This paper focuses on networks in which power is distributed around a local area at LV, as is commonplace in Europe. In a typical network, as described in [1], a primary substation supplies several medium voltage (MV) feeders, each routed to a number of LV distribution transformers. Power from these transformers is routed via underground mains cables or on overhead lines. Customer service connections are attached where required along the distribution route. The details of LV network design practices can vary significantly between different countries and even local regions with different operating companies and these variations present additional challenges in the modelling.

Some of the assumptions typically used in LV network modelling are adopted from experience of modelling higher voltage networks. Others may stem from limitations in data available to describe the actual network configurations, and the varying characteristics of the loads and generators that are connected. This paper aims to provide a general review of all assumptions and uses the following categories:

***Loads, generators and substation nodes*—** The characteristics of power import and export at the nodes on the LV network.

***Network topology*—** The connectivity between the nodes of the LV network. This covers the cables and overhead lines and the connections between them.

***Conductor impedances*—** The impedances of the cables or overhead lines between the junction nodes.

In each section, a summary table provides a list of assumptions, including references to examples where they have been applied. Brief comments are included to review the impacts of the assumptions on modelling results. The text then describes some of the more questionable assumptions which are reviewed in further detail. The paper concludes by identifying the modelling assumptions that appear to be the most critical for further evaluation.

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TABLE I. MODELLING ASSUMPTIONS FOR LOADS GENERATORS AND SUBSTATION NODES

Assumption	Review of risks and impacts
Customer demand is represented by national mean profiles [2].	Assuming a common demand profile for all customers may neglect regional variations, and differences between individual customers (e.g. for shift workers) or attitudes to energy use. ***See section II.A.***
Individual customer demand generated from statistical distributions [2].	Impacts of phase unbalance depend on adequate representation of the deviations of individual customer demands from the mean . ***See section II.A***
All PV installations on the LV network are subject to the same irradiance variation	Simulations of PV on an LV network showed significantly different voltage rise if the effect of cloud movement was applied to the irradiance data, rather than all customers having the same irradiance [21].The peak irradiance data were noted as occurring on cloudy days.
Commercial demand is not modelled [3]	Phase balance and loads may be inaccurate, due to differing demand and use of three phase supplies.
Loads on each phase are balanced [22].	If unbalanced networks are simulated as being balanced, neutral currents and losses will be under-estimated. Voltage extremes are under-represented if currents are averaged between phases [23]. ***See section II.B.***
Demand is unbalanced due to load variations but mean demand on each phase is balanced [1], [24].	Effects of unbalance on voltage range and losses are under-estimated if simulations only model scenarios with equal mean demand and generation on each phase, for example with customers allocated sequentially to each phase and having the same mean demand profile. ***See section II.B***
Loads and generation can be represented by time averaged samples [1], [2], [8].	If currents are averaged over too long a period, short term voltage deviations will not be represented and losses will be under-estimated.The proportion of exported power from generators will be over-estimated if demand from loads is time averaged [10]. ***See section II.C.***
Loads have constant power vs. voltage [1][8], or a constant current model [22] [9].	Risks in accuracies in customer node voltages, unbalance and neutral currents [13]. ***See section II.D.***
Generators have a constant power output vs. voltage variation.	Generators driven by renewable energy are commonly assumed to provide output power dependent on the renewable resource available, and that this is independent of the network voltage [22].
The network can be simulated as sinusoidal 50/60 Hz with no harmonics [24].	This neglects the voltage drops due to increased reactance at higher frequency, and assumes a greater cancellation of three phase currents in the neutral than will occur in practice. ***See section II.E.***
A constant power factor is assumed for loads, for example 0.9 as in [22].	Variation with customer loads and throughout the day would be omitted. All phases are balanced in terms of phase angle relative to the three phase voltage. The approach in [3] addresses this by assigning a power factor for each appliance. ***See section II.E.***
A constant power factor is assumed.for generators, typically unity [22].	A low risk assumption as this may be defined by grid connection regulations.
Non-metered demand such as due to street lighting is neglected [25].	Estimated losses and voltages would be inaccurate if the full demand is not modelled, for example if based on data from customer smart meters [26].
The distribution transformer (or primary substation, if the MV feeder is modelled) are a constant voltage source [9], [1].	The On Load Tap Changer (OLTC) at the primary substation maintains the voltage within a specified bandwidth, adjusted in fixed steps. Tap changes only occur if the bandwidth is exceeded for a defined time period. A voltage uncertainty of 2% has been allowed for the OLTC accuracy [27].

II. LOADS, GENERATORS AND SUBSTATION NODES

Table I lists assumptions relating to loads, generators and the substation.

A. Demand profiles

The variation of demand in time can be characterized according to standard load profiles. These profiles are based on many aggregated customers so do not reflect the stochastic variation of individual customer loads.

One approach to creating individual customer demand samples is to assume that a particular statistical distribution applies. In [2], a normal distribution was used with assumptions made for the standard deviation.

An alternative approach is to build up the customer demands based on individual appliance use, and then scale the total power to match known demand profiles [3]. The loads are based on statistics relating to active occupancy of homes and data describing the appliances. An assumption is needed to determine the correlation between daily occupancy patterns. Assuming the same profile each day ignores potential variations, but using an independent profile each day models every customer according to the same statistical distribution. In [3] the model provided good agreement with measured data but underestimated the low and high extremes of the average demand per customer.

B. Phase balance

If unbalanced networks are simulated as being balanced, neutral currents and losses will be under-estimated. Voltage extremes may be under-represented if currents are averaged.

With single phase service connections, the time varying characteristics of demand and generation cause currents in the three phase mains to be unbalanced. Where three phase connections are provided, as in Germany, high power heating loads will be balanced across all three phases, but lower power appliances are still on single phase circuits so some unbalance remains.

In addition to short term unbalance due to appliance activity, the mean demand on each phase may be unbalanced. This could be due to the fact that the network serves a mixture of residential and commercial customers with different demand profiles, or just due to the differences between customers. Single phase connections may not be equally shared between phases. A possible cause of this where 4-core main cables are used is that it is less easy to separate the bundle to select the core opposite the neutral.

Where customers have single phase connections, distributed generation is likely to be unbalanced, as installations build up in a randomized pattern on each phase. Where customers have three phase connections, smaller PV inverters still have single phase operation. Since all three phases are available, the phase allocation is selected by the installer. The balance of the aggregated generation depends on the evenness of these phase selection decisions.

Phase unbalance has been highlighted by results from a program of LV substation monitoring [4]. A distribution substation (on a feeder with a high penetration of PV) was monitored over two months, with mean currents on the three phases of 69 A, 99 A and 126 A. Similar results were found in [5] where the mean current unbalance was 27%, (ratio of peak phase to average phase current).

This suggests that modelling should include scenarios in which the mean demand and generation are unbalanced.

C. *Variation with time*

Detailed simulations often use a time step approach in which each sample represents the demand or generation over a fixed time interval. A high resolution is needed to represent 'spiky' demand characteristics. If currents are averaged over too long a period, short term voltage deviations will not be represented. Since power dissipated is proportional to the square of the current, losses are under-estimated if calculated using an average current.

The required time resolution could be determined by standards, for example where the power delivered to customers is required to conform to EN 50160. This defines that voltage magnitude, unbalance and harmonic distortion should be averaged over 10 minutes [6]. The resolution can also be considered in relation to the typical activity periods of appliances. Thermostatically controlled heating has a significant impact on residential demand models due to the high power required and short time periods. An example was given in [7] of a cooker hob on a low setting, modelled as a 2 kW load switched on for 30 s then off for 120 s.

Simulation models have used a wide range of time resolutions, including 1 minute [1], 15 minutes [8], and 30 minutes [2]. The impact of selecting different time intervals has been reviewed for periods of 1 to 30 minutes [9]. For a single customer, the maximum demand with 30 minute averaging was 16% below that for 1 minute samples. When the demand from 16 customers was aggregated (and therefore more balanced), there was only a 6% difference.

The proportion of energy imported for a house with a hypothetical constant power generator was reviewed in [10]. If the demand is smoothed by 30 minute averaging then it appears that on-site generation meets a greater proportion of the demand than if 1 minute samples were used.

D. *Load power variation with voltage*

Typical domestic appliances have been reviewed in [11] in which an aggregated residential load model is proposed. During most of the day, the demand is approximately 20% constant impedance, and 80% constant power. In the evening, the proportion with constant impedance increases to 40% due to resistive heating loads. At night, when the resistive power peaks are absent, the characteristic reverts to a constant power model. Work on conservation voltage reduction also suggests that a constant power model is not fully representative, with one study suggesting that demand reduces by 0.5% to 1% for a 1% reduction in voltage [12].

Simulations of loads with constant power and constant impedance were compared in [13]. The neutral currents and voltage unbalance for the constant power model were doubled compared to the constant impedance model. The end node customer voltage also varied by up to 7%.

A constant power model is probably not representative of real domestic appliances, and making this approximation does have impacts to the overall simulation accuracy.

E. *Harmonic distortion and power factor*

Analysis often uses voltage and current phasors, assuming sinusoidal operation with no harmonics. In practice, significant harmonic distortion appears to occur.

Harmonics from domestic appliances have been reviewed in [11], where a combined model of the appliances in a house includes 3rd and 5th harmonics at 20% and 8% relative to the fundamental. Tests of PV inverters indicate a lower current distortion, with Total Harmonic Distortion (THD) between 2.1% and 4.8% [14].

Monitoring at a UK distribution substation on a network with a high penetration of PV connected has shown voltage THD between 2% and 3.5% [4]. This also showed that current harmonics vary between the three phases, with up to 25%, 13%, 9% and 8% for the 3rd, 5th, 7th and 9th harmonics.

Currents harmonics propagate through the network, causing voltage distortion. Loads and generators may simply be treated as a source of harmonic currents, with no dependency on the voltage. Alternatively, loads could also accept active power at harmonic frequencies and act as a sink for the distortion.

The power factor allows for both distortion (since the average power delivered is zero if voltage and current have different frequencies) and reactance. Simulations at the fundamental frequency represent the power factor entirely as a phase displacement between current and voltage.

III. Network Connectivity

Table II lists assumptions related to the connectivity. The following sections provide further discussion on two assumptions that affect the network simulation method.

A. *Connectivity of Neutrals and Ground*

Where the currents in the three phases are unbalanced or include harmonics, currents will flow in the neutral

TABLE II. Modelling Assumptions for Network Connectivity

Assumption	Review of risks and impacts
Cable types, routes and connectivity are as described in the network database.	There is relatively high confidence in the database for HV networks, but the accuracy of data describing the LV network is less certain. In [24] service cables were approximated by straight line routes from the house centre to the nearest point on the LV main.
Phase allocations for customers with single phase phase supplies are known.	Where customers are provided with single phase service connections, the records of phase allocations may be missing or incorrect, so that the current balance between phases may not be correctly modelled.
Service cables are omitted from the model, as in [2], [9].	The impact of neglecting the voltage drop in the service cable may be minimal for customers in urban areas, but could be greater for older installations in rural areas.
Neutral conductors, concentric neutral or sheath are grounded at each node.	Risk of inaccuracies in voltage calculations, proportion of current in neutral and ground calculations, and therefore for losses in neutral conductors, if ground connections are not as modelled. ***See section III.A.***
Neutral and earth connections at link boxes can be ignored, as in [1], [28].	Simulations would not include circulating currents that may exist bewteen separate LV network branches, even though the branches are radial for phase conductors. ***See section III.B.***

conductor. The accuracy of simulations of network voltages therefore depends on the modelling of the routes available for the neutral current to flow back to the sub-station.

As described in section IV.D, it is common practice to simplify the cable impedance matrix to a 3×3 form on the basis that the voltage between neutrals and the earth at each end of a line is zero, i.e. that the neutrals are grounded.

In European networks, it is common for the LV side of the distribution transformer to have a wye configuration with a grounded neutral. The connection is provided by electrodes designed to ensure a low earth potential rise in the presence of fault currents. Metallic sheaths and concentric neutrals of underground cables are also connected to ground.

At the customer premises, the regulations governing the earthing system define several different connection configurations [15]. For TT earthing, there is no protective earth provided by the network and so the customer must install an earth electrode. For TN configurations, the earth conductor provided by the network is connected to pipes at the customer meter point. This creates an equipotential zone within the customer's premises, but may also provide a ground connection if the pipes are metallic. For TN-S earthing, there is no connection between neutral and ground.

LV mains include branched joints and junctions between different cable types. Junction boxes connect the cable cores and concentric neutral or sheath, but have no connection between the neutral and the sheath, and neutrals are not grounded. Similarly, neutral cores may not be grounded at link boxes or where service cables are attached to mains.

However, where a combined neutral/earth conductor is provided, it is important for safety that the earth does not become broken. Additional earth electrodes are added at nodes within the LV network such as cable joints and at the ends of feeder mains. It is also possible for LV networks to include a combination of sections with separate neutral and earth and sections with combined neutral and earth.

In summary, there is a wide variation in earth configurations, each with different ground connections for the neutral. This issue is usually considered carefully in regard to safety and fault conditions but treated less rigorously in simulation models. There is no concern if the network is balanced and has negligible harmonics, but the consequences for real networks require investigation.

B. Links Between Radial Branches

Link boxes between radial branches allow the supply to be re-routed in the case of faults. Where the link box is used as a normally open point, the phase conductors links are removed but the neutral and sheath conductors may remain connected through. This creates loops within the neutrals or sheaths and allows circulating currents to flow.

The forward/backward sweep algorithm provides an efficient means of solving the network power flow, but is most easily implemented if the network has radial branches. Assuming that neutrals and sheaths are disconnected at the link boxes (in addition to the phase conductors) allows the network to be simplified to a radial structure. However, the impact of this assumption is not clear.

IV. CONDUCTOR IMPEDANCES

Table III lists assumptions related to the conductor impedances, with further discussion below.

A. Full conductor model

A full model of these conductors would include the series impedance and shunt admittance, plus the lumped impedances of neutral or sheath connections to ground. Typical underground LV mains cables include the three phase conductors, a conductive sheath or concentric neutral, and in some cases an additional neutral core. If connections to ground exist, the earth provides a further conductor.

Carson's equations are frequently used to provide a matrix $\hat{\boldsymbol{z}}$, containing the self-impedance and mutual impedances for each conductor in a circuit with a ground return path [16]. For a 3-core cable with concentric neutral, this would be a 4×4 matrix.

TABLE III. MODELLING ASSUMPTIONS FOR CONDUCTOR IMPEDANCES

Assumption	Review of risks and impacts
Shunt admittance is neglected [13]	Calculations in [17] conclude that neglecting capacitance has minimal impact, at least in an example for overhead line. Shunt admittatance is also assumed to be negligible.
The simplified Carson's equations are valid for cable and overhead line impedances.	Comparison between impedance matrices obtained with the full Carson's equations and with the simplified equations suggests that little inaccuracy is caused by this approximation, at least for the fundamental mains frequency [29]. ***See section IV.A.***
Ground resistivity is a constant, e.g. 100 Ωm [17].	Althougjh negligible impact on voltage was found for resistivity varying from 10 to 1000 Ωm in [29], it would be useful to review this for a case with significant unbalance and for underground cables.
End effects are negelected in calculating cable impedances for LV cables	Carson's equations assume that end effects are negligible so that the current distribution in the ground and cable is the same all along the cable [16]. This appears questionable, as discussed in ***section IV.C***.
Carson's equations define separate conductor and earth voltages.	Calculated earth voltage drops are dependent on arbitrary assumptions made when separating Carson's equatons into terms for the conductor and for the earth. See ***section IV.B***.
Sum of currents equals zero in each line segment.	Where there are loops in neutral conductors due to link boxes, circulating currents may exist. Circulating currents can flow in the ground if branches with unequal earth potentials are co-located.
Conductor impedances can be reduced to 3×3 form using the Kron reduction.	Assuming a multi-grounded network, if this is not the case in practice, introduces an approximation to neutral voltages if impedances are reduced to 3×3 phase impedance matrices. ***See section IV.D***.
Impedances defined by positive and zero sequence impedance values [1].	This approach neglects any coupling between sequence modes, and makes an approximation that the cables are fully balanced. Any asymmetry due to the cable is not modelled. ***See section IV.E.***
Zero sequence impedance can be estimated to be a multiple of the positive sequence impedance [1], [2].	Results in [1] where found to be insensitive to the scaling factor used, but results in [2] with more unbalanced currents showed that the zero sequence impedance significantly affected the proportion of voltage range and unbalance constraint violations.
Conductors represented by phase and neutral conductor impedances [9].	The model does not include mutual coupling between the conductors or currents in the earth. ***See section IV.E.***
Neutral to ground impedance is zero.	The grounding impedance at LV substations may be up to 20 Ω [27]. Results in [30] compared a model with different grounding resistances and noted significant difference in the neutral voltage.

The full equations include an infinite summation term and so a simplification is generally made in which only one resistive term and two reactive terms are retained [17].

B. Ground impedance derived from Carson's equations

Carson's equations define the impedance of a conductor together with an earth return path. In order to model the conductor and earth voltage drops separately, terms within Carson's equations have been partitioned in order to provide separate ground and conductor impedance equations.

The ground resistance can easily be isolated from the combined circuit resistance since the conductor resistance is known. However, different approaches have been developed to separate the reactance given by Carson's equations into terms due to the ground and conductor self-inductances and the mutual inductance between them.

The approach by Ciric [13] is described for overhead lines and follows the physical concept from Carson [16]. The earth return path is modelled as an equivalent conductor that is the image of the conductor above ground.

Anderson [18] uses a re-arranged form of Carson's equations in which the earth return is represented as a wire with a specified geometric mean radius (GMR) and depth within the ground. The conceptual earth return wire is arbitrarily selected to have a GMR of unity and the depth in the ground then calculated accordingly.

The two approaches give different equations for the self-reactance of the earth return path, and for the mutual reactance between this and the wire conductor. This seems to be an area in which there is some uncertainty as both approaches require an arbitrary definition of the reactance.

C. Axial current flow assumed in cable direction

Based on the modified Carson's equations from [17], the resistance of the earth return path is $r_d = 0.0592$ Ω/km and is independent of the ground resistivity. For $\rho = 100$ Ωm, this suggests an effective ground conductor with area 1.67×10^6 m^2, equivalent to a semi-circular profile with radius 1037 m. This appears surprising, and is a dimension much greater than the typical node to node distance in an LV network. A key assumption in Carson's equations is that currents only flow in the axial direction [16], but this might be questioned where the implied current distribution is of a scale greater than the actual length of the cable.

A different assumption is made when calculating the earth potential rise due to fault currents, for which the potential reduces approximately in inverse proportion to the radius from the fault point, rather than linearly along a cable.

D. Kron reduction

Simulation methods may utilize the Kron reduction in order to reduce cable impedance data from an $n \times n$ matrix $\boldsymbol{\hat{z}}$, to a 3×3 phase impedance matrix $\boldsymbol{z_{abc}}$ as in [17]. This allows the cable models to be integrated into a network simulation with components such as transformers that are also modelled by a 3×3 matrix. The Kron reduction applies a constraint that the multiple neutral or earth paths are connected together at each end of a line. However, the technique is questionable if the neutral to ground connections absent at some junction nodes in the network.

Using the notation from [17], the Kron reduction provides the phase impedance matrix:

$$\boldsymbol{z_{abc}} = \boldsymbol{\hat{z}_{ij}} - \boldsymbol{\hat{z}_{in}}.\boldsymbol{\hat{z}_{nn}}^{-1}.\boldsymbol{\hat{z}_{nj}} \tag{1}$$

The neutral and ground currents can also be determined:

$$\mathbf{I_n} = -\mathbf{\hat{z}_{nn}}^{\mathbf{-1}}.\mathbf{\hat{z}_{nj}}.\mathbf{I_{abc}} \tag{2}$$

$$I_g = -(I_n + I_a + I_b + I_c) \tag{3}$$

For example, if two cable segments of different types are connected in series, and if the neutrals are grounded between the segments, then the combined impedance can be represented by the sum of the 3×3 phase impedance matrices for each line section. Alternatively, if there is no ground between the two sections, the $n \times n$ conductor impedances must be combined first, and the Kron reduction applied to the result. This gives a different result to that with the Kron reduction performed first and the results combined.

In both cases, the sum of currents in each line is assumed to be zero. Phase currents are the same in both segments. If the neutral is grounded between the two line segments, then the proportion of current in the ground varies depending on the self- and mutual impedances of the conductors. If the neutral is not grounded between the two segments, the currents in the neutral and ground are constant throughout.

Assuming additional neutral to ground connections makes an approximation that each section can be treated independently. A comparative study would be needed to determine the significance of this approximation.

E. Approximated cable impedances

In the absence of data to provide the full $n \times n$ conductor impedance matrix, an approximate model may be defined based on only the self-impedance data.

One approach is to use the impedances for the conductors from manufacturers' datasheets. These define the conductor alone, without an earth return path (as would be given by Carson's equations). Mutual coupling data is not normally available so is assumed to be zero. Since no earth currents are included, it is implied that the circuits are isolated from the ground. If all the phase conductor impedances are equal, these are also equal to the positive sequence impedance. Since there is no coupling between the sequence impedances, the positive and zero sequence modes could be simulated separately.

The impact of approximating the impedances using only the positive sequence value was reviewed in [19] and shown to introduce considerable error into voltage calculations.

Alternatively, the impedances might be defined by positive and zero sequence values. These are used to populate the leading diagonal of a 3×3 sequence impedance matrix $\boldsymbol{z_{012}}$ which can be transformed to give a phase impedance matrix $\boldsymbol{z_{abc}}$. As the sequence matrix contained no coupling, the corresponding phase impedance matrix is fully balanced. If the cable being modelled is asymmetrical, this is equivalent to making an approximation that the phases are transposed. Since there is no information available to expand the 3×3 matrix into a 4×4 matrix, if z_{012} represents a cable with an earth path, it is implied that the neutral to ground voltage is zero.

The impact of this approximation was shown to have minimal impact on voltage magnitudes [19]. However, there is a greater error on estimates of voltage unbalance and losses [20].

V. CONCLUSIONS

This paper has presented a broad review of assumptions made when modelling power flow in LV networks, particularly where the impact of distributed generation is to be assessed. Many of these are more questionable for LV networks than for higher voltages as the demand is more subject to the stochastic variations of customer loads and as the networks are less well characterized.

A formal comparative study would be needed to fully assess the impact of all of these assumptions. However, the following appear to present some level of risk as they clearly affect the numeric results:

- The use of time averaged demand samples for periods much longer than the typical on-time of appliances
- Assuming a constant power vs. voltage model for loads.
- Assuming mean demand is balanced across each phase.
- Modelling the network as sinusoidal without harmonics.
- Assuming one earthing scheme throughout, when many configurations and combinations may occur in practice.
- Applying the Kron reduction technique when ground connections may not exist, or have non-zero impedance.
- The use of separate terms from Carson's equations to provide an impedance model for the earth currents.

Assumptions regarding time resolution, current balance and harmonics, all have impacts on the models of neutral currents, losses and voltage unbalance. These assumptions are particularly questionable when combined.

Further work is planned to evaluate the impact of these simulation assumptions, initially addressing the questions of harmonics and grounding assumptions.

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