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Internal Forces in the Spine Modelled as an Arch

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Abstract

The work reported here presents a parametric solid model of the spine, loading systems of the spine and a extracted arch spine model in which the body weight and external loads are treated as forces applied at appropriate points on the spine and muscle and ligament forces are treated as reaction forces applied to both ends of the spine. The model extends the arch spine approach by using optimisation techniques to find a better fitting thrust line compared with the previous arch model in the literature, and calculates the internal forces between the vertebrae in the spine. Case studies show the reasonable values of the internal forces (1.1-1.5 kN) in the arch spine. The values are much less than that of lever models (up to 6.6 kN).

Keywords: spine, model, internal forces, arch

1. Introduction

The human spine consists of the cervical spine, thoracic spine, lumbar spine and sacrum in the form of a column surrounded by ligaments and muscles. It is the main structure to support human body weight and external loads, to allow the torso to reach to a variety of positions and to protect the spinal nervous system. Lumbar back pain and disorders may be related to spine curvature and disc pressure. The human spine is a statically indeterminate structure which is held in equilibrium by a complex set of internal and external forces, is composed of a variety of materials and is subject to deformation in normal use. An understanding of the mechanical behaviour of the spine is important in the study of its normal function and the prevention, diagnosis and treatment of pathological conditions. Understanding is limited due to the structural complexity of the spine and the difficulty of conducting experiments in vivo. Mathematical analogs become a useful tool for manipulating and analysing such a complex system. Many attempts to model the spine have been made. Four categories of the models can be made according to classification of mechanics of materials which deals with the behaviour of solid bodies subjected to various types of loading, axially or laterally. Those are levers, simple beams, cantilever beams and arches. Lever models (Morris, 1973; Morris et al, 1961) typically describe the whole spine as a rigid lever in two dimensions with no proper consideration of spinal curvature. Loads arc applied to the lever spine with balancing reaction forces at the sacrum. This static analysis is simple but can produce unacceptably large reaction forces at the sacrum. Elastic analysis is used in lever, simple beam and cantilever beam models while plastic analysis is used in arch model. The arch model (Aspden, 1989) describes the spine as an arch in two dimensions in the sagittal plane. The stability of the spine under a variety of loading conditions can then be determined using plastic analysis methods in compliance with the criterion that stability, or safety of the arch, requires that a thrust line should be completely located within all the cross sections of the arch (Heyman, 1982). The spine curvature can therefore be considered in the arch model. Comparisons between the arch, lever, simple beam and cantilever beam models can be found (Xiao et al, 1996).

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2. A parametric model of the spine and loading system of the spine

A parametric solid spine model was generated (Stepney, et al, 1996) as shown in Figure 1. A biomechanical loading system of the spine consists of body weight (concentrated or distributed), external loads, muscle forces, ligament forces, abdominal pressure etc. as shown in Figure 2. The loading system of the human spine may be simplified as a mechanical structure like a tower with independent layers which are fixed through a number of strings to the base and linked between the layers in three dimensions. This is a statically indeterminate structure as the number of variables is greater than the number of equations available. The tension in the strings, the frictional forces between the layers of the tower and the connection conditions between the string ends and the base are unknown. The vertebra and discs of the spine are modelled as rigid blocks in order to generate an arch spine.

3. The arch model using optimisation techniques

A masonry arch, loaded as shown in Figure 3 (a), is a statically indeterminate structure, because the reaction forces HI, H2. R1 and R2 cannot he found from consideration of equilibrium of the applied loads F1, F2 and F3 using force and moment equations such as:

$$\Sigma F_x=0,\ \Sigma F_y=0,\ \Sigma F_z=0.$$

The four unknown reactive forces, H1, H2, Rl and R2, cannot be found from the above three independent equations of statics, as the number of variables is larger than the number of equations, i.e. there are an infinite number of solutions. To obtain a fourth equation, deformation of the arch must be considered. The horizontal reaction H2 can be taken as the redundant element, then its magnitude will be determined from the condition that the horizontal displacement of the hinge E in Figure 3 vanishes. As a general approach to the problem of the analysis of statically indeterminate arches, we shall use the second theorem of Castigliano or the theorem of least work which requires the establishment of an expression for total strain energy stored in the arch under load (Timoshenko, 1965).

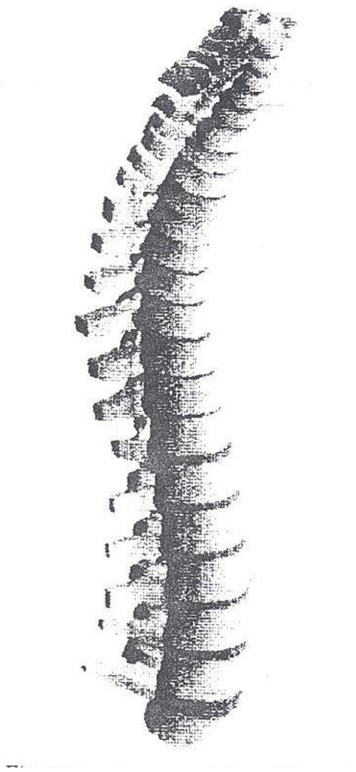


Figure 1: A parametric solid model of the spine

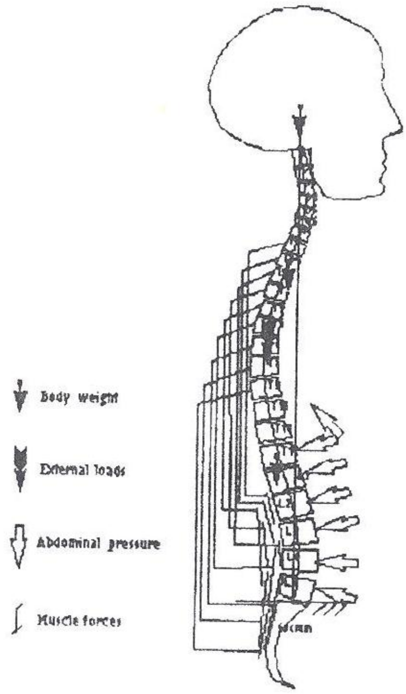


Figure 2: Loading system of the spine with partial muscle forces

A well-known tool for the analysis of arches is the funicular polygon (thrust line) shown in Figure 3 (b,c) (Heyman, 1982). The problems of the hanging string in Figure 3 (b) and the arch which is modelled as hinged rigid rods in Figure 3 (c), are the same to statics. The Figure 3 (b) inverted becomes Figure 3 (c). The rods work in compression while the strings are in tension. Compressive force, thrust, is transmitted along a line called the thrust line. The thickness of the arch voussoirs which contain the thrust line, and the funicular polygon (thrust line) determines the stability of the arch The reaction forces R1 and R2 in Figure 3 can be found using $\Sigma M_A=0$, $\Sigma Fy=0$ if it is assumed that the horizontal component H (H1=H2 in this case) is known, then the funicular polygon (thrust line) in Figure 3 (b,c) can be drawn from the force polygon at once and only one thrust line is existent But in most cases, the horizontal components H1 and H2 are unknown, so the pole O of the force polygon could be placed anywhere, and an infinite number of thrust lines in an arch corresponding to different pole positions O could be found. The position and shape of the thrust line depends on the position of the pole O of the force polygon and changes with the position of the pole O. A similar situation is found in the spine due to difficulty of finding all the muscle and ligament forces applied to the neck and sacrum.

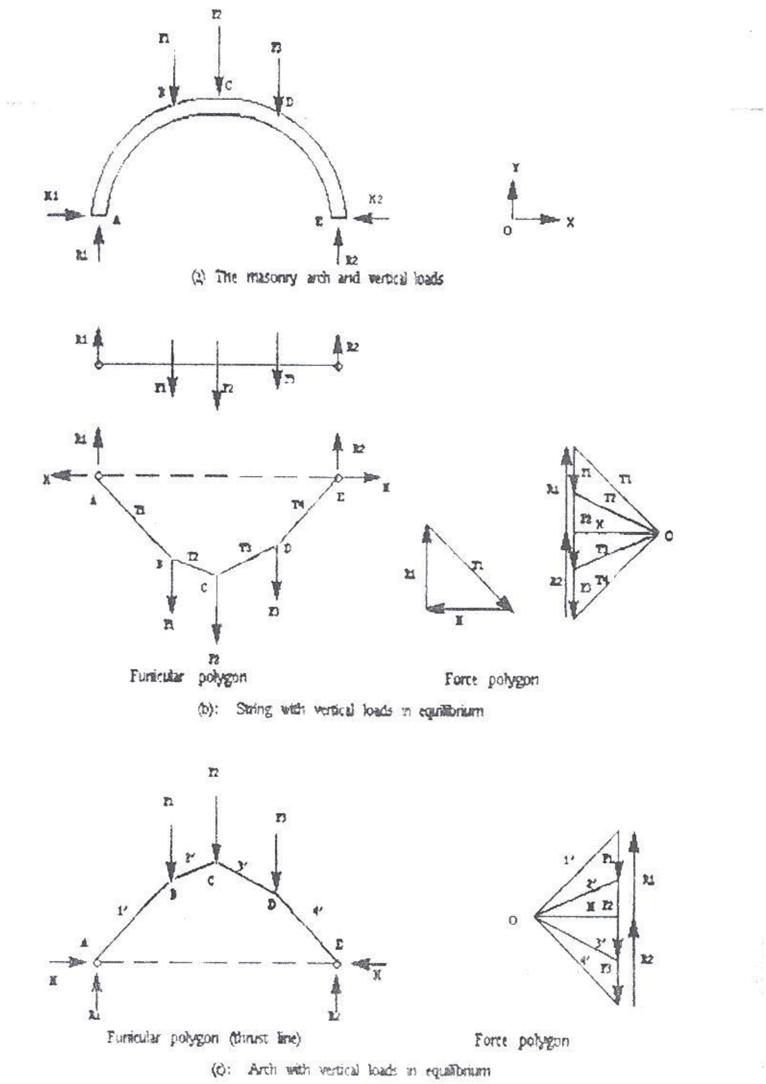
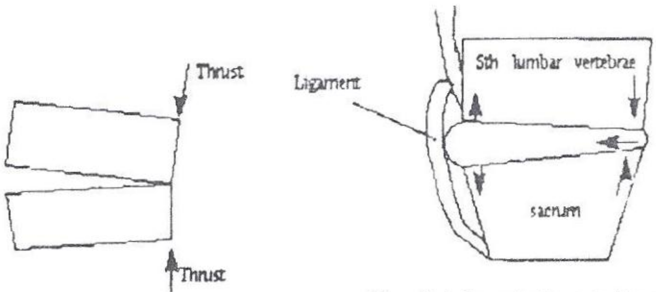


Figure 3: String and arch with loads in equilibrium

There are three assumptions for the masonry arch, and thus for the spine model, as follows: (1) Masonry has no tensile strength; that is no tensile forces can be transmitted in arches. For the spine, loads are transmitted by compressive forces along the spine length even though ligaments, muscles, vertebrae and discs have some tensile strength. (2) Masonry has an infinite compressive strength; that is there is no danger of crushing the materials of arches. For the spine, compressive damage to the vertebrae or discs does not lead the spine to collapse and normally compressive forces are lower than the crushing strength of vertebrae and discs. (3) Sliding failure cannot occur in arches, that is friction between voussoirs is high enough to prevent sliding. For the spine, the vertebral disc is sufficiently strong to resist shear forces between the vertebrae, as are the ligaments.

According to the "Middle-Third Rule" hinges (Heyman, l982), if the applied load (thrust line) stays within a core of the section, stresses across the whole section will be compressive. For a rectangular section the core has a depth of one-third of the total depth. This means that an assurance of satisfactory design of an arch would be obtained if a thrust line is found to lie within the core of the whole arch. This is the geometrical factor of safety with respect to the core of the arch. In other words, a thrust line for the arch must lie within the masonry. This means that the pole O of the force polygon must be located in such a position that the corresponding funicular polygon (thrust line) does indeed lie within the masonry. If any other thrust line can be found for the whole arch, which is in equilibrium with external loads (including self weight), and which lies totally within the masonry of the arch, then the arch is safe. In other words, the existence of one satisfactory thrust line ensure that the arch cannot collapse. Thrust lines lie outside the masonry (or critical core) may cause the arch to collapse due to the formation of four hinges (Heyman, 1982). Posterior extrusion of fibrocartilage from the disc, caused by hydraulic wedging pressure and the stretching of ligaments may be a cause of back pain as shown in Figure 4(b) (Keegan, 1953).



(a) A hinge forms as thrust line approaches to the surface of the arch

(b) Posterior extrusion of fibrocartilage from disc caused by hydraulic wedging pressure. The stretched ligament may be a cause of back pain (Keegan,1953)

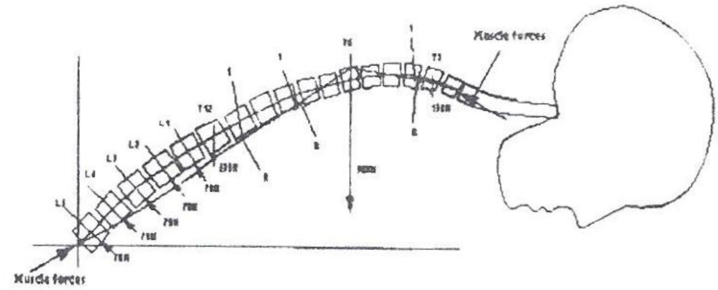
Figure 4: Thrust line and lumbar disorders

An infinite number of thrust lines for a single arch can be obtained due to it being a statically indeterminate structure However there is one thrust line which is the best for the arch or spine. Hence this is an optimization problem. However there may be some difficulty in finding the best fitting thrust line. A better fitting thrust line which is as close as possible to the reference line (or the central line) of the spine can be found. The question of finding a better fitting thrust line in an arch spine is the question of finding a better position for the pole O of the force polygon which generates a better fitting thrust line. This is a non-linear optimization problem.

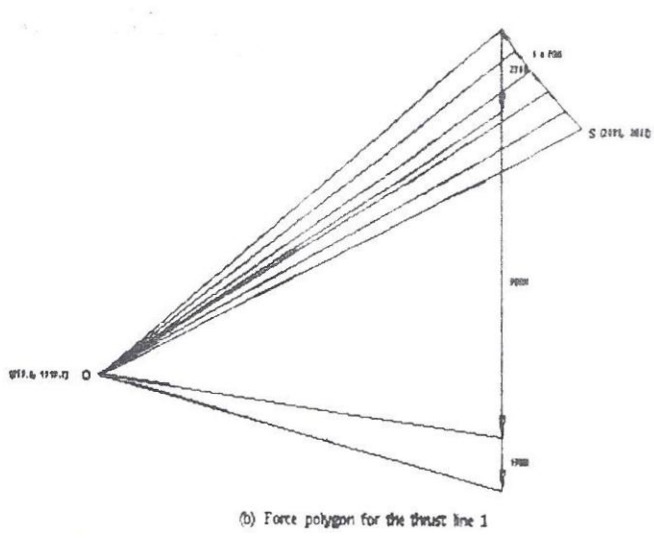
A 77 kg (755 N) man in a stooped, forward bending, position holding a 91.8 kg (900 N) weight in both hands which is assumed to act on T6 (thorax 6). The head and shoulders weight of 130 N (l8% of body weight) acts on T1, the trunk weight of 230 N (30% of body weight) acts on T12, and intra-abdominal pressure of 70 N acts perpendicularly to each of the vertebrae L1 to L5, as shown in Figure 5(a) (Aspden, 1989). The body weight, intra-abominal pressure and additional loads are treated as the external forces.

The ligament and muscle force are treated as reaction forces acting on both ends of the spine (sacrum and head or neck.). Lever model leads to a predicted reaction force at the sacrum of 6.6 kN (Morris et al, 1961) which is comparable with that of fracture of the end plates of the vertebral body (whose bearing strength is about 5-8 kN). This is unlikely to be the real situation.

The optimised thrust line (line 1) calculated from the force polygon, for this case are shown in Figure *5*. Internal forces between the vertebrae in the spine can be calculated from the thrust line 1 as shown in Table 1. It should be noticed from Table 1 that the reaction force at sacrum is only about 1.5 kN which is much smaller than the one predicted by the lever model (6.6 kN). This simple comparison between the lever model and our arch model shows clear differences in the predicted reaction forces. Further work is required to explore the validity and utility of our approach.



(a) Optimised thrust line 1



(b) Force polygon for the thrust line 1

Figure 5: Calculation of the optimization arch model

Table 1: Internal forces

Locations	Internal forces (compression) N
Top of T1	1156.92
T1/T2	1156.92
T2-T6	1129.43
T6-T12	1329.45
T12/L1	1467.33
L1/L2	1460.44
L2/L3	1461.58
L3/L4	1469.86
L4/L5	1485.52
L5/S1 (sacrum)	1505.86

4. Discussion

1. A better fitting thrust line which is as close as possible to the reference (or central) line of the spine in the complete arch spine can be obtained using optimisation techniques. The optimised arch model in this paper produces a better predictor which is closer to the central line of the spine than the one from the literature and represents a better fitting thrust line in the spine.
2. If the result thrust line lies outside the spine (in the lumbar region in the case study shown in Figure 5) the risk of the spine disorder may occur. If a lordosis can be introduced in this case, this kind of lumbar disorder may be avoided. So a good spinal posture under certain load conditions might be suggested by the optimised thrust line.
3. The internal forces in the vertebrae and discs could be found through the optimised thrust line and the force polygon, which provides for the strength check. The reaction force at sacrum calculated by the plastic arch model with optimisation is about 1.5 kN which is much smaller than that by elastic lever model (6.6 kN) and may be more realistic in practice.
4. The calculations and analysis show that intra-abdominal pressure and the lordosis work together to strengthen the spine which is the experience of the weightlifter.

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References

Aspden R.M., (1989). The Spine as an Arch: A New Mathematical Model, Spine, 14, pp266-274.

Goel V.K., Kong W., Jung H., Han S., Weinstein J.N., Gilbertson L.G., (1993), A. Combined Finite Element and Optimisation Investigation of the Lumbar Spine. Mechanics with and without Muscles, Spine, 18, 11, pp1531-1541.

Heyman, J., (1982), The Masonry Arch, Ellis Horwood Series in Engineering Science, Ellis Horwood Ltd, Publishers, Chichester, New York, Brisbane.

Keegan J.J., Nebask O., (1953), Alterations of the Lumbar Curve Related to Posture and Seating, The Journal of Bone and Joint Surgery, V35-A, No.3, July, pp589-603.

Morris J.M., (1973), Biomechanics of the Spine, Arch Surg, 107, pp418-423.

Morris J.M., Lucas D.R., Bresler B., (1961). Role of the Trunk in Stability of the Spine, The Journal of Bone and Joint Surgery, April, 1961, 43-A. (American Volume), pp327-351.

Stepney, P.D., Acar, B.S., Case, K., Porter, J.M., Computer Aided Parametric Solid Modelling of the Spine, Proc. of Eurographics UK Chapter, 14th Annual Conference, V2, pp235-244, Imperial College, London, 26-28 March 1996

Timoshenko, S.P., Young, D.H., (1965), Theory of Structures, McGraw Hill Co., New York, ppl9-45.

Xiao, D.C., Case, K, Acar, B.S., Porter, J.M., (1996), Mathematical Modelling of the Human Spine (Appendix II), Internal Report for EPSRC project "Spine Modelling: A Tool for Safe to Use Equipment Design" Grant Number GB/K58241, May, 1996.