Resource Allocation and Optimization Techniques in Wireless Relay Networks

by

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A doctoral thesis submitted in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy (PhD), at Loughborough University.

June 2013

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CERTIFICATE OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this thesis, that the original work is my own except as specified in acknowledgements or in footnotes, and that neither the thesis nor the original work contained therein has been submitted to this or any other institution for a degree.

.... (Signed)

.... (candidate)

I dedicate this thesis to my parents and all my friends.

Abstract

Relay techniques have the potential to enhance capacity and coverage of a wireless network. Due to rapidly increasing number of smart phone subscribers and high demand for data intensive multimedia applications, the useful radio spectrum is becoming a scarce resource. For this reason, two way relay network and cognitive radio technologies are required for better utilization of radio spectrum. Compared to the conventional one way relay network, both the uplink and the downlink can be served simultaneously using a two way relay network. Hence the effective bandwidth efficiency is considered to be one time slot per transmission. Cognitive networks are wireless networks that consist of different types of users, a primary user (PU, the primary license holder of a spectrum band) and secondary users (SU, cognitive radios that opportunistically access the PU spectrum). The secondary users can access the spectrum of the licensed user provided they do not harmfully affect to the primary user. In this thesis, various resource allocation and optimization techniques have been investigated for wireless relay and cognitive radio networks.

The first contribution, consists of an optimization technique for the use of two way relays to forward signals and perform spatial multiplexing. The problem has been formulated using a Semidefinite Programming (SDP) framework that can be solved using interior point methods. Also, we extend the SDP technique to a two way asynchronous relay network, which utilizes the filter-based relays to overcome inter-symbol interference and enhance the signal to interference-plus-noise ratio (SINR). The simulation results indicate that the proposed two-way-relay technique has the ability to enhance the spectrum efficiency and filter-based relays can effectively mitigate the interference and reduce the bit error rate (BER) of signal transmission.

The second contribution is the proposal of a minimum mean square error (MMSE) based design technique for a two way relay network that serves multiple peer-to-peer users. The algorithm aims at minimizing the sum of the mean square errors associated with the retrieval of symbols at multiple destinations subject to a total transmission power constraint at the relays. The problem is solved using a Lagrangian formulation of the constrained optimization problem. Through certain mathematical manipulations of the optimization problem, we suggest a method to choose the optimum Lagrange multiplier that enables full use of the transmission power at the relay.

The third contribution is on a power allocation technique for an orthogonal frequency division multiplexing (OFDM)-based cognitive radio wireless relay network. In particular, the sum rate maximization problem for the SU network subject to constant total power and interference leakage constraints is considered. This multiple-constraint maximization problem is transformed into an equivalent single constraint problem using two auxiliary variables. Simulation results demonstrate the convergence of the algorithm and the satisfaction of multiple constraints.

Finally the third contribution has been extended to a Multiple-Input Multiple-Output (MIMO)-OFDM based multiple user wireless relay network. The aim is to maximize the capacity of the wireless relay network whist satisfying the total transmission power budget.

Statement of Originality

The contributions of this thesis are mainly on the development of various resource allocation algorithms for two way relay and cognitive radio networks. The novelty of the contributions is supported by the following international journal and conference papers.

In Chapter 4, spatial multiplexing based two relay network that has multiple transmitter-receiver pairs has been proposed. The relaying matrix has been designed using semidefinite programming based optimization for minimising the total transmission power at the relay while satisfying SINR constraints for each user. The original contribution has been supported by the following publications:

1. J. C. Hu, Z. Xiong, Y. Rahulamathavan, K. Cumanan and S. Lambotharan, 'Optimization Techniques for Two-Way Relaying Based Multiuser Multiplexing' IEEE Eleventh International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), Marrakech, Morocco, June. 2010, pp. 1-5.

2. J. C. Hu, W. Wang, S. Lambotharan and Z. Xiong 'Optimization Techniques for Asynchronous Two-Way Relaying Based Multiuser Multiplexing', IEEE International Conference on Communication Systems (ICCS), Singapore, November. 2010, pp. 545-548.

In Chapter 5, an MMSE based relaying technique for multiple peer-to-peer users has been proposed. An efficient scheme for determining the optimum Lagrangian multiplier for full use of the transmission power resource has been proposed. This work will be submitted for possible publication in a conference.

3. J. C. Hu and S. Lambotharan, 'A Cooperative MMSE Strategy for Two-Way Relaying Based Multiuser Multiplexing', to be submitted.

A power allocation technique for an OFDM-based cognitive radio wireless relay network has been proposed in Chapter 6. The work has been published in:

4. J. C. Hu, S. Lambotharan and X. Zhu, 'A Power Optimization Technique for an OFDMA-Based Cognitive Radio Relay Network', IEEE Asia Pacific Wireless Communication Symposium (IEEE APWCS), Singapore, August. 2011.

In Chapter 7, a joint optimization technique for a MIMO relay network with multiple users has been proposed. The scheme is based on singular value decomposition and water filling power allocation for the signals from the source to relay and a set of beamforming for signals from relay to multiple users. The work proposed optimal time sharing for transmissions from source to relay and relay to multiple users, beamforming based spatial multiplexing and power allocation to maximise overall throughput of the network. This work will be submitted to IET Signal Processing.

5. J. C. Hu, J. Tang and S. Lambotharan, 'Downlink Resource Allocation Techniques for OFDMA-Based Wireless Relay Networks', to be submitted to IET Signal Processing.

Acknowledgements

I AM DEEPLY INDEBTED to my supervisor Professor Sangarapillai Lambotharan for his kind interest, generous support and constant advice throughout the past couples of years. I have benefitted tremendously from his rare insight, his ample intuition and his exceptional knowledge. This thesis would never have been written without his tireless and patient mentoring. It is my very great privilege to have been one of his research students. Also, I would like to thank Professor Jonathan Chambers for his support and encouragement.

I wish to take this opportunity to thank my father and mother, Xianzhao Hu and Yongqun Zeng, for their moral and financial support throughout my studies. No word can be used to fully express my appreciation and love to them, and I hope my family will be happy every day.

I would like to extend my appreciations to my academic colleagues Mohsen, Cumanan, Rahul, Ranaji, Amod, Jie, Gorgia, Ziming, Yu, Peter, Miao, Gaojie, Yanfeng, Lulu, Ata, Adel, Mustafa, Salman and so on, for making my stay at Loughborough pleasant. I also wish to acknowledge colleagues in Imago and all friends of my life who gived me support in these amazing six years in UK.

Last, but most importantly, I wish to express my deepest gratitude and love to my beloved girlfriend, Yoyo Zhou, for her endless and selfless love and support. She makes me confident and brave to face and solve every single difficulty in all my life, and I am quite sure that will also be in the future.

Juncheng Hu

August, 2013

List of Acronyms

1G	First Generation
2G	Second Generation
3G	Third Generation
3GPP	3rd Generation Partnership Project
4G	Fourth Generation
AF	Amplify and Forward
AMPS	Advanced Mobile Phone Service
AWGN	Additive White Gaussian Noise
BC-MAC	Broadcast Channel-Multiple-Access Channel
BER	Bit Error Rate
BNetzA	Bundesnetzagentur
BS	Base Station
BPSK	Binary Phase Shift Keying
CCI	Co-Channel Interference
CDMA	Code Division Multiple Access
CF	Compress and Forward

CP	Cyclic Prefix
CSI	Channel State Information
DF	Decode and Forward
EDGE	Enhanced Data rates for GSM Evolution
EV-DO	Evolution-Date Optimised
FCC	Federal Communications Commission
FDD	Frequency Division Duplexing
FDMA	Frequency Division Multiple Access
FM	Frequency Modulation
GP	Geometric Programming
GPRS	General Packet Radio Service
GSM	Global System for Mobile
HC-SDMA	High Capacity-Spatial Division Multiple Access
HD	High-Definition
IDFT	Inverse Discrete Fourier Transform
IFFT	Inverse Fast Fourier Transform
ISI	Intersymbol Interference
ISM	Industrial, Scientific, and Medical
LTE	Long Term Evolution
LTU	International Telecommunication Union
KKT	Karush-Kuhn-Tucker

MA	Margin Adaptive
M-ary FSK	M-ary Frequency Shift Keying
M-ary PSK	M-ary Frequency Phase Keying
M-ary QAM	M-ary Quadrature Amplitude Modulation
MA	Margin Adaptive
MIMO	Multiple-Input-Multiple-Output
MISO	Multiple-Input-Single-Output
MMR-BS	Mobile Multihop Relay-Base Station
MMSE	Minimum Mean Square Error
MS	Mobile Station
MU	Mobile User
NMT	Nordic Mobile Telephony
NRT	Non-Realtime
Ofcom	Office of Communications
OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
PU	Primary User
PUSC	Partial Usage of Subcarriers
QoS	Quality-of-Service
QPSK	Quadrature Phase Shift Keying
RA	Rate Adaptive

RF	Radio-Frequency
RKRL	Radio Knowledge Representation Language
RS	Relay Station
SC-FDMA	Single-Carrier Frequency Division Multiple Access
SDMA	Space Division Multiple Access
SDP	Semi Definite Programming
SIMO	Single-Input-Multiple-Output
SINR	Signal-to-Noise-plus-Interference Ratio
SISO	Single-Input-Single-Output
SMS	Short Message Service
SU	Secondary User
SVD	Singular Value Decomposition
TACS	Total Access Communication System
TDMA	Time Division Multiple Access
TDD	Time Division Duplex
TD-SCDMA	Time Division-Synchronous Code Division Multiple Access
TWRN	TwoWay Relay Network
UMTS	Universal Mobile Telecommunications System
UT	User Terminal
WCDMA	Wideband Code Division Multiple Access
WiMAX	Worldwide Interoperability for Microwave Access

WLAN	Wireless Local Area Network
WLS	Weighted Least Square
WMAN	Wireless Metropolitan Area Network
WPAN	Wireless Personal Area Network
WRAN	Wireless Regional Area Network

List of Symbols

Scalar variables are denoted by plain lower-case letters, (i.e., x), vectors by bold-face lower-case letters, (i.e., $\mathbf{x}$), and matrices by upper-case bold-face letters, (i.e., $\mathbf{X}$). Some frequently used notations are as follows:

$\mathrm{E}\{\cdot\}$	Statistical expectation
$(\cdot)^T$	Transpose
$(\cdot)^H$	Hermitian transpose
$(\cdot)^*$	Complex conjugate
$(\cdot)^{(q)}$	q^{th} iteration
$\|.\|_1$	L_1 norm
$\|.\|_2$	Euclidean norm
$(.)^{-1}$	Matrix inverse
$\mathrm{Re}\{\cdot\}$	Real part
$\mathrm{Im}\{\cdot\}$	Imaginary part
$\mathrm{Tr}\{\cdot\}$	Trace operator
$\mathbf{I}$	Identity matrix
dom	Domain
$\mathbf{diag}(\mathbf{x})$	Diagonal matrix with vector $\mathbf{x}$

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Chapter 1

INTRODUCTION

1.1 Evolution of Wireless Communication Systems

Wireless communication has become one of the most irreplaceable areas in the communication field today, and it has been advancing at an incredible speed over the last decades. Wireless local area network (WLAN) is one of the most important and successful wireless communication applications, and because of the high-speed data transmission and simplicity of installation, it has become very popular in the home, university and commercial districts offering wireless access to customers.

The federal communications commission (FCC) authorized the public use of the Industrial, Scientific, and Medical (ISM) frequency bands for wireless LAN products to enable the commercial development of wireless LANs in 1985. The ISM bands are all unlicensed, and located at 900 MHz, 2.4 GHz and 5.8 GHz, and the Unlicensed National Information Infrastructure (U-NII) band at 5 GHz. Due to the unlicensed spectrum, different systems can operate in these bands; however, they must tolerate any interference from other ISM equipments. And for the same reason, the ISM band is also very absorbing to wireless LAN users.

Institute of Electrical and Electronic Engineering (IEEE) 802.11 is a set of standard to realize WLAN communication, in which the 802.11a and 802.11b protocols define the most popular original standard in order to satisfy the requirement of high data rate on limited channel bandwidth. Building upon previous 802.11 standards, new standards are evolved to meet different demand. For example, 802.11e is a wireless standard that supports LAN by defining the Quality of Service (QoS), and it is an enhancement to the 802.11a and 802.11b WLAN specifications; Multiple-Input Multiple-Output (MIMO) has been included in the 802.11n standards, which support for wider channels and beamforming abilities to achieve higher data throughput. Thanks for the help of these standards, WLAN is being used widely with low cost and easy fast access.

The mobile communication develops and evolves rapidly. The first generation (1G) of wireless data modems was exploited in the early 1980's by amateur communication groups, which was based on the analogue system. It works within the 26 MHz spectrum of the 900 MHz ISM band employing direct sequence spread spectrum and the data rate is approximately 1-2 Mbps, therefore, the focus is on the service of voice information. Advanced Mobile Phone System (AMPS), Nordic Mobile Telephone (NMT), and Total Access Communication System (TACS) are mobile phone system standards based on analog technology in the first generation (1G).

The second generation (2G) telecommunication networks were commercially launched in the early 1990s in Europe. They can be divided into Time Division Multiple Access (TDMA)-based and Code Division Multiple Access (CDMA)-based techniques. TDMA divide transmission of signals to various users into time slots and CDMA allocates each user a special code to communicate over a multiplex physical channel. 2G was launched on the Global System for Mobile communications (GSM) standard. GSM uses digital modulation for improved audio quality and offers the customers with voice and limited data services, such as Short Message Service (SMS). This also brought significant benefit to users as digital technology requires substantially low battery power consumption and yields unprecedented improvement in the quality of services. After that, 2.5G was introduced by applying General Packet Radio Service (GPRS). It provides moderate-speed data transfer, by using unused TDMA channels in, for example, the GSM system. GPRS was able to send and receive data at the speed of 56 - 115 kbps.

The third generation (3G) mobile telecommunications technology provides even much better performance. Thanks for the result of groundbreaking research and development carried out by the International Telecommunication Union (ITU), 3G technology is still used nowadays. Different countries employ different 3G standards such as Wideband Code Division Multiple Access (WCDMA), Time Division-Synchronous Code Division Multiple Access (TD-SCDMA), Worldwide Interoperability for Microwave Access (WiMAX) and CDMA2000 including CDMA2000 1x, CDMA2000 3x, CDMA2000 Evolution-Date Optimised (EV-DO) and CDMA2000 Evolution-Date/Voice (EV-DV). 3G provides an information transfer rate of at least 200 kbit/s, and because of that, it can support the service of wireless voice telephony, mobile internet access, fixed wireless internet access, video calls and mobile TV. However, 3G performance may not be sufficient to meet needs of emerging high performance services like multi-media, full motion video and teleconferecencing, which appeals a new generation of technologies with higher performance.

As a successor of 3G standards, the fourth generation (4G) of mobile telecommunications technology has been researched and applied. 4G technologies include the 802.16e mobile WiMax, High Capacity-Spatial Division Multiple Access (HC-SDMA), and Long Term Evolution (LTE). It provides much higher data rate and streaming media, ranging from the Ethernet-grade 100Mbps between any two points in the world to 1Gbps, if the customers are in comparatively fixed positions. Otherwise, much wider bandwidth, coverage and range can be increased with a lower cost. Due to good QoS, people can enjoy the high-quality media services like gaming services, high-definition (HD) mobile TV, video conferencing, 3D television, and many more.

In 4G, transmission techniques such as MIMO, OFDM and Orthogonal Frequency Division Multiple Access (OFDMA) are the key features, which is also the focus of this thesis. MIMO uses multiple antennas at both the transmitter and the receiver to realize the spatial diversity gain and to increase the reliability of the wireless link. MIMO is an important part of modern wireless communication standards such as IEEE 802.11n (Wi-Fi), 3rd Generation Partnership Project (3GPP), Long Term Evolution and WiMAX. On the other hand, OFDM technique splits the radio spectrum into multiple smaller narrowband channels enabling simultaneous transmission of multiple streams of data to users. OFDMA is a form of OFDM for user multiplexing. These technologies within the context of relay networks are investigated in this thesis.

1.2 Development of Wireless Techniques

The use of relay in wireless network improves the performance such as the throughput, data rate and coverage. Relay techniques have been treated as one of the most important standardization process for the next-generation (4G) mobile broadcast communication systems. These communication systems include the 3GPP LTE-Advanced [2], IEEE 802.16j, and IEEE 802.16m.

Different global standards of WiMAX adopt relay communication, especially the first-mile/last-mile broadband wireless access in big cities, and backhaul services for voice/data communication. WiMAX has become one of the 3G standards in 2007, and the relay-based multihop technique has been developed by the 802.16j working group.

There are three possible relay schemes, namely amplify and forward (AF) [3, 4], decode and forward (DF) [5, 6], and compress and forward (CF) [7, 8] strategies.

In the AF scheme (analog repeaters) the symbols are amplified and re-transmitted by the relay nodes. In this scheme, the relay link fading and additive receiver noise could deteriorate the received signal. In the DF scheme (digital repeaters or layer 2 relays) the received signal is demodulated and decoded by the relay before retransmission. In this situation, the additional degradation is not contained in the forwarded signal, but only symbol errors resulting from it. The CF strategy allows the relay station to compress the received signal from the source node and forwards it to the destination without decoding the signal.

Relays can be also consist of multiple antennas providing spatial diversity for transmission and reception. The advantages of relay networks can be summarized as follows:

1.) Extension of coverage: Relay network supports the wireless communications in relatively complex terrains and buildings. Base stations in subway or areas under shadow of big buildings may employ relay nodes to assist transmission.

2.) Improvement in the capacity: Relay network provides more than one paths for communicating the symbols. The purpose is to transmit the same data at the same time, thorough relays which could provide spatial diversity increasing the system capacity.

3.) Reduction of transmit power: With the increasing number of users, base station cannot transmit signal effectively to each destination because it will require very high power to ensure QoS to overcome the attenuation in the transmission channel. The excessive power will introduce undue interference in the network. Relay nodes can help BS to disperse most of the transmit power, which will improve the capacity indirectly and enhance frequency re-usage.

4.) Low deployment cost. Compared to the deployment of a base station, the cost of relay deployment is relatively low. Hence, the coverage can be enhanced without significant additional investment.

1.3 Motivation for Cognitive Radio Techniques

The spectrum is considered as one of the most important optimization parameters in wireless networks. Fig.1.1 shows the distribution of spectrum occupancy in Europe. These bands are filling up very fast, and most of these radio spectrums have already been licensed. As seen, the spectrum region between 5150 MHz and 5350 MHz, as well as that between 5470 MHz to 5850 MHz can be used for WLAN. It can be seen that in the same region of spectrum for WLAN, there are two other network services appear and share the frequencies. The most of the spectrum bands have been licensed, and the unlicensed ones will also be filled up soon resulting in spectrum scarcity.

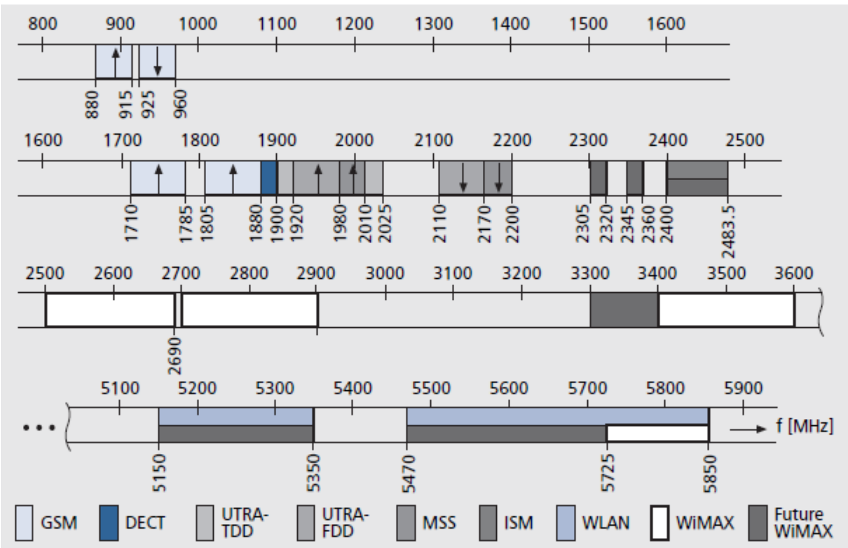

Figure 1.1. The mobile spectrum in Europe [1].

The government agencies restraint the right of licensing spectrum resources who implement a nation's rights in spectrum usage, for example, by the U.K. Office of Communications (Ofcom), by the FCC of the United States, or by Germany's Bundesnetzagentur (BNetzA), and the others. This conservative approach of licensing spectrum exclusively for different operators and services is inefficient. It is reflected in a recent survey of spectrum utilization made by the FCC [17], which shows that, for example, maximum total spectrum occupancy is only 13.1% from 30 MHz to 3 GHz in the spectrum measurement taken in New York City. The same situation appears in the most crowded area of downtown Washington, D.C.. The occupancy of frequency bands is less than 35% of the radio spectrum below 3 GHz. Ways of enhancing the spectrum utilization is a very hot research topic, in particular within the context that the spectrum usage varies continually in time, frequency and geographic locations.

Cognitive radio is considered as one of the most effective techniques to improve the electromagnetic radio spectrum utilization [9–12]. The idea of "cognitive radio" was first offered by Joseph Mitola in a seminar at KTH in 1998 [13], which was later published in 1999. Mitola presented it based on a new language called the Radio Knowledge Representation Language (RKRL). A cognitive radio could develop the flexibility of personal wireless services.

Cognitive radios have the ability to sense radio spectrum for spectrum holes and transmit signals through the spectrum that is not occupied. The definition of "spectrum hole" can be presented as [14]:"A spectrum hole is a band of frequencies assigned to the primary user, but, at a particular time and specific geographic location, the band is not being utilized by that user". Spectrum utilization can be enhanced significantly if the secondary user (who is not been serviced) can employ the "spectrum hole" which has been made vacant by the primary user at the right location and the time in question. Cognitive radio, inclusive of software-defined radio, has been projected as the means to encourage the efficient use of the spectrum by exploiting the existence of spectrum holes. Other possible arrangement in cognitive radio is to enable the SUs to use the spectrum provided the interference due to SU transmissions do not harmfully affect the PU receivers. This arrangement is considered as part of the relay technique proposed in the thesis.

1.4 Thesis Outline

Spatial multiplexing techniques and adaptive resource allocation techniques significantly enhance the spectrum utilization. The techniques developed for conventional wireless networks cannot be directly applied to a cognitive radio relay network due to the additional interference constraints on the primary users. Hence the work in this thesis mainly focuses on performing spatial multiplexing using two way relay network and the resource allocation techniques for cognitive radio networks using various mathematical optimization techniques.

The Chapter 2 provides a survey on various resource allocation techniques used in conventional wireless networks. Initially, multi-carrier modulation technique, which is the basis of OFDM and OFDMA techniques is discussed. Following on from this, static and adaptive resource allocation techniques for wireless networks are described. Multiple antenna techniques such as beamforming and spatial multiplexing are also discussed briefly. Finally, relay network is introduced and cognitive radio techniques are surveyed.

The Chapter 3 briefly introduces the fundamentals of convex optimizations, including definition of convex functions, convex sets problems. These are the basis of the resource allocation techniques developed in this thesis. The novel contributions in this thesis are covered in Chapters 4, 5, 6 and 7 as described below.

Contribution Chapters

In Chapter 4, we have used two way relays to perform spatial multiplexing for multiple transmitter-receiver pairs using a semidefinite programming (SDP) technique. The SDP formulation considered has the advantage of using minimum possible transmission power while ensuring desired Signal to Interference-plus-Noise Ratio (SINR) for each user is attained. The SDP technique is extended to a two way asynchronous relay network, which utilizes the filter-based relays to overcome inter-symbol interference. The simulation results indicate that the proposed two-way-relay technique enhances the spectrum efficiency and filter-based relays can effectively mitigate the interference from the various users and decrease the BER. In Chapter 5, we extend our earlier work on MMSE-based precoder design [15] to a two way cooperative relay network. Single and multiple pairs of source and destination are considered to demonstrate the potential of the two-way relay networks for forwarding signals while performing spatial multiplexing. We propose a new approach to determine the optimum signal scaling factor required to use all available transmission power at the relay though certain manipulation in the Lagrangian formulation.

The focus of Chapter 6 is on a new resource allocation technique to a cognitive radio wireless relay network consisting of transmissions from the base station to a relay station and from the relay station to multiple destinations. This optimization problem has both the sum power constraint and the interference leakage constraint and is solved by combining multiple constraints into a single constraint using auxiliary variables and by adapting the auxiliary variables using a sub-gradient method.

In Chapter 7, a joint beamforming and power allocation technique to maximize the total capacity of the whole network has been proposed. In the MIMO transmission between BS and RS, Singular Value Decomposition

(SVD) is utilized to design the transmit and receive beamforming vectors and the power allocation is performed using waterfilling algorithm. In the MISO transmission between RS and multiple-users with single antenna, transmit beamforming vectors are designed using BC-MAC duality. The waterfilling based power allocation at BS and the user power allocation at the RS needs to be jointly optimized so that the sum rate of users should match the back-haul BS-RS data throughput. We solved this problem using the Lagrangian optimization.

Finally, conclusions are drawn in Chapter 8 together with a discussion on possible future directions.

Chapter 2

RESOURCE ALLOCATION TECHNIQUES FOR WIRELESS COMMUNICATION NETWORKS

2.1 Introduction

The exponential growth of wireless communication systems has opened up new challenges in the way in which communication network should be designed and optimized. For wireless communication systems, one of the major challenges is the increasing demand for spectral resources created by high data rate services such as rich multimedia and interactive services. Various resource allocation techniques have been proposed to utilize the scarce resources efficiently [10,16–20]. These techniques involve strategies and algorithms for controlling transmission power, frequency allocation, modulation scheme and channel coding. The main objective of the resource allocation scheme is to make the best use of the scarce radio resources to increase spectrum efficiency as much as possible [21].

In this chapter, multi-carrier modulation techniques as well as multi-antenna techniques for general wireless communication system are introduced. First OFDM technique is discussed, followed by beamforming techniques, including receiver beamforming and transmitter beamforming. Spatial multiplexing techniques can also improve the system performance against fading. The MIMO OFDM is also studied. After that, the relay networks are introduced, specifically two way relay network and cooperative relay network are discussed in details. Finally, the concept cognitive radio is produced for high spectrum efficiency.

Multi-carrier modulation technique is an attractive solution for providing higher data throughput on limited bandwidth channels. The basic idea of multi-carrier modulation is to divide the transmitted signal into many different substreams, send over many different subchannels [16, 18, 22, 23]. Typically the subchannels are orthogonal under ideal propagation conditions. The frequency selective wide band channel is converted into frequency non-selective multiple subchannels, meaning the data stream modulated on each sub-band is facing narrowband thus improving the system's performance.

Beamforming is a signal processing technique for directional signal transmission and reception. By combining elements in a phased array, the signals at a particular angle experience constructive interference while others experience destructive interference. Beamforming takes advantage of interference to change the directionality of the array thus improves the system's performance [16, 24–26]. Spatial multiplexing technique is a transmission technique in MIMO wireless communications to transmit and receive signal using multiple antennas. The technique supports enhanced data throughput even under conditions of interference, signal fading, and multipaths [27, 28].

Relay transmission can enhance the reliability, increase the rate and save the transmission power in a wireless network. There has been many works on relay networks [15,24,26,29–43]. Two Way Relay Network (TWRN) has been considered as a good way of saving the spectrum in the communications as it significantly reduces the spectral loss caused by the half-duplex constraint in practical systems.

Cognitive radio is a form of wireless communications in which the availability of communication channels can be intelligently detected. This optimizes the use of available radio-frequency (RF) spectrum while minimizing interference to other users. White space has been introduced as an important aspect of cognitive radio in which the unused spectrum in the television frequencies are utilized [44, 45].

2.2 Multi-Carrier Modulation Techniques

Multi-carrier modulation is a technique for data-transmission by dividing a high-bit rate data stream into several parallel low bit-rate data streams and using these low bit-rate data streams to modulate several carriers. The basic idea of multi-carrier modulation is to divide the whole channel bandwidth into a number of smaller frequency bands. The number of substreams is chosen to make the symbol time on each substream much greater than the delay spread of the channel or, equivalently, to make the substream bandwidth less than the channel coherence bandwidth. This ensures that the substreams will not experience significant intersymbol interference (ISI).

Consider a linearly-modulated system with data rate R and passband bandwidth B. The signal experiences frequency-selective fading, if the coherence bandwidth for the channel is assumed to be $B_c < B$. The basic premise of multicarrier modulation is to break this wideband system into N linearly-modulated subsystems in parallel, each with subchannel bandwidth $B_N = B/N$ and data rate $R_N \approx R/N$. For N sufficiently large, the subchannel bandwidth $B_N = B/N << B_c$, which ensures relatively flat fading on each subchannel. This can be seen in the time domain: the symbol time T_N of the modulated signal in each subchannel is proportional to the subchannel bandwidth $1/B_N$. So $B_N << B_c$ implies that $T_N \approx 1/B_N >> 1/B_c \approx T_m$, where T_m denotes the delay spread of the channel. Thus, if N is sufficiently large, the symbol time is much bigger than the delay spread, so each subchannel does not experience any ISI degradation.

There are lots of useful properties such as delay-spread tolerance and spectrum efficiency in multi-carrier modulation technique. OFDM is a multi-carrier modulation technique with densely spaced subcarriers, that has gained a lot of popularity among the broadband community in the last few years.

2.2.1 Introduction to OFDM

OFDM is a multi-channel modulation scheme employing Frequency Division Multiplexing (FDM) of orthogonal subcarriers, each modulating a low bit-rate digital stream, maintaining total data rates similar to conventional single-carrier modulation schemes in the same bandwidth. Compared to FDM, which divides the bandwidth into N non-overlapping frequency subchannels, OFDM divides the bandwidth into N overlapping (but orthogonal) subcarriers using an Inverse Fast Fourier Transform (IFFT) block instead, which can overcome the problem of bandwidth wastage (the value of N is a power of 2). As seen in Fig 2.1, OFDM can save the bandwidth as compared to FDM. This ensures that the carriers are spaced at the Nyquist minimum, which in turn maximizes the bandwidth efficiency. The orthogonality of OFDM comes from the precise relationship between the subcarriers that make up one OFDM symbol. In an OFDM system, each subcarrier has exactly an integer number of cycles in a given T time interval. In other words, the number of cycles between any two adjacent subcarriers differs exactly by one. This implies that each subcarrier frequency is an integer multiple of a base frequency (that is, $f_1 = f_0$, $f_2 = 2 * f_0$, $f_3 = 3 * f_0$, and so on). These properties allow each subcarrier to be individually and independently

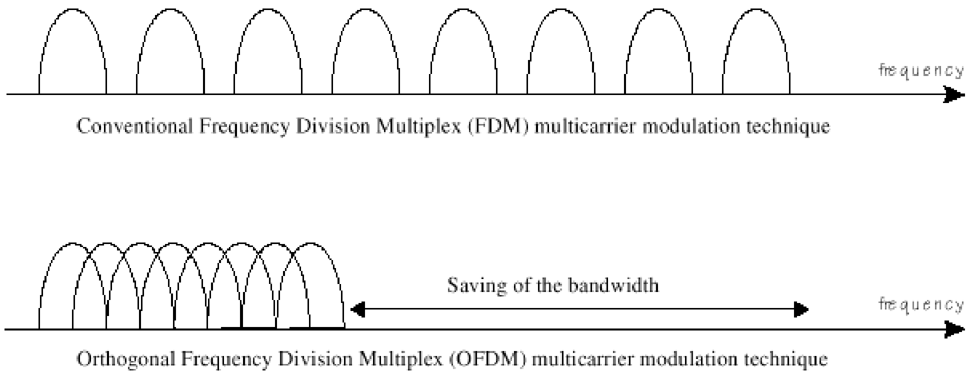

Figure 2.1. Comparison of spectrum efficiency between conventional FDM and OFDM.

demodulated from any other adjacent subcarriers. Orthogonality between each subcarrier is satisfied by

$$\int_0^T e^{j2\pi f_m t} e^{-j2\pi f_n t} f(t)dt = \begin{cases} 0, & m \neq n \\ T, & m = n, \end{cases} \tag{2.2.1}$$

In order to maintain orthogonality in an OFDM system, we have to make sure that

$$\frac{1}{T_s} = \Delta f \tag{2.2.2}$$

where Δf is the sub-carrier spacing and T_s is the symbol duration, as seen in Fig. 2.2.

If N-point Inverse Discrete Fourier Transform (IDFT) (or FFT) is used, the total bandwidth (in Hz) is $W = N\Delta f$. An equivalent frequency domain representation of Fig. 2.3 can be seen in Fig 2.4, in which we assume that it is a 512-point FFT, the frequency of different signals $f_1 = f_0 = 1Hz$, $f_2 = 2 * f_0 = 2Hz$, $f_3 = 3 * f_0 = 3Hz$, $f_4 = 4 * f_0 = 4Hz$, respectively, and the sampling frequency $f_s = 80Hz$. Later an extended symbol period is introduced which adds the duration of Cyclic Prefix (CP) T_{cp} into symbol duration T_s in order to eliminate the ISI from the previous symbol.

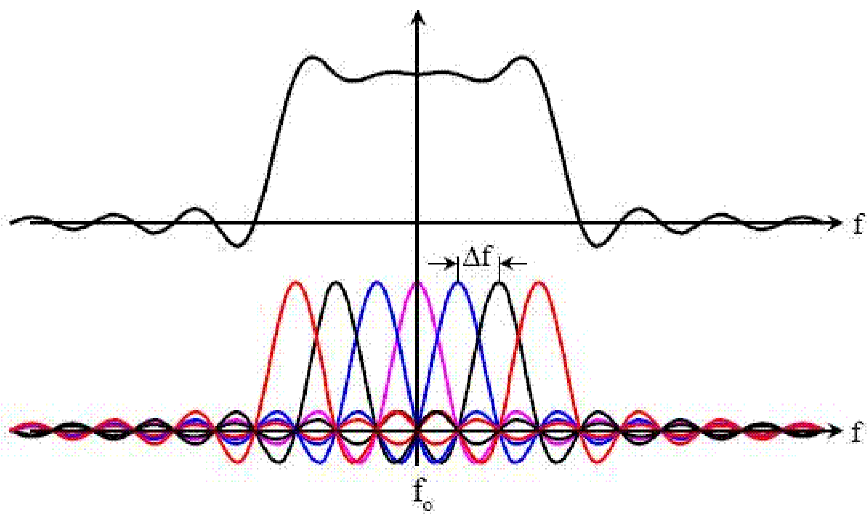

Figure 2.2. OFDM signal consisting of several narrow sub-carriers.

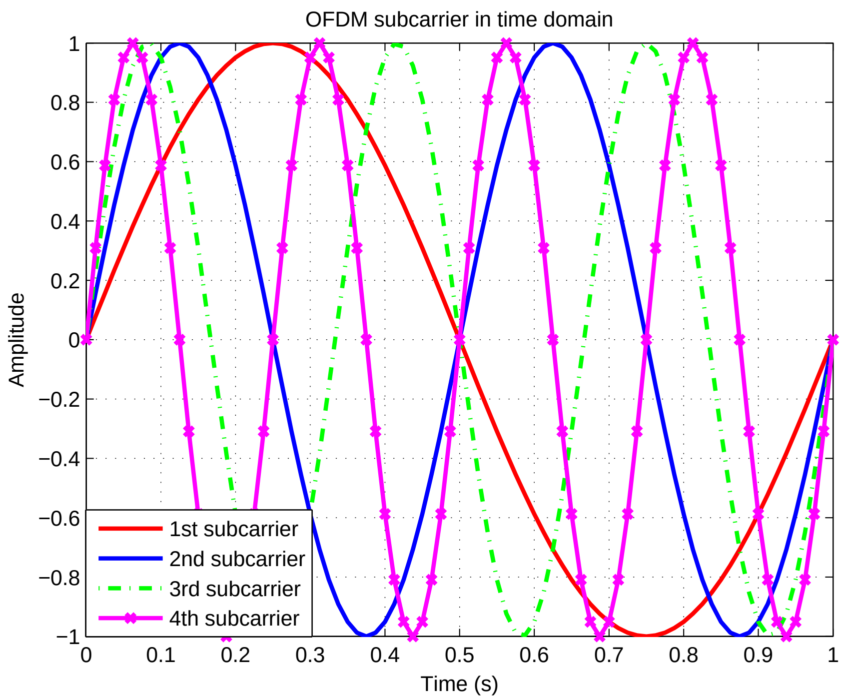

Figure 2.3. OFDM subcarrier in the time domain.

2.2.2 The Cyclic Prefix

The robustness against delay spread is one of the main advantages of OFDM over a single-carrier system. Channel delay spread causes ISI which causes irreducible error floor, hence limiting the maximum data rate. Since symbol

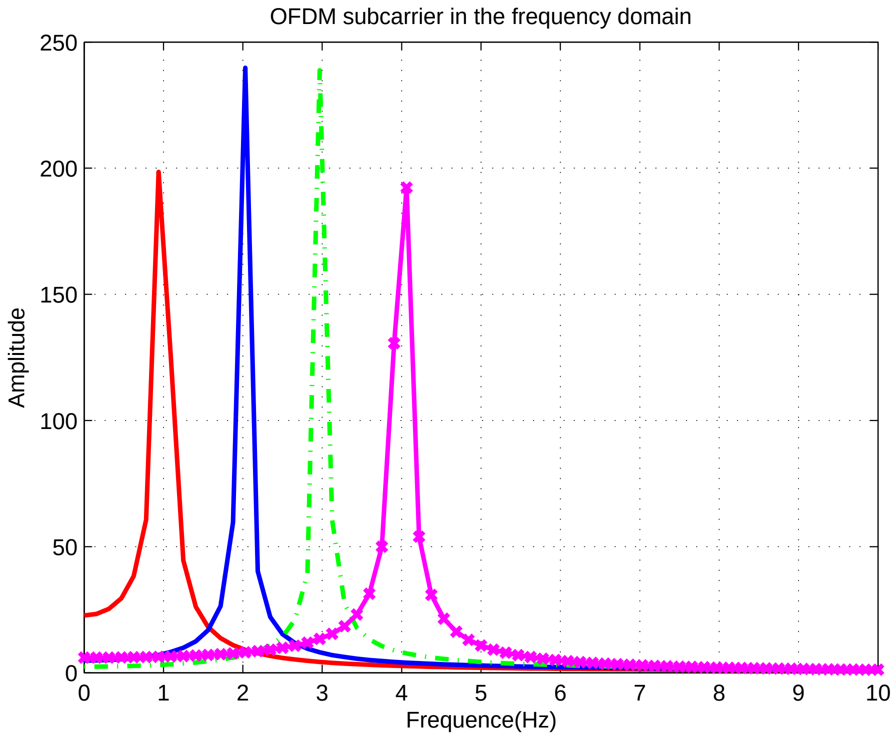

Figure 2.4. The equivalent frequency domain subcarrier representation of Fig. 2.3

duration of each subcarrier in OFDM is N times longer than that of a single carrier system, OFDM is more robust to delay spread. However, OFDM could still suffer from ISI. CP can be used to avoid ISI. The use of CP allows to represent the linear convolution of the channel impulse response in terms of cyclic convolution. The length of CP is equal to or longer than maximum channel delay spread. The extended OFDM symbol is showen in Fig. 2.5.

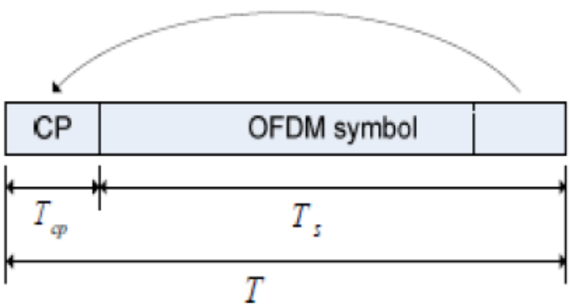

Figure 2.5. An OFDM symbol and cyclic prefix.

2.2.3 Matrix Representation of OFDM

In order to explain how the CP in OFDM can help to eliminate ISI, a matrix representation is produced. We assume that the input symbols $\{s_k(i)\}_{i=1}^{N}$ denote the transmit symbols for the k^{th} OFDM block. N denotes the number of OFDM subcarriers (the number of constellation symbols to be transmitted in one OFDM block). An N-point IFFT is taken to get $\{x_k(i)\}_{i=1}^{N}$ after serial to parallel conversion of the input symbol stream. We add a cyclic redundancy of length ν (the number of CP samples) as a prefix in such a way that $x(k)(-i) = x(k)(N-i)$ for $i = 1, 2, \ldots, \nu$ in front of the first symbol $x_k(0)$.

The signal is then transmitted on a multipath channel with the Channel Impulse Response (CIR) of the multipath channel of length L denoted by the vector as

$$\boldsymbol{h} = [h_0 \;\; h_1 \;\; \cdots \;\; h_L]^T \in \mathbb{C}^{L\times 1}.$$

The k^{th} channel output symbol

$$\boldsymbol{y}^k = \begin{bmatrix} y_0^k & y_1^k & \cdots & y_{N-1}^k \end{bmatrix}^T \in \mathbb{C}^{N\times 1}$$

can be expressed using matrix notation in terms of the transmitted samples $x_k(i)$ and noise vector $\eta_k \in \mathbb{C}^{N\times 1}$ as,

$$\begin{bmatrix} y_k(0) \\ y_k(1) \\ \vdots \\ \vdots \\ \vdots \\ y_k(N-1) \end{bmatrix} = \begin{bmatrix} h_{L-1} & \ldots & h_\nu & \ldots & h_0 & 0 & \ldots & \ldots & \ldots & 0 \\ 0 & \ddots & & \ddots & & \ddots & \ddots & & & \vdots \\ \vdots & & \ddots & & \ddots & & \ddots & \ddots & & \vdots \\ \vdots & & & \ddots & & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & & & \ddots & & \ddots & & \ddots & 0 \\ 0 & \ldots & \ldots & \ldots & \ldots & h_{L-1} & \ldots & h_\nu & \ldots & h_0 \end{bmatrix} \begin{bmatrix} x_{k-1}(N-E) \\ \vdots \\ x_{k-1}(N-1) \\ x_k(-\nu) \\ \vdots \\ x_k(0) \\ \vdots \\ \vdots \\ x_k(N-1) \end{bmatrix} + \tilde{\eta}_k$$

where $E = L - \nu - 1$ is the channel length exceeding the duration of CP ν. We can see that the entities marked in red color appear only if CIR length L exceeds the duration of CP, i.e. $E > 0$ and thereby contribute to what is called ISI.

For the reason stated above, we consider the CP length to be greater than CIR length, and with incorporation of CP property $x_k(-i) = x_k(N-i)$ for $i = 1, 2, \ldots, \nu$, the k^{th} channel output symbol can be expressed as

$$\begin{bmatrix} y_k(0) \\ y_k(1) \\ \vdots \\ \vdots \\ \vdots \\ y_k(N-1) \end{bmatrix} = \begin{bmatrix} h_0 & 0 & \ldots & 0 & h_{L-1} & \ldots & h_1 \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & h_{L-1} \\ h_{L-1} & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \ldots & 0 & h_{L-1} & \ldots & \ldots & h_0 \end{bmatrix} \begin{bmatrix} x_k(0) \\ x_k(1) \\ \vdots \\ \vdots \\ \vdots \\ x_k(N-1) \end{bmatrix} + \tilde{\eta}_k$$

As can be seen, by using long enough CP, the linear convolution of the channel impulse response and the signal becomes cyclic convolution, and the previous symbols will not affect the current one any more and ISI has been eliminated. Since any IDFT and DFT matrix can diagonalize a circulant matrix, IDFT at the transmitter and DFT at the receiver can diagonalize the channel yielding orthogonal subcarrier for OFDM transmission.

2.2.4 Overview of OFDMA

Earlier OFDM has been used as a modulation scheme for wireless systems, where all the subchannels of OFDM are assigned to a single user at any given time (i.e., IEEE 802.11a/g). Later TDMA or FDMA has been used with OFDM in order to support multiple users. This kind of static resource allocation cannot provide a good performance. Disadvantage of this static resource allocation is that multi-user diversity is not exploited (i.e., different users have different channel gains on same subchannel). OFDMA has been developed to exploit the multi-user diversity where multiple users are allowed to transmit simultaneously on different subchannels per OFDM symbol. The probability that all users experience worst channel gain in a particular subchannel is typically quite low. Hence, adaptive resource allocation algorithms can be developed to efficiently allocate resources to multiple users by exploiting the multi user diversity.

2.2.5 Resource allocation techniques

Adaptive resource allocation techniques allocate radio resources to various users according to users channel gain and QoS requirements. The problem of assigning the subchannels and transmission power to different users in an OFDMA system has been intensively studied over the past decade (i.e., [46–51] and references therein). All of these studies can be divided into two categories namely Margin Adaptive (MA) and Rate Adaptive (RA) resource allocation problems [46–48]. The objective of the MA problem is to minimize the total transmission power subject to users' individual data rate constraint BER requirements. The objective of the RA is to maximize system data throughput subject to a total transmission power constraint. Various resource allocation techniques available for OFDMA can also be used in cognitive radio networks to allocate radio resources to secondary users while maintaining the interference leakage to the primary user below a threshold. Various work proposed for cognitive radio networks based on OFDMA technique and/or multiple antennas technique are reviewed as follows.

The OFDMA has been considered as one of the powerful techniques for cognitive radio network due to its natural ability to use different portions of the spectrum. The work in [20, 52] introduced the resource allocation technique used in OFDMA-based cognitive radio networks. The work in [52] proposed an adaptive radio resource allocation algorithm for a MIMO OFDMA based uplink cognitive radio network with multiple secondary users (SUs) and multiple primary users (PUs). The aim is to admit as many SUs as possible in various subcarriers while ensuring interference is not leaked to PUs. In [20], resource allocation problem has been considered for a multiuser OFDM-based cognitive radio system under a non-realtime (NRT) user scheme. In this work, the user data rates have been maintained proportionally while at the same time providing an improved system throughput. In [53], power is allocated to each subchannel, by considering the received interference as a fairness metric which has been solved using Lagrangian dual function. Due to high computational complexity of determining optimal solution for OFDMA based resource allocation problem, some low complexity algorithms were proposed in [18, 54]. The work in [18, 54] considered a single user cognitive radio network and maximized the cognitive radio network throughput by allocating data bits to various subchannels, optimally.

The work in [55], presents a systematic method of distributed algorithms for power control based on geometric-programming, which forms important resource allocation technique for beamforming in chapter 7. Asymmetric resource allocations for the temporal power in different time slots have been proposed in the works of [56] and [43], respectively. In these works, Lagrangian optimization and the "waterfilling" algorithm are utilized to determine power allocations in different subchannels for OFDMA. These are basis for the techniques proposed in chapters 6 and 7.

In this thesis, resource allocation for wireless network based on beamforming technique and spatial multiplexing technique is considered. Hence, a brief description about beamforming technique, spatial multiplexing technique and resource allocation based on both these techniques are provided in the following sections.

2.3 Beamforming Techniques

Applications of beamforming can be found in many disciplines including radars, sonars, and communications. It can be considered as a marriage between antenna technology and digital technology. In communication systems, the duty of an antenna is to receive or transmit electromagnetic waves. For beamforming, as can be seen in Fig. 2.6, there are several antennas beside each other.

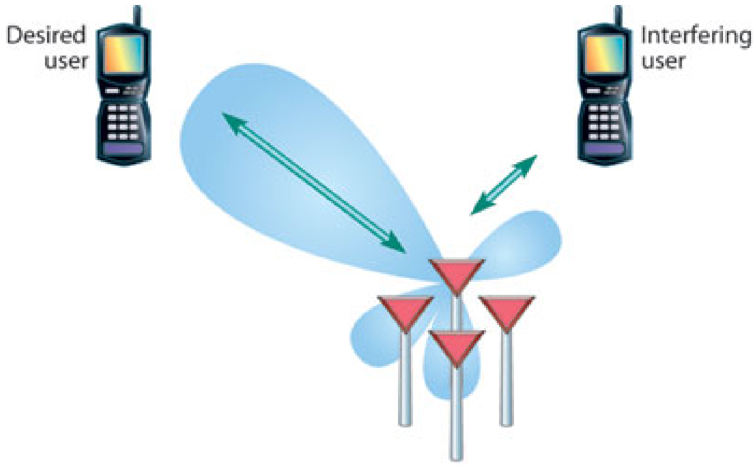

Figure 2.6. The beamforming techniques.

Each antenna on its own is omni directional. By applying phase shifts on the signals of each antenna and summing the results, a beam pattern can be produced as a response to the total array of antennas. By changing the phase or time delay of the signals the beam pattern can be steered toward different directions and this is called beamforming. Beamforming is a signal processing technique used in the physical layer of a communication network to control the directionality of transmission or reception of a signal using antenna array at the transmission or at the reception [57]. Beamforming has many benefits:

1. Higher SNR: The highly directional transmission improves the link budget. This can be used to increase coverage.

2. Interference avoidance and rejection: Beamforming overcomes ex-

ternal and internal co-channel interference (CCI) by exploiting the spatial properties of the antennas. Since the interference comes from a certain direction, the beamformer can be applied to place nulls towards interference.

3. Higher network efficiency: By significantly reducing the CCI, beamforming can allow much denser deployments than single antenna systems. Due to higher link budget, the likelihood of running high-order modulations (64QAM, 16QAM) is much higher even at the edges of the cell. Overall capacity is greatly improved.

The structure of beamforming for reception and transmission of signals has been introduced as follow:

2.3.1 Receiver Beamforming Techniques

In receiver side beamforming, multiple antennas are deployed at the receiver. When only one antenna is used at the transmitter, this scheme is referred to as Single-Input-Multiple-Output (SIMO) in literature. In the receiver beamforming design, the objective is to estimate the desired signal in the presence of noise and interference. Fig. 2.7 depicts a receiver beamformer structure. The output of the beamformer can be written as

$$y(n) = \mathbf{w}^H \mathbf{r}(n), \tag{2.3.1}$$

where n is the time index, $\mathbf{r}(n) = [r_1(n) \;\cdots\; r_M(n)]^T$ is the $M \times 1$ received signal vector and $\mathbf{w} = [w_1 \;\cdots\; w_M]^T$ is the complex beamforming weight vector. The received signal vector is given by

$$\mathbf{r}(n) = \mathbf{d}(n) + \mathbf{i}(n) + \boldsymbol{\eta}(n), \tag{2.3.2}$$

where $\mathbf{d}(n)$, $\mathbf{i}(n)$ and $\boldsymbol{\eta}(n)$ are the desired signal, interference and receiver noise respectively. Hence, the received signal-to-noise-plus-interference ratio

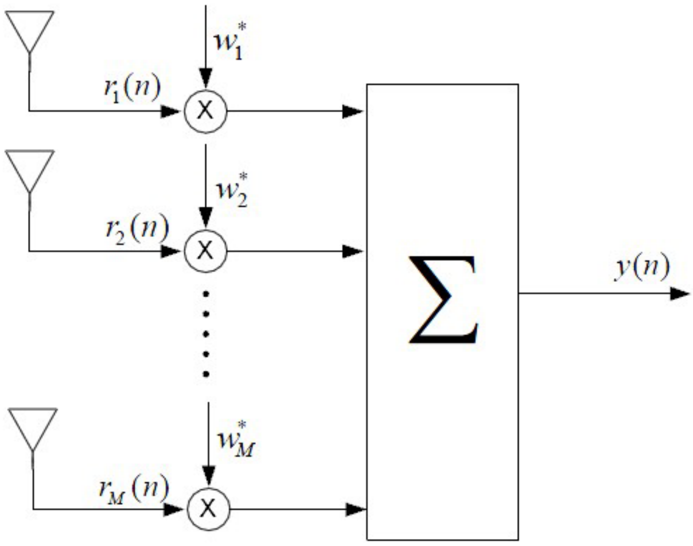

Figure 2.7. The receiver beamformer design.

(SINR) at the receiver can be given as follows [58]:

$$\text{SINR} = \frac{\mathbf{w}^H \mathbf{R}_d \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}, \tag{2.3.3}$$

where $\mathbf{R}_d = \mathrm{E}\left\{\mathbf{d}(n)\mathbf{d}(n)^H\right\}$ and $\mathbf{R}_{i+n} = \mathrm{E}\left\{[\mathbf{i}(n) + \boldsymbol{\eta}(n)]\,[\mathbf{i}(n) + \boldsymbol{\eta}(n)]^H\right\}$ are the signal and interference-plus-noise covariance matrices. The optimum beamforming weight vector that maximizes the SINR of received signal can be obtained as a generalized eigenvalue solution as follows [28]

$$\mathbf{R}_{i+n}^{-1} \mathbf{R}_d \mathbf{w} = \lambda_{\max} \mathbf{w}, \tag{2.3.4}$$

where $\lambda_{\max}$ is the maximum eigenvalue of the matrix $\mathbf{R}_{i+n}^{-1}\mathbf{R}_d$.

Proof: See Appendix A. ■

Hence, the optimal beamformer weight vector is equivalent to generalized eigenvector of the matrices $[\mathbf{R}_d,\ \mathbf{R}_{i+n}]$ [28].

2.3.2 Transmitter Beamforming Techniques

For the transmit beamforming multiple antennas are deployed at the transmitter. When the receiver employs only a single antenna, the scheme is also known as Multiple-Input-Single-Output (MISO) system. Beamforming at the transmitter is substantially different in several aspects as compared to a beamformer at the receiver. In the latter, the design will only determine the performance of a specific user whereas the transmit beamformer will affect not only the desired user but also all the users in the coverage area. Hence, the transmit beamforming design should ideally take into consideration the system level performance, i.e., all the users in the reception area rather than a specific set of users. Another fundamental difference is the channel knowledge. For receiver beamformer design, the receiver could estimate the channel coefficients using the training signal. For transmitter beamformer design, the channel knowledge could be made available to the transmitter by sending the estimates of the Channel State Information (CSI) from the receiver through a finite rate feedback channel [59–61] .

The transmitter beamforming can be used to enhance SINR of a particular user in the network. However, by steering multiple beams, multiple users can access the same frequency band simultaneously. This is known as spatial multiplexing. The focus of this section is on multiuser spatial multiplexing. The transmit beamformers can be designed to satisfy QoS requirements for each user i.e., received SINR for each user. Consider a wireless network base station equipped with N_t transmit antennas serving K users. Each user is equipped with a single antenna. The signal transmitted by the base station is given by

$$\mathbf{x}(n) = \mathbf{W}\mathbf{s}(n), \tag{2.3.5}$$

where $s(n) = [s_1(n) \cdots s_K(n)]^T$, $s_k(n) \quad (k = 1, 2, \cdots, K)$ is the symbol intended for the k^{th} user, $\mathbf{W} = [\mathbf{w}_1 \ \cdots \ \mathbf{w}_K]$ and $\mathbf{w}_k \in \mathbb{C}^{N_t \times 1}$ is the transmit beamforming weight vector for the k^{th} user. The received signal at the k^{th} receiver can be written as

$$y_k(n) = \mathbf{h}_k^H \mathbf{x}(n) + \eta_k(n), \tag{2.3.6}$$

where $\mathbf{h}_k$ is the channel coefficient vector between the base station and the k^{th} user and $\eta_k(n)$ is receiver noise. By defining $\mathbf{R}_k \triangleq \mathbf{h}_k \mathbf{h}_k^H$, the SINR of the k^{th} user can be written as

$$\mathrm{SINR}_k = \frac{\mathbf{w}_k^H \mathbf{R}_k \mathbf{w}_k}{\sum_{i \neq k} \mathbf{w}_i^H \mathbf{R}_k \mathbf{w}_i + \sigma_k^2}, \tag{2.3.7}$$

where σ_k^2 is the noise variance at the i^{th} receiver.

The transmit beamforming problem based on SINR requirements can be formulated as minimization of the transmitted power at the base station subject to each user SINR being greater than a target value [62, 63].

$$\begin{aligned} \underset{\mathbf{w}_i}{\text{minimize}} \quad & \sum_{i=1}^{K} \|\mathbf{w}_i\|_2^2 \\ \text{subject to} \quad & \mathrm{SINR}_i \geq \gamma_i \quad i = 1, \cdots, K. \end{aligned} \tag{2.3.8}$$

This problem can be converted into a SDP with Lagrangian relaxation and can be efficiently solved using convex optimization toolboxes [64–66]. However, it is quite difficult to predict in advance whether the problem in (2.3.8) with a given set of target SINRs and total transmit power at the base station is feasible.

To overcome this infeasibility issue, this problem can be formulated into a more attractive framework based on a max-min fairness approach, where the worst-case user SINR is maximized while using the available total transmission power [28]. This is known as the SINR balancing technique and it can be formulated as [28, 67–70]

$$\begin{aligned} \underset{\mathbf{U}, \mathbf{p}}{\text{maximize}} \quad & \min_{1 \le i \le K} \cdot \frac{\mathrm{SINR}_i(\mathbf{U}, \mathbf{p})}{\gamma_i}, \ i = 1, \ldots, K \\ \text{subject to} \quad & \mathbf{1}^T \mathbf{p} \le \mathrm{P}_{\max}, \end{aligned} \tag{2.3.9}$$

where $\mathbf{U} = [\mathbf{u}_1 \ \cdots \ \mathbf{u}_K]$, $\|\mathbf{u}_k\|_2 = 1$, and $\mathbf{p} = [p_1 \ \cdots \ p_K]^T$. Here $\mathbf{u}_k \in \mathbb{C}^{N_t \times 1}$ and p_k are the transmit beamforming weight vector and the corresponding allocated power for the k^{th} user respectively. In [28], an iterative algorithm has been proposed using uplink-downlink duality, where the solution balances the ratio between the achieved SINR and the target SINR for all users while using all the available transmission power at the base station.

2.3.3 Related works on Beamforming

The power allocation and beamforming problems have been widely studied to control interference among users in [62, 63]. In [71], an optimal downlink power assignment technique has been proposed for a given set of beamforming weight vectors. This power allocation problem is formulated into an eigenvector matrix equation and the optimal power allocations have been obtained by finding the eigenvector corresponding the largest eigenvalue of the matrix. The property that all elements of the eigenvector corresponding to the largest eigenvalue of a non-negative matrix are always positive [72], has been exploited in [71]. In [73], an iterative algorithm has been proposed to jointly design the beamforming weight vectors and the power allocation vectors in the uplink and the downlink. This design ensures that SINR of each user is above a threshold while minimizing the total transmission power. The same problem has been formulated into an SDP in [62, 63] by using Lagrangian relaxation and it has been solved using interior point methods [74]. This relaxed problem provides a rank-one solution for each user and the optimal beamforming weight vector has been determined by extracting the eigenvector corresponding to the positive eigenvalue of the matrix. In addition, it has been proved that the relaxed problem always yields an optimal rank-one solution. In [75], a special scenario has been considered where the transmitter sends the same data to multiple users known as multicasting. In the multicasting setup, the SDP formulation might not always provide a rank-one solution. To overcome this problem, a randomization technique [76] has been recommended to find an optimal solution. The problem of transmit beamforming to multiple cochannel multicast groups is considered in [77,78], where QoS and the max-min fairness approaches have been presented using convex optimization and randomization techniques.

2.4 Spatial Multiplexing Techniques

The spatial multiplexing facilitates simultaneous transmission of different signals or data bits through several independent (spatial) communication channels by multiple antennas. The receiving side also uses multiple antennas for decoding the signals. Since data is transmitted simultaneously in the same frequency band, spatial multiplexing enhances the spectrum efficiency substantially.

2.4.1 MIMO Techniques

The technique using multiple antennas at both the transmitter and the receiver is known as spatial multiplexing or MIMO array processing. The MIMO array processing can improve the system performance by adding additional diversity against fading, as compared to the use of only multiple receive antennas or multiple transmit antennas [79–81]. Spatial multiplexing can be obtained by decomposing the MIMO channel matrix into various independent spatial subchannels that are used to transmit different data streams independently. This has the potential to increase the data rate up to a factor that is the same as the rank of the MIMO matrix [82, 83]. Consider a point-to-point MIMO channel with M_t transmit antennas and M_r receive antennas as shown in Fig. 2.8. The received signal is given by

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \boldsymbol{\eta}(n), \tag{2.4.1}$$

where $\mathbf{y} = [y_1(n) \ \cdots \ y_{M_r}(n)]^T$ and $y_r(n)$ is the received signal at the r^{th} receiver antenna. $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ and h_{ij} is the complex channel gain between the i^{th} transmitter antenna and j^{th} receiver antenna. $\mathbf{x}(n) \in \mathbb{C}^{M_t \times 1}$ and $\boldsymbol{\eta}(n) \in \mathbb{C}^{M_r \times 1}$ are the transmitted symbol vector and the noise vector at the receiver end respectively. It is assumed that the channel gain matrix $\mathbf{H}$ is

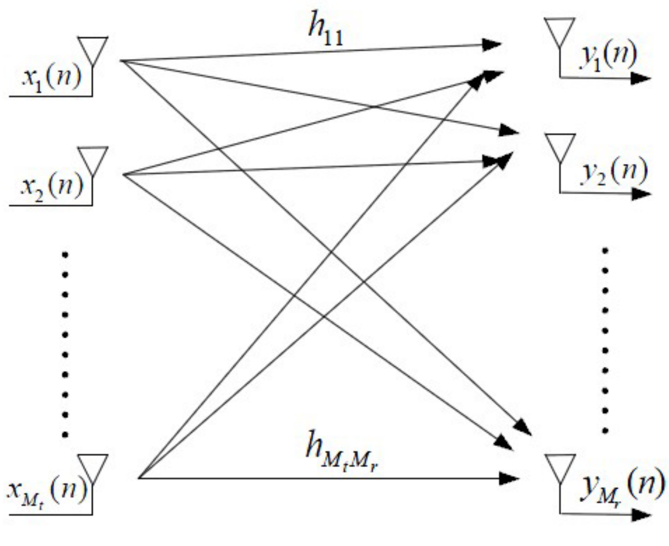

Figure 2.8. A MIMO system with M_t transmit antennas and M_r receive antennas..

known to both the transmitter and the receiver. The MIMO channel matrix $\mathbf{H}$ can be decomposed using the Singular Value Decomposition (SVD) as [84]

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H, \tag{2.4.2}$$

where $\mathbf{U} \in \mathbb{C}^{M_r \times M_r}$ and $\mathbf{V} \in \mathbb{C}^{M_t \times M_t}$ are unitary left and right singular matrices of $\mathbf{H}$. $\boldsymbol{\Sigma} \in \mathbb{R}^{M_r \times M_t}$ is a diagonal matrix consisting of the singular values (υ_i) of $\mathbf{H}$. R_H number of singular values are nonzero, so that R_H is the rank of the matrix $\mathbf{H}$. The singular value satisfies the property $\upsilon_i = \sqrt{\lambda_i}$, where λ_i is the i^{th} eigenvalue of $\mathbf{H}\mathbf{H}^H$. These MIMO spatial subchannels are obtained using linear transformation of the input signal and the output signal through transmit precoding and receiver shaping. In transmit precoding, the modulated symbol is precoded as

$$\mathbf{x} = \mathbf{V}\tilde{\mathbf{x}}, \tag{2.4.3}$$

where $\tilde{\mathbf{x}}$ is the modulated symbol stream. Similarly, the received signal is shaped as

$$\tilde{\mathbf{y}} = \mathbf{U}^H\mathbf{y} \tag{2.4.4}$$

as shown in Fig. 2.9. Such transmit precoding and receiver shaping de-

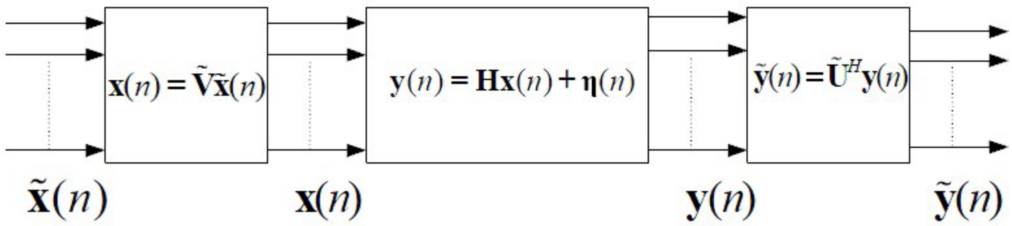

Figure 2.9. Transmit precoding and receiver shaping.

compose the MIMO channel into R_H number of independent single-input single-output (SISO) channels as follows:

$$\begin{aligned}
\tilde{\mathbf{y}} &= \mathbf{U}^H\left(\mathbf{H}\mathbf{x} + \boldsymbol{\eta}\right) \\
&= \mathbf{U}^H\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathbf{H}}\mathbf{V}\tilde{\mathbf{x}} + \mathbf{U}^H\boldsymbol{\eta} \\
&= \boldsymbol{\Sigma}\tilde{\mathbf{x}} + \tilde{\boldsymbol{\eta}}
\end{aligned} \tag{2.4.5}$$

where $\tilde{\boldsymbol{\eta}} = \mathbf{U}^H\boldsymbol{\eta}$. The resulting parallel spatial subchannels are shown in Fig. 2.10. They are independent from each other in the sense that signals through each spatial subchannels do not interfere with each other. Hence

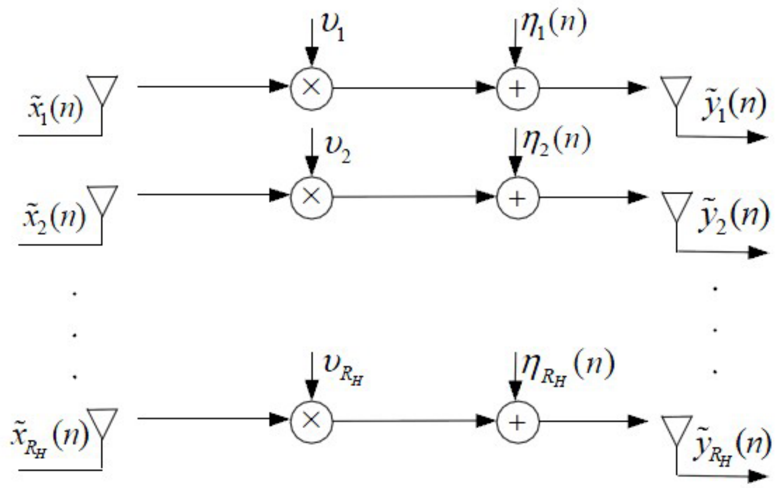

Figure 2.10. Parallel decomposition of the MIMO channel.

this MIMO channel can support upto R_H times the data rate of a SISO channel.

2.4.2 MIMO-OFDM

MIMO wireless technology in combination with orthogonal frequency division multiplexing (MIMO-OFDM) is an attractive air-interface solution for next-generation wireless local area networks (WLANs), wireless metropolitan area networks (WMANs), and fourth-generation mobile cellular wireless systems [85–88]. The main motivation for using OFDM in a MIMO channel is the fact that OFDM modulation turns a frequency-selective MIMO channel into a set of parallel frequency-flat MIMO channels. MIMO-OFDM combines OFDM and MIMO techniques thereby achieving spectral efficiency and increased throughput. A MIMO-OFDM system transmits independent OFDM modulated data from multiple antennas simultaneously. At the receiver, after OFDM demodulation, MIMO decoding on each of the subchannels extracts the data from all the transmit antennas on all the subchannels, as depicted in Fig. 2.11.

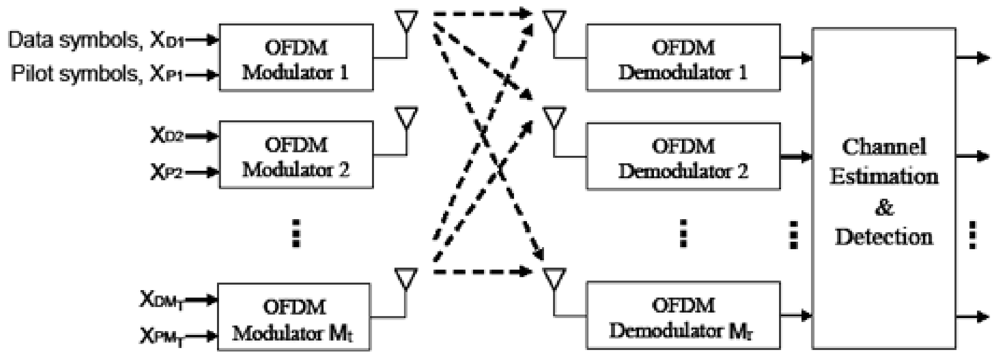

Figure 2.11. MIMO-OFDM system

Because the OFDM system effectively provides numerous parallel narrowband channels, MIMO-OFDM is considered a key technology in emerging high-data rate systems such as 4G, IEEE 802.16, and IEEE 802.11n [52, 87–89]. For a MIMO system with N_T transmit antennas and N_R receive antennas, the channel equation can be expressed as $N_T \times N_R$ matrix for each OFDM subcarrier. Each element is denoted by $h_{i,j,k}$, where i denotes the i^{th} transmitting antenna, j denotes the j^{th} receiving antenna and k denotes the k^{th} subcarrier. For simplicity, the subscript of the k^{th} subcarrier is ignored. Therefore, the MIMO channel matrix shown in Fig. 7.2.2 can be express as:

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{2,1} & \cdots & h_{N_T,1} \\ h_{1,2} & h_{2,2} & \cdots & h_{N_T,2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{1,N_R} & h_{2,N_R} & \cdots & h_{N_T,N_R} \end{bmatrix} \tag{2.4.6}$$

The transmit signal vector from N_T antennas is shown below:

$$x = [x_1 x_2 \cdots x_{N_T}] \tag{2.4.7}$$

Thus the received signal vector can be expressed using matrix multiplication in the frequency domain, as shown below:

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{N_R} \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{2,1} & \cdots & h_{N_T,1} \\ h_{1,2} & h_{2,2} & \cdots & h_{N_T,2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{1,N_R} & h_{2,N_R} & \cdots & h_{N_T,N_R} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_T} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{N_T} \end{bmatrix} \tag{2.4.8}$$

The MIMO technique explained earlier can be applied to each subcarrier.

2.4.3 Related works on Spatial Multiplexing Technique

The MIMO-OFDM techniques have been widely studied in the literature from different perspectives including transceiver design [90, 91], resource allocations [34, 92, 93] and estimation [94–96] and synchronization [97, 98]. In the work of [95,96], the channel estimation has been considered and the channel capacity have been increased in a MIMO-OFDM system. Rate balancing has been achieved in [22]. An adaptive MMSE multiuser detection scheme has been combined with prior information of the channel and interference cancelation in the spatial domain in [99].

Resource allocation technique based on combined MIMO and OFDM techniques has been identified as one of the most promising physical-layer feature for achieving high capacity, high spectral efficiency, and good performance in dispersive channels [100–102]. Multiuser wireless system based on MIMO-OFDM is considered in [100]. In [100] multiple users are allocated in each OFDM subchannel based on their spatial correlation between the users. The spatial correlation between two users were obtained using the vector multiplication of both users' spatial subchannel gains. High spatial correlation between users can cause high inter-user interference. Hence, users with lower spatial correlation are allocated in the same subchannel. After user-subchannel allocation, greedy algorithm is used to allocate transmission power in order to satisfy the QoS requirement for each user.

2.5 Wireless Relay Networks

A relay network refers to a broad class of network topology commonly used in wireless networks, where the source and destination are interconnected by means of some nodes. In such a network the source and destination cannot communicate to each other directly because the distance between the source and destination is greater than the transmission range of both of them, hence the need for intermediate node(s) known as relays.

2.5.1 One Way Relay Network

Various relay communication techniques have been studied to improve the capacity and/or expand the coverage of wireless networks [3,15,26,29,37,92]. A classical three-node relaying model has been originally proposed by Van der Meulen in 1971. This network as shown in Fig. 2.12 includes three basic factors: only one source, such as Mobile Multihop Relay-Base Station (MMR-BS); a relay, that may be fixed, nomadic, or mobile, and only one destination, for instance, Mobile Station (MS), or User Terminal (UT).

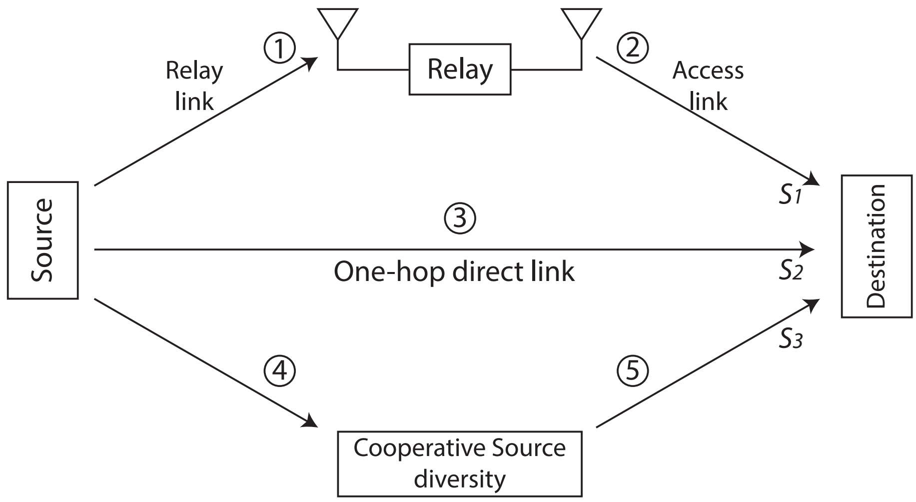

Figure 2.12. Three-node relay network and cooperative relaying.

The physical channels between source and relay, as well as the one between relay and destination are called relay link and access link respectively.

The relay does not receive and transmit using the same channel simultaneously. In other words, the relay works in half-duplex mode. This is because the received signal is difficult to be separated from the transmitted signal. Therefore, there are two orthogonal subchannels with the channel which is allocated for the relay operation. Take Time Division Duplex (TDD) as an example. There are two orthogonal subchannels of consecutive time slots. The relay receives during the first time slot and uses the second time slot for retransmission.

The concept of cooperative relaying is based on the fact that a signal can be received by multiple terminals and transmitted to destination. The inherent diversity of the relay channel is frequently exploited by the combining of received signal constructively at the destination. The details of various level of cooperation will be discussed later.

A cooperative relay network that models transmission between one pair of source-destination through a set of N relay nodes is shown in Fig. 2.13.

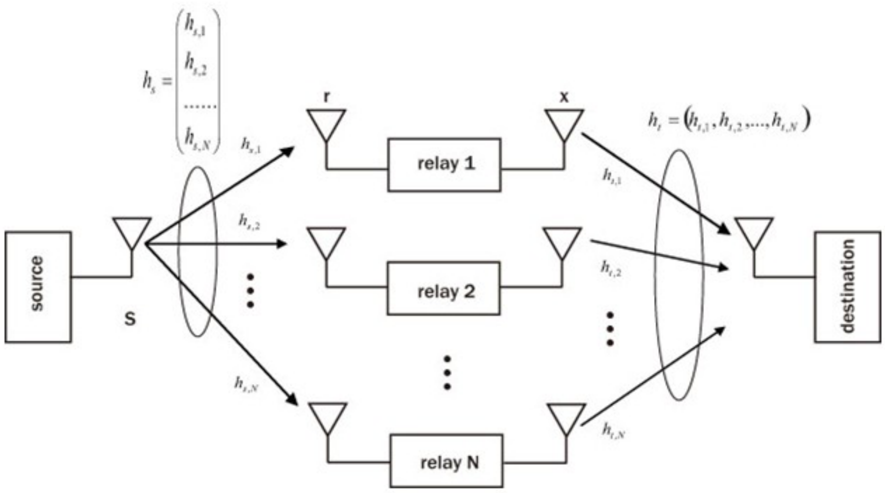

Figure 2.13. One way relay network

A two-phase protocol is employed for transmission of data where the relays in the network observe signal **s** transmitted by the source in the first phase (broadcasting phase). The relays transmit their signals to the destination during the second phase (relaying phase). Let $\mathbf{h}_s \in \mathbb{C}^{N\times 1}$ denote the channel vector consisting of complex channel coefficients between the source and the relay nodes

$$\mathbf{h}_s = [h_{s,1}, h_{s,2}, \cdots, h_{s,N}]^T, \tag{2.5.1}$$

The received signal vector at the relay nodes is given as

$$\mathbf{r} = \mathbf{h}_s s + \mathbf{v}_s, \tag{2.5.2}$$

where $\mathbf{v}_s \in \mathbb{C}^{N\times 1}$ is zero-mean circularly symmetric complex additive white Gaussian noise (AWGN) vector with covariance matrix $\sigma^2_{v_s} I$. The channel is assumed to be quasi-static fading so that the channel realizations stay fixed for the duration of a number of frames. We assume relays have perfect knowledge of the forward channels through feedbacks from the destination sensors. In the second phase of transmission, the relays rebroadcast a transformed signal vector as

$$\mathbf{x} = \mathbf{F}\mathbf{r}, \tag{2.5.3}$$

where $\mathbf{F} \in \mathbb{C}^{N\times N}$ is a linear transformation matrix (relay transceiver) to be determined in order to optimize receiver performance. The received signal at the destination can be written as

$$y = \mathbf{h}_t^T \mathbf{x} + v_t, \tag{2.5.4}$$

where $\mathbf{h}_t \in \mathbb{C}^{N\times 1}$ denotes the channel vector consisting of complex channel coefficients between the relay nodes and the destination sensors,

$$\mathbf{h}_t = [h_{t,1}, h_{t,2}, \cdots, h_{t,N}]^T, \tag{2.5.5}$$

where v_t is a zero-mean circularly symmetric AWGN scalar with covariance

$\sigma^2_{v_t}$. Substituting (2.5.3) into (2.5.4) and using (2.5.2), we obtain

$$y = \mathbf{h}_t^T \mathbf{F} \mathbf{h}_s \mathbf{s} + \mathbf{h}_t^T \mathbf{F} \mathbf{v}_s + v_t. \tag{2.5.6}$$

One way relay transmission is not spectrally efficient as it requires two time slots for the overall transmission. However, this inefficiency can be avoided by using two way relay as discussed in the next section.

2.5.2 Two Way Relay Network (TWRN)

To reduce the use of extra channel resources and to improve spectral efficiency, TWRN methods (bidirectional communications) between two users have been studied. The TWRN is one of the basic elements in decentralized/centralized wireless networks. The simplest TWRN consists of two source nodes, $S1$ and $S2$, which exchange information through a relay node. TWRN communications complete the data exchange between two users through two phases: receive-phase (first-phase) and transmit-phase (second-phase). In the receive-phase, the relay receives data from the two users simultaneously, and in the transmit-phase, it retransmits the received signals to the two users.

A TWRN with one pair of source nodes, A and B, and a set of N relay nodes is shown in Fig. 2.14. For simplicity, only one antenna at the source nodes and relay nodes is considered. In the first phase (broadcasting phase) signals s and b are transmitted from a set of source nodes A and B respectively. The variances of the signals at the A and B sides are denoted by σ_s^2 and σ_b^2 respectively. The signals are received by a set of relay nodes. The relay nodes transmit the received data to the destination nodes in the second phase (relaying phase). Let us denote the vectors consisting of complex channel coefficients from the source nodes A to the relay nodes, $\mathbf{h} = [h_1, h_2, \cdots, h_N]^T$ and from the source nodes B to the relay nodes,

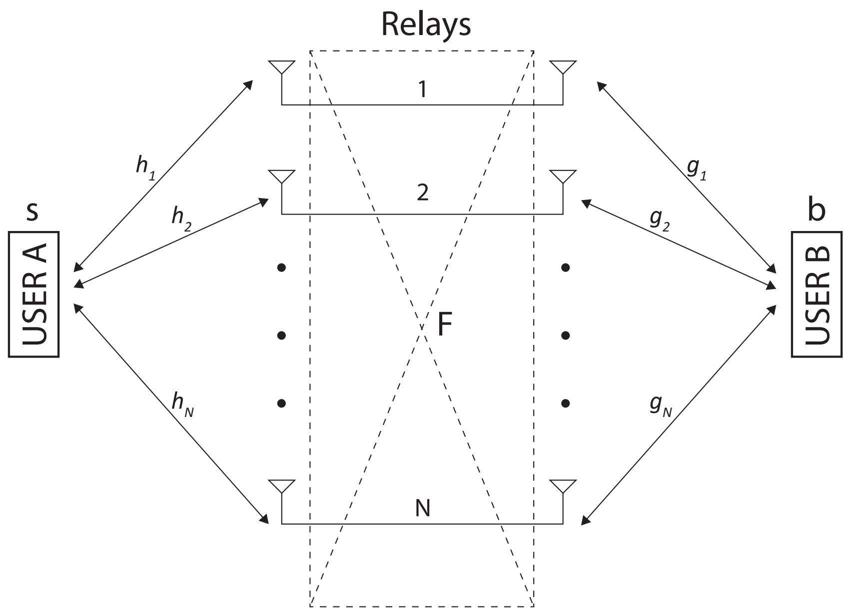

Figure 2.14. Two way relay network

$\mathbf{g} = [g_1, g_2, \cdots, g_N]^T$, respectively. The received signal vector at the relay nodes in the first transmission phase is given as

$$\mathbf{x} = \mathbf{h}s + \mathbf{g}b + \mathbf{n}_r, \tag{2.5.7}$$

where $\mathbf{n}_r \in \mathbb{C}^{N\times 1}$ is zero-mean circularly symmetric complex AWGN vector with covariance matrix $\sigma_r^2 I$. The channel is assumed to be quasi static, i.e. channel realizations are assumed to be fixed for the duration of a number of frames. In the second transmission phase, the relays rebroadcast a transformed signal vector as

$$\mathbf{z} = \mathbf{F}\mathbf{x}, \tag{2.5.8}$$

where $\mathbf{F} \in \mathbb{C}^{N\times N}$ is a linear transformation matrix (relay transceiver) to be determined in order to optimize receiver performance. The received signal vectors at the destination sides Y and C can be written, respectively, as

$$y = \mathbf{h}^T\mathbf{z} + n_y, \tag{2.5.9}$$

and

$$c = \mathbf{g}^T \mathbf{z} + n_c, \tag{2.5.10}$$

where n_y and n_c are noise scalars containing zero-mean circularly symmetric AWGN components of the users at receivers with covariance σ_y^2 and σ_c^2, respectively. Substituting (2.5.8) into (2.5.9) and (2.5.10), respectively, and using (2.5.7) we obtain

$$y = \mathbf{h}^T \mathbf{F} \mathbf{h} s + \mathbf{h}^T \mathbf{F} \mathbf{g} b + \mathbf{h}^T \mathbf{F} \mathbf{n}_r + n_y, \tag{2.5.11}$$

where $\mathbf{h}^T \mathbf{F} \mathbf{g} b$ is the desired signal at side Y. Similarly we obtain the received signal vector at the destination side C as

$$c = \mathbf{g}^T \mathbf{F} \mathbf{h} s + \mathbf{g}^T \mathbf{F} \mathbf{g} b + \mathbf{g}^T \mathbf{F} \mathbf{n}_r + n_c, \tag{2.5.12}$$

where $\mathbf{g}^T \mathbf{F} \mathbf{h} s$ is the desired signal at side C. Various methods can be applied to decode the signal, as discussed in Chapter 4 and 5.

2.5.3 Related Works on TWRN

TWRN has been studied and applied widely in recent years. In [16, 94, 103–107], the TWRN has been applied with OFDM-based wireless network. [16, 103, 104] discussed the resource allocation for OFDM-based TWRN. In [16], a new transmission protocol, named hierarchical OFDMA, has been proposed to support two way communications between the base station (BS) and each mobile user (MU) with or without an assisting relay station (RS) to achieve the optimal allocation of transmit resources such as power levels, bit rates, and OFDM subcarriers at the BS, RSs, and MUs. The work in [103] presented a novel two way DF relay strategy which employs multi-subcarrier joint channel coding to leverage frequency selective fading. Hence, it can achieve a higher data rate than the conventional per-subcarrier DF relay strategies. Channel estimation has been studied in the work of [94,105, 106]. In the work of [106], a two-phase training protocol has been proposed for the channel estimation, which is compatible with the two-phase data transmission scheme. Two different types of training methods were proposed. A method of two way relay sparse channel estimation based on compressive sensing (CS) has been discussed in [105]. The work in [107] combined a multihop relay system and OFDM modulation as a promising way to increase capacity and coverage.

Beamforming was also used in TWRN [25,108–110]. In the work of [25], new approaches to distributed cooperative beamforming for two way relay networks with frequency selective channels were proposed. In [108], MIMO relay transceiver processing has been proposed for multiuser two way relay communications. Various transmitter and receiver beamforming methods including eigen beamforming, antenna selection, random beamforming, and modified equal gain beamforming have been studied. [109, 110] focussed on the power allocation for the beamforming with TWRN.

A work based on the minimum mean-square-error (MMSE) criterion has been considered for relay design in [111]. The work in [112] addresses a non-regenerative two way MIMO relay system where two sources exchange information via a common relay.

2.6 Cognitive Radio Networks

With rapid development of wireless communication technologies, particularly those of WLAN, Wireless Personal Area Network (WPAN) and Wireless Metropolitan Area Network (WMAN), radio spectrum becomes increasingly scarce. Obviously, the spectrum resources have become a bottleneck that restrains the development of future wireless communication systems because the totality of spectrum is limited and the number of users is increasing rapidly. However, the allocated spectrums are far from full use, and most of them are more or less left unused in time or space. As the survey of American FCC at the end of 2003 showed that the spectrum has been used only from 15% to 85% [113], some bands (e.g. the bands assigned to mobile networks) were overloaded, but a large number of bands, such as amateur radio, were not exploited.

2.6.1 Concept of Cognitive Radio Technology

Cognitive radio network is introduced to solve the problem of low spectrum utilization by using the under utilized licensed spectrum for unlicensed users (also known as secondary users) [114]. In cognitive radio networks as shown in Fig. 2.15, secondary users share the radio spectrum bands, that are licensed to primary users. By detecting and employing the "spectrum hole", secondary users could finish their communication without harmfully affecting the primary user's communication process [114, 115]. Sharing the licensed spectrum by secondary users improves the overall spectrum utilization and at the same time the transmission power of the secondary user causes interference to primary user. Therefore, secondary user network should be designed in a way to allocate its radio resources to satisfy its own quality of service (QoS) requirements while ensuring that the interference caused to the primary users is below a predefined threshold level. The main functions of a cognitive radio network are spectrum sensing and exploitation of available spectrum by adjusting the transmission parameters such as frequency and transmission power.

In order to use the licensed spectrum, cognitive radio networks should detect the under-utilized licensed frequency bands. The performance of the detection schemes is mainly affected by channel fading and shadowing. There are difficulties in differentiating the attenuated primary signal from a white noise spectrum. This spectrum sensing problem has been widely studied and

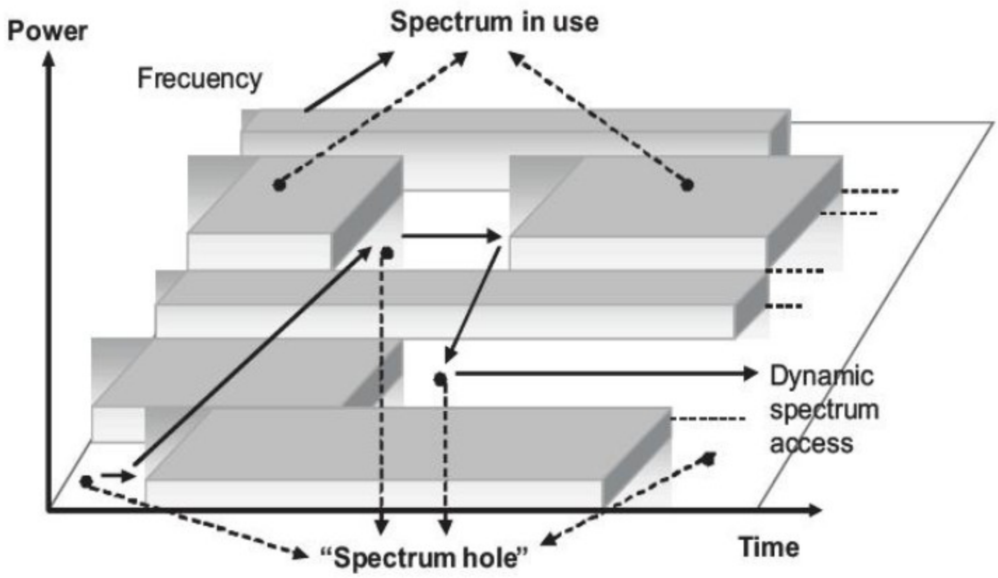

Figure 2.15. Spectrum hole.

different spectrum sensing schemes have been proposed to improve the detection performance [116,117]. Spectrum detection techniques can be classified based on the type of detection techniques employed at the receiver: energy detection [118], coherent detection [119] and cyclostationary feature detection [120]. Energy detection is optimal when the information on the primary signal is limited. Coherent detection can be efficiently employed when the primary pilot signal is known, whereas a cyclostationary detector has the potential to distinguish the primary signal energy from the local noise energy.

There are three different types of spectrum sharing arrangements, namely, interweave, overlay, and underlay [121]. The interweave approach is motivated by the idea of opportunistic communication. In this scheme, cognitive transmitters are required to sense availability of spectrum and transmit signals only when frequency holes are available. This is also known as white space filling, as shown in Fig. 2.16.

In the overlay approach, the secondary users coexist with primary users and use part of the transmission power to relay the primary users' signals

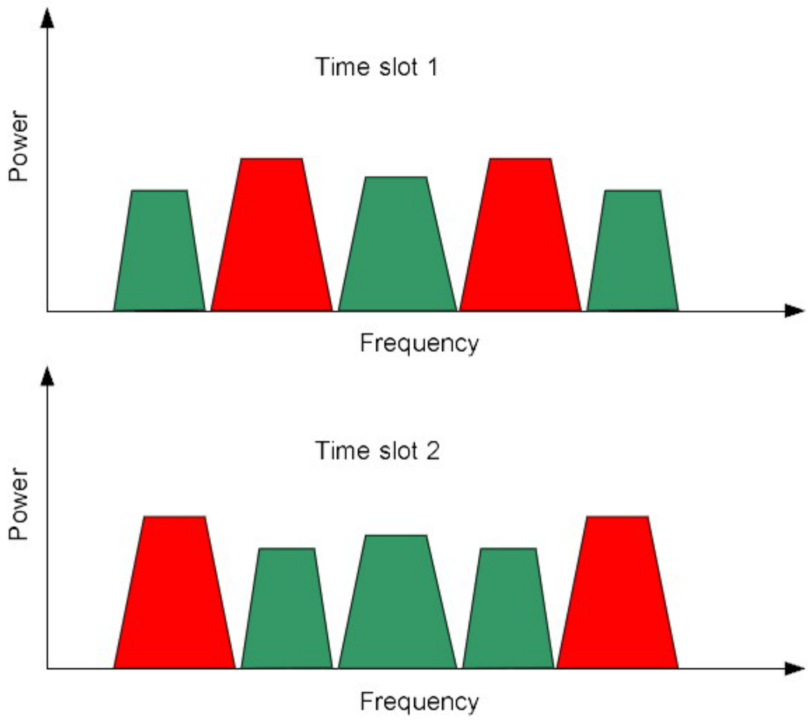

Figure 2.16. Interweave spectrum scheme. Green and red represent the spectrum occupied by the primary users and secondary users respectively.

to the primary receiver. This assistance will offset the interference caused by the secondary user transmissions to the primary users' receiver. Hence, there is no loss in primary users' signal-to-noise ratio by secondary users spectrum access.

In the underlay approach, the secondary users access the licensed spectrum without causing harmful interference to primary users' communications. In this method, the secondary users ensure that interference leakage to the primary users is below an acceptable level as shown in Fig. 2.17, and this approach will be considered in this thesis.

2.6.2 Cognitive Radio Standards

IEEE 802.22, is a standard for Wireless Regional Area Network (WRAN) using white spaces in the TV frequency spectrum. The white spaces are

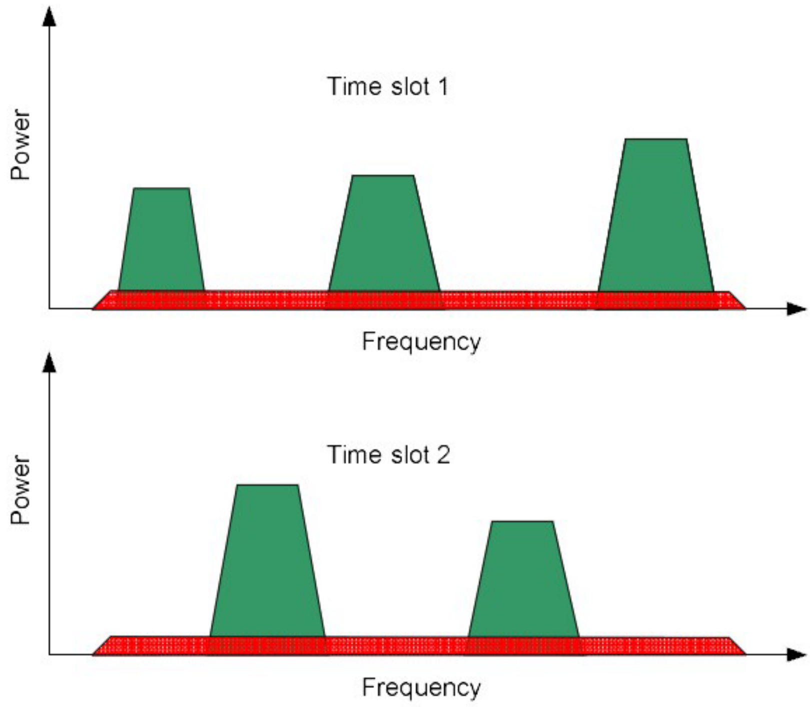

Figure 2.17. Underlay spectrum paradigm. Green and red represent the spectrum occupied by the primary users and the secondary users respectively.

channels within the licensed TV spectrum that are not used for TV services at a given location. Different frequency bands have been assigned by national and international bodies for specific uses. Some unused radio spectrum which has either never been used, or is becoming free as a result of technical changes, for example, the switchover to digital television frees up large portions of spectrum between about 50 MHz and 700 MHz. The development of the IEEE 802.22 WRAN standard is aimed at using cognitive radio techniques to allow sharing of geographically unused spectrum allocated to the Television Broadcast Service, on a non-interfering basis, to bring broadband access to hard-to-reach, low population density areas, typical of rural environments. It is the first worldwide effort to define a standardized air interface based on cognitive radio techniques for the opportunistic use of TV bands on a non-interfering basis. IEEE 802.22 WRANs are designed to operate in the TV broadcast bands while assuring that no harmful interference is caused to the incumbent operation, i.e., digital TV and analog TV broadcasting, and low power licensed devices such as wireless microphones.

2.6.3 Resource Allocation Techniques for Cognitive Radio Networks

Due to the natural ability of utilizing different portions of the spectrum, the OFDMA has been regarded as one of the best candidates for cognitive radio networks [122] and various resource allocation techniques have been studied [123–126]. In the work in [123], a cognitive OFDMA network has been proposed to maximize the multiple cognitive users' weighted sum rate by jointly adjusting their rate, frequency and power, under the constraints of multiple primary users' interference temperatures. A single user cognitive radio network has been considered in [124] to maximize the cognitive radio network throughput by allocating the bit and power to each subcarrier. The conventional greedy algorithm has been modified to allocate resources to the cognitive radio networks based on efficiency factors. In [125], iterative waterfilling and iterative multilevel waterfilling are exploited for the power allocation. It can be seen that the proposed technique offers fast convergence, and complexity is reduced by a distributed implementation. New design formulations have been presented in [126], aiming at optimizing the performance of an OFDMA ad hoc cognitive radio network through joint subcarrier assignment and power allocation.

Beamforming has been used in cognitive radio network for resource allocation in [127–129]. In [127], two iterative algorithms have been considered that using an weighted least square (WLS) approach and other based on admission control for secondary users. An underlay cognitive radio network has been proposed in [128], in which a beamforming and power allocation problem in the downlink with mixed QoS requirements has been solved using uplink-downlink duality and subgradient method. The work in [129] proposed a joint fast optimal resource allocation and beamforming algorithm to accommodate maximum possible number of secondary users while satisfying QoS requirement for each admitted secondary users.

MIMO-OFDMA techniques have been combined with multiuser cognitive radio network for resource allocation in [52, 130, 131]. The work in [130] proposed a MIMO-OFDMA structure for the multiple user access in cognitive radio networks. The resource allocation problem included allocation of sub-carriers to users and optimal power allocation to maximize capacity. In the work [52], an adaptive radio resource allocation algorithm for a MIMO-OFDMA based uplink cognitive radio network has been proposed. In this thesis, various resource allocation techniques based on convex optimization are proposed within the context of wireless relay and cognitive radio networks.

Chapter 3

CONVEX OPTIMIZATION THEORY

The use of optimization methods has become vital in numerous problems in signal processing and communication [132–134]. Many problems in communications can be appropriately formulated into a constrained optimization framework. These constrained problems are either naturally convex or can be expressed in convex form after some mathematical manipulations [133, 134]. Once it has been formulated into convex form, it can be efficiently solved using interior point methods [74]. Convex optimization has influenced most problems of practical interest, since a local optimum is also the global optimum for convex problems and they can be solved with polynomial time complexity. One of the attractive features of convex problems is that they allow verification of the optimality of the solutions using KKT conditions and duality gaps. The widely available software and tool boxes to solve convex problems also make convex optimization more attractive in many engineering applications [64–66]. However, most problems are naturally not in convex form. Therefore, recognizing the problems which can be solved using convex optimization and formulating the problem into a convex form are the major challenges in the application of convex optimization.

3.1 Fundamentals of Convex Optimizations

In this section, the fundamentals of convex optimization are introduced briefly.

3.1.1 Convex Sets

A convex set $\mathcal{S} \in \mathbb{R}^n$ can be expressed as follows [132]:

$$\theta\mathbf{x} + (1-\theta)\mathbf{y} \in \mathcal{S}, \qquad \forall\theta \in [0,1] \text{ and } \mathbf{x}, \mathbf{y} \in \mathcal{S}. \tag{3.1.1}$$

A set can be classified as a convex set if all the points of a line segment, which is formed by connecting two points from the set by a straight line, should be in the same set.

3.1.2 Convex Cones

A set $\mathcal{K}$ is said to be a convex cone, if for each $\mathbf{x} \in \mathcal{K}$ and each $\alpha \geq 0$, $\alpha\mathbf{x} \in \mathcal{K}$ and $\mathcal{K}$ is convex [132]. This can be mathematically expressed as

$$\theta_1\mathbf{x} + \theta_2\mathbf{y} \in \mathcal{K}, \qquad \forall\theta_1 \geq 0, \;\; \theta_2 \geq 0 \text{ and } \mathbf{x}, \mathbf{y} \in \mathcal{K}. \tag{3.1.2}$$

Convex cones arise in various forms in engineering applications. The most common convex cones are

1. Nonnegative orthant $\mathbb{R}^n_+$
2. Second-order cone (ice cream cone)
   $\mathcal{K} = \{(t, \mathbf{x}) \; t \geq \parallel \mathbf{x} \parallel\}$
3. Positive semidefinite matrix cone
   $\mathcal{K} = \mathbb{S}^n_+ = \{\mathbf{X} \mid \mathbf{X} \text{ symmetric and } \mathbf{X} \succeq 0\}$

3.1.3 Convex Functions

A function $f(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ is convex if $\mathbf{dom}\ f(\mathbf{x})$ is a convex set and if for all $\mathbf{x}, \mathbf{y} \in \mathbf{dom}\ f(\mathbf{x})$ the following inequality is satisfied [132]:

$$f(\theta\mathbf{x} + (1-\theta)\mathbf{y}) \leq \theta f(\mathbf{x}) + (1-\theta)f(\mathbf{y}), \qquad \forall\theta \in [0,1]. \tag{3.1.3}$$

In other words, along any line segment in $\mathbf{dom}\ f(\mathbf{x})$, $f(\mathbf{x})$ is less than or equal to the value of the linear function agreeing with $f(\mathbf{x})$ at the end points. The function $f(\mathbf{x})$ is concave if $-f(\mathbf{x})$ is convex. If $f(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ is continuously differentiable, the convexity of $f(\mathbf{x})$ is equivalent to

$$f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) \ \ \forall\mathbf{x}, \mathbf{y} \in \mathbb{R}^n. \tag{3.1.4}$$

Moreover, if $f(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ is twice continuously differentiable, then the convexity of $f(\mathbf{x})$ is equivalent to

$$\nabla^2 f(\mathbf{x}) \succeq 0 \qquad \forall\mathbf{x} \in \mathbb{R}^n. \tag{3.1.5}$$

i.e. its Hessian is positive semidefinite on its domain [132]. Thus, for example a linear function is always convex, while a quadratic function $\mathbf{x}^T\mathbf{P}\mathbf{x}+\mathbf{a}^T\mathbf{x}+b$ is convex if and only if $\mathbf{P} \succeq 0$.

3.2 The Conversion of Convex Optimization

The non-convex optimization problem can be converted into the convex optimization and this is the key for solving convex optimization. However, a systematic method to achieve this does not exist. There are two main methods for formulating problems into a convex form:

- First, change of variables. By using change of variables, a non-convex optimization problem may be formulated into a convex problem which is equivalent to the original one. For instant, we minimize the ℓ_2-norm of a vector, i.e. $\min \|\mathbf{w}\|_2$, by changing of variable, $\mathbf{W} = \mathbf{w}\mathbf{w}^H$ formulates the problem into minimizing the trace of the new variable $\mathbf{W}$, i.e. $\min\ trace\{\mathbf{W}\}$.

- Secondly, removing the constraints. By removing some of the constraints, a non-convex problem could be relaxed into a convex one. This technique is sufficient as long as both the non-convex problem and the related convex problem are equivalent, i.e. they share the same set of optimal solutions (related by some mapping). Semidefinite relaxation (SDR) is a good example of this technique. By using this, a non-convex constraint restricting the rank of the optimization variable matrix may be dropped, and a Semidefinite Programming (SDP) can be formed. As the example above, the trace of matrix $\mathbf{W}$ could be minimized, which is equivalent of minimizing the ℓ_2-norm of the vector $\mathbf{w}$. However, in the process of changing of variable, an additional non-convex constraint has been introduced, i.e. $rank\{\mathbf{W}\} = 1$, making the whole optimization problem non-convex. As our explanation later in the thesis, this non-convex constraint $rank\{\mathbf{W}\} = 1$ need to be dropped to guarantee a convex form.

3.3 Convex Optimization Problems

A convex optimization problem can be defined in the following standard form:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \le 0, \qquad i = 1, \dots, m, \\ & h_i(\mathbf{x}) = 0, \qquad i = 1, \dots, p, \end{aligned} \tag{3.3.1}$$

where the vector $\mathbf{x} \in \mathbb{R}^n$ is the *optimization variable* of the problem, the functions $f_0, \cdots, f_m$ are convex functions and the functions $h_1, \cdots, h_p$ are linear functions. The function f_0 is the *objective function* or *cost function.* The inequalities $f_i(\mathbf{x}) \leq 0, \;\; i = 1, \cdots, m$ are called the *inequality constraints* and equalities $h_i(\mathbf{x}) = 0, \;\; i = 1, \cdots, p$ are called the *equality constraints.* If there are no constraints, then the problem can be classified as an unconstrained problem. The *domain* of the optimization problem (3.3.1) is the set of points for which the objective and the constraints are defined and is denoted as

$$D = \bigcap_{i=0}^{m} \mathrm{dom} f_i \cap \bigcap_{i=0}^{p} \mathrm{dom} h_i. \tag{3.3.2}$$

A point $\mathbf{x} \in D$ is feasible, if it satisfies all the constraints $f_i(\mathbf{x}) \leq 0 \;\; i = 1, \cdots, m$ and $h_i(\mathbf{x}) = 0 \;\; i = 1, \cdots, p$. Problem (3.3.1) is said to be *feasible* if there exists a feasible point and is *infeasible* otherwise. The *optimal value* or the *solution* of the optimization problem is achieved at the optimal point $\mathbf{x}^\star$ if and only if

$$f_0(\mathbf{x}^\star) \leq f_0(\mathbf{x}) \quad \forall \mathbf{x} \in D. \tag{3.3.3}$$

3.4 Canonical Optimization Problems

In this section, the most general form of the canonical optimization problem formulations are provided.

3.4.1 Linear Programming

A convex optimization problem is called linear programming (LP), when the objective and constraint functions are all affine [132]. A general LP can be defined as follows :

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \mathbf{c}^T\mathbf{x} + d \\ \text{subject to} \quad & \mathbf{G}\mathbf{x} \preceq \mathbf{h}, \\ & \mathbf{A}\mathbf{x} = \mathbf{b}, \end{aligned} \tag{3.4.1}$$

where $\mathbf{G} \in \mathbb{R}^{m\times n}$ and $\mathbf{A} \in \mathbb{R}^{p\times n}$.

3.4.2 Quadratic Programming

When the objective function is quadratic (convex) and the constraint functions are affine, then the convex optimization problem is called QP. A QP can be expressed as follows:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \mathbf{x}^T\mathbf{P}\mathbf{x} + \mathbf{q}^T\mathbf{x} + r \\ \text{subject to} \quad & \mathbf{G}\mathbf{x} \preceq \mathbf{h}, \\ & \mathbf{A}\mathbf{x} = \mathbf{b}, \end{aligned} \tag{3.4.2}$$

where $\mathbf{P} \in \mathbb{S}_+^n$, $\mathbf{G} \in \mathbb{R}^{m\times n}$, and $\mathbf{A} \in \mathbb{R}^{p\times n}$. In QP, a convex quadratic function is minimized over a polyhedron. QP includes LP as a special case. This may be obtained by setting $\mathbf{P} = 0$ in the objective of (3.4.2).

3.4.3 Quadratically Constrained Quadratic Programming

The convex optimization problem is called a QCQP, when both objective and constraint functions are quadratic. This has the form

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \mathbf{x}^T\mathbf{P}_0\mathbf{x} + \mathbf{q}_0^T\mathbf{x} + r_0 \\ \text{subject to} \quad & \mathbf{x}^T\mathbf{P}_i\mathbf{x} + \mathbf{q}_i^T\mathbf{x} + r_i \le 0, \qquad i = 1, 2, \ldots, m, \\ & \mathbf{A}\mathbf{x} = \mathbf{b}, \end{aligned} \tag{3.4.3}$$

where $\mathbf{P}_i \in \mathbb{S}_+^n,\ i = 1, 2, \ldots, m$. In a QCQP, a quadratic convex function is minimized over a feasible region that is the intersection of ellipsoids. In QCQP, by setting $\mathbf{P}_i = 0,\ i = 0, 1, 2, \ldots, m$ in the constraints of (3.4.3), an LP can be obtained.

3.4.4 Second-Order Cone Programming

An SOCP can be written as [132]

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \mathbf{f}^T\mathbf{x} \\ \text{subject to} \quad & \|\mathbf{A}_i\mathbf{x} + \mathbf{b}_i\|_2 \le \mathbf{c}_i^T\mathbf{x} + \mathbf{d}_i, \qquad i = 1, 2, \ldots, m, \\ & \mathbf{F}\mathbf{x} = \mathbf{g}, \end{aligned} \tag{3.4.4}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the optimization variable, $\mathbf{A}_i \in \mathbb{R}^{n_i \times n}$ and $\mathbf{F} \in \mathbb{R}^{p \times n}$. The first constraint in (3.4.4) is a second order cone constraint in $\mathbb{R}^{k+1}$. Setting $\mathbf{c}_i = \mathbf{0},\ i = 1, 2, \ldots, m$ and squaring both sides of the constraints, a QCQP will be obtained. Similarly, if $\mathbf{A}_i = \mathbf{0},\ i = 1, 2, \ldots, m$, then the SOCP reduces to a LP. In general, SOCPs are more widely used in convex optimization applications.

3.4.5 Semidefinite Programming

Every canonical optimization problem can be considered as a special case of SDP. The most general of all the forms is an SDP. An SDP can be written as [132, 135]

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & \mathbf{x}_1\mathbf{F}_1 + \mathbf{x}_2\mathbf{F}_2 + \ldots + \mathbf{x}_n\mathbf{F}_n + \mathbf{G} \preceq \mathbf{0}, \\ & \mathbf{A}\mathbf{x} = \mathbf{b}, \end{aligned} \tag{3.4.5}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the optimization variable and $\mathbf{G}, \mathbf{F}_1, \ldots, \mathbf{F}_n \in \mathbb{S}^{k\times k}$ are symmetric matrices and $\mathbf{A} \in \mathbb{R}^{p\times n}$. The inequality constraints in (3.4.5) are also called linear matrix inequality (LMI). An SDP reduces to an LP if the matrices $\mathbf{G}, \mathbf{F}_1, \ldots, \mathbf{F}_n$ are all diagonal.

3.4.6 Geometric Programming

A geometric programming (GP) problem consists of monomial functions and posynomial functions. A monomial function is defined as $f(\mathbf{x}) : \mathbb{R}^n_{++} \to \mathbb{R}$

$$f(\mathbf{x}) = c x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}, \tag{3.4.6}$$

where $c > 0$ and $a_i \in \mathbb{R},\ i = 1, \cdots, n$. A posynomial is a sum of monomials and can be defined as

$$f(\mathbf{x}) = \sum_{k=1}^{K} c_k x_1^{a_{1k}} x_2^{a_{2k}} \cdots x_n^{a_{nk}}, \tag{3.4.7}$$

where $c_k > 0$. A standard GP can be expressed as follows:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_i(\mathbf{x}) \le 1, \quad i = 1, \cdots, m, \\ & h_i(\mathbf{x}) = 1, \quad i = 1, \cdots, p, \end{aligned} \tag{3.4.8}$$

where $f_0, \cdots f_m$ are posynomials and $h_1, \cdots h_p$ are monomials. The domain of this problem is $D = \mathbb{R}^n_{++}$.

3.5 Duality and KKT Conditions

The basic idea in Lagrangian duality is to take the constraints in (3.3.1) into account by augmenting the objective function with a weighted sum of the constraint functions. The *Lagrangian* $L : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p \to \mathbb{R}$ for the original problem in (3.3.1) can be defined as [132]

$$L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{p} \nu_i h_i(\mathbf{x}), \tag{3.5.1}$$

where λ_i and ν_i are the *Lagrange multipliers* associated with the i^{th} inequality $f_i(\mathbf{x}) \leq 0$ and equality $h_i(\mathbf{x}) = 0$ constraints respectively. The objective $f_0(\mathbf{x})$ in (3.3.1) is called the *primal objective* and the optimization variable $\mathbf{x}$ is termed the *primal variable.* Lagrange multipliers $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$ associated with the problem (3.5.1) are called the *dual variables.* The *Lagrange dual objective* or the *Lagrange dual function* $g : \mathbb{R}^m \times \mathbb{R}^p \to \mathbb{R}$ is defined as the minimum value of the Lagrangian over $\mathbf{x}$: for $\boldsymbol{\lambda} \in \mathbb{R}^m$, $\boldsymbol{\nu} \in \mathbb{R}^p$ [132]

$$g(\boldsymbol{\lambda}, \boldsymbol{\nu}) = \inf_{\mathbf{x} \in \mathcal{D}} \left(f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{p} \nu_i h_i(\mathbf{x}) \right). \tag{3.5.2}$$

The Lagrange dual function is always concave regardless of whether the original problem is convex or not. This is because the dual function is the pointwise infimum of a family of affine functions of $(\boldsymbol{\lambda}, \boldsymbol{\nu})$ [132]. The dual function $g(\boldsymbol{\lambda}, \boldsymbol{\nu})$ yields a lower bound on the optimal value $f_0(\mathbf{x}^*)$ of the problem (3.3.1) [132]. For any $\boldsymbol{\lambda} \succeq 0$ and any $\boldsymbol{\nu}$,

$$g(\boldsymbol{\lambda}, \boldsymbol{\nu}) \leq f_0(\mathbf{x}^*) \tag{3.5.3}$$

This can be shown for any feasible set $(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu})$ as follows:

$$\begin{aligned} f_0(\mathbf{x}) &\geq f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{p} \nu_i h_i(\mathbf{x}) \\ &\geq \inf_{\mathbf{z} \in \mathcal{D}} \left(f_0(\mathbf{z}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{z}) + \sum_{i=1}^{p} \nu_i h_i(\mathbf{z}) \right) \\ &= g(\boldsymbol{\lambda}, \boldsymbol{\nu}) \end{aligned} \tag{3.5.4}$$

Duality gap is the measure of the difference between the primal objective $f_0(\mathbf{x})$ and the dual objective $g(\boldsymbol{\lambda}, \boldsymbol{\nu})$. When the inequality in (3.5.3) is satisfied with strict inequality, then it holds a *weak duality*. If the inequality in (3.5.3) is satisfied with equality, it holds *strong duality* between the primal problem and the dual problem. To obtain the best lower bound of the original problem, the following dual problem is solved:

$$\begin{aligned} \underset{\boldsymbol{\lambda}, \boldsymbol{\nu}}{\text{maximize}} \quad & g(\boldsymbol{\lambda}, \boldsymbol{\nu}) \\ \text{subject to} \quad & \boldsymbol{\lambda} \geq 0. \end{aligned} \tag{3.5.5}$$

The *Lagrange dual problem* is always a convex problem, since the objective function (concave function) in (3.5.5) is maximized with convex constraints. This always holds regardless of the nature of the primal problem (3.3.1) [132]. The following conditions are called KKT conditions which provide the facility to validate the optimality of the solutions.

1. Primal constraints: $f_i(\mathbf{x}) \leq 0 \qquad i = 1, 2, \ldots, m,$
   $h_i(\mathbf{x}) = 0 \qquad i = 1, 2, \ldots, p,$

2. Dual constraints: $\boldsymbol{\lambda} \succeq 0$

3. Complementary slackness $\lambda_i f_i(\mathbf{x}) = 0 \qquad i = 1, 2, \ldots, m,$

4. Gradient of Lagrangian with respect to $\mathbf{x}$ vanishes:

$$\nabla f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i \nabla f_i(\mathbf{x}) + \sum_{i=1}^{p} \nu_i \nabla h_i(\mathbf{x}) = 0. \tag{3.5.6}$$

These KKT conditions are necessary conditions for optimality in general but not sufficient conditions. For convex and non-convex problems, if strong duality holds, then the KKT conditions will be satisfied. But if the KKT conditions are satisfied, it does not mean that strong duality holds. However, for convex optimization problems, if the KKT conditions hold, then the strong duality holds between the primal problem and the dual problem. In addition, primal and dual variables are optimal [132].

3.6 Summary

In this chapter, various convex optimization problems have been discussed briefly. These problems can be effectively solved using interior point methods. The concepts of Lagrange duality and KKT conditions have also been presented. However, in this thesis, the focus will be on SOCP, SDP and GP to solve the optimization problems in wireless relay networks.

Chapter 4

OPTIMIZATION TECHNIQUE FOR TWO WAY RELAYS AND SPATIAL MULTIPLEXING

In this chapter, optimization techniques for the use of two way relays to forward signals is proposed. Two way relays has been widely used to transmit signal for saving the radio frequency. The multiuser transmitter and receiver pairs employ single antennas, however, the two way relays can be used to forward signals to each other while performing spatial multiplexing by appropriately changing the phase and the amplitude of the signals at the relays. The problem is formulated into a SDP framework that can be solved using interior point methods. The simulation results demonstrate that the proposed technique has the ability to perform spatial multiplexing using two way relays for a wide range of target signal to interference and noise ratio requirements.

4.1 Introduction

The spectrum resources are quite limited due to ever increasing demand for data intensive wireless applications. The transmission power should also be kept at minimum possible level in order to mitigate interference to neighbouring cells and to enhance frequency re-use factor. One way of optimizing power usage while increasing the range of wireless coverage is to employ wireless relays. A one-way relay network would require two time slots for transmission through relays. However, this inefficiency can be resolved using two way relay networks.

In a two-relay network, a pair of nodes that wish to communicate each other transmit signals simultaneously to the relays in the first phase. The relays receive these composed signals and forward them towards the receiver destinations in the second phase [5, 16, 42, 136–146]. In the case of a single source on each side, a simple detection scheme is used at the receiver to decode the signal from other side by subtracting the originally transmitted signal from the received signal. For multiple users on both sides, a similar interference cancelation at the receiver is required to cancel self interference, but interference from other sources will be mitigated at the relays using the proposed relay transceiver.

Even though this would require two time slots, since both the uplink and the downlink is served simultaneously, the effective bandwidth efficiency is considered to be one time slot per transmission. There have been a lot of works performed recently on two way relay networks in terms of efficient channel estimation [138], [5], [147], [148], signal encoding [136], [137], [143] and receiver design [146]. A MMSE method has been considered for spatial multiplexing for a given transmission power at the relays in [139]. However the SDP formulation proposed in this chapter has the merit of using minimum possible transmission power while attaining SINR for each user.

The use of relays for forwarding signals while performing spatial multiplexing has been studied for a one-way relay network in [15,29,149–151]. Recent work also considered optimum power allocation at the transmitter and at the relays to maintain desired SNR at the receiving end for a pair of transmitter-receiver terminals with a number of relays [140]. Potential of two way relay networks for forwarding signals while performing spatial multiplexing. Later on, this problem will be extended to an asynchronous one, which means that the asynchronous relays will be employed, where the signals from various sources and relays experience different delays.

4.2 Application of SDP for One Way Relay Network

SDP is an extension of linear programming, with real symmetric matrices replacing vector variables and positive semidefinite constraints replacing component wise inequalities. It can be solved with reasonable efficiency, both theory and practice, which constitute one of the largest classes of optimization problem. Due to the replacing the vector variable with the real symmetric matrix, it need to be ensured that the rank of the matrix is one which can solve the optimal solution of the original problem, however, this additional constraint, i.e. rank is one, is not a convex constraint. In this case, this rank-one constraint need to be dropped, which is called Semidefinite Relaxation (SDR). By relaxing the rank-one constraint, the objective function and the rest constraints are convex and can be solved using interior point methods.

SDP has been widely used for one way relay network. The work in [31] and [92] has presented the combination of SDP and MIMO. The work [31] considered a MIMO relay system where the transmitter multicasts a common message to multiple receivers with the aid of a relay node. Given the power constraints at the source and the relay nodes, aiming at minimizing the maximal mean-squared error (MSE) of the signal waveform estimation among the destination nodes through joint source, relay, and receiver matrices optimization. [92] consider an interference MIMO relay system where multiple source nodes communicate with their desired destination nodes concurrently with the aid of distributed relay nodes all equipped with multiple antennas. The work minimized the total source and relay transmit power such that a minimum SINR threshold is maintained at each receiver.

Beamforming has been employed with SDP [152–154]. In the work [152], a new approach to distributed beamforming in a relay network was proposed, which maximized the destination QoS under the total relay transmitted power constraint. The work [153] develops a nonsmooth optimization algorithm, which provides the optimal solution at low computational complexity. The paper [154] investigates the interference alignment solution for a fully connected symmetric interference network with single-antenna relays. And an iterative algorithm is proposed to alternatively optimize the precoders at transmitters, decoders at receivers and relay beamforming weights. In this works, the original problems were turned into a convex SDP problems by using SDR technique, respectively.

SDP can be used in one way spatial multiplexing, for instants, the works [149, 155] propose cooperative signal-forwarding schemes for wireless networks. An SDP is used in [155] to ensure target SINRs at the destination are achieved with minimum possible transmit power at the relay layers. In [149], an SDP framework allows us to impose various QoS constraints for each source-destination pair.

In this chapter, the SDP is employed for two way relay network based multiuser multiplexing, which improves the frequency efficiency.

4.3 Optimization for Two Way Relaying Based Multiuser Multiplexing

In this section, we propose to use two way relays to perform spatial multiplexing for multiple transmitter-receiver pairs using an SDP technique. An MMSE method has been considered in [139] for spatial multiplexing based on maximizing system desired SNR for a given transmission power. However the SDP formulation considered in this chapter has the advantage of using minimum possible transmission power while attaining SINR for each user. The use of relays for forwarding signals while performing spatial multiplexing has been studied for a one way relay network in [15, 29, 149–151].

4.3.1 System Model

As shown in Fig. 4.1, consider M pairs of source nodes and receiver nodes. There are N two way relays between sources and destinations with one antenna in each. Bidirectional relaying strategies facilitate information exchange between the M pairs of source nodes and receiver nodes in two time phases via a half-duplex relay. In the first phase (broadcasting phase) signal vectors $\mathbf{s}$ and $\mathbf{b}$ are transmitted from a set of source nodes Y and C respectively, where $\mathbf{s} = [s_1(n), s_2(n), \cdots, s_M(n)]^T$ and $\mathbf{b} = [b_1(n), b_2(n), \cdots, b_M(n)]^T$. The variance of the signal at the Y and C sides are denoted as σ_s^2 and σ_b^2 respectively. The signals are received by a set of relay nodes. The relay nodes transmit the received data to the destination nodes in the second phase (relaying phase).

Let us denote the channel matrix consisting of complex channel coefficients between source nodes Y and the relay nodes as

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_M] \in \mathbb{C}^{N\times M} \tag{4.3.1}$$

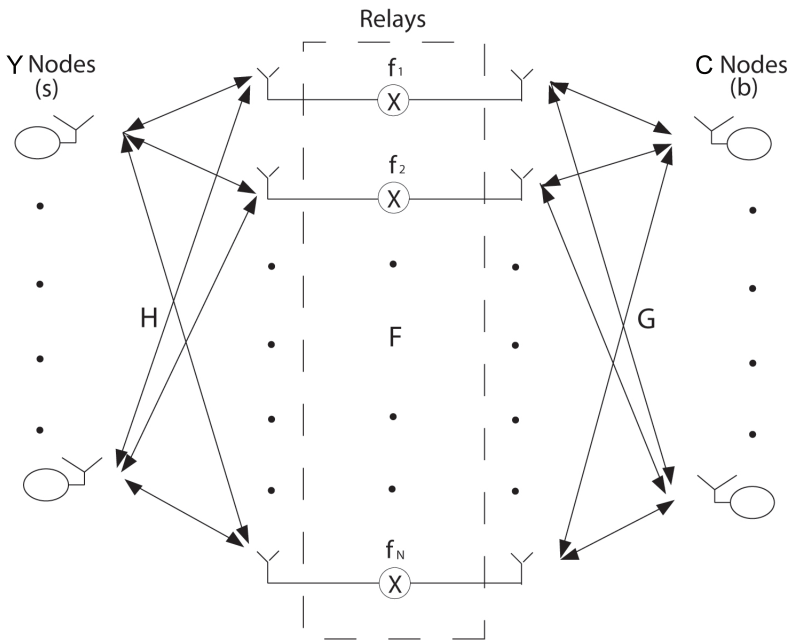

Figure 4.1. A relay network of M pairs of nodes and N relay nodes.

where $\mathbf{h}_m = [h_{m,1}, h_{m,2}, \cdots, h_{m,N}]^T$ for $m = 1, \cdots, M$, is a column vector consisting of channel coefficients between the m^{th} source y_m and the observing relays. Similarly, let us denote the channel matrix consisting of complex channel coefficients between source nodes C and the relay nodes as

$$\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2, \cdots, \mathbf{g}_M] \in \mathbb{C}^{N \times M} \tag{4.3.2}$$

where $\mathbf{g}_m = [g_{m,1}, g_{m,2}, \cdots, g_{m,N}]^T$ for $m = 1, \cdots, M$, is a column vector consisting of channel coefficients between the m^{th} source c_m and the observing relays. The source nodes Y and nodes C transmit the signals simultaneously to the relays, and the received signal vector at the relay nodes in the first transmission phase is given as

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{G}\mathbf{b} + \mathbf{n}_r \tag{4.3.3}$$

where $\mathbf{n}_r \in \mathbb{C}^{N \times 1}$ is zero-mean circularly symmetric complex additive white Gaussian noise (AWGN) vector with covariance matrix $\sigma_r^2 \mathbf{I}$. The channel is assumed to be quasi static, i.e. channel realizations are assumed to be fixed for the duration of a number of frames. In the second transmission phase, the relays rebroadcast a transformed signal vector as

$$\mathbf{z} = \mathbf{F}\mathbf{x} \tag{4.3.4}$$

where $\mathbf{F} \in \mathbb{C}^{N\times N}$ is a diagonal matrix whose elements are drawn from the relay coefficient vector $\mathbf{f} \triangleq [f_1, f_2, \cdots, f_N]^T$. The received signal vector can be written as (consider destination side Y as an example)

$$\mathbf{y} = \mathbf{H}^T\mathbf{z} + \mathbf{n}_y \tag{4.3.5}$$

where $\mathbf{n}_y \in \mathbb{C}^{M\times 1}$ is a zero-mean circularly symmetric AWGN vector with covariance matrix $\sigma_y^2\mathbf{I}$. Substituting (4.3.4) into (4.3.5) and using (4.3.3) we obtain

$$\mathbf{y} = \mathbf{H}^T\mathbf{F}\mathbf{H}\mathbf{s} + \mathbf{H}^T\mathbf{F}\mathbf{G}\mathbf{b} + \mathbf{H}^T\mathbf{F}\mathbf{n}_r + \mathbf{n}_y \tag{4.3.6}$$

where $\mathbf{H}^T\mathbf{F}\mathbf{G}\mathbf{b}$ is the desired signal and $\mathbf{H}^T\mathbf{F}\mathbf{H}\mathbf{s} + \mathbf{H}^T\mathbf{F}\mathbf{n}_r + \mathbf{n}_y$ is the interference and noise terms for node Y. Similarly we obtain the received signal vector at the destination side C as

$$\mathbf{c} = \mathbf{G}^T\mathbf{F}\mathbf{H}\mathbf{s} + \mathbf{G}^T\mathbf{F}\mathbf{G}\mathbf{b} + \mathbf{G}^T\mathbf{F}\mathbf{n}_r + \mathbf{n}_c \tag{4.3.7}$$

where $\mathbf{n}_c \in \mathbb{C}^{M\times 1}$ is a zero-mean circularly symmetric AWGN vector with covariance matrix $\sigma_c^2\mathbf{I}$, $\mathbf{G}^T\mathbf{F}\mathbf{H}\mathbf{s}$ is the desired signal and $\mathbf{G}^T\mathbf{F}\mathbf{G}\mathbf{b}+\mathbf{G}^T\mathbf{F}\mathbf{n}_r+\mathbf{n}_c$ is the interference and noise terms for note C.

4.3.2 Semidefinite Programming (SDP) Framework

Formulation of the SINR cost function

An optimum $\mathbf{F}$ is determined that minimizes the total power usage at the relay nodes, p_{pow}, subject to QoS constraints $\text{SINR}_m \geq \gamma_m$ imposed at each destination node m on both sides, i.e.

$$\begin{aligned} \min_{\mathbf{F}} \quad & p_{\text{pow}} \\ \text{s.t.} \quad & \text{SINR}_m^y \geq \gamma_m, \quad m = 1, \cdots, M \\ & \text{SINR}_m^c \geq \gamma_m, \quad m = 1, \cdots, M \end{aligned} \tag{4.3.8}$$

where γ_m is the target SINR value for the m^{th} destination at both the Y and C sides. Power usage by the relays in the network is given by

$$\text{E}\{\mathbf{z}^H\mathbf{z}\} = \text{tr}(\mathbf{F}\mathbf{R}\mathbf{F}^H) = p_{\text{pow}} \tag{4.3.9}$$

where $\text{tr}(\cdot)$ is a trace operator, $\mathbf{R} = \mathbf{H}\mathbf{H}^H\sigma_s^2 + \mathbf{G}\mathbf{G}^H\sigma_b^2 + \sigma_r^2\mathbf{I}$. We now derive the SINR_m at the Y side, however, the SINR at the C side can be derived similarly. The received signal power at the m^{th} destination node is determined as

$$\begin{aligned} p_{\text{sig,y,m}} &= \text{E}\left\{(\mathbf{h}_m^T\mathbf{F}\mathbf{g}_m b_m)(\mathbf{h}_m^T\mathbf{F}\mathbf{g}_m b_m)^H\right\} \\ &= \mathbf{h}_m^T\mathbf{F}\mathbf{g}_m\mathbf{g}_m^H\mathbf{F}^H\mathbf{h}_m^* \ \sigma_{bm}^2 \end{aligned} \tag{4.3.10}$$

Similarly the received interference power at the m^{th} destination node y_m due to signal contribution of all other source nodes is given as

$$\begin{aligned} p_{\text{int,y,m}} &= \sum_{k=1,k\neq m}^{M} \text{E}\left\{(\mathbf{h}_m^T\mathbf{F}\mathbf{h}_k s_k)(\mathbf{h}_m^T\mathbf{F}\mathbf{h}_k s_k)^H\right\} \\ &+ \sum_{k=1,k\neq m}^{M} \text{E}\left\{(\mathbf{h}_m^T\mathbf{F}\mathbf{g}_k b_k)(\mathbf{h}_m^T\mathbf{F}\mathbf{g}_k b_k)^H\right\} \\ &= \sum_{k=1,k\neq m}^{M} (\mathbf{h}_m^T\mathbf{F}\mathbf{h}_k\mathbf{h}_k^H\mathbf{F}^H\mathbf{h}_m^*)\ \sigma_{sk}^2 + \\ &\sum_{k=1,k\neq m}^{M} (\mathbf{h}_m^T\mathbf{F}\mathbf{g}_k\mathbf{g}_k^H\mathbf{F}^H\mathbf{h}_m^*)\ \sigma_{bk}^2 \qquad (4.3.11) \end{aligned}$$

The noise power transferred to the m^{th} destination node, from the noise present at the relay nodes can be written as

$$\begin{aligned} p_{\text{nse,y,m}} &= \text{E}\left\{(\mathbf{h}_m^T\mathbf{F}\mathbf{n}_r)(\mathbf{h}_m^T\mathbf{F}\mathbf{n}_r)^H\right\} \\ &= \mathbf{h}_m^T\mathbf{F}\mathbf{F}^H\mathbf{h}_m^*\sigma_r^2 \qquad (4.3.12) \end{aligned}$$

Hence the SINR at the m^{th} destination node can be written as

$$\text{SINR}_{\text{m}}^{y} = \frac{p_{\text{sig,y,m}}}{p_{\text{int,y,m}} + p_{\text{nse,y,m}} + \sigma_{ym}^2} \qquad (4.3.13)$$

Therefore the optimization problem is formulated as

$$\begin{aligned} \min_{\mathbf{F}} \quad & p_{\text{pow}} \\ \text{s.t.} \quad & \frac{p_{\text{sig,y,m}}}{p_{\text{int,y,m}} + p_{\text{nse,y,m}} + \sigma_{ym}^2} \geq \gamma_m \\ & \frac{p_{\text{sig,c,m}}}{p_{\text{int,c,m}} + p_{\text{nse,c,m}} + \sigma_{cm}^2} \geq \gamma_m, \\ & m = 1, \cdots, M \qquad (4.3.14) \end{aligned}$$

Formulation of objective and constraints functions

We formulate the objective function and the constraint functions in (4.3.14) so that the optimization problem can be solved using SDP. A diagonal $\mathbf{F} = \text{diag}(\mathbf{f})$ is considered such that each relay receives the signal, multiples it by a complex coefficient f_i and forwards the signal. Please note that if the relays are located in a close neighbourhood, it is also possible for the relays to forward the signals between themselves before forwarding the processed signals to the destinations. In this case, a non-diagonal matrix $\mathbf{F}$ should be considered as in [149] and [15]. This has the potential to provide a better performance, however with a relatively higher complexity. Equation (4.3.4) can be rewritten as

$$\begin{aligned} \mathbf{z} &= \text{diag}(\mathbf{x})\mathbf{f} \\ &= \left(\sum_{m=1}^{M} \text{diag}(\mathbf{h}_m) s_m + \right. \\ &\quad \left. \sum_{m=1}^{M} \text{diag}(\mathbf{g}_m) b_m \right) \mathbf{f} \end{aligned} \tag{4.3.15}$$

Define $\mathbf{P}_{\text{pow}} \triangleq \sum_{m=1}^{M} \left(\text{diag}(\mathbf{h}_m)\text{diag}(\mathbf{h}_m^*)\right) + \sum_{m=1}^{M} \left(\text{diag}(\mathbf{g}_m)\text{diag}(\mathbf{g}_m^*)\right) + \sigma_r^2(\mathbf{I}) \in \mathbb{C}^{N\times N}$. The power transmitted by the relay nodes can be written as

$$\begin{aligned} p_{\text{pow}} &= \text{E}\{\mathbf{z}^H \mathbf{z}\} && (4.3.16) \\ &= \mathbf{f}^H \left(\sum_{m=1}^{M} \left(\text{diag}(\mathbf{h}_m)\text{diag}(\mathbf{h}_m^*)\right) + \right. \\ &\quad \left. \sum_{m=1}^{M} \left(\text{diag}(\mathbf{g}_m)\text{diag}(\mathbf{g}_m^*)\right) + \sigma_r^2(\mathbf{I}) \right) \mathbf{f} \\ &= \mathbf{f}^H \mathbf{P}_{\text{pow}} \mathbf{f} && (4.3.17) \end{aligned}$$

The received signal power at the m^{th} destination node y_m can be expressed as

$$\begin{aligned} p_{\text{sig,y,m}} &= \mathbf{h}_m^T \mathbf{F} \mathbf{g}_m \mathbf{g}_m^H \mathbf{F}^H \mathbf{h}_m^* \ \sigma_{bm}^2 \\ &= \mathbf{f}^H \text{diag}(\mathbf{h}_m^*) \mathbf{g}_m^* \mathbf{g}_m^T \text{diag}(\mathbf{h}_m) \mathbf{f} \sigma_{bm}^2 \\ &= \mathbf{f}^H \mathbf{P}_{\text{sig,ym}} \mathbf{f} \end{aligned} \tag{4.3.18}$$

where $\mathbf{P}_{\text{sig,ym}} \triangleq \text{diag}(\mathbf{h}_m^*) \mathbf{g}_m^* \mathbf{g}_m^T \text{diag}(\mathbf{h}_m) \in \mathbb{C}^{N \times N}$. The total interference power at the m^{th} destination node y_m from all other source nodes shown in (4.4.15) can be expressed as

$$\begin{aligned} p_{\text{int,y,m}} &= \sum_{k=1, k \neq m}^{M} (\mathbf{h}_m^T \mathbf{F} \mathbf{h}_k \mathbf{h}_k^H \mathbf{F}^H \mathbf{h}_m^*) \ \sigma_{sk}^2 + \\ &\quad \sum_{k=1, k \neq m}^{M} (\mathbf{h}_m^T \mathbf{F} \mathbf{g}_k \mathbf{g}_k^H \mathbf{F}^H \mathbf{h}_m^*) \ \sigma_{bk}^2 \\ &= \sum_{k=1, k \neq m}^{M} \mathbf{f}^H \text{diag}(\mathbf{h}_m^*) \mathbf{h}_k^* \mathbf{h}_k^T \text{diag}(\mathbf{h}_m) \mathbf{f} \sigma_{sk}^2 + \\ &\quad \sum_{k=1, k \neq m}^{M} \mathbf{f}^H \text{diag}(\mathbf{h}_m^*) \mathbf{g}_k^* \mathbf{g}_k^T \text{diag}(\mathbf{h}_m) \mathbf{f} \sigma_{bk}^2 \\ &= \mathbf{f}^H \mathbf{P}_{\text{int,ym}} \mathbf{f} \end{aligned} \tag{4.3.19}$$

where $\mathbf{P}_{\text{int,ym}} \triangleq \sum_{k=1, k \neq m}^{M} \text{diag}(\mathbf{h}_m^*) \mathbf{h}_k^* \mathbf{h}_k^T \text{diag}(\mathbf{h}_m) \sigma_{sk}^2 +$ $\sum_{k=1, k \neq m}^{M} \text{diag}(\mathbf{h}_m^*) \mathbf{g}_k^* \mathbf{g}_k^T \text{diag}(\mathbf{h}_m) \sigma_{bk}^2 \in \mathbb{C}^{N \times N}$. The total receiver noise power transferred to the m^{th} destination node y_m from the relay nodes described in (4.4.16) can be written as

$$\begin{aligned} p_{\text{nse,y,m}} &= \mathbf{h}_m^T \mathbf{F} \mathbf{F}^H \mathbf{h}_m^* \ \sigma_r^2 \\ &= \mathbf{f}^H \text{diag}(\mathbf{h}_m^*) \text{diag}(\mathbf{h}_m) \mathbf{f} \sigma_r^2 \\ &= \mathbf{f}^H \mathbf{P}_{\text{nse,ym}} \mathbf{f} \end{aligned} \tag{4.3.20}$$

where $\mathbf{P}_{\text{nse,ym}} \triangleq \text{diag}(\mathbf{h}_m^*) \text{diag}(\mathbf{h}_m) \sigma_r^2 \in \mathbb{C}^{N \times N}$. Therefore the optimization problem is formulated as

$$\begin{aligned}
\min_{\mathbf{F}} \quad & \mathbf{f}^H \mathbf{P}_{\text{pow}} \mathbf{f} \\
\text{s.t.} \quad & \frac{\mathbf{f}^H \mathbf{P}_{\text{sig,ym}} \mathbf{f}}{\mathbf{f}^H (\mathbf{P}_{\text{int,ym}} + \mathbf{P}_{\text{nse,ym}}) \mathbf{f} + \sigma_{ym}^2} \geq \gamma_m, \\
& m = 1, \cdots, M
\end{aligned} \tag{4.3.21}$$

We now proceed to formulate the optimization scheme within a SDP framework [156]. Defining $\mathbf{D} = \mathbf{f}\mathbf{f}^H$, the optimization problem can be written as

$$\begin{aligned}
\min_{\mathbf{D}} \quad & \operatorname{tr}(\mathbf{P}_{\text{pow}} \mathbf{D}) \\
\text{s.t.} \quad & \frac{\operatorname{tr}(\mathbf{P}_{\text{sig,ym}} \mathbf{D})}{\operatorname{tr}(\mathbf{P}_{\text{int,ym}} \mathbf{D}) + \operatorname{tr}(\mathbf{P}_{\text{nse,ym}} \mathbf{D}) + \sigma_{ym}^2} \geq \gamma_m \\
& m = 1, \cdots, M \\
& \operatorname{rank}(\mathbf{D}) = 1, \mathbf{D} \succeq 0, \ \mathbf{D} = \mathbf{D}^*
\end{aligned} \tag{4.3.22}$$

The above scheme is not convex due to the constraint on rank, i.e. $\operatorname{rank}(\mathbf{D}) = 1$. The problem can however be relaxed into a convex problem using a standard technique known as semidefinite relaxation by dropping the constraint $\operatorname{rank}(\mathbf{D}) = 1$ [156]. Also, by including the SINR constraint for C side, one could write the overall optimization problem with rank relaxation as

$$\begin{aligned}
\min_{\mathbf{D}} \quad & \operatorname{tr}(\mathbf{P}_{\text{pow}} \mathbf{D}) \\
\text{s.t.} \quad & \operatorname{tr}(\mathbf{P}_{\text{sig,ym}} \mathbf{D}) - \gamma_m \operatorname{tr}(\mathbf{Z}_{m,y} \mathbf{D}) \geq \gamma_m \ \sigma_{ym}^2 \\
& \operatorname{tr}(\mathbf{P}_{\text{sig,cm}} \mathbf{D}) - \gamma_m \operatorname{tr}(\mathbf{Z}_{m,c} \mathbf{D}) \geq \gamma_m \ \sigma_{cm}^2 \\
& m = 1, \cdots, M \\
& \mathbf{D} \succeq 0, \ \mathbf{D} = \mathbf{D}^*
\end{aligned} \tag{4.3.23}$$

where $\mathbf{Z}_{m,y} = \mathbf{P}_{\text{int,ym}} + \mathbf{P}_{\text{nse,ym}}$ and $\mathbf{Z}_{m,c} = \mathbf{P}_{\text{int,cm}} + \mathbf{P}_{\text{nse,cm}}$. The above problem can be solved using interior point methods.

4.3.3 Simulation results

The performance of the proposed scheme is investigated for a relay network with two sources, two destinations at each side and a relay layer comprising of eight relay nodes, i.e. $M = 2$ and $N = 8$. The channels $\mathbf{h}$ and $\mathbf{g}$ have been generated using zero-mean unity variance complex Gaussian variables. The simulation discards the infeasible solutions. However, this occurs with a small probability, even at high target SINRs, as shown in Table 4.1.

Table 4.1. Table of outage probabilities (O.P.) for varying SINRs. The outage probability represents the probability that the target SINRs can not be achieved.

SINR (dB)	0	5	10	15	20	25
O.P.	0	0	0	0.0033	0.0086	0.0117

The required average power at the relay nodes versus SINR targets is depicted in Fig. 4.2. In this case, the channel is affected by AWGN, and the signal passes through it. As can be seen, the required average total power increases with increasing target SINR values. Fig. 4.3 depicts the relationship between the number of relays and the outage probability. The target SINR is assumed to be 15dB. The outage probability is reduced with increasing number of relays. The outage probability value reaches zero when more that six relays are used. As expected, the outage is one when two or three relays are used. This is because three relays are inadequate to suppress interference from two sources while achieving the desired SINR.

4.4 Optimization for Asynchronous Two Way Relaying Based Multiuser Multiplexing

In this section, the focus is on asynchronous relays where the signals from various sources and relays experience different delays, hence a simple spatial

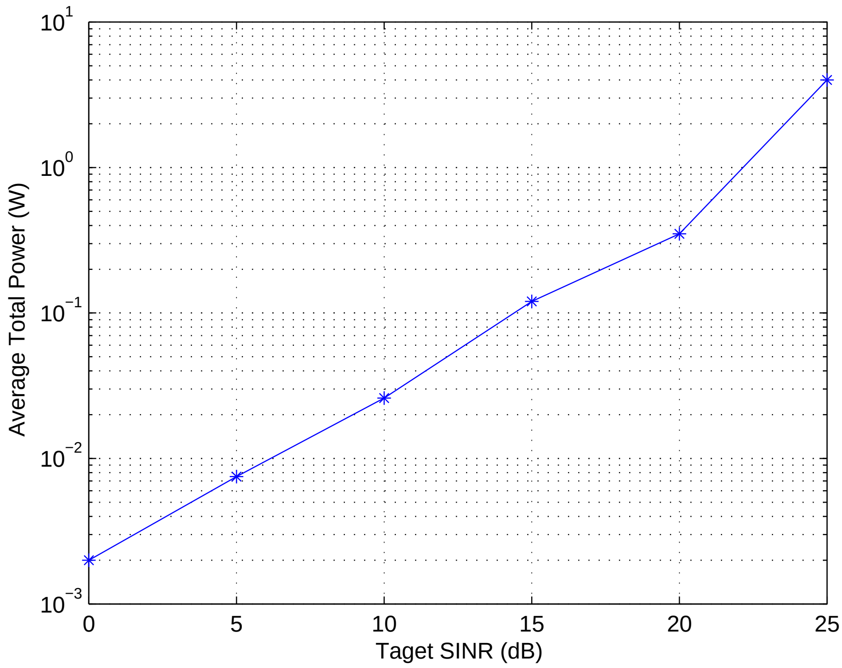

Figure 4.2. Relay average total power versus SINR targets.

only processing at the relays is inadequate. Hence space-time operation is used at the relays using finite impulse response (FIR) filters at the relays for this two-way relay network. It should be noted that FIR based processing at the relays have been used in [157] for a one-way relay network for mitigating ISI while performing spatial multiplexing.

4.4.1 System Model

As shown in Fig. 4.4, consider M pairs of source nodes and receiver nodes. There are R relays between sources and destinations. In the first phase (broadcasting phase) signal vectors $\mathbf{s}$ and $\mathbf{b}$ are transmitted from a set of source nodes A and B respectively, where $\mathbf{s} = [s_1(n), s_2(n), \cdots, s_M(n)]^T$ and $\mathbf{b} = [b_1(n), b_2(n), \cdots, b_M(n)]^T$. The variances of the signals at the A and B sides are denoted by σ_s^2 and σ_b^2 respectively. The signals are received by a set of relay nodes. The relay nodes transmit the received data to the destination nodes in the second phase (relaying phase).

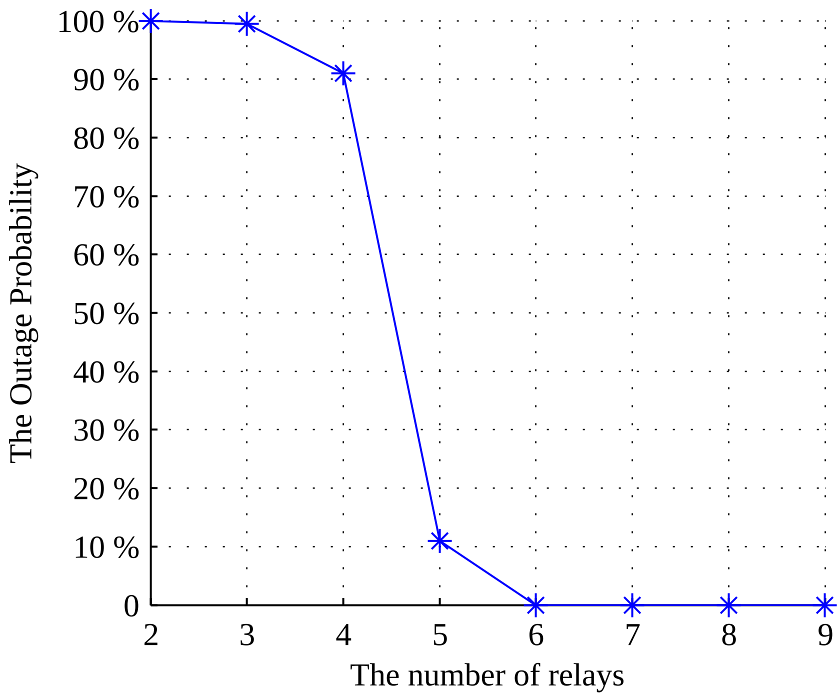

Figure 4.3. The outage probability (infeasible problem) versus number of relays.

Let us denote the matrix consisting of complex channel coefficients between the source nodes A and the relay nodes as $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_M] \in \mathbb{C}^{R\times M}$, where $\mathbf{h}_m = [h_{m,1}, h_{m,2}, \cdots, h_{m,R}]^T$ for $m = 1, \cdots, M$, is a column vector consisting of channel coefficients between the m^{th} source s_m and the observing relays. Similarly, let us denote the channel matrix consisting of complex channel coefficients between the source nodes B and the relay nodes as $\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2, \cdots, \mathbf{g}_M] \in \mathbb{C}^{R\times M}$, where $\mathbf{g}_m = [g_{m,1}, g_{m,2}, \cdots, g_{m,R}]^T$ for $m = 1, \cdots, M$, is a column vector consisting of channel coefficients between the m^{th} source b_m and the observing relays. Each transmission path is assumed to have different transmission delay. Let us define the delay matrix between the source nodes A and the relay nodes as $\boldsymbol{\tau}^f = \left[\boldsymbol{\tau}_1^f, \boldsymbol{\tau}_2^f, \cdots, \boldsymbol{\tau}_M^f\right] \in \mathbb{C}^{R\times M}$, where $\boldsymbol{\tau}_m^f = [\tau_{m1}^f, \tau_{m2}^f, \cdots, \tau_{mR}^f]^T$ for $m = 1, \cdots, M$, is a column vector consisting of delay values between the m^{th} source s_m and the observing relays. Similarly, let us denote the delay matrix between the source

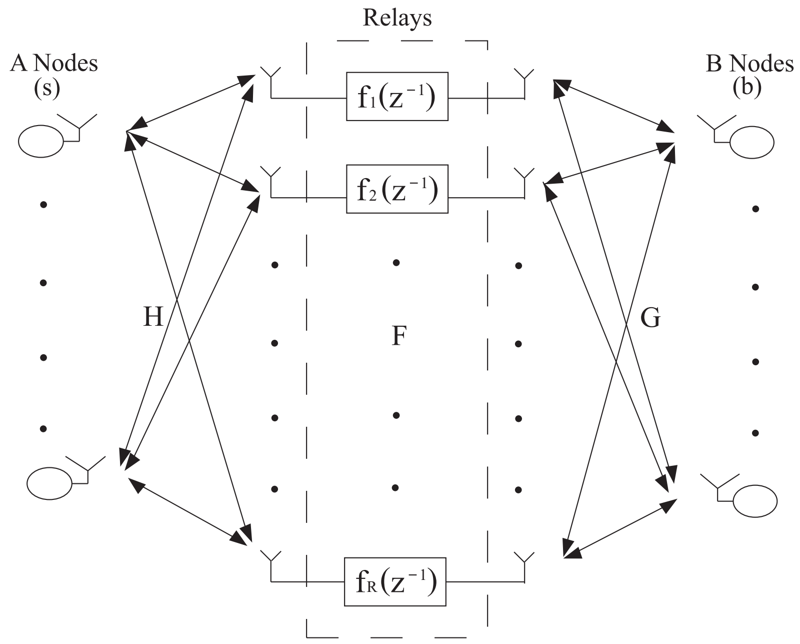

Figure 4.4. A relay network consisting of M pairs of source nodes and R relay nodes.

nodes B and the relay nodes as $\boldsymbol{\tau}^b = \left[\boldsymbol{\tau}_1^b, \boldsymbol{\tau}_2^b, \cdots, \boldsymbol{\tau}_M^b\right] \in \mathbb{C}^{R\times M}$, where $\boldsymbol{\tau}_m^b = [\tau_{m1}^b, \tau_{m2}^b, \cdots, \tau_{mR}^b]^T$ for $m = 1, \cdots, M$, is a column vector consisting of delay coefficients between the m^{th} source b_m and the observing relays. We assume that the maximum delay is τ_{max}. The received signal at the r^{th} relay node in the first transmission phase is given as

$$\begin{aligned} x_r(n) &= \sum_{i=1}^{M} h_{ir} s_i(n - \tau_{ir}^f) + \sum_{i=1}^{M} g_{ir} b_i(n - \tau_{ir}^b) \\ r &= 1, \cdots, R \end{aligned} \tag{4.4.1}$$

Since the transmission paths between the sources and the relays have different delays, it is important to employ FIR filter structure instead of a complex scaling operation at the relays. The transfer function of the FIR filter at the i^{th} relay is denoted as $f_i(z^{-1})$. The length of the FIR filter is assumed to be L. Using convolution operation, the received signal vector at the relay nodes is written as

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{G}\mathbf{b} + \mathbf{n} \tag{4.4.2}$$

where $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \cdots, \mathbf{x}_R^T]^T$ and $\mathbf{x}_r = [\mathbf{x}_r(n), \mathbf{x}_r(n-1), \cdots, \mathbf{x}_r(n-L+1)]^T$ is the input regressor vector for the FIR filter at the r^{th} relay. The transmitted signal vectors are given by,
$\mathbf{s} = [s_1(n), s_1(n-1), \cdots, s_1(n-\tau_{max}-L+1), s_2(n), \cdots, s_2(n-\tau_{max}-L+1), \cdots, s_M(n), \cdots, s_M(n-\tau_{max}-L+1)]^T$ and $\mathbf{b} = [b_1(n), b_1(n-1), \cdots, b_1(n-\tau_{max}-L+1), b_2(n), \cdots, b_2(n-\tau_{max}-L+1), \cdots, b_M(n), \cdots, b_M(n-\tau_{max}-L+1)]^T$. $\mathbf{n} \in \mathbb{C}^{RL\times 1}$ is a zero-mean circularly symmetric complex additive white Gaussian noise (AWGN) vector with covariance matrix $\sigma_r^2\mathbf{I}$. The channel is assumed to be quasi static, i.e. channel realizations are assumed to be fixed over the duration of a number of frames. In the second transmission phase, the r^{th} relay transmits the processed signal as

$$z_r(n) = \mathbf{f}_r^H \mathbf{x}_r(n) \tag{4.4.3}$$

where $\mathbf{f}_r \triangleq [f_{r1}^*, f_{r2}^*, \cdots, f_{rL}^*]^T$ is the FIR filter coefficient vector of the r^{th} relay. Hence the signal picked up by the m^{th} terminal on the A side is

$$y_m(n) = \sum_{r=1}^{R} h_{mr} z_r(n - \tau_{mr}^f) \tag{4.4.4}$$

Similarly the received signal at the m^{th} destination on the B side is obtained as

$$c_m(n) = \sum_{r=1}^{R} g_{mr} z_r(n - \tau_{mr}^b) \tag{4.4.5}$$

Substituting (4.4.3) into (4.4.4), we obtain

$$y_m(n) = \begin{bmatrix} h_{m1} & h_{m2} & \dots & h_{mR} \end{bmatrix} \begin{bmatrix} \mathbf{f}_1^H \mathbf{x}_1(n-\tau_{m1}^f) \\ \mathbf{f}_2^H \mathbf{x}_2(n-\tau_{m2}^f) \\ \vdots \\ \mathbf{f}_R^H \mathbf{x}_R(n-\tau_{mR}^f) \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{f}_1^H & \mathbf{f}_2^H & \dots & \mathbf{f}_R^H \end{bmatrix} \begin{bmatrix} h_{m1}\mathbf{x}_1(n-\tau_{m1}^f) \\ h_{m2}\mathbf{x}_2(n-\tau_{m2}^f) \\ \vdots \\ h_{mR}\mathbf{x}_R(n-\tau_{mR}^f) \end{bmatrix} \tag{4.4.6}$$

Similar to (4.4.2), we could write the vector

$$\begin{bmatrix} \mathbf{x}_1(n-\tau_{m1}^f) \\ \mathbf{x}_2(n-\tau_{m2}^f) \\ \vdots \\ \mathbf{x}_R(n-\tau_{mR}^f) \end{bmatrix} = \widetilde{\mathbf{H}}\widetilde{\mathbf{s}} + \widetilde{\mathbf{G}}\widetilde{\mathbf{b}} + \widetilde{\mathbf{n}} \tag{4.4.7}$$

where $\widetilde{\mathbf{s}} = [s_1(n), s_1(n-1), \cdots, s_1(n-2\tau_{max}-L+1), s_2(n), \cdots, s_2(n-2\tau_{max}-L+1), \cdots, s_M(n), \cdots, s_M(n-2\tau_{max}-L+1)]^T$ and $\widetilde{\mathbf{b}} = [b_1(n), b_1(n-1), \cdots, b_1(n-2\tau_{max}-L+1), b_2(n), \cdots, b_2(n-2\tau_{max}-L+1), \cdots, b_M(n), \cdots, b_M(n-2\tau_{max}-L+1)]^T$ respectively.

Let us define $\mathbf{H}_f$ such that the first L rows of $\mathbf{H}_f$ are element by element multiplication of the first L rows of $\widetilde{\mathbf{H}}$ by h_{m1}. Similarly, the other block rows are multiplied by $h_{m2}, \cdots, h_{mR}$. Therefore,

$$y_m(n) = \mathbf{f}^H(\mathbf{H}_f\widetilde{\mathbf{s}} + \mathbf{G}_f\widetilde{\mathbf{b}} + \widetilde{\mathbf{n}}) \tag{4.4.8}$$

where $\mathbf{f} \triangleq [\mathbf{f}_1^H, \mathbf{f}_2^H, \cdots, \mathbf{f}_R^H]^H \in \mathbb{C}^{RL\times 1}$ is the FIR filter coefficient vector for all the relays. Similarly the following is obtained

$$c_m(n) = \mathbf{f}^H(\mathbf{H}_b\widetilde{\mathbf{s}} + \mathbf{G}_b\widetilde{\mathbf{b}} + \widetilde{\mathbf{n}}) \tag{4.4.9}$$

Part of the vector $\widetilde{\mathbf{s}}$ consists of the signal transmitted by the m^{th} source $[s_m(n), \cdots, s_m(n-2\tau_{max}-L+1)]$, the rest of the elements are the interference from all other sources at A side, which can be written as $\widetilde{\widetilde{\mathbf{s}}} = [s_1(n), \cdots, s_1(n-2\tau_{max}-L+1), s_{m-1}(n), \cdots, s_{m-1}(n-2\tau_{max}-L+1), s_{m+1}(n), \cdots, s_{m+1}(n-2\tau_{max}-L+1), \cdots, s_M(n), \cdots, s_M(n-2\tau_{max}-L+1)]^T$. In the same way, part of the vector $\widetilde{\mathbf{b}}$ consists of the signal transmitted by the m^{th} source at B side $[b_m(n), \cdots, b_m(n-2\tau_{max}-L+1)]$, which is the desired signal for y_m. The rest of the elements account for interference for y_m, i.e.
$\widetilde{\widetilde{\mathbf{b}}} = [b_1(n), \cdots, b_1(n-2\tau_{max}-L+1), b_{m-1}(n), \cdots, b_{m-1}(n-2\tau_{max}-L+1), b_{m+1}(n), \cdots, b_{m+1}(n-2\tau_{max}-L+1), \cdots, b_M(n), \cdots, b_M(n-2\tau_{max}-L+1)]^T$. Let us denote the columns of $\mathbf{H}_f$ which correspond to $(s_m(n), \cdots, s_m(n-2\tau_{max}-L+1))$ as $\mathbf{H}_f^m$. All other elements of $\mathbf{H}_f$ are given by $\widetilde{\mathbf{H}}_f^m$. Defining $\mathbf{G}$ in the same way, we could write

$$y_m(n) = \mathbf{f}^H(\mathbf{H}_f^m\widetilde{\mathbf{s}}_m + \widetilde{\mathbf{H}}_f^m\widetilde{\widetilde{\mathbf{s}}_m} + \mathbf{G}_f^m\widetilde{\mathbf{b}}_m + \widetilde{\mathbf{G}}_f^m\widetilde{\widetilde{\mathbf{b}}_m} + \widetilde{\mathbf{n}}) \tag{4.4.10}$$

$$c_m(n) = \mathbf{f}^H(\mathbf{H}_b^m\widetilde{\mathbf{s}}_m + \widetilde{\mathbf{H}}_b^m\widetilde{\widetilde{\mathbf{s}}_m} + \mathbf{G}_b^m\widetilde{\mathbf{b}}_m + \widetilde{\mathbf{G}}_b^m\widetilde{\widetilde{\mathbf{b}}_m} + \widetilde{\mathbf{n}}) \tag{4.4.11}$$

4.4.2 SDP apply to filter-based relays

We determine an optimum FIR filter coefficient vector $\mathbf{f}$ that minimizes the total power usage at the relay nodes, p_{pow}, subject to QoS constraints

$\mathrm{SINR}_m \geq \gamma_m$ imposed at each destination node m on both sides, i.e.

$$\begin{aligned} \min_{\mathbf{F}} \quad & p_{\text{pow}} \\ \text{s.t.} \quad & \mathrm{SINR}_m^A \geq \gamma_m, \quad m = 1, \cdots, M \\ & \mathrm{SINR}_m^B \geq \gamma_m, \quad m = 1, \cdots, M \end{aligned} \tag{4.4.12}$$

where γ_m is the target SINR for the m^{th} destination at both the A and B sides. The power usage by the relays in the network is given by

$$\begin{aligned} p_{\text{pow}} &= \sum_{l=1}^{R} \mathbf{f}_l^H (\mathbf{H}_l \mathbf{H}_l^H \sigma_s^2 + \mathbf{G}_l \mathbf{G}_l^H \sigma_b^2 + \sigma_r^2 \mathbf{I}) \mathbf{f}_l \\ &= \mathbf{f}^H \mathbf{P}_{\text{pow}} \mathbf{f} \end{aligned} \tag{4.4.13}$$

where $\mathbf{H}_l$ and $\mathbf{G}_l$ are the channel coefficients matrix corresponding to the l^{th} relay.

$\mathbf{P}_{\text{pow}} \triangleq \text{diag}[\mathbf{H}_1\mathbf{H}_1^H + \mathbf{G}_1\mathbf{G}_1^H + \sigma_r^2\mathbf{I}, \mathbf{H}_2\mathbf{H}_2^H + \mathbf{G}_2\mathbf{G}_2^H + \sigma_r^2\mathbf{I}, \cdots, \mathbf{H}_R\mathbf{H}_R^H + \mathbf{G}_R\mathbf{G}_R^H + \sigma_r^2\mathbf{I}] \in \mathbb{C}^{RL\times RL}$. We derive the equation for SINR_m at the A side, however, the SINR at the B side can also be derived similarly. The received signal power at the m^{th} destination node is determined as

$$\begin{aligned} p_{\text{sig,y,m}} &= \mathrm{E}\left\{(\mathbf{f}^H \mathbf{G}_f^m \widetilde{\mathbf{b}}_m)(\mathbf{f}^H \mathbf{G}_f^m \widetilde{\mathbf{b}}_m)^H\right\} \\ &= \mathbf{f}^H \mathbf{P}_{\text{sig,ym}} \mathbf{f} \end{aligned} \tag{4.4.14}$$

where $\mathbf{P}_{\text{sig,ym}} \triangleq \mathbf{G}_f^m (\mathbf{G}_f^m)^H \mathbf{f} \sigma_b^2 \in \mathbb{C}^{RL\times RL}$. Similarly, the received interference power at the m^{th} destination node y_m due to signal contribution of all other source nodes is given as

$$\begin{aligned} p_{\text{int,y,m}} &= \mathrm{E}\left\{(\mathbf{f}^H \widetilde{\mathbf{H}}_f^m \widetilde{\widetilde{\mathbf{s}}}_m)(\mathbf{f}^H \widetilde{\mathbf{H}}_f^m \widetilde{\widetilde{\mathbf{s}}}_m)^H\right\} + \\ &\quad \mathrm{E}\left\{(\mathbf{f}^H \widetilde{\mathbf{G}}_f^m \widetilde{\widetilde{\mathbf{b}}}_m)(\mathbf{f}^H \widetilde{\mathbf{G}}_f^m \widetilde{\widetilde{\mathbf{b}}}_m)^H\right\} \\ &= \mathbf{f}^H \mathbf{P}_{\text{int,ym}} \mathbf{f} \end{aligned} \tag{4.4.15}$$

where $\mathbf{P}_{\text{int,ym}} \triangleq \widetilde{\mathbf{H}}_f^m(\widetilde{\mathbf{H}}_f^m)^H\sigma_s^2 + \widetilde{\mathbf{G}}_f^m(\widetilde{\mathbf{G}}_f^m)^H\sigma_b^2 \in \mathbb{C}^{RL\times RL}$. The noise power transferred to the m^{th} destination node, from the noise present at the relay nodes can be written as

$$\begin{aligned} p_{\text{nse,y,m}} &= \mathrm{E}\,\{(\mathbf{f}^H\mathbf{H}_{ym}\mathbf{n})(\mathbf{f}^H\mathbf{H}_{ym}\mathbf{n})^H\} \\ &= \mathbf{f}^H\mathbf{P}_{\text{nse,ym}}\mathbf{f} \end{aligned} \tag{4.4.16}$$

where $\mathbf{P}_{\text{nse,ym}} \triangleq \mathbf{H}_{ym}(\mathbf{H}_{ym})^H\sigma_r^2 \in \mathbb{C}^{RL\times RL}$ and $\mathbf{H}_{ym} = \text{diag}([h_{m1}, h_{m1}, \cdots, h_{m1}, h_{m2}, \cdots, h_{mR}, \cdots, h_{mR}]^T) \in \mathbb{C}^{RL\times 1}$. Hence the SINR at the m^{th} destination node at A side can be written as $\text{SINR}_{\text{m}}^A = \frac{p_{\text{sig,ym}}}{p_{\text{int,ym}}+p_{\text{nse,ym}}+\sigma_{ym}^2}$. By defining $\mathbf{D} \triangleq \mathbf{ff}^H$, we write the optimization problem using the SDP framework [156] is written as

$$\begin{aligned} \min_{\mathbf{D}} \ & \text{tr}(\mathbf{P}_{\text{pow}}\mathbf{D}) \\ \text{s.t.} \ & \frac{\text{tr}(\mathbf{P}_{\text{sig,ym}}\mathbf{D})}{\text{tr}(\mathbf{P}_{\text{int,ym}}\mathbf{D}) + \text{tr}(\mathbf{P}_{\text{nse,ym}}\mathbf{D}) + \sigma_{ym}^2} \geq \gamma_m \\ & m = 1, \cdots, M \\ & \text{rank}(\mathbf{D}) = 1, \mathbf{D} \succeq 0,\ \mathbf{D} = \mathbf{D}^* \end{aligned} \tag{4.4.17}$$

The above scheme is not convex due to the constraint on rank, i.e. $\text{rank}(\mathbf{D}) = 1$. The problem can be relaxed into a convex problem using a standard technique known as semidefinite relaxation by dropping the constraint $\text{rank}(\mathbf{D}) = 1$ [156]. Also, by including the SINR constraint for B side, we could write the overall optimization problem with rank relaxation is written as

$$\begin{aligned} \min_{\mathbf{D}} \ & \text{tr}(\mathbf{P}_{\text{pow}}\mathbf{D}) \\ \text{s.t.} \ & \text{tr}(\mathbf{P}_{\text{sig,ym}}\mathbf{D}) - \gamma_m\ \text{tr}(\mathbf{Z}_{m,y}\mathbf{D}) \geq \gamma_m\ \sigma_{ym}^2 \\ & \text{tr}(\mathbf{P}_{\text{sig,cm}}\mathbf{D}) - \gamma_m\ \text{tr}(\mathbf{Z}_{m,c}\mathbf{D}) \geq \gamma_m\ \sigma_{cm}^2 \\ & m = 1, \cdots, M \\ & \mathbf{D} \succeq 0,\ \mathbf{D} = \mathbf{D}^* \end{aligned} \tag{4.4.18}$$

where $\mathbf{Z}_{m,y} = \mathbf{P}_{\text{int,ym}} + \mathbf{P}_{\text{nse,ym}}$ and $\mathbf{Z}_{m,c} = \mathbf{P}_{\text{int,cm}} + \mathbf{P}_{\text{nse,cm}}$.

4.4.3 Simulation Results

A a two-way relay network with two sources at each side and six relay nodes is considered, i.e. $M = 2$ and $R = 8$. The channels were generated using zero mean Gaussian random variables with unity variance. Fig. 4.5 depicts the average transmission power required at the relays for various SINR targets. The outage probability shown on the plot indicates the probability that the target SINRs can not be achieved. As can be seen, the average total power of the relay nodes and the outage probability increase with increasing SINR targets. Fig. 4.6 depicts the BER performance of the proposed scheme. It is compared with the BER performance of AWGN channel. AWGN channel is the digital wired communication channel whose SNR is $\frac{p}{\sigma^2}$, where p is the transmit power, and σ^2 is the noise power. In this simulation scenario, the channel is not affected by the fading, therefore AWGN channel scenario is considered as an ideal channel scenario and the bench mark for performance comparison. The proposed relay network performs close to the performance of a simple AWGN channel.

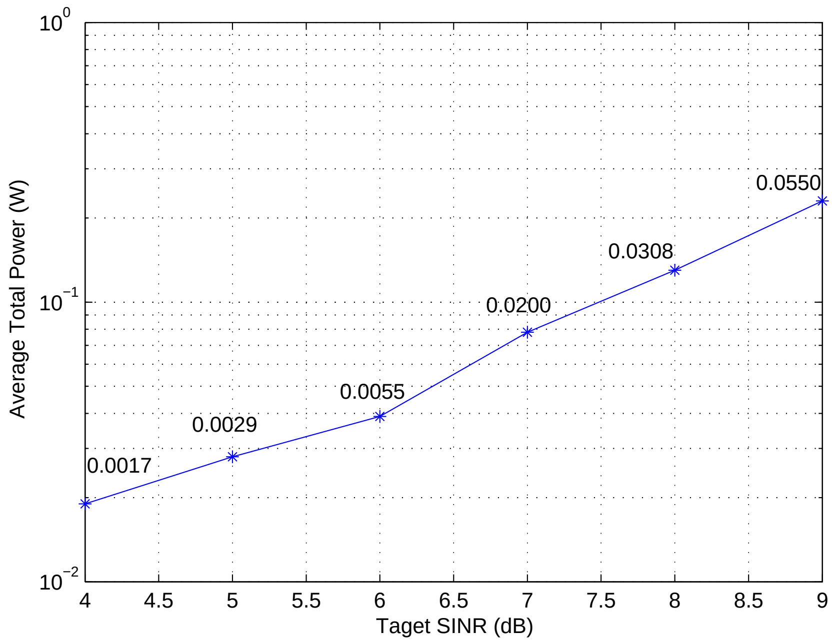

Figure 4.5. Relay average total power and the outage probability against SINR targets.

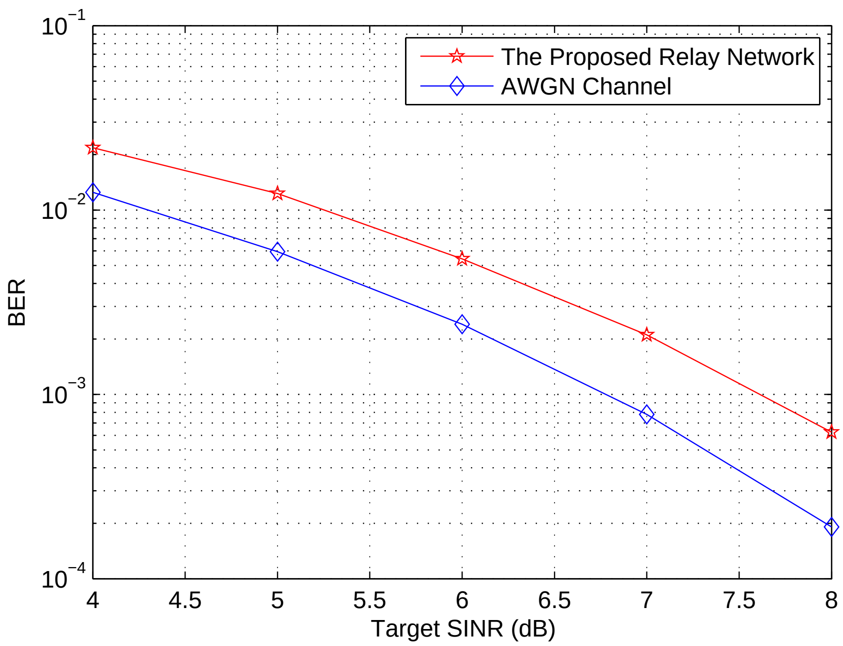

Figure 4.6. The BER comparison between the proposed relay network and AWGN channel for various values of SINR targets.

Chapter 5

DISTRIBUTED BEAMFORMING FOR PEER-TO-PEER TWO WAY RELAY NETWORK

In this chapter, a minimum mean square error (MMSE) based design technique for a two way relay network is proposed that serves multiple peer-to-peer users. The algorithm aims at minimizing sum of the mean square error associated with the retrieval of symbols at multiple destinations subject to a total transmission power constraint at the relays. The problem is solved using a Lagrangian formulation of the constrained optimization problem. Through certain mathematical manipulations of the optimization problem, a method to choose the optimum Lagrange multiplier that enables full use of the transmission power at the relay is proposed.

5.1 Introduction

The wireless relays have the potential to improve coverage, data throughput and energy usage of wireless networks [15, 24, 26, 29–43]. In [15, 26, 29, 30], MMSE has been considered as the design criterion for wireless relay design. By using multiple antennas at the transmit or receive nodes [24, 31–35], beamforming can be used and the relay network can take advantage of MIMO techniques. In addition, interference constraints can be included in the design within the context of cognitive radios as in [36–39, 41]. Compared to an ordinary one way relay network that requires two time slots for transmission through the relays, the two way relay network [106, 136, 148, 158] can improve spectral efficiency through simultaneous bidirectional transmissions. In a two way amplify and forward relay network, a pair of nodes wish to communicate each other simultaneously. In the first phase, both sides transmit signals simultaneously. After receiving the information, in the second phase, combining with a linear transformation matrix $\boldsymbol{F}$, the relay transmits the signals again towards each destination node. The effective bandwidth efficiency of the two way relay network is considered to be one time slot per transmission, because both the uplink and the downlink are served simultaneously, even though this will require two time slots. There has been a considerable amount of works published in the literature recently, for example, efficient channel estimation [5, 138, 147, 148], signal encoding [136, 137, 143] and receiver design [146]. The work in [140] considered optimum power allocation at the transmitter and the relays to maintain desired SNR at the receiving end for a pair of transmitter-receiver terminals with a number of relays. An MMSE method has been considered for spatial multiplexing for a given transmission power at the relays in [139]. The work in [158] utilized the filter-based relays to overcome inter-symbol interference in order to enhance the signal to SINR. In this chapter, work on MMSE-based precoder design in [15] is extended to a two way cooperative relay network. Single and multiple pairs of source and destination are considered to demonstrate the potential of the two-way relay networks for forwarding signals while performing spatial multiplexing. A new approach is proposed to determine the optimum signal scaling factor required to use all available transmission power at the relay through certain manipulations in the Lagrangian formulation.

5.2 System Model

As shown in Fig. 5.1, consider M pairs of source nodes and receiver nodes. There are N cooperative relays between sources and destinations. In the first phase (broadcasting phase) signal vectors $\boldsymbol{s} \in \mathbb{C}^{M\times 1}$ and $\boldsymbol{b} \in \mathbb{C}^{M\times 1}$ are transmitted from a set of source nodes Y and C respectively, where $\boldsymbol{s} = [s_1(n), s_2(n), \cdots, s_M(n)]^T$ and $\boldsymbol{b} = [b_1(n), b_2(n), \cdots, b_M(n)]^T$. The variance of the signals at the Y and C sides are denoted by σ_s^2 and σ_b^2 respectively. The signals are received by a set of relay nodes. The relay nodes transmit the received data to the destination nodes in the second phase (relaying phase).

Let us denote the channel matrix consisting of complex channel coefficients between source nodes Y and the relay nodes as

$$\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2, \cdots, \boldsymbol{h}_M] \in \mathbb{C}^{N\times M} \tag{5.2.1}$$

where $\boldsymbol{h}_m = [h_{m,1}, h_{m,2}, \cdots, h_{m,N}]^T \in \mathbb{C}^{N\times 1}$ for $m = 1, \cdots, M$, consists of the channel coefficients between the m^{th} source y_m and the observing relays. Similarly, let us denote the channel matrix consisting of complex channel coefficients between source nodes C and the relay nodes as

$$\boldsymbol{G} = [\boldsymbol{g}_1, \boldsymbol{g}_2, \cdots, \boldsymbol{g}_M] \in \mathbb{C}^{N\times M} \tag{5.2.2}$$

where $\boldsymbol{g}_m = [g_{m,1}, g_{m,2}, \cdots, g_{m,N}]^T \in \mathbb{C}^{N\times 1}$ for $m = 1, \cdots, M$, consists of

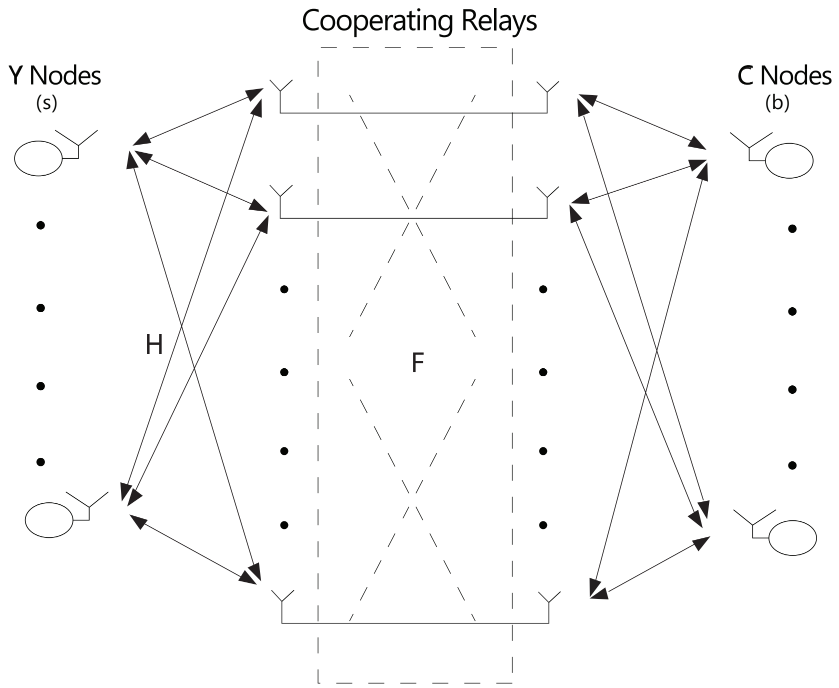

Figure 5.1. A relay network of M pairs of nodes and N relay nodes.

the channel coefficients between the m^{th} source c_m and the observing relays. The received signal vector at the relay nodes in the first transmission phase is given as

$$\boldsymbol{x} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{G}\boldsymbol{b} + \boldsymbol{n}_r \tag{5.2.3}$$

where $\boldsymbol{n}_r \in \mathbb{C}^{N \times 1}$ is zero-mean circularly symmetric complex additive white Gaussian noise (AWGN) vector with covariance matrix $\sigma_r^2 \boldsymbol{I}$. The channel is assumed to be quasi static, i.e. channel realizations are assumed to be fixed for the duration of a number of frames. In the second transmission phase, the relays rebroadcast a transformed signal vector as

$$\boldsymbol{z} = \boldsymbol{F}\boldsymbol{x} \tag{5.2.4}$$

where $\boldsymbol{F} \in \mathbb{C}^{N \times N}$ is a linear transformation matrix (relay transceiver) to be determined in order to optimize receiver performance. The received signal vectors at the destination sides Y and C can be written, respectively, as

$$\boldsymbol{y} = \boldsymbol{H}^T \boldsymbol{z} + \boldsymbol{n}_y \tag{5.2.5}$$

and

$$\boldsymbol{c} = \boldsymbol{G}^T \boldsymbol{z} + \boldsymbol{n}_c \tag{5.2.6}$$

where $\boldsymbol{n}_y \in \mathbb{C}^{M\times 1}$ and $\boldsymbol{n}_c \in \mathbb{C}^{M\times 1}$ are noise vectors containing zero-mean circularly symmetric AWGN components of the users at receivers with co-variance matrix $\sigma_y^2\boldsymbol{I}$ and $\sigma_c^2\boldsymbol{I}$, respectively. Substituting (5.2.4) into (5.2.5) and (5.2.6), respectively, and using (5.2.3) we obtain

$$\boldsymbol{y} = \boldsymbol{H}^T\boldsymbol{F}\boldsymbol{H}\boldsymbol{s} + \boldsymbol{H}^T\boldsymbol{F}\boldsymbol{G}\boldsymbol{b} + \boldsymbol{H}^T\boldsymbol{F}\boldsymbol{n}_r + \boldsymbol{n}_y \tag{5.2.7}$$

where $\boldsymbol{H}^T\boldsymbol{F}\boldsymbol{G}\boldsymbol{b}$ is the desired signal at side Y.

Similarly the received signal vector at the destination side C is obtained as

$$\boldsymbol{c} = \boldsymbol{G}^T\boldsymbol{F}\boldsymbol{H}\boldsymbol{s} + \boldsymbol{G}^T\boldsymbol{F}\boldsymbol{G}\boldsymbol{b} + \boldsymbol{G}^T\boldsymbol{F}\boldsymbol{n}_r + \boldsymbol{n}_c \tag{5.2.8}$$

where $\boldsymbol{G}^T\boldsymbol{F}\boldsymbol{H}\boldsymbol{s}$ is the desired signal at side C.

5.3 MMSE Cooperative Relay Strategy

We determine an optimum linear relay transceiver $\hat{\boldsymbol{F}}$ that minimizes the mean-square error (MMSE) between the uncorrupted received signals $\boldsymbol{H}^T\boldsymbol{z}$, $\boldsymbol{G}^T\boldsymbol{z}$ at both sides and the transmitted signals $\boldsymbol{b}$, $\boldsymbol{s}$, respectively, i.e.

$$\hat{\boldsymbol{F}} = \arg\min_{\boldsymbol{F}} J(\boldsymbol{F}) \tag{5.3.1}$$

where the cost function $J(\boldsymbol{F})$ is

$$\begin{aligned} J(\boldsymbol{F}) &= \sum_{m=1}^{M} (E\{|\boldsymbol{h}_{m,n}\boldsymbol{z} - b_m(n)|^2\} \\ &\quad + E\{|\boldsymbol{g}_{m,n}\boldsymbol{z} - s_m(n)|^2\}). \end{aligned} \tag{5.3.2}$$

The optimum $\boldsymbol{F}$ is determined subject to the relay power constraint expressed as

$$E\{\boldsymbol{z}^H\boldsymbol{z}\} = \text{tr}\ (\boldsymbol{F}\boldsymbol{R}_r\boldsymbol{F}^H) \leq P \tag{5.3.3}$$

where P is the total maximum possible power that can be used at the relay nodes and it controls the ability of the relays to amplify the received signals and forward them to the destinations of both the sides, respectively. The above optimization is solved using a Lagrangian formulation. It should be observed that when there is more than adequate power is available at the relays, the optimization of cost in (5.3.2) may not use all available power at the relays. Hence, a signal scaling factor α is introduced in the cost function and optimized so that the relay power constraint is satisfied with equality. Hence the cost function in (5.3.2) is modified as

$$\begin{aligned} J(\boldsymbol{F}) = &\ \text{tr}\ (E\{(\boldsymbol{H}^T\boldsymbol{z} - \alpha\boldsymbol{b})(\boldsymbol{H}^T\boldsymbol{z} - \alpha\boldsymbol{b})^H\} + \\ &\ \text{tr}\ (E\{(\boldsymbol{G}^T\boldsymbol{z} - \alpha\boldsymbol{s})(\boldsymbol{G}^T\boldsymbol{z} - \alpha\boldsymbol{s})^H\} \\ = &\ \text{tr}\ (\gamma - \alpha(\boldsymbol{H}^T\boldsymbol{F}\boldsymbol{G} + \boldsymbol{G}^H\boldsymbol{F}^H\boldsymbol{H}^*)\sigma_b^2 + \alpha^2\sigma_b^2\boldsymbol{I}) + \\ &\ \text{tr}\ (\xi - \alpha(\boldsymbol{G}^T\boldsymbol{F}\boldsymbol{H} + \boldsymbol{H}^H\boldsymbol{F}^H\boldsymbol{G}^*)\sigma_s^2 + \alpha^2\sigma_s^2\boldsymbol{I}) \end{aligned} \tag{5.3.4}$$

where tr $(\cdot)$ is a trace operator, and

$$\gamma = \boldsymbol{H}^T\boldsymbol{F}\boldsymbol{R}_r\boldsymbol{F}^H\boldsymbol{H}^*,$$

$$\xi = \boldsymbol{G}^T \boldsymbol{F} \boldsymbol{R}_r \boldsymbol{F}^H \boldsymbol{G}^*,$$

$$\boldsymbol{R}_r = \boldsymbol{H}\boldsymbol{H}^H \sigma_s^2 + \boldsymbol{G}\boldsymbol{G}^H \sigma_b^2 + \sigma_r^2 \boldsymbol{I}.$$

Adding total relay power constraint in (5.3.3), the constrained optimization is formulated using a Lagrangian multiplier $\tilde{\lambda}$ as

$$\begin{aligned} J(\boldsymbol{F}) = \text{tr } (&\boldsymbol{H}^T \boldsymbol{F} \boldsymbol{R}_r \boldsymbol{F}^H \boldsymbol{H}^* - \alpha(\boldsymbol{H}^T \boldsymbol{F} \boldsymbol{G} + \boldsymbol{G}^H \boldsymbol{F}^H \boldsymbol{H}^*)\sigma_b^2 \\ &+\alpha^2 \sigma_b^2 \boldsymbol{I}) + \text{tr } (\boldsymbol{G}^T \boldsymbol{F} \boldsymbol{R}_r \boldsymbol{F}^H \boldsymbol{G}^* - \alpha(\boldsymbol{G}^T \boldsymbol{F} \boldsymbol{H} + \\ &\boldsymbol{H}^H \boldsymbol{F}^H \boldsymbol{G}^*)\sigma_s^2 + \alpha^2 \sigma_s^2 \boldsymbol{I}) + \tilde{\lambda}(\text{tr } (\boldsymbol{F} \boldsymbol{R}_r \boldsymbol{F}^H) - P). \end{aligned} \quad (5.3.5)$$

Differentiating with respect to $\boldsymbol{F}^*$, the optimum $\boldsymbol{F}$ is determined in terms of $\tilde{\lambda}$ as

$$\begin{aligned} \boldsymbol{F}_{opt} = \alpha(&\boldsymbol{H}^* \boldsymbol{H}^T + \boldsymbol{G}^* \boldsymbol{G}^T + \tilde{\lambda} \boldsymbol{I})^{-1} \\ &(\boldsymbol{H}^* \boldsymbol{G}^H \sigma_b^2 + \boldsymbol{G}^* \boldsymbol{H}^H \sigma_s^2) \boldsymbol{R}_r^{-1}. \end{aligned} \quad (5.3.6)$$

Substituting (5.3.6) into (5.3.3), we obtain

$$\begin{aligned} P = \alpha^2 \text{tr } [((&\boldsymbol{H}\boldsymbol{H}^H + \boldsymbol{G}\boldsymbol{G}^H)^T + \tilde{\lambda} \boldsymbol{I})^{-1} (\boldsymbol{H}^* \boldsymbol{G}^H \sigma_b^2 + \boldsymbol{G}^* \boldsymbol{H}^H \sigma_s^2) \\ &\boldsymbol{R}_r^{-1} (\boldsymbol{G}\boldsymbol{H}^T \sigma_b^2 + \boldsymbol{H}\boldsymbol{G}^T \sigma_s^2)((\boldsymbol{H}\boldsymbol{H}^H + \boldsymbol{G}\boldsymbol{G}^H)^T + \tilde{\lambda} \boldsymbol{I})^{-1}]. \end{aligned} \quad (5.3.7)$$

The Lagrangian multiplier $\tilde{\lambda}$ can be determined by using (5.3.7). Using the approach proposed in [15], the following eigendecomposition is used to solve (5.3.7) and to avoid matrix inversion at every stage.

$$\begin{aligned} (\boldsymbol{H}\boldsymbol{H}^H + \boldsymbol{G}\boldsymbol{G}^H)^T &= \boldsymbol{Q} \Lambda \boldsymbol{Q}^H \\ \tilde{\lambda} \boldsymbol{I} &= \boldsymbol{Q} \tilde{\Lambda} \boldsymbol{Q}^H \end{aligned} \quad (5.3.8)$$

where $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \cdots, \lambda_{2M}, 0, \cdots, 0\}$ consists of eigenvalues of rank $2M$ matrix $(\boldsymbol{H}\boldsymbol{H}^H + \boldsymbol{G}\boldsymbol{G}^H)^T$, and $\tilde{\Lambda} = \mathrm{diag}\{\tilde{\lambda}, \tilde{\lambda}, \cdots, \tilde{\lambda}\}$. The matrix $((\boldsymbol{H}\boldsymbol{H}^H + \boldsymbol{G}\boldsymbol{G}^H)^T + \tilde{\lambda}\boldsymbol{I})^{-1}$ in (5.3.7) can be expressed as

$$((\boldsymbol{H}\boldsymbol{H}^H + \boldsymbol{G}\boldsymbol{G}^H)^T + \tilde{\lambda}\boldsymbol{I})^{-1} = \boldsymbol{Q}(\Lambda + \tilde{\Lambda})^{-1}\boldsymbol{Q}^H \tag{5.3.9}$$

where $(\Lambda+\tilde{\Lambda})^{-1} = \mathrm{diag}\{(\lambda_1+\tilde{\lambda})^{-1}, (\lambda_2+\tilde{\lambda})^{-1}, \cdots, (\lambda_{2M}+\tilde{\lambda})^{-1}, \tilde{\lambda}^{-1}, \tilde{\lambda}^{-1}, \cdots, \tilde{\lambda}^{-1}\}$. Hence, the power usage of the relay (5.3.7) can be expressed as

$$\mathrm{tr}\ (\boldsymbol{Q}(\Lambda + \tilde{\Lambda})^{-1}\boldsymbol{Q}^H\boldsymbol{B}\boldsymbol{Q}(\Lambda + \tilde{\Lambda})^{-1}\boldsymbol{Q}^H) = \frac{P}{\alpha^2} \tag{5.3.10}$$

where $\boldsymbol{B} = (\boldsymbol{H}^*\boldsymbol{G}^H\sigma_b^2 + \boldsymbol{G}^*\boldsymbol{H}^H\sigma_s^2)\boldsymbol{R}_r^{-1}(\boldsymbol{G}\boldsymbol{H}^T\sigma_b^2 + \boldsymbol{H}\boldsymbol{G}^T\sigma_s^2)$. Using the approach proposed in [15] and defining that $\boldsymbol{C} = \boldsymbol{Q}^H\boldsymbol{B}\boldsymbol{Q} = \boldsymbol{Q}^H(\boldsymbol{H}^*\boldsymbol{G}^H\sigma_b^2 + \boldsymbol{G}^*\boldsymbol{H}^H\sigma_s^2)\boldsymbol{R}_r^{-1}(\boldsymbol{G}\boldsymbol{H}^T\sigma_b^2 + \boldsymbol{H}\boldsymbol{G}^T\sigma_s^2)\boldsymbol{Q}$, (5.3.10) can be expressed as

$$\sum_{k=1}^{2M} C_{k,k}(\lambda_k + \tilde{\lambda})^{-2} = \frac{P}{\alpha^2}. \tag{5.3.11}$$

Using the same arguments as in [15], it can be proved that the last $(N - 2M)$ columns of $(\boldsymbol{H} + \boldsymbol{G})^T\boldsymbol{Q}$ are zero vectors. Therefore, (5.3.11) is a $4M$th-order polynomial in $\tilde{\lambda}$.

5.4 Algorithm Description

The cost in (5.3.4) and the constraint in (5.3.3) are convex function and convex set, respectively. Hence the overall optimization is a convex problem. The Lagrangian multiplier $\tilde{\lambda}$ should be either zero or positive at the optimum solution. The minimum of (5.3.4) is achieved when the power constraint becomes active, i.e. at optimum $\tilde{\lambda} \geq 0$. For this reason, it should be ensured that an appropriable α needs to be chosen so that $\tilde{\lambda} \geq 0$. To proceed, we need the following Lemma is needed.

Lemma 1. *A positive Lagrangian multiplier $\tilde{\lambda}$ can be ensured if*

$$\alpha = \sqrt{\frac{P}{\sum_{k=1}^{2M} C_{k,k}\lambda_k^{-2}}} + \theta = \alpha_0 + \theta, \tag{5.4.1}$$

for any arbitrary small positive value θ, where

$$\alpha_0 = \sqrt{\frac{P}{\sum_{k=1}^{2M} C_{k,k}\lambda_k^{-2}}}. \tag{5.4.2}$$

□

Proof. From (5.3.11) we can obtain a function α in terms of $\tilde{\lambda}$ as

$$\alpha(\tilde{\lambda}) = \sqrt{\frac{P}{\sum_{k=1}^{2M} C_{k,k}(\lambda_k + \tilde{\lambda})^{-2}}}. \tag{5.4.3}$$

Hence

$$\frac{\partial\tilde{\lambda}}{\partial\alpha} = \frac{\alpha[\sum_{k=1}^{2M} C_{k,k}(\lambda_k + \tilde{\lambda})^{-2}]^2}{P\sum_{k=1}^{2M} C_{k,k}(\lambda_k + \tilde{\lambda})^{-3}}. \tag{5.4.4}$$

Since $\boldsymbol{C}$ is positive semidefinite, $C_{k,k} \geqslant 0, \forall k$. Also since $(\boldsymbol{H}\boldsymbol{H}^H + \boldsymbol{G}\boldsymbol{G}^H)$ is positive semidefinite, $\lambda_k \geqslant 0, \forall k$. Therefore, $\frac{\partial\tilde{\lambda}}{\partial\alpha} \geq 0$ for $\alpha \geq 0$ and $\tilde{\lambda} \geq 0$. When $\alpha = \alpha_0$, $\tilde{\lambda} = 0$. Since $\alpha_0 > 0$ and $\frac{\partial\tilde{\lambda}}{\partial\alpha} \geq 0$ for $\alpha > 0$, $\tilde{\lambda}$ for $\alpha = \alpha_0 + \theta$ must be positive, i.e. any value of α greater than α_0 will ensure that $\tilde{\lambda} \geqslant 0$ and the power constraint is satisfied with equality, i.e. α is chosen as follows:

$$\alpha = \alpha_0 + \theta = \sqrt{\frac{P}{\sum_{k=1}^{2M} C_{k,k}\lambda_k^{-2}}} + \theta, \tag{5.4.5}$$

where θ is a very small positive value. □

Substituting (5.4.5) into (5.3.11), it can be ensured that a positive Lagrangian multiplier $\tilde{\lambda}$ can be obtained by solving

$$f(\tilde{\lambda}) = \sum_{k=1}^{2M} C_{k,k}(\lambda_k + \tilde{\lambda})^{-2} - \frac{P}{\alpha^2} = 0. \tag{5.4.6}$$

Since

$$\frac{\partial f(\tilde{\lambda})}{\partial \tilde{\lambda}} = -2\sum_{k=1}^{2M} C_{k,k}(\lambda_k + \tilde{\lambda})^{-3} \leq 0, \tag{5.4.7}$$

$f(\tilde{\lambda})$ is a monotonically decreasing function of $\tilde{\lambda}$. The minimum value of $\tilde{\lambda}$ considered should be zero, to satisfy the nonnegative condition for the Lagrangian multiplier. The solution of (5.4.6) for $\tilde{\lambda}$ is always bounded below

$$\tilde{\lambda} \leq \sqrt{\frac{2M\alpha^2}{P}\max_k(C_{k,k})} - \min_k(\lambda_k). \tag{5.4.8}$$

Hence a bisection search is performed to determine the positive $\tilde{\lambda}$ within the range of $\tilde{\lambda}_L = 0$ and $\tilde{\lambda}_R = \sqrt{\frac{2M\alpha^2}{P}\max_k(C_{k,k})} - \min_k(\lambda_k)$, where $\tilde{\lambda}_L$ is a lower bound and $\tilde{\lambda}_R$ is an upper bound. The search algorithm for the optimal Lagrangian multiplier $\tilde{\lambda}$ is the following. Firstly, we find out the range of $\tilde{\lambda}$. Secondly, update $\tilde{\lambda} = (\tilde{\lambda}_L + \tilde{\lambda}_R)/2$, substitute $\tilde{\lambda}$ into $f(\tilde{\lambda})$, if $f(\tilde{\lambda}) \neq 0$, adjust $\tilde{\lambda}_L = \begin{cases} \tilde{\lambda}_L & = & f(\tilde{\lambda}) < 0 \\ \tilde{\lambda} & = & f(\tilde{\lambda}) > 0 \end{cases}$, $\tilde{\lambda}_R = \begin{cases} \tilde{\lambda}_R & = & f(\tilde{\lambda}) > 0 \\ \tilde{\lambda} & = & f(\tilde{\lambda}) < 0 \end{cases}$, and update $\tilde{\lambda} = (\tilde{\lambda}_L + \tilde{\lambda}_R)/2$. Repeat these procedures until $f(\tilde{\lambda}) = 0$. The optimal Lagrangian multiplier $\tilde{\lambda}$ is obtained from this solution.

5.5 Simulation Results

The performance of the proposed algorithm is studied for a two way relay network with single and multiple pairs of sources and destinations. The total transmit power of relays is set to $P = 1$W, and the channels have been generated using zero mean complex Gaussian random variables with unity variance.

Firstly, a system with one source and one destination is considered, i.e. $M = 1$. The BER performance is depicted in Fig. 5.2 for various number of relays against SNR. The BER decreases with increasing SNR for a fixed number of relays. Also BER decreases with increasing number of relays. For the simulation purpose, the SNR is defined as the ratio between the total relay power P and the variance of the noise at the destination.

Fig. 5.3 depicts the BER performance with multiple pairs of sources and destinations. 6 relays were used, i.e. $N = 6$. It can be seen that the BER decreases with increasing SNR. The BER increases with increasing number of source-destination pairs.

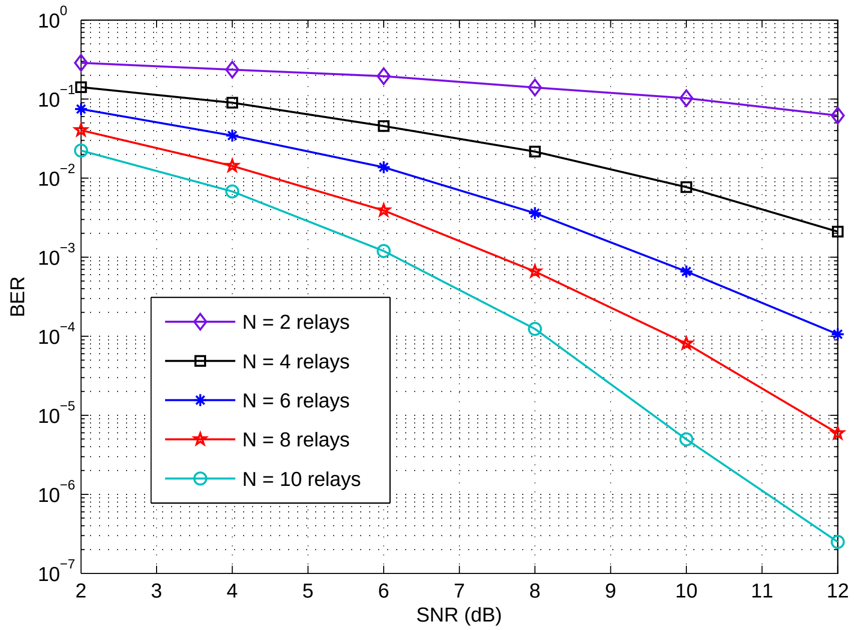

Figure 5.2. Comparison of the BER performance for the two-way relay network with one source and one destination for various numbers of relays.

5.6 Conclusion

A signal forwarding algorithm for a two way cooperative relay network has been proposed, whose optimal receiver has been designed using an MMSE criterion. The Lagrangian multiplier was employed to solve the optimization problem, and a bisection method simplified the calculation of the optimal Lagrangian multiplier instead of solving a high order polynomial function.

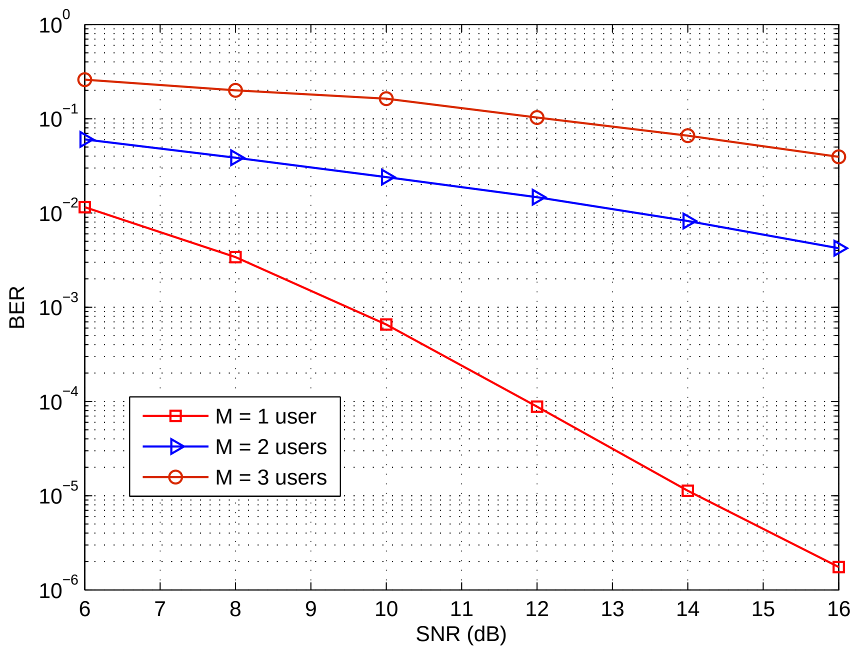

Figure 5.3. Comparison of the BER performance for the two-way relay network with six relays for various numbers of source and destination pairs.

Chapter 6

RESOURCE OPTIMIZATION TECHNIQUES FOR OFDMA-BASED COGNITIVE RADIO RELAY NETWORK

In this chapter, a power allocation technique is proposed for an OFDM-based cognitive radio wireless relay network. In particular, the sum rate maximization problem is studied for the SU network subject to constant total power and interference leakage constraints. This multiple-constraint maximization problem is transformed into an equivalent single constraint problem using two auxiliary variables. The auxiliary variables are adapted using a sub-gradient technique while the optimal Lagrangian multipliers are obtained using a bisection method. Simulation results demonstrate the convergence of the algorithm and the satisfaction of multiple constraints.

6.1 Introduction

When a receiver is not in the coverage area of a transmitter, a wireless relay can help the transmitter to forward signals to the receiver. Employment of relays could also result into lower overall transmission power and signal diversity [159]. Such a relay network is considered within a spectrum sharing context namely cognitive radio networks. The cognitive radio is considered as one of the promising techniques to improve the radio spectrum utilization [13, 14] and [160]. This is because a secondary network could access the primary spectrum provided the transmissions of the signal from the secondary network do not harmfully affect the primary receivers. One possible arrangement is to enable the secondary transmitter to sense the availability of spectrum and to transmit only when the spectrum is unoccupied [98, 161–163] and [164]. Another possibility is to transmit signals while ensuring the interference leakage to the primary receiver is below a specific threshold. This arrangement is known as underlay cognitive radio network. Such an underlay secondary network is considered with multiple users and a relay in the presence of a primary receiver. There have been significant interests in the combined relay and the cognitive radio networks due to power and spectrum efficiencies [41, 165–167] and [168]. There are recent works on the optimization techniques for relay networks under a spectrum sharing set up. For example, in [165], a cooperative relaying scheme for an underlay cognitive radio network was proposed, but from a beamforming perspective. The work in [166] considered a two way relay network under a cognitive radio setup, again from the perspective of relay beamforming. In this chapter, an OFDM-based relay network is considered. Resource allocation for OFDM-based networks has been studied in [17, 169–171] and [56] which improves the network performance by allocating the subcarriers and power optimally. A power allocation (PA) technique at the source and the relay separately has been proposed in [170]. The work in [170] is based on a suboptimal joint source-relay PA scheme. In [56, 172] and [43], a dynamic resource allocation technique for an OFDM-based wireless relay network was proposed which considered asymmetric transmission durations from the source to relay and from relay to the destinations. Since the work in [43, 56, 172] considered only a conventional wireless network without primary network, the optimization problem for power allocation considered only one constraint that is the total transmission power constraint. However, in this paper, a cognitive radio-based wireless relay network is considered and a power allocation technique for OFDM-subcarriers from the base station to the relay and from the relay to multiple destinations is proposed to maximize the sum rate. This optimization problem has both the sum power constraint and the interference leakage constraint and is solved by combining multiple constraints into a single constraint using auxiliary variables and by adapting the auxiliary variables using a sub-gradient method.

6.2 System model

A cognitive radio relay network is considered where one base station (BS) transmits signal to K users through a regenerative half-duplex relay, as shown in Fig. 7.2. An OFDM Access (OFDMA) scheme is employed to serve multiple-users. It is assumed that the BS cannot transmit the signals to the users directly due to long distance and obstacles. A total bandwidth of B is divided into N independent subchannels. The downlink and the uplink transmissions between BS and the users cover two time slots. In the first time slot T_1, the BS transmits signal to the relay, where the signal can be received and decoded. In the second time slot T_2, the relay re-encodes the signal and forwards it to the users. The total end-to-end transmission time can be expressed as $T = T_1 + T_2$. The total transmit power allocated to the whole network is p_T. It is assumed that the single-sided power spectral density for additive white Gaussian noise (AWGN) N_0 is identical at the relay and the receiver terminals. Define h_{SR}^m as the channel gain from the BS to the relay at subcarrier m and $h_{RD_k}^n$ is the channel gain from the relay to the k^{th} user terminal at subcarrier n $(m, n \in \{1, \cdots, N\})$. The channel-to-noise power ratio (CNR) for the two consecutive slots can be written as $\gamma_{SR}^m = |h_{SR}^m|^2/(N_0 B/N)$ and $\gamma_{RD_k}^n = |h_{RD_k}^n|^2/(N_0 B/N)$. In the meantime, h_{SP}^m and h_{RP}^m are the channel gains from the BS to the PU receiver and from the relay to the the PU receiver respectively, at subcarrier m.

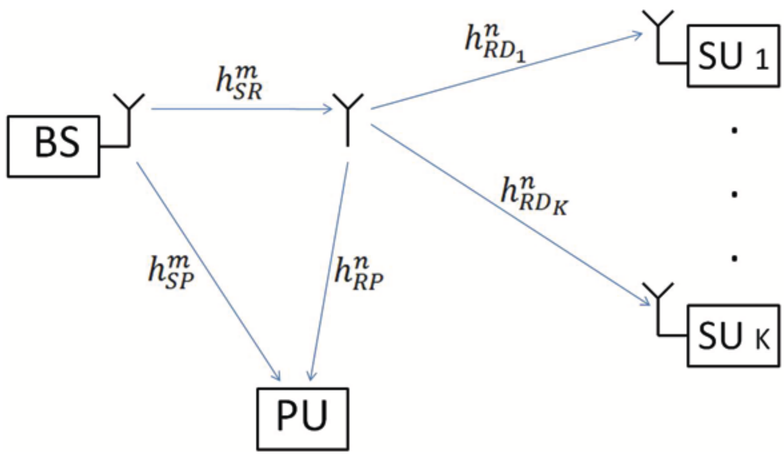

Figure 6.1. A cognitive radio relay network with K SUs and one PU.

6.3 Optimal Asymmetric Resource Allocation for a Cognitive Relay Network

6.3.1 Formulation of the cost function

An asymmetric resource allocation similar to [56] and [43], but for a cognitive relay network is considered. The instantaneous capacities of the two consecutive slots over the subchannel m and n can be written respectively as

$$C_{SR}^m = \frac{T_1}{T} \log_2(1 + p_{SR}^m \gamma_{SR}^m) \tag{6.3.1}$$

$$C_{RD_k}^n = \frac{T_2}{T}\log_2(1 + p_{RD_k}^n \gamma_{RD_k}^n) \tag{6.3.2}$$

where p_{SR}^m and $p_{RD_k}^n$ are the power allocated to the subchannel m in the first time slot and the subchannel n to the k^{th} user in the second time slot, respectively. In this case, the power will be allocated to the user whose n^{th} subchannel is the best for gaining the highest capacity for the whole network, i.e., the best $\gamma_{RD_k}^n$ will be selected from K users. Therefore, the channels for the K users in n^{th} subchannel in the second time slot can be considered as an equivalent single channel whose gain is $\gamma_{RD}^n = \max\{\gamma_{RD_1}^n, \cdots, \gamma_{RD_k}^n\}$. p_T and T are total transmission power and a fixed total transmission duration. The aim is to maximize the sum data rate subject to a sum power constraint and an interference constraint. The interference constraint will limit the interference leakage from the source and the relay to the primary terminal below a specific threshold ξ. The sum data rate is limited by the minimum of the data rate from the source to the relay and that from the relay to all destinations. Therefore, the optimization problem can be formulated as follows:

$$\underset{p_{SR}^m, p_{RD}^n, T_1, T_2}{\arg\max} \left(\min\left\{ \sum_{m=1}^{N} C_{SR}^m, \sum_{n=1}^{N} C_{RD}^n \right\} \right) \tag{6.3.3}$$

subject to

$$\sum_{m=1}^{N} p_{SR}^m + \sum_{n=1}^{N} p_{RD}^n \leq p_T \tag{6.3.4}$$

$$\sum_{m=1}^{N} p_{SR}^m |h_{SP}^m|^2 + \sum_{n=1}^{N} p_{RD}^n |h_{RP}^n|^2 \leq \xi \tag{6.3.5}$$

$$p_{SR}^m > 0, p_{RD}^n > 0 \tag{6.3.6}$$

and

$$T_1 + T_2 = T, \quad T_1 > 0, \quad T_2 > 0 \tag{6.3.7}$$

It can be proved that (6.3.3) is maximized only when $\sum_{m=1}^{N} C_{SR}^m = \sum_{n=1}^{N} C_{RD}^n$. Also using the approach of [39], the sum power and the interference leakage constraints in (6.3.4) and (6.3.5) can be combined into one constraint using auxiliary dual variables α and β. Hence, the optimization in (6.3.3) can be written as

$$\begin{aligned} \underset{p_{SR}^m, p_{RD}^n, T_1, T_2}{\arg\max} \quad & \sum_{m=1}^{N} \frac{T_1}{T} \log_2(1 + p_{SR}^m \gamma_{SR}^m) + \\ & \sum_{n=1}^{N} \frac{T_2}{T} \log_2(1 + p_{RD}^n \gamma_{RD}^n) \end{aligned} \tag{6.3.8}$$

subject to

$$\sum_{m=1}^{N} \frac{T_1}{T} \log_2(1 + p_{SR}^m \gamma_{SR}^m) = \sum_{n=1}^{N} \frac{T_2}{T} \log_2(1 + p_{RD}^n \gamma_{RD}^n) \tag{6.3.9}$$

$$\begin{aligned} \alpha \left(\sum_{m=1}^{N} p_{SR}^m + \sum_{n=1}^{N} p_{RD}^n \right) + \\ \beta \left(\sum_{m=1}^{N} p_{SR}^m |h_{SP}^m|^2 + \sum_{n=1}^{N} p_{RD}^n |h_{RP}^n|^2 \right) \leq \alpha p_T + \beta \xi \end{aligned} \tag{6.3.10}$$

6.3.2 Algorithm Description

Using a similar approach as in [56] and [43], it can be proven that the optimization problem in (7.3.5) is convex. It can be solved using the Karush-Kuhn-Tucker (KKT) conditions [132]. The optimization in the primal form is written as

$$
\begin{aligned}
L = & \sum_{m=1}^{N} \frac{T_1}{T} \log_2(1 + p_{SR}^m \gamma_{SR}^m) \\
& + \sum_{n=1}^{N} \frac{T_2}{T} \log_2(1 + p_{RD}^n \gamma_{RD}^n) \\
& - \lambda \left[\sum_{m=1}^{N} \frac{T_1}{T} \log_2(1 + p_{SR}^m \gamma_{SR}^m) \right. \\
& \left. - \sum_{n=1}^{N} \frac{T_2}{T} \log_2(1 + p_{RD}^n \gamma_{RD}^n) \right] \\
& - \mu \left[\alpha(\sum_{m=1}^{N} p_{SR}^m + \sum_{n=1}^{N} p_{RD}^n) \right. \\
& \left. + \beta(\sum_{m=1}^{N} p_{SR}^m |h_{SP}^m|^2 + \sum_{n=1}^{N} p_{RD}^n |h_{RP}^n|^2) - \alpha p_T - \beta \xi \right] \\
& - \sigma(T_1 + T_2 - T)
\end{aligned}
\tag{6.3.11}
$$

where λ, μ and σ are the Lagrangian multipliers. T_1, T_2, p_{SR}^m and p_{RD}^n can be obtained using the KKT conditions, where

$$
T_1 = \frac{1-\lambda}{2} T \tag{6.3.12}
$$

$$
T_2 = \frac{1+\lambda}{2} T \tag{6.3.13}
$$

$$
p_{SR}^m = \left[\frac{(1-\lambda)^2}{2\mu(\alpha + \beta |h_{SP}^m|^2)} - \frac{1}{\gamma_{SR}^m} \right]^+ \tag{6.3.14}
$$

$$
p_{RD}^n = \left[\frac{(1+\lambda)^2}{2\mu(\alpha + \beta |h_{RP}^n|^2)} - \frac{1}{\gamma_{RD}^n} \right]^+ \tag{6.3.15}
$$

where $[x]^+ = \begin{cases} x & = & x > 0 \\ 0 & = & x < 0 \end{cases}$. The Lagrangian multipliers can be obtained by solving the following two simultaneous equations:

$$\begin{aligned} & f(\mu,\lambda) = \alpha p_T + \beta\xi \\ & -(\alpha + \beta|h_{SP}^m|^2)\sum_{m=1}^{N}\left[\frac{(1-\lambda)^2}{2\mu(\alpha+\beta|h_{SP}^m|^2)} - \frac{1}{\gamma_{SR}^m}\right]^+ \\ & -(\alpha + \beta|h_{RP}^n|^2)\sum_{n=1}^{N}\left[\frac{(1+\lambda)^2}{2\mu(\alpha+\beta|h_{RP}^n|^2)} - \frac{1}{\gamma_{RD}^n}\right]^+ \\ & = 0 \end{aligned} \tag{6.3.16}$$

$$\begin{aligned} g(\mu,\lambda) = (1+\lambda)\sum_{n=1}^{N}\left[\log_2\left(\frac{(1+\lambda)^2\gamma_{RD}^n}{2\mu(\alpha+\beta|h_{RP}^n|^2)}\right)\right]^+ \\ -(1-\lambda)\sum_{m=1}^{N}\left[\log_2\left(\frac{(1-\lambda)^2\gamma_{SR}^m}{2\mu(\alpha+\beta|h_{SP}^m|^2)}\right)\right]^+ = 0 \end{aligned} \tag{6.3.17}$$

The overall algorithm is solved as shown in the pseudo code below:

1. Initialize α and β to positive values.
2. Repeat a)-b) until convergence of α and β.
   (a) Solve (6.3.16) and (6.3.17) using a bisection search method.
   (b) Determine the optimum T_1, T_2, p_{SR}^m and p_{RD}^n using (6.3.12)-(6.3.15).
   (c) Adapt α and β using the subgradient method shown in (6.3.18) and (6.3.19).
3. End for repeat 2).

The values of α and β are initialized to arbitrary positive values, for example $\alpha = 1$ and $\beta = 1$. For a given set of auxiliary variables α and β, the optimal Lagrangian multipliers can be obtained by solving (6.3.16) and (6.3.17) using a bisection method as described in [43]. The optimal time durations for each time slot and the optimal power allocation to each subchannel can be obtained by substituting the optimal Lagrangian multipliers into (6.3.12), (6.3.13), (6.3.14) and (6.3.15). After determining the optimal power allocation p_{SR}^m and p_{RD}^n at the end of each iteration, we can update α and β using a subgradient algorithm until convergence as,

$$\alpha^{n+1} = \alpha^n + t(\sum_{m=1}^{N} p_{SR}^m + \sum_{n=1}^{N} p_{RD}^n - p_T) \tag{6.3.18}$$

and

$$\beta^{n+1} = \beta^n + t(\sum_{m=1}^{N} p_{SR}^m |h_{SP}^m|^2 + \sum_{n=1}^{N} p_{RD}^n |h_{RP}^n|^2 - \xi) \tag{6.3.19}$$

This is repeated until the values of α and β converge either to zero or to an asymptotic value. The convergence to zero means that the corresponding constraint is inactive at the optimum point, and this is the mostly case as the optimization is limited by either the total power or the interference leakage [39].

6.4 Simulation results

The performance of the proposed scheme is investigated for an OFDM regenerative CR relay network with one BS, one relay, two SUs and one PU. The total bandwidth is $B = 5$MHz, which is divided into $N = 64$ subchannels. The power spectral density of AWGN $N_0 = 0.1\mu$W/Hz, and the total time frame is assumed as $T = 1$s.

For the results in Fig. 6.2, the total transmit power for the whole network is assumed as $p_T = 1$W. As seen in Fig. 6.2 that the total capacity increases with increasing total interference threshold. From Fig. 6.3, we can see that the capacity increases with increasing total power when the total interference from transmitters to primary user is fixed to $\xi = 0.2$W. Fig. 6.4 shows the adaptation of the auxiliary variables α and β corresponding to the total power and total interference, respectively. It is assumed that $p_T = 1$W and $\xi = 0.2$W. It can be seen that β is increasing and α is decreasing. Either α or β should converge to zero indicating only one constraint will be active as observed in [39].

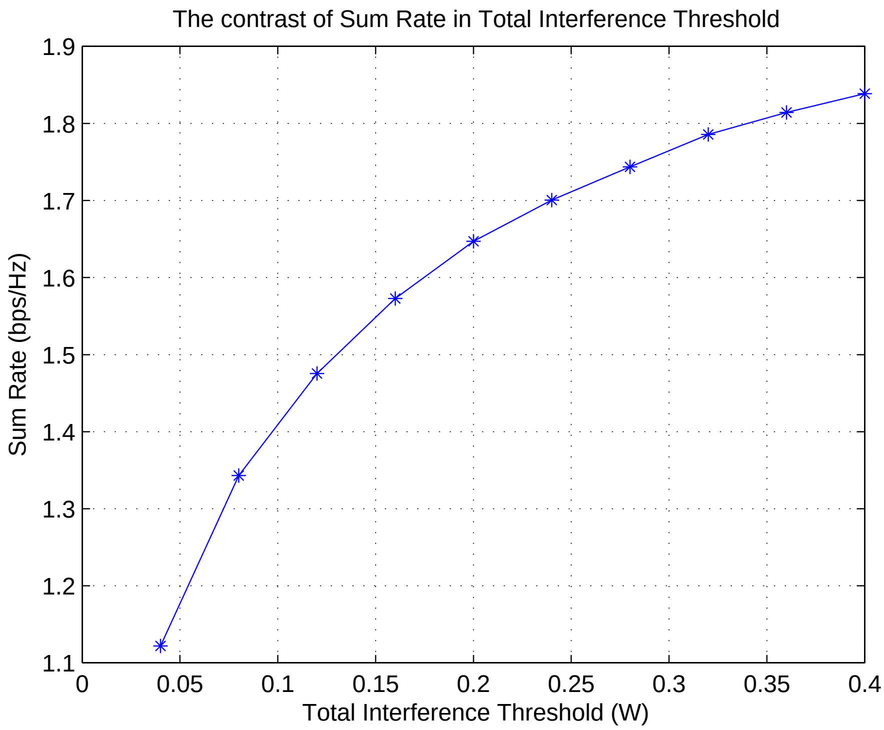

Figure 6.2. The total capacity against total interference threshold.

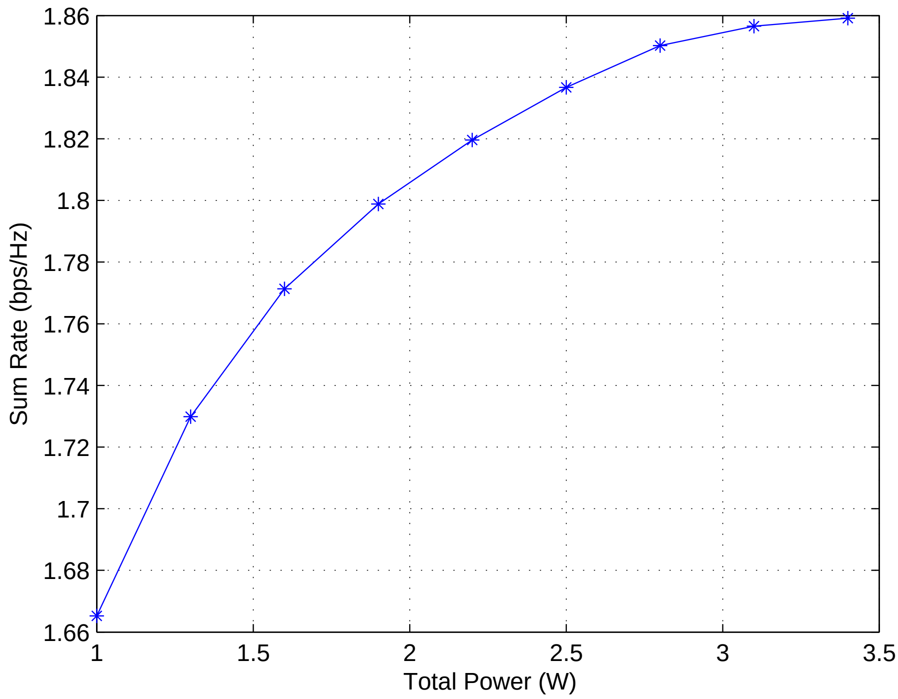

Figure 6.3. The total capacity versus total power.

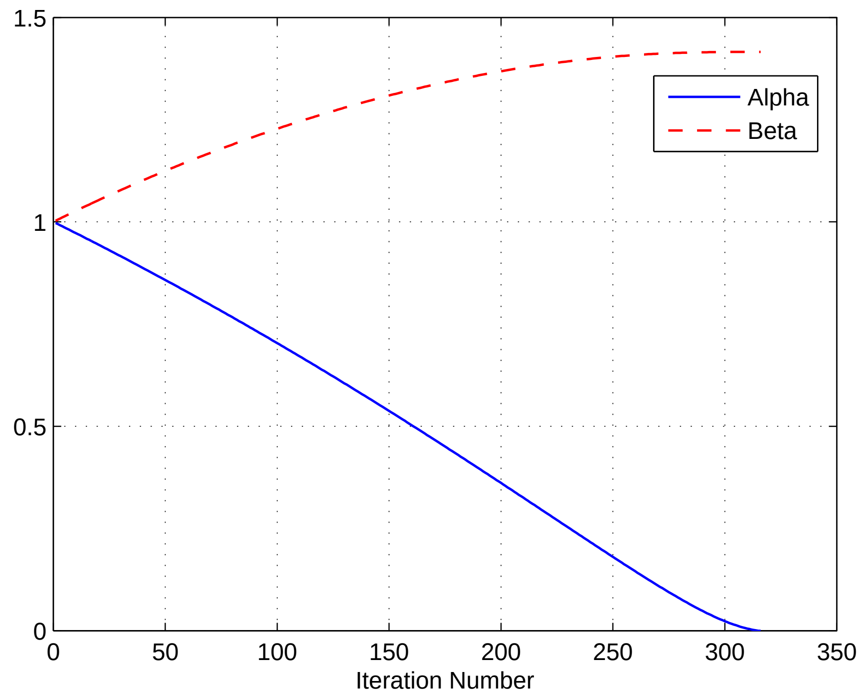

Figure 6.4. Convergence of the auxiliary variables α and β.

Chapter 7

DOWNLINK RESOURCE ALLOCATION TECHNIQUES FOR OFDMA-BASED WIRELESS RELAY NETWORKS

In this chapter, we investigate a joint optimization problem for beamforming and temporal power allocation for an OFDM-based wireless relay network. Both the BS and the relay employ multiple antennas for transmission whilst the multiple destinations (users) employ single antenna. The aim is to maximize the capacity of the wireless relay network while satisfying the total power budget. We solve this problem by asymmetrically allocating the temporal power to different time slots and using the principle of broadcast channel-multiple access channel (BC-MAC) duality. The proposed algorithm in its dual form has been solved using bisection-based Geometric Programming (GP).

7.1 Introduction

Employment of wireless relays between a source node and the destination nodes can enhance capacity and coverage of a wireless network and it has the potential to reduce the required transmission power significantly [173, 174], [159]. On the other hand, OFDM enables transmission of information over multiple orthogonal narrowband subcarriers, hence has the potential to enhance efficient use of the radio spectrum. Also, through insertion of cyclic prefix [175], OFDM could convert a frequency-selective channel into a number of frequency flat channels resulting into considerable benefits in terms of complexity of transmission and reception of the signals, [29, 159, 176, 177].

7.1.1 Related Work

Wireless relay can be generally classified into three groups: amplify and forward, compress and forward and decode and forward. In the amplify and forward scheme, the relay receives, amplifies and forwards the signals transmitted by the source nodes to the destinations [149, 178–180]. In compress and forward scheme, the received signals can be compressed at the relay by exploiting the statistical dependencies between the signals at the nodes [181, 182]. In decode and forward scheme, relay decodes and re-encodes the received signals, before forwarding them to the destination nodes [183, 184].

In [17, 159, 169–171], and [56], various resource allocation techniques for OFDM-based networks were studied. A power allocation (PA) technique at the source and the relay has been proposed in [170]. The work in [170] is based on a suboptimal joint source-relay PA scheme. A dynamic resource allocation technique for an OFDM-based wireless relay network was proposed in [56] and [172], which considered asymmetric transmission durations from the source to relay and from relay to the destination nodes. Recently, resource allocation techniques for OFDM-based cognitive radio have also been studied, for example, [185] proposed a power allocation technique for OFDM-based cognitive radio wireless relay network in order to maximize the sum rate for the secondary user network subject to sum power and interference leakage constraints.

Many works in the literature considered the principle of multiple access channel (MAC) and broadcast channel (BC) duality for designing precoders for wireless networks with multiple users. For example, the work in [186] showed that BC and the dual MAC share the same achievable capacity region, and vice versa. In [187], the uplink-downlink SINR duality was used to design SINR balancing based multiple-user downlink beamforming. In [188], the duality was extended to multiuser MIMO Gaussian BC channels, in which the optimal solutions for the MIMO-BC sum rate problems have been obtained under a single sum power constraint. By employing the general BC-MAC duality, [39] proposed an efficient algorithm to transform a non-convex BC problem into a sequence of convex MAC problem, in order to solve weighted sum rate maximization problem for the cognitive radio MIMO-BC, subject to the sum power and the interference power constraints, respectively.

7.1.2 Our Main Contributions

In this chater, we consider a wireless relay network as depicted in Fig. 7.1. The network consists of a BS with multiple antennas, a RS with multiple antennas and a number of users, each equipped with a single antenna. Both the links from the BS to the RS and the RS to the users employ OFDM. We propose a joint beamforming and power allocation technique to maximize the total capacity of the whole network. In the MIMO transmission between the BS and the RS, SVD is utilized to design the transmit and receive

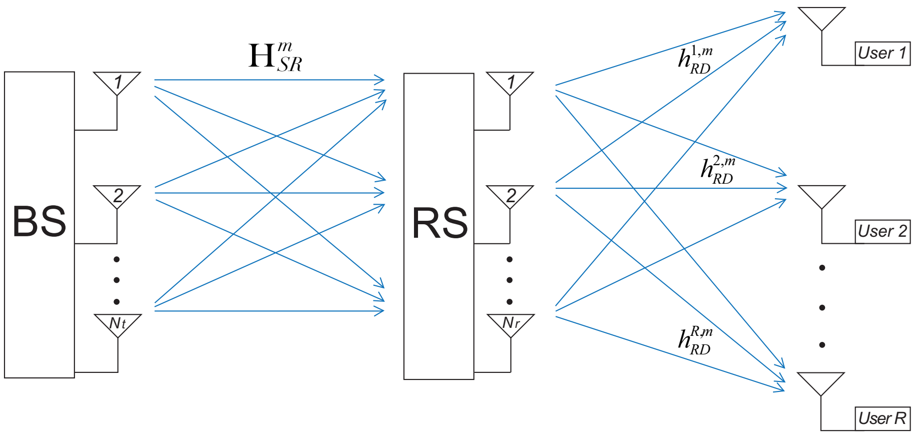

Figure 7.1. A relay network with an N_t-antenna BS, an N_r-antenna RS and R users with one antenna for each. The channel gains are shown for the m^{th} subcarrier.

beamforming vectors and the power allocation is performed using "waterfilling" algorithm. In the MISO transmission between RS and multiple-users, transmit beamforming vectors are designed using BC-MAC duality. The waterfilling based power allocation at the BS and the user power allocation at the RS needs to be jointly performed so that the sum rate of users should be matched to the backhaul BS-RS data throughput. We solve this problem using Lagrangian optimization.

The rest of this chapter is organized as follows. Section 7.2 presents the system model and the problem formulation. In Section 7.3, overall transceiver design has been formulated using an asymmetric resource allocation optimization framework. In Section 7.4, an algorithm is proposed to solve the above optimization problem, and the performance is compared with a simple transmitter design scheme. Simulation results are presented in Section 7.5.

7.2 System Model

We consider a downlink OFDM-based relay network which consists of a base station (BS), a relay station (RS) and R multiple-users. As depicted in Fig. 7.1, the BS and RS are equipped with N_t and N_r antennas, respectively, and each of the multiple-users is equipped with single antenna. It is assumed that the signals can not be transmitted from BS to multiple-users directly, due to long distance and obstacles, hence the channel gains from BS to multiple-users are very small. As a result, BS will require very high power to transmit the signal to the multiple-users, unless a relay is employed as considered in this paper. There are two time slots to cover the transmissions from BS to multiple-users. In the first time slot T_1, the signal is transmitted from BS, which is received and decoded at the RS. In the second time slot T_2, RS re-encodes the signal and forwards it to multiple-users. The total end-to-end transmission time can be indicated as $T = T_1 + T_2$. The total bandwidth B is also separated into N subcarriers using OFDM for both sides of the transmissions. The total available transmit power is P_T.

7.2.1 MIMO-OFDMA downlink system from BS to RS.

Before formulating the optimization problem, transmissions in both the time slots are described. In the first time slot T_1, the MIMO channel gain for the m^{th} subcarrier from the BS to the RS is expressed as an $N_r \times N_t$ matrix $\mathbf{H}_{SR}^m$, which is assumed to be known either at the BS or at the RS depending on where the optimization is performed. Normally, optimization is performed at the RS, as RS can estimate the channel from BS using training sequences, the channels from RS to users can be obtained through feedback from user terminals. It is more convenient to perform optimization at the RS. The received signal vector at the RS $\boldsymbol{y}_{SR}^m \in \mathbb{C}^{N_r \times 1}$ in the m^{th} OFDM subcarrier can be written in terms of the corresponding transmit signal vector $\boldsymbol{x}_{SR}^m \in \mathbb{C}^{N_t \times 1}$ as

$$\boldsymbol{y}_{SR}^m = \mathbf{H}_{SR}^m \boldsymbol{x}_{SR}^m + \boldsymbol{\eta}_m, \quad m = 1, 2, \cdots, N. \tag{7.2.1}$$

where $\boldsymbol{\eta}_m \in \mathbb{C}^{N_r \times 1}$ is the Additive White Gaussian Noise (AWGN) vector with covariance $E\{\boldsymbol{\eta}_m \boldsymbol{\eta}_m{}^H\} = \sigma_m^2 I_{N_r}$. The channel gain matrix for the m^{th} subcarrier $\mathbf{H}_{SR}^m$ can be decomposed into parallel independent subchannels using SVD as:

$$\mathbf{H}_{SR}^m = \mathbf{U}_{SR}^m \mathbf{\Sigma}_{SR}^m (\mathbf{V}_{SR}^m)^H, \tag{7.2.2}$$

where $\mathbf{U}_{SR}^m \in \mathbb{C}^{N_r \times N_r}$ and $(\mathbf{V}_{SR}^m)^H \in \mathbb{C}^{N_t \times N_t}$ are unitary matrices and $\mathbf{\Sigma}_{SR}^m \in \mathbb{C}^{N_r \times N_t}$ is a rectangular matrix whose diagonal elements are the singular values of $\mathbf{H}_{SR}^m$. Let $K_{min} = min\{N_r, N_t\}$. It is well known that the optimal transmitting beamforming matrix and the receiving beamforming matrix corresponding to the MIMO channel matrix $\mathbf{H}_{SR}^m$ are the first K_{min}

columns of $\mathbf{V}_{SR}^m$ and $(\mathbf{U}_{SR}^m)^H$ [189]. Therefore, the transmit signal vector $\boldsymbol{x}_{SR}^m$ is given as $\boldsymbol{x}_{SR}^m = \mathbf{V}_{SR}^m \tilde{\boldsymbol{x}}_{SR}^m$, where $\tilde{\boldsymbol{x}}_{SR}^m$ is the vector of modulated symbol constellations. The transmit covariance matrix for the m^{th} subcarrier is given by,

$$\mathbf{V}_{SR}^m E\{\tilde{\boldsymbol{x}}_{SR}^m (\tilde{\boldsymbol{x}}_{SR}^m)^H\} \mathbf{V}_{SR}^{m\,H} = \mathbf{V}_{SR}^m \mathrm{diag}(\boldsymbol{p}_{SR}^m)(\mathbf{V}_{SR}^m)^H, \tag{7.2.3}$$

where $\mathrm{diag}(\boldsymbol{p}_{SR}^m)$ is a diagonal matrix with the principle diagonal elements given by $\boldsymbol{p}_{SR}^m = [p_{SR}^{1,m}, p_{SR}^{2,m}, \ldots, p_{SR}^{k,m}, \ldots, p_{SR}^{K_{min},m}]^T \in \mathbb{R}^{K_{min} \times 1}$, and $p_{SR}^{k,m}$ denotes the power allocation to the k^{th} mode of the MIMO channel corresponding to the m^{th} subcarrier from the BS to the RS.

The non-zero diagonal elements in $\mathbf{\Sigma}_{SR}^m$, $\lambda_{SR}^{1,m} \geq \lambda_{SR}^{2,m} \geq \ldots \lambda_{SR}^{k,m} \ldots \geq \lambda_{SR}^{K_{min},m}$ can be treated as the "virtual" independent subchannel gains between the BS and RS for the subcarrier m. Using the power allocation vector $\boldsymbol{p}_{SR}^m$, the instantaneous data rate for the m^{th} subcarrier in the first time slot T_1 can be written as:

$$C_{SR}^m = \sum_{k=1}^{K_{min}} \frac{T_1}{T} \log_2(1 + p_{SR}^{k,m} \gamma_{SR}^{k,m}), \tag{7.2.4}$$

where $p_{SR}^{k,m}$ is the power allocated to the k^{th} subchannel of the m^{th} subcarrier, $\gamma_{SR}^{k,m} = (\lambda_{SR}^{k,m})^2/\sigma_{k,m}^2$, and $\sigma_{k,m}^2 = \sigma_m^2$ is the variance of zero mean AWGN noise present at the k^{th} mode of the MIMO channel. With these new "virtual" independent channel gains, for a given total power limit at the BS, the "waterfilling" algorithm can be employed to determine the optimal power allocation [189].

7.2.2 Beamforming for the MISO-OFDMA downlink network from the RS to multiple-users.

In the second time slot T_2, after receiving and re-encoding the signals from the BS, the RS forwards the signal to various users. User-specific radiation patterns (beams) can be formed and multiple users can share each subcarrier simultaneously. The individual signal-to-interference-plus-noise ratios (SINR) are controlled by the beamformer vectors and the corresponding power allocation.

The beamforming vector for the i^{th} user at the m^{th} subcarrier is defined as $\boldsymbol{w}_{i,m} = [w_{i,m}^1, w_{i,m}^2, \ldots, w_{i,m}^r, \ldots, w_{i,m}^{N_r}]^H \in \mathbb{C}^{N_r \times 1}$, where $1 \leq i \leq R$, respectively. The matrix $\mathbf{W}_m = [\boldsymbol{w}_{1,m}, \boldsymbol{w}_{2,m}, \ldots, \boldsymbol{w}_{i,m}, \ldots, \boldsymbol{w}_{R,m}]$ indicates the collection of all beamformers for the m^{th} subcarrier. Each beamformer will be normalized, so that

$$\|\boldsymbol{w}_{i,m}\|_2 = 1, \quad 1 \leq i \leq R. \tag{7.2.5}$$

We define $s_{RD}^{i,m}$ and $p_{RD}^{i,m}$ as the re-encoded independent information symbol and the corresponding power allocation, respectively, at the relay, for the i^{th} user in the m^{th} subcarrier. $p_{RD}^{i,m}$ can be stacked into a vector $\boldsymbol{p}_{RD}^m = [p_{RD}^{1,m}, p_{RD}^{2,m}, \ldots, p_{RD}^{i,m}, \ldots p_{RD}^{R,m}]^T \in \mathbb{R}^{R \times 1}$, and the $\|\boldsymbol{p}_{RD}^m\|_1$ denotes the total transmission power allocation for subcarrier m at the RS. Define $\boldsymbol{h}_{RD}^{i,m} = [h_{RD}^{1,i,m}, h_{RD}^{2,i,m}, \ldots, h_{RD}^{r,i,m}, \ldots h_{RD}^{N_r,i,m}]^T$ as the channel gain between the RS and the i^{th} user in the m^{th} subcarrier. The received signal at the i^{th} user can be expressed as:

$$\begin{aligned} y_{RD}^{i,m,DL} &= \sqrt{p_{RD}^{i,m}} \boldsymbol{w}_{i,m}^H \boldsymbol{h}_{RD}^{i,m} s_{RD}^{i,m} \\ &+ \sum_{j=1, j \neq i}^{R} \sqrt{p_{RD}^{j,m}} \boldsymbol{w}_{j,m}^H \boldsymbol{h}_{RD}^{i,m} s_{RD}^{j,m} + \zeta_i^m, \end{aligned} \tag{7.2.6}$$

where ζ_i^m is the AWGN noise component at the i^{th} user in the m^{th} subcarrier, with the variance $\sigma^2_{i,m}$. Defining the following spatial covariance matrices

$$\mathbf{Q}^{i,m}_{RD} = E\{\boldsymbol{h}^{i,m}_{RD}(\boldsymbol{h}^{i,m}_{RD})^H\}, \quad 1 \leq i \leq R, \tag{7.2.7}$$

the downlink SINR for the i^{th} user in m^{th} subcarrier can be written as

$$\mathrm{SINR}^{i,m,DL}_{RD} = \frac{p^{i,m}_{RD}\boldsymbol{w}^H_{i,m}\mathbf{Q}^{i,m}_{RD}\boldsymbol{w}_{i,m}}{\sum_{j=1,j\neq i}^{R} p^{j,m}_{RD}\boldsymbol{w}^H_{j,m}\mathbf{Q}^{i,m}_{RD}\boldsymbol{w}_{j,m} + \sigma^2_{i,m}}. \tag{7.2.8}$$

Uplink-Downlink SINR Duality

As seen in (7.2.8), the downlink SINR is coupled in terms of the beamforming vectors of all users. Hence the optimization is relatively difficult to solve as compared to the beamformer design in the uplink, where each user's SINR is a function of only its beamformer.

Fortunately, there exists a result known as uplink-downlink SINR duality [187] which states that the SINRs that can be achieved for the users in the downlink can also be achieved in the virtual uplink with the same set of beamformers and identical total power constraint, i.e. for a given beamforming matrix $\mathbf{W}$ and the total power P_{RD}, the uplink and the downlink could achieve the same SINR as

$$C^{UL}(\mathbf{W}, P_{RD}) = C^{DL}(\mathbf{W}, P_{RD}). \tag{7.2.9}$$

Where C^{UL} and C^{DL} denotes the set of SINRs that can be achieved in the uplink and downlink respectively. More specifically, if a set of target SINRs $\vartheta^{1,m,DL}_{RD}, \vartheta^{2,m,DL}_{RD}, \ldots, \vartheta^{R,m,DL}_{RD}$ can be achieved in the downlink on m^{th} subcarrier between the RS and the multiple-users, then the same set of SINRs can be achieved in the uplink and vice versa. Therefore, the downlink beamforming problem can be solved more easily through the uplink beamformer design. In the virtual uplink, the received signal in the m^{th} subcarrier at the

RS can be expressed as

$$\begin{aligned} y_{RD}^{i,m,UL} &= \sqrt{q_{RD}^{i,m}}\boldsymbol{w}_{i,m}^{H}\boldsymbol{h}_{RD}^{i,m}d_{RD}^{i,m} \\ &+ \sum_{j=1,j\neq i}^{R} \sqrt{q_{RD}^{j,m}}\boldsymbol{w}_{i,m}^{H}\boldsymbol{h}_{RD}^{j,m}d_{RD}^{j,m} + \theta_{i}^{m}, \end{aligned} \tag{7.2.10}$$

where θ_i^m is the AWGN noise component at RS for the i^{th} user in the m^{th} subcarrier in the virtual uplink, with variance $\sigma_{i,m}^2$. $d_{RD}^{i,m}$ and $q_{RD}^{i,m}$ denote the transmit signal and the corresponding power for the i^{th} user in the uplink. It can be stacked into a vector as $\boldsymbol{q}_{RD}^{m} = [q_{RD}^{1,m}, q_{RD}^{2,m}, \ldots, q_{RD}^{i,m}, \ldots q_{RD}^{R,m}]^T \in \mathbb{R}^{R\times 1}$. The SINR of the i^{th} user in the virtual uplink can be written as

$$\mathrm{SINR}_{RD}^{i,m,UL} = \frac{q_{RD}^{i,m}\boldsymbol{w}_{i,m}^{H}\mathbf{Q}_{RD}^{i,m}\boldsymbol{w}_{i,m}}{\boldsymbol{w}_{i,m}^{H}(\sum_{j=1,j\neq i}^{R} q_{RD}^{j,m}\mathbf{Q}_{RD}^{j,m} + \sigma_{i,m}^{2}I)\boldsymbol{w}_{i,m}}. \tag{7.2.11}$$

For the uplink-downlink duality to hold the noise variance at each user terminal should be identical [187]. However, if $\sigma_{i,m}^2 \neq \sigma_{l,m}^2$, $i \neq l$, both the numerator and the denominator of (7.2.8) can be divided by $\sigma_{i,m}^2$ to obtain virtual spatial covariance matrix $\hat{\mathbf{Q}}_{RD}^{i,m} = \mathbf{Q}_{RD}^{i,m}/\sigma_{i,m}^2$, and the corresponding noise variance $\sigma_{i,m}^2 = 1, 1 \leq i \leq R$. The achieved set of target SINRs will not be affected by this scaling [187], and in this case the SINRs in the downlink and the uplink can be written as

$$\hat{\mathrm{SINR}}_{RD}^{i,m,DL} = \frac{p_{RD}^{i,m}\boldsymbol{w}_{i,m}^{H}\hat{\mathbf{Q}}_{RD}^{i,m}\boldsymbol{w}_{i,m}}{\sum_{j=1,j\neq i}^{R} p_{RD}^{j,m}\boldsymbol{w}_{j,m}^{H}\hat{\mathbf{Q}}_{RD}^{i,m}\boldsymbol{w}_{j,m} + 1} \tag{7.2.12}$$

and

$$\hat{\mathrm{SINR}}_{RD}^{i,m,UL} = \frac{q_{RD}^{i,m}\boldsymbol{w}_{i,m}^{H}\hat{\mathbf{Q}}_{RD}^{i,m}\boldsymbol{w}_{i,m}}{\boldsymbol{w}_{i,m}^{H}(\sum_{j=1,j\neq i}^{R} q_{RD}^{j,m}\hat{\mathbf{Q}}_{RD}^{j,m} + I)\boldsymbol{w}_{i,m}}, \tag{7.2.13}$$

The beamformers can be designed using the virtual uplink SINR formulation as in (7.2.13) rather than that in (7.2.12). The power allocation for the 2^{nd} slot is obtained using geometric programming (GP) as described below.

Power allocation

Using the principle of uplink-downlink SINR duality, the power allocation problem in the downlink can be transferred into the power allocation problem of the virtual uplink for a given set of beamformers. We assume that the power of received signal is much larger than that of the interference plus noise, so that the SINR is much greater than 1. In this case, we could ignore the "1" in $(1+\hat{\mathrm{SINR}}_{RD}^{i,m,UL})$ in the capacity equation $\log_2(1+\mathrm{SINR}_{RD}^{UL})$. Hence, the sum rate can be approximated to

$$\begin{aligned} C_{RD}^{m,UL} &= \sum_{i=1}^{R} \frac{T_2}{T} \log_2(\hat{\mathrm{SINR}}_{RD}^{i,m,UL}) \\ &= \frac{T_2}{T} \log_2(\prod_{i=1}^{R} \hat{\mathrm{SINR}}_{RD}^{i,m,UL}). \end{aligned} \tag{7.2.14}$$

Defining $\hat{\mathbf{G}}_{RD}^{ii,m} = \boldsymbol{w}_{i,m}^{H}\hat{\mathbf{Q}}_{RD}^{i,m}\boldsymbol{w}_{i,m}$ and $\hat{\mathbf{G}}_{RD}^{ji,m} = \boldsymbol{w}_{i,m}^{H}\hat{\mathbf{Q}}_{RD}^{j,m}\boldsymbol{w}_{i,m}$, we can rewrite (7.2.13) as

$$\hat{\mathrm{SINR}}_{RD}^{i,m,UL} = \frac{q_{RD}^{i,m}\hat{\mathbf{G}}_{RD}^{ii,m}}{\sum_{j=1,j\neq i}^{R} q_{RD}^{j,m}\hat{\mathbf{G}}_{RD}^{ji,m}+1}. \tag{7.2.15}$$

It can be seen that the numerator and the denominator of $\hat{\mathrm{SINR}}_{RD}^{i,m,UL}$ are monomial and posynomial functions of transmit power variables $\boldsymbol{q}_{RD}^{m}$. Therefore, the reciprocal SINRs $1/\hat{\mathrm{SINR}}_{RD}^{i,m,UL}$ and their product of $\prod_{i=1}^{R}(1/\hat{\mathrm{SINR}}_{RD}^{i,m,UL})$ are posynomials. The optimization problem is to maximize the total capacity subject to a given total power constraint P_{RD} which is equivalent to the following minimization problem:

$$\min_{q_{RD}^{i,m}} \quad \prod_{m=1}^{N}\prod_{i=1}^{R}(1/\hat{\mathrm{SINR}}_{RD}^{i,m,UL}), \tag{7.2.16}$$

subject to

$$\sum_{m=1}^{N}\sum_{i=1}^{R} q_{RD}^{i,m} \le P_{RD}, \quad q_{RD}^{i,m} > 0. \tag{7.2.17}$$

The above optimization problem falls into GP and can be solved using convex optimization techniques [132].

Iterative Beamformer Design and Power Allocation

For a given set of beamforming, power allocation is performed $q_{RD}^{i,m}$ using the GP as in (7.2.16) and (7.2.17), and the beamforming vectors are designed in the uplink for the power allocation $q_{RD}^{i,m}$. The same beamformers will be used in the downlink with a different set of power allocations. The $\hat{\text{SINR}}_{RD}^{i,m,UL}$ can be written in terms of the beamforming vector $\boldsymbol{w}_{i,m}$ as

$$\hat{\text{SINR}}_{RD}^{i,m,UL} = \frac{\boldsymbol{w}_{i,m}^{H}\mathbf{A}\boldsymbol{w}_{i,m}}{\boldsymbol{w}_{i,m}^{H}\mathbf{B}\boldsymbol{w}_{i,m}}, \tag{7.2.18}$$

where $\mathbf{A} = q_{RD}^{i,m}\hat{\mathbf{Q}}_{RD}^{i,m}$ and $\mathbf{B} = \sum_{j=1,j\neq i}^{R} q_{RD}^{j,m}\hat{\mathbf{Q}}_{RD}^{j,m} + I$. The beamformer $\boldsymbol{w}_{i,m}$ is obtained as the generalized eigenvector corresponding to the largest generalized eigenvalue of the matrix pair $(\mathbf{A}, \mathbf{B})$ and expressed as $\tau_{max}(\hat{\mathbf{Q}}_{RD}^{i,m}, q_{RD}^{i,m})$. Here, we propose an iterative algorithm to optimize the beamformers $\boldsymbol{w}_{i,m}$. Each iteration for solving the beamformers $\boldsymbol{w}_{i,m}$ consists of two steps: first, optimize the power allocation, i.e. calculate $\boldsymbol{q}_{RD}^{m}$ using GP with previous normalized $\boldsymbol{w}_{i,m}$. Then, substitute $\boldsymbol{q}_{RD}^{m}$ into (7.2.18) and obtain optimum $\boldsymbol{w}_{i,m}$. This iteration is repeated until the stopping criterion $\|\tilde{\boldsymbol{w}}_{i,m}(\phi) - \boldsymbol{w}_{i,m}(\phi)\|_2 \le \epsilon$ is satisfied as summarized in Table 7.1.

According to the uplink-downlink SINR duality in (7.2.9), by solving the optimal power allocation $\boldsymbol{q}_{RD}^{m}$ and the beamformers $\boldsymbol{w}_{i,m}$ for each user, a set of uplink SINR target is obtained which can also be achieved in the downlink. By substituting the beamformers $\boldsymbol{w}_{i,m}$ obtained in the virtual

Table 7.1. Proposed algorithm for beamforming vector using BC-MAC duality.

1) Initialize feasible beamformers $\boldsymbol{w}_{i,m}$ randomly, and normalize them: $\boldsymbol{w}_{i,m} = \boldsymbol{w}_{i,m}/\|\boldsymbol{w}_{i,m}\|_2,\ 1 \leq i \leq R$;
2) Keep each noise variance $\sigma_{i,m}^2 = 1$, and create new virtual scaled spatial covariance matrices $\hat{\mathbf{Q}}_{RD}^{i,m} = \mathbf{Q}_{RD}^{i,m}/\sigma_{i,m}^2,\quad 1 \leq i \leq R$;
3) Repeat a) - f) [but it is start from c) in the first running];
   - a) $\phi = \phi + 1$;
   - b) $\boldsymbol{w}_{i,m}(\phi) = \tilde{\boldsymbol{w}}_{i,m}(\phi - 1)$;
   - c) Substitute $\boldsymbol{w}_{i,m}(\phi)$ into the SINR of uplink (7.2.13);
   - d) Determine the optimum power allocation of uplink using GP;
   - e) $\tilde{\boldsymbol{w}}_{i,m}(\phi) = \tau_{max}(\hat{\mathbf{Q}}_{RD}^{i,m}, q_{RD}^{i,m})$;
   - f) Calculate $\|\tilde{\boldsymbol{w}}_{i,m}(\phi) - \boldsymbol{w}_{i,m}(\phi)\|_2$;
4) End for repeat 3) when $\|\tilde{\boldsymbol{w}}_{i,m}(\phi) - \boldsymbol{w}_{i,m}(\phi)\|_2 \leq \epsilon$, where ϵ is the error tolerance for exit condition.

uplink into (7.2.8), the SINR for the downlink can be expressed as:

$$\mathrm{SINR}_{RD}^{i,m,DL} = \vartheta_{RD}^{i,m,UL} = \frac{p_{RD}^{i,m}\mathbf{G}_{RD}^{ii,m}}{\sum_{j=1,j\neq i}^{R} p_{RD}^{j,m}\mathbf{G}_{RD}^{ij,m} + \sigma_{i,m}^2} \tag{7.2.19}$$

where $\vartheta_{RD}^{i,m,UL}$ is the achieved SINR in the uplink which should be the same as in the downlink for each user, $\mathbf{G}_{RD}^{ii,m} = \boldsymbol{w}_{i,m}^H\mathbf{Q}_{RD}^{i,m}\boldsymbol{w}_{i,m}$ and $\mathbf{G}_{RD}^{ij,m} = \boldsymbol{w}_{j,m}^H\mathbf{Q}_{RD}^{i,m}\boldsymbol{w}_{j,m}$. Then from (7.2.19), we obtain:

$$p_{RD}^{i,m}\mathbf{G}_{RD}^{ii,m} - \vartheta_{RD}^{i,m,UL}\sum_{j=1,j\neq i}^{R} p_{RD}^{j,m}\mathbf{G}_{RD}^{ij,m} = \vartheta_{RD}^{i,m,UL}\sigma_{i,m}^2,$$
$$i = 1,2,\cdots,R,\quad m = 1,2,\cdots,N. \tag{7.2.20}$$

By stacking equations in (7.2.20) for $i = 1,2,\cdots,R$, we formulate a matrix equation for the m^{th} subcarrier as follows

$$\mathbf{V}_m\boldsymbol{x}_m = \boldsymbol{b}_m. \tag{7.2.21}$$

where

$$\mathbf{V}_m = \begin{bmatrix} \mathbf{G}_{RD}^{11,m} & -\mathbf{G}_{RD}^{12,m}\vartheta_{RD}^{1,m,UL} & \cdots & -\mathbf{G}_{RD}^{1R,m}\vartheta_{RD}^{1,m,UL} \\ -\mathbf{G}_{RD}^{21,m}\vartheta_{RD}^{2,m,UL} & \mathbf{G}_{RD}^{22,m} & \cdots & -\mathbf{G}_{RD}^{2R,m}\vartheta_{RD}^{2,m,UL} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{G}_{RD}^{R1,m}\vartheta_{RD}^{R,m,UL} & -\mathbf{G}_{RD}^{R2,m}\vartheta_{RD}^{2,m,UL} & \cdots & \mathbf{G}_{RD}^{RR,m} \end{bmatrix} \in \mathbb{C}^{R\times R},$$

$\boldsymbol{x}_m = [p_{RD}^{1,m},\ p_{RD}^{2,m},\ \cdots,\ p_{RD}^{R,m}]^T \in \mathbb{C}^{R\times 1}$ and

$\boldsymbol{b}_m = [\vartheta_{RD}^{1,m,UL}\sigma_{1,m}^2,\ \vartheta_{RD}^{2,m,UL}\sigma_{2,m}^2, \cdots,\ \vartheta_{RD}^{R,m,UL}\sigma_{R,m}^2]^T \in \mathbb{C}^{R\times 1}$. Hence, the optimal power allocation for the downlink for the m^{th} subcarrier can be obtained as follows

$$\boldsymbol{x}_m = \mathbf{V}_m^{-1}\boldsymbol{b}_m. \tag{7.2.22}$$

Since $\mathbf{V}_m$ is an $R\times R$ square matrix, and all the elements $\mathbf{G}_{RD}^{ii,m}$ are formed using independent channel matrices, the inverse of $\mathbf{V}$, denoted by $\mathbf{V}^{-1}$, should exist with a probability reaching one. To summarize, in order to solve the optimal power allocation $\boldsymbol{p}_{RD}^m$ in downlink, we determine the beamformers $\boldsymbol{w}_{i,m}$ and power allocation $\boldsymbol{q}_{RD}^m$ in the uplink first, solve the SINR values for each user and then substitute them into downlink to obtain downlink power allocation. In the following section, we propose an asymmetric resource allocation for maximizing the total throughput of the relay network.

7.3 Optimal Asymmetric Resource Allocation

We consider an asymmetric resource allocation as similar to the works in [56] and [172]. The instantaneous downlink capacity of the m^{th} subcarrier from RS to multiple-users in the second time slot T_2 is formulated as:

$$C_{RD}^{m,DL} = \sum_{i=1}^{R} \frac{T_2}{T} \log_2(1 + \mathrm{SINR}_{RD}^{i,m,DL}). \tag{7.3.1}$$

The instantaneous capacities of the two consecutive slots over all subcarriers should be maximized. Our optimization is to allocate the power at the base station and the relay through different subcarriers optimally in addition to choosing the optimal subcarriers for different users. In order to maximize the capacity from the base station to multiple users, it is important that the capacities of the channel from the base station to the relay, and that from the relay to multiple users should be matched. Otherwise the relays will be forced to drop the data packet. Therefore, the optimization criterion is to maximize the minimum of the data rates of the channel from the base station to the relay, and from the relay to the multiple users. The optimization problem can be formulated from (7.2.4) and (7.3.1) as

$$\arg\max_{p_{SR}^{k,m}, p_{RD}^{i,m}, T_1, T_2} \left(\min \left\{ \sum_{m=1}^{N} C_{SR}^{m}, \sum_{m=1}^{N} C_{RD}^{m,DL} \right\} \right), \tag{7.3.2}$$

subject to

$$\begin{aligned} &\sum_{m=1}^{N} \sum_{k=1}^{K_{min}} p_{SR}^{k,m} + \sum_{m=1}^{N} \sum_{i=1}^{R} p_{RD}^{i,m} \leq P_T, \\ &p_{SR}^{k,m} > 0, p_{RD}^{i,m} > 0 \end{aligned} \tag{7.3.3}$$

and

$$T_1 + T_2 = T, \quad T_1 > 0, \quad T_2 > 0. \tag{7.3.4}$$

Using a similar argument as in [56] and [172], it can be proved that (7.3.2) is maximized only when $\sum_{m=1}^{N} C_{SR}^{m} = \sum_{n=1}^{N} C_{RD}^{m,DL}$. In order to approximate the problem into GP, we assume high SINRs in the downlink from the RS to multiple-users, and ignore the term "1" in calculating the capacity. Hence, the optimization in (7.3.2) can be written as

$$\begin{aligned}\arg\max_{p_{SR}^{k,m},p_{RD}^{i,m},T_1,T_2} \quad & \sum_{m=1}^{N}\sum_{k=1}^{K_{min}}\frac{T_1}{T}\log_2(1+p_{SR}^{k,m}\gamma_{SR}^{k,m})+ \\ & \sum_{m=1}^{N}\sum_{i=1}^{R}\frac{T_2}{T}\log_2(\mathrm{SINR}_{RD}^{i,m,DL}),\end{aligned} \tag{7.3.5}$$

subject to

$$\begin{aligned}&\sum_{m=1}^{N}\sum_{k=1}^{K_{min}}\frac{T_1}{T}\log_2(1+p_{SR}^{k,m}\gamma_{SR}^{k,m}) \\ &= \sum_{m=1}^{N}\sum_{i=1}^{R}\frac{T_2}{T}\log_2(\mathrm{SINR}_{RD}^{i,m,DL})\end{aligned} \tag{7.3.6}$$

and constraints in (7.3.3) and (7.3.4). In the following section, we use the algorithm proposed in [56] and [172] to solve this optimization problem using Lagrangian duality and bisection method.

7.4 Algorithm Description

Using a similar approach as in [56] and [172], we can prove that the optimization problem in (7.3.5) is convex. It can be solved using the Karush-Kuhn-Tucker (KKT) conditions [132]. The optimization in the primal form is written as

$$\begin{aligned}L = \; & \sum_{m=1}^{N}\sum_{k=1}^{K_{min}}\frac{T_1}{T}\log_2(1+p_{SR}^{k,m}\gamma_{SR}^{k,m})+\sum_{m=1}^{N}\sum_{i=1}^{R}\frac{T_2}{T}\log_2(\mathrm{SINR}_{RD}^{i,m,DL}) \\ & -\lambda\left[\sum_{m=1}^{N}\sum_{k=1}^{K_{min}}\frac{T_1}{T}\log_2(1+p_{SR}^{k,m}\gamma_{SR}^{k,m}) - \sum_{m=1}^{N}\sum_{i=1}^{R}\frac{T_2}{T}\log_2(\mathrm{SINR}_{RD}^{i,m,DL})\right] \\ & -\mu\left[\sum_{m=1}^{N}\sum_{k=1}^{K_{min}}p_{SR}^{k,m}+\sum_{m=1}^{N}\sum_{i=1}^{R}p_{RD}^{i,m}-P_T\right]-\sigma(T_1+T_2-T),\end{aligned} \tag{7.4.1}$$

where λ, μ and σ are the Lagrangian multipliers. From the KKT condition and constraints (7.3.3) and (7.3.4) it can be derived that $\mu \geq 0$ and $\lambda \in$

$(-1, 1)$.

$$\left.\frac{\partial L}{\partial T_1}\right|_{T_1=T_1^*, T_2=T_2^*, p_{SR}^{k,m}=p_{SR}^{k,m^*}, p_{RD}^{i,m}=p_{RD}^{i,m^*}}$$
$$= \frac{1-\lambda^*}{T}\sum_{m=1}^{N}\sum_{k=1}^{K_{min}}\log_2(1+p_{SR}^{k,m^*}\gamma_{SR}^{k,m}) - \sigma^*$$
$$\begin{cases} <0 & T_1^* = 0 \\ =0 & T_1^* > 0, \end{cases} \tag{7.4.2}$$

$$\left.\frac{\partial L}{\partial T_2}\right|_{T_1=T_1^*, T_2=T_2^*, p_{SR}^{k,m}=p_{SR}^{k,m^*}, p_{RD}^{i,m}=p_{RD}^{i,m^*}}$$
$$= \frac{1+\lambda^*}{T}\sum_{m=1}^{N}\sum_{i=1}^{R}\log_2(\mathrm{SINR}_{RD}^{i,m,DL^*}) - \sigma^*$$
$$\begin{cases} <0 & T_2^* = 0 \\ =0 & T_2^* > 0, \end{cases} \tag{7.4.3}$$

$$\left.\frac{\partial L}{\partial p_{SR}^{k,m}}\right|_{T_1=T_1^*, T_2=T_2^*, p_{SR}^{k,m}=p_{SR}^{k,m^*}, p_{RD}^{i,m}=p_{RD}^{i,m^*}}$$
$$= \frac{T_1^*(1-\lambda^*)\gamma_{SR}^{k,m}}{T(1+p_{SR}^{k,m^*}\gamma_{SR}^{k,m})} - \mu^*$$
$$\begin{cases} <0 & p_{SR}^{k,m^*} = 0 \\ =0 & p_{SR}^{k,m^*} > 0. \end{cases} \tag{7.4.4}$$

T_1, T_2 can be obtained by using (7.3.6), (7.4.2) and (7.4.3) that

$$T_1 = \frac{1-\lambda^*}{2}T, \tag{7.4.5}$$

$$T_2 = \frac{1+\lambda^*}{2}T. \tag{7.4.6}$$

Substituting (7.4.5) into (7.4.4), it can be shown that the power allocation of the BS for k^{th} mode at m^{th} subcarrier is

$$p_{SR}^{k,m} = \left[\frac{(1-\lambda^*)^2}{2\mu^*} - \frac{1}{\gamma_{SR}^{k,m}}\right]^+, \tag{7.4.7}$$

where$[x]^+ = \begin{cases} x & = & x > 0 \\ 0 & = & x < 0 \end{cases}$. From (7.4.5) and (7.4.6), it can be seen that the time constraint controlled by σ in the optimization problem (7.4.1) has been satisfied and expressed in terms of λ. The constraints on power and capacity controlled by μ and λ need also to be satisfied simultaneously. Substituting (7.4.5), (7.4.6) and (7.4.7) into the power constraint $\sum_{m=1}^{N}\sum_{k=1}^{K_{min}} p_{SR}^{k,m} + \sum_{m=1}^{N}\sum_{i=1}^{R} p_{RD}^{i,m} - P_T = 0$ and capacity constraint $\sum_{m=1}^{N}\sum_{k=1}^{K_{min}} \frac{T_1}{T}\log_2(1+p_{SR}^{k,m}\gamma_{SR}^{k,m}) - \sum_{m=1}^{N}\sum_{i=1}^{R}\frac{T_2}{T}\log_2(\mathrm{SINR}_{RD}^{i,m,DL}) = 0$, equations f and g in terms of μ and λ can be obtained as shown below. The Lagrangian multipliers can be obtained by solving the following two simultaneous equations:

$$\begin{aligned} f(\mu,\lambda) &= \sum_{m=1}^{N}\sum_{k=1}^{K_{min}}\left[\frac{(1-\lambda)^2}{2\mu} - \frac{1}{\gamma_{SR}^{k,m}}\right]^+ \\ &+ \sum_{m=1}^{N}\sum_{i=1}^{R} p_{RD}^{i,m} - P_T \\ &= 0, \end{aligned} \tag{7.4.8}$$

$$\begin{aligned} g(\mu,\lambda) &= (1+\lambda)\sum_{m=1}^{N}\sum_{i=1}^{R} C_{RD}^{i,m} \\ &-(1-\lambda)\sum_{m=1}^{N}\sum_{k=1}^{K_{min}}\left[\log_2(\frac{(1-\lambda)^2}{2\mu}\gamma_{SR}^{k,m})\right]^+ \\ &= 0. \end{aligned} \tag{7.4.9}$$

It can be shown that $g(\mu,\lambda)$ is a monotonically increasing function not only for a fixed μ but also for a fixed λ, with the increasing λ and μ, respectively. We define $\gamma_{min} = \min\{\gamma_{SR}^{1,1},\cdots,\gamma_{SR}^{K_{min},N}\}$ and $\gamma_{max} =$

$\max\{\gamma_{SR}^{1,1}, \cdots, \gamma_{SR}^{K_{min},N}\}$. It can be determined that the lower and upper bounds of μ are $\mu_L = \frac{NK_{min}\gamma_{min}}{2(P_T\gamma_{min}+NK_{min})}$ and $\mu_R = 2\gamma_{max}$, respectively. Similarly, the lower and upper bounds of λ are $\lambda_L = -1$ and $\lambda_R = 1$, respectively. Using an approach similar to the one presented in [56] and [172], the optimal Lagrangian multipliers μ^* and λ^* can be obtained using a bisection search algorithm. For example, after finding out the ranges of μ and λ, we can substitute $\mu = (\mu_L + \mu_R)/2$ and $\lambda = (\lambda_L + \lambda_R)/2$ into $f(\mu, \lambda)$, and the total transmission power $\sum_{i=1}^{R} p_{RD}^{i,m}$ in T_2 can be determined. By using the algorithm in Table 7.1 and the known total power constraint, the total throughput $\sum_{m=1}^{N}\sum_{i=1}^{R} C_{RD}^{i,m}$ in T_2 can be determined, and whether $g(\mu, \lambda)$ is zero or not can be tested. The values of μ and λ will be updated until $f(\mu, \lambda) = 0$ and $g(\mu, \lambda) = 0$. The search algorithm for Lagrangian multipliers has been shown in Table 7.2.

Table 7.2. Proposed algorithm for Lagrangian multipliers using bisection method.

1) Initialize feasible $\mu = (\mu_L + \mu_R)/2$ and $\lambda = (\lambda_L + \lambda_R)/2$;
2) Repeat a) - f);
   a) Substitute μ and λ into $f(\mu, \lambda)$, and calculate $\sum_{i=1}^{R} p_{RD}^{i,m}$ in $f(\mu, \lambda)$;
   b) Calculate $\sum_{m=1}^{N}\sum_{i=1}^{R} C_{RD}^{i,m}$ in $g(\mu, \lambda)$ with obtained $\sum_{i=1}^{R} p_{RD}^{i,m}$ by using Table 7.1. Substitute μ and λ into $g(\mu, \lambda)$, if $g(\mu, \lambda) \neq 0$, adjust $\mu_L = \begin{cases} \mu_L & = & f(\mu,\lambda) > 0 \\ \mu & = & f(\mu,\lambda) < 0 \end{cases}$, $\mu_R = \begin{cases} \mu_R & = & f(\mu,\lambda) < 0 \\ \mu & = & f(\mu,\lambda) > 0 \end{cases}$, and update $\mu = (\mu_L + \mu_R)/2$; repeat these procedures until $g(\mu, \lambda) = 0$;
   c) Substitute μ and λ into $f(\mu, \lambda)$, and calculate $\sum_{i=1}^{R} p_{RD}^{i,m}$ in $f(\mu, \lambda)$;
   d) Calculate $\sum_{m=1}^{N}\sum_{i=1}^{R} C_{RD}^{i,m}$ in $g(\mu, \lambda)$ with obtained $\sum_{i=1}^{R} p_{RD}^{i,m}$ by using Table 7.1. Substitute μ and λ into $g(\mu, \lambda)$, if $g(\mu, \lambda) \neq 0$, adjust $\lambda_L = \begin{cases} \lambda_L & = & g(\mu,\lambda) > 0 \\ \lambda & = & g(\mu,\lambda) < 0 \end{cases}$, $\lambda_R = \begin{cases} \lambda_R & = & g(\mu,\lambda) < 0 \\ \lambda & = & g(\mu,\lambda) > 0 \end{cases}$, and update $\lambda = (\lambda_L + \lambda_R)/2$; repeat these procedures until $g(\mu, \lambda) = 0$;
3) End for repeat 2) until $f(\mu, \lambda) = 0$ and $g(\mu, \lambda) = 0$ both.

Having determined the total transmission power $\sum_{i=1}^{R} p_{RD}^{i,m}$, the optimal time durations T_1 and T_2, the optimal power allocations in T_1 and T_2 can be obtained by substituting the optimal Lagrangian multipliers into (7.4.5), (7.4.6), and (7.4.7).

The scheme proposed so far facilitates multiple users accessing every subcarriers simultaneously using directional beamforming scheme. However, for comparison, we also propose a relatively simple scheme, where each subcarrier is allocated to only one user. However, the relay will still perform beamforming to each user; hence one beamformer per subcarrier needs to be designed. Since only one user is allowed in each subcarrier, the required beamformer is a simple coherent transmitter. Users are allocated in each subcarrier as the one that provides highest possible gain after beamforming. For example, in the m^{th} subcarrier between RS and multiple-users, the user i will be selected as $\arg\max_i \left\{(\boldsymbol{h}_{RD}^{i,m})^H \boldsymbol{h}_{RD}^{i,m}\right\}$, where $\boldsymbol{h}_{RD}^{i,m} = [h_{RD}^{i,m,1}, h_{RD}^{i,m,2}, \cdots, h_{RD}^{i,m,r}, \cdots, h_{RD}^{i,m,N_r}]^T$ denotes the channel gain for the i^{th} user in the m^{th} subcarrier. The beamformer for the i^{th} user is defined as $\boldsymbol{v}_{RD}^{i,m} = [v_{RD}^{i,m,1}, v_{RD}^{i,m,2}, \cdots, v_{RD}^{i,m,r}, \cdots, v_{RD}^{i,m,N_r}]^T = \frac{\boldsymbol{h}_{RD}^{i,m*}}{\|\boldsymbol{h}_{RD}^{i,m}\|}$. Therefore, the instantaneous capacity of the m^{th} subcarrier in the second time slot T_2 is written as:

$$C_{RD}^{m} = \frac{T_2}{T} \log_2(1 + p_{RD}^{i,m} \gamma_{RD}^{i,m}), \tag{7.4.10}$$

where $p_{RD}^{i,m}$ expresses the allocated transmit power for the i^{th} user in the m^{th} subcarrier of the RS, and the total power at the RS is limited by P_{RD}; $\gamma_{RD}^{i,m} = [(\boldsymbol{v}_{RD}^{i,m})^H \boldsymbol{h}_{RD}^{i,m}]^2 / \sigma^2{}_{i,m}$, and $\sigma^2{}_{i,m}$ is the variance of the zero mean AWGN present at the the selected user in each subcarrier.

We consider the same asymmetric resource allocation presented in Section III, then the optimization problem can be formulated as

$$\underset{p_{SR}^{k,m}, p_{RD}^{i,m}, T_1, T_2}{\arg\max} \left(\min\left\{ \sum_{m=1}^{N} C_{SR}^{m}, \sum_{m=1}^{N} C_{RD}^{m} \right\} \right), \tag{7.4.11}$$

subject to

$$\sum_{m=1}^{N}\sum_{k=1}^{K_{min}} p_{SR}^{k,m} + \sum_{m=1}^{N} p_{RD}^{i,m} \leq P_T,$$
$$p_{SR}^{k,m} > 0, p_{RD}^{i,m} > 0 \tag{7.4.12}$$

and (7.3.4).

The objective function in (7.4.11) can be maximized only when $\sum_{m=1}^{N} C_{SR}^{m} = \sum_{m=1}^{N} C_{RD}^{m}$. Hence, the optimization in (7.4.11) can be written as

$$\begin{aligned}\arg\max_{p_{SR}^{k,m}, p_{RD}^{i,m}, T_1, T_2} \quad & \sum_{m=1}^{N}\sum_{k=1}^{K_{min}} \frac{T_1}{T}\log_2(1 + p_{SR}^{k,m}\gamma_{SR}^{k,m}) + \\ & \sum_{m=1}^{N} \frac{T_2}{T}\log_2(1 + p_{RD}^{i,m}\gamma_{RD}^{i,m}),\end{aligned} \tag{7.4.13}$$

subject to

$$\begin{aligned}&\sum_{m=1}^{N}\sum_{k=1}^{K_{min}} \frac{T_1}{T}\log_2(1 + p_{SR}^{k,m}\gamma_{SR}^{k,m}) \\ &\quad = \sum_{m=1}^{N} \frac{T_2}{T}\log_2(1 + p_{RD}^{i,m}\gamma_{RD}^{i,m})\end{aligned} \tag{7.4.14}$$

and (7.4.12), (7.3.4).

Like (7.4.1), KKT conditions [132] can be also employed to solve this and the optimization in the primal form is written as

$$\begin{aligned}\tilde{L} = \quad & \sum_{m=1}^{N}\sum_{k=1}^{K_{min}} \frac{T_1}{T}\log_2(1 + p_{SR}^{k,m}\gamma_{SR}^{k,m}) + \sum_{m=1}^{N} \frac{T_2}{T}\log_2(1 + p_{RD}^{i,m}\gamma_{RD}^{i,m}) \\ & -\tilde{\lambda}\left[\sum_{m=1}^{N}\sum_{k=1}^{K_{min}} \frac{T_1}{T}\log_2(1 + p_{SR}^{k,m}\gamma_{SR}^{k,m}) - \sum_{m=1}^{N} \frac{T_2}{T}\log_2(1 + p_{RD}^{i,m}\gamma_{RD}^{i,m})\right] \\ & -\tilde{\mu}\left[\sum_{m=1}^{N}\sum_{k=1}^{K_{min}} p_{SR}^{k,m} + \sum_{m=1}^{N} p_{RD}^{i,m} - P_T\right] - \tilde{\sigma}(T_1 + T_2 - T),\end{aligned} \tag{7.4.15}$$

where $\tilde{\lambda}$, $\tilde{\mu}$ and $\tilde{\sigma}$ are the Lagrangian multipliers. By differentiating $\tilde{L}$ with

T_1, T_2, $p_{SR}^{k,m}$ and $p_{RD}^{i,m}$, the results in (7.4.2), (7.4.3), it can be obtained that

$$\frac{\partial \tilde{L}}{\partial p_{SR}^{k,m}}\Bigg|_{T_1=T_1^*,T_2=T_2^*,p_{RD}^{k,m}=p_{RD}^{k,m^*},p_{RD}^{i,m}=p_{RD}^{i,m^*}} = \frac{T_1^*(1-\tilde{\lambda}^*)\gamma_{SR}^{k,m}}{T(1+p_{SR}^{k,m^*}\gamma_{SR}^{k,m})} - \tilde{\mu}^* \begin{cases} <0 & p_{SR}^{k,m^*}=0 \\ =0 & p_{SR}^{k,m^*}>0, \end{cases} \tag{7.4.16}$$

$$\frac{\partial \tilde{L}}{\partial p_{RD}^{i,m}}\Bigg|_{T_1=T_1^*,T_2=T_2^*,p_{SR}^{k,m}=p_{SR}^{k,m^*},p_{RD}^{i,m}=p_{RD}^{i,m^*}} = \frac{T_1^*(1+\tilde{\lambda}^*)\gamma_{RD}^{i,m}}{T(1+p_{RD}^{i,m^*}\gamma_{RD}^{i,m})} - \tilde{\mu}^* \begin{cases} <0 & p_{RD}^{i,m^*}=0 \\ =0 & p_{RD}^{i,m^*}>0. \end{cases} \tag{7.4.17}$$

T_1 and T_2 can be solved as

$$T_1 = \frac{1-\tilde{\lambda}^*}{2}T, \tag{7.4.18}$$

$$T_2 = \frac{1+\tilde{\lambda}^*}{2}T, \tag{7.4.19}$$

respectively. The optimal power allocation for BS and RS can be determined as

$$p_{SR}^{k,m} = \left[\frac{(1-\tilde{\lambda}^*)^2}{2\tilde{\mu}^*} - \frac{1}{\gamma_{SR}^{k,m}}\right]^+ \tag{7.4.20}$$

and

$$p_{RD}^{i,m} = \left[\frac{(1+\tilde{\lambda}^*)^2}{2\tilde{\mu}^*} - \frac{1}{\gamma_{RD}^{i,m}}\right]^+, \tag{7.4.21}$$

where$[x]^+ = \begin{cases} x & = & x > 0 \\ 0 & = & x < 0 \end{cases}$. We define $\gamma_{min} = \min\{\gamma_{SR}^{1,1}, \cdots, \gamma_{SR}^{K_{min},N}, \gamma_{RD}^{i,1}, \cdots, \gamma_{RD}^{i,N}\}$ and $\gamma_{max} = \max\{\gamma_{SR}^{1,1}, \cdots, \gamma_{SR}^{K_{min},N}, \gamma_{RD}^{i,1}, \cdots, \gamma_{RD}^{i,N}\}$. It can be determined that the lower and upper bounds of $\tilde{\mu}$ are $\tilde{\mu}_L = \frac{N(K_{min}+1)\gamma_{min}}{2[P_T\gamma_{min}+N(K_{min}+1)]}$ and $\tilde{\mu}_R = 2\gamma_{max}$, respectively, and the lower and upper bounds of $\tilde{\lambda}$ are $\tilde{\lambda}_L = -1$ and $\tilde{\lambda}_R = 1$, respectively. And using "waterfilling" algorithm for both sides, the Lagrangian multipliers $\tilde{\mu}$ and $\tilde{\lambda}$ can be solved by two equations $\tilde{f}$ and $\tilde{g}$ both in terms of $\tilde{\mu}$ and $\tilde{\lambda}$ that satisfy the constraints of power and capacity in the optimization problem (7.4.15), where $\tilde{f}$ and $\tilde{g}$ can be expressed as

$$\begin{aligned} \tilde{f}(\tilde{\mu}, \tilde{\lambda}) &= \sum_{m=1}^{N}\sum_{k=1}^{K_{min}}\left[\frac{(1-\tilde{\lambda})^2}{2\tilde{\mu}} - \frac{1}{\gamma_{SR}^{k,m}}\right]^+ \\ &+ \sum_{m=1}^{N}\left[\frac{(1+\tilde{\lambda})^2}{2\tilde{\mu}} - \frac{1}{\gamma_{RD}^{i,m}}\right]^+ - P_T \\ &= 0, \end{aligned} \tag{7.4.22}$$

$$\begin{aligned} \tilde{g}(\tilde{\mu}, \tilde{\lambda}) &= (1+\tilde{\lambda})\sum_{m=1}^{N}\left[\log_2(\frac{(1+\tilde{\lambda})^2}{2\tilde{\mu}}\gamma_{RD}^{i,m})\right]^+ \\ &-(1-\tilde{\lambda})\sum_{m=1}^{N}\sum_{k=1}^{K_{min}}\left[\log_2(\frac{(1-\tilde{\lambda})^2}{2\tilde{\mu}}\gamma_{SR}^{k,m})\right]^+ \\ &= 0. \end{aligned} \tag{7.4.23}$$

7.5 Simulation Results

The performance of the proposed scheme is investigated for an OFDM-based relay network which consists of one BS with $N_t = 4$ antennas, one RS with $N_r = 4$ antennas and different number multiple-users employing single antenna each. The total bandwidth is $B = 5MHz$ which is divided into $N = 16$ subcarriers, as specified in IEEE 802.16e downlink Partial Usage of Subcarriers (PUSC) channel [190], and the total time frame is assumed to be $T = 1s$.

For the results in Fig. 7.2, the total transmit power of the whole network P_T is varied from 2w to 12w. The network employs single-user and multiple-users to compare the total throughput. As can be seen in Fig. 7.2, the optimal achievable sum rate for 3 users is much higher than that obtained with single user. Also the total optimal throughput increases with the increasing total power.

Fig. 7.3 shows the comparison of the optimal achievable sum rates obtained using the proposed beamforming for all users and employing only one user per each subcarrier. The total power is assumed as $P_T = 10$w. As seen the optimal achievable sum rates increase when the number of users grows for both cases. The total throughput of using multiple user beamforming is much higher than that choosing only single user with the best channel gain. However, in the case of multiple user beamforming, the sum rate saturates beyond 4 users. This is because interuser interference is increased and multiuser multiplexing returns diminishing increase in thoughput.

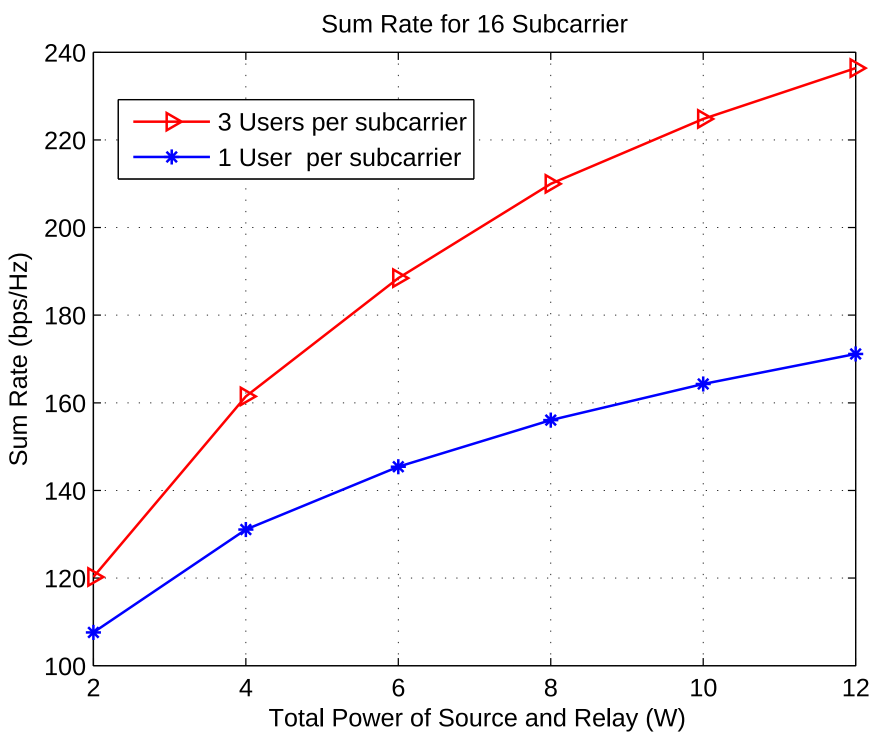

Figure 7.2. Comparison of the optimal achievable sum rates for 16 subcarriers obtained by serving 3 users and 1 user in each subcarrier.

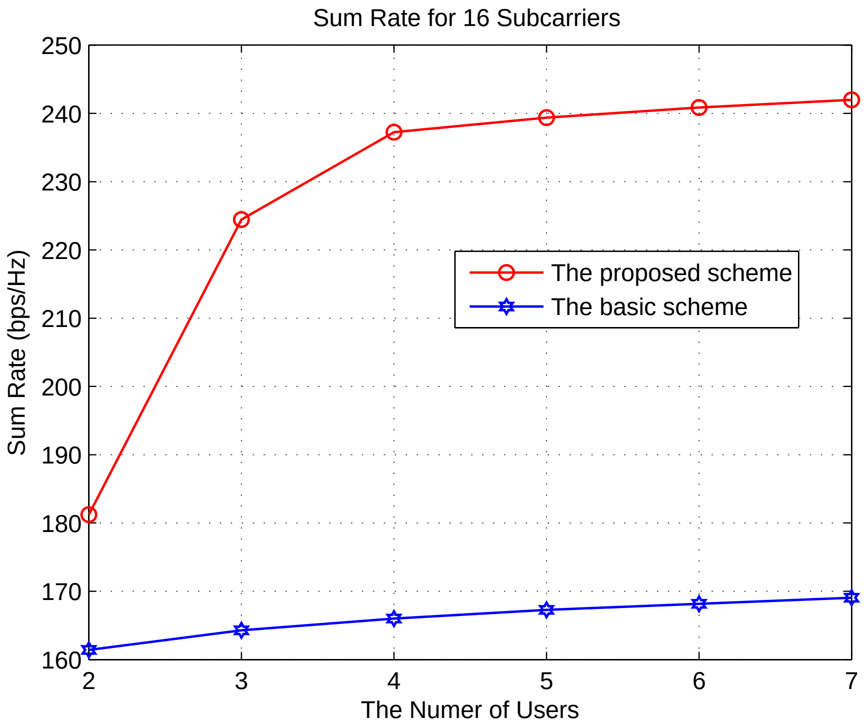

Figure 7.3. Comparison of the optimal achievable sum rates for 16 subcarriers obtained by using beamforming for different users in each subcarrier.

Chapter 8

SUMMARY, CONCLUSION AND FUTURE WORK

8.1 Summary and Conclusions

In this thesis, various resource allocation and spatial multiplexing techniques have been investigated for wireless relay networks. In chapter 4, a spatial multiplexing technique has been proposed for a two way relay network using convex optimization techniques. The proposal considered multiple transmitter-receiver pairs and a MIMO relay transceiver has been optimized to satisfy a set of target SINRs for every users while minimizing the total transmission power at the relays. The optimization in its original form has not been convex; however, the semidefinite relaxing has been applied for transforming the non-convex problem into convex problem. Accordingly, the non-convex rank-one constraint has been dropped to form the optimization problem using SDP. The proposed SDP technique has also been extended to a two way asynchronous relay network which utilized FIR filter-based relays to overcome intersymbol interference. The simulation results showed that the proposed two way relay technique has the ability to enhance spectrum efficiency and the filter based relays can effectively mitigate the interference from the various users and reduce BER.

In chapter 5, an MMSE based design criterion has been proposed for the design of two way relay network. The work considered both a single pair of users and multiple pair users. A new approach has been proposed to determine the optimum signal scaling factor required to use all available transmission power at the relay though certain manipulation in the Lagrangian formulation. A bisection method has also been used to reduce the computational complexity.

Power allocation technique for an OFDM-based cognitive radio wireless relay network has been proposed in Chapter 6. The work considered asymmetric resource allocation between source to relay and relay to destination of a secondary wireless network while ensuring interference leakage to primary user receiver is below a threshold. The asymmetric resource allocation is to ensure that the capacity of the channel from source to relay and the relay to destination is matched in order to maximise the overall throughput.

Finally, a MIMO-OFDM relay network with multiple users has been considered in chapter 7. Accordingly, the base station and the relay employ multiple antennas forming a MIMO channel while the user terminals employ single antenna forming MISO channels between the relay and the user terminals. Singular value based channel decomposition and water filing algorithm has been proposed to optimise the transmission from source to relay while a set of beamformers and power allocation have been designed to optimise transmission from relays to destination. As in chapter 6, the throughputs of the channel from source to relay and relay to destination have been equated to maximise the overall capacity. The beamformers have been designed using the principle of BC-MAC duality. The proposed algorithm in its dual form has been solved using bisection-based and GP.

8.2 Future Work

The potential areas for future research have been recognized. All the algorithms proposed in this thesis, were developed based on the assumption that the CSI is well known. However, in reality, CSI may have errors due to feedback quantization, estimation error etc. Therefore, the resource allocation and beamforming techniques proposed in various chapters in this thesis can be extended to include possibility of imperfect or incomplete CSI and robust designs can be performed.

The work in Chapters 4 and 5 on two-way relay network can be extended to a cognitive radio network scenario where interference constraints have to be imposed in addition to the optimization of SINR or MMSE criterion in the design framework. Also, the work in these chapters considered single antenna based peer-to-peer users, and only the relays employed multiple antennas. This work has the potential for possible extension towards employing multiple antennas at the transmitter and the receiver. In this case, the beamforming vectors at the transmitter and the receivers as well as the relay transceiver matrix have to be designed jointly. This problem is very likely to be non-convex; however, convex approximations or iterative design methods can be employed to solve this joint optimization problem.

Also, the works in Chapter 6 and 7 considered single antenna for the receiver terminals. These works can also be extended to multiple antennas based receiver terminals. Hence both the channels from the source to relay and relay to multiple destinations will be MIMO and joint optimization of source precoder, relay transceiver and receiver filters needs to be performed over all OFDM subcarriers. Finally the work in Chapter 7 considered only one channel in time slot 1 that is from source to relay. However, it is possible to have multiple relays as well as other user terminals in addition to relays in the first time slot. Hence, the optimization framework can be extended to cater this requirement as part of the future work. It is also possible to consider multiple hops for all the relay designs considered in the thesis.

Appendix

A. Proof: Maximization of SINR

$$\underset{\mathbf{w}}{\text{maximize}} \quad \frac{\mathbf{w}^H\mathbf{R}_d\mathbf{w}}{\mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w}} = \underset{\mathbf{w}}{\text{maximize}} \quad \frac{\mathbf{w}^H\mathbf{R}_d\mathbf{w}}{\mathbf{w}^H\mathbf{R}_{i+n}^{1/2}\mathbf{R}_{i+n}^{1/2}\mathbf{w}} \tag{9.2.1}$$

This maximization is equivalent to

$$\begin{aligned} \underset{\mathbf{u}}{\text{maximize}} \quad & \mathbf{u}^H\mathbf{R}_{i+n}^{-1/2}\mathbf{R}_d\mathbf{R}_{i+n}^{-1/2}\mathbf{u} \\ \text{subject to} \quad & \mathbf{u}^H\mathbf{u} = 1, \end{aligned} \tag{9.2.2}$$

where $\mathbf{u} = \mathbf{R}_{i+n}^{1/2}\mathbf{w}$ and the solution of (9.2.2) will be the eigenvector corresponding to the largest eigenvalue of $\mathbf{R}_{i+n}^{-1/2}\mathbf{R}_d\mathbf{R}_{i+n}^{-1/2}$.

$$\begin{aligned} \mathbf{R}_{i+n}^{-1/2}\mathbf{R}_d\mathbf{R}_{i+n}^{-1/2}\mathbf{u} &= \lambda_{max}\mathbf{u} \\ \Rightarrow \mathbf{R}_{i+n}^{-1}\mathbf{R}_d\mathbf{R}_{i+n}^{-1/2}\mathbf{u} &= \lambda_{max}\mathbf{R}_{i+n}^{-1/2}\mathbf{u} \\ \Rightarrow \mathbf{R}_{i+n}^{-1}\mathbf{R}_d\mathbf{w} &= \lambda_{max}\mathbf{w} \end{aligned} \tag{9.2.3}$$

This completes the proof. ■

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