Mathematical equations as Durkheimian social facts?

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Introduction

It is widely assumed that one of the main fracture lines within social thought is between those who affirm and those who deny the reality of 'social facts' or their equivalent, between those who insist that action is constrained by objective social structures and those who deny that there are such constraints and adopt what is variously called 'subjectivism' or 'voluntarism' instead. Emile Durkheim's definition of the expression 'social facts' is widely viewed as a key precursor of the former view and thus remains relevant to contemporary sociological discussions about whether society is essentially an 'objective' or a 'subjective' reality (or a Janus-faced combination of the two).

Responses to Durkheim's (1982 [1895]) recommendation that we should "consider social facts as things" (60) are often taken as critical for assigning those who accept the existence of social facts to the objective side of the argument and those who deny this to the subjective side[1]. We are sceptical of the suitability of social facts as the primary demarcation for the articulation of diverse sociological

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[1] Since ethnomethodology is typically placed on the denial side, it might seem bewildering that Garfinkel would subtitle his *Ethnomethodology's Program* (2002) as "Working out Durkheim's Aphorism" and would endorse Anne Rawls' re-reading of Durkheim (1996, 2004). See the papers by Lynch and Rawls (THIS VOLUME) for further discussion of Garfinkel's relation to 'Durkheim's aphorism' (to treat social facts as things).

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positions, because we do not think that questions about the existence of social facts are as clear cut as might seem[2].

Durkheim uses the model of natural facts to derive what he sees as the fundamental characteristics of social facts (that they are external to and independent of individual will). This extension from natural to social facts may make it seem that social facts must be strange things indeed, but if we consider the sort of social phenomena Durkheim includes under his definition we might see the notion rather differently. Consequently, we will treat Durkheim's idea of social facts as allowing two different estimates of significance. The first one, based on what Durkheim has to say about the general nature of social facts, suggests that the identification of social facts and relations between them provides the basis for an ambitious new science that deals with a new kind of entity and whose results will overturn our existing understandings of life in society. As Garfinkel and Sacks (1970) put it many years ago, this 'inflationary' reading takes Durkheim's statement as sociology's "slogan, […] task, aim, achievement, brag, sales pitch, justification, discovery, […] or research constraint" (339).

The other reading relates to the examples of social facts that Durkheim gives and suggests that the idea of social facts does no more than trace the outline of existing understandings that inform the life of the society, pointing to features that are integral to anyone's conduct of their daily affairs. This 'deflationary' reading is not sceptical of the existence of social facts (since on this reading there is nothing to be sceptical about), but is cautious about the use of the notion of social facts as a platform for sociology's self-promotion.

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[2] And we should note that the objective—subjective dichotomy does not even strictly apply to Durkheim himself (cf., Rawls 2004).

We will review these issues in relation to the argument that mathematical equations (such as '2 + 2 = 4') are social facts, developed by one of one of the most resolute of contemporary Durkheimians, David Bloor (e.g., Bloor 1982). Our disagreement with Bloor is not so much about whether mathematical equations are social facts, but more about what saying that they are might amount to. For Bloor, it is the start of the project of a professional sociology, which will yield a new understanding of the real nature of mathematics and which is at odds with the understandings that – allegedly – underpin practical dependence on mathematical calculations. In contrast, we question whether Bloor has pinned down the understandings involved in mastery and use of mathematical equations, and whether, therefore, showing that and how mathematical equations *are* social facts involves any more than clarificatory reference to an assortment of familiar mathematical considerations.

Two readings of 'social facts'

Durkheim (1982 [1895], 50-51) answers his question "What is a Social Fact?" (Chapter 1) thus:

> When I perform my duties as a brother, a husband or a citizen and carry out the commitments I have entered into, I fulfil obligations which are defined in law and custom and which are external to myself and my actions. Even when they conform to my own sentiments and when I feel their reality within me, that reality does not cease to be objective, for it is not I who have prescribed these duties; I have received them through education. Moreover, how often does it happen that we are ignorant of the details of the obligations that we must assume, and that, to know them, we must consult the legal code and its authorised interpreters!

Further:

> Not only are these types of behaviour and thinking external to the individual, but they are endued with a compelling and coercive power by virtue of which, whether he wishes it or not, they impose themselves upon him. […] If I attempt to violate the rules of law they react against me so as to

> forestall my action, if there is still time. Alternatively, they annul it or make my action conform to the norm if it is already accomplished but capable of being reversed; or they cause me to pay the penalty for it if it is irreparable. […] In other cases, although it may be indirect, constraint is no less effective. I am not forced to speak French with my compatriots, nor to use the legal currency, but it is impossible for me to do otherwise. If I tried to escape the necessity, my attempt would fail miserably. As an industrialist nothing prevents me from working with the processes and methods of the previous century, but if I do I will most certainly ruin myself.

The important elements in Durkheim's account of social facts are:

- that they are *external* to individuals;
- that they are endowed with *coercive power* (they causally compel individuals to do things); and
- that individuals are often *ignorant* of them (and therefore must consult "authorised interpreters").

The more common 'inflationary' reading takes the identification of social facts as the basis for a new kind of science that is modelled after the natural sciences (which deal with natural facts):

> Social phenomena must therefore be considered in themselves, detached from the conscious beings who form their own mental representations of them. They must be studied from the outside, as external things, because it is in this guise that they present themselves to us. (70)

It is further assumed that members of society have only limited understanding of social facts and it is up to social scientists to discover the hidden realities of society. On this inflationary reading, sociology will yield, for the first time, a true understanding of the nature of social facts and the ways they affect human lives.

Note that the challenging part of Durkheim's proposal "to consider social facts as things" (60) is not the suggestion that there *are* social facts, but the claim that social facts are *things*. The argument for social facts itself is almost indisputable, since it is an argument from truisms: that I speak the native tongue to communicate with native speakers; that I employ economically viable technologies in perpetuating a competitive economic enterprise; or that I founded my household, but not the institution of the family. This opens the possibility for a different reading, one that does not dispute the existence of the substantive social facts (such as the currency or the family), but sees the injunction to treat them as things as unnecessary and misleading. This reading takes Durkheim's innovation as one in nomenclature, not ontology. Rather than denying the reality of social facts, this reading follows up on the idea that they "present themselves to us". Rather than pointing *away* from the "mental representations" "conscious beings" make of social facts, Durkheim's own account of social facts is seen as pointing attention *toward* them – toward understandings available to anyone.

Durkheim takes the externality and coercive power of social facts as the basis for an argument that a new kind of understanding of our actions is being introduced, in which our actions are occasioned not by our individual thoughts but by the demand and compulsion of social facts. Hence the label 'inflationary', since on this reading members of society are being told something *new* (in particular: that many things that they consider as 'natural' are in truth 'social', i.e., conventional). However, both externality and coercive power could equally be seen as expressions of incontestable truisms that members of society *are* aware of. The constraint involved in Durkheim's own examples is more conditional than causal: *if* I want to speak to someone else and be understood by them, then I should speak the language they understand; *if* I want to engage in valid economic exchanges, I should employ a valid currency; *if* I want to prosper economically, I should take measures to avoid operating permanently at a loss (and therefore adopt up-to-date business practices). Alternatively, the constraint is constitutive in nature (cf., Garfinkel 1963), i.e., defines what counts as valid actions: whilst individuals decide to found a family, individuals do not invent the idea of 'the family'. Finally, the constraint of social facts may involve a variety of practical and ethical considerations.

Just as Freud liked to flatter himself that psychoanalysis achieved a triumph over the most profound resistance, though it could equally well be understood as having pandered to a ready audience, so sociology admires itself for having to overthrow pervasive resistance to the truth that human reality is social. Durkheim certainly thought in that way, but we have tried to show that there are only two things about his case that could possibly be objected to: firstly, the terminological innovation 'social facts' (though in truth there is little enough to be objected to here); secondly, the association of the innovation with superfluous realist ontology. The substantive basis for this ontology is something that anyone could agree with – indeed, something that everybody acknowledges in practice. It is only if social facts are treated as in a certain sense beyond the comprehension of ordinary members of society that Durkheim has opened an entire realm accessible *only* to the sociologist. On the deflationary reading of social facts, sociologists are more akin to Monsieur Jourdain's teacher who explains how a familiar practice – speaking – is identified as 'speaking prose' in the discourse of grammarians (cf. Molière's *The Bourgeois Gentleman* [1670], Act 2, Scene 4). It is only in the inflationary reading that there is a suggestion that social science has a more general and complete perspective on social life that it aims to share with ordinary members – possibly against their resistance.

The point can be reinforced by a juxtaposition of Durkheim's statements with some remarks of Alfred Schutz. These two are commonly seen as embodying fundamental oppositions, representing objectivism/realism and subjectivism/constructivism respectively. However, there is a striking similarity in their remarks about the social world:

> […] in the natural attitude of everyday life the following is taken for granted without question: […] that a stratified social and cultural world is historically pregiven as a frame of reference for me and my fellow-men, indeed in a manner as taken for granted as the 'natural world'; […] that therefore the situation in which I find myself at any moment is only to a small extent purely created by me. […] The life-world is thus a reality which we modify through our acts and which, on the other hand, modifies our actions. (Schutz and Luckmann 1973, 5-6)

Both Durkheim and Schutz argue that the social world is – for the most part – not created by individuals and that individuals are often constrained by it. The main difference between them is the intended status of their respective remarks. Schutz is not stating his own sociological doctrines, but is engaged in a descriptive enterprise, delineating the quality of commonplace experience under the natural attitude, the matter-of-course, taken-for-granted orientation of individuals to their practical circumstances. We are not suggesting that there is no difference in their treatments of social facts, but that their projects differ less in respect of the substantive content than might first appear. Durkheim's arguments are substantively in harmony with Schutz's, i.e., there seems to be little or no disagreement in their statements about happens in the world (e.g., about the legal code, about our interactions with others, about our social institutions, etc.). Thus while there are considerable differences in philosophical convictions between realists and phenomenologists, there is no need to introduce the notion of social facts to give rise to those differences. Whatever the intellectual gulfs between Durkheim and Schutz, it is not because one is asserting and the other rejecting the existence of social facts.

We thus want to suggest that much of the recent debates (e.g., objectivism-subjectivism, structure-agency) are misplaced. These debates treat the disagreements between the respective positions as a matter of affirming or denying the existence of social facts (or the constraining character of them). Resistance to Durkheim's sociological programme is thus commonly understood as also entailing doubt about whether there are any social facts (i.e., that there are subjective states only). This is the reason why ethnomethodology and phenomenology are typically taken to be on the subjective (or agency) side of the traditional dualisms. In contrast, we have been arguing that many of the disagreements with Durkheim's project are not about whether there *are* social facts, but about the character of social facts – and its implications for the project of social science.

Mathematical facts

Durkheim's influence is very directly felt in one of the central contributions to the sociology of mathematics, that of David Bloor (cf., 1973, 1976, 1983, 1994). To Bloor, mathematical statements (such as '2 + 2 = 4') invite a neo-Durkheimian approach, since they are experienced as Durkheimian facts, being external to individuals and endowed with a coercive power. If mathematical expressions can be treated as facts, what species of facts are they? Philosophers have tried to establish that they are natural facts (empiricism) or that they are logical facts (logicism). A neo-Durkheimian perspective insists that they are, rather, social facts. As White (1947) put it very early on:

> Mathematics does have objective reality. And this reality, as Hardy insists, is *not* the reality of the physical world. But there is no mystery about it. Its reality is cultural: the sort of reality possessed by a code of etiquette, traffic regulations, the rules of baseball, the English language or rules of grammar. (302-303)

There is a relatively straightforward way in which everyone could acknowledge White's point. The mathematical symbols that we use today are conventional in the sense that other symbols could (and have) been used. Rather than using the symbol '5' to signify the number five, we could use other symbols – and most of us are aware of other symbols to signify this number, e.g., the Roman numeral 'V' or five strokes '|||||'. Similarly, rather than using a decimal system (base 10), we could use a duodecimal system (base 12), i.e., be counting in dozens. In that sense, the rules of arithmetic are as conventional as the rules of traffic.

In this context, sociologists often draw on Wittgenstein's (1976, 1978) reflections on mathematics, which are seen as demonstrations of the conventional character of mathematics. For example, Wittgenstein's discussion of a pupil continuing a number series taught by his teacher (1953, §§185-242) or the visual proof that 2 + 2 + 2 = 4 (1978, I, §38) are often taken as 'proof' that mathematics is social in character. However, just as there were two different ways of reacting to Durkheim's 'discovery' of social facts, so there are in the case of Wittgenstein's reflections on mathematics. We will compare Bloor's understanding with our own, which is intended to be more in accord with Wittgenstein's aims in philosophy.

Bloor sees Wittgenstein as enabling an extension of Mannheim's (1936) sociology of knowledge to mathematics. Mannheim famously excluded mathematics (and the natural sciences) from his sociology of knowledge, since the truths of mathematics seem to be eternal, true in all times and places, and do not seem to vary with socio-cultural formations. For Bloor, Wittgenstein's demonstrations of the conventional character of mathematics open the way for a sociological treatment of mathematics. However, for Bloor, Wittgenstein does not go far enough, since he did not develop a theory to explain causally the social character of mathematics.

In our reading of Wittgenstein, Bloor's sociology of mathematics does not actually add anything to the initial claim that mathematics is social. Bloor is thus an example of an inflationary treatment, since he puts the sociologist in a position to see something about mathematics that ordinary members of society are supposedly unaware of. In contrast, we see Wittgenstein as being engaged in a deflationary project, where reminding people of what they already know (e.g., that other arithmetics could, and have, been used) is enough to answer – or dissolve – any questions that one might have, eliminating the impression that the 'problem' of mathematical objectivity calls for the creation of an explanatory theory.

Bloor's sociology of mathematics

If mathematical statements are facts, then what endows them with factual status? For Bloor, this entails asking what *causes* mathematical statements to obtain. He recognises three possible causes: natural-empirical, logical, or social. Like Durkheim, Bloor's argument is eliminatory: if it can be shown that the objectivity of mathematical facts is neither a natural-empirical nor a logical necessity, then mathematical statements must be social facts.

Bloor (1994)[3] takes the equation 2 + 2 = 4 as an example of a mathematical fact, since it is typically taken to be an eternal truth (2 + 2 cannot equal 0, 3, or 5, but *must* equal 4). Bloor also thinks that most people when asked to explain why 2 + 2 = 4 would argue that it expresses an empirical fact (i.e., a statement about reality) or a logical fact (i.e., a fundamental logical principle). Therefore Bloor aims to establish that 2 + 2 = 4 is neither an empirical nor a logical but a social fact. He does so by showing that there are alternatives, i.e., by giving examples in which 2 + 2 does not equal to 4:

> Sociologists are professionally concerned with the conventional aspects of knowledge. So I will try to identify the conventional components of the concepts '2' and '4' and 'addition'. Conventions are shared ways of acting that could in principle be otherwise. They are contingent arrangements, not necessary ones. Thus it is conventional that we drive on the side of the road that we do, and (if proof were needed) we could point to others who drive on the other side. Even if everybody, as a

[3] See Greiffenhagen and Sharrock (2006) for a discussion of Bloor's (1976) earlier arguments about alternative mathematics.

> matter of fact, drove on the same side, we could easily imagine the alternative. Demonstrating conventionality therefore involves demonstrating alternative possibilities. Although this necessary condition is easy to state it isn't always easy to satisfy in practice. For one thing, our imaginations are limited. Another reason is that candidate alternatives often meet objections. Reasons are found to sideline, trivialize or re-interpret them so that their character as alternatives is disguised. (Bloor 1994, 21)

Bloor thinks that there will be strong objections to the idea that mathematical equations are conventional, which sociologists have to overcome. One of the most common objections to the idea that mathematics is (only) a convention is that mathematics seems to apply to reality. If it is just a convention that 2 + 2 = 4, then why do two sheep added to two sheep always make four sheep?

Bloor is not denying that two sheep added to two sheep results in four sheep, but wants to undermine the idea that this could contribute to an empiricist *justification* of mathematics (i.e., that mathematical equations are true *because* they correspond to reality). Consequently, he gives another purportedly empirical example, in which 2 + 2 does not equal 4. He asks us to imagine a number wheel with four segments, labelled '0', '1', '2', and '3', where turning the wheel in one direction "we think of ourselves as adding" (25):

> Just as we felt a naturalness about carrying out our counting technique from one empirical circumstance to another in the past, so we feel a naturalness about this. […] Then, of course, we make the inevitable discovery: we set the wheel at 2, and then turn it so as to add a further 2, and we get back to zero. 2 + 2 = 0. (26)

Turning the number wheel is as 'empirical' as adding sheep. So Bloor has given us two examples, one in which 2 + 2 = 4 and one in which 2 + 2 = 0. For Bloor, this does not show that mathematical equations aren't applicable to reality, but that their applicability cannot be used to justify them[4]. Having demonstrated that empiricism cannot be used to justify mathematical facts, Bloor dismantles logicism (justifying mathematical equations as derivative consequences of fundamental logical principles) by showing that logicist justifications will be circular, i.e., will have to presuppose the principles they aim to deduce. Having eliminated the possibilities that mathematical equations are natural or logical facts, Bloor sees himself as having successfully demonstrated that they are social facts.

The conventional character of mathematics

In order to clarify our differences with Bloor, let us reflect on what Bloor is trying to do. Bloor is asking whether 2 + 2 must equal 4, but he is not really asking about this one equation, but rather uses that single (iconic) equation as a proxy for a whole *system* of arithmetic. This system of arithmetic is one that has no ready name to identify it, but we will refer to it as the 'default' system (which is briefer than other correct designations, e.g., 'arithmetic with base ten and Arabic numerals'). It is the default arithmetic in the sense that it is the one taught in schools and employed across a wide diversity of practices.

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[4] We might notice that in the attempt to make the number wheel out as one that does not fit the '2 + 2 = 4' case, Bloor conflates *cumulatively adding up* the total number of points passed through on the number wheel with using the points on the number wheel to *track the location* of the pointer. That is, if we ask how many points on the number wheel we have moved through as we rotate it, then we will correctly pronounce that – having moved through initially two points and then two further points – we have moved through four points. It is a different question to ask what numbered point on the wheel will be reached – if we make two moves and then two more – the answer is, of course, the fourth point in the rotation, the one identified as '0'. The number wheel is thus not counter-evidence to an empiricist view.

The question "Must 2 + 2 = 4?" can be asked as a question *about* the default mathematical system or as a question *within* the default system. If asked *within* the default number system, the answer is "yes", since any other answer is wrong. That this is so is part of what identifies the system, and, also, what identifies the equation as belonging to the system. If asked *about* the default system, then it is not entirely clear what the question is asking: "Can you think of mathematical systems in which 2 + 2 is not 4?"; "Are there empirical situations in which 2 + 2 is not 4?"; or: "Are there historical reasons why we have this arithmetic system?". Asked *about* a system (rather than within one), '2 + 2' does not *yet* have a definite sense, since it only constitutes a notational form. For example, '+' in the case of adding sheep has the sense of 'accumulating a totality', whereas in the case of the number wheel it has the sense of 'tracking positions in a cycle'. All that Bloor demonstrates is another mathematical triviality: the same symbols can be used differently in different calculi (cf., Ambrose 1955, 208).

The difficulty in specifying what exactly the question "Must 2 + 2 = 4?" is asking points to a second issue, namely: Of *whom* is this question being asked? Asked of a professional mathematician, understood as asking "Is there more than one arithmetic system?", it is mathematically trivial that there are. Asked of ordinary users in the street, unprepared for it, the question may well create difficulties, since it is not clear in what sense those using the default system understand that it is a distinctive system, let alone what 'other systems of arithmetic' might be. This is not because they are unaware or unfamiliar with 'other systems of arithmetic' (most people are familiar with twelve-hour clocks in which 11 + 3 = 2), but because the sense of the question is unclear. It is easier to know how to respond to "Could one drive on an alternative side of the road?" then "Are there alternative systems of arithmetic?"

Of course, Bloor is not so much asking *whether* 2 + 2 = 4, but *why* 2 + 2 = 4, i.e., what kind of justification one might give for this mathematical fact. Bloor wants to argue that it is true *by convention* that 2 + 2 = 4 (which is why he is often taken to be a relativist). In contrast, we want to ask: In which sense does convention make it true that if we add two things and two things, it always comes out as four?

It is certainly the case that the development of a specific arithmetic system is contingent (a different 'default' system could have been institutionalised). However, this does entail that the operations within a particular system are contingent. There is a difference between the necessity in adopt*ing* a system and the necessity imposed by an adopt*ed* system. As Wittgenstein, (1976, 241) remarks: "We must distinguish between a necessity in the system and a necessity of the whole system". The observation that 2 + 2 is 'always' 4 is thus not a false generalisation, rebutted by the existence of other arithmetics, but the expression of the necessity imposed by a particular system of arithmetic. Whenever one is operating in *this* number system, then if one is given 2 + 2 one *must* sum it to 4.

The origin of confusing the different kinds of necessity is to take mathematical expressions as propositions, i.e., as descriptions of states of affairs. As Wittgenstein emphasised, equations should not be treated as representations of any kind of realities (natural, logical, or social), but rather as *means* of representation, i.e., as providing a framework within which description of states of affairs may be constructed (Wittgenstein 1978, VII, §2). That is to say, Wittgenstein argued that mathematical statements should (in order to avoid confusion) be considered as rules – and rules although 'conventional' are *in themselves* neither true nor false.

Bloor, in a sense, is asking where the correctness of an arithmetic system resides. He rebuts the two traditional answers (in nature or in logic), which leaves him, through a process of elimination, with the answer: in society. In contrast, Wittgenstein questions whether it makes sense to ask about the correctness of an arithmetic system without specifying a particular context or purpose. For Wittgenstein, ‘correct’ is not constant across different systems of conventions, but has a different sense, a different application, depending upon the system of conventions. It cannot be that all systems of conventions are equally correct, because there is no general, independent standard of ‘correctness’. The puzzle as to how conventions can apply to reality is thereby dissolved, for compliance with the rules of the arithmetic *establishes* what we mean by ‘correspondence with reality’.

Why ‘the sociologist’?

Wittgenstein, by characterising mathematical expressions as rules, is making a ‘sociological turn’ that reminds us of the roles that arithmetical expressions play in our practices and the kinds of questions that can be sensibly asked of them. Need the sociological turn go further? Bloor thinks so. Our above claim to be more in tune with Wittgenstein does not mean that Bloor misguidedly thinks of himself as following through Wittgenstein’s approach. Bloor is aware that Wittgenstein thinks philosophical inquiries require no controversial empirical evidence, but Bloor rejects this idea and reproaches Wittgenstein for chickening out on the necessity to move on from philosophical to empirical (scientific) investigations. It is this claim that is the nub of our disagreement with him.

In our view, pointing out that mathematical equations are rules is sufficient to undermine traditional philosophical positions. Furthermore, the observation that there is no necessary arithmetical system points toward mathematical truisms, not to either sociological findings or the need for an explanatory sociological theory. Wittgenstein argues that rather than making extensive historical and anthropological investigations into the different kinds of mathematics one can find (although such enterprises may be interesting), we pay more careful attention to the *form* of the mathematical statements that are to be found in the mathematics we are already familiar with. The status of '2 + 2 = 4' as a universal truth is not undermined by showing that there are circumstances in which it is false, but by realising that the idea of 'universal truth' is misleading, because '2 + 2 = 4' does not function as an empirical proposition but more like a rule and, as such, can be *neither* true *nor* false[5].

Bloor is insistent that the sociologist plays a part, but the idea that this is necessary arises, we think, from the fact that Bloor has taken an argument addressed to the source of philosophical problems and treated it as a diagnosis of the functional structure of the practical understanding of arithmetic. Bloor construes the 'problem' in the following way:

> Food is a cultural universal, because everybody has to eat to survive. Does this preclude the sociologist having significant things to say about food? Clearly not, because there is still the

[5] If there is any difficulty in understanding why '2 + 2 = 4' is said to be a rule, recognise that one is taught it as an injunction, saying what one has to do: "whenever you see '2 + 2 = ' then write '4'". In other words, arithmetical equations can be seen as rules for the transformation of notations. In applied calculations, they are rules for the transformation of empirical propositions – if you have two sheep and you buy two sheep you can then (correctly) say you have four sheep. There is no opportunity here to venture into the background debates between David Bloor and Michael Lynch over Wittgenstein and rules (Bloor, 1992; Lynch 1992a, b), but our views are much closer to Lynch's than Bloor's, and agree with Friedman's (1998, 266) observation that "Lynch's Wittgenstein is both closer to the actual Wittgenstein and more sophisticated philosophically than Bloor's".

> question of how people eat, who eats what, and when, and with whom. We might say that while 'nutrition' is a biological category, 'the meal' is a sociological category. […] We must see if analogous ideas and distinctions apply in the case of 2 + 2 = 4. Can numbers be divided into their physical, biological and social aspects in the way that the ingesting of food can? (Bloor 1994, 22)

The term 'sociological' can be applied in two ways: firstly, to identify a certain genre of observations, which note the connection of activities to social groups, cultural traditions, social relations, etc.; secondly, to characterise remarks originating with those who are sociologists by occupational title and theoretical affiliation (thereby implying that it is through specific sociological tools – methods or theories – that these remarks were discovered or are validated). That there are local cuisines and local culinary practices is 'sociological' in the first sense, for such observations did not and need not originate as deliverances of sociological professionals. Although travellers would not necessary label these observations as 'sociological', observations about cultural diversity can and are made by non-sociologists. In contrast, a comparison of the birth rates among the lower, middle, and upper classes more clearly belongs to the sociologist as a professional.

Now remember that Bloor sees his project as establishing "the conventional aspects of knowledge", i.e., "to identify the conventional components of the concepts '2' and '4' and 'addition'" (21). We have argued that Bloor's observations about the "conventional components" of arithmetic are 'sociological' in our first sense. It is not that people are always aware that there are a multitude of arithmetic systems, but rather that 'demonstrating' the conventional aspects of knowledge resembles the neighbour coming back from a trip to China and telling everyone: "They eat rice rather than potatoes over there, and they use chopsticks rather than forks and knives." – thereby 'demonstrating' the cultural diversity of the meal and cutlery.

Bloor believes that such observations may be acceptable to non-sociologists in the case of traffic rules or meals, but not in the case of mathematics. In other words, the attempt to show that there are alternative mathematics will meet with resistance: “candidate alternatives often meet objections. Reasons are found to sideline, trivialize or re-interpret them so that their character as alternatives is disguised” (21). This is why Bloor thinks that demonstrating the conventional aspects of knowledge is the task of the professional sociologist, since it is presumably only the sociologist who is able to see the implications of the availability of alternatives and who is able to prevent those implications being sidelined, trivialized, or re-interpreted. Our disagreement with Bloor is thus not on whether there are alternatives, but whether there are those reactions to those alternatives.

In our view, Bloor does not adequately clarify the implications of the availability of alternatives for the default system. In other words, once we have read his paper on 2 + 2 = 0, what has *changed*? That there are alternative arithmetics does not make any difference to everyday practical computations using the default system. In particular, the number wheel system does not have an impact on most practical calculations: buying two pairs of apples is still four apples – and not ‘0’ apples (since this is not a situation to which the number wheel system is applicable). For many everyday calculations, it simply does not matter that the default arithmetic system is not a unique system. Nor do we think that it is appropriate to conceive everyday users of the default arithmetic as though they are in denial about the conventionality of mathematics. Just as we did with Durkheim, we would argue for a deflationary reading of Bloor, which puts members of society in M. Jourdain’s situation: his difficulty is not that he does not know how to speak prose (for that is what he does before and after being taught by the philosopher), but that he simply isn’t familiar with the name that grammarians give to the activity of which he is a practical master. Equally, there are grounds to suppose that many members of the society are *practically* familiar with more than a single arithmetic system and that they can and do, in practice, comfortably switch from using one form to using another when it is appropriate (e.g., when using a twelve-hour clock or counting in dozens). In that sense, Bloor does not demonstrate something new to members of society, but simply reminds them of something that they are already familiar with (but perhaps incapable of recognising as the intended referent of 'conventional').

None of this really matters to Bloor, since his target is not the computational validity of calculations that people perform, but the 'justificatory underpinnings' of those calculations. We say this, because we are trying to make sense of the fact that the availability of alternative arithmetics for us is more like a triviality, whereas for Bloor it has epistemological implications. Bloor seems to be engaged in a therapy of ideology, which supposedly accompanies, even underpins, the practical mastery of arithmetic. Bloor seems to suppose that children being drilled in arithmetic are *additionally* supplied with justifications for using this practice as a means of binding them into it (i.e., they will act in accord with it, because they are under the delusion that they could not do otherwise). In line with critique of ideology more generally, Bloor supposes that in the case of arithmetic ideology takes the form of naturalising a practice, i.e., as treating arithmetic as though it originates in the natural order of things and *therefore* is immutable. For Bloor, the typical reaction to 2 + 2 = 4 (that it is impossible to do otherwise) is evidence for this ideology.

The rather standard sociological assumption that practices are sustained by virtue of associated justifications is perhaps too easily made, and only serves to ensure a sociological surplus above what is required to understand the problem at issue. The idea is that the naturalisation of a practice *prevents* its participants for calling it into question (since they cannot doubt what must inevitably be the case). If the practice is de-naturalised, then it will be apparent that they *could* do otherwise, enabling them to withdraw support from this practice. However, even if people with practical mastery of arithmetic did labour under such a naturalising illusion, it does not follow that they could simply resign from the default arithmetic and make up their own, since many practices that they engage in have the default arithmetic installed in their organisational and technological infrastructures. More importantly, we have been arguing that mastery of arithmetic does not depend upon any such naturalising justification. Practical mastery of calculation in our society involves a fundamental familiarity with the default arithmetic, but the practical familiarity with it as the default one does not exclude practical participation with other, non-default, arithmetic practices. In other words, the conventions of the default system are not destabilised by the fact that its users 'could do otherwise' when, in fact, they cannot, because, in fact, people have no difficulty in already 'doing otherwise' in the sense that they frequently operate according to conventions other than those of the default system.

Conclusion

We have tried to suggest that buying into the idea of 'social facts' does not involve any very momentous decision, since it can be seen as simply a nomenclatural matter, i.e., as only naming a range of unremarkable, entirely familiar, and in a practical way very well understood affairs. It is not the recognition that one lives amongst collectively accumulated institutions and practices that causes the trouble, but that Durkheim moves from this observation to ontological doctrines about social facts.

Rafanell (THIS VOLUME, XX) quotes Durkheim arguing that we are under "an illusion [if] we believe that we ourselves have produced what has been imposed upon us externally". On what we have termed the inflationary reading, ordinary members of society commonly believe that they have freely created what is in reality imposed upon them. However, Durkheim's documentations of the existence of social facts allow for a deflationary interpretation, where 'having to' use the local language and local currency is a practical rather than causal necessity: living in society does consist in acting within its local institutions and customs, but this does not entail believing that these are the only institutions and customs there could possibly be.

Using David Bloor's campaign for a Durkheimian sociology of mathematics as an illustration, we argued that the notion that people are subject to illusions about the objectivity of the practices they engage in becomes an *a priori* assumption that is projected onto the workings of practices with the effect of distorting the sense that attaches to the ways of these practices[6]. Bloor's whole argument on mathematical equations assumes that their users are under such an illusion, believing that when asked "2 + 2 = ?" they have no choice but to answer "4". Bloor seeks to demonstrate users' delusional state by establishing that they 'could do otherwise', i.e., that alternative mathematical systems might accommodate different answers to "2 + 2 = ?". However, it is not so much the idea that equations are conventions that propels Bloor's argument as ambiguity about 'could do otherwise'. One certainly could do otherwise then write 4, since one could refuse to complete the question or reinterpret the question. However, giving a different answer to a different question (e.g., interpreting the equation as part of a different arithmetic system), says nothing about what one 'has' to do to if one wants to complete *this* equation correctly. Bloor's

[6] Sociologists are apt, at crucial moments, to think of individual consciousness as solipsist, which is perhaps why they have had such a hard time recognising that positions which they keep (critically) designative as 'subjective' actually emphasise intersubjectivity.

attempt to invoke alternate mathematical systems itself depends upon the very thing that it purports to explain, namely that if one understands (for example) the number wheel system properly, then one understands that if asked “2 + 2 = ?” *one has no choice* but to say “0”. The compulsion does not result from any causal force, but from the identifying requirements of that system delimiting what are valid actions within it. Bloor has not shown that there is space for a sociological explanation as to why the users of arithmetic need to operate under a socially necessary illusion, since there is no such illusion.

Bloor sees the sociologist’s job as therapeutic, i.e., as convincing members of society of the conventional aspects of knowledge in the face of their reluctance to accept this. However, this therapeutic strategy only works by assuming that members’ practical doings (their use of the default system) are underpinned by a naturalised ideology of that practice (universalistic beliefs). In contrast, Wittgenstein’s therapeutic method (cf., Anderson *et al.* 1986, Chapter 6) cures us of the idea that practices are founded in theoretical presuppositions. With respect to mathematics, Wittgenstein does not replace empiricism or universalism with social constructivism or relativism, since he argues that the understanding of philosophical problems does not come from any theory, but from careful reflection on our mastery of the relevant practice, e.g., of elementary arithmetic. While Bloor’s therapy is a therapy *from* one (ideological) theory *to* another (correct) theory, Wittgenstein’s therapy is a therapy *away from theory.*

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