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A Method of Visualising the 3D Dimensional Variation Behaviour of Kinematic Systems

Leslie C. Sleath[1], Denis M. Sleath[2] and Paul G. Leaney[1]

[1]*Department of Mechanical and Manufacturing Engineering, Loughborough University, Leicestershire, LE11 3TU, UK*
[2] *i-dmsolutions Ltd, Progress House, 4 Linden Road, Loughborough, Leicestershire LE11 1PG, UK.*

Abstract

At present, the output from the dimensional variation of an assembly is often in the form of a statistical process control chart or histogram. Such output, while informative, is not particularly suited to the evaluation of dimensional variation behaviour in complex kinematic assembly systems. This paper presents a graphical method of visualising the 3D dimensional variation behaviour of any chosen point feature in a kinematic assembly system by the use of 3D bounding surfaces. The bounding surfaces describe a volume such that there is a known probability that the chosen point is contained within the volume. The bounding surfaces and the volume they describe visualise the dimensional variation behaviour of the chosen point feature as it traverses one or more of the kinematic assembly system's movement ranges. This paper demonstrates that the visualisation allows the dimensional variation behaviour of the chosen point to be presented in a compact and readily comprehensible manner that enables evaluation of the analysis output within the context of the original simulation model geometry. An example of the method applied to a simple kinematic system is also shown.

Keywords; Dimensional variation analysis, Dimensional variation behaviour, Information visualisation.

Introduction

A kinematic assembly is one that has one or more continuous ranges of movement. The assembly can occupy any position between the limits of the movement range(s). A frequently used method of simulating the dimensional variation behaviour of kinematic systems is to divide the movement range(s) into a series of increments.

Where the assembly system only has a single movement range then each increment will form a separate configuration of the simulation model. Where there are two or more movement ranges each unique combination of increments will form a separate simulation model configuration. The number of simulation model configurations will rise exponentially with the number of movement ranges in the assembly system. Each of the simulation model configurations is analysed separately. The method is analogous to the individual frames in a cinema film which when recombined give the impression of movement on the screen. One disadvantage of the method is that it produces large volumes of numeric data.

Juster et.al. [1] make the point that tolerance analysis systems often produce data in the form of histograms that require interpretation by an experienced analyst to establish the dimensional variation behaviour of the assembly. A basic tenet of information visualisation [2] is that information structures are more easily interpreted, if they can be visualised. Hansen and Johnson [3] make the point that data visualisation is an indispensible part of the scientific discovery process. The benefits of visualising medical data such as ultrasound, CT and MRI scans are well known. However, the data produced by the scan still requires significant computer manipulation using dedicated software to produce the visualisation [4]. The visualisation of the effects of variation on the perceived quality of products and in particular automobiles is well established [5] as are the financial benefits. Unlike the real time dynamic visualisations found in medicine the visualisation of perceived quality is applied to essentially static systems. Given the current economic climate manufacturing companies are reluctant to purchase dedicated software unless the financial benefits have been demonstrated. Thus, a method of visualising the dimensional variation behaviour of a kinematic assembly that only utilises software one could reasonably expect to find in a company undertaking dimensional variation analysis would be beneficial.

Depending on the assembly under investigation, visualisation of the analysis output may not be necessary, it may be possible to interpret the information by the simple expedient of collating and tabulating the analysis results from each individual configuration. However, where the intrinsic information is represented by more than one parameter such as polar or Cartesian co-ordinates it may still be difficult to interpret the data. It would of course be possible to visualise such data as a 3D graph. Unfortunately, in doing so the data is completely divorced from the assembly system, making it difficult to relate the data to the kinematic assembly that produced it. What is required is a method of introducing the analysis data back into the simulation model of the assembly system that produced it. This would allow the visualisation and evaluation of the dimensional variation behaviour within the context of the original kinematic system. Such a method would also in part alleviate a limitation of most CAD systems [1] which normally only display nominal geometry.

This paper proposes just such a method whereby post processed dimensional variation analysis data is introduced into the simulation model of the assembly system. The data is used to create bounding prismatic surfaces that visualise the dimensional variation behaviour of a specific point in the assembly within the context of the simulation model.

## Visualisation of the effect of variation on a point

The proposed method is intended to visualise the dimensional variation behaviour of a selected point in the assembly. The dimensional variation behaviour of an assembly is determined by means of dimensional variation analysis (DVA) software. This simulates the propagation of variation through an assembly system and determines the effect of this variation on selected assembly level measurements or system attributes within the assembly. The first stage of the process is to select the point of interest and locate it in 3D space.

Point location

A point feature P within the simulation model is selected, or if necessary added, which is appropriate for the dimensional variation behaviour to be visualised. The point feature is located in space by means of three directed assembly level measurements. As the feature is a mathematical point it does not require orientation. The three measurements commence at the global origin, and are aligned to the principal axes (X, Y, Z) of the global co-ordinate system (Figure 1).

The values of the directed measurements give the Cartesian co-ordinates of the point P. The global origin is used as the reference feature for the measurement as it is one of the few features in the assembly system known to remain fixed. Potentially, most assembly features in the simulation model are subject to relative motion as the assembly traverses its movement range and/or due to the effects of component part variation.

The dimensional variation behaviour of the point P, as the assembly traverses its movement range, is simulated by dividing the movement range into a series of incremental positions. Each of the positions represents a simulation model configuration, which is analysed separately. The incremental direction, in which the model configurations increment is aligned to the axis of the global co-ordinate system that contains the largest component of the movement range direction, in this instance the Z axis. The planes in Figure 1, which, represent the incremental configurations, are aligned normal to the incremental direction, and positioned by fixing, in this instance, the Z co-ordinate so as to pass through the point of interest, P. The position of the point P can move within the incremental plane but by definition, it cannot move out of the plane as, in this instance, the Z co-ordinate is fixed to align the simulation model into the correct configuration. The X and Y co-ordinate values are defined by the dimensional variation behaviour of the point P.
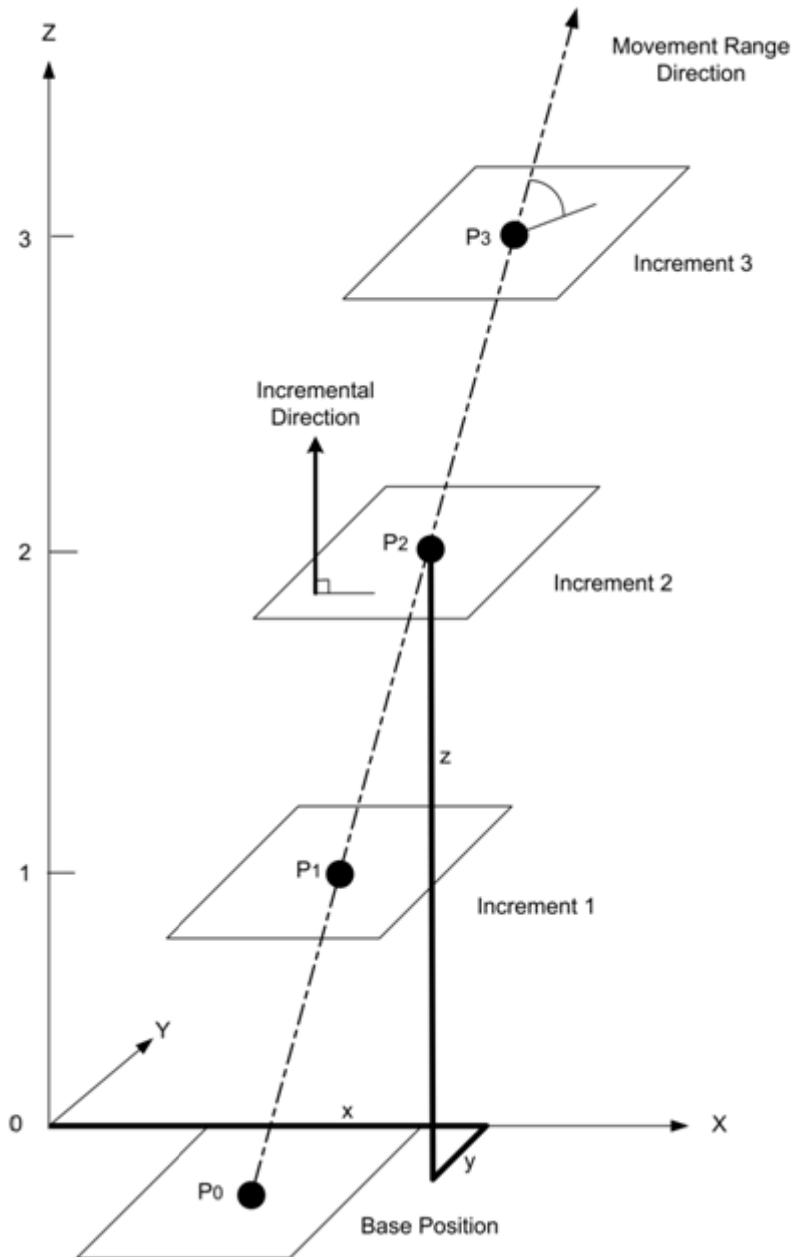
Figure 1 Relationship between the movement range direction, incremental direction and the point P

Generation of variation points

Since the simulation model takes into account, a significant number of input variables when simulating the system behaviour the output is usually in the form of a normal distribution curve. The X and Y co-ordinates are represented by a mean value (μ) and the standard deviation (σ) of the variation distribution curve. The probability that the X co-ordinate has a value of X less than or equal to x is given by the cumulative

distribution function $\Phi(x)$ (equation 1)

$$\Phi(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \int_{-\infty}^{x} e^{-(x-\mu_x)^2/2\sigma_x^2}$$

(eq 1)

By the appropriate use of integration limits (lower limit x, upper limit ∞) equation 1 can give the probability that X is greater than or equal to x. This is known as the Q function where,

$$Q(x) = 1 - \Phi(x)$$

(eq 2)

Thus for any given value X or Y it is possible to calculate the probability that X>x or Y>y. However, the probability that the point (X, Y) lies outside the point (x, y) is given by equation 3.

$$P(X,Y) = Q(x)Q(y) \quad \text{where} \quad X > \mu_x, Y > \mu_y$$

(eq 3)

To determine the probability that the point (X, Y) lies outside the locus of points on the incremental plane requires three additional equations one for each quadrant of the incremental plane.

$$P(X,Y) = Q(x)\Phi(y) \quad \text{where} \quad X > \mu_x, Y < \mu_y$$ (eq 4)

$$P(X,Y) = \Phi(x)Q(y) \quad \text{where} \quad X < \mu_x, Y > \mu_y$$ (eq 5)

$$P(X,Y) = \Phi(x)\Phi(y) \quad \text{where} \quad X < \mu_x, Y < \mu_y$$ (eq 6)

Consider equation 3, if P(X,Y) is fixed at a value of, for example, 10^{-3} then there is a 1:1000 possibility that the point (X, Y) lies outside the point (x,y). The probability that each individual co-ordinate exceeds a given value is controlled by three variables (eq 1), μ the mean value of the variation distribution, σ the standard deviation of the distribution and the value of x or y. By substituting these parameters into equation 3 equation 7 is obtained.

$$Q(x,\mu_x,\sigma_x)Q(y,\mu_y,\sigma_y) = 10^{-3}$$

(eq 7)

The output from the dimensional variation analysis provides values for μ_x, μ_y, σ_x and σ_y. Thus, y can be determined for any given value of x contained by the variation distribution curve. However, this solution only applies where both x and y are greater than their mean values. Co-ordinate pairs for the remaining three quadrants of the incremental plane are generated by solutions based on equations 4, 5 and 6.

Visualisation of generated point data

If a series of co-ordinate pairs generated by the above method is plotted on the incremental plane (Figure 2) a bounding line is produced. By generating series of co-

ordinate pairs for each increment in the movement range, linking them both within the incremental planes and between the incremental planes it is possible to generate a 3D bounding surface (Figure 2). In this instance, the probability that the point (X, Y) lies outside the bounding surface is 10^{-3} but bounding surfaces can be generated for any probability value or series of probability values. The points so generated could be used to plot a 3D graph as in Figure 2. However, by importing the points into the CAD platform used to create the original CAD geometry a 3D part can be created and then added to the original simulation model.
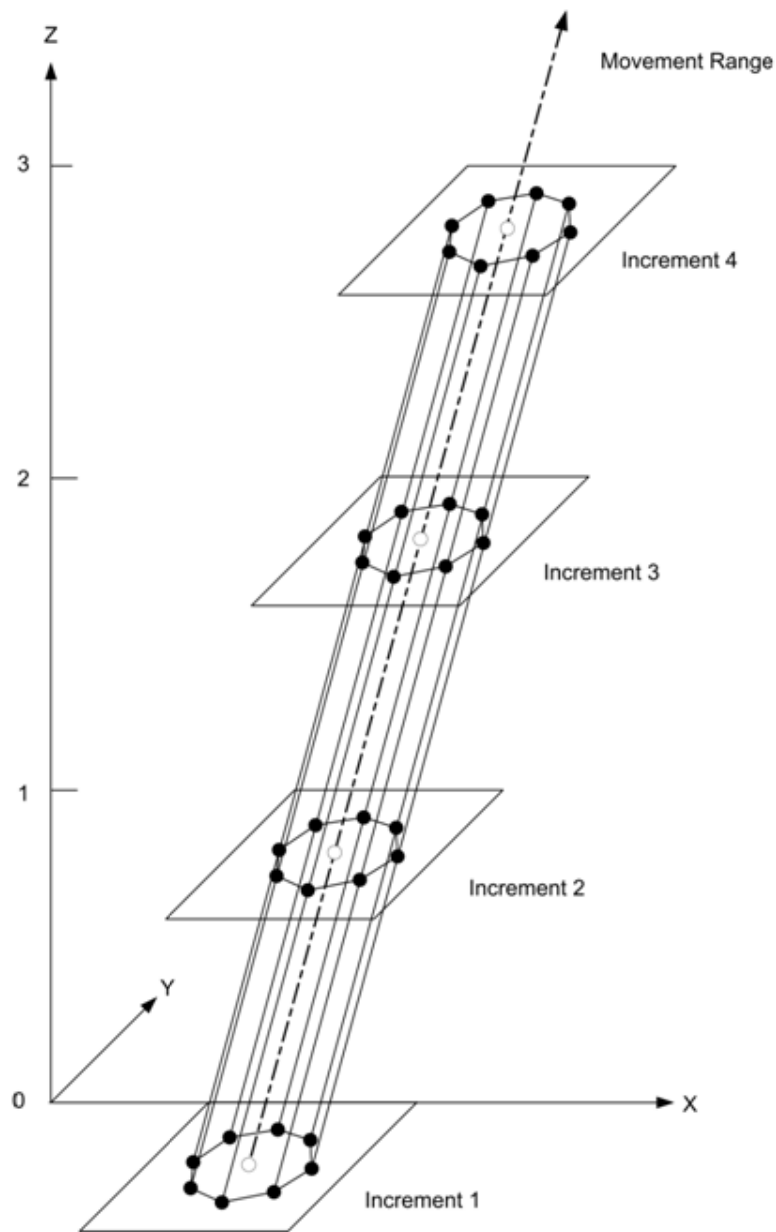
Figure 2 Creation of a prismatic bounding surface

Figure 3 shows a simple kinematic assembly with a single movement range. A point P is located at the right hand end of the round axle in Figure 3. The output data from the dimensional variation analysis of the three directed measurements along the global axes of the assembly to point P is given in Table 1.
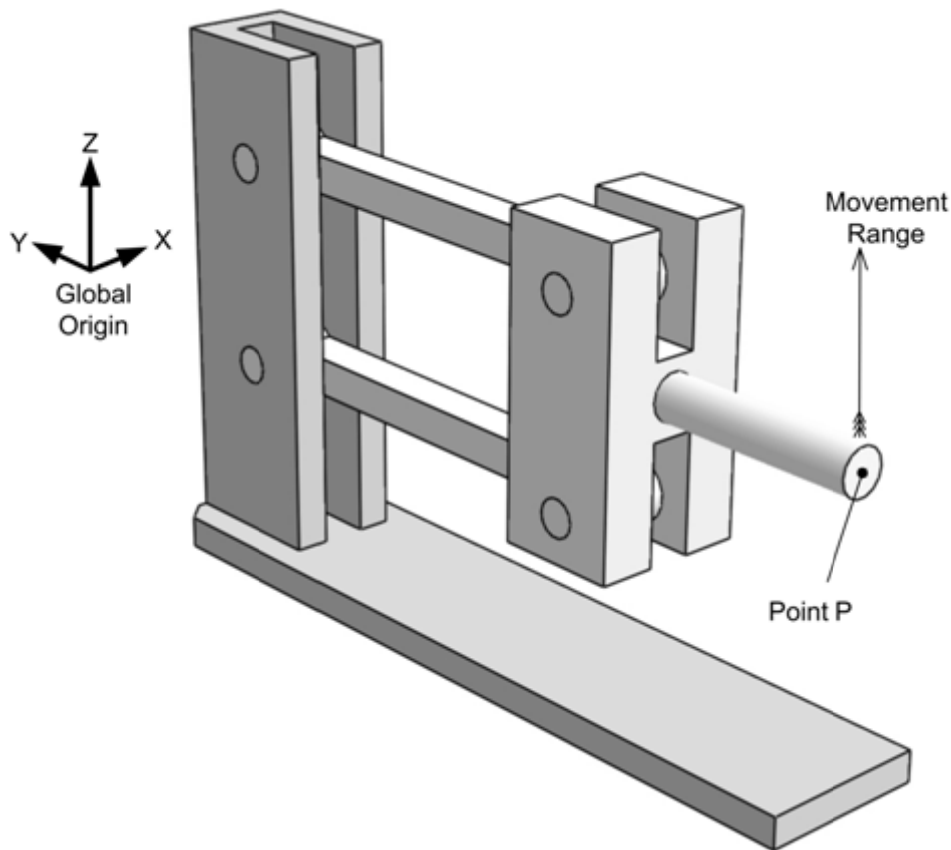
Figure 3 Simple kinematic assembly

Table 1 dimensional variation analysis output

μ_x	σ_x	μ_y	σ_y	μ_z	σ_z
35	1.84926	-281.417	0.156651	-30	0
35	1.72479	-283.970	0.139571	-20	0
35	1.61848	-285.807	0.126759	-10	0
35	1.52890	-286.954	0.118029	0	0
35	1.45542	-287.427	0.113403	10	0
35	1.39808	-287.232	0.112983	20	0
35	1.35752	-286.368	0.116858	30	0
35	1.33486	-284.821	0.125085	40	0
35	1.33167	-282.570	0.137768	50	0

Table 1 shows that the standard deviation of the X co-ordinate σ_x is approximately one order of magnitude larger than that of the Y co-ordinate σ_y but the dimensional variation behaviour of the point P is obscured by the need to combine the three co-ordinates. The Z co-ordinate has, in this instance, a standard deviation σ_z of zero because the Z co-ordinate of the point P is fixed to align the simulation model configurations. The mean values of Y and μ_y are all negative to ensure directional consistency between the global axes and the measurement direction. The apparently fixed mean value μ_x of the X co-ordinate is purely coincidental and due to the fact that the assembly is not designed to move in the X direction. Thus any movement in X is due to the dimensional variation behaviour of the assembly which is symmetrical about, but does not in this instance affect, μ_x.

The data in Table 1 when processed by the method previously described and reinserted into the simulation model permits the construction of a prismatic bounding surface. The volume defined by the bounding surface renders the dimensional variation behaviour of the point P readily discernable within the context of the simulation model (Figure 4). The probability that the point P lies outside the volume contained by the bounding surface is, in this instance, 10^{-3}.
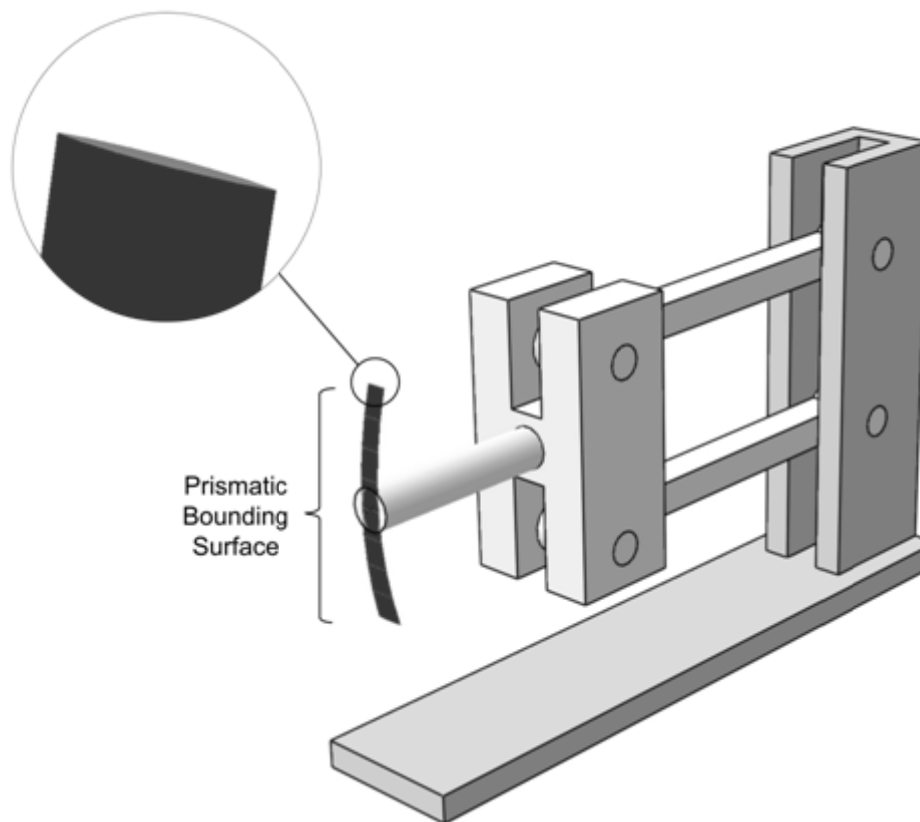
Figure 4 Dimensional variation behaviour of the point P at a probability of 1:1000

Visualisation of the effect of variation on an angle
It is a necessary condition of this method that the user be able to define, in the DVA software, the direction from which the angle to be measured is viewed (Figure 5).
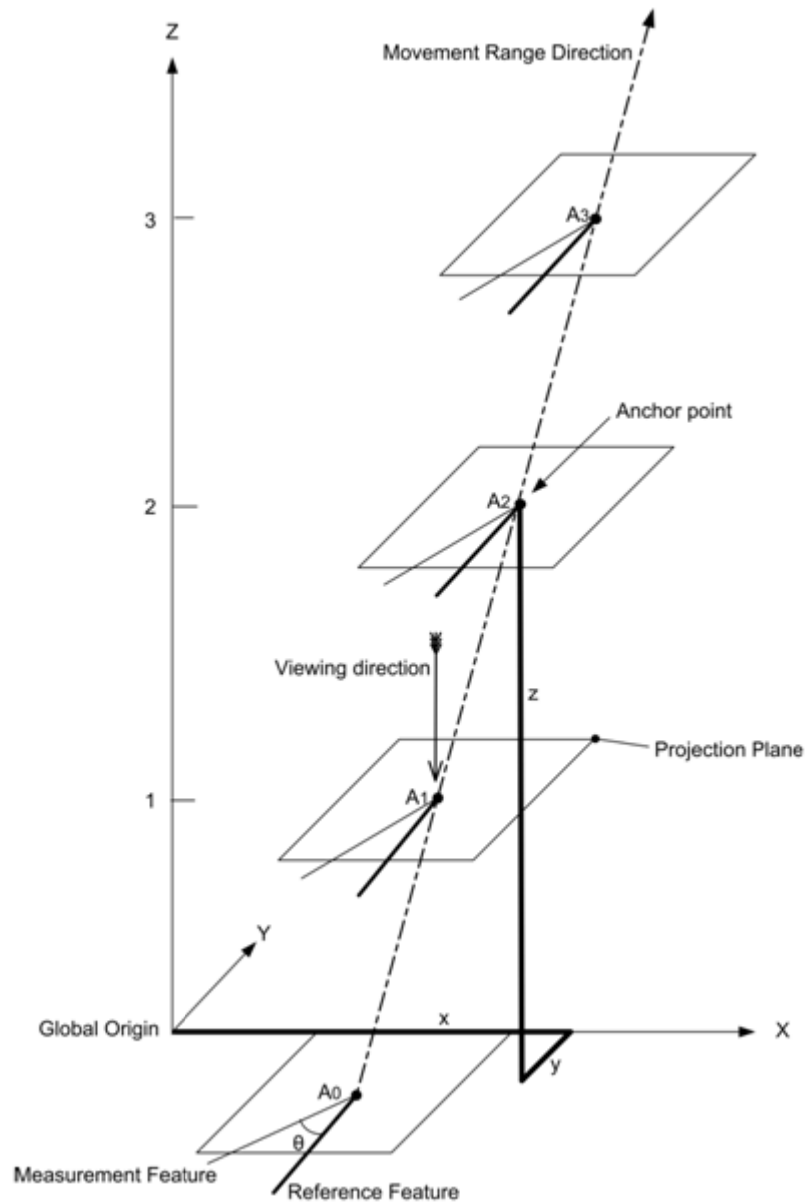
Figure 5 The relationship between the global co-ordinates, anchor point, reference and measurement features and projection plane

The visualisation of angular data is a two-stage process. The first stage is to choose a point in the simulation model to which the visualisation will be anchored. The chosen anchor point is the one most appropriate for, and that facilitates

interpretation of, the information being visualised. This is a largely subjective decision. Once chosen the anchor point is located in 3D space relative to the global co-ordinate system by exactly the same method used to locate the point P described above. The second stage of the process is to generate a reference frame. This describes how the chosen angle is to be measured and locates the visualisation in 3D space relative to the anchor point

Angular visualisation reference frame

The anchor point occupies the same relative position in each configuration of the simulation model and, although its absolute position may vary in each configuration of the simulation model, it acts as the origin for the orthogonal reference frame used to visualise angular data. In a 3D scenario, the angle of a feature (the measurement feature) is measured relative to a reference feature. If the two features are not co-planar, it is necessary to specify a viewing direction to complete the angle definition. The effect of defining a viewing angle is to project the reference and measurement features onto a plane normal to the viewing direction that passes through the anchor point. To reduce possible ambiguities when projecting planar features subject to compound angles, linear measurement and reference features are preferred. Where a suitable linear measurement feature does not exist, it will be necessary to add one to the simulation model. One axis of the reference frame is defined by the viewing direction while the projection of the reference feature onto the projection plane defines a second (Figure 5).

The third axis of the orthogonal reference frame will depend on whether a left or right handed axis system is used. One point that must be born in mind when defining the reference frame is that, as the angle between the movement range direction and the projection plane (Figure 5) decreases, then the quality of the visualisation will also decrease. When the movement range direction lies in the projection plane the visualisation will be reduced to a two dimensional entity and thereby rendered ineffective.

Generation of visualisation data

The output from a dimensional variation analysis (DVA) of an assembly system is usually in the form of a mean value and standard deviation for each system attribute analysed. The basic principle used to generate the visualisation data is the same as that for a point.

The mean value and standard deviation values produced by the dimensional variation analyses are manipulated mathematically to generate two points (B & C Figure 6) that bound the angular variation in the chosen system attribute for a given probability or number of standard deviations. The use of the reference frame reduces the manipulation of the DVA output to a 2D exercise in trigonometry. If the line 0A (Figure 6) which represents the mean value of the angle θ is of unit length then:
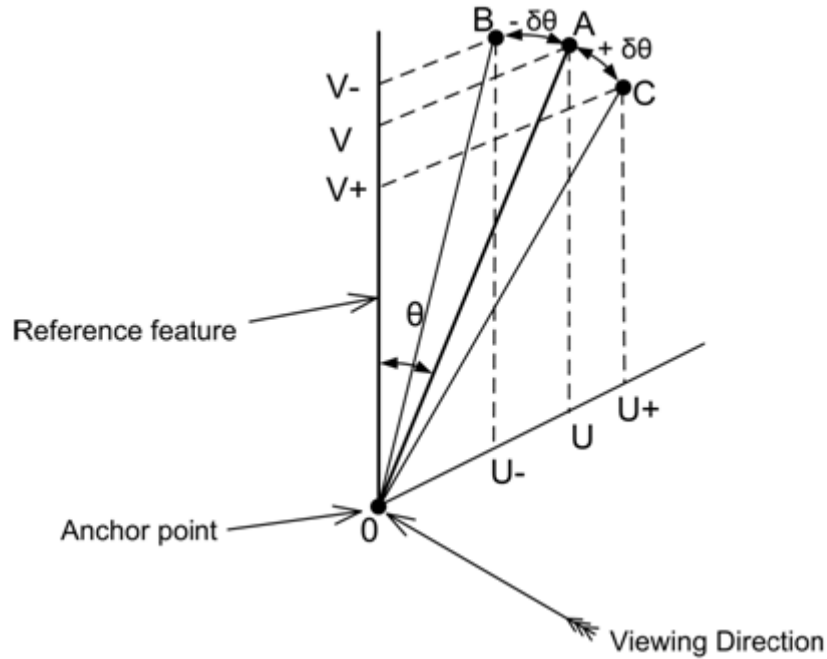
$$V\text{-} = \cos(\theta\text{-}\delta\theta) \qquad\qquad U\text{-} = \sin(\theta\text{-}\delta\theta)$$
$$V = \cos(\theta) \qquad\qquad U = \sin(\theta)$$

$$V+ = \cos(\theta+\delta\theta) \qquad\qquad\qquad\qquad U+ = \sin(\theta+\delta\theta)$$

Figure 6 Arrangement for generating the limiting values for angular variation

If the reference frame is aligned to the global co-ordinate system it is only necessary to add the co-ordinates of the points A, B and C to those of the anchor point to arrive at the co-ordinate for the points A, B and C relative to the global co-ordinate system. If the viewing direction is not aligned to the global co-ordinate system, a further trigonometric manipulation will be required to convert the reference frame co-ordinates to those of the global co-ordinate system. The global co-ordinates of the points 0,A, B and C are then collated for all the configurations that comprise the movement range and imported into the simulation model where they are used to create three surface features (Figure 7). The surface comprised of points 0 and B (Figure 6) bounds the lower value of the angle θ while the surface created from the points 0 and C bounds the upper value of the angle. A third surface created from the points 0 and A represents the mean value of the angle. Unlike the closed prismatic surfaces formed when visualising a point, the surfaces of the angular visualisation remain open. The angular visualisation of variation has one advantage over the visualisation of point variation in that the bounding surfaces can be extended to any desired extent in the direction of the measurement reference feature, in this instance the centreline of the axle.
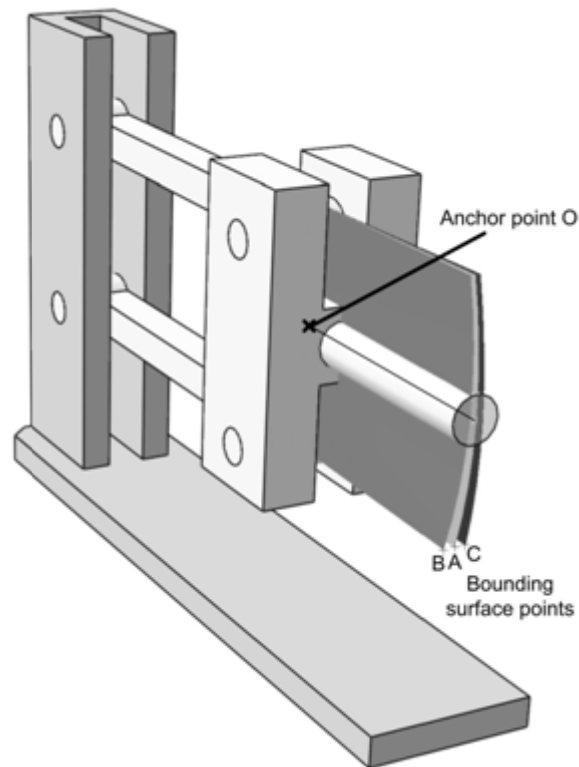
Figure 7 Visualisation of angular variation at a probability of 1:1000

The bounding planes shown in Figure 7 are extended from their nominal unit length to the same length as the axle in the simulation model. In this condition the bounding surfaces not only show the amount of angular variation in the axle centreline but also give an indication of the lateral movement of the outboard end of the axle due to angular variation.

Application

It is envisaged that the main application of the methods described above will be the presentation of complex DVA data to multi disciplinary audiences such as those found in product development or design review in a concurrent or simultaneous engineering environment. The ability to present complex analysis data in a simple manner will enable increased knowledge transfer and thus strengthen the concurrent or simultaneous engineering approach. To assist in the preparation of the visualisations, Two Excel based spreadsheets have been developed to automatically generate the data points used to construct the bounding surfaces. The spreadsheets are known to be compatible with Catia V5R16, Catia V5R19 and ProEngineer Wildfire 3 CAD platforms. A secondary application of the visualisation method is the detection of unexpected dimensional variation behaviour. Consider Figure 7, a visual

examination of the bounding surfaces shows that when the axle is in its lowest position the angular variation is at its maximum but decreases as the axle moves upwards. The same information is present in the tabulated data from which Figure 7 was derived, but the dimensional variation behaviour is not nearly so apparent and thus might not be recognised as such. While the cause of the unexpected behaviour is unknown, once detected it can be investigated further if necessary

The bounding surfaces are created within the simulation model and are thus CAD entities in their own right. It may therefore be possible, depending on the CAD platform used, to employ some of the native CAD interrogative functions, such as clash detection, to perform additional analyses on the bounding surfaces and their relationship with adjoining component parts of the assembly. Such functionality might be particularly useful where more than one set of bounding surfaces are present in the simulation model.

Conclusions

The two method described in this paper provide a means of presenting complex dimensional variation behaviour of kinematic assembly systems in a simple and easily comprehensible manner to audiences with a wide range of technical expertise. This ability enhances the concurrent or simultaneous engineering process by aiding knowledge transfer between the participants.

Further work

At present, the output from the Excel spreadsheet that generates the data points used to construct the bounding surfaces is converted manually from Excel format into a format readable by the CAD platform. Automation of this process would significantly enhance the functionality of the visualisation methods. A second area requiring further development is the means by which the generated data is imported into the CAD platform. At present the importation technique is very software specific. The development of a more generic importation technique applicable to a wider range of CAD platforms would be of considerable benefit. Additional work is required to develop a method capable of visualising the dimensional variation behaviour of linear features in addition to the method for point features described in the main body of this paper. This will require the consideration of the position of the two end points of the linear feature as well as the length of the feature. These parameters are not necessarily independent of each other and will thus complicate any visualisation technique.

References

[1] Juster, N.P. Fitchie, S. Taylor, S. Dew, P. Maxfield, J. Zhao, J. (2001) "Visualising the impact of tolerances on cosmetic product quality". International Conference on Engineering Design ICED 01. Glasgow, 21-23 August 2001.

[2] Ware,C. Mitchell, P. (2008) "Visualising graphs in three dimensions". ACM Transactions on applied perception (TAP), 2008

[3] Hansen, C.D. Johnson, C.R. (2005) The visualisation handbook Elsevier Academic Press, 2005.

[4] Sakos,G. Schreyer L. Grimm, M. (1994) "Case study: Visualisation of 3D ultrasonic data". Visualisation 1994, Proceedings of the IEEE conference on visualisation, Washington DC, 17-24 October1994.

[5] Quality Manufacturing Today (2009) "Perceived quality". Quality Manufacturing Today, September, 2009.