




This item was submitted to Loughborough's Institutional Repository (<https://dspace.lboro.ac.uk/>) by the author and is made available under the following Creative Commons Licence conditions.



C O M M O N S D E E D


Attribution-NonCommercial-NoDerivs 2.5


You are free:

- to copy, distribute, display, and perform the work

Under the following conditions:

 **Attribution.** You must attribute the work in the manner specified by the author or licensor.


 **Noncommercial.** You may not use this work for commercial purposes.

 **No Derivative Works.** You may not alter, transform, or build upon this work.

- For any reuse or distribution, you must make clear to others the license terms of this work.
- Any of these conditions can be waived if you get permission from the copyright holder.

Your fair use and other rights are in no way affected by the above.

This is a human-readable summary of the [Legal Code \(the full license\)](#).

[Disclaimer](#) 

For the full text of this licence, please go to:
<http://creativecommons.org/licenses/by-nc-nd/2.5/>

Stability Analysis and H_∞ Control for Hybrid Complex Dynamical Networks with Coupling Delays ^{*}

Zhi-Hong Guan¹†, Shuang-Hua Yang², and Jing Yao¹

¹*Department of Control Science and Engineering
Huazhong University of Science and Technology, Wuhan, 430074, P. R. China*

²*Department of Computer Science
Loughborough University, Loughborough, LE11 3TU, UK*

Abstract

This paper formulates and studies a model of complex dynamical networks with switching topology and coupling delays. Based on the hybrid control and Lyapunov function, the stability and robust H_∞ control of such networks with impulsive and switching effects, which have not been studied before, are addressed with some criteria derived. Examples are given to illustrate the effectiveness of the theoretical results.

Keywords complex dynamical networks; impulsive control; hybrid systems; coupling delays.

1 INTRODUCTION

In recent years, research on complex networks has attracted increasing attention in many areas of science and engineering [23, 26]. Basically, complex networks consist of nodes and links, where nodes may represent routers in the Internet [4, 7, 21], document files in the World Wide Web [1, 4, 15], and individuals, organizations or countries in the social networks [25], etc., while links represent the connections or interactions between any pair of the nodes within a network. These universal features in science and technology have stimulated the current intensive studies of topologies (i.e., architectures) of complex networks and their dynamical processes based on the network structures [2, 5].

In order to emulate different features of some real-world complex networks, several models have been proposed in the literature. Network modeling goes back at least as far as the well-known random graph theory, initiated by Erdős and Rényi (ER) [6]. Small-world networks, introduced recently [27], play the role as the middle ground between regular and random networks. However, many large-scale complex networks such as the World Wide Web, the Internet, various social networks, etc., belong to yet another large class of inhomogeneous networks, called scale-free networks [24]. It has been shown that these structural properties strongly influence the network dynamics [22, 23, 24].

Many complex networks are hybrid in the sense that they have both discrete-state and continuous-state dynamics. In addition, these two aspects of the system behaviors often interact to a significant extent that they cannot be decoupled and must be analyzed simultaneously. One example is a network of mobile agents studied in [22], in which agents are communicating with each other and need to agree upon a certain objective

^{*}Supported by the National Natural Science Foundation of China under Grants 60834002, 60873021, 60973012, 60973039, 61073025, the Doctoral Foundation of Ministry of Education of China under Grant 20090142110039, and the Royal Society of UK under Grant 2005/R1.

[†]Corresponding author. Tel.: +86-27-8754 2145. E-mail address: zhguan@mail.hust.edu.cn (Z.-H. Guan).

of interest or to perform synchronously. Since the nodes of the network are moving around in space, some of the existing links may fail to stay connected due to the existence of obstacles between two communicating agents. On the other hand, new links between nearby agents may be created, when two agents come to an effective range of signal detection with respect to each other. This means that a certain number of links will be added or be removed from the graph at times. This is a typical scenario of a network with a switching topology. Internet is also a typical hybrid switching network since its nodes and links are always variable dynamically [4, 21]. Furthermore, some real-world complex dynamical networks commonly have communication time-delays due to finite speed of information processing. They are commonly represented by coupling delays in complex dynamical networks [16, 30]. Based on this observation, we propose a new model of complex dynamical network with switching topology and time-varying coupling delays in this paper, and then study its control and stability. Close to our work, Belykh etc. [3] considered a model of small-world networks with a time-dependent on-off coupling between any other pair of cells. At each moment, the coupling structure corresponds to a small-world graph, but the shortcuts randomly change from time interval to time interval. The blinking model is indeed of practical importance in biology as well as in technology. It is to be distinguished from networks whose topology changes according to a switching function in this paper.

In the field of control systems, various control techniques based on hybrid impulses and switches have been extensively studied in recent years [8, 10, 11, 12, 17, 19, 20, 28, 29], due to their theoretical and practical importance. Compared to the continuous methods, hybrid impulsive control methods can increase the efficiency of bandwidth usage, among other advantages such as it allows achieving stabilization of a complex network by using only small control impulses acting at certain necessary instants. To our knowledge there are very few reports [14] dealing with hybrid impulsive and switching complex dynamical networks and the corresponding control problem.

This paper studies the stability and robust H_∞ control problems of the complex network with switching topology, by using hybrid impulsive control. A class of hybrid complex dynamical networks with coupling delays is formulated, and a new hybrid impulsive and switching control strategy for complex network control is developed. Based on the Lyapunov function and technique of average dwell-time for the impulsive and switching interval, some criteria of exponential stability and robust H_∞ control criteria of the resulting closed-loop network are established. A typical example, using Lorenz chaotic system as the dynamical node in networks, is given to visualize the satisfactory control performance.

The paper is organized as follows: In Section 2, an impulsively controlled complex dynamical network model with switching topology is proposed. Section 3 presents some sufficient conditions for network stability. Then, in Section 4, the theory and approach of H_∞ control for the hybrid complex dynamical network with coupling delays are investigated. Numerical results and conclusions are given in Section 5 and 6, respectively.

2 PROBLEM FORMULATION

Let $R_+ = [0, +\infty)$, and R^n denote the n -dimensional Euclidean space. For $x = (x_1, \dots, x_n)^\top \in R^n$, the norm of x is $\|x\| := \left(\sum_{i=1}^n x_i^2 \right)^{\frac{1}{2}}$, and $|x|$ denotes the sup norm $|x| = \max\{|x_1|, \dots, |x_n|\}$. Correspondingly, for $A = (a_{ij})_{n \times n} \in R^{n \times n}$, $\|A\| := \lambda_{\max}^{\frac{1}{2}}(A^T A)$. The identity matrix of order n is denoted as I_n (or simply I if no confusion arises).

Notice that, most real-world complex dynamic networks have the following characteristics: different nodes have different dynamic behavior, different edges have different and variable coupling strengths, as well as the time delay connection between nodes, and so on. Consider a complex dynamical network consisting of N coupled nodes, with each node being an different n -dimensional dynamical system

$$\dot{x}_i = A_i x_i + f_i(t, x_i).$$

The coupled dynamical network with switching topology and time delay connections is described by

$$\dot{x}_i = A_i x_i + f_i(t, x_i) + \sum_{j=1}^N D_{ij}^{\sigma} \Gamma x_j(t - \tau) + B_i u_i, \quad i = 1, 2, \dots, N, \quad (2.1)$$

with initial value conditions

$$x_i(t) = \phi_i(t), \quad t_0 - \tau \leq t \leq t_0, \quad i = 1, \dots, N, \quad (2.2)$$

where $t \in R_+, t_0 \geq 0$, $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^\top \in R^n$ are the state variables of node i , $\phi_i \in R^n$, $\Phi(t) = \text{col}(\phi_1(t), \dots, \phi_n(t))$ is continuous on $[t_0 - \tau, t_0]$. A_i and B_i are known matrices with appropriate dimensions, $f_i : R_+ \times R^n \rightarrow R^n$ is the nonlinear vector-valued function with $f_i(t, 0) \equiv 0$, $t \in R_+$, u_i is the control input, and $\tau \geq 0$ is the time delay. Switching signal $\sigma : R_+ \rightarrow \{1, 2, \dots, m\}$, which is represented by $\{\sigma_k\}$ according to $(t_{k-1}, t_k] \rightarrow \sigma_k \in \{1, 2, \dots, m\}$, is a piecewise constant function, the time sequence $\{t_k\}$ satisfies

$$t_1 < t_2 < \dots < t_k < \dots, \quad \lim_{k \rightarrow \infty} t_k = \infty, \quad (2.3)$$

where $t_1 > t_0 \geq 0$. $\Gamma \in R^{n \times n}$ is a constant inner coupling matrix between nodes and $D^r = (D_{ij}^r)_{N \times N}$ ($r = 1, \dots, m$) are the switched coupling configuration matrices, where D_{ij}^r represents the coupling strength, that is, if there is a connection from node i to node j ($j \neq i$), then $D_{ij}^r \neq 0$; otherwise, $D_{ij}^r = 0$ ($j \neq i$), and the diagonal elements of matrix D^r are defined by

$$D_{ii}^r = - \sum_{\substack{j=1 \\ j \neq i}}^N D_{ij}^r, \quad i = 1, 2, \dots, N, \quad r = 1, 2, \dots, m. \quad (2.4)$$

In this case, the system (2.1) can be rewritten as

$$\dot{x}_i = A_i x_i + f_i(t, x_i) + \sum_{j=1}^N D_{ij}^{\sigma_k} \Gamma x_j(t - \tau) + B_i u_i, \quad t \in (t_{k-1}, t_k], k = 1, 2, \dots, i = 1, 2, \dots, N, \quad (2.5)$$

which implies that the network is switching between m different coupling modes.

The set of candidate controllers $u_i = u_{i1} + u_{i2}$ for the network (2.1) are assumed to be of the form

$$\begin{cases} u_{i1}(t) = \sum_{k=1}^{\infty} E_i x_i(t) l_k(t), \\ u_{i2}(t) = \sum_{k=1}^{\infty} C_{ik} x_i(t) \delta(t - t_k), \end{cases} \quad i = 1, 2, \dots, N, \quad (2.6)$$

where E_i and C_{ik} are constant matrices with appropriate dimensions, $\delta(\cdot)$ is the Dirac impulse function, and $l_k(t)$ is given by

$$l_k(t) = \begin{cases} 1, & t_{k-1} < t \leq t_k, \\ 0, & \text{otherwise,} \end{cases}$$

with discontinuity points (2.3).

Clearly, from (2.6) we have

$$u_{i1}(t) = E_i x_i(t), \quad t \in (t_{k-1}, t_k], k = 1, 2, \dots, i = 1, \dots, N.$$

On the other hand, $u_{i2} = 0$ as $t \neq t_k$, from (2.5) and (2.6) we have

$$x_i(t_k + h) - x_i(t_k) = \int_{t_k}^{t_k+h} \left[A_i x_i(s) + f_i(s, x_i(s)) + B_i u_i(s) + \sum_{j=1}^N D_{ij}^{\sigma_k} \Gamma x_j(s - \tau) \right] ds, \quad i = 1, 2, \dots, N,$$

where $h > 0$ is sufficiently small, as $h \rightarrow 0^+$, which reduces to

$$\Delta x_i(t)|_{t_k} = x_i(t_k^+) - x_i(t_k) = B_i C_{ik} x_i(t_k), \quad (2.7)$$

where $x_i(t_k^+) = \lim_{h \rightarrow 0^+} x_i(t_k + h)$, and, for simplicity, it is assumed that $x_i(t_k) = x_i(t_k^-) = \lim_{h \rightarrow 0^+} x_i(t_k - h)$. This implies that the controller $u_i(t)$ has the effect of suddenly changing the state of system (2.1) at the points t_k .

From the hybrid impulsive and switching control (2.6), it can be seen that, one has more flexible design strategy and more choices, such as the switching times t_k , the impulsive gains C_{ik} , the switching gains E_i , to achieve control purpose than the single continuous control, discrete control, impulsive control, and switching control does, separately.

Accordingly, under hybrid control (2.6), the closed-loop system of (2.1) becomes

$$\begin{cases} \dot{x}_i = \tilde{A}_i x_i(t) + f_i(t, x_i(t)) + \sum_{j=1}^N D_{ij}^{\sigma_k} \Gamma x_j(t - \tau), & t \in (t_{k-1}, t_k] \\ \Delta x_i = \tilde{C}_{ik} x_i, & t = t_k, \quad k = 1, 2, \dots, \\ x_i(t_0^+) = x_{i0}, & i = 1, 2, \dots, N, \end{cases} \quad (2.8)$$

where $\tilde{A}_i = A_i + B_i E_i$, $\tilde{C}_{ik} = B_i C_{ik}$, $x_{i0} = \phi_i(t_0)$, $\{t_k\}$ satisfies (2.3), and Δx_i is given by (2.7). For convenience, let $t_0 = 0$.

3 STABILITY OF COMPLEX DYNAMICAL NETWORKS WITH SWITCHING TOPOLOGY AND COUPLING DELAYS

The following lemmas are need to facilitate the development of the subsequent results of this paper.

Lemma 3.1 *If $x, y \in R^h$, $a > 0$ is a constant, then*

$$-ax^\top x + x^\top y \leq -\frac{a}{2}x^\top x + \frac{1}{2a}y^\top y.$$

Lemma 3.2 *If $A = (a_{ij})_{m \times n} \in R^{m \times n}$, $x \in R^n$, then*

$$x^\top A^\top A x \leq \|A\|_F^2 x^\top x,$$

where

$$\|A\|_F := \left\{ \sum_{i=1}^m \sum_{j=1}^n a_{ij}^2 \right\}^{\frac{1}{2}}.$$

Lemma 3.3 [9] *For any $x \in R^n$, if $P \in R^{n \times n}$ is a positive definite matrix, $Q \in R^{n \times n}$ is an symmetric matrix, then*

$$\lambda_{\min}(P^{-1}Q)x^\top P x \leq x^\top Q x \leq \lambda_{\max}(P^{-1}Q)x^\top P x.$$

Lemma 3.4 [13] *Let $v(t)$ be a continuous function with $v(t) \geq 0$ for $t \geq t_0$. If*

$$v'(t) \leq -av(t) + bv(t - \tau), \quad t \geq t_0$$

with the initial condition $v(t) = \phi(t)$, $t \in [t_0 - \tau, t_0]$, where $\phi(t)$ is piecewise continuous, a and b are positive constants with $a > b > 0$, then

$$v(t) \leq v(t_0)e^{-\lambda(t-t_0)}, \quad t \geq t_0,$$

where λ is a positive solution of the equation $\lambda - a + be^{\lambda\tau} \leq 0$.

In the subsequent discussion, the concept of “average dwell-time” introduced by Hespanha and Morse [8] will be used. That is, for each switching signal σ and each $t \geq t_0 \geq 0$, let $N_\sigma(t_0, t)$ denote the number of discontinuities of σ over the interval $[t_0, t)$. For given $N_0, \tau_a > 0$, let $S_a[\tau_a, N_0]$ denote the set of all switching signals satisfying $N_\sigma(t_0, t) \leq N_0 + \frac{(t-t_0)}{\tau_a}$. The constant τ_a is called the “average dwell-time” and N_0 the “chatter bound”. This implies that, for a given switching signal $\sigma \in S_a[\tau_a, N_0]$ over $[t_0, t)$, there may exist some consecutive discontinuities with interval separated by less than τ_a , but the average interval between consecutive discontinuities is no less than τ_a .

We now consider the asymptotic properties of the controlled hybrid complex dynamic network (2.8). For system (2.8), assume that, for $t \in R_+, x \in R^n, i = 1, 2, \dots, N$, there exist continuous functions $\varphi_i(t) \geq 0$, constants $\lambda_i > 0$, and positive definite matrices P_i , such that

$$f_i^\top(t, x)P_i x \leq \varphi_i(t)x^\top P_i x \quad (3.1)$$

and

$$P_i \tilde{A}_i + \tilde{A}_i^\top P_i + 2\varphi_i(t)P_i \leq -\lambda_i I_i. \quad (3.2)$$

Remark 3.1 For nonlinear function $f_i(t, x)$, the inequality (3.1) is less conservative than the Lipschitz condition, see Remark 3.1 in [12]. It is well known that if we assume that the matrix pair (A_i, B_i) ($i = 1, 2, \dots, N$) are controllable, then there exist gain matrices E_i such that $\text{Re}\lambda(A_i - B_i E_i) < 0$, which implies that there must exist a unique symmetric positive definite matrix P_i subject to (3.2).

Furthermore, for convenience, define the following:

$$\left\{ \begin{array}{l} a_r = \max_{1 \leq i \leq N} \left\{ \frac{1}{2} \left(-\lambda_i + \sum_{j=1}^N \frac{1}{\tilde{\lambda}_j} \|P_i D_{ij}^r \Gamma\|_F^2 \right) \right\}, \\ b_r = \max_{1 \leq i \leq N} \left\{ \frac{1}{2} \left(N \tilde{\lambda}_i + \sum_{j=1}^N \frac{1}{\tilde{\lambda}_j} \|P_j \Gamma D_{(j)}^r\|_F^2 \right) \right\}, \\ \lambda_m = \min_{1 \leq i \leq N} \left\{ \lambda_{mi}(P_i) \right\}^{\frac{1}{2}}, \lambda_M = \max_{1 \leq i \leq N} \left\{ \lambda_{Mi}(P_i) \right\}^{\frac{1}{2}}, \varepsilon = \min_{1 \leq r \leq m} \left\{ \varepsilon_r \right\}, \\ \beta_k = \max_{1 \leq i \leq N} \lambda_{\max}[P_i^{-1}(I + \tilde{C}_{ik})^\top P_i (I + \tilde{C}_{ik})], D_{(i)}^r = (D_{i1}^r I_n, \dots, D_{iN}^r I_n) \end{array} \right. \quad (3.3)$$

in which, $r = 1, 2, \dots, m$, $\lambda_{mi}(P_i)$ and $\lambda_{Mi}(P_i)$ are the minimum and maximum eigenvalues of the positive definite matrix P_i , respectively, $\tilde{\lambda}_i$ and ε_r are to be determined in Theorem 3.1.

Theorem 3.1 Assume that (3.1) and (3.2) hold, and for $r = 1, 2, \dots, m, i = 1, 2, \dots, N$, there exist constants $\tilde{\lambda}_i > 0$ and $\varepsilon_r > 0$ such that

$$a_r + \varepsilon_r \lambda_m^2 + b_r \exp(\varepsilon_r \tau) \leq 0, \quad (3.4)$$

where a_r, b_r and λ_m are defined by (3.3).

- (i) If $\beta_k \leq 1, k = 1, 2, \dots$, then the trivial solution of network (2.8) is globally exponentially stable, where β_k is given by (3.3).
- (ii) If there exists a constant $0 < \alpha < \varepsilon_r, r = 1, 2, \dots, m$, such that

$$\ln \beta_k - \alpha(t_k - t_{k-1}) \leq 0, \quad k = 1, 2, \dots, \quad (3.5)$$

then the conclusion of (i) holds.

(iii) If $\beta_k \leq \beta$ with $\beta > 1$, and

$$\frac{\ln \beta}{\tau_a} - \varepsilon < 0, \quad (3.6)$$

then the conclusion of (i) holds for any switching signal $\sigma = \{\sigma_k\} \in S_a[\tau_a, N_0]$, where $N_0, \tau_a > 0$ are given constants satisfying $k-1 \leq N_0 + \frac{t-t_0}{\tau_a}$ for any $t \in (t_{k-1}, t_k]$, $k = 1, 2, \dots$, and ε is given by (3.3). Specifically, if $t_k - t_{k-1} \geq \delta > 0$, $k = 1, 2, \dots$, and the average dwell time τ_a in (3.6) is replaced with δ , then the conclusion of (i) holds for arbitrary switching.

Proof. Construct the following Lyapunov function

$$v(x) = \sum_{i=1}^N x_i^\top P_i x_i, \quad (3.7)$$

where P_i are positive-definite matrices satisfying (3.1) and (3.2). Clearly,

$$\lambda_m^2 \|x\|^2 \leq v(x) \leq \lambda_M^2 \|x\|^2, \quad (3.8)$$

where $x \in R^{nN}$, λ_m and λ_M are given by (3.3).

For $t \in (t_{k-1}, t_k]$, the total derivative of $v(x(t))$ with respect to (2.8) is

$$\begin{aligned} \dot{v}(x(t)) \Big|_{(2.8)} &= \sum_{i=1}^N \left\{ \dot{x}_i^\top P_i x_i + x_i^\top P_i \dot{x}_i \right\} \\ &= \sum_{i=1}^N \left\{ x_i^\top (\tilde{A}_i^\top P_i + P_i \tilde{A}_i) x_i + 2f_i^\top(t, x_i) P_i x_i + x_i^\top P_i \sum_{j=1}^N D_{ij}^{\sigma_k} \Gamma x_j(t - \tau) \right. \\ &\quad \left. + \sum_{j=1}^N x_j^\top(t - \tau) \Gamma^\top D_{ij}^{\sigma_k} P_i x_i \right\} \\ &\leq \sum_{i=1}^N \left\{ x_i^\top \left[\tilde{A}_i^\top P_i + P_i \tilde{A}_i + 2\varphi_i(t) P_i \right] x_i + x_i^\top P_i \sum_{j=1}^N D_{ij}^{\sigma_k} \Gamma x_j(t - \tau) \right. \\ &\quad \left. + \sum_{j=1}^N x_j^\top(t - \tau) \Gamma^\top D_{ij}^{\sigma_k} P_i x_i \right\} \\ &\leq \sum_{i=1}^N \left\{ -\lambda_i x_i^\top x_i + x_i^\top P_i \sum_{j=1}^N D_{ij}^{\sigma_k} \Gamma x_j(t - \tau) + \sum_{j=1}^N \left[x_j^\top(t - \tau) \Gamma^\top D_{ij}^{\sigma_k} P_i x_i \right. \right. \\ &\quad \left. \left. - \tilde{\lambda}_j x_j^\top(t - \tau) x_j(t - \tau) \right] + \sum_{j=1}^N \tilde{\lambda}_j x_j^\top(t - \tau) x_j(t - \tau) \right\}, \end{aligned}$$

where constants $\tilde{\lambda}_j > 0$ satisfy (3.4). It follows from Lemma 3.1 that

$$\begin{aligned}
\dot{v}(x(t)) \Big|_{(2.8)} &\leq \sum_{i=1}^N \left\{ -\frac{\lambda_i}{2} x_i^\top x_i + \frac{1}{2\lambda_i} \left[P_i \sum_{j=1}^N D_{ij}^{\sigma_k} \Gamma x_j(t-\tau) \right]^\top \left[P_i \sum_{j=1}^N D_{ij}^{\sigma_k} \Gamma x_j(t-\tau) \right] \right. \\
&\quad + \sum_{j=1}^N \left[-\frac{\tilde{\lambda}_j}{2} x_j^\top(t-\tau) x_j(t-\tau) + \frac{1}{2\tilde{\lambda}_j} (\Gamma^\top D_{ij}^{\sigma_k} P_i x_i)^\top (\Gamma^\top D_{ij}^{\sigma_k} P_i x_i) \right] \\
&\quad \left. + \sum_{j=1}^N \tilde{\lambda}_j x_j^\top(t-\tau) x_j(t-\tau) \right\} \\
&= \sum_{i=1}^N \left\{ -\frac{\lambda_i}{2} x_i^\top x_i + \frac{1}{2\lambda_i} x^\top(t-\tau) (D_{(i)}^{\sigma_k})^\top \Gamma^\top P_i P_i \Gamma D_{(i)}^{\sigma_k} x(t-\tau) \right. \\
&\quad \left. + \sum_{j=1}^N \left[\frac{\tilde{\lambda}_j}{2} x_j^\top(t-\tau) x_j(t-\tau) + \frac{1}{2\tilde{\lambda}_j} x_i^\top P_i D_{ij}^{\sigma_k} \Gamma \Gamma^\top D_{ij}^{\sigma_k} P_i x_i \right] \right\},
\end{aligned}$$

where $x(t-\tau) = (x_1^\top(t-\tau), \dots, x_N^\top(t-\tau))^\top$, $D_{(i)}^{\sigma_k} = (D_{i1}^{\sigma_k} I_n, \dots, D_{iN}^{\sigma_k} I_n)$. By Lemma 3.2, it follows that

$$\begin{aligned}
\dot{v}(x(t)) \Big|_{(2.8)} &\leq \sum_{i=1}^N \left\{ -\frac{\lambda_i}{2} x_i^\top x_i + \frac{1}{2\lambda_i} \|P_i \Gamma D_{(i)}^{\sigma_k}\|_F^2 x^\top(t-\tau) x(t-\tau) \right. \\
&\quad \left. + \sum_{j=1}^N \left[\frac{\tilde{\lambda}_j}{2} x_j^\top(t-\tau) x_j(t-\tau) + \frac{1}{2\tilde{\lambda}_j} \|P_i D_{ij}^{\sigma_k} \Gamma\|_F^2 x_i^\top x_i \right] \right\} \\
&= \sum_{i=1}^N \left\{ -\frac{\lambda_i}{2} x_i^\top x_i + \frac{1}{2\lambda_i} \|P_i \Gamma D_{(i)}^{\sigma_k}\|_F^2 \sum_{j=1}^N x_j^\top(t-\tau) x_j(t-\tau) \right. \\
&\quad \left. + \sum_{j=1}^N \left[\frac{\tilde{\lambda}_j}{2} x_j^\top(t-\tau) x_j(t-\tau) + \frac{1}{2\tilde{\lambda}_j} \|P_i D_{ij}^{\sigma_k} \Gamma\|_F^2 x_i^\top x_i \right] \right\} \\
&= \sum_{i=1}^N \left(-\frac{\lambda_i}{2} + \sum_{j=1}^N \frac{1}{2\tilde{\lambda}_j} \|P_i D_{ij}^{\sigma_k} \Gamma\|_F^2 \right) x_i^\top x_i + \sum_{i=1}^N \left(\frac{N\tilde{\lambda}_i}{2} + \sum_{j=1}^N \frac{1}{2\tilde{\lambda}_j} \|P_j \Gamma D_{(j)}^{\sigma_k}\|_F^2 \right) x_i^\top(t-\tau) x_i(t-\tau) \\
&\leq a_{\sigma_k} \|x(t)\|^2 + b_{\sigma_k} \|x(t-\tau)\|^2, \quad t \in (t_{k-1}, t_k],
\end{aligned}$$

where a_{σ_k} and b_{σ_k} are defined in (3.3), which yields from (3.8) that

$$\dot{v}(x(t)) \Big|_{(2.8)} \leq \frac{1}{\lambda_m^2} [a_{\sigma_k} v(t) + b_{\sigma_k} v(t-\tau)], \quad t \in (t_{k-1}, t_k] \quad (3.9)$$

where $v(t) := v(x(t))$. Using comparison equation method Lemma 3.4, from (3.4), inequality (3.9) leads to

$$\dot{v}(x(t)) \Big|_{(2.8)} \leq v(t_{k-1}^+) e^{-\varepsilon_{\sigma_k}(t-t_{k-1})}, \quad t \in (t_{k-1}, t_k]. \quad (3.10)$$

On the other hand, it follows from (2.8), (3.7) and Lemma 3.3 that

$$\begin{aligned}
v(t_k^+) &= \sum_{i=1}^N x_i^\top(t_k^+) P_i x_i(t_k^+) \\
&= \sum_{i=1}^N [(I + \tilde{C}_{ik}) x_i(t_k)]^\top P_i [(I + \tilde{C}_{ik}) x_i(t_k)] \\
&\leq \sum_{i=1}^N \lambda_{\max}[P_i^{-1} (I + \tilde{C}_{ik})^\top P_i (I + \tilde{C}_{ik})] x_i^\top(t_k) P_i x_i(t_k) \\
&\leq \beta_k v(t_k), \quad k = 1, 2, \dots,
\end{aligned} \tag{3.11}$$

where β_k are defined by (3.3).

The following can be reduced from (3.10) and (3.11). For $t \in (t_0, t_1]$,

$$v(t) \leq v(t_0) e^{-\varepsilon_{\sigma_1}(t-t_0)},$$

which leads to

$$v(t_1) \leq v(t_0) e^{-\varepsilon_{\sigma_1}(t_1-t_0)},$$

and

$$v(t_1^+) \leq \beta_1 v(t_1) \leq \beta_1 v(t_0) e^{-\varepsilon_{\sigma_1}(t_1-t_0)}.$$

Similarly, for $t \in (t_1, t_2]$,

$$v(t) \leq v(t_1^+) e^{-\varepsilon_{\sigma_2}(t-t_1)} \leq \beta_1 v(t_0) e^{-\varepsilon_{\sigma_1}(t_1-t_0) - \varepsilon_{\sigma_2}(t-t_1)}.$$

In general, for $t \in (t_{k-1}, t_k]$,

$$\begin{aligned}
v(t) &\leq v(t_0) \beta_1 \cdots \beta_{k-1} \exp \left[-\varepsilon_{\sigma_1}(t_1 - t_0) - \varepsilon_{\sigma_2}(t_2 - t_1) - \cdots - \varepsilon_{\sigma_{k-1}}(t_{k-1} - t_{k-2}) - \varepsilon_{\sigma_k}(t - t_{k-1}) \right] \\
&\leq v(t_0) \beta_1 \cdots \beta_{k-1} e^{-\varepsilon(t-t_0)}, \quad t \in (t_{k-1}, t_k]
\end{aligned} \tag{3.12}$$

where ε is defined by (3.3).

Case (i). Obviously, it is easy to get from (3.12) and $\beta_k \leq 1$ that

$$v(t) \leq v(t_0) e^{-\varepsilon(t-t_0)}, \quad t \geq t_0.$$

Moreover, by (3.7) and (3.8), it leads to

$$\|x(t)\| \leq \frac{\lambda_M}{\lambda_m} \|x(t_0)\| \exp \left[-\frac{\varepsilon}{2}(t - t_0) \right], \quad t \geq t_0,$$

which immediately implies that the trivial solution of closed-loop network (2.8) is globally exponentially stable.

Case (ii). Since $0 < \alpha < \varepsilon_r$, it follows from (3.12) and (3.5) that

$$\begin{aligned}
v(t) &\leq v(t_0) \beta_1 \cdots \beta_{k-1} e^{-\alpha(t-t_0)} e^{-(\varepsilon-\alpha)(t-t_0)} \\
&\leq v(t_0) \beta_1 \cdots \beta_{k-1} e^{-\alpha(t_{k-1}-t_0)} e^{-(\varepsilon-\alpha)(t-t_0)} \\
&= v(t_0) \beta_1 e^{-\alpha(t_1-t_0)} \cdots \beta_{k-1} e^{-\alpha(t_{k-1}-t_{k-2})} e^{-(\varepsilon-\alpha)(t-t_0)} \\
&\leq v(t_0) e^{-(\varepsilon-\alpha)(t-t_0)}, \quad t \in (t_{k-1}, t_k]
\end{aligned}$$

namely,

$$v(t) \leq v(t_0)e^{-(\varepsilon-\alpha)(t-t_0)}, \quad t \geq t_0,$$

furthermore,

$$\|x(t)\| \leq \frac{\lambda_M}{\lambda_m} \|x(t_0)\| \exp \left[-\frac{1}{2}(\varepsilon - \alpha)(t - t_0) \right], \quad t \geq t_0,$$

which implies the conclusion of (i) holds.

Case (iii). Since $\beta_k \leq \beta, \beta > 1$, and $k - 1 \leq N_0 + \frac{(t-t_0)}{\tau_a}$ for $t \in (t_{k-1}, t_k]$, by (3.12), it arrives at

$$\begin{aligned} v(t) &\leq v(t_0)\beta^{k-1}e^{-\varepsilon(t-t_0)} \\ &\leq v(t_0)\beta^{N_0}\beta^{\frac{t-t_0}{\tau_a}}e^{-\varepsilon(t-t_0)} \\ &= v(t_0)\beta^{N_0} \exp \left[\left(\frac{\ln \beta}{\tau_a} - \varepsilon \right) (t - t_0) \right], \quad t \in (t_{k-1}, t_k] \end{aligned}$$

that is,

$$v(t) \leq v(t_0)\beta^{N_0} \exp \left[\left(\frac{\ln \beta}{\tau_a} - \varepsilon \right) (t - t_0) \right], \quad t \geq t_0$$

or

$$\|x(t)\| \leq \frac{\lambda_M}{\lambda_m} \beta^{\frac{N_0}{2}} \|x(t_0)\| \exp \left[\frac{1}{2} \left(\frac{\ln \beta}{\tau_a} - \varepsilon \right) (t - t_0) \right], \quad t \geq t_0$$

which implies from (3.6) that the conclusion of (ii) holds.

For the special case, $t_k - t_{k-1} \geq \delta > 0$, it follows from $\beta_k \leq \beta, \beta > 1$, and (3.12) that, for $t \in (t_{k-1}, t_k]$,

$$\begin{aligned} v(t) &\leq v(t_0)\beta_1 \cdots \beta_{k-1}e^{-\varepsilon(t-t_0)} \\ &\leq v(t_0)e^{\frac{\ln \beta}{\delta}(t_1-t_0)} \cdots e^{\frac{\ln \beta}{\delta}(t_{k-1}-t_{k-2})}e^{-\varepsilon(t-t_0)} \\ &= v(t_0)e^{\frac{\ln \beta}{\delta}(t_{k-1}-t_0)}e^{-\varepsilon(t-t_0)} \\ &\leq v(t_0) \exp \left[\left(\frac{\ln \beta}{\delta} - \varepsilon \right) (t - t_0) \right], \quad t \in (t_{k-1}, t_k]. \end{aligned}$$

Namely,

$$v(t) \leq v(t_0) \exp \left[\left(\frac{\ln \beta}{\delta} - \varepsilon \right) (t - t_0) \right], \quad t \geq t_0$$

which leads to

$$\|x(t)\| \leq \frac{\lambda_M}{\lambda_m} \|x(t_0)\| \exp \left[\frac{1}{2} \left(\frac{\ln \beta}{\tau_a} - \varepsilon \right) (t - t_0) \right], \quad t \geq t_0.$$

Accordingly, $\frac{\ln \beta}{\delta} - \varepsilon < 0$ implies the conclusion holds. This completes the proof. \diamond

4 ROBUST H_∞ CONTROL OF COMPLEX DYNAMICAL NETWORKS WITH SWITCHING TOPOLOGY AND COUPLING DELAYS

In this section, we discuss the robust H_∞ control of hybrid dynamical networks with coupling delays.

Consider the following uncertain dynamical network with switching topology and coupling delays

$$\begin{cases} \dot{x}_i = A_i x_i(t) + f_i(t, x_i(t)) + \sum_{j=1}^N D_{ij}^\sigma \Gamma x_j(t - \tau) + B_i u_i + G_i w_i, \\ z_i = H_i x_i, \quad i = 1, 2, \dots, N, \end{cases} \quad (4.1)$$

with the initial value condition (2.2), where $w_i \in R^p$ is the disturbance input, $z_i \in R^q$ is the controlled output, G_i and H_i are known matrices of appropriate dimensions. The rest of the notations are introduced in Section 2.

For the disturbance signal $w_i(\cdot) \in R^p$, define

$$\|w\|_T = \left[\int_0^T \|w(t)\|^2 dt \right]^{\frac{1}{2}} = \left[\int_0^T \sum_{i=1}^N w_i(t)^\top w_i(t) dt \right]^{\frac{1}{2}}, \quad (4.2)$$

where $T > 0$ is an arbitrary constant and $w = \text{col}(w_1, w_2, \dots, w_N)$. Then, w_i is said to belong to $L_2[0, T]$, if $\|w\|_T < \infty$. Throughout the paper, it is assumed that the disturbance input $w_i \in L_2[0, T]$.

Now, one is in a position to consider the robust H_∞ control problem for system (4.1). When the controller (2.6) is used, the closed-loop system of (4.1) becomes

$$\begin{cases} \dot{x}_i = \tilde{A}_i x_i(t) + f_i(t, x_i(t)) + \sum_{j=1}^N D_{ij}^{\sigma_k} \Gamma x_j(t - \tau) + G_i w_i, & t \in (t_{k-1}, t_k], \quad k = 1, 2, \dots, \\ z_i = H_i x_i, & i = 1, 2, \dots, N, \\ \Delta x_i = \tilde{C}_{ik} x_i, & t = t_k \\ x_i(t_0^+) = x_{i0}, \end{cases} \quad (4.3)$$

where $\tilde{A}_i = A_i + B_i E_i$ and $\tilde{C}_{ik} = B_i C_{ik}$, $x_{i0} = \phi_i(t_0)$, $\{t_k\}$ satisfies (2.3), and Δx_i is given by (2.7). For convenience, let $t_0 = 0$.

With the above preliminaries, the robust H_∞ control problem to be addressed here can be formulated as a problem of using an impulsive controller to achieve the following objectives:

- (i) The closed-loop system (4.3) is exponentially stable when $w = 0$.
- (ii) Under the zero-initial condition, the controlled output z satisfies

$$\|z\|_T \leq \gamma \|w\|_T,$$

for any nonzero $w_i(\cdot) \in L_2[0, T]$, where $\gamma > 0$ is a prescribed scalar and $z = \text{col}(z_1, z_2, \dots, z_N)$.

The above conditions are also called robust H_∞ criteria for the closed-loop system (4.3).

In the following discussion, define $\Phi_r(P_i, t)$ and $\Psi_r(P_i, t)$ as

$$\Phi_r(P_i, t) = \begin{bmatrix} \psi_i(t) & P_i D_{i1}^r \Gamma & \cdots & P_i D_{iN}^r \Gamma \\ \Gamma^\top D_{i1}^r P_i & 0 & \cdots & 0 \\ \Gamma^\top D_{i2}^r P_i & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma^\top D_{iN}^r P_i & 0 & \cdots & 0 \end{bmatrix}, \quad (4.4)$$

$$\Psi_r(P_i, t) = \Phi_r(P_i, t) + \text{diag}(p_r P_i, -q_r P_1, \dots, -q_r P_N), \quad (4.5)$$

$$\Lambda_r(P_i, t) = \begin{bmatrix} \psi_i(t) + \frac{1}{\gamma} H_i^\top H_i & P_i D_{i1}^r \Gamma & \cdots & P_i D_{iN}^r \Gamma & P_i G_i \\ \Gamma^\top D_{i1}^r P_i & 0 & \cdots & 0 & 0 \\ \Gamma^\top D_{i2}^r P_i & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \Gamma^\top D_{iN}^r P_i & 0 & \cdots & 0 & 0 \\ G_i^\top P_i & 0 & \cdots & 0 & -\gamma I \end{bmatrix}, \quad (4.6)$$

where p_r and q_r are constants to be determined,

$$\psi_i(t) = (\tilde{A}_i)^\top P_i + P_i \tilde{A}_i + 2\varphi_i(t) P_i,$$

$i = 1, 2, \dots, N$, $r = 1, 2, \dots, m$, P_i are positive definite matrices, $\varphi_i(t)$ are continuous functions on R_+ , which satisfy (3.1).

Theorem 4.1 Assume that for $r = 1, 2, \dots, m$ and $i = 1, 2, \dots, N$, there exist positive-definite matrices P_i and constants $p_r, q_r > 0$, such that (3.1) holds, and

$$\Psi_r(P_i, t) \leq 0, \quad \Lambda_r(P_i, t) \leq 0, \quad (4.7)$$

where $\Psi_r(P_i, t)$ and $\Lambda_r(P_i, t)$ are defined by (4.5) and (4.6), respectively.

If $\beta_k \leq 1$ and ϵ_r are positive solutions of the inequality $\epsilon_r - p_r + q_r N \exp(\epsilon_r \tau) \leq 0$, then the closed-loop system (4.3) has the property of robust H_∞ criteria (i) and (ii).

Proof. Construct a Lyapunov function in the form of

$$v(x) = \sum_{i=1}^N x_i^\top P_i x_i, \quad (4.8)$$

where P_i are positive-definite matrices satisfying (3.1) and (4.7), and let $v(t) := v(x(t))$. For $t \in (t_{k-1}, t_k]$, the total derivative of $v(x(t))$ with respect to (4.3) is

$$\begin{aligned} \dot{v}(x(t)) \Big|_{(4.3)} &= \sum_{i=1}^N \left\{ \dot{x}_i^\top P_i x_i + x_i^\top P_i \dot{x}_i \right\} \Big|_{(4.3)} \\ &= \sum_{i=1}^N \left\{ x_i^\top (\tilde{A}_i^\top P_i + P_i \tilde{A}_i) x_i + 2f_i^\top(t, x_i) P_i x_i + w_i^\top G_i^\top P_i x_i + x_i^\top P_i G_i w_i \right. \\ &\quad \left. + \sum_{j=1}^N \left[x_i^\top P_i D_{ij}^{\sigma_k} \Gamma x_j(t - \tau) + x_j^\top(t - \tau) \Gamma^\top D_{ij}^{\sigma_k} P_i x_i \right] \right\} \\ &\leq \sum_{i=1}^N \left\{ x_i^\top \left[\tilde{A}_i^\top P_i + P_i \tilde{A}_i + 2\varphi_i(t) P_i \right] x_i + w_i^\top G_i^\top P_i x_i + x_i^\top P_i G_i w_i \right. \\ &\quad \left. + \sum_{j=1}^N \left[x_i^\top P_i D_{ij}^{\sigma_k} \Gamma x_j(t - \tau) + x_j^\top(t - \tau) \Gamma^\top D_{ij}^{\sigma_k} P_i x_i \right] \right\} \\ &= \sum_{i=1}^N \left\{ y_i^\top \Phi_{\sigma_k}(P_i, t) y_i + w_i^\top G_i^\top P_i x_i + x_i^\top P_i G_i w_i \right\}, \quad t \in (t_{k-1}, t_k] \end{aligned} \quad (4.9)$$

where $y_i(t) = \text{col}(x_i(t), x_1(t - \tau), \dots, x_N(t - \tau))$, $\Phi_{\sigma_k}(P_i, t)$ is given by (4.4).

When $w_i = 0$, from (4.5) and (4.7), we obtain

$$\begin{aligned} \dot{v}(x(t)) &\leq \sum_{i=1}^N \left\{ -p_{\sigma_k} x_i^\top P_i x_i + q_{\sigma_k} \sum_{j=1}^N \left[x_j^\top(t - \tau) P_j x_j(t - \tau) \right] \right\} \\ &= -p_{\sigma_k} v(t) + q_{\sigma_k} N v(t - \tau), \quad t \in (t_{k-1}, t_k], \quad k = 1, 2, \dots \end{aligned} \quad (4.10)$$

Following a similar argument as that given in the proof of Theorem 3.1, one can further prove that the closed-loop system (4.3) is globally exponentially asymptotically stable when $w = 0$.

Suppose that the initial condition is zero. For any given $T \in (t_{k-1}, t_k]$, we introduce

$$J = \int_0^T \left(\frac{1}{\gamma} z^\top z - \gamma w^\top w \right) dt. \quad (4.11)$$

Since

$$\begin{aligned}
\int_0^T \dot{v}(t)dt &= \int_0^{t_1} \dot{v}(t)dt + \int_{t_1}^{t_2} \dot{v}(t)dt + \cdots + \int_{t_{k-2}}^{t_{k-1}} \dot{v}(t)dt + \int_{t_{k-1}}^T \dot{v}(t)dt \\
&= v(t_1) - v(0^+) + v(t_2) - v(t_1^+) + \cdots + v(t_{k-1}) - v(t_{k-2}^+) + v(T) - v(t_{k-1}^+) \\
&= v(t_1) - v(t_1^+) + \cdots + v(t_{k-2}) - v(t_{k-2}^+) + v(t_{k-1}) - v(t_{k-1}^+) + v(T) \\
&\geq \sum_{i=1}^{k-1} (1 - \beta_i)v(t_i) + v(T) \\
&\geq 0,
\end{aligned}$$

for any nonzero $w_i(\cdot) \in L_2[0, T]$,

$$J \leq \int_0^T \left[\frac{1}{\gamma} z^\top z - \gamma w^\top w + \dot{v} \right] dt.$$

Following (4.9), we have

$$\begin{aligned}
J &\leq \sum_{i=1}^N \int_0^T \left[\frac{1}{\gamma} x_i^\top(t) H_i^\top H_i x_i(t) - \gamma w_i^\top(t) w_i(t) + w_i^\top(t) G_i^\top P_i x_i(t) \right. \\
&\quad \left. + x_i^\top(t) P_i G_i w_i(t) + y_i^\top(t) \Phi_{\sigma_k}(P_i, t) y_i(t) \right] dt \\
&= \sum_{i=1}^N \int_0^T Y_i(t)^\top \Lambda_{\sigma_k}(P_i, t) Y_i(t) dt, \quad T \in (t_{k-1}, t_k], \tag{4.12}
\end{aligned}$$

where $Y_i(t) = \text{col}(x_i(t), x_1(t-\tau), \dots, x_N(t-\tau), w_i(t))$, $\Lambda_{\sigma_k}(P_i, t)$ is given by (4.6). It follows from (4.7) that

$$J \leq 0,$$

i.e.,

$$\|z\|_T^2 - \gamma^2 \|w\|_T^2 \leq 0,$$

which immediately reduces to $\|z\|_T \leq \gamma \|w\|_T$. The proof is thus completed. \diamond

In Theorem 4.1, the inequality (4.7) gives the connection between the network structure, parameters and controllers, which guarantees the property of robust H_∞ for the network with disturbance. Since there are many parameter choices in (4.7), such as the switching times t_k , the impulsive gains C_{ik} , the switching gains E_i , and matrices P_i , the inequality (4.7) can be easily satisfied.

5 NUMERICAL EXAMPLE

Example 1. In the following, Theorem 3.1 is illustrated by using Lorenz chaotic system as the dynamical node in networks (2.1). Consider every network (2.1) with ten nodes as shown in Fig.1. The chaotic system of node i is described by

$$\begin{cases} \dot{x}_{i1} = a(x_{i2} - x_{i1}) \\ \dot{x}_{i2} = cx_{i1} - x_{i1}x_{i3} - x_{i2} \\ \dot{x}_{i3} = x_{i1}x_{i2} - bx_{i3}, \end{cases} \tag{5.1}$$

where a , b and c are the parameters. If we choose $a = 10$, $b = \frac{8}{3}$, $c = 28$, it exhibits the chaotic behavior [18].

There are three typical network structures shown in Fig.1. Network (2.1) periodically switches from (a) to (b) then to (c) every 4 seconds, i.e. dwell-time $\tau_a = 4$. Let time delay $\tau = 3$, $B_i = I$, $\Gamma =$

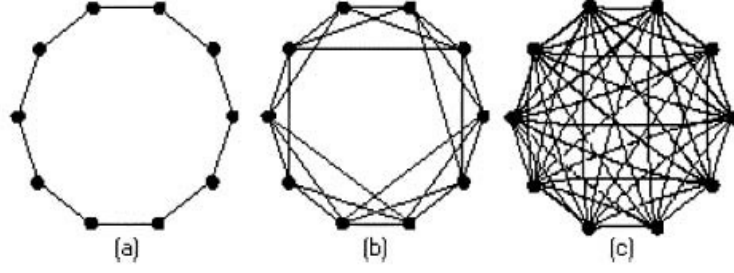


Figure 1: (a) Simplest nearest-neighbor coupling network. (b) Small-world network. (c) Fully connected network.

$\text{diag}\{0.88, 0.52, 0.65\}$. When there is a connection from node i to node j ($j \neq i$), then $D_{ij}^r = 1$; otherwise, $D_{ij}^r = 0$ ($j \neq i$) ($r = 1, 2, 3$). Note that for system (4.1) if take $P_i = I$, then in (3.1), $\varphi_i(t) = 0$. We can find control gain matrices as

$$\begin{aligned}
 E_1 &= \begin{pmatrix} -100 & -2 & 0 \\ 1 & -580 & 0 \\ -1 & 0 & -300 \end{pmatrix}, & E_2 &= \begin{pmatrix} -180 & 0 & 0 \\ 0 & -220 & 0 \\ 0 & 0 & -100 \end{pmatrix}, & E_3 &= \begin{pmatrix} -80 & 5 & 0 \\ -28 & -120 & 10 \\ 20 & 0 & -120 \end{pmatrix}, \\
 E_4 &= \begin{pmatrix} -92 & -1 & 0 \\ -10 & -135 & 8 \\ -12 & 0 & -90 \end{pmatrix}, & E_5 &= \begin{pmatrix} -87 & -41 & 0 \\ 0 & -121 & 0 \\ 0 & 0 & -99 \end{pmatrix}, & E_6 &= \begin{pmatrix} -98 & -21 & 0 \\ 15 & -138 & 0 \\ 0 & 0 & -110 \end{pmatrix}, \\
 E_7 &= \begin{pmatrix} -128 & -20 & 0 \\ 0 & -155 & 0 \\ 0 & 0 & -101 \end{pmatrix}, & E_8 &= \begin{pmatrix} -130 & 0 & 0 \\ 0 & -119 & 0 \\ 0 & 0 & -124 \end{pmatrix}, & E_9 &= \begin{pmatrix} -102 & -22 & 0 \\ 0 & -125 & 0 \\ 8 & 0 & -108 \end{pmatrix}, \\
 E_{10} &= \begin{pmatrix} -113 & -32 & 0 \\ 0 & -129 & 0 \\ 0 & 0 & -112 \end{pmatrix},
 \end{aligned}$$

and

$$\begin{aligned}
 C_{1k} &= \text{diag}\{0.02, -0.65, -0.75\}, & C_{2k} &= \text{diag}\{-0.2, -0.52, -0.80\}, & C_{3k} &= \text{diag}\{-0.91, -0.69, 0.01\}, \\
 C_{4k} &= \text{diag}\{-0.76, -0.62, -0.99\}, & C_{5k} &= \text{diag}\{-0.48, -0.68, -0.33\}, & C_{6k} &= \text{diag}\{-0.37, -0.91, -0.42\}, \\
 C_{7k} &= \text{diag}\{-0.55, 0.018, 0.01\}, & C_{8k} &= \text{diag}\{-0.86, 0.02, -0.90\}, & C_{9k} &= \text{diag}\{-0.6, -0.67, -0.47\}, \\
 C_{10k} &= \text{diag}\{0.01, -0.64, 0.01\}.
 \end{aligned}$$

Then, from (3.2) and (3.3), we have

$$\begin{aligned}
 \lambda_1 &= 150, & \lambda_2 &= 162, & \lambda_3 &= 170, & \lambda_4 &= 178, & \lambda_5 &= 185, \\
 \lambda_6 &= 192, & \lambda_7 &= 199, & \lambda_8 &= 206, & \lambda_9 &= 211, & \lambda_{10} &= 220. \\
 \beta_k &= \max_{1 \leq i \leq 10} \lambda_{\max}[P_i^{-1}(I + \tilde{C}_{ik})^\top P_i(I + \tilde{C}_{ik})] = 1.0404 = \beta > 1.
 \end{aligned}$$

For $r = 1, 2, 3$, $i = 1, 2, \dots, 10$, and $k = 1, 2, \dots$, take $\tilde{\lambda}_i = 1$, $\varepsilon_1 = 0.8$, $\varepsilon_2 = 0.7$, $\varepsilon_3 = 0.01$. It is easy to get that

$$a_1 = -70.5981; \quad b_1 = 5.2382; \quad a_2 = -58.9905; \quad b_2 = 6.1066; \quad a_3 = -8.9715; \quad b_3 = 8.5733;$$

Moreover,

$$a_1 + \varepsilon_1 \lambda_m^2 + b_1 \exp(\varepsilon_1 \tau) = -12.0565 < 0,$$

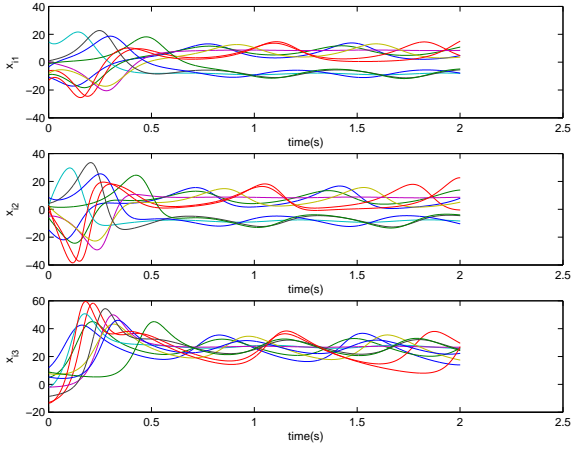


Figure 2: The states of each nodes in dynamical network (2.1) when $u_i = 0$

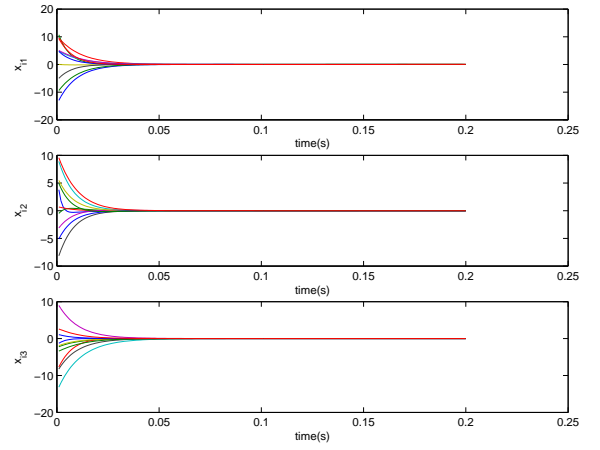


Figure 3: The trajectories of hybrid impulsive controlled network (2.8)

$$a_2 + \varepsilon_2 \lambda_m^2 + b_2 \exp(\varepsilon_2 \tau) = -8.4230 < 0,$$

$$a_3 + \varepsilon_3 \lambda_m^2 + b_3 \exp(\varepsilon_3 \tau) = -0.1271 < 0.$$

Since $\varepsilon = \min_{1 \leq r \leq 3} \{\varepsilon_r\} = 0.01$, $\frac{\ln \beta}{\tau_a} - \varepsilon = \frac{\ln 1.0404}{4} - 0.01 = -0.0001 < 0$ implies from Theorem 3.1 that the trivial solution of system (2.8) is globally exponentially asymptotically stable. Fig.2. shows that when $u_i = 0$, the hybrid complex dynamical network with coupling delays is unstable. The hybrid impulsive controlled network is asymptotically stable in Fig.3.

Example 2. Consider the hybrid complex dynamical network in Example 1 with $\tau_a = 0.4$, $\Gamma = I$, $G_i = \text{diag}\{0.5, 0.5, -0.5\}$ and

$$H_i = \begin{pmatrix} -12 & 5 & 0 \\ 0 & 1 & -20 \\ 2 & 0 & -10 \end{pmatrix}.$$

Other parameters are the same as Example 1. If we choose control gain matrices as

$$E_i = \begin{pmatrix} -900 & -20 & 0 \\ 1 & -2000 & 0 \\ -1 & 0 & -1000 \end{pmatrix},$$

and

$$C_{ik} = \text{diag}\{-0.58, -0.65, -0.75\},$$

for $r = 1, 2, 3$, $i = 1, 2, \dots, 10$, and $k = 1, 2, \dots$, there exist positive definite matrices $P_i = I$ and scalars $p_r = 10.2$, $q_r = 1$ and $\gamma = 10$ such that inequalities (4.7) are satisfied. It is easy to verify that $\beta_k < 1$ and $\varepsilon_r - p_r + q_r N \exp(\varepsilon_r \tau) = -0.1690 < 0$. Then the closed-loop system (4.3) has the property of robust H_∞ criteria (i) and (ii). Fig 4. and Fig 5. show the states and the controlled output of the closed-loop system with $w_i = \text{col}(1/3 \sin(\pi t - 1), 1/2 \cos(2\pi t + 2), 1/2 \sin(\pi t))$.

It can be seen that the trajectories of the controlled switching complex dynamical network (2.8) and (4.3) converge to zero after a very short period of time. And, if the impulsive switching interval is further reduced, then even less time is required for suppressing the network chaos and disturbance. The design process, and simulation results, indicate that, for the new complex dynamical networks with switching topology and coupling delays, the new hybrid impulsive and switching control strategy has many possible advantages, such as less time, less energy, and more flexible design, to achieve global stabilization and H_∞ robust stability.

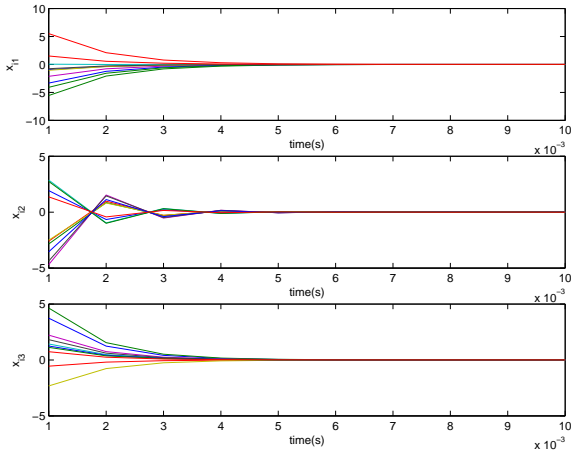


Figure 4: States of the closed-loop system

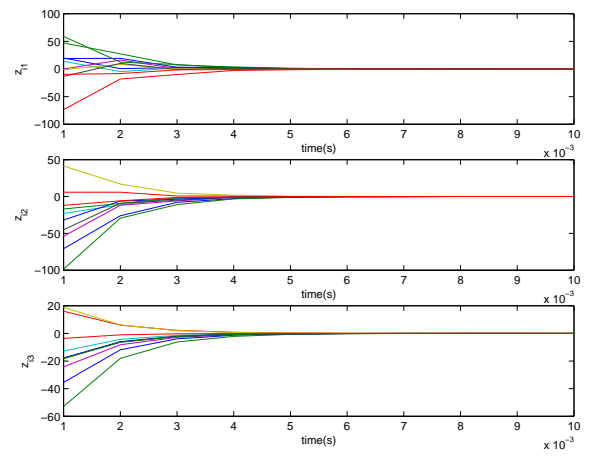


Figure 5: Controlled output of the closed-loop system

6 CONCLUSIONS

This paper has formulated a class of hybrid complex dynamical networks with coupling delays. The problem of stability and robust H_∞ control for the dynamical networks using hybrid impulsive control are taken into account for the first time. Some criteria of exponential stability and H_∞ criteria of the resulting closed-loop system are obtained. The examples of the dynamical networks have been provided to verify the effectiveness of the proposed approach.

References

- [1] Albert R, Jeong H, Barabási AL. Diameter of the world wide web. *Nature* 1999; **401**:130-131.
- [2] Albert R, Jeong H, Barabási AL. Error and attack tolerance of complex networks. *Nature* 2000; **406**:378-382.
- [3] Belykh IV, Belykh VN, Hasler M. Blinking model and synchronization in small-world networks with a time-varying coupling. *Physica D* 2004; **195**:188-206.
- [4] Baldi P, Frasconi P, Smyth P. *Modeling the Internet and the Web: Probabilistic Method and Algorithm*. Chichester: John Wiley & Sons Ltd., 2003.
- [5] Cohen R, Erez K, Ben-Avraham D, Havlin S. Resilience of the internet to random breakdowns. *Phys. Rev. Lett.* 2000; **85**(21):4626-4628.
- [6] Erdős P, Rényi A. On the evolution of random graphs. *Publ. Math. Inst. Hung. Acad. Sci.* 1960; **5**:17-60.
- [7] Faloutsos M, Faloutsos P, Faloutsos C. On power-law relationships of the internet topology. *Comput. Commun. Rev.* 1999; **29**:251-263.
- [8] Hespanha JP, Morse AS. Stability of switched systems with average dwell-time. *Proceedings of 38th IEEE Conference on Decision and Control*, Phoenix, AZ, December 1999, 2655-2660.
- [9] Huang L. *Linear Algebra in Systems and Control Theory*. Beijing: Science Press, 1984. 211-214.
- [10] Guan ZH, Chen G, Yu X, Qin Y. Robust decentralized stabilization for a class of large-scale time-delay uncertain impulsive dynamical systems. *Automatica* 2002; **38**(11):2075-2084.

- [11] Guan ZH, Liu YQ, Wen XC. Decentralized stabilization of singular and time-delay large scale control systems with impulsive solutions. *IEEE Trans. Automat. Contr.* 1995; **40**(8):1437-1441.
- [12] Guan ZH, Hill DJ, Shen XM. On hybrid impulsive and switching systems and application to nonlinear control. *IEEE Trans. Automat. Contr.* 2005; **50**(7):1058-1062.
- [13] Guan ZH, Chen G. On nonlinear delayed impulsive neural networks with interval uncertainties. *Neural Networks* 2010 (to appear).
- [14] Guan ZH, Liu ZW, Feng G, Wang YW. Synchronization of complex dynamical networks with time-varying delays via impulsive distributed control. *IEEE Trans. on Circuits and Systems, I* 2010; **57**(8):2182-2195.
- [15] Lawrence S, Giles CL. Accessibility of information on the web. *Nature* 1999; **400**:107-109.
- [16] Li C, Chen G. Synchronization in general complex dynamical networks with coupling delays. *Physica A* 2004; **343**:263-278.
- [17] Liberzon D. *Switching in Systems and Control*. Boston, MA : Birkhauser, 2003.
- [18] Lorenz EN. Deterministic nonperiodic flow. *J. Atmos. Sci.* 1963; **20**:130-141.
- [19] Lu J, Ho DWC, Cao J. A unified synchronization criterion for impulsive dynamical networks. *Automatica* 2010; **46**(7):1215-1221.
- [20] Khadra A, Liu XZ, Shen XM. Analyzing the robustness of impulsive synchronization coupled by linear delayed impulses. *IEEE Trans. On Automatic Control* 2009; **54**:923-928.
- [21] Pastor-Satorras R, Vespignani A. *Evolution and Structure of the Internet: A Statistical Physics Approach*. Cambridge: Cambridge University Press, 2004.
- [22] Saber RO, Murray RM. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Trans. Automat. Contr.* 2004; **49**(9):1520-1533.
- [23] Strogatz SH. Exploring complex networks. *Nature* 2001; **410**:268-276.
- [24] Wang XF, Chen G. Complex networks: small-world, scale-free and beyond. *IEEE Circuits and Systems Magazine* 2003; **3**(1):6-20.
- [25] Wasserman S, Faust K. *Social Network Analysis*. Cambridge: Cambridge Univ. Press, 1994.
- [26] Wang L, Dai H, Kong X, Sun Y. Synchronization of uncertain complex dynamical networks via adaptive control. *Int. J. of Robust and Nonlinear Control* 2009; **19**:495-511.
- [27] Watts DJ, Strogatz SH. Collective dynamics of 'small-world' networks. *Nature* 1998; **393**:440-442.
- [28] Yang T. *Impulsive Control Theory*. Berlin: Springer, 2001.
- [29] Yao J, Guan ZH, Hill DJ. Passivity-based control and synchronization of general complex dynamical networks. *Automatica* 2009; **45**(9):2107-2113.
- [30] Zhou J, Chen T. Synchronization in General Complex Delayed Dynamical Networks. *IEEE Trans. Circuits Syst. I* 2006; **53**:733-744.